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7 Dynamic Considerations

7.1 Dynamic Sequencing of Static Equilibria

Thus far we have discussed the economic model and data with which we can calculate a static general equilibrium for 1973. However, we think that tax policy evaluations based on single-period, static equilibria can be misleading. In particular, meaningful welfare calculations can be based on neither *H*—the utility of present consumption in 1973—nor *U*—the overall utility measure that includes expected consumption in later years.

First, consider a welfare measure based on *H* and consider a tax change that increases the net return to saving. If consumers respond to increasing saving incentives, then the utility *H* from current consumption will fall. Only over time will the additional savings provide enough capital deepening to allow for higher future consumption. Thus a policy that looks harmful in the short run can provide substantial welfare gains in the long run.

Second, consider a welfare measure, based on *U*, from 1973. This utility function includes expected future consumption for each household group. The expectations are myopic in that our consumers assume that the current price of capital will remain unchanged in the future.[1] Thus a tax change that raises the net rate of return also raises expectations about the amount of future consumption that can be obtained from a given amount of current saving. Actual capital deepening will bring the net return down; thus expected future consumption overstates actual future consumption. Expected utility, *U*, therefore overstates the utility from actual consumption.

1. In fact, the price of capital can change substantially over time, as we shall see when we consider some large policy changes in chapters 8 and 9.

The myopic expectations turn out to be correct if the economy is on a balanced growth plan. They are incorrect in any transition to a higher or lower capital/labor ratio, and they become more accurate as the economy settles down to a new steady-state path. Consequently, it is appropriate to look explicitly at the future path of the economy. In so doing we base our welfare measures on current consumption H from each year in a sequence of static equilibria. Preferences based on U and expected future consumption are used only insofar as they generate actual consumption and savings for any particular year.

The first equilibrium in every sequence is for the 1973 benchmark year. The later equilibria in a sequence can represent any later years. It would be possible to calculate one equilibrium for every year. However, computational expenses increase with the number of equilibria to be calculated. We usually calculate equilibria that are five or ten years apart, and we usually calculate enough equilibria to look fifty years beyond 1973. For example, we frequently calculate a sequence of equilibria representing the years 1973, 1983, 1993, 2003, 2013, and 2023.[2] Later in this chapter we discuss the sensitivity of our results to such choices.

The equilibria in any sequence are connected to each other through capital accumulation. Each single-period equilibrium calculation begins with an initial capital service endowment. Saving in the current period will augment the capital service endowment available in the next period. As we move through our sequence, the capital stock grows because of saving. When the capital endowment grows at the same rate as the effective labor force, the economy is on a *balanced growth path.*

In fact, we *assume* that, in a base-case sequence, the economy is on a steady-state, balanced growth path. This assumption is crucially important. Just as the assumption that the economy is in equilibrium in 1973 is central to the development of the static version of the model, so the assumption of a balanced growth path is central to the development of the dynamic version.

To be more precise, the definition of a steady-state growth path is a situation where tax policy is unchanging and

2. When we make our dynamic welfare calculations (as discussed below in section 7.2), we need to have the values of certain variables for every year, even though we do not usually make an equilibrium calculation for every year. In order to calculate the values of these variables for the intermediate years, we assume that the path between equilibria is characterized by smooth exponential growth. For example, assume that we have calculated the value of the H function (defined in chapter 3) for 1973 and 1983. If we assume that the growth rate between the two years is constant, then it must be that $H_{1983} = H_{1973}(1 + GR)^{10}$ where GR is the growth rate. We can then solve for GR as

$$GR = \left[\frac{H_{1983}}{H_{1973}}\right]^{\frac{1}{10}} - 1.$$

Then, for the intermediate years, we calculate H as $H_{1973+t} = H_{1973}(1 + GR)^t$.

$$\frac{\dot{E}}{E} = \frac{\dot{K}}{K} = n, \tag{7.1}$$

where

E = labor endowment,
$\dot{E}$ = increase in labor endowment,
K = capital endowment,
$\dot{K}$ = increase in capital endowment, and
n = growth rate of effective units of labor.

Moreover, we separate the growth of effective labor units into components that reflect population growth and Harrod-neutral technical change (increase in productivity of existing labor). Thus,

$$n = (1 + h)(1 + g) - 1, \tag{7.2}$$

where

g = growth rate of natural units of labor,
h = growth rate of output per worker hour.

