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The Road Not Taken - What Is The "Appropriate" Path to Development When Growth is Unbalanced?

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Abstract

This paper develops a model that endogenizes both directed technologies and demography. Potential innovators decide which technologies to develop after considering available factors of production, and individuals decide the quality and quantity of their children after considering available technologies. This interaction allows us to evaluate potentially divergent development paths. We find that exogenous unskilled-labor biased technological growth can induce higher fertility and lower education, inhibiting overall growth in per person income. However, if technical progress is *locally endogenous*, increases in the overall workforce caused by unskilled intensive technological progress can make R&D more profitable; this can actually induce more income growth can the alternative, skill-intensive path.

- *Keywords*: directed technical change, demography, human capital, fertility
- *JEL Codes*: O31, O33, J13, J24

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1 Introduction

The last half century has seen great divergence in living standards among the countries of the developing world; while rich nations have maintained fairly consistent rates of growth (2 or 3% per annum), poorer nations have traversed widely different growth paths (between -1 and 7%). This paper suggests a possible source of this divergence by producing a model emphasizing the interdependence between directed technical change and demography. In this model, potential innovators decide which technologies to develop after considering available factors of production, and individuals decide the quality and quantity of their children after considering available technologies. This interaction allows us to analyze the macroeconomic effects of "unbalanced growth," where a country develops *either* labor-intensive techniques and expands the pool of unskilled labor, or skill-intensive techniques and expands the pool of human capital. Which path will lead to greater prosperity is the primary focus of this paper.

The model emphasizes how economic growth can often be an unbalanced process, where choices are made between alternative modes of production. A farm can be maintained either with uneducated farmers wielding hand tools, or with farmers skilled in using agronomic instruments and automated machinery. A factory can be structured as an assembly line run mainly with unskilled workers supervised by a few skilled ones, or as a computer-controlled facility mainly run by skilled workers with a few unskilled janitors.[1] A road can be built using lots of manual labor physically laying down stone and brick by hand, or construction workers trained in using bulldozers and steamrollers. These examples highlight not only that technologies can be directed towards particular factors, but also that each country can take its own unique development path, producing similar things in very different ways.

This paper boils down all these considerations into a simple question - would greater aggregate wealth be generated with skilled-labor biased technological growth (the "skill-intensive path") or unskilled-labor biased technological growth (the "unskilled-intensive path")? The answer for each country, of course, is that it depends. It depends on how *productive* skilled and unskilled labor already are. It depends on how *abundant* skilled and unskilled labor are. And it depends on how technological changes can affect *future* supplies of skilled and unskilled labor.

By exploring the simultaneity between technological changes directed towards particular factors and the factors themselves, we can explore some of these issues. This approach constitutes a notable departure from the existing literature on technologies that augment specific factors or sectors.[2] These works often highlight the "inappropriateness" of growth in technologies that can be implemented by only a small portion of the economy. For example, Basu and Weil (1998) and Acemoglu and Zilibotti (2001) illustrate how technologies designed for capital-intensive (physical

[1] This example comes from Caselli and Coleman (2006)

[2] Katz and Murphy 1992; Acemoglu 1998; Kiley 1999; Acemoglu and Zilibotti 2001; Xu 2001; Acemoglu 2002, to name but a few papers.

or human) societies that diffuse to developing regions are used ineffectually there, if at all. And Mokyr (1999) explains that the British Industrial Revolution initially produced only minor improvements in living standards because technical progress occurred in just a few small industries. These papers suggest that technologies catered for the abundant factors of production are more appropriate for the economy and will provide robust future growth. Thus poorer, labor abundant countries should develop labor-intensive technologies to make their large uneducated workforce more productive.

But these works typically do not take into account that these factors can evolve, and will adjust to changing economic circumstances.[3] If factors do change in these models, they typically do so exogenously. But this partial equilibrium approach may mislead us, particulary when it comes to *long-run* growth. Allowing for the co-evolution of factors and technologies can alter our perspective of the "appropriate" technological path - that is, the path that generates more macroeconomic growth. Two new considerations emerge with this approach. The first is that if unbalanced growth also promotes growth in the more productive factor, overall growth will be higher than the alternative path. The other consideration is that different technological paths can produce different rates of population growth. All else equal, the path with the lower population growth rate will produce more income per person.[4]

With simulations of the model, we discover a number of things. First, by raising the returns to education, exogenous skill-biased technological growth can induce higher education and (through quality-quantity tradeoffs in child-rearing) lower fertility; this provides an additional boost to per person income. This case highlights that, contrary to the lessons of the appropriate technology literature, the overall size of the factor or sector may not be the most important consideration; even though unskilled labor is plentiful, making them more productive can produce dynamically harmful effects like fertility increases and education decreases.

However, if technical progress is *locally endogenous*, the increase in the overall workforce caused by unskilled intensive technological progress can make R&D more profitable by raising the scale of the market for innovators; this can actually induce more income growth can the alternative, skill-intensive path. Thus we see that the *source* of technological growth may be important in answering our titular question - skill-biased technologies can indeed be appropriate for development if they exogenously flow from other economies like manna, even if skilled labor is in relative short supply. On the other hand, unskill-biased technologies can be appropriate for development if they are home-grown, even if they produce some ostensibly negative side effects like population growth and a de-skilled workforce.

[3] Papers that do consider interactions between technology and human capital include Stokey 1988, Chari and Hopenhayn 1991, Grossman and Helpmann 1991, Young 1993, Redding 1996, Galor and Weil 2000, and Galor and Moav 2000. None however assess the appropriate path to development for an economy in the context of such simultaneity.

[4] Galor and Mountford (2006) stress this point in explaining divergent growth paths in history.

This paper heavily borrows from Acemoglu's important work on directed technological change (Acemoglu 1998, 2002). But this work departs from that literature in two fundamental ways. First, the literature relies almost exclusively on analyzing *balanced* growth paths, while here we look solely on the *unbalanced* case (where technological growth occurs only in one sector of the economy), implicitly assuming that countries often face a choice in its overall growth direction. Second, as already mentioned, the literature also almost always treats factors of production as exogenously determined,[5] whereas here they are endogenous in the model.

The rest of the paper is organized as follows. Section 2 motivates the paper by looking at some cross-country data. Section 3 presents the model in steps, first presenting a model of semi-endogenous biased technological growth, and then merging this with a simple theory of demography. This model then motivates our simulation experiments in section 4. Section 5 provides some concluding remarks.

2 Some Data

2.1 A Cross-Section of Factor-Specific Technologies

We begin by taking account of estimated factor-specific productivities of a cross-section of countries. Consider the following production function for country i:

$$Y = [(A_{l,i}L_i)^{\sigma} + (A_{h,i}H_i)^{\sigma}]^{1/\sigma} \tag{1}$$

Here we specify production as one with a constant elasticity of substitution between skilled and unskilled labor aggregates (this elasticity being $1/(1-\sigma)$. $A_{l,i}$ is the efficiency of unskilled labor in country i and $A_{H,i}$ is the efficiency of skilled labor in country i.[6]

If factors of production are paid their marginal products, the "skill-premium" can be written as:

$$\frac{w_{h,i}}{w_{l,i}} = \left(\frac{A_{H,i}}{A_{L,i}}\right)^{\sigma}\left(\frac{H_i}{L_i}\right)^{\sigma-1} \tag{2}$$

Caselli and Coleman (2006) note that one can study cross-country productivity differences using equations (1) and (2), for these represent two equations with two unknowns. That is,

[5] Acemoglu 1998 relegates the possibility of endogenously determined human capital in the appendix to his paper, while he does not discuss the possibility either in Acemoglu 2002 or in the chapter on directed technical change in his recent growth textbook (Acemoglu 2008).

[6] This functional form resembles the production function used in section 3, where we endogenize technological growth; efficiency coefficients will proxy for the breadth and depth of factor-complementary machines.

given data on Y_i, L_i, H_i, and $\frac{w_i^h}{w_i^l}$, we can back out each country's implied pair of technological coefficients and compare them.[7]

Key to this exercise is our parameter choice for $\sigma \leq 1$. Careful empirical labor studies such as Autor et al (1998) and Ciccone and Peri (2005) have found that the elasticity of factoral substitution between more and less skilled workers most likely lies between 1 and 2.5 (consistent with a value of σ between 0 and 0.6). Both for this exercise and the simulations in section 4, we choose a benchmark value of $\sigma = 0.5$ for a proxy elasticity parameter most applicable for a wide range of countries and for a wide variety of skilled and unskilled labor categories.[8]

Figure 1 depicts the relationships between relative technical skill-bias (A_h/A_l), relative skill-endowments (H/L), and income per capita across a broad array of countries. Immediately clear is the positive associations between technical skill-bias and skill endowment, and between technical skill-bias and income levels. These positive relationships hold whether we consider a skilled worker as someone with primary schooling, or someone with secondary schooling, or even someone with a college education. This was precisely one of the main points behind Caselli and Coleman's study. Not only do wealthy nations enjoy large pools of human capital, but they also employ this capital far more effectively than poorer nations.

But from these static pictures it is not clear which technological path would produce more output for any particular country *over time*. On the one hand, a country with a relative abundance of unskilled labor should greatly benefit by making them more productive. On the other hand, unskilled labor's *level* of productivity may already be fairly low; unskilled-bias technical change that induces a rise in L and a fall in H would then lower the relatively-more productive factor and raise the relatively-less productive factor.

We begin exploring these issues by allowing the factors of production to *respond* to biased technological changes, first in a comparative static experiment in section 2.2, and then in a fully specified general equilibrium model in section 3.

