Factor Classification and the Theory of the Firm: A Simplification and Synthesis of Recent Contributions

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Light has recently been cast on some hitherto dark corners of the standard textbook theory of the firm. The illumination comes in the form of a more thorough comparative static analysis of factor price changes. Ferguson and Saving (1969) analyzed a firm's demand for factors and the long-run scale adjustments of firms in a competitive industry, and demonstrated precisely how a factor price change affects a firm's marginal and average cost curves and the output at which the latter curve is minimized. Bassett and Borcherding (1970b) provided a similar though less elaborate analysis. Building on the Ferguson-Saving paper, Maurice (1971) investigated the relationship between a change in input prices and profit, entry, and the optimal number of firms in a competitive industry and demonstrated that, when all factors are variable, profits (losses) resulting from an input price change provide a correct entry (exit) signal. Portes (1968) extended the analysis of factor demand to firms having goals other than profit maximization, investigated "income effects" in the theory of the firm, and discovered the possibility of a Giffen input.

At the heart of each of these papers is the special attention devoted to factor classification, by which a factor is defined to be normal, superior, or inferior in a manner strictly analogous to the definition of normal, superior and inferior goods in the theory of consumer behavior. Classification of inputs apparently originated with Hicks' (1946, Chap. 7) distinction between ordinary factors of production and "regressive" (inferior) factors, meaning inputs that are used in smaller amounts as output expands with constant input prices. Scott (1962), Bear (1965, 1972), Bishop (1967), Ferguson (1968), Bassett and Borcherding (1969), Truett (1971), Maurice (1972), and Truett and Roberts (1973), when taken together with the four papers referred to above, provide an extensive treatment of the role of factor classification in the theory of the firm.

The purpose of this paper is to provide a synthesis of this recent work and to prove the major results, usually presented in highly mathematical form and only *illustrated* geometrically, with the geometric and elementary mathematical tools commonly used in intermediate theory courses. In addition to the pedagogical usefulness of giving shorter and perhaps more intuitively appealing proofs of the effects of factor price change, the paper also provides a graphical method of analyzing a firm's demand for factors of production that is in some respects simpler and more enlightening than the

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usual technique employing value of marginal product curves.

To facilitate graphical analysis, discussion is limited to the case of a good produced with only two inputs, but the conclusions reached, with slight modification, can and have been shown in the papers above to hold for the case of n factors. Section I defines the factor classifications in terms of which the remaining analysis is couched. The effects of an input price change on a firm's marginal and average cost curves are examined in Section II, while in Section III, it is proved geometrically that a profit-maximizing firm necessarily has a downward-sloping demand curve for a factor of production. Section IV examines the demand for inputs by sales- and output-maximizing firms and identifies the possibility of a Giffen input. The long-run and short-run effects of a factor price change on a competitive industry are examined in Sections V and VI, respectively, with particular attention being given to the so-called "factor price paradox." Section VII concludes the paper by offering some remarks about applications of the theoretical results.

I. Classifying Factors of Production: Normal, Superior, and Inferior Factors

To clear the way for the analysis that follows, each of the two factors of production used by a single-product firm and purchased at fixed prices will be classified as being either normal, superior, or inferior within the relevant range of the firm's output and factor usage levels. With two factors, A and B, the isoquant diagram in Figure 1 shows the meaning of each of the three categories. Quantities of A and B are measured on the horizontal and vertical axes respectively. An isoquant having the usual properties of negative slope and convexity to the origin and derived from the firm's production function, $Q = f(A, B)$, is drawn for output Q_o.[1] Desiring to produce this output at minimum cost (or desiring to produce the largest output possible while spending only C_o on the inputs), the

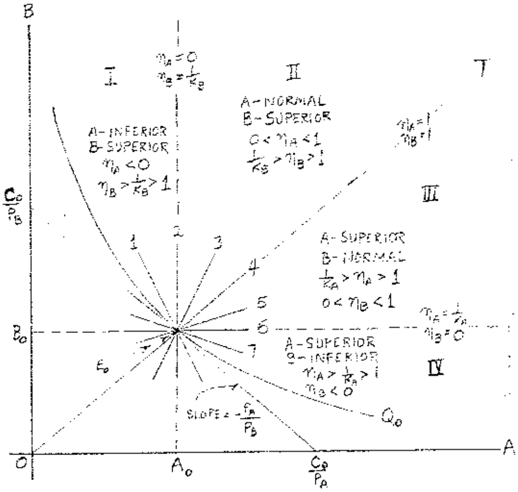

Figure 1

firm selects input combination (A_o, B_o) at point E_o, where isoquant Q_o is tangent to the isocost line with intercepts C_o/P_A and C_o/P_B and slope $-P_A/P_B$. To produce larger or smaller outputs, holding factor prices constant, the firm would select tangency points between other isocost lines and isoquants, and the common slope at all such points would be $-P_A/P_B$, the same slope as that of isoquant Q_o at E_o. When these points are connected, they trace out the firm's expansion path. Seven alternative expansion paths are given by the lines numbered 1 through 7 in Figure 1.

A factor of production is classified as normal, superior, or inferior on the basis of the variation in its use as output, (or, equivalently, total cost) varies, holding both factor prices constant.

[1] The situation for which factors are classified can be regarded either as the long-run for a firm using only two factors, or as a short-run period in which only two factors are variable, the rest being fixed. In the latter case, the production function $Q = f(A, B)$ is the short-run production function derived from the long-run function $Q = g(A, B, C, \ldots)$ by holding other inputs $(C, D, \ldots)$ at fixed levels. The short-run cost equation is then $C = P_A A + P_B B + Z$, where Z is the cost of "fixed factors." A factor's classification may depend on whether other factors are fixed. This possibility will be considered further in Section VI.

Consider factor A. If the expansion path of the firm is given by line 1, the use of A varies *inversely* with output and the total expenditure on factors, and A is classified as inferior.[2] For the expansion paths indicating that the use of A varies directly with output, A is defined as being either normal or superior, according to whether the slope of the expansion path is greater or smaller than the slope of a ray (OT) from the origin to the initial factor combination (A_o, B_o). If the expansion path is given by line 3, A is a normal factor, since the slope of line 3 exceeds the slope of the ray OT; if line 5, 6, or 7 is the expansion path, A is a superior factor because the slope of each of these expansion paths is smaller than the slope of OT. In other words, if A is a normal factor, absolutely more is used as output expands, but the ratio A/B falls as compared to the ratio at E_o; if A is superior more is used as output expands and the A/B ratio rises compared to the ratio at E_o. Borderline expansion paths for A are given by lines 2 and 4. Along the former the use of A remains constant as output varies, and A is neither normal nor inferior; along the latter, the use of A varies directly with output, the A/B ratio remains constant, and A is neither normal nor superior.

An equivalent way of classifying factor A is by the value of its expenditure elasticity, η_A, defined as the percentage change in the use of A divided by the percentage change in expenditure, C, on both factors.

$$\eta_A = \frac{dA}{A} \Big/ \frac{dC}{C} = \frac{C}{A}\frac{dA}{dC} \qquad (1)$$

Factor A is inferior if $\eta_A < 0$, normal if $0 < \eta_A < 1$, and superior if $\eta_A > 1$. Following the analysis of Truett (1971), the expenditure elasticity of A can be determined directly from Figure 1. By differentiating the equation of the firm's isocost line, $C = P_A A + P_B B$, with respect to A

$$dC/dA = P_A + P_B\,(dB/dA), \qquad (2)$$

and dividing the isocost equation through by A

$$C/A = P_A + P_B\,(B/A), \qquad (3)$$

the expression for η_A can be written as

$$\eta_A = \frac{P_A + P_B \dfrac{B}{A}}{P_A + P_B \dfrac{dB}{dA}} = \frac{\dfrac{P_A}{P_B} + \dfrac{B}{A}}{\dfrac{P_A}{P_B} + \dfrac{dB}{dA}} \qquad (4)$$

The ratio B/A in the numerator of (4) is the slope of a ray OT in Figure 1, and the term dB/dA in the denominator of (4) is the slope of the firm's expansion path. For the expansion path represented by line 3, A is a normal factor since $B/A < dB/dA$, and right-hand side of (4) must have a value between zero and one. Alternatively, if line 5 gives the firm's expansion path, A is superior since $B/A > dB/dA$, and η_A must exceed unity, since the value of the numerator of (4) exceeds that of the denominator.

Turn now to the two expansion paths which have negative slopes (lines 1 and 7). For line 1, the use of A varies inversely with output and A is inferior. Since $dB/dA < 0$, the denominator of (4) will be negative if $|dB/dA| > P_A/P_B$ and positive if this inequality is reversed. For line 1, however, $|dB/dA|$ *must* exceed P_A/P_B. Given isoquant convexity, movement to higher outputs than Q_o along expansion path 1 requires moving to points to the right and above isoquant Q_o. This can occur when moving along line 1 only if the absolute value of its slope at E_o, given by $|dB/dA|$, exceeds the absolute value of the slope of the isoquant at E_o, given by P_A/P_B. Hence, for line 1, the denominator of the right-hand side of (4) is negative, and

[2] In his insightful analysis of inferior factors, Scott (1962) argues that inferiority is represented not by a smooth decline in factor usage as output expands, but rather by a complete cessation of a factor's use. The expansion path is therefore discontinuous. The other literature has ignored this approach, perhaps sacrificing greater realism for the convenience of mathematics. Since the principal purpose here is a simplification and synthesis of this other literature, it is assumed that expansion paths are continuous.

$\eta_A < 0$. By analogous reasoning, if the expansion path is line 7, $|dB/dA| < P_A/P_B$, and $\eta_A > 1$.

Factor B can, of course, be classified by this same procedure. But when there are only two factors, once A is classified, so is B. The expenditure elasticities of the two factors are related by the formula[3]

$$K_A \eta_A + K_B \eta_B = 1, \qquad (5)$$

where $K_A = P_A A/C$ and $K_B = P_B B/C$ are the shares of total cost devoted to expenditures on A and B, respectively. Solving for η_B from (5),

$$\eta_B = \frac{1 - K_A \eta_A}{K_B} \qquad (6)$$

From (6), it follows that when A is either inferior ($\eta_A < 0$) or normal ($0 < \eta_A < 1$), B must be superior ($\eta_B > 1$);[4] conversely, when A is superior ($\eta_A > 1$), B must be either normal ($0 < \eta_B < 1$) or inferior ($\eta_B < 0$). Hence, both factors cannot be inferior, for this not only violates (5), but also implies that output can be expanded using less of both factors and, therefore, that cost is not being minimized.[5]

[3]Equation (5) is derived by taking the total differential of $C = P_A A + P_B B$ to get $dC = P_A dA + dP_A A + P_B dB + dP_B B$. Holding factor prices constant, $dP_A = dP_B = 0$, and $dC = P_A dA + P_B dB$. Dividing through by dC, $1 = P_A(dA/dC) + P_B(dB/dC)$. Multiplying the first term on the right by $(A/C)(C/A)$ and the second term by $(B/C)(C/B)$, $1 = (P_A A/C)(C/A)(dA/dC) + (P_B B/C)(C/B)(dB/dC)$. Letting $K_A = P_A A/C$ and $K_B = P_B B/C$ and noting that $(C/B)(dB/dC) = \eta_B$, (5) is obtained.

[4]When $0 < \eta_A < 1$, it follows from (6) that $\eta_B > 1$, because $(1 - K_A)/K_B = K_B/K_B = 1$ and $1 - K_A \eta_A > 1 - K_A$. Hence, $(1 - K_A \eta_A)/K_B > (1 - K_A)/K_B = 1$.

[5]Inferiority of both factors is inconsistent with isoquant convexity to the origin. In the two-factor case, denoting partial derivatives of the production function as $f_A = \partial f/\partial A$, $f_{AA} = \partial^2 f/\partial A^2$, $f_{AB} = \partial^2 f/\partial A \partial B$, etc., factor A is inferior if $f_B f_{AB} - f_A f_{BB} < 0$ and factor B is inferior if $f_A f_{AB} - f_B f_{AA} < 0$. The condition for isoquant convexity is given by $f_A^2 f_{BB} - 2f_{AB} f_A f_B + f_B^2 f_{AA} < 0$. If both factors are inferior, $f_{AB} < f_B f_{AA}/f_A$ and $f_{AB} < f_A f_{BB}/f_B$. Isoquant convexity requires $(f_A f_{BB}/f_B) + (f_B f_{AA}/f_A) < 2f_{AB}$. But if both factors are inferior $(f_A/f_{BB}/f_B) + (f_B f_{AA}/f_A) > 2f_{AB}$, a contradiction.

II. The Effect of a Factor Price Change on Average and Marginal Cost

A change in the price of one of the inputs used by a firm will cause a change in its total and average cost of production at every level of output (for which some positive quantity of the input is used). Except in a special case, its marginal cost at each output level will also be altered. The nature of the changes that are of particular interest depend upon the classification, as specified in the previous section, of the factor undergoing the change in price.

Consider first the effect of a factor price change on average cost (AC). The firm's total cost is $C = P_A A + P_B B$, and the change in total cost, as say, P_A changes is

$$dC/dP_A = A + P_A(dA/dP_A) + P_B(dB/dP_A). \qquad (7)$$

Cost minimization requires $\lambda P_A = MP_A$ and $\lambda P_B = MP_B$, where λ is the reciprocal of marginal cost (MC), and MP_A and MP_B are respective marginal products of A and B. Holding output fixed, $dQ = 0$, and

$$dQ/dP_A = MP_A(dA/dP_A) + MP_B(dA/dP_B) = 0.$$

Substituting $MP_A = \lambda P_A$ and $MP_B = \lambda P_B$ shows that $P_A(dA/dP_A) + P_B(dB/dP_A) = 0$, and (7) becomes:

$$dC/dP_A = A. \qquad (7')$$

If $A > 0$, total cost must vary directly with P_A. Moreover, at a given output the change in AC as P_A changes is the change in total cost divided by Q:

$$d(C/Q)/dP_A = d(AC)/dP_A = A/Q, \qquad (8)$$

and average cost must also vary directly with P_A.

Turn now to the effect of a change in P_A on marginal cost (MC). If (7′) is differentiated with respect to Q,

$$d(dC/dP_A)/dQ = dA/dQ = d(dC/dQ)/dP_A, \qquad (9)$$

since the order of differentiation is irrelevant. The extreme right-hand term is simply the change in MC as P_A varies. Therefore,

$$d(MC)/dP_A = dA/dQ, \qquad (10)$$

and MC varies directly with P_A if A is normal or superior (varies directly with output) and inversely if A is inferior (varies inversely with output).

The results in (8) and (10) can be used to determine the answer to one additional question: what effect does the change in P_A have on the output, Q^*, at which the firm's average cost is minimized? If at the initial value of Q^*, the extent of the shift in MC relative to that in AC can be found, the direction of the change in Q^* can be ascertained as well. (For example, if a fall in P_A reduces MC by more than AC at Q^*, the new value of Q^* must exceed the old.)

The changes in MC and AC resulting from a change in P_A can be related to the expenditure elasticity of A, η_A. Taking the ratio of (10) to (8),

$$d(MC)/dP_A/d(AC)/dP_A$$
$$= (dA/dQ)/(A/Q) = (Q/A)/(dQ/dA). \qquad (11)$$

At the output Q^* where AC is minimized the firm's production function exhibits constant returns to scale. Therefore by Euler's theorem, $Q = MP_A A + MP_B B$. Dividing the Euler equation through by A,

$$Q/A = MP_A + MP_B(B/A) \qquad (12)$$

Differentiating the Euler equation with respect to A[6]

$$dQ/dA = MP_A + MP_B(dB/dA). \qquad (13)$$

The right-hand side of (11) can be written as the ratio of (12) and (13):

$$d(MC)/dP_A/d(AC)/dP = \frac{MP_A + MP_B(B/A)}{MP_A + MP_B(dB/dA)}. \qquad (11')$$

Since $MP_A = \lambda P_A$ and $MP_B = \lambda P_B$ as the required conditions for cost minimization,

$$\frac{MP_A + MP_B \dfrac{B}{A}}{MP_A + MP_B \dfrac{dB}{dA}} = \frac{\lambda\left(P_A + P_B \dfrac{B}{A}\right)}{\lambda\left(P_A + P_B \dfrac{dB}{dA}\right)} = \eta_A, \qquad (14)$$

as can be seen by comparing (14) with (4). Therefore,

$$\eta_A = d(MC)/dP_A/d(AC)/dP_A. \qquad (15)$$

If $\eta_A < 1$, a fall in P_A reduces MC by less than AC (if $\eta_A < 0$, MC actually rises) and the output at which AC is minimized, Q^*, is reduced. If $\eta_A = 1$, AC and MC fall by the same amount and Q^* remains unchanged. Finally, if $\eta_A > 1$, MC falls by more than AC and Q^* increases.[7]

[6] $dQ/dA = MP_A + MP_B(dB/dA) + A\,d(MP_A)/dA + B\,d(MP_B)/dA$. But the last two terms on the right can be shown to equal

$$\left(\frac{B}{A} + \frac{MP_A}{MP_B}\right)\left[A\left(\frac{A}{B}\frac{\partial(MP_A)}{\partial A} - \frac{B}{A}\frac{\partial(MP_B)}{\partial B}\right)\right]$$

Since at the point of constant returns scale $(A/B)(\partial(MP_A)/\partial A) = (B/A)(\partial(MP_B)/\partial B)$ these last two terms cancel out, leaving the expression for dQ/dA given in (13).

[7] This proof of the way Q^* varies with a factor price change is necessarily valid only if the factor classification associated with the new Q^* (which may be the same as the old) and the new factor price ratio remains the same as it was before the factor price change. While factor classification must remain the same if the price change is infinitesimal, as the proof assumes, it may not, as Truett and Roberts (1973) demonstrate, for a finite price change. Just as a consumer good may have a different (point) income elasticity of demand in finitely different price-income situations, so may a factor of production have a different (point) expenditure elasticity in finitely different factor-price-output situations. In other words, arc and point elasticities may differ. These two elasticities will not differ if each expansion path of the firm, for each alternative set of factor prices, is a ray through the origin, so that $\eta_A = \eta_B = 1$. Imagine that a finite change in factor prices is broken into infinitesimal steps. For each step (15) shows that no change occurs in Q^*, and Q^* must be invariant to finite, as well as infinitesimal, factor price changes. In the more general case where the firm's expansion paths are not rays through the origin, (15) shows that the effect of a factor price change, say, in P_A also remains unambiguous if the value of η_A always stays below or above unity at each value of P_A between its old and its new value. Ambiguity can only arise when the value of η_A changes from below to above unity or *vice versa* for some values of P_A between its old and new levels.

TABLE 1

Region	Expansion Path	Value of η_A	Value of η_B	$\frac{d(MC)}{dP_A} = \frac{dA}{dQ}$	$\frac{d(MC)}{dP_B} = \frac{dB}{dQ}$	$\frac{dQ^*}{dP_A}$	$\frac{dQ^*}{dP_B}$
I	1	$\eta_A < 0$	$\eta_B > 1/K_B > 1$	<0	>0	>0	<0
Borderline I, II	2	$\eta_A = 0$	$\eta_B = 1/K_B > 1$	0	>0	>0	<0
II	3	$0 < \eta_A < 1$	$1/K_B > \eta_B > 1$	>0	>0	>0	<0
Borderline II, III	4	$\eta_A = 1$	$\eta_B = 1$	>0	>0	0	0
III	5	$1/K_A > \eta_A > 1$	$0 < \eta_B < 1$	>0	>0	<0	>0
Borderline III, IV	6	$\eta_A = 1/K_A > 1$	$\eta_B = 0$	>0	=0	<0	>0
IV	7	$\eta_A > 1/K_A > 1$	$\eta_B < 0$	>0	<0	<0	>0

The results of Sections I and II are collected in Table 1.

III. Factor Price Changes and a Firm's Factor Demand

This section presents a geometric proof that—in contrast to a consumer's demand curve for a particular commodity—it is theoretically impossible for a profit-maximizing firm's demand curve for an input to be positively sloped at any point. A competitive firm's demand for a factor for which $\eta > 0$ is analyzed first, and this is followed by a treatment of the case where $\eta < 0$. The similar analysis for a monopolistic firm is then briefly summarized.

The Demand for a Normal or Superior Factor. The equilibrium position of a firm when the competitive industry to which it belongs is in long-run equilibrium is given by Figure 2, panels (a) and (b). Associated with its optimal output rate of Q_o in panel (a) is an optimal level of usage of inputs, A and B, given by point E_o in panel (b).

Suppose now that the price of input A falls to this firm only, say, through a government subsidy of its purchases of A.[8] Having the price of A fall only to the firm in question and not to other firms in the industry makes it possible to maintain the assumption that product price remains constant at OP_o, since only this firm and no others will change output. The effect of this fall in P_A on the firm's use of A can be broken into two parts, a cost-minimizing adjustment which is called the *substitution effect* and profit-maximizing output adjustment which can be called the *output effect.* The substitution effect represents the substitution of A for B holding output constant while the output effect is the change in the use of A resulting from the firm's decision to change its rate of output because P_A falls. The output effect itself can be viewed heuristically as composed of three linked relationships: the change in MC when P_A is reduced, the change in output due to the change in MC, and the change in the use of A associated with the change in output.

The substitution and output effects of the fall in P_A are represented in Figure 2, panel (b). From the initial position at E_o, the fall in P_A induces the firm to move to E_1, increasing its output from Q_o to Q_1 and the use of A from A_o to A_1. The substitution effect of the fall in P_A is represented by the move from E_o to F', the point at which a line with a slope indicating the new input price ratio is tangent to the isoquant Q_o representing the original output level. The output effect is represented by the move from F' to E_1 along the expansion path $F'E_1$ which reflects the new input price ratio. The output effect reflects the three linked relationships mentioned above: when P_A falls, MC falls from MC_o to MC_1 (in panel (a)) since $\eta_A > 0$; this fall in MC induces the firm to increase output to the new profit-maximizing level at Q_1;

[8]This procedure, along with the heuristic description of the output effect below, is employed in a lucid, unpublished paper by Welch (1968).

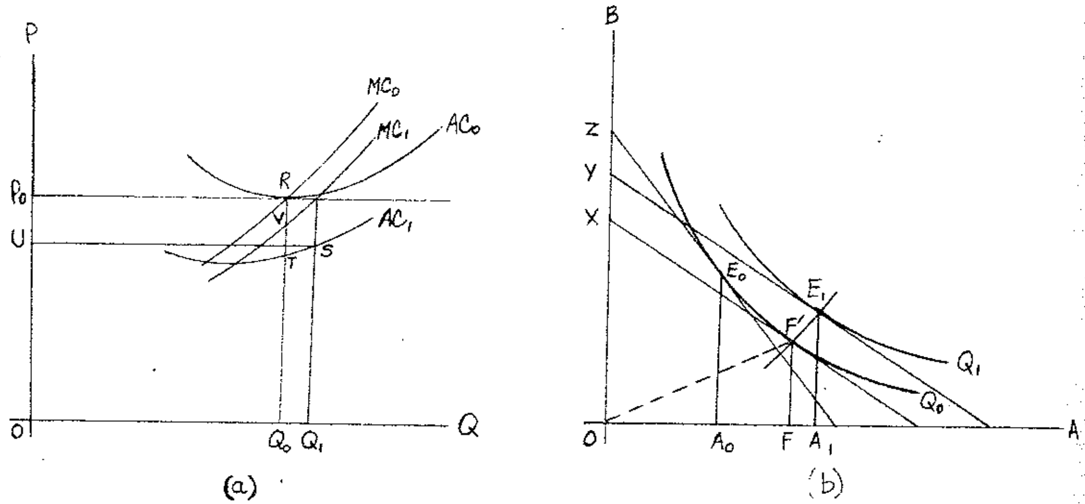

Figure 2

associated with this higher output is an increase (again because $\eta_A > 0$) in the use of A from F to A_1. Both the substitution and output effects lead to an increase in the use of A when P_A declines.

As a result of the fall in P_A, two points on the firm's demand curve for A are generated. To get other points P_A can be varied by other amounts with the firm being allowed to make the appropriate cost-minimizing substitution and profit-maximizing output adjustments. It is clear that the demand curve for a normal or superior factor must be downward-sloping, since the substitution and output effects produce adjustments in input usage that are in the same direction—always opposite to the direction of the input price change.[9] It is now shown that this same conclusion holds if the factor is inferior over the relevant range.

The Demand for an Inferior Factor. To determine the effect of a fall in P_A when $\eta_A < 0$, consider Figure 3, panels (a) and (b). Since $\eta_A < 0$, marginal cost rises when the price of A falls, and the firm's profit-maximizing output declines. In panel (a) of Figure 3, this is represented by the shift in the marginal cost curve from MC_o to MC_1 and by the consequent reduction in output from Q_o to Q_1. Panel (b) indicates the effect of the change in P_A on the amount of factor A the firm employs to produce the new profit-maximizing output, Q_1. The firm's factor combination shifts from that indicated by point E_o to point E_1. The substitution effect is indicated by the movement from E_o to F'. With output held constant at Q_o, the increase in the usage of A resulting solely from the fall in its price is given by A_oF. The output effect is indicated by the movement from F' to E_1, the latter point representing the profit-maximizing output level and input combination for the firm. The increase in the employment of A due to the output effect is FA_1. This movement is the adjustment the firm makes along the negatively sloped input

[9]As panels (a) and (b) of Figure 4 are drawn, η_A must lie between zero and unity. The expansion path E_1F' has a steeper slope at F' than the dashed ray OF'. Hence, the fall in P_A must reduce MC but by less than AC ($RV < RT$ in panel (a)) and the output at which average cost is minimized shifts to the left. It should be clear, however, that in the case where $\eta_A > 1$, the substitution and output effects must still be in the same direction, for MC must also fall in this case as well.

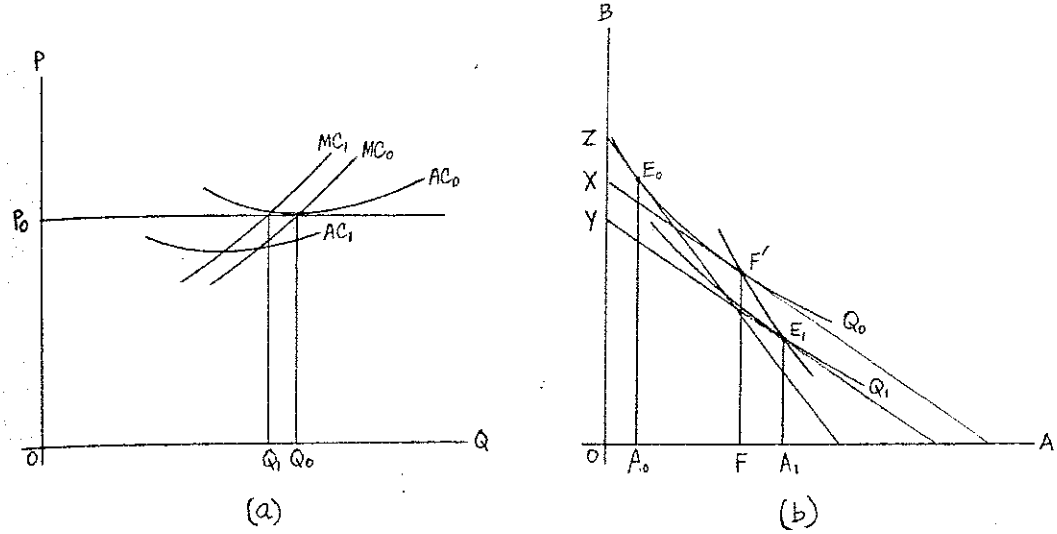

Figure 3

expansion path, E_1F'. Again, both of the substitution and output effects work in the *same* direction: both lead to an increase in the employment of A.

A reduction in the price of an input always leads, therefore, to an increase in its use by a profit-maximizing, competitive firm regardless of how the factor is classified. The substitution effect is always negative and the output effect is also. Where $\eta_A > 0$, a fall in P_A *reduces* MC, increases output, and causes the use of A to increase; if, on the other hand, $\eta_A < 0$, a fall in P_A *increases* MC, reduces output, and again causes the use of A to increase.

The conclusion that a competitive firm's demand curves for inputs will be downward sloping readily generalizes to the firm facing a downward-sloping product demand curve. The substitution effect of an input price change is clearly negative for any firm independently of demand conditions. In addition, the output effect is again always in the same direction as the substitution effect, since a change in marginal cost must change the output of a monopolistic firm in the same direction as that of a competitive firm, except in the trivial case where the product demand curve is perfectly inelastic and there is no output effect at all.

These results can be used to explain why there is no reference to "Giffen inputs" in the theory of the profit-maximizing firm. Input price changes do not produce anything analogous to the income effects discussed in the theory of consumer behavior because, unlike the consumer who is postulated to have a certain fixed budget, the firm is not viewed as being constrained to spend only a certain amount of money on inputs and no more. Hence, the output effect for the firm is derived in a different way than the income effect for a consumer. Following a change in an input price, the firm's new equilibrium may involve it spending more, less, or the same amount on inputs than before, depending on what adjustment is required to maximize profits given the altered situation. As illustrated in Figure 2, panel (b), the firm spends less on inputs (OY) in its new equilibrium at E_1 than it spent (OZ) at E_o. However, the fall in P_A, given that $\eta_A > 0$, could have induced it to spend the same amount or more. As Figure 3 shows, if $\eta_A < 0$, the firm would definitely spend less on inputs than before since OY must be less than OZ.

IV. Maximizing Sales or Output and Giffen Inputs

If a non-competitive firm has a goal of maximizing not profit but sales revenue subject to a profit constraint, Portes (1968) has pointed out the theoretical possibility of an upward sloping input demand curve. The essence of the argument is that, because of the profit constraint, sales may not be at the maximum level where marginal revenue (MR) is zero. A fall in the price of an input reduces average cost and allows the firm to increase its output and sales by lowering price, dissipating the extra profits the fall in the input price creates. While the firm substitutes the now cheaper input for others in order to minimize cost, the increase in output that the input price reduction allows causes *less* of the input to be employed, if the input in question is inferior. Hence, this output effect could conceivably offset the substitution effect, less of the input being employed at the lower price. This perverse result can occur, however, only if the input is inferior; for normal and superior inputs both the output and substitution effects must be in the direction of greater employment.

Figure 4, panels (a) and (b), illustrates the argument. Panel (a) shows the demand, marginal revenue, and average cost curves of the firm, where average cost includes the required profit per time period. (Ignore for now the dashed curve $\overline{AC}_o$.) Initially, the firm is producing at Q_o where AC_o intersects the demand curve. The corresponding point in panel (b) is at E_o. The firm is not able to attain the unrestrained sales maximum at Q^* where $MR = 0$ since this would require it to violate the minimum profit constraint. (AC_o is above AR at point Q^*.) Suppose now the price of A falls and that $\eta_A < 0$. The substitution effect of this price reduction is represented in panel (b) as the move from E_o to F' along isoquant Q_o. The use of A increases from A_o to F. However, the output effect of the reduction in P_A, repre-

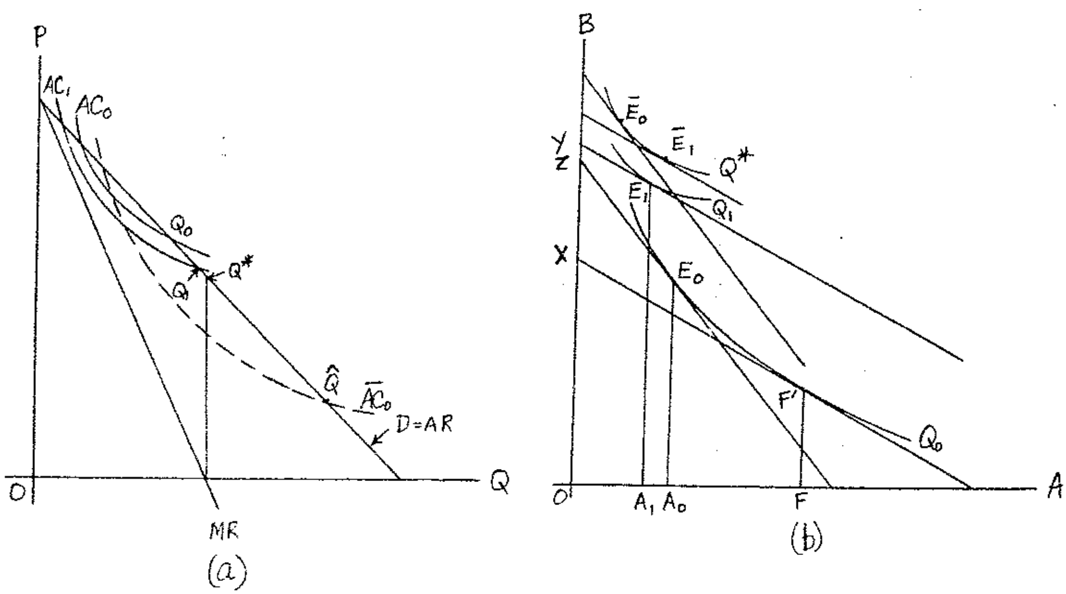

Figure 4

sented by the move from F' to E_1, involves a smaller employment of A. Output increases when P_A falls because the postulated goal of the firm is the maximization of sales rather than profit. The fall in P_A allows the firm to move nearer to the unrestrained sales maximum at Q^*. As illustrated in panel (b), the change in A used due to the output effect, FA_1 (which is necessarily negative since $\eta_A < 0$), exceeds the substitution effect A_oF and the use of A declines.[10]