On the steady-state path, all relative prices remain constant. Tax policy changes will alter the steady-state path and set the economy on a transition path (which may be rather lengthy). Eventually the economy approaches a new steady state. Figure 7.1 illustrates the transition for a tax policy change which results in increased saving. Without the change, consumption per capita is growing at a constant rate. With the change, consumption is at first lower than it would have been. Consumption will then rise at a faster rate, however, as a result of the greater amount of capital accumulation. The level of consumption under the new regime

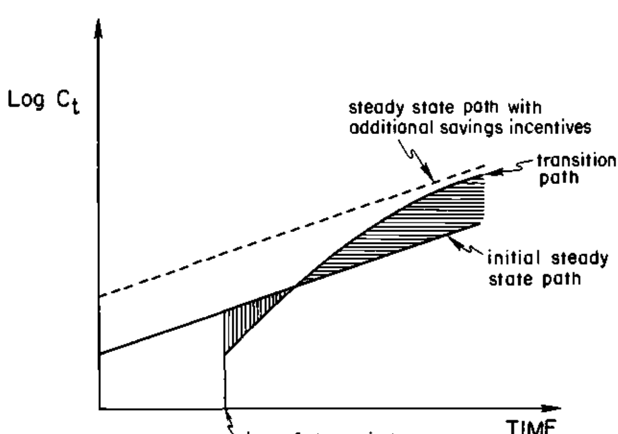

Fig. 7.1 The transition for a tax policy change that stimulates savings.

eventually surpasses the level under the old set of taxes, and consumption approaches a new balanced growth path asymptotically. By calculating a sequence of equilibria, we are able to study the transition in detail.

We choose the parameters for the dynamic version of our model in the following way. First, we observe the amount of saving and the size of the capital stock in the benchmark year. This gives us the rate of growth of capital. We then assume that the effective labor force grows at the same rate.

Using the notation in chapter 3, the expression $n = \dot{K}/K$ in equation (7.1) can be rewritten as

$$n = \frac{S\gamma}{CAP(1 - \bar{f}\tau)}, \tag{7.3}$$

where S is total savings, γ is the factor of conversion from stock to flow units, and CAP is capital income net of corporate and property taxes.[3] The multiplication by $(1 - \bar{f}\tau)$ is necessary because we define units of capital as being net of personal factor taxes.

If we plug the appropriate values from the consistent 1973 data set into equation (7.3), we can calculate n, the steady-state growth rate:

$$n = \frac{(105441.7)(.04)}{(189672.9)(1 - (.816058)(.278288))} = 0.028770. \tag{7.4}$$

The endowment of effective labor must also grow at this rate. For simplicity we assume that the growth of effective labor is divided evenly between population growth and increased productivity. We therefore assume that $g = h$ in equation (7.2), and we solve the equation:

$$g = h = \sqrt{1.02877} - 1 = 0.014283. \tag{7.5}$$

The importance of the division of n between g and h will become clearer later in this chapter, when we describe our dynamic welfare calculations. In particular, we are able to consider the effect of tax changes on a population that is the same size as the initial population. If we were to include the utility of additional individuals in later years, our welfare measure would be weighted in favor of future periods.

When all endowments and incomes increase at the rate n, and when demand functions are homogeneous of degree one in income, the government's tax receipts also grow proportionately. Transfer income will grow at the same rate because consumers are each given a share of government revenue in the benchmark. The only problem is presented by progressive personal income taxes. This problem is solved by scaling the (B_j) intercept of each linear tax schedule by the steady-state growth rate n. With a larger negative intercept and larger incomes subject to the same marginal

3. We choose γ = .04. We will discuss this choice later in this chapter.

tax rate, each consumer will experience a constant average rate of tax as income grows through time. The personal income tax remains progressive, however, in the sense that high-income consumers still have higher average and marginal tax rates than low-income consumers.

7.2 The Comparison of Dynamic Sequences

In this section we describe our procedures for the evaluation of alternative sequences of equilibria.

A well-known concept in static welfare analysis is the *compensating variation*, defined as the amount of additional income at new prices that would be necessary in order to allow the consumer to reach his old level of utility. The *equivalent variation* is defined as the amount of additional income at old prices that would enable the consumer to reach the new utility level.

We can derive the static compensating and equivalent variations for the H utility function (the CES composite of current consumption and leisure). The H function might be called an *evaluation function*, distinguishing it from the overall utility function, U, which we use to determine saving behavior. The H function is the appropriate one to use in our dynamic welfare evaluations. Since savings are used to buy future consumption, we would be double counting if we included savings in our evaluation of utility in the current period.

