UNIVERSIDAD CARLOS III DE MADRID working papers

Working Paper 10-05
Statistics and Econometrics Series 02
January 2010

Departamento de Estadística
Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)
Fax (34) 91 624-98-49

OUTLIERS IN GARCH MODELS AND THE ESTIMATION OF RISK MEASURES

Aurea Grané(1) and Helena Veiga(2)

Abstract

In this paper we focus on the impact of additive level outliers on the calculation of risk measures, such as minimum capital risk requirements, and compare four alternatives of reducing these measures' estimation biases. The first three proposals proceed by detecting and correcting outliers before estimating these risk measures with the GARCH(1,1) model, while the fourth procedure fits a Student's *t*-distributed GARCH(1,1) model directly to the data. The former group includes the proposal of Grané and Veiga (2010), a detection procedure based on wavelets with hard- or soft-thresholding filtering, and the well known method of Franses and Ghijsels (1999). The first results, based on Monte Carlo experiments, reveal that the presence of outliers can bias severely the minimum capital risk requirement estimates calculated using the GARCH(1,1) model. The message driven from the second results, both empirical and simulations, is that outlier detection and filtering generate more accurate minimum capital risk requirements than the fourth alternative. Moreover, the detection procedure based on wavelets with hard-thresholding filtering gathers a very good performance in attenuating the effects of outliers and generating accurate minimum capital risk requirements out-of-sample, even in pretty volatile periods.

Keywords: Minimum Capital Risk Requirements, Outliers, Wavelets.

JEL classification: C22, C5, G13.

(1) A. Grané, Statistics Department, Universidad Carlos III de Madrid, C/ Madrid 126, 28903 Getafe (Madrid), Spain, e-mail: aurea.grane@uc3m.es . Corresponding author.
(2) H Veiga, Universidad Carlos III de Madrid, C/ Madrid 126, 28903 Getafe (Madrid), Spain; ISCTE Business School, Avenida das Forças Armadas, 1600-083 Lisboa, Portugal, e-mail: mhveiga@est-econ.uc3m.es
Work partially supported by Spanish grants SEJ2007-64500, ECO2009-08100 and MTM2009-13985-C02-01 (Ministry of Education and Science- FEDER).

Outliers in GARCH models and the estimation of risk measures

Aurea Grané[(1)] Helena Veiga[(2)]

(1),(2) Statistics Department. Universidad Carlos III de Madrid.

(2) Finance Research Center. ISCTE Business School.

Abstract

In this paper we focus on the impact of additive level outliers on the calculation of risk measures, such as minimum capital risk requirements, and compare four alternatives of reducing these measures' estimation biases. The first three proposals proceed by detecting and correcting outliers before estimating these risk measures with the GARCH(1,1) model, while the fourth procedure fits a Student's t-distributed GARCH(1,1) model directly to the data. The former group includes the proposal of Grané and Veiga (2010), a detection procedure based on wavelets with hard- or soft-thresholding filtering, and the well known method of Franses and Ghijsels (1999). The first results, based on Monte Carlo experiments, reveal that the presence of outliers can bias severely the minimum capital risk requirement estimates calculated using the GARCH(1,1) model. The message driven from the second results, both empirical and simulations, is that outlier detection and filtering generate more accurate minimum capital risk requirements than the fourth alternative. Moreover, the detection procedure based on wavelets with hard-thresholding filtering gathers a very good performance in attenuating the effects of outliers and generating accurate minimum capital risk requirements out-of-sample, even in pretty volatile periods.

JEL classification: Minimum Capital Risk Requirements, Outliers, Wavelets

Keywords: C22, C5, G13.

1 Introduction

The increase in volatility over the recent years and the cataclysm involving financial markets across the world, specially from September 2008, has created a urgent need to protect the finance and banking system against large trading losses. With the Basel Accord of 1988 the first measure to tackle the problem was taken by demanding the financial institutions to reserve part of the capital to absorb a pre-specified percentage of these unforseen losses, denoted minimum capital risk requirements (MCRRs).

Financial support from research projects MTM2009-13985-C02-01, SEJ2007-64500 and ECO2009-08100 by the Spanish Ministry of Education and Science are gratefully acknowledged.
Authors' address: (1),(2) Statistics Department, Universidad Carlos III de Madrid, C/ Madrid 126, 28903 Getafe, Spain. (2) ISCTE Business School, Avenida das Forças Armadas, 1600-083 Lisboa, Portugal. E–mails: A. Grané, `aurea.grane@uc3m.es`; H. Veiga, `mhveiga@est-econ.uc3m.es`. Corresponding author: A. Grané; Date: January 29, 2010.