This result could not be a theoretical possibility if $\eta_A > 0$, because the expansion path of the firm would be positively sloped, and the increase in output accompanying the fall in P_A would involve a larger employment of A and reinforce the substitution effect. Nor could the firm's demand for A be positively sloped if its initial average cost curve were $\overline{AC}_o$. The firm would choose point Q^* in panel (a) and point $\overline{E}_o$ in panel (b) and attain the unconstrained sales maximum since the profit constraint would not be binding. A fall in P_A allows the firm to remain at Q^*. There is, therefore, no output effect, and the substitution effect, represented by the movement from $\overline{E}_o$ to $\overline{E}_1$, involves increasing the use of A.

In view of the criticism of the sales-maximizing model as implying that firm managers or enterpreneurs have a lexicographic ordering (Alchian, 1965; Rosenberg, 1970), the theoretical possibility of an upward sloping factor demand curve may have little practical relevance due to the scarcity of sales-maximizers. However, this possibility also arises if the firm is owned by the state and the reward structure for the managers is such that his (her) goal is maximizing, not sales revenue, but production (i.e., output) subject to the constraint that revenues cover costs (which may include some minimum profit requirement). Such a firm will always produce the largest output at which AC equals AR, and a fall (rise) in P_A, would always raise (reduce) output and involve an output effect, if $\eta_A < 0$, in the opposite direction to the substitution effect.[11] Again, a Giffen input is a theoretical possibility.

V. Factor Price Changes and Long-Run Adjustment by a Competitive Industry

This section turns from the analysis of the effects of input price changes on the individual firm to the effects of such changes on the equilibrium of a competitive industry. It is assumed that the supply curves of the inputs used by the industry, A and B, are perfectly elastic and that the firms in the industry and all potential entrants have the same production function. The industry, therefore, is assumed to be one of constant costs. Attention is confined to the analysis of a factor price decrease, but what follows can easily be reconstructed by the reader to deal with a price increase. This section deals with the long-run effects. Section VI considers the modifications required when one factor is "fixed."

Reduction in the Price of a Normal Input. Suppose the price of A, a normal input, falls because of a technological innovation in its production. The effects are illustrated in panels (a), (b), and (c) of Figure 5. Panel (a) represents both the industry supply and demand curves and the horizontal summation of the marginal and average cost curves of all the firms (the number of which is, say, N_o) in the industry in its initial long-run equilibrium, with

[10] A fall in P_A would not reduce the amount of A used by the firm if its goal were profit maximization. Since $\eta_A < 0$, MC would be increased when P_A falls. The higher MC curve would intersect MR at a lower output, and the output effect would lead to the use of *more* A just as the substitution effect does.

[11] This is presumably the model that Portes (1968) had in mind when constructing his Figure 1. Though he assumes sales maximization he fails to put an MR curve on his diagram and he argues that the firm will always choose the output where $AC = AR$. This is true for an output-maximizer, but not necessarily for a sales-maximizer, who would select the output Q^* where $MR = 0$ if the profit constraint is satisfied at that output. If $\overline{AC}_o$ were the relevant cost curve, the sales-maximizer would select Q^* while the output-maximizer would select $\hat{Q}$.

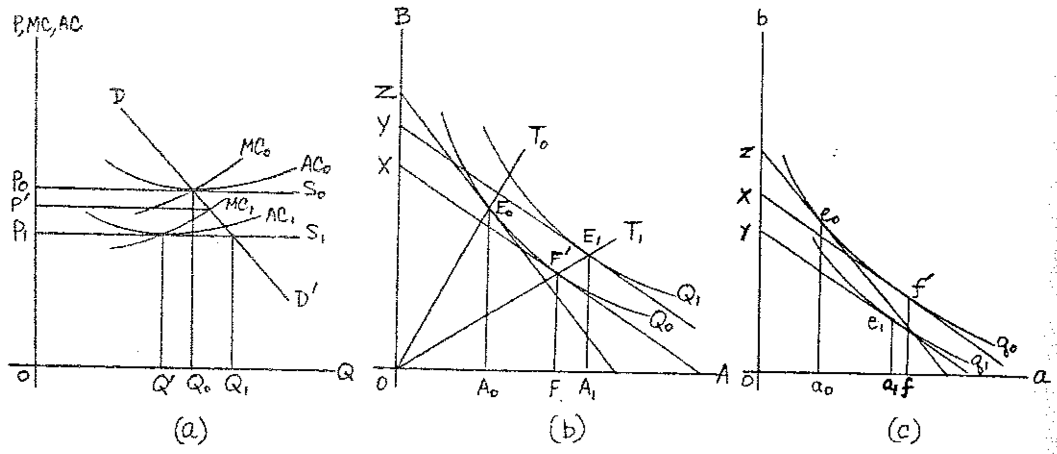

Figure 5

output Q_o and price P_o.[12] Panel (b) represents an isoquant map for the *industry*, the quantity of A used by the industry measured horizontally and the quantity B measured vertically. The factor combination used by the industry is initially given by point E_o, with line ZE_o reflecting the initial factor price ratio and industry isoquant Q_o showing initial industry output. Panel (c) is an isoquant diagram for an individual firm and represents the situation of every firm since all are identical. The initial position of each firm is e_o, where isocost line ze_o (having the same slope as ZE_o in panel (b)) is tangent to isoquant q_o. With N_o identical firms, A_o and Q_o in panel (b) are equal, respectively, to $N_o a_o$ and $N_o q_o$ in panel (c).

Suppose now that P_A falls and that sufficient time is allowed for all factors to be variable but insufficient time for any change in the number of firms in the industry. Since $0 < \eta_A < 1$, the fall in P_A shifts downward the marginal and average cost curves of each firm, the latter falling by more than the former at output q_o, and each firm's minimum average cost output is reduced. Since the number of firms is held fixed, the aggregate curves in panel (a) shift in the same way, viz., from MC_o to MC_1 and AC_o to AC_1. The industry supply curve is now MC_1 and product price falls to P'. At this price each firm must be making above-normal profits because $P' = MC_1$ and MC_1 exceeds AC_1 for outputs greater than Q'. New firms are induced to enter, and as entry occurs, MC_1 and AC_1 shift horizontally to the right along long-run supply curve S_1 until output has increased to Q_1 and price has fallen to P_1, at which point above-normal profits are eliminated and the industry is in a new long-run equilibrium. The aggregate output of the original firms is now Q'; the remainder, $Q'Q_1$, is produced by new firms.

Consider now the change in the factor mix utilized by the industry and by each firm. In panel (b) point E_1 shows the new industry factor mix while point e_1 in panel (c) shows the factor mix for each firm, new and old alike; the industry's employment of A has increased from A_o to A_1. This increase can be broken into industry substitution and output effects. The former is represented by the move from E_o to F'. This would be the only adjustment made by the industry if the product demand curve

[12] A diagram similar to panel (a) is used by Maurice (1971) to which the analysis of this and Section VI is heavily indebted.

happened to be perfectly inelastic. It necessarily involves an increase in the number of firms in the industry, since the fall in P_A reduces the minimum average cost output and, therefore, requires more firms if the given output Q_o is to be produced at minimum cost. Hence, at F', $\overline{N}(>N_o)$ firms each produce q_1 (given by point e_1 in panel (c)), and $\overline{N}q_1 = Q_o$. The move from F' to E_1 is the industry output effect and results solely from the addition of new firms, each of which is producing an output of q_1 and using the combination of factors indicated by point e_1 in panel (c). This industry output effect is along the new industry expansion path OT_1, which, like the expansion path OT_o (relevant before the fall in P_A), must be a ray from the origin since the *industry* must be regarded as having a production function exhibiting constant returns to scale. Therefore, from the *industry's* standpoint, the expenditure elasticity of A and B, η_A^I and η_B^I, must equal unity, regardless of the values of these elasticities for the individual firms.

Panel (c) indicates the long-run adjustment made by each old firm, which is from e_o to e_1, and shows a reduction in output from q_o to q_1. There is no point in panel (b) to which point f' in panel (c) corresponds. In particular, the move from e_o to f' in panel (c) does not correspond to the move from E_o to F' in panel (b), since in making the latter move the number of firms is increased from N_o to $\overline{N}$, to produce industry output Q_o at minimum cost, while the move from e_o to f' simply represents the adjustment that would have to be made by each individual firm to minimize the cost of producing the old individual firm output, q_o, an output which no firm continues to produce once P_A has fallen. The quantity of A represented by F in panel (b) equals $\overline{N}$ times quantity a_1 in panel (c), and the quantity A_1 in panel (b) equals N_1 times a_1 in panel (c), where N_1 $(>\overline{N})$ is the number of firms in the industry in the new long-run equilibrium.

Panel (c) reveals that for each of the existing firms in the industry prior to the fall in P_A, the substitution and output effects of this price change are in opposite directions.[13] Though not illustrated in panel (c), it is theoretically possible for the output effect to offset the substitution effect, so that each old firm uses less A at its new long-run equilibrium position, e_1, than at its old position, e_o.[14] Consistency with panel (b), however, requires greater employment of A by the industry as a whole. Hence, if there is a reduction in the use of A by the group of old firms, this must be more than offset by the additional A used by the new firms. This is easily proven. By showing that it must be true when considering only the industry substitution effect, the desired result is obtained, since the industry output effect necessarily involves entry of more firms and a greater use of A.

For the industry substitution effect, industry output is constant at Q_o; therefore, $Ndq + qdN = 0$, where dN is the increase in firms and dq is the decrease in output per firm. Hence, $dN/N = |dq/q|$. Suppose the amount of A used in old firms actually falls, the reduction being Nda. The increase in A used by new firms is adN. For a greater amount of A to be used, it is required that $adN > |Nda|$, or

$$\frac{dN}{N} = \left|\frac{dq}{q}\right| > \left|\frac{da}{a}\right|. \tag{16}$$

Equations (11) and (15), taken together, show that for a small output change from the point at which average cost is minimized (such as e_1 in panel (c)), $\eta_A = (da/a)/(dq/q)$. With $0 < \eta_A < 1$

[13] Only when $0 < \eta_A < 1$ can these effects be in opposite directions. If $\eta_A > 1$, each firm increases output and the use of A when P_A falls, since the output at which average cost is minimized increases. If $\eta_A < 0$, the output of each old firm contracts and more A is used at this lower output.

[14] Points e_o and e_1 are not, however, points on a competitive firm's demand curve for A, since such a curve is derived by holding product price, as well as the prices of other inputs, except P_A, constant. This procedure was followed in Section IV. The move from e_o to e_1 is the result of a variation in both P_A and the product price.

by assumption, the inequality in (16) must hold, and the industry substitution effect involves a greater use of A. And, because the industry output effect simply involves the addition of more firms, all using a positive amount of A, the total effect of a fall in P_A must be an increase in the quantity of A used by the industry.

Reduction in the Price of an Inferior Input. The analysis of a fall in P_A when $\eta_A < 0$ differs only slightly from that of the previous case, and a separate diagram is not provided. (The change in the cost curves of a firm can be seen in panel (a) of Figure 3.) When P_A falls, the average cost of each firm falls producing a downward shift in the long-run industry supply curve. However, marginal cost of each firm *rises,* and, prior to entry of new firms, price rises *above* the initial equilibrium price. The above-normal profits of each existing firm induces entry, and price eventually falls to the minimum of the new average cost curve. Industry output and the number of firms is greater and output per firm is smaller than in the previous equilibrium. The only other significant difference between the analysis when $\eta_A < 0$ and when $0 < \eta_A < 1$ is that with $\eta_A < 0$, the substitution and output effects for each firm are in the same rather than opposite directions. Since each existing firm uses more A, there is no need to go through any further analysis (like that culminating in equation (16)) to establish that the industry demand curve for A is downward sloping.

Reduction in the Price of a Superior Input. The effects of a change in the price of an input for which $\eta > 1$ has been a source of controversy in the literature because of the possibility that each existing firm in an industry may suffer losses (earn profits) as a result of a reduction (rise) in the price of such an input. This "paradox" was first analyzed as a short-run phenomenon by Nelson (1957) and has caused one economist (Meyer, 1967 and 1968) to conclude that the competitive model contains an inconsistency. A factor price reduction surely means (since average cost is reduced) that the product using the factor is cheaper to produce and that the amount produced ought to expand. Yet, if a factor price reduction causes existing firms to suffer losses, this suggests that firms will exit from the industry, causing a fall in the amount supplied at any given price.

This seeming paradox is, however, illusory, as Figure 6, indicating the effects of a fall in P_A when $\eta_A > 1$, reveals. Panel (a) has been "rigged" to show the borderline situation in

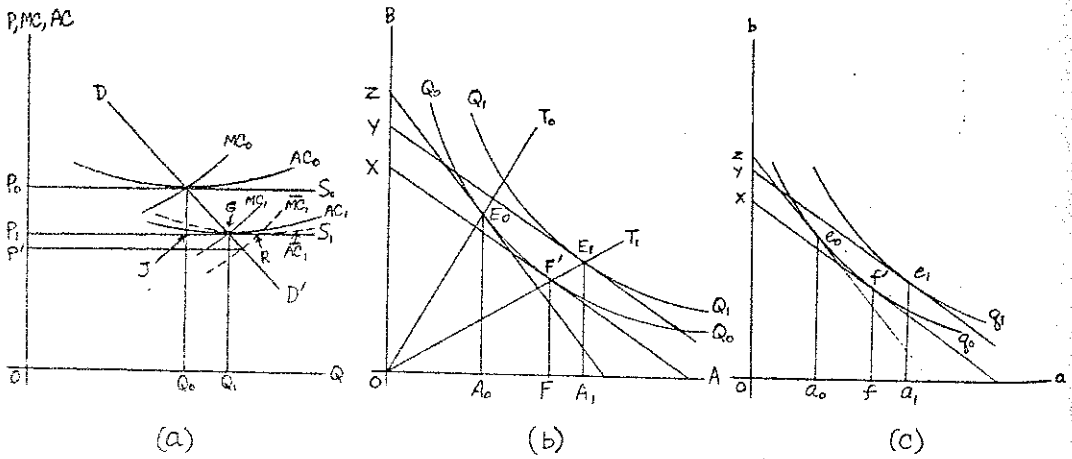

Figure 6

which the fall in P_A creates neither above- nor below-normal profits (i.e., losses). The shift down and to the right in the aggregate average cost curve, from AC_0 to AC_1 is such that the new long-run equilibrium (Q_1, P_1) is attained with no need for a change in the number of firms in the industry. Price falls to the new level of minimum average cost of each existing firm, and the increase in output per firm, indicated in panel (c) by the move from isoquant q_o to q_1, when multiplied by N_o, the existing number of firms, is just equal to the increase in quantity demanded when price falls from P_o to P_1. Each firm, of course, uses more A, and the quantity used by the industry rises from $A_o(=N_oa_o)$ to $A_1(=N_oa_1)$.

The "paradox" arises when the fall in P_A shifts the minimum point on the aggregate average cost curve to a position that falls to the right of the demand curve, such as point R in panel (a) of Figure 6. At R each firm is incurring a loss because product price, P', given by the intersection of the aggregate marginal cost curve $\overline{MC}_1$ and DD', must necessarily be less than average cost. Given the increase in the minimum cost output of each firm, the original number of firms, N_o, is now too great to provide the output demanded at price P_1 (which equals minimum average cost) at the smallest possible total cost. Hence, the "paradox" is no paradox at all. The occurrence of losses is a *correct* signal that too many firms are in the industry. The losses cause enough firms to exit to insure that price rises to P_1. At this point the losses are no longer being made and the industry is again in long-run equilibrium with fewer firms, a larger output and a lower price. The "paradox" is resolved by recognizing that losses can occur because of a fall in a factor price only if this reduction causes the existing number of firms in an industry to be too great to produce a long-run equilibrium output at minimum industry cost.[15]

Panel (b) of Figure 6 indicates the adjustments in factor use by the industry as a whole. When P_A declines, the industry substitution effect involves an adjustment from E_o to F', and at F' the output Q_o would necessarily be produced by a smaller number of firms, each of which would be using the factor combination indicated by e_1 in panel (c). The industry output effect is represented by the move from F' to E_1 and represents an increase in output via the addition of enough firms, each producing q_1, to raise industry output to Q_1. Whether there are more, less, or the same number firms at E_1 than E_o depends on the extent of the shift in the output at which aggregate AC is minimized in relation to the industry demand curve. For AC_1 there is the same number; for $\overline{AC}_1$ there are fewer firms at E_1.

Panel (c) also indicates that the substitution and output effects both cause each firm that is in the industry in both the old and new equilibrium to use more A than before. Hence, if the number of firms either increases or remains constant, it is clear that more A is used by the industry, as indicated in panel (b). However, if the number of firms declines, can it still be asserted that the industry uses more A? The answer is "yes," and the proof is exactly analogous to that used in discussing the similar problem arising where $0<\eta_A<1$, and firm output and substitution effects go in opposite directions. Here, however, these effects for each firm go in the same direction and the relevant question is whether the increase in each remaining firm's use of A more than offsets the reduction in A used by the industry because some firms exit. Considering only the industry substitution effect, the change in industry output is zero, and the relative decrease in the number of firms $|dN/N|$ equals dq/q, the relative increase in output per firm. If the industry is to use more A, the increase in A used by remaining firms, Nda, must exceed the absolute value of the reduction in use of A by exiting

[15] For the opposite case of a factor price *increase* that causes *profit*, such a result can only occur when the existing number of firms is too few to produce the new long run output at minimum cost. Profits (losses) only occur when entry (exit) is appropriate.

firms, $|adN|$. This implies:

$$\left|\frac{dN}{N}\right| = \frac{dq}{q} < \frac{da}{a}. \qquad (17)$$

At the minimum AC output, $(da/a)/(dq/q) = \eta_A$. Since $\eta_A > 1$, the inequality in (17) holds and the industry substitution effect reinforces the industry output effect. More A is used by the industry as P_A falls.

VI. Short-Run Effects of Input Price Changes and the Dynamics of Competitive Industry Adjustment

When a firm uses only two inputs, the conventional analysis of the short run assumes that the use of one input can be varied while the quantity of the other available for use is fixed. The firm can adjust to factor or product price changes only by changing the amount of the variable input. If B is the fixed factor and A is variable, the firm's adjustment to changed conditions, as represented in an isoquant diagram, is limited to movements back and forth along a horizontal line representing the fixed quantity of B, such as line 6 in Figure 1. Therefore, the short-run expenditure elasticity of A is $\eta_A^S = (1/K_A) > 1$, and that of B, η_B^S, is, of course, zero. Starting from a position of long-run equilibrium for the firm and industry, the short-run effects of factor price changes can be readily determined by applying the results of previous sections. Specifically, when A is the variable factor, it must be superior ($\eta_A^S > 1$), and a change in P_A affects the short-run cost curves of the firm in the same way that the long-run cost curves are affected by a change in P_A if the long-run expenditure elasticity of A, which may now be denoted η_A^L, is greater than unity. Similarly, if A is the fixed factor ($\eta_A^S = 0$), a change in P_A affects the short-run cost curves in the same way that the long-run curves are affected by a change in P_A when $\eta_A^L = 0$. No particular relationship must exist, however, between η_A^S and η_A^L. While $\eta_A^S > 1$ if A is variable in the short run and $\eta_A^S = 0$ if A is fixed, A may be either normal, inferior, or superior in the long run when full adjustments can be made by choosing different cost-minimizing input combinations on the expansion path.[16] A change in P_A may thus affect the short-run and long-run curves differently. Rather than examine each case, the analysis is confined to the situation in which A is a normal factor in the long run. The reader can easily alter the analysis to fit the cases where A is inferior or superior in the long run.

Consider first the effect of a fall in P_A to only one firm in a competitive industry and suppose that A is the variable factor and B is fixed. Figure 7, panel (a), shows the effect on the firm's cost curves. With $\eta_A^S > 1$ and $0 < \eta_A^L < 1$, both the short and long-run marginal cost curves shift downward and to the right from SMC_o to SMC_1 and LMC_o to LMC_1, respectively. The average cost curves also shift downward, but the output at which long-run average cost is minimized shifts to the left, from Q_o to Q', while for the short-run average cost curve, this output moves to the right from Q_o to Q''. Since P_A falls to this firm only, product price remains constant at P_o and the firm's short-run response is an increase in output to Q_1^S, where $SMC_1 = P_0$. When the firm has sufficient time to alter the amount of B employed and adjust its factor combination in

[16] When there are more than two factors and more than one is variable in the short run, there is no necessity for the short-run expenditure elasticity of any one of these variable factors to be greater than its expenditure elasticity in the long run when all factors can be varied. For example, with three factors, A, B, and C, and C the only fixed factor in the short run, equation (5) indicates that $K_A\eta_A^S + K_B\eta_B^S = 1$ in the short run, while in the long run, $K_A\eta_A^L + K_B\eta_B^L + K_C\eta_C^L = 1$. Subtracting the first equation from the second, $K_A(\eta_A^L - \eta_A^S) + K_B(\eta_B^L - \eta_B^S) = -K_C\eta_C^L$. Even if $\eta_C^L > 0$, both A and B could have greater expenditure elasticities in the long run than in the short run. Hence, the discussion of the two-factor case in which $\eta_A^S > 1$ is a rather special case. If A is not the only variable factor it would not necessarily be a superior factor in the short run.

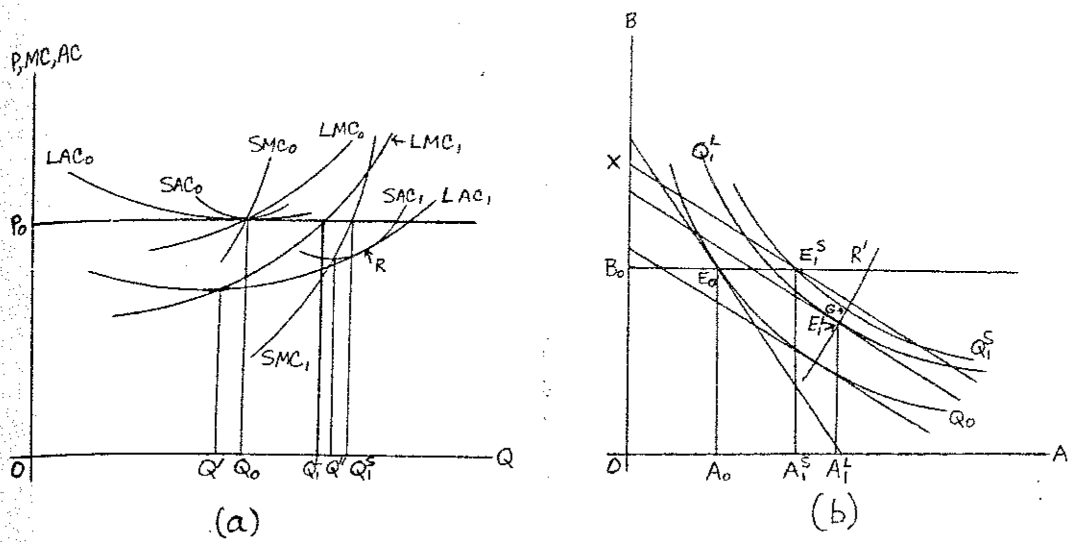

Figure 7

light of the new factor prices, its long-run output is Q_1^L, the output for which $P_o = LMC_1$.

Panel (b) indicates the firm's short- and long-run input adjustments. From its initial input combination at E_o, the short-run adjustment is to point E_1^S, as the firm increases its use of the variable factor from A_o to A_1^S. The cost of producing Q_1^S is not being minimized at this point because the factor price ratio, P_A/P_B, is less than the marginal product ratio, MP_A/MP_B. The total cost of this output is X (measured in units of B). If the firm could vary its use of B and move to the cost-minimizing point G, output Q_1^S could be produced at a lower cost. This explains why in panel (a) the short-run average cost of producing Q_1^S is greater than the long-run average cost of producing this output.[17] Given sufficient time to vary the amount of B employed, the firm moves to point E_1^L on the expansion path appropriate to the lower relative price of A, the use which expands further to A_1^L.

If A is the fixed factor ($\eta_A^S = 0$), the analysis is rather less complicated. A fall in P_A leaves short-run marginal cost unaffected and the firm does not change its output rate in the short run. However, with $0 < \eta_A^L < 1$, the fall in P_A does shift downward the firm's long-run marginal cost curve and in the long run the firm expands its output rate.

Consider now the short- and long-run adjustments of a competitive industry as the price of A falls to all firms. An interesting divergence between short- and long-run profit signals can arise if A is the variable factor in the short run. As demonstrated in the previous section, a fall in P_A could lead to losses by all firms if $\eta_A^L > 1$. A similar possibility arises in the short run, since, with only two factors and A variable, $\eta_A^S = 1/K_A > 1$. If at the same time the long-run expenditure elasticity of A is less than unity, it is necessarily true that once existing firms have had time to adjust fully and prior to entry of new firms, above-normal profits must be accruing to existing firms. The new long-run

[17] SAC_1 equals LAC_1 at point R and exceeds it at all other outputs, both larger and smaller. Point R corresponds to point R' in panel (b) where the expansion path intersects the horizontal line drawn from B_O. This would be the cost-minimizing quantity of B to use if the firm desired to produce the output level associated with point R in panel (a), given the new lower price of A.

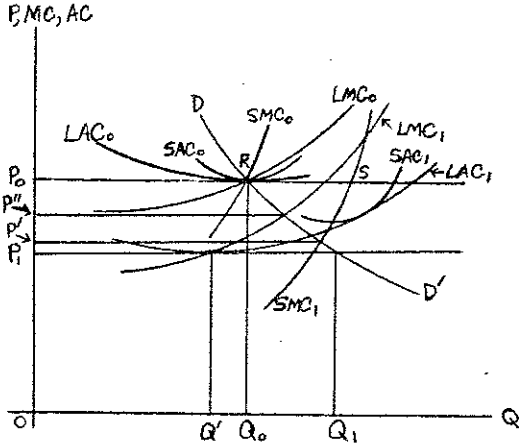

Figure 8

equilibrium requires more firms but in the short run each firm may suffer below-normal profits, meaning that the short-run signal is incorrect. If, alternatively, the price of A rises, a similar perverse short-run signal is possible. The rise in P_A shifts the aggregate of short-run marginal cost curves to the left and it is possible for each existing firm to earn above-normal profits at higher short-run industry price. At the same time, the rise in P_A must cause all firms to earn losses when they have fully adjusted, given that $0 < \eta_A^L < 1$. The minimum average cost output of each firm increases, and the new long-run equilibrium necessarily requires fewer firms. Again, the short-run profit signal is incorrect.

The situation where a fall in P_A causes short-run losses but long-run profits, holding constant the initial number of firms, is represented in Figure 8. This figure is identical to Figure 7, panel (a), except that it represents the whole industry rather than a single firm, and all the cost curves are horizontal summations of the cost curves of the firms in the industry in its initial equilibrium. The industry demand curve is DD'. Beginning from the initial equilibrium with price P_o and quantity Q_o, a fall in P_A causes the aggregate short-run marginal cost curve to shift from SMC_o to SMC_1. Product price falls to P', the price at which SMC_1 intersects the demand curve. Since SMC_1 is below SAC_1 at the output associated with this price, each firm sustains losses. These losses are an incorrect signal, however, since entry is necessary to restore long-run industry equilibrium.

The crucial question here is whether this perverse short-run signal will prevent the attainment of the new long-run equilibrium where the industry contains a larger number of firms, the price is P_1, and the output produced is Q_1. Answering this question rigorously would require a dynamic (as opposed to a comparative static) analysis that focuses on the time path of the product price and which makes assumptions about such hitherto neglected matters as possible differences in the information about costs possessed by existing as opposed to potential entrants, price expectations of the firms, the age and durability of the short-run fixed factor B (plant and equipment), and the time required to construct new plants. While such a dynamic analysis is beyond the scope of this paper, the tentative conclusion can be offered that the short-run loss will not serve to prevent the eventual attainment of the new long-run equilibrium. At most, the loss will only serve to delay the time at which this equilibrium will be reached.

To explain the basis for this conclusion, two alternative sets of circumstances are analyzed. The first is illustrated in Figure 8, in which the short-run product price, P', that prevails after the fall in P_A is greater than the minimum value of average cost on the new long-run average cost curve, LAC_1. (The analysis of this case also applies if P' equals the minimum of LAC_1.) In the second, the short-run price P' is assumed to be less than the minimum value of LAC_1.

Suppose first that P' is greater than the minimum of LAC_1. Though the fall in P_A has caused short-run losses for existing firms, each will recognize that, given sufficient time to adjust its durable plant and equipment, profits can be earned at the short-run price P' since this price exceeds LAC_1 in Figure 8 over some range of output. Hence, even if new firms do not immediately enter the industry because of the losses observed and because, more fundamentally, these potential entrants do not have the information on the new long-run cost conditions available to existing firms, it remains true that existing firms have an incentive to adjust their plant scales. As they do so, profits will eventually appear in the industry. After these profits appear, additional firms are induced to enter, and price will approach the new long-run level of P_1. Short-run losses serve here to delay the attainment of this equilibrium by retarding the time at which new firms enter the industry. This retardation occurs, however, only because potential entrants are assumed to be ignorant of the new cost conditions known to existing firms. If potential entrants also know these cost conditions, the short-run losses do not retard their entry. In essence, the effect of the short-run losses depends on the information available to potential entrants and the existing firms. If the information in Figure 8 is known to all, then the actual short-run losses are of no real consequence, since the real signal is provided by the *potential* profits which all firms can expect to make when using the correct scale of plant. Only if the cost information is not available to all firms is the process of adjustment somewhat delayed.

The second and only other possible circumstance that can exist following the fall in P_A is that the short-run price P' is below the minimum of LAC_1 (not illustrated in Figure 8). In this case, neither existing firms nor potential entrants (even if they know the new cost conditions) will find construction of new (smaller) scales of plant desirable, given the existing short-run price. Existing firms will continue to produce at $P' = SMC_1$ but will leave the industry as old plants wear out. As firms exit, product price will eventually rise above the minimum of LAC_1, profits will appear, new firms will enter the industry, and long-run equilibrium will be attained.[18] The short-run losses retard the attainment of long-run equilibrium in this second case, not because potential entrants are assumed to lack cost information, but because of the initial unprofitability of new plant construction by any firm, existing or potential. In neither the first nor the second case, however, do the short-run losses appear to prevent the ultimate attainment of the new long-run equilibrium. It must be emphasized again, however, that the importance of finding an incorrect profit signal in the short run can be completely assessed only in light of a dynamic adjustment model that includes a more thorough analysis of behavior out of equilibrium (in the spirit of Arrow, 1959) with the accompanying errors that uncertainty may cause.

VII. Application to Real World Events

Testing many of the foregoing results requires knowledge of expenditure elasticities. While discussion of the methods and difficulties of obtaining empirical estimates would take us too far afield, some suggestive applications can be mentioned. The papers in which the major comparative static results have appeared (Ferguson and Saving, and Maurice) have no tests or applications, but two other papers (Meyers and Nelson) both motivate their analysis of factor price changes by proposing what could be actual examples of the "paradox" that higher factor prices lead to higher firm profits. Nelson suggests that incomes of farm landowners could be enlarged by an increase in the price of seed, that incomes of coal mine owners might rise due to a rise in miner wages, and that the foreign trade balance of an economy processing an imported raw material for export might improve when the raw material rises in price. Similarly, Meyer suggests as an example the rise in farm profits after the 1965 curtailment of entry of foreign farm labor into the United States led to a rise in agricultural wages rates. If in each of these examples either the short- or long-run expendi-

[18] If DD' intersects SMC_1 at a price less than the minimum level of average variable cost associated with the old scale of plant, enough firms will immediately cost down to cause the short-run price to fall no lower than the minimum level of the average variable cost of the old plants. Again, as these remaining plants wear out, price will rise above the minimum of LAC_1, entry will occur and the new long-run equilibrium will be attained.

ture elasticity of the factor undergoing the price change exceeds unity, the theory presented in Sections V and VI clearly indicates that a rise in profits of existing firms is possible. Suggestive examples, of course, are not a substitute for more careful testing. Perhaps more thorough empirical investigations will be forthcoming in the future.

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