The first step in deriving compensating and equivalent variations is to solve for the expenditure function that corresponds to H. The budget constraint was provided in equation (3.8), which we reproduce here:

$$I = P_H H + P_S S. \tag{7.6}$$

In equation (7.6), I is expanded income, as defined in equation (3.39). When we subtract $P_S S$ (the amount spent on saving), we have the income left over for current consumption and leisure. Let us call this I_H. Substitution implies

$$I_H = H \cdot P_H. \tag{7.7}$$

The evaluation function H is a combination of goods and leisure, similar to a utility function. For present purposes, however, it is also useful to think of H as a composite commodity, a physical combination or aggregation of goods and leisure. Each unit of H costs P_H, provided in equation (3.27) as a composite of prices for goods and leisure. The expenditure function is $H \cdot P_H$, a function of the required utility level and prices.[4]

4. This equation does not hold exactly for the first of our twelve consumer groups, because we use a negative minimum purchase requirement to account for their negative savings.

Assume we have old values H^o, I_H^o, and P_H^o and new values H^n, I_H^n, and P_H^n. The compensating variation for any single period is defined as the additional income required to obtain old utility levels at new prices:

$$CV = H^o P_H^n - H^n P_H^n = (H^o - H^n) P_H^n. \tag{7.8}$$

It is customary to reverse the sign of this welfare measure, such that the CV is positive for a welfare gain.

In similar fashion, we write the equivalent variation as

$$EV = (H^n - H^o) P_H^o. \tag{7.9}$$

These measures can be applied to any consumer in any period, since we calculate each consumer's income and utility in each time period under each tax regime. A problem arises, however, in trying to sum a stream of compensating or equivalent variations, because these are measured in prices of different years from different sequences. Fortunately, the benchmark sequence is on a steady-state path, so its prices remain stable with no tax change. Thus a natural choice is to measure welfare gains in benchmark prices, equal to actual 1973 prices. For the i^{th} period, then, the constant dollar difference between revise-case consumption and benchmark consumption is $(H_i^n - H_i^o)P_H^o$. If we use PV to denote a present value operator to be described below, then the present value of this stream of welfare gains is

$$PVWG \equiv PV\,[(H_i^n - H_i^o)P_H^o] = P_H^o \cdot [PV(H_i^n) - PV(H_i^o)], \tag{7.10}$$

since P_H^o is a constant. Also, since $(H_i^n - H_i^o)P_H^o$ is just the EV for period i in unchanged prices, this measure might be interpreted as the present value of equivalent variations. This is the measure used to report all welfare gains in results below.

Because this measure is so important for our welfare evaluations, let us further describe and justify the choice in several ways. First, suppose that we want the present value of a CV measure that is comparable to the EV measure above. Since we need to add together the compensating variations for different periods, and since these are measured in all different prices P_{Hi}^n, we could convert them to constant prices P_H^o through the use of a price index P_{Hi}^n / P_H^o. Then the present value of these "real" CVs (with sign reversal) is:

$$PV\left[\frac{(H_i^n - H_i^o)P_{Hi}^n}{P_{Hi}^n / P_{Hi}^o}\right], \tag{7.11}$$

which reduces to $PVWG$ from (7.10). The point is that compensating and equivalent variations are defined so as to express welfare changes in different prices, so the conversion to common prices eliminates any useful distinction between the two concepts. The welfare change $(H_i^n -$

H_i^o) could be multiplied by a price such as P_H^o or any price level from the revise-case sequence. Thus we merely use P_H^o.

Second, since *PVWG* is in base-period prices, the welfare gains from one tax proposal can be compared directly to the welfare gains from a different tax proposal. That is, since *PVWG* reduces to the present value of equivalent variations from each period, we have an argument analogous to that of John Kay (1980). The *EV* is preferred to the *CV* for comparing alternative replacement policies, since the *CV* are each measured in the new prices of each different replacement equilibrium.

Third, suppose that our model contained a lifetime utility function of the form $U(H_0, \ldots, H_T)$, where consumers could substitute consumption among the various periods in a CES function for U with a composite price P_U. Then there would exist a comprehensive expenditure function of the form $U \cdot P_U$ and a comprehensive *CV* or *EV* of the form $(U^n - U^o)P_U^n$ or $(U^n - U^o)P_U^o$. Additional discounting in this case would not be necessary. Either the *CV* or *EV* would be a valid measure of the number of additional dollars in one set of lifetime prices that would be required to attain a different total lifetime utility. We do not have a lifetime utility function, however. Our welfare measure provides enough additional dollars in one set of prices to allow the consumer to reach a particular pattern of $H_0, \ldots, H_T$, more than enough to reach the same lifetime utility if there were substitution among the H_i. Our welfare measure for each consumer may be biased for this reason.

Finally, since the *PVWG* are dollar measures of welfare change, they can be summed across consumer groups to provide an estimate of the total welfare effect (efficiency gain or loss) of any policy change.[5] Because the utility from government expenditures is held constant, as described in section 3.5, these measures represent all welfare changes for U.S. consumers. The gains and losses of foreigners are excluded. Therefore, the *PVWG* measures represent changes in domestic welfare levels, rather than worldwide levels. We should recognize that a tax change could have terms of trade effects, such that worldwide welfare increased while U.S. consumer *PVWG* measures were all negative.

7.3 Present Value Calculations

Now let us discuss the details of the calculations of present values in equation (7.10). We have a problem in evaluating the welfare gains of a growing economy because an overly high weight might be placed on outcomes occurring in the distant future. For example, if the economy is

5. We take the unweighted sum of the *PVWG* for the various consumers, so we might be said to be using a Benthamite social welfare function. It would be possible to employ a concave social welfare function by weighting poorer consumers more highly, but we have not done so.

growing at 3 percent per year (due to labor force growth), and we are discounting our social welfare measure at 4 percent, then a 2 percent improvement in consumption seventy-five years hence is weighted at half the importance of an immediate 2 percent improvement. We believe this does not sufficiently discount the distant future because of the spurious effect of population growth. The approach we have taken is to consider the impact of the tax policy change on a population the size of the original 1973 population. That is, we do not consider population growth per se as a reason to assign additional weight to the outcomes of future years. Rather, we determine the effect of our plan on a per capita basis and then multiply by our initial population size.

In base- and revise-case calculations we compute a value of H for each year for each consumer. If we consider a sequence of equilibria extending fifty years past our benchmark (i.e., from 1973 to 2023), the present value of the stream of H values for those five decades is given by:

$$PV = \sum_{t=0}^{50} \frac{H_t}{(1+g)^t(1+\rho)^t}, \tag{7.12}$$

where g is the population growth rate and ρ is the consumer's rate of time preference. We discount by g because we want our calculations to consider only the initial-sized population.

Equation (7.12) is fine for the first fifty years. However, it neglects the utility accruing more than fifty years after the tax change. Before we can use equation (7.12), we must add a *termination term*. In general, we will have a series of equilibrium calculations that extend T years past the starting year, 1973. Over the course of the T years in a revise-case calculation, the economy asymptotically approaches a new steady state. However, it will not actually reach the new steady state. In the case of a policy designed to increase saving, the economy will still be experiencing a small amount of capital deepening, even after many years. In order to calculate the termination term, we *assume* that the capital in the economy has reached the steady state in period T. In the case of a capital-deepening policy, we thus assume a slight decrease in saving and increase in H from the amount that we actually calculated for period T.

The actual saving in period T might be called S_T, but we get the steady-state level of saving for period T, S', by rearranging equation (7.3) to obtain

$$S' = \frac{nCAP_T(1-\bar{f}_T)}{\gamma}. \tag{7.13}$$

Actual consumption expenditure in period T is I_{HT}, but steady-state expenditure for period T is then calculated as $I'_H = I_T - P_S S'$. From these values we need to calculate the H' that would result from expenditure I'_H.

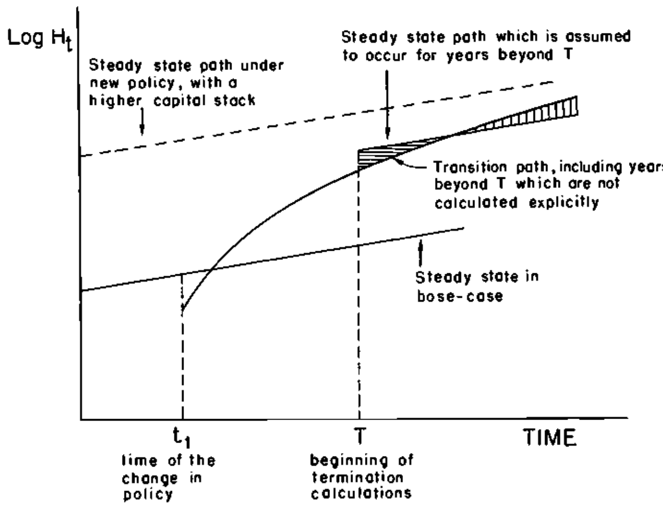

Fig. 7.2 The approximation involved in calculations of the termination term.