Following the 1995 amendment to the Basel Accord, banks were allowed to use internal models to calculate their risk measures' thresholds. This amendment tried to ratify the fact that the standard approach to the estimation of the minimum capital risk requirements led to very conservative estimates and consequently to a wasting of valuable resources by financial institutions that used the standard approach. Nevertheless, the lately poor evolution of financial markets emphases the bad protection of financial institutions to extreme events and the importance of forecasting volatility accurately for providing good estimates of these risk measures.
The accurate estimation of minimum capital risk requirements depends crucially on the accuracy of parameter estimates and volatility forecasts. Several models have been proposed in the literature to capture the main features of financial time series and forecast volatility. The ARCH model by Engle (1982) and the GARCH model by Bollerslev (1986) became seminal models in financial econometrics, specially due to their easy applicability, effectiveness in parameterizing the higher order dependence and good ability in forecasting volatility (see for instance Lunde and Hansen, 2005). Since their introduction to the literature they have been extended in several directions. The first extension, proposed by Bollerslev (1987), allowed the error of the GARCH model to follow a Student's t distribution in order to accommodate the high kurtosis of the data. However, it has been observed that the estimated residuals from this extended model still register excess kurtosis (see Baillie and Bollerslev, 1989; Teräsvirta, 1996). One possible reason for this occurrence is that some observations on returns are not fitted by a Gaussian GARCH model and not even by a t-distributed GARCH model. These observations may be influential (see Zhang, 2004, for a detailed definition of influential observation) since they can affect undesirably the estimation of parameters (see for example Fox, 1972; Van Dijk et al., 1999; Verhoeven and McAleer, 2000), the tests of conditional homoscedasticity (see Carnero et al., 2007; Grossi and Laurini, 2009) and the out-of-sample volatility forecasts (see for instance Ledolter, 1989; Chen and Liu, 1993a; Franses and Ghijsels, 1999; Carnero et al., 2008). When this is the case, some authors denote them by outliers and distinguish between additive and innovational (or innovations) outliers. The first type is classified in two categories: additive level outliers (ALO), which exert an effect on the level of the series but not on the evolution of the underlying volatility, and additive volatility outliers (AVO), that also affect the conditional variance (see Hotta and Tsay, 1998; Sakata and White, 1998).
This paper focuses mainly on the study of the effects of additive level outliers on the estimation of MCRRs for short and long trading investment positions and different day horizons. The effects of innovational outliers on the dynamic properties of the series are less important because they are propagated by the same dynamics, as in the rest of the series (see for example Peña, 2001). Through an intensive Monte Carlo experiment we study the outlier effects on the calculation of MCRRs and compare four proposals for reducing the MCRR estimation biases. In particular, the methods under evaluation consist in computing the MCRRs using a Gaussian GARCH(1,1) model fitted to the outlier filtered data with the procedure of Grané and Veiga (2010) based on wavelets, either with hard- or soft-thresholding, using a Gaussian GARCH(1,1) model fitted to the outlier filtered data with the procedure of Franses and Ghijsels (1999) and, finally, using the t-distributed GARCH(1,1) model fitted directly to the data. These alternatives are exhaustively tested for out-of-

sample conditional coverage, whenever it is possible.
The most important findings in this paper are: first, outliers affect seriously the estimates of MCRRs and the effects depend on their magnitudes. Often, the larger the outlier magnitudes are the larger the biases. Second, the detection proposal by Grané and Veiga (2010) with hard-thresholding correction almost eliminates the biases on the MCRR estimates and generates out-of-sample more accurate MCRRs, even in pretty volatile periods. This is due to a more accurate detection and correction of outliers that leads, consequently, to a reduction of parameter biases and more accurate volatility forecasts. Finally, fitting a t-distributed GARCH(1,1) model directly to the data often generates MCRR biases larger than those obtained with the proposals of Grané and Veiga (2010) and Franses and Ghijsels (1999). Similar results were found by Charles (2008), who detected that after correcting the series for outliers with the method proposed by Franses and Ghijsels (1999), the parameter estimates that governed the volatility dynamics were almost free of biases and the volatility forecasts were much more accurate than those obtained with fat tail models, such as the t-distributed GARCH(1,1) model.
The organization of this paper is as follows. In Section 2 we present the volatility model used in the paper and recall the concept of additive level outlier introduced by Hotta and Tsay (1998). In Section 3 we present the algorithms for outlier detection proposed by Grané and Veiga (2010) and Franses and Ghijsels (1999). In Section 4 we perform the Monte Carlo study in order to evaluate the outlier effects on the calculation of MCRRs and compare four different proposals for reducing the MCRR estimation biases. In Section 5 we test the presented proposals on three daily stock market indexes and we conclude in Section 6.