2.2 Unbalanced Growth - A Comparative Static Experiment

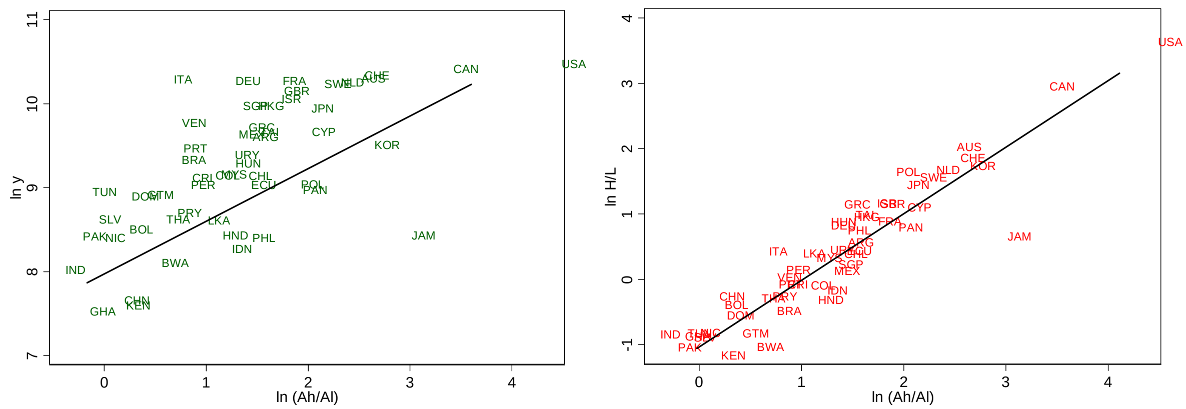
Here we consider changes in output, Y, that can occur when we have the factors of production respond to exogenous unbalanced technological growth. First, let us totally differentiate the production function given by (1):

[7]The data is also from Caselli and Coleman (2006). Y is average GDP per capita for 1985-1990, taken from the Penn World Tables. Labor levels are constructed using the implied Mincerian coefficients from Bils and Klenow (2000). Wages for skilled and unskilled are constructed using Mincerian coefficients and the duration in years of the various schooling levels. See their paper for more details.

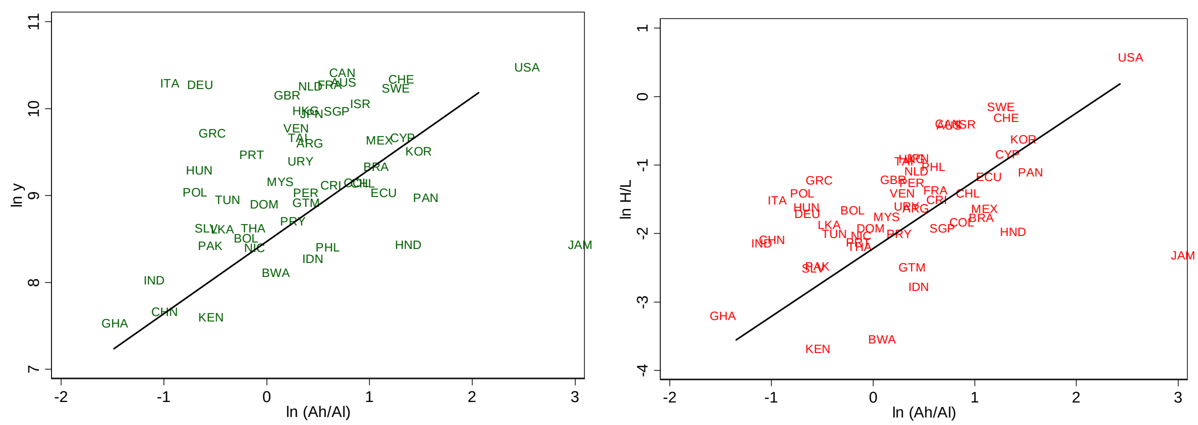
[8]Ciconne and Peri (2005) themselves estimate σ to be 0.5 when considering U.S. high school dropouts as unskilled labor and high school graduates as skilled labor (although their preferred measure is 0.33).

Figure 1: Relative Technologies versus Relative Factors and Output ($\sigma = 0.5$)

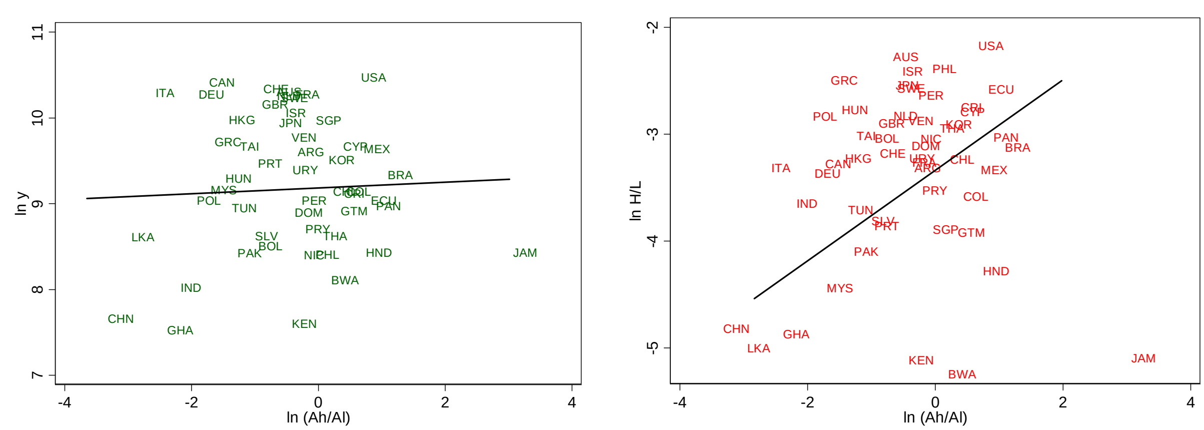
H = primary school or more completed



H = secondary school or more completed



H = college or more completed



$$dY = \left(\frac{\partial Y}{\partial A_l}\right) dA_l + \left(\frac{\partial Y}{\partial A_h}\right) dA_h + \left(\frac{\partial Y}{\partial L}\right) dL + \left(\frac{\partial Y}{\partial H}\right) dH \tag{3}$$

Both types of technologies and both types of factors have the potential to change. Let us assume that when technologies are *biased* towards factor L, it induces L to rise and H to fall (higher unskilled-intensive productivity makes some people become unskilled laborers instead of skilled ones). On the other hand, technological growth that is biased towards H induces L to fall and H to rise (higher skilled-intensive productivity makes some erstwhile unskilled laborers become skilled ones). That is, $dA_l > 0 \Rightarrow -dH = dL > 0$. And $dA_h > 0 \Rightarrow -dL = dH > 0$.[9] Let us consider two possibilities. The first is where $dA_l = 1$ and $dA_h = 0$. This is the case of unskilled-bias technological change (where the total change in output can be written as dy_{unsk}). The second case is where $dA_h = 1$ and $dA_l = 0$. This is the case of skilled-bias technological change (where the total change in output can be written as dy_{sk}).

When there is unskilled-biased technological change, the total change in income per capita can be written as

$$dY_{unsk} =$$

$$[(A_l L)^\sigma + (A_h H)^\sigma]^{\frac{1-\sigma}{\sigma}} \cdot \left((A_l L)^{\sigma-1} (L \cdot dA_l + A_l \cdot dL) + (A_h H)^{\sigma-1} (A_h \cdot (-dH))\right) \tag{4}$$

where dY_{unsk} is the total change in income per capita with unskilled intensive growth. Note that here $dA_h = 0$ and the change to H is negative. On the other hand, when there is skilled-biased technological change, the total change in income per capita can be written as

$$dY_{sk} =$$

$$[(A_l L)^\sigma + (A_h H)^\sigma]^{\frac{1-\sigma}{\sigma}} \cdot \left((A_l L)^{\sigma-1} (A_l \cdot (-dL)) + (A_h H)^{\sigma-1} (A_h \cdot dA_h + (A_h dH))\right) \tag{5}$$

where dy_{sk} is the total change in income per capita with skilled intensive growth. Note that here $dA_l = 0$ and the change to L is negative.

Does skilled labor-biased technological growth produce more output than unskilled labor-biased technological growth? If labor is strictly fixed, the answer is no. With Caselli and Coleman (2006)'s categorization and calculations of L and H, $L > H$ even for wealthy nations. Since factors are grossly substitutable, technologies used by the more abundant factor will generate the greater aggregate gain.

[9] Note that only when $\sigma > 0$ can we consider A_l unskilled *biased* and A_h skilled *biased*. This is a reasonable assumption given previous estimates of σ. See Acemoglu 2002 for a fuller discussion.

However, the more responsive are factors to biased technological changes, the greater are the relative output gains from skill-biased technological change. This follows simply from the fact that H is inherently the more productive factor. This comes both from its relative scarcity (so its marginal productivity tends to be higher even if technologies are symmetrical) and from the higher productivity coefficient on H compared to the one for L. So if labor tends to readily switch from one type to the other with unbalanced technical progress, skill-intensive growth tends to produce more output.

Combining both observations, we see that each country has a *threshold level* of factoral responsiveness, whereby $dY_{sk} = dY_{unsk}$. Figure 2 plots $dY_{sk} - dY_{unsk}$ against the degree of factoral response for two illustrative countries, Argentina and Great Britain. If we consider H to be those with at least some secondary schooling, we can see that a one-unit change in A_h would require a 0.64 unit shift from L to H to produce more output than a similar change in A_l in Argentina, while it would require only a 0.35 unit shift from L to H in Britain.

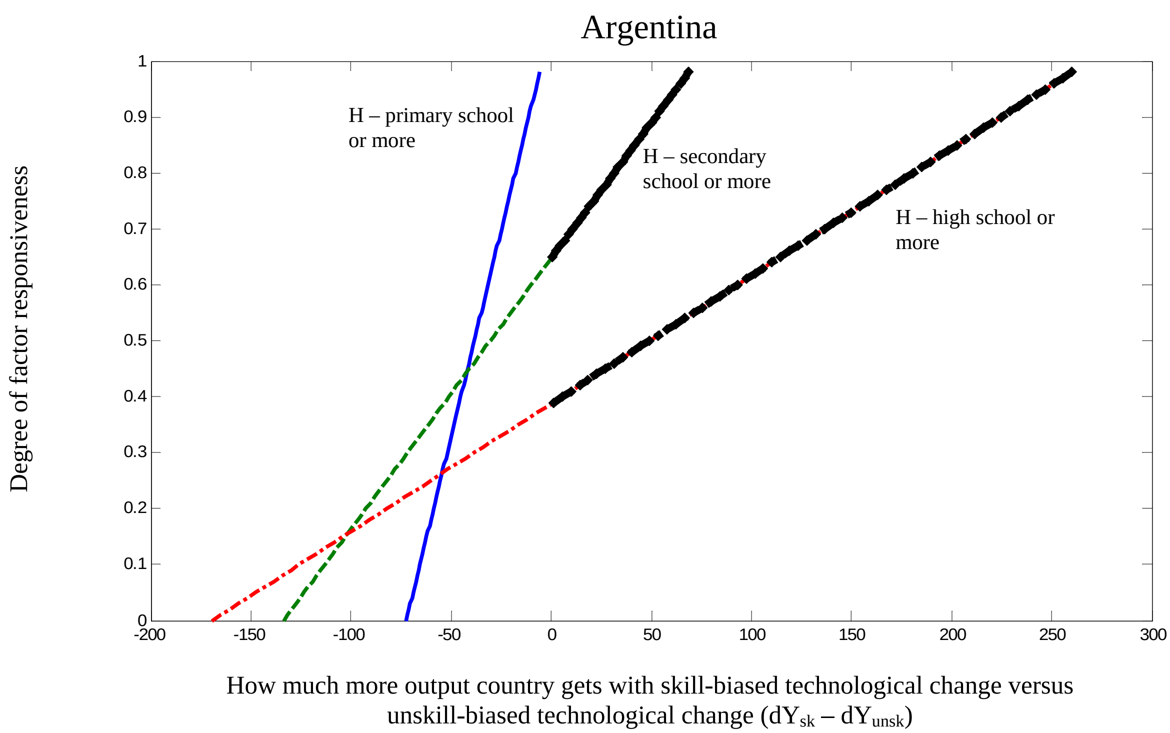
Thus we see that because countries have their own unique pairs of factor supplies and productivities, they will have different factoral response threshold levels. Figure 3 plots each country's threshold level of factor responsiveness (where $dY_{sk} = dY_{unsk}$) against its GDP per capita. We can see that the poorer the nation is on average, the greater will factors need to respond to technological changes for skill-intensive growth to be the superior path to development. Because poorer nations tend to have greater relative quantities of unskilled labor, and also tend to have relatively less productive skilled labor, factors need to respond with greater magnitude in order for skill-intensive technical growth to produce relatively more output.

However, as we compare the top and bottom scatterplots we can see that the more narrow is our definition of H, the *smaller* is the threshold factor responsiveness. This is simply because increases from the relatively more scarce factor produces greater benefits, for the marginal productivities of the more scarce factor tends to be larger. This in effect flips Acemoglu's discussion of so-called "market-size effects" on its head: if factors are allowed to respond to technological change, such change that augments the *less* abundant factor may produce *more* output in the longer run.

Thus we need to better understand how elastically these factors of production respond to directed technical change. On top of this, if we believe there exists a quality-quantity tradeoff in child-rearing (Becker and Lewis 1973; Becker and Barro 1988),[10] educational changes will generate fertility changes, forming another channel that affects *per capita* income. And finally, factor changes may themselves lead to subsequent changes in biased technologies. So we should move beyond this comparative static analysis to a model that endogenizes both factors and technologies in a general equilibrium framework. That is, by actually endogenizing the micro-

[10] This is also stressed by Galor and Mountford (2006, 2008). In these papers population changes come from trade specialization patters. Technological changes however can affect demographic patterns in very similar ways.

Figure 2: Skilled versus Unskilled Technological Growth When Factors Respond - Comparing Two Countries



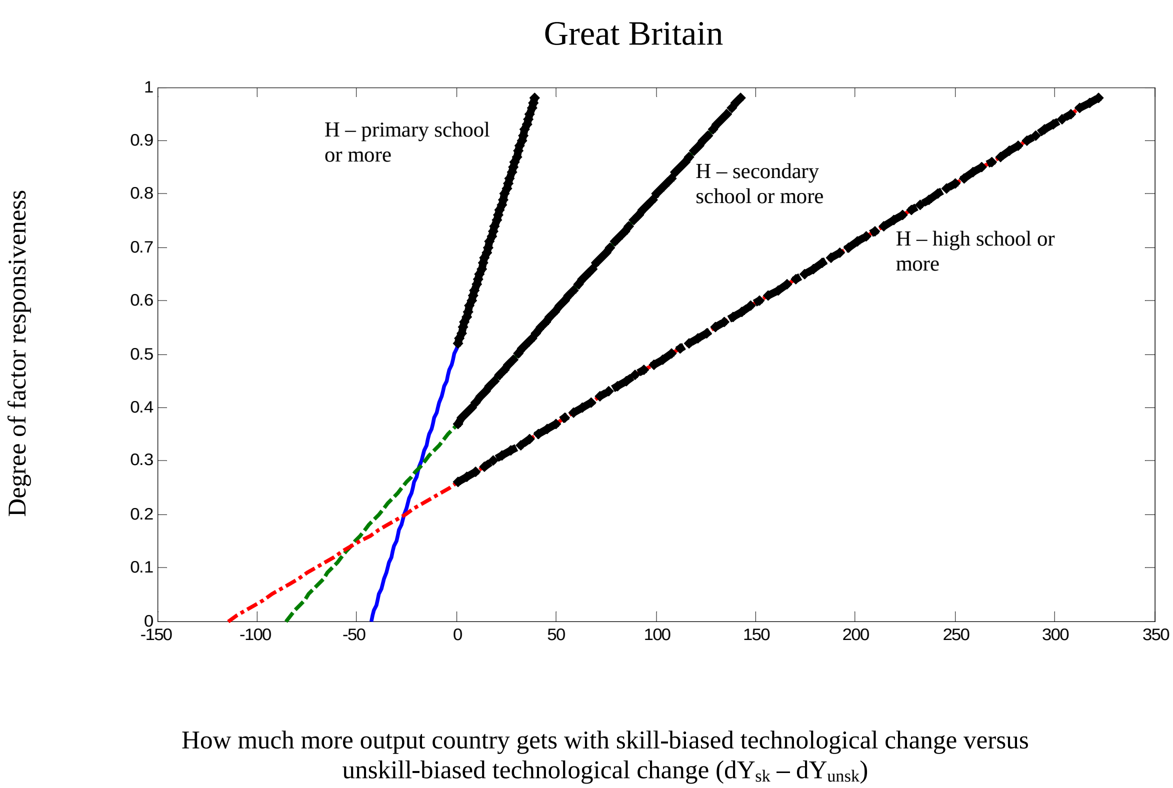
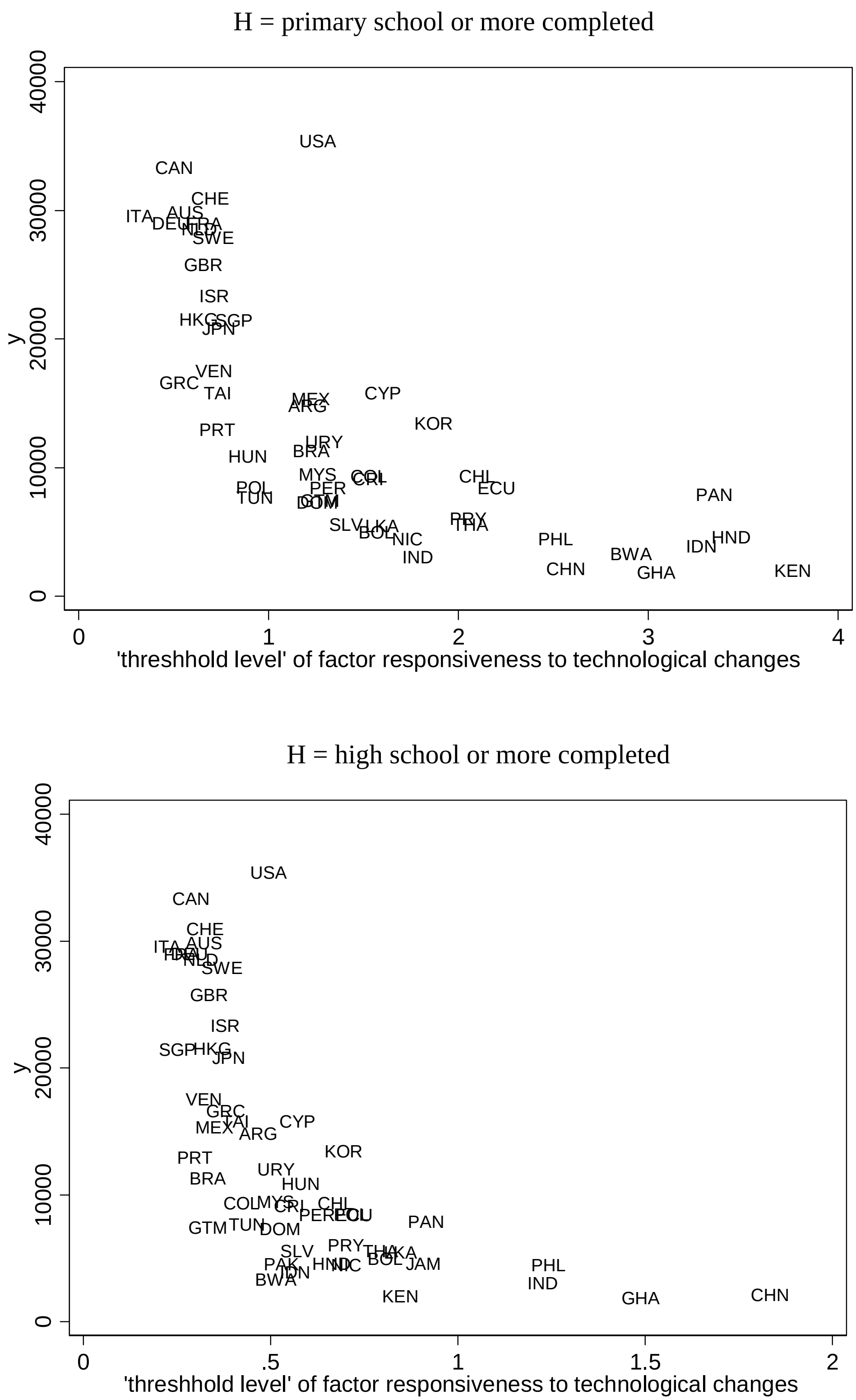


Figure 3: How Much Must Factors Respond To Make Skilled-Biased Technological Growth "Better?" - Cross Country Comparison



economic incentives for researchers and families, we can simulate values for dA_l, dA_h, dL and dH for a hypothetical economy over time.

3 The Model

3.1 Production

Consider a discrete-time economy. We use the production function given by (1) but now we explicitly specify factor-specific technologies. Specifically production is specified as the following.

$$Y = [(A_l L)^\sigma + (A_h H)^\sigma]^{1/\sigma} \tag{6}$$

$$A_l \equiv \int_0^{M_l} \left(\frac{x_l(j)}{L}\right)^\alpha dj \qquad A_h \equiv \int_0^{M_h} \left(\frac{x_h(k)}{H}\right)^\alpha dk \tag{7}$$

Here both types of labor (unskilled L, and skilled H) work with intermediate "machines" to produce a homogenous final output. A machine is designed for use either by skilled labor or unskilled labor, but not both. Machines (of type j) which complement unskilled labor are denoted by $x_l(j)$, while machines (of type k) which complement skilled labor are denoted by $x_h(k)$.

The parameter σ indicates the degree of substitutability between the skilled and unskilled-intensive "sectors" in aggregate production. When $\sigma = 1$, the production function is linear; when it is 0, the production function is Cobb-Douglas; when it is $-\infty$, the production function is Leontieff. As mentioned in section 2.1, estimates of this elasticity clearly place σ above zero; thus we will assume that these sectors are grossly substitutable.

Echoing the assumptions of Kiley(1999) and Acemoglu (2002), technological advance is assumed to come in two varieties. In the "unskilled labor sector," technical advance comes about from an expansion in the number of intermediate machines specialized for unskilled labor (that is, an increase in M_l). Similarly, in the "skilled labor sector," technical advance means an expansion in the number of intermediate machines specialized for skilled labor (an increase in M_h).

Final goods output produced by different firms is identical, and can be used for consumption, for the production of different intermediate machines, and for research and development to expand the varieties of skill-augmenting and unskilled-augmenting machines. For each time period (suppressing time subscripts) these firms endeavor to maximize:

$$\max_{\{L,H,x_l(j),x_h(k)\}} Y - w_l L - w_h H - \int_0^{M_l} p(j)x_l(j)dj - \int_0^{M_h} p(k)x_h(k)dk \tag{8}$$

where $p(j)$ is the price of machine $x_l(j)$ and $p(k)$ is the price of machine $x_h(k)$.

Endogenous growth theory suggests that research is generally profit motivated. However, modeling purposive research and development effort becomes difficult when prices and factors change over time, as they certainly do when growth is unbalanced. Endogenous growth theory typically assumes that the gains from innovation flow to the innovator throughout her lifetime, and this flow will depend on the price of the product being produced and the factors required for production at each moment in time.[11] If prices and factors are constantly changing (as they may in an economy where factors evolve endogenously), a calculation of the expected discounted profits from an invention may be impossibly complicated.

To avoid this complication but still gain from the insights of endogenous growth theory, we assume that the gains from innovation last one time period only. More specifically, we assume that intermediate machines are produced either in monopolistic or competitive environments. An inventor of a new machine at time t enjoys monopoly profits for machine production only at t. After this patent rights expire, and subsequent production of this brand of machine is performed by many competitive manufacturers. Whether a machine is produced monopolistically or competitively will be conveyed in its rental price, denoted either as $p(j)$ for a unskilled-labor using machine j or $p(k)$ for a skilled-labor using machine k, and explained in the next sub-section. Also for simplicity, we assume that all machines depreciate completely after use, and that the marginal cost of production is simply unity in terms of the final good.

Given technology levels M_l and M_h and labor types L and H, an equilibrium can be characterized as machine demands for $x_l(j)$'s and $x_h(k)$'s that maximize final-good producers' profits (from equation 8), machine prices $p(j)$ and $p(k)$ that maximize machine producers' profits, and factor prices w_l and w_h that clear the labor market.

The first-order conditions for final-good producers yield intermediate-machine demands:

$$x_l(j) = [(A_l L)^\sigma + (A_h H)^\sigma]^{\frac{1-\sigma}{(1-\alpha)\sigma}} A_l^{\frac{1-\sigma}{\alpha-1}} \left(\frac{p(j)}{\alpha}\right)^{\frac{1}{\alpha-1}} L^{\frac{\sigma-\alpha}{1-\alpha}}$$

$$x_h(k) = [(A_l L)^\sigma + (A_h H)^\sigma]^{\frac{1-\sigma}{(1-\alpha)\sigma}} A_h^{\frac{1-\sigma}{\alpha-1}} \left(\frac{p(k)}{\alpha}\right)^{\frac{1}{\alpha-1}} H^{\frac{\sigma-\alpha}{1-\alpha}} \tag{9}$$

Note that greater levels of employment of a factor raise the demand for intermediate goods augmenting that factor so long as $\sigma > \alpha$, an idea consistent with Acemoglu's so-called "market-size" effect. We will assume throughout the analysis that this condition is met.

The other first-order conditions for final-good producers illustrate that workers receive their marginal products:

[11] For example, the seminal Romer (1990) model describes the discounted present value of a new invention as a positive function of $L - L_R$, where L is the total workforce and L_R are the number of researchers. Calculating this value function is fairly straight-forward if labor supplies of production workers and researchers are constant. If they are not, however, calculating the true benefits to the inventor may be difficult.

$$w_l = \left[(A_l L)^\sigma + (A_h H)^\sigma\right]^{\frac{1-\sigma}{\sigma}} A_l^\sigma L^{\sigma-1} \tag{10}$$

$$w_h = \left[(A_l L)^\sigma + (A_h H)^\sigma\right]^{\frac{1-\sigma}{\sigma}} A_h^\sigma H^{\sigma-1} \tag{11}$$

3.2 Research

In this section we describe the growth paths of M_l and M_h. Researchers expend resources to develop new types of machines, and these resource costs can change over time. We make this modeling choice to stress that unbalanced growth can occur when research costs differ between different sectors. We will assume that these costs will depend both on the number of machine types already extant (indexed by M_l and M_h), and on some factor-specific policy variable (denoted by z_l and z_h, and discussed below). Specifically, the up-front cost of developing the blueprint of a new machine, c, is given simply by

$$c\left(\frac{M_l}{z_l}\right) = \frac{M_l}{z_l}$$

for an unskilled labor augmenting machine, and

$$c\left(\frac{M_h}{z_h}\right) = \frac{M_h}{z_h}$$

for a skilled labor augmenting machine. These functional forms illustrate that the costs of invention are negligible when there is little machine variety. As factor-specific technologies grow, however, costs can become increasingly prohibitive.[12]

Given these costs of technological advance, innovating firms must receive some profits from the development of a new technology in order to make research worth the expense. As mentioned above, we assume that developers of new machines receive monopoly rights to the production and sale of their machines for only one period. As a result, we must make a distinction between *old* machines (those invented before t) and *new* machines (those invented at t).

Assuming unitary marginal costs of machine production, the revenue generated from new machines of both types are given by the 'value' functions:

$$V_l = (p(j) - 1)\, x_l(j)$$

$$V_h = (p(k) - 1)\, x_h(k)$$

[12]This approach of varying the cost of research echoes the leader-follower model illustrated in Barro and Xala-i-Martin 2003), where costs depend on the distance from the frontier of general knowledge.

Because demand is isoelastic, the price which maximizes monopolists' profits equals $1/\alpha$ for both skill- and unskilled-augmenting machines, so that demand for *new* intermediate machines (those invented at t) are:

$$x_{l,new}(j) = x_{l,new} = \alpha^{\frac{2}{1-\alpha}} \left[(A_l L)^{\sigma} + (A_h H)^{\sigma} \right]^{\frac{1-\sigma}{(1-\alpha)\sigma}} A_l^{\frac{1-\sigma}{\alpha-1}} L^{\frac{\sigma-\alpha}{1-\alpha}}$$

$$x_{h,new}(j) = x_{h,new} = \alpha^{\frac{2}{1-\alpha}} \left[(A_l L)^{\sigma} + (A_h H)^{\sigma} \right]^{\frac{1-\sigma}{(1-\alpha)\sigma}} A_h^{\frac{1-\sigma}{\alpha-1}} H^{\frac{\sigma-\alpha}{1-\alpha}} \tag{12}$$

On the other hand, because older machines are competitively produced, their prices equal unitary marginal costs, so that demand for *old* intermediate machines (those invented before t) are simply:

$$x_{l,old}(j) = x_{l,old} = \alpha^{\frac{1}{1-\alpha}} \left[(A_l L)^{\sigma} + (A_h H)^{\sigma} \right]^{\frac{1-\sigma}{(1-\alpha)\sigma}} A_l^{\frac{1-\sigma}{\alpha-1}} L^{\frac{\sigma-\alpha}{1-\alpha}}$$

$$x_{h,old}(j) = x_{h,old} = \alpha^{\frac{1}{1-\alpha}} \left[(A_l L)^{\sigma} + (A_h H)^{\sigma} \right]^{\frac{1-\sigma}{(1-\alpha)\sigma}} A_h^{\frac{1-\sigma}{\alpha-1}} H^{\frac{\sigma-\alpha}{1-\alpha}} \tag{13}$$

Thus factor-specific TFPs given by equation (6) can be re-written as an aggregation of two kinds of machines, illustrating the cumulation of all past and current innovation. If $M_{z,old}$, $M_{z,new}$, and M_z are, respectively, the number of existing old, new and total machine-types used by factor z, we can write factor productivity as:

$$A_l \equiv \int_0^{M_l} \left(\frac{x_l(j)}{L} \right)^{\alpha} dj = \left[\int_0^{M_{l,old}} x_{l,old}\,(j)^{\alpha}\, dj + \int_{M_{l,old}}^{M_l} x_{l,new}\,(j)^{\alpha}\, dj \right] (1/L)^{\alpha} =$$
$$\frac{M_{l,old}\; x_{l,old}^{\alpha} + M_{l,new}\; x_{l,new}^{\alpha}}{L^{\alpha}} \tag{14}$$

$$A_h \equiv \int_0^{M_h} \left(\frac{x_h(k)}{H} \right)^{\alpha} dk = \left[\int_0^{M_{h,old}} x_{h,old}\,(k)^{\alpha}\, dk + \int_{M_{h,old}}^{M_h} x_{h,new}\,(k)^{\alpha}\, dk \right] (1/H)^{\alpha} =$$
$$\frac{M_{h,old}\; x_{h,old}^{\alpha} + M_{h,new}\; x_{h,new}^{\alpha}}{H^{\alpha}} \tag{15}$$

Substituting the monopoly price into our value functions yield:

$$V_l = \left[\frac{1-\alpha}{\alpha} \right] x_{l,new}$$

$$V_h = \left[\frac{1-\alpha}{\alpha} \right] x_{h,new}$$

where $x_{l,new}$ and $x_{h,new}$ are given by (12). Finally, an individual is free to research, guaranteeing that:

$$V_l(L, H, A_l, A_h) \leq c\left(\frac{M_{l,old} + M_{l,new}}{z_l}\right) \tag{16}$$

$$V_h(L, H, A_l, A_h) \leq c\left(\frac{M_{h,old} + M_{h,new}}{z_h}\right) \tag{17}$$

If resource costs of research were actually *less* than discounted profits, entry into research would occur, driving technology levels, and hence costs, up. We assume this happens quick enough so that valuations never exceed costs in any time period. Further, since applied research is irreversible (a society cannot forget how to make something once it is learned), the variety of machines remains unchanged when the inequalities in (16) or (17) do not bind with equality.

The levels of our policy variables z_l and z_h in the economy are key determinants of the costs of developing new "production processes;" higher levels of z_u lower the costs of developing intermediate machines which complement factor u. Conceivably these variables are functions of many possible factors, such as government policies, or technological diffusion from other countries. For now let us assume that these variables simply grow at an exogenously steady rate:

$$g = \frac{z_{l,t+1} - z_{l,t}}{z_{l,t}} = \frac{z_{h,t+1} - z_{h,t}}{z_{h,t}}$$

where $g > 0$ is the growth factor. Thus we see that (16) and (17) can also illustrate barriers to technology adoption - if z_l and/or z_h are too small, factor-specific technological growth cannot happen.

The steady-state can be characterized as one where the share of labor devoted to each sector (skilled and unskilled) remains fixed, while output, the policy variables z_l and z_h, the varieties of skilled and unskilled machines, and wages all grow at the same rate, g. This will occur so long as equations (16) and (17) hold with strict equality. But as these inequalities imply there may be a considerable period of time when growth is *unbalanced*; this would occur if only one of the equations held with equality. What kind of unbalanced growth is likely to unfold will depend on a number of things, including the available supply of different factors (a relatively large L for example raises V_l and thus increases the chance that growth will be unskill-biased) and the relative "skewness" of the policy variables (a relatively large z_l for example lowers c_l and likewise increases the chance for unskill-biased growth).

No doubt unbalanced growth will be slower than balanced steady-state growth, but it seems logical that growth in the *bigger* sector will produce faster growth than growth in the smaller sector.[13] This indeed is the essence of the appropriate technology story - typically it involves a

[13] If $\frac{\Delta a}{a} = g$ and $\frac{\Delta b}{b} = 0$, $\frac{\Delta(a+b)}{a+b} = \frac{\Delta a}{a+b}$, which is smaller than, but converges to, g. The smaller is b relative to a, the closer will this growth be to g.

story of factor abundance. By its logic, a country awash with throngs of unskilled labor would do well to develop and adopt technologies readily employable by them. The tragedy stressed in this tale often involves the nature of the "technology frontier" - because cutting-edge technologies produced by wealthy nations tend to be skill-intensive, developing nations often inherit a lot of skill-intensive technologies (Acemoglu and Zilibotti 2001). In our simple model this may be reflected by a large (z_h/z_l); the consequence of this is that poor countries end up developing technologies for which they are structurally ill-suited, resulting in anemic macro growth.[14]

At the same time, there is recognition among development economists of the importance of skill accumulation in economic growth. The centrality of human capital in economic development is so established that most economists now treat education and modernity as going hand in hand.[15] From this perspective, a country's relative abundance in unskilled labor scarcely matters; the skill-intensive path is the *only* viable path to sustainable progress.

This paper suggests that forces that change the factors of production themselves are an important part of our answer to the question of which is the more appropriate growth path. Specifically, changes in the relative rewards to factors due to technological developments surely will alter the incentives to become educated or to remain an unskilled laborer. From the model we can write the "skill premium," the skilled wage relative to the unskilled wage, as

$$\frac{w_h}{w_l} = \left(\frac{\alpha^{\frac{\alpha}{1-\alpha}} M_{h,old} + \alpha^{\frac{2\alpha}{1-\alpha}} M_{h,new}}{\alpha^{\frac{\alpha}{1-\alpha}} M_{l,old} + \alpha^{\frac{2\alpha}{1-\alpha}} M_{l,new}} \right)^{\frac{\sigma-\sigma\alpha}{1-\sigma\alpha}} \cdot \left(\frac{H}{L} \right)^{\frac{\sigma-1}{1-\sigma\alpha}} \tag{18}$$

In the absence of any demographic response, skill-bias technological growth will raise the skill premium (by raising $M_{h,new}$), while unskill-bias technological growth will lower it (by raising $M_{l,new}$). But surely if unskill-intensive growth lowers the relative returns to skill, this will induce some people to remain unskilled. Conversely, increases in the returns to skills should induce individuals to increase human capital, and thus lower fertility rates through the quality-quantity tradeoff. Indeed, from the last section we suggest that the more responsive these factors are to changes in their relative returns, the more likely will skill-biased technological growth yield greater income per capita growth. These considerations compel us to merge this growth model with a simple theory of demography. The next sections do precisely that.

[14]The development literature is filled with anecdotal evidence of this technology-skill mismatch, highlighted in Todaro and Smith's seminal text. "Gleaming new factories with the most modern and sophisticated machinery and equipment are a common feature of urban industries while idle workers congregate outside the factory gates." (pp. 256 in Todaro and Smith 2006).

[15]By one recent paper's account, "Anything that harms the accumulation of human capital harms our economic well-being" (Remler and Pema 2009). For a brief history of the study of human capital see Ehrlich and Murphy 2007.

3.3 Endogenous Demography

To capture the symbiotic relationship between technologies and factors, we introduce households into the model in an over-lapping generations framework, where individuals have two stages of life: young and old. Only old people are allowed to make any decisions regarding demography. Specifically, the representative household is run by an adult who maximizes her utility by deciding two things: how many children to have (denoted by n) and the fraction of these children who will receive an education (denoted by e).

An individual born at time t works either as an unskilled laborer (earning the unskilled wage w_l), or as a skilled laborer (earning the skilled wage w_h). The individual becomes old at $t+1$. At this point she decides how many children to have herself, and the fraction of these children that will get an education and work as skilled workers.

Specifically, individuals wish to maximize both their own income and the income of their young.[16] Let utility for the household planner be described by the function

$$U = w_j\,(1 - c(n,e)) + ln\,[w_l(1-e)n + w_h en]$$

where $c(\cdot)$ is the function denoting child-rearing costs, and w_j is the wage of the parent (who could be either a skilled worker or an unskilled worker, depending on what *her* parent chose for her last time period, so $j = l, h$). Fraction $(1-e)$ of young work as unskilled workers, while fraction e of young work as skilled workers. This quasi-linear utility form[17] simply conveys that adults face diminishing returns to enjoyment in their children's income, but not in their own.

The first-order condition for the number of children is:

$$\frac{1}{n} = w_j c_n \tag{19}$$

where c_n is the derivative of the cost function with respect to fertility. The left-hand side illustrates the marginal benefit of an additional child (which falls with the total number of children), while the right-hand side denotes the marginal cost (the income foregone to raise an additional child).

The first order condition for education is:

$$\frac{w_h}{w_l(1-e) + w_h e} = \frac{w_l}{w_l(1-e) + w_h e} + w_j c_e \tag{20}$$

where c_e is the derivative of the cost function with respect to fertility. Again, the left-hand side is the marginal benefit and the right-hand side the marginal cost. At the optimum, the gains

[16] This echoes Moav (2005), who models parents that decide both the number of children and the level of human capital of each child in order to simply maximize their potential income.

[17] Such quasi-linear utility functions to model demography have been used by, among other works, Kremer and Chen (2002) and Weisdorf (2007).

received from the added skilled income offsets the foregone unskilled- and adult-income requisite for giving more children an education.

Note that the results of this simple optimization problem is consistent with the negative correlations between income and fertility and between education and fertility that are observed in developing countries (Kremer and Chen 2002). For example, rising skilled wages induces households to increase education; the rise in child-rearing costs this produces however will also incentivise households to lower fertility.

Completing the model requires us to relate fertility and education rates to aggregate levels of unskilled labor and skilled labor. At time t, labor-types are given by:

$$L = N_t(1 - e_{t-1}) + N_t n_t(1 - e_t) \tag{21}$$

$$H = N_t e_{t-1} + N_t n_t e_t \tag{22}$$

where N is simply the adult population. Note that each type of labor is comprised of both young and old workers. Finally, population growth is given by

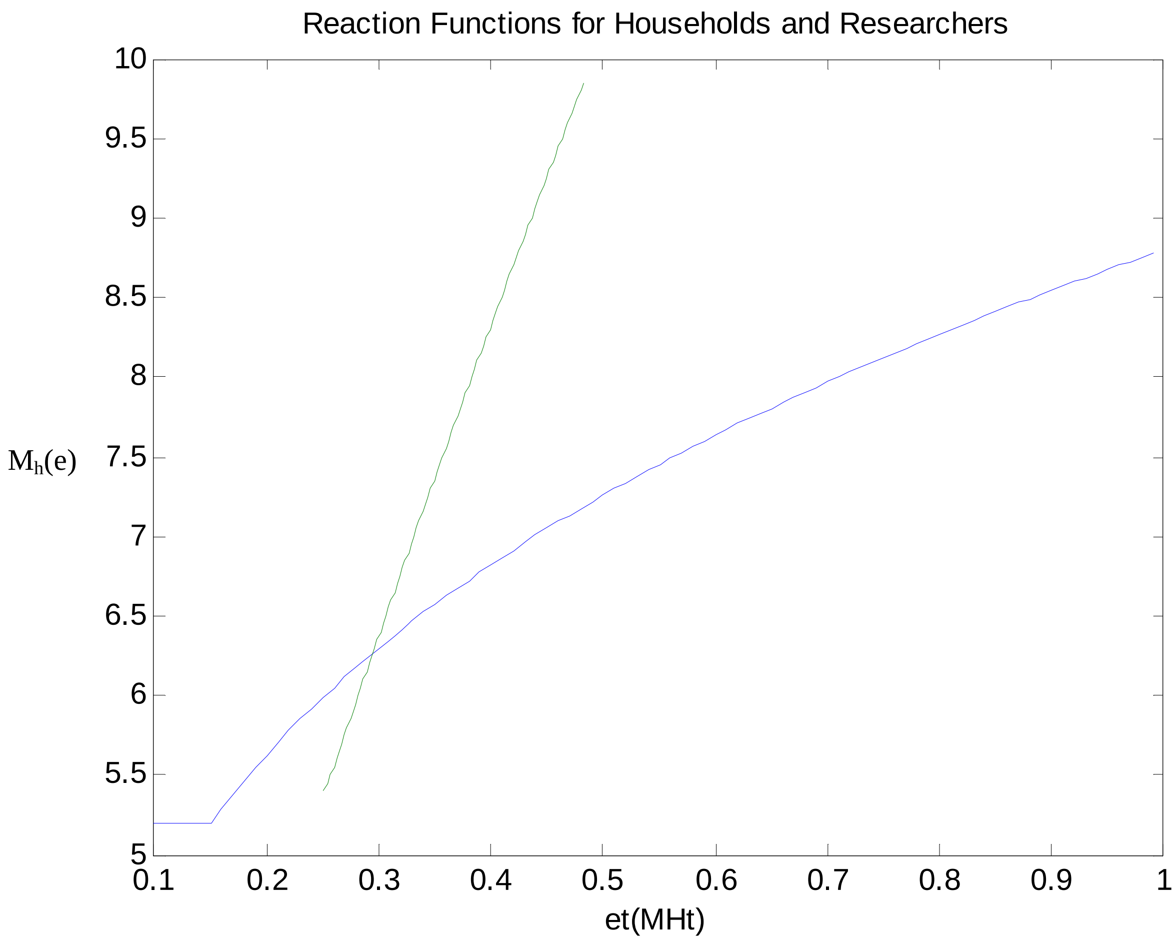
$$N_t = n_{t-1} N_{t-1} \tag{23}$$

Combining this model of demography with our model of biased technologies is straightforward. Through the simultaneous solving of (10), (11), (14), (15), (16), (17), (19), (20), (21), and (22), a unique set of variables w_l, w_h, A_l, A_h, M_l, M_h, n, e, L, and H can be determined for every time period.[18] We can perhaps synopsize our findings by initially focusing only on the economy's choice of e and M_h. If an adult expects researchers to develop new skill-biased technologies (and so to increase w_h), she will want to endow her children with more human capital. Similarly, if researchers anticipate a larger pool of human capital, they may wish to invent and build new skill-intensive machines, raising $M_{h,new}$ and thus M_h overall. Consequently we can plot the two "reaction functions" of each group as two upward-sloping curves; the development of new skill-using machines and the accumulation of skills are strategic complements. From the intersection of these reaction curves we find the unique simultaneous solution of the level of education and the new skill-biased technical coefficient. This is done in Figure 4. We can similarly plot two upward-sloping curves to determine an economy's choice of n and M_l.

To summarize, potential researchers look to the skill composition of the workforce (something influenced by households) to determine the direction and scope of technical change. Households look to wages (something influenced by researchers) to determine the levels of skilled and unskilled workers. Together they jointly determine the overall composition of the economy.

[18]This 10-by-10 system is reiterated with more detail in the Appendix.

Figure 4: "Reaction Curves"



- The steeper line represents the fraction of educated young a parent would choose for a given technological parameter M_h. The flatter curve represents the skill-biased technical coefficient that would result from a given fraction of educated young.

4 "Appropriate" Growth Paths for a Developing Country - Some Simulations[19]

With a model that endogenizes both technologies and factors, we may better assess the appropriateness of alternative development paths. Let us consider a hypothetical developing country endowed with a fairly sizeable amount of unskilled labor and a modest amount of skilled labor. We can then test the effects of unbalanced growth by allowing either only unskilled-labor technology or skilled-labor technology to rise, run the "horse-race," and compare the two paths. Each simulation is run for ten time periods.

4.1 Case 1 - Simulation with Exogenous Unbalanced Growth[20]

Our first horse-race is where we simply have either A_l or A_h grow exogenously, and compare the two paths. In other words, we ignore our discussion about endogenous technical growth in section 3.2 for the moment, and assume that unbalanced growth happens simply as some exogenous process, such as through technological diffusion from other countries (see the appendix for the system of equations being solved each time period). Specifically, each technological parameter grows 5% each period.

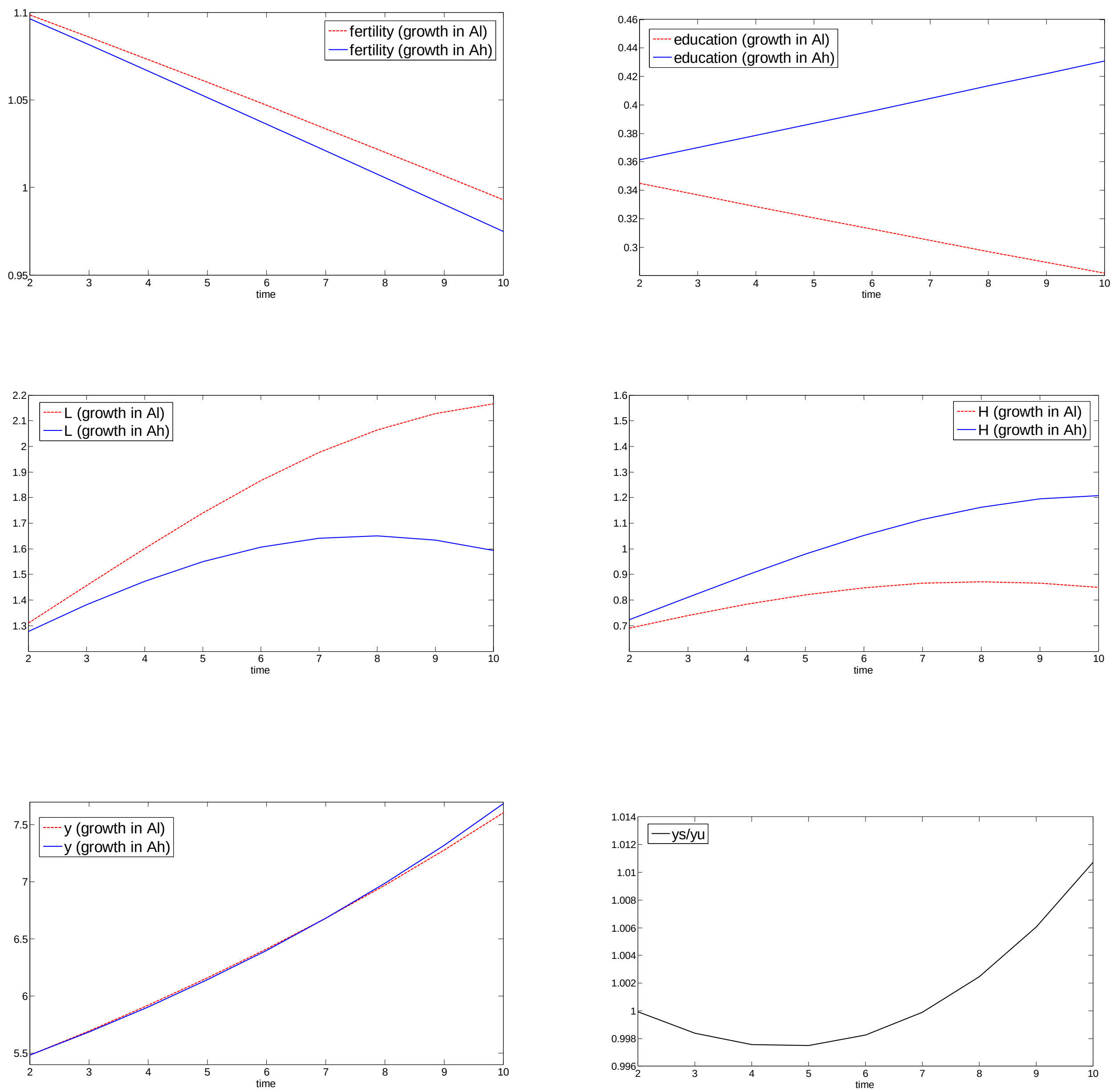
Figure 5 illustrates the results of these simulations. Red dotted lines are where only A_l grows; blue solid lines are where only A_h grows. Growth in both cases lowers fertility, since it raises the opportunity costs to raise children. However, it is clear that skilled-biased growth lowers fertility more dramatically, since it induces families to provide more of their offspring with education; this raises the costs of children even further. Unskilled-biased growth on the other hand puts downward pressure on skill premia, exerting upward pressure on fertility and downward pressure on education.

We can see how these demographic shifts affect the factors of production. Initial fertility rates above one induce increases in both labor types; once fertility falls below one, both labor types begin to fall. However, we can see that changes in H are more muted than changes in L. Why is this? Basically, the quality-quantity tradeoff affects H in opposing ways (lower fertility lowers H, while higher education raises H). This same tradeoff however reinforces changes in L (higher fertility raises L, and lower education *also* raises L). Unbalanced growth thus is likely to change the unskilled work force more dramatically than the skilled work force.

[19]Note that for the lessons of the simulations to hold, we require only that $0 < \alpha < \sigma < 1$. This simply means that factors of production must be substitutable "enough." Specifically we assume that $\alpha = 0.33$ and $\sigma = 0.5$. We also specify the simple cost function $c(n, e) = n^2 + (1 + ne)^2$ to ensure that costs rise in both n and e.

[20]For this case initial values of A_l and A_h are set to 3 and N is normalized to 1. With n set to 1.1 to represent a growing population, $e = 0.4$, and this gives us initial factor endowments of $L = 1.3$ and $H = 0.7$, and initial wages of $w_l = 5.2$ and $w_h = 7.05$.

Figure 5: Simulation of Economy with Exogenous Unbalanced Growth



- Exogenous skill-biased technical growth augments a smaller workforce and so generates relatively less output growth at first; the lower fertility and higher education that this growth provokes however generates relatively more output later on.

It is also clear that unbalanced growth creates changes in *relative* factors. We can see that H/L falls with unskilled-bias growth, and rises with skilled-bias growth. Recall from our discussion in section 2.2 that the latter means there is relative growth in the more productive factor (H is more productive even though we start with $A_l = A_h$ because it is scarcer than L), and this should be a boost to overall income. On top of this, the overall population grows faster with unskilled-bias growth than with skilled-bias growth.

The final two graphs compare simulated per capita GDPs ($y = Y/N$) for the two paths. While there is only a small difference between the two, we can see that skill-biased growth slightly underperforms early on, and outperforms later on. Consistent with our earlier discussions, skill-biased technologies are not "appropriate" in the sense that they augment a relatively smaller workforce, so unskilled-biased technical growth generates relatively more overall growth at first. However, if factors themselves react to technological changes, we see that growth in unskilled labor generates greater growth in the entire population, with no beneficial feedback through market-size effects for researchers (since technologies grow exogenously in this case). A skill-intensive technological path can produce more income per person *over time* than the alternative path.

4.2 Case 2 - Simulation with Endogenous Unbalanced Growth Starting with $M_l \approx M_h$[21]

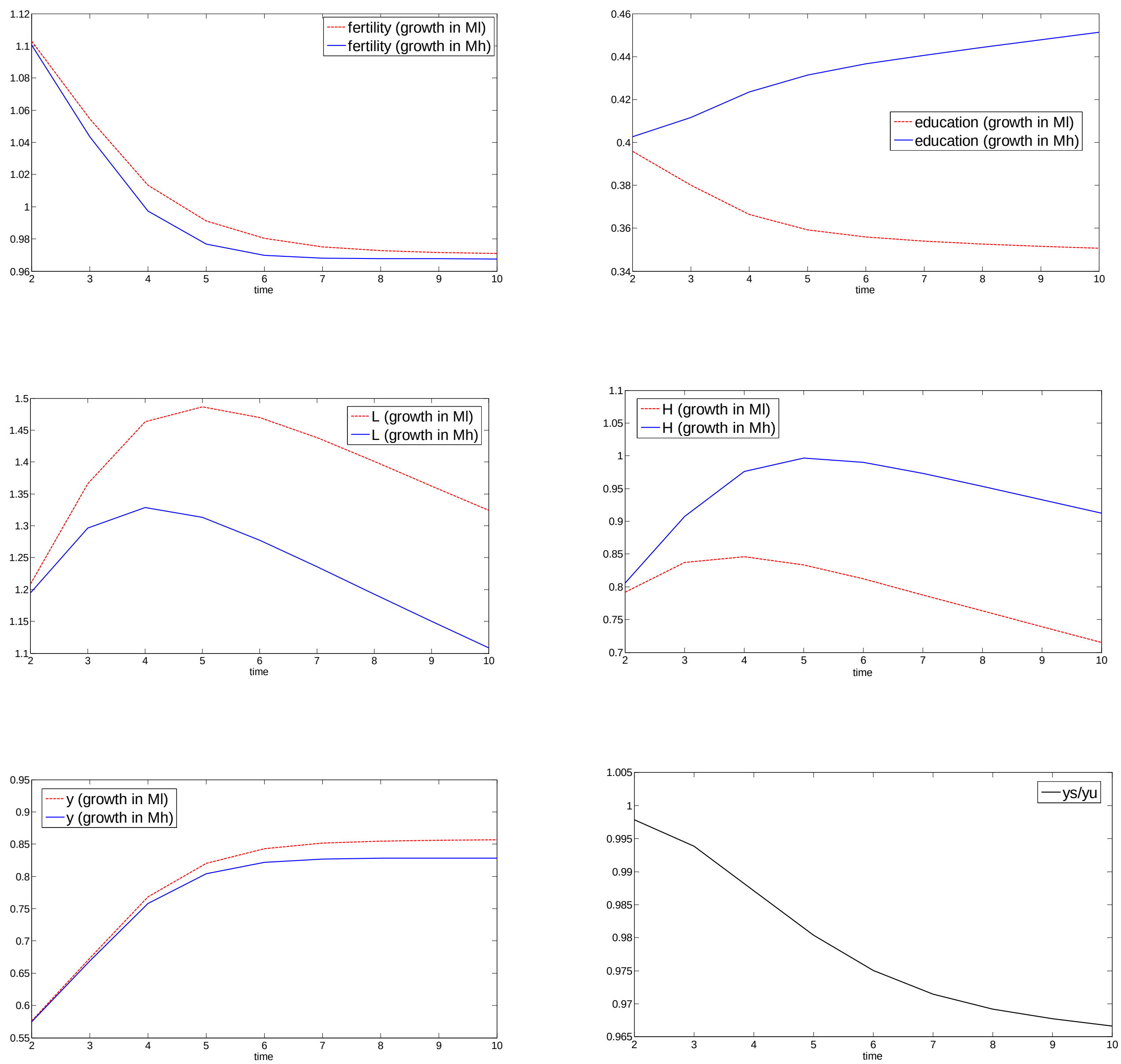
In this case we use the full system of equations that jointly solve for technological levels and for demographic variables. Technologies in this case are "semi-endogenous," in that we have research costs exogenously fall in order to observe the endogenous technological and demographic responses (see the appendix for the full system of equations). Specifically, for unskilled-intensive growth, we set z_l such that $V_l = c_l$ at the start of the simulation. Then we simply have z_l grow 5% each period, inducing research that produces new varieties of unskilled labor-using machines. For skill-intensive growth we do the same to z_h, V_h and c_h.

Figure 6 illustrates the results of these two simulations. Immediately we see that here *unskilled*-labor intensive growth produces more per capita output than skill-intensive growth. This reversal from the findings of Case 1 may be surprising at first, because like Case 1 unskilled-intensive growth lowers H/L, and we have said that H tends to be the more productive factor. And it bucks conventional wisdom, which suggests that decreases in education can be destructive for the overall macro economy.

In this case, however, the upward pressure on fertility that results from unskilled-bias technological growth allows unskilled labor to expand. This contributes to even more unskilled-biased technological progress in two ways. First, it induces final-good producers to demand and use more *existing* unskilled-using machines, making unskilled labor more productive (equation 9).

[21] For this case initial levels of machines are set to $M_l = 0.3$ and $M_h = 0.35$. This gives us initial values of A_l and A_h of 0.3, $L = 1.2$, $H = 0.8$, $w_l = 0.54$ and $w_h = 0.75$.

Figure 6: Simulation of Economy with Semi-endogenous Unbalanced Growth ($M_h \approx M_l$)



- Semi-endogenous skill-biased technical growth produces a slightly faster drop in fertility than unskilled-biased technical growth. This actually generates *less* per capita income over time because the productivity slowdown caused by falling population is more severe in this case.

Second, it induces more research to produce *new* brands of unskilled-using machines (equation 16). With skill-biased technological growth, the quality-quantity tradeoff in child rearing affect H in opposing ways; as a result H does not rise as much, and so cannot generate beneficial market-size effects to the same extent.

One lesson here is that fertility declines, while inevitable in the process of economic development, can hurt subsequent growth when technologies are endogenous. Scale matters in this case, since researchers require a large group of workers to purchase and use their new machines in order to recoup their fixed costs. Because unskilled-labor biased growth puts upward pressure on fertility, it limits such negative effects to market size. This case suggests that the high fertility rates we observe for poorer nations may actually contribute to future economic prosperity, not hinder it as the prior case suggests.

4.3 Case 3 - Simulation with Endogenous Unbalanced Growth Starting with $M_l < M_h$

While the above case implies that unskilled-intensive growth produces more per capita income in the long-run than the alternative, we should acknowledge that this case is based on the extreme assumption that $A_h = A_l$ at the start. But we could very well have the case where $A_h > A_l$. This is especially true if we have a fairly broad definition of H. For example, when H is considered those with primary education or more, skilled-labor productivity tends to be larger for unskilled-labor productivity for most countries (see Table 1). As mentioned in section 2.1, wealthier nations also tend to have higher relative productivity levels for skilled workers.