Since H is homogeneous of degree one in expenditure I_H, we can multiply actual H by the ratio I'_H/I_{HT} to obtain steady state H' as:[6]

$$H' = H_T\left(\frac{I_T - P_S S'}{I_T - P_S S_T}\right). \tag{7.14}$$

In the case of a capital-deepening policy, I'_H/I_{HT} exceeds unity, so we increase H_T. We then assume that H grows at the steady-state rate from period T to infinity. These assumptions are illustrated in figure 7.2.

We are now able to calculate the present value of a sequence of values of H. The present value formula consists of equation (7.12) plus the termination term:

$$PV(H) = \sum_{t=0}^{T} \frac{H_t}{(1+g)^t(1+\rho)^t} + \frac{H'}{(\rho - h)(1+g)^T(1+\rho)^T}. \tag{7.15}$$

Note that the second term on the right-hand side of equation (7.15) (the termination term), is divided by $(\rho - h)$ where ρ is the discount rate and h is the growth rate of labor efficiency. When we divide by $(\rho - h)$,

6. An alternative would be simply to assume that H_T was close enough to the new steady state. Since H approaches the new steady state from below, the direct use of H_T would understate welfare gains. Instead, we assume that capital is close enough to the steady state. Savings S' (enough to increase that capital at rate n) is less than savings S_T (enough to continue towards the true steady state). Since $S' < S_T$, equation (7.14) scales *up* H_T to approximate the higher steady-state welfare. It thus avoids understating the gains as would occur with the use of H_T.

we are in effect taking a stream growing at the rate of h per person but discounted at the rate ρ, and transforming it into a present value at time T. This present value in period T is also discounted to the present by the $(1 + \rho)^T$ division. Furthermore, the initial population becomes a smaller fraction of the economy through time because of population growth at the rate g. We reduce the termination term to the initial population size when we divide by $(1 + g)^T$. This fixed population size still enjoys ever-increasing absolute economic power due to the exponential growth in labor productivity at rate h.

When we evaluate a tax policy change, we use equation (7.15) to calculate the present value of the base-case sequence and the present value of the revise-case sequence. This gives us $PV(H^o)$ and $PV(H^n)$, which we plug into equation (7.10) to get our present value of welfare gains or losses for each consumer.

Our procedures do *not* guarantee that our measures of the change in welfare will be invariant with respect to the number of years between the first and last equilibrium. In general, the longer is our calculated sequence of equilibria, the closer are the final values of income, consumption, and utility to the actual steady-state values. In table 7.1 we test the sensitivity of the model to changes in the number of periods. In each of these runs the policy change is the switch to a consumption tax. (This policy proposal and the way in which we model it are discussed in much greater detail in chapter 9.) All of the runs use additive replacement for equal revenue yield, and all of the equilibria are spaced five years apart.

As seen in table 7.1, the results become fairly robust when we carry out the calculations for fifty years or more. A fifty-year simulation of the consumption tax provides a welfare gain of $556 billion, while a hundred-year simulation provides $569 billion, which is not much different.

Our welfare measure also depends somewhat on the number of years between any two equilibria in a sequence, although our assumption of smooth, exponential growth between equilibria helps to reduce the size of this effect (see footnote 2, this chapter). Table 7.2 shows our tests for this type of sensitivity. Since all of these simulations cover a total of fifty years, the $556 billion for five-year spacing matches the number from the previous table. When equilibria are separated by only one year, the same tax change implies a $491 billion welfare gain.[7]

7.4 Important Parameters for Dynamic Sequencing

Several of the parameters necessary to implement our dynamic sequencing procedures were presented and described in earlier chapters or in earlier sections of this chapter. These parameters include the growth

7. See Ballard and Goulder 1982 for further discussion of these effects.

Table 7.1 **Effect of Number of Equilibrium Periods on Welfare Gain from Adoption of a Consumption Tax (in billions of 1973 dollars)**

Number of Equilibria	Number of Years beyond 1973 Covered by Those Calculations	Welfare Gain (Loss)
2	5	$(−61)
3	10	144
4	15	272
5	20	365
6	25	431
7	30	476
8	35	507
9	40	530
10	45	545
11	50	556
12	55	563
13	50	567
14	65	570
15	70	572
16	75	572
17	80	572
18	85	572
19	90	571
20	95	570
21	100	569

Table 7.2 **Effect of Number of Years between Equilibria on Welfare Gain from Adoption of a Consumption Tax (in billions of 1973 dollars)**

Number of Equilibria	Number of Years between Equilibria	Welfare Gain (Loss)
6	10	$641
11	5	556
26	2	505
51	1	491

Note: Because the first equilibrium represents the initial year, each of these sequences covers exactly fifty years.

rates of population and technical progress, (g and h), set in equation (7.5), and r, the real net-of-tax rate of return. While the average net-of-tax rate of return is set to 4 percent, each consumer group has a net rate of return that depends on its own marginal tax rate. Any group with a marginal tax rate less than the average earns a net return higher than 4 percent, while any group with a marginal tax rate greater than the average earns a net return of less than 4 percent.

We must now discuss two more parameters: γ, the annual rate at which capital services are augmented as a result of additional saving, and ρ, the discount rate. We assign to γ the same value that we assign to r in the benchmark—0.04. If each unit of capital earns a 4 percent rate of return net of taxes in the benchmark, then an additional unit of capital acquired through savings can be expected to earn the same return. That is, S will contribute .04 units of capital services, each earning a dollar per period net of taxes in the benchmark.

Now let us consider the discount rate. In order to calculate present values for consumption sequences, we want to use the discount rate, ρ, implied by the utility function evaluation of present and future consumption. In general, for each consumer, this will also be equal to the real net rate of return, r. To see this, represent the choice between present consumption (C_0) and future consumption (C_1) by the indifference-curve diagram of figure 7.3. Suppose that the individual has I_0 of income in the present period, and faces consumption prices of P in either period. Any income not consumed in the present period can earn the return r before being spent in the future period, so the effective price of C_1 is $P/(1 + r)$, which is less than P. The individual maximizes utility subject to his income I_0 by setting the ratio of marginal utilities equal to the price ratio. His marginal rate of substitution at the tangency is therefore $(1 + r)$, implying a discount rate of r in the benchmark. As a result, each consumer's H stream is discounted at its own rate of return to measure dollar present values that can be added up across groups.

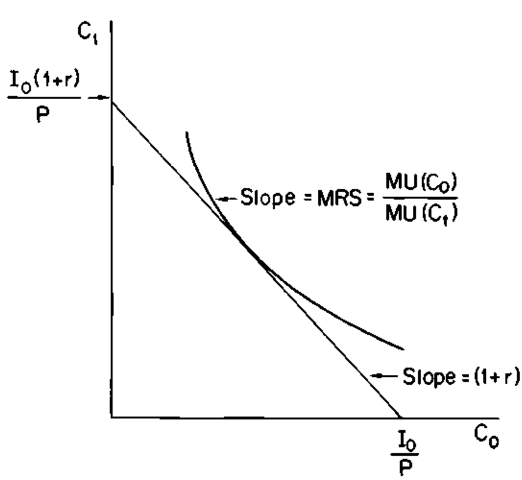

Fig. 7.3 The choice between present and future consumption.

7.4 Revenue Yield Equality in the Dynamic Model

In order to analyze tax policy changes intelligently, it is important to maintain the assumption of equal revenue yield. This is true in the dynamic case as well as in the static case. In the dynamic case, however, we could apply more than one standard of yield equality. We invoke a strong form of yield equality in that we require that the government collect the same revenue in each period of the revise sequence as it collected in the corresponding period of the base sequence. This requirement can cause some problems, most notably with a switch from an income tax to a consumption tax. (See chapter 9 for more discussion of the consumption tax.) Since the consumption tax base is less than the income tax base for at least the first few periods of a consumption tax, unchanged rates of tax would provide substantially reduced revenue in those periods. Our strong form of yield equality implies that the tax rates must be substantially higher with a consumption tax in the initial periods. These higher tax rates exacerbate the already distorted choice between work and leisure. As time goes on, however, the economy will grow faster under the consumption tax, since this type of tax policy leads to faster capital accumulation. In fact, the consumption tax base might ultimately exceed the income tax base.

We could consider a weaker form of yield equality under which the present value of the revenues under the two systems would be equated. The differences in the pattern of tax collections would be made up with deficits or surpluses. In order to do this, however, we would need to deal explicitly with the government bond market. We have not done so, although we should note that the treatment of financial assets in general equilibrium models is receiving more and more attention. (For example, see Auerbach and Kotlikoff 1983, Feltenstein 1984, or Slemrod 1983).