2 Additive level outliers in GARCH(1,1) model

Return series of financial assets, although uncorrelated, are not independent because they contain higher order dependence. One way of parameterizing this dependence is using models of autoregressive conditional heteroscedasticity such as the GARCH(1,1) model proposed by Bollerslev (1986). This model is given by:

$$\begin{aligned} y_t &= \mu + \varepsilon_t, \\ \sigma_t^2 &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \end{aligned} \tag{1}$$

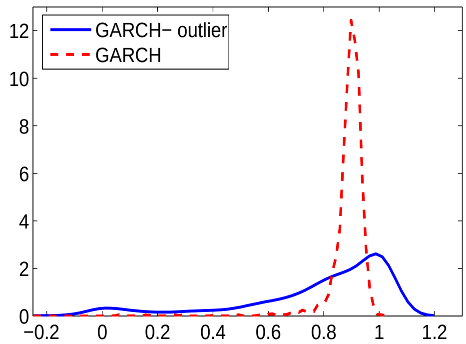
where μ is the conditional mean of the asset return y_t, $\varepsilon_t = \sigma_t \epsilon_t$ is the prediction error, $\sigma_t > 0$ is the conditional standard deviation of the underlying asset return (denoted volatility) and the error $\epsilon_t \sim NID(0,1)$. Furthermore, $\alpha_0 > 0$, $\alpha_1 \geq 0$ and $\beta_1 \geq 0$ to guarantee the positiveness of the conditional variance and $\alpha_1 + \beta_1 < 1$ to assure its stationarity. Under this model, volatility evolves in a continuous manner over time, it is stationary and $|\varepsilon_t|$ tends to assume a large value, which means that large errors tend to be followed by a large error (see Tsay, 2005). Moreover, the conditional variance defined by equation (1) satisfies the property of slow decay of the unconditional autocorrelation function of ε_t^2, when it exists, mimicking the behavior of the autocorrelation functions of squared returns (see Teräsvirta, 2006). The first extension of this model is due to Bollerslev (1987) and consists in allowing the error ϵ_t to follow a Student's t distribution.

Additive level outliers can be caused by an institutional change or a market correction that does not affect volatility. The conditional mean equation of the GARCH(1,1) model with an ALO is defined as:

$$y_t = \mu + \omega_{AO}\, I_T(t) + \varepsilon_t,$$

where ε_t is defined as before, ω_{AO} represents the magnitude (or size) of the additive level outlier and $I_T(t) = 1$ for $t \in T$ and 0 otherwise, representing the presence of the outlier at a set of times T. The equation of the conditional variance for the GARCH(1,1) model remains the same, since this type of outlier only affects the series' level. In order to illustrate the importance of outlier detection, in Figure 1 we depict the anomalies caused by one outlier of moderate magnitude on the distribution of $\hat{\beta}_1$, the estimator of the β_1 parameter.

Figure 1: Kernel density of $\hat{\beta}_1$ obtained from 1000 samples of size $n = 500$ from a GARCH(1,1) with true parameter value $\beta_1 = 0.912$.



3 Outlier detection and correction procedures

In this section we sketch the outlier detection proposals by Grané and Veiga (2010) and Franses and Ghijsels (1999). We address the reader to the original references for the details.

3.1 Wavelet-based detection and correction procedures

Grané and Veiga (2010) proposed a general detection and correction method based on wavelets that can be applied to a large class of volatility models. The effectiveness of these authors' proposal was tested applying it to several volatility models, such as the GARCH(1,1), the GJR(1,1) by Glosten et al. (1993) and the autoregressive stochastic volatility model, ARSV(1) by Taylor (1986), with errors following a Gaussian or a Student's t distribution and comparing it with the proposals of Bilen and Huzurbazar (2002), Franses and Ghijsels (1999) and Doornik and Ooms (2005). The intensive Monte Carlo study revealed that Grané and Veiga (2010)'s proposal is not only as good as these alternatives in detecting different type of outliers (isolated ALOs, multiple ALOs, AVOs and patches of ALOs), but it is also much more reliable, since it detectes a significantly smaller number of false outliers.

The algorithm uses the notions of discrete wavelet transform (DWT) and inverse discrete wavelet transform (IDWT) (see Percival and Walden, 2000, for a complete guide to wavelet methods for time series). In particular, the proposal is based on the detail coefficients resulting from the discrete wavelet transform of the series of residuals, which are obtained after fitting a particular volatility model. The outliers are identified as those observations in the original series whose detail coefficients are greater (in absolute value) than a certain threshold.

Next we briefly describe the steps of the procedure for detecