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Uncertainty and the Durability of Machinery

ERIC W. BOND*

I. Introduction

The purpose of this paper is to study the effect of uncertainty concerning the lifetime of the firm on the durability of machinery installed by the firm. This problem arises in several contexts. One example is a multinational firm operating a subsidiary that may be nationalized at some future date. The importance of this problem has been documented in a study by Williams (1975) of events of nationalization occurring over the period 1956–72. He found that nationalizations had occurred in forty developing countries and had affected an amount of assets equal to nearly 25% of the total foreign-owned capital stock in developing countries in 1972. A second example of lifetime uncertainty arises where a firm is facing the possibility of regulatory action that will ban either the product the firm produces or the machinery that is used by the firm.

The results of this paper have implications for the demand for capital goods by firms, since an increase in lifetime uncertainty may lead to a change in the durability of capital goods demanded by firms. Furthermore, if capital goods of different durabilities are used with different factors of production, then the demand for other factors may be altered as well.

The effect of lifetime uncertainty on decision-making has appeared in connection with other economic problems. Yaari (1965) found that the existence of lifetime uncertainty for the consumer acted like an increase in the interest rate, and led to higher consumption in the early period of the consumer's life. Long (1975) examined the effect of uncertainty regarding future nationalization on the optimal extraction rate for a non-renewable resource, and found that the existence of uncertainty generally led to an increase in the extraction rate. In these problems, the existence of lifetime uncertainty reduces the attractiveness of future returns relative to current returns, leading to a preference for current consumption or production.

In the case of lifetime uncertainty for a competitive industry, an increase in the probability of shutdown will reduce the expected profits of the firm and lead to an exit of firms from the industry. The returns to firms will rise until equilibrium with zero expected profits is restored. It is shown in Section II of this paper that the increase in returns from capital resulting from the exit of firms will tend to increase the durability, but that this effect will be dominated by the decline in durability resulting from the increased probability of shutdown in the case where probability of shutdown takes an exponential form.[1]

*Assistant Professor of Economics, The Pennsylvania State University. I would like to thank Walter Oi, Roger McCain, Douglas Mitchell, and an anonymous referee for comments on earlier drafts of this paper.

[1]It should be noted that the increases in uncertainty discussed in this paper refer to an increase in the probability that the firm will be shut down before a particular time period. This is the same type of uncertainty treated by Yaari and Long, and it involves a reduction in the expected life of the firm. It has been pointed out by Levhari and Mirman (1977) in the context of the con-

II. The Model

This section develops a model of a competitive industry in which firms choose the durability of machinery that will maximize the expected value of returns from machinery. The industry is assumed to be small in the sense that the interest rate and the cost of capital are exogenous to the industry, and free entry is assumed to ensure zero expected profits in the long run equilibrium.[2]

Machines are assumed to be of the one-hoss shay variety that yield one unit of machine services at each point in time until they reach age N. The durability of the machine is then represented by N, the age at which the machine breaks down. The durability of machines is fixed at the time of purchase, and cannot be extended once the machine is in place. The quasi-rent earned from a machine at time t is denoted $R(t)$.

The uncertainty for the firm in this model concerns whether or not the firm will be in operation in the future. The firm is assumed to have a probability distribution, $f(t)$, representing the probability that the firm will cease operations at time t. This formulation can be given several interpretations, corresponding to the types of uncertainty discussed in Section I. In the case of nationalization uncertainty, $f(t)$ represents the probability that the event of nationalization occurs at time t. In the case of regulatory uncertainty, two interpretations are possible. If the regulatory action is to prohibit the use of machinery or ban the product the machine produces, then $f(t)$ represents the probability that the action occurs at time t. Alternatively, the firm can be viewed as operating in a stochastic environment, where there are some states of the world with returns that are negative and large enough that the firm becomes bankrupt. $R(t)$ then represents the expected value of quasi-rents if the firm is in operation, and $f(t)$ is the probability of bankruptcy at time t. If regulatory action increases the variance of returns from machinery, it could increase the probability of bankruptcy for the firm.