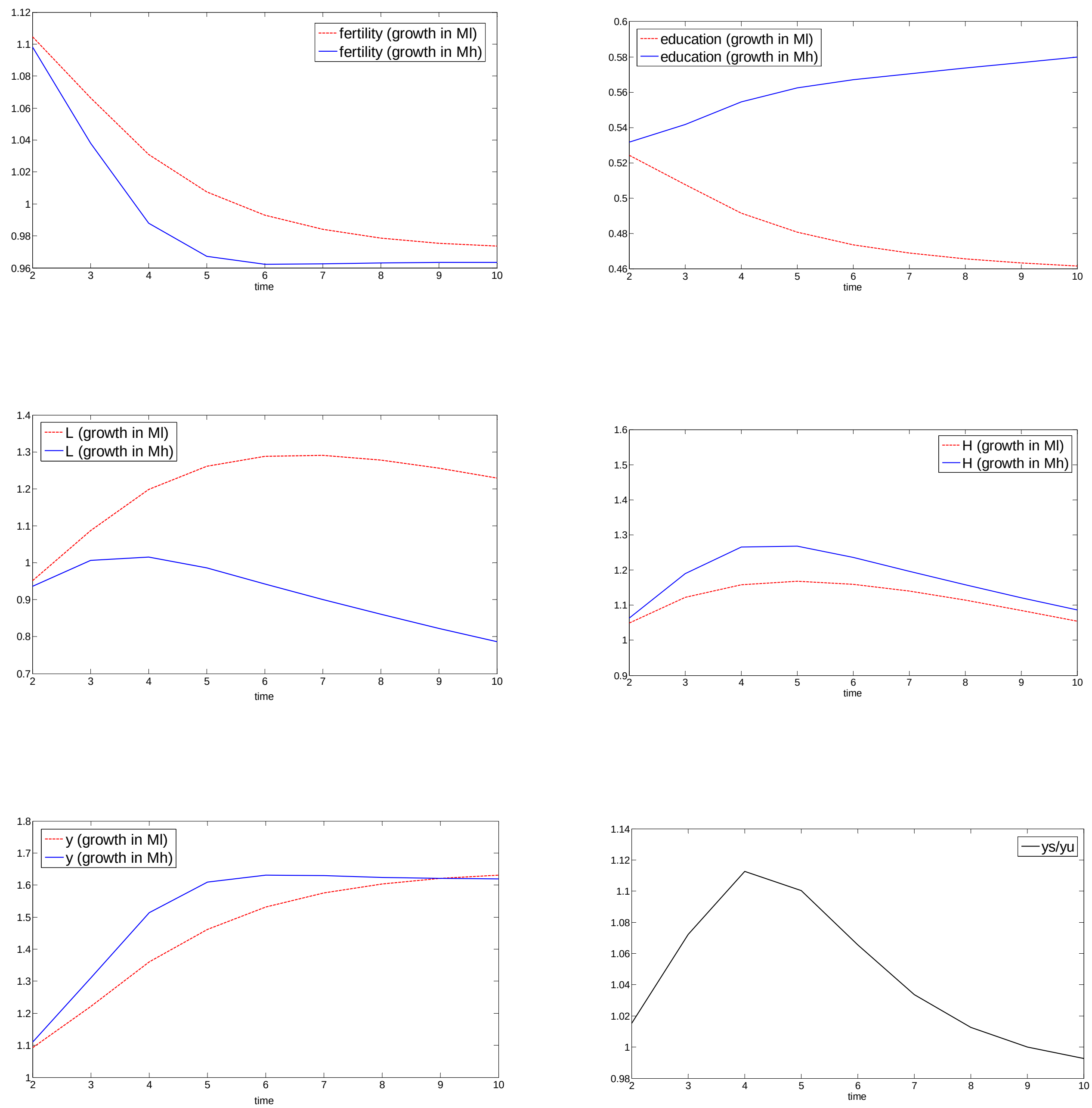
Our final case explores this possibility by endowing skilled-labor with many more machine varieties than unskilled labor at the start of the simulations, thus making skilled labor inherently more productive.[22] Figure 7 illustrates this case.

The per capita GDP comparison between the two paths in this case produces the mirror image to the one in Case 1 - the skilled-intensive path produces relatively more early on, but relatively less later on. Why? Given the productivity advantage of skilled workers here, increases in education generate a lot of macroeconomic growth. The rapid fall in fertility that this path generates, however, contributes to a faster productivity slowdown.

Thus, even though skilled labor is much more productive than unskilled labor, the unskilled technological path will eventually win the horse race if we let it run long enough. With skill-intensive growth, the quantity-quality tradeoff and the forces of demographic momentum sow the seeds for this path's eventual slowdown. The fall in fertility that inevitably results (as kids get more and more expensive) anchors the growth in H so the market size effects with this path inevitably peter away.

[22]Specifically, we set initial levels of M_l and M_h to 0.3 and 1, respectively. Again, after setting N to 1 and n to 1.1, we get initial values of $e = 0.53$, $L = 0.95$, $H = 1.05$, $w_l = 0.9$, and $w_h = 1.4$.

Figure 7: Simulation of Economy with Semi-endogenous Unbalanced Growth ($M_h > M_l$)



- Semi-endogenous skill-biased technical growth generates a more rapid drop in fertility and rise in education, same as in the previous cases. This generates relatively more output growth *at first* because skilled labor is so much more productive in this case; but it still generates *less* output later on due to the more rapid productivity slowdown from fertility declines.

5 Conclusion

This paper models the simultaneity of factors and technologies to evaluate different growth paths. Unlike approaches that credit either technological progress (Christensen and Cummings 1981) or factor accumulation (Young 1995) alone for economic success, the interaction of both can lend us new insights on which development path will breed the greatest rewards.

We see that the answer depends on the structure of the macro economy. Generally, a skill-intensive path will generate more benefits the more productive skilled labor is. It also produces more benefits the more *responsive* are factors to technological changes, provided there is no or limited feedback from these changes on technologies. This is because the falling population growth caused by skill-intensive growth, normally a boom to income per capita, would hurt economic growth if technologies are locally endogenous. Thus skill-biased technological diffusion, of the kind generated by the world-wide pervasiveness of skill-intensive technologies (Berman and Machin 2000; Berman, Bound and Machin 1998) can generate robust growth *because* of its exogenous nature; and this despite its apparent inappropriateness due to a low endowment of H.

The bottom line is that the proper path to macro prosperity depends on lots of things - here we provide only the broadest brush-strokes delineating some major concerns. This subject however is relevant to all developing nations. Should India focus more on labor-heavy manufacturing or skill-heavy services? Should China's fiscal stimulus channel resources to build infrastructure using skill-intensive or labor-intensive techniques? Questions such as these dominate discussions over macro economic strategy in these countries; the answers will depend on some of the issues raised here.

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Appendix A: Simulations

If technologies are exogenously determined, the simulation pre-determines A_l and A_h, and solves the following system of equations for w_l, w_h, n_t, e_t, L, and H for each time period t, given n_{t-1}, e_{t-1}, and N_t.

$$w_l = [(A_l L)^\sigma + (A_h H)^\sigma]^{\frac{1-\sigma}{\sigma}} A_l^\sigma L^{\sigma-1} \tag{24}$$

$$w_h = [(A_l L)^\sigma + (A_h H)^\sigma]^{\frac{1-\sigma}{\sigma}} A_h^\sigma H^{\sigma-1} \tag{25}$$

$$\frac{1}{n_t} = 2w_j \left(n_t + e_t + n_t e_t^2\right) \tag{26}$$

$$\frac{w_h}{w_l(1-e_t) + w_h e_t} = \frac{w_l}{w_l(1-e_t) + w_h e_t} + 2w_j \left(n_t + n_t^2 e_t\right) \tag{27}$$

$$L = N_t(1-e_{t-1}) + N_t n_t (1-e_t) \tag{28}$$

$$H = N_t e_{t-1} + N_t n_t e_t \tag{29}$$

If on the other hand technologies are endogenously determined by the process discussed in section 3.2, the following equations are solved for $M_{l,new}$, $M_{h,new}$, A_l, A_h, w_l, w_h, n_t, e_t, L, and H for each time period t, given n_{t-1}, e_{t-1}, and N_t.

$$\left(\frac{1}{1+r}\right)\left(\frac{1-\alpha}{\alpha}\right)\alpha^{\frac{2}{1-\alpha}} [(A_l L)^\sigma + (A_h H)^\sigma]^{\frac{1-\sigma}{(1-\alpha)\sigma}} A_l^{\frac{1-\sigma}{\alpha-1}} L^{\frac{\sigma-\alpha}{1-\alpha}} \leq \left(\frac{M_{l,old} + M_{l,new}}{z_l}\right) \tag{30}$$

$$\left(\frac{1}{1+r}\right)\left(\frac{1-\alpha}{\alpha}\right)\alpha^{\frac{2}{1-\alpha}} [(A_l L)^\sigma + (A_h H)^\sigma]^{\frac{1-\sigma}{(1-\alpha)\sigma}} A_h^{\frac{1-\sigma}{\alpha-1}} H^{\frac{\sigma-\alpha}{1-\alpha}} \leq \left(\frac{M_{h,old} + M_{h,new}}{z_h}\right) \tag{31}$$

$$A_l = \left[\alpha^{\frac{\alpha}{1-\alpha}} M_{l,old} + \alpha^{\frac{2\alpha}{1-\alpha}} M_{l,new}\right] \left((A_l L)^\sigma + (A_h H)^\sigma\right)^{\frac{(1-\sigma)\alpha}{(1-\alpha)\sigma}} A_l^{\frac{(\sigma-1)\alpha}{1-\alpha}} L^{\frac{\alpha(\sigma-1)}{1-\alpha}} \tag{32}$$

$$A_h = \left[\alpha^{\frac{\alpha}{1-\alpha}} M_{h,old} + \alpha^{\frac{2\alpha}{1-\alpha}} M_{h,new}\right] \left((A_l L)^\sigma + (A_h H)^\sigma\right)^{\frac{(1-\sigma)\alpha}{(1-\alpha)\sigma}} A_h^{\frac{(\sigma-1)\alpha}{1-\alpha}} H^{\frac{\alpha(\sigma-1)}{1-\alpha}} \tag{33}$$

$$w_l = \left[(A_l L)^\sigma + (A_h H)^\sigma\right]^{\frac{1-\sigma}{\sigma}} A_l^\sigma L^{\sigma-1} \tag{34}$$

$$w_h = \left[(A_l L)^\sigma + (A_h H)^\sigma\right]^{\frac{1-\sigma}{\sigma}} A_h^\sigma H^{\sigma-1} \tag{35}$$

$$\frac{1}{n_t} = 2w_j \left(n_t + e_t + n_t e_t^2\right) \tag{36}$$

$$\frac{w_h}{w_l(1-e_t) + w_h e_t} = \frac{w_l}{w_l(1-e_t) + w_h e_t} + 2w_j \left(n_t + n_t^2 e_t\right) \tag{37}$$

$$L = N_t(1-e_{t-1}) + N_t n_t (1-e_t) \tag{38}$$

$$H = N_t e_{t-1} + N_t n_t e_t \tag{39}$$

(30) and (31) illustrate the benefits and costs of innovation; (32) and (33) are factor-specific TFP levels as functions of the demand for old and new machines and factors of production; (34) and (35) are wages; (36) and (37) are the benefits and costs of having children and educating them; (38) and (39) describe how fertility and education choices translate into aggregate factors of production. Note that if either of the first two equations holds with strict inequality, the algorithm sets the value of M_{new} to zero and simply solves the the rest of the system.