Once the firm is shut down, it is assumed that no future returns will be earned. The probability that the firm is in operation in period t, denoted $P(t)$, will be the probability that shutdown has not occurred prior to t. Thus,

$$P(t) = 1 - \int_0^t f(s)\, ds. \qquad (1)$$

$P(t)$ will represent the expected value of a dollar of potential machine earnings at time t. All uncertainty regarding the maintenance costs of machinery and the life of machinery has been assumed away to focus on the role of uncertainty regarding the firm's lifetime.[3]

The purchase price of a machine of durability N is denoted $C(N)$. It is assumed that the cost of producing a machine of greater durability increases at the margin, so that the price schedule for machinery to the firm will have the characteristics $C', C'' > 0$ if the machinery producers are competitive. Maintenance costs for machinery have not been explicitly allowed for, but can be taken to have been subtracted from gross returns to yield R.

sumer problem that such an increase in uncertainty does not satisfy the Rothschild and Stiglitz definition of an increase in risk. Levhari and Mirman argue that Yaari's result is due to the shorter expected life of the consumer under uncertainty, rather than to the consumer's risk aversion. In light of this point, it should be emphasized that there is no claim that the firm's response to lifetime uncertainty is due to risk aversion. It is assumed that the firm is risk neutral throughout this paper. The objective here is to point out the additional considerations resulting from the competitive profit conditions that come into play in the case of lifetime uncertainty.

[2]As a result of these assumptions, the price of output is supply determined in the long run. The assumption that the industry is small also means that changes in uncertainty for the industry will not alter the market interest rate.

[3]The case of uncertainty regarding the time at which a machine will break down is treated by Jorgenson, McCall, and Radner (1967).

If the firm has an infinite horizon, the objective of the firm will be to maximize the returns from an infinite series of machines. Let σ_i denote the time at which the i^{th} machine is installed and N_i the durability of the i^{th} machine. Assuming no installation lag, we have $\sigma_{i+1} = \sigma_i + N_i$. The expected value at time σ_i of the returns from the i^{th} machine will be

$$V(N_i, \sigma_i) = \int_0^N R(\sigma_i + s)\, P(\sigma_i + s) e^{-rs}\, ds - C(N_i)\, P(\sigma_i) \qquad (2)$$

and the expected value of the firm at time 0 will be

$$W = \sum_{i=1}^{\infty} V(N_i, \sigma_i) e^{-r\sigma_i}. \qquad (3)$$

The firm's optimal policy will be a series of machine durabilities $\{N_1, N_2, \ldots\}$ that maximize (3). In general, machine durabilities will vary over the life of the firm and will depend on the entire time path of $P(t)$, so that it will be extremely difficult to derive general conclusions about the effect of lifetime uncertainty.[4]

We shall concentrate in this section on the special case where uncertainty takes the exponential form

$$f(t) = \beta e^{-\beta t}$$
$$P(t) = e^{-\beta t} \qquad (4)$$

The special characteristic of this form of uncertainty is that the probability of shutdown at a point in time, given that the firm is still in operation, is a constant (β). The probability that the firm will live for an additional interval of time T is independent of calendar time, since $P(T)/P(0) = P(t + T)/P(t) = e^{-\beta T}$. Therefore, a firm making plans at time t will face the same probabilities of shutdown over the life of a machine as a firm planning at time 0. This means that if quasi-rents are independent of calendar time, durability decisions will also be independent of calendar time.[5]

In this case, it is possible to treat the industry as being in a steady state. If $R(t) = R$ for all t, the optimal durability and the level of profits will be the same for each investment cycle. Entry or exit of firms will then lead to adjustments in R to ensure zero expected profits for firms. Since all cycles are identical, this implies zero expected profits for each investment cycle and we can analyze the steady state equilibrium by considering a representative cycle.

A steady state equilibrium will be characterized by two conditions in this model. First, firms will choose N to maximize the value of a machine, for a given level of R. This is the profit-maximizing condition, which is

$$V'(N) = \mathrm{R}e^{-(r+\beta)N} - C'(N) = 0. \qquad (5)$$

This condition states that the firm chooses durability by equating the marginal cost of durability to the expected return from extending the life of the machine. The assumption of increasing marginal cost of durability ensures that (5) will be a maximum. Second, the level of R will adjust as a result of entry or exit of firms until the expected present value of a machine is equal to zero for a representative machine. This is the competitive profit condition, which is that

$$V(N) = \int_0^N \mathrm{R}e^{-(r+\beta)s}\, ds - C(N) = 0 \qquad (6)$$

[4]This problem can be solved by standard dynamic maximization techniques where N_i is the control variable and σ_i the state variable. It should be noted that since the probability of shutdown depends on calendar time, there will be no dynamic inconsistency of the type discussed by Strotz (1956) in this problem as a result of lifetime uncertainty. Once firms make plans, there will be no desire to revise plans unless new information is received. A proof of this point is given by Long (1975).

[5]This can be seen by substituting (5) into (3) and observing that the expected value of a machine, conditional on the firm surviving until the starting time of the cycle, is independent of the starting time.

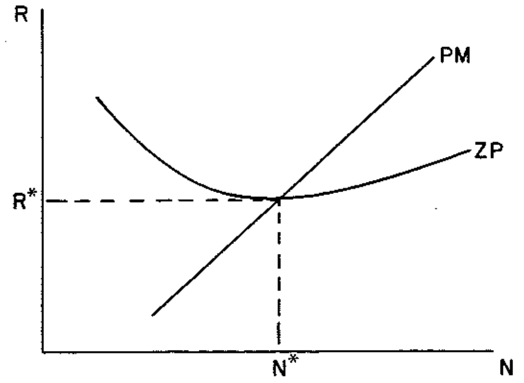

Figure 1

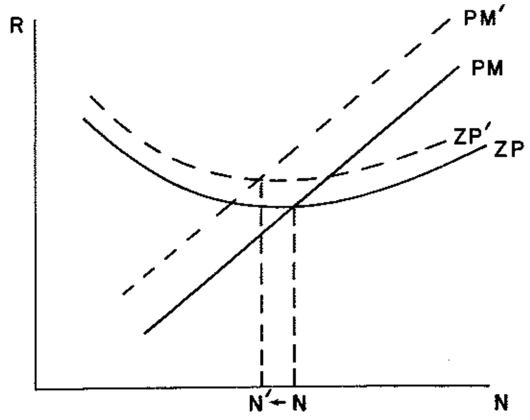

Figure 2

These two conditions will simultaneously determine the equilibrium values of R and N in the steady state.

The equilibrium conditions are illustrated in Figure 1. The profit-maximizing (PM) curve is the locus of values of R and N that satisfy $V'(N) = 0$. An increase in R will lead to an increase in the optimal life of the machine, since it raises the benefit from extending its life. The zero profit (ZP) curve is the locus of points consistent with $V(N) = 0$. By implicit differentiation of (6) we have

$$\left.\frac{\partial R}{\partial N}\right|_{V=0} = \frac{\mathrm{Re}^{-(r+\beta)N} - C'(N)}{\left(\dfrac{1 - e^{-(r+\beta)N}}{r + \beta}\right)}.$$

The numerator of this expression is the profit-maximizing condition, so that the $V(N) = 0$ curve will be horizontal where it crosses the PM curve. To the left of the intersection, the ZP curve will be downward sloping, and to the right of the intersection it will be upward sloping. It should be noted that since competitive firms will earn a return at the minimum point of the ZP curve, competitive firms are efficient in the sense that they choose the durability of machinery that leads to the minimum cost of producing output.

An increase in uncertainty about future operations will affect both the profit-maximizing and zero profit conditions. An increase in β results in a greater discount on future machinery, so that the optimal value of N will be reduced for each level of R. This effect is shown by the upward shift of the PM curve in Figure 2, and tends to reduce the optimal machine life. However, the increased value of β also means that a higher value of R will be necessary to maintain 0 expected profits. This is shown by the upward shift of the ZP curve, which tends to offset the previous effect. The ultimate effect of an increase in β can be obtained by total differentiation of the equilibrium conditions (5) and (6)

$$\begin{pmatrix} \dfrac{1 - e^{-(r+\beta)N}}{(r+\beta)} & 0 \\ e^{-(r+\beta)N} & -(r+\beta)\mathrm{Re}^{-(r+\beta)N} - C''(N) \end{pmatrix} \begin{pmatrix} dR \\ dN \end{pmatrix} = \begin{pmatrix} \dfrac{R(A+1)}{(r+\beta)^2} \\ NRe^{-(r+\beta)N} \end{pmatrix} d\beta$$

where $\Delta \equiv -(1 - e^{-(r+\beta)N})(\mathrm{Re}^{-(r+\beta)N} + (C''(N)/(r + \beta))) < 0$ and $A = -(1 + (r + \beta)N)e^{-(r+\beta)N}$. Solving for $dN/d\beta$ and simplifying we obtain

$$\frac{dN}{d\beta} = \frac{(r+\beta)^{-2}\mathrm{Re}^{-(r+\beta)N}}{\Delta} [(r+\beta)N + e^{-(r+\beta)N} - 1] < 0$$

The shift of the profit-maximizing curve will always dominate the shift of the zero profit curve, so that increased β will reduce the optimal durability of machinery.

A closed form solution for the optimal value of N can be obtained if we consider the special case where the cost function takes the form

$$C = \alpha e^{\gamma N}$$

Substituting into the equilibrium conditions and solving simultaneously for N, we obtain

$$N^* = -\left(\frac{1}{r+\beta}\right)\ln\left(\frac{\gamma}{r+\gamma+\beta}\right)$$

and

$$\frac{dN^*}{d\beta} = \left(\frac{1}{r+4\beta}\right)^2\left[1 - \frac{\gamma}{\gamma+r+\beta} + \ln\left(\frac{\gamma}{\gamma+r+\beta}\right)\right] < 0$$

Some additional insight into the determinants of durability can be gained by considering the effects of several types of changes in the cost function in this case. An increase in the cost function that raises total cost, but leaves the marginal cost schedule unchanged, will result in an upward shift in the ZP curve (higher values of R are needed to maintain zero profits). The profit-maximizing curve will be unchanged, so the optimal machine life will be increased. An increase in γ will shift both curves, since both total and marginal costs will be increased. Differentiating (8) we obtain

$$\frac{dN^*}{d\gamma} = -\frac{1}{\gamma(\gamma+r+\beta)} < 0.$$

Optimal machine life will be reduced since the increase in γ will have a larger percentage impact on marginal costs than on total costs. A change in α, on the other hand, will have no effect on N^* since it changes marginal and total costs of durability by the same proportion. These results indicate that an increase in the probability of shutdown will reduce the durability of machinery where the probability of shutdown takes an exponential form.

The following example illustrates a perverse result that could occur if uncertainty is not sufficiently "smooth." With no possibility of nationalization, let the optimal life of each machine in the cycle be denoted by $\hat{N}$. Now suppose that the firm finds that there is a probability of ½ that the firm will be nationalized at time $\hat{N} + \epsilon$ (where ϵ is small), and a probability ½ that it will never be nationalized. That is,

$$P(t) = 1 \quad t < \hat{N} + \epsilon$$
$$P(t) = \tfrac{1}{2} \quad t \geq \hat{N} + \epsilon$$

All machines installed after time $\hat{N} + \epsilon$ will have durability $\hat{N}$, since future returns are certain if the firm survives beyond time $\hat{N} + \epsilon$. The durability of machines purchased prior to $\hat{N} + \epsilon$ will depend on the relationship between the average and the marginal costs. If fixed costs are high, then the firm will choose to purchase a single machine of life $\hat{N} + \epsilon$, and then will purchase machines of life $\hat{N}$ afterwards. In this example, the firm extends the life of the first machine (and leaves that of all others unchanged) in order to avoid the fixed costs of installing a second machine prior to the time at which the firm may be closed down. The discontinuity in the P function leads to the possibility that replacement of machinery will be postponed until the firm knows whether nationalization will take place, so that the existence of uncertainty actually leads to an increase in the optimal machine life.

III. Summary

This paper has shown that if the uncertainty takes an exponential form, then an increase in the probability of shutdown reduces the durability of machinery. If the lifetime uncertainty results from the possibility of nationalization, then the reduction in the durability of machinery gives rise to a deadweight loss, since there is no social cost that correponds to the private cost of potential nationalization. In the case of regulation, the issue is more complex. If the possibility of regulation arises in a situation where the social cost of a firm's operation exceeds its private cost, then the comparison should be between the choice of durability with potential regulation and the choice of durability when the firm incurs the full social cost of the operation.

[6]In the case where the firm receives some compensation in the event of nationalization, these results will continue to hold as long as the compensation is not complete. If θ is the portion of the value of assets received in the event of nationalization, it can be shown that the expected value of a machine will be

$$W(N) = \int_0^N [P(t) + \theta(1 - P(t))] \, \mathrm{Re}^{-rt} \, dt - C(N)$$

Changes in P will affect firm valuation of machinery as long as $\theta \neq 1$. If the firm is able to sell its machinery in used markets in the event of shutdown, the above expression will also apply where θ is interpreted to be the portion of the machine's value that can be recovered by resale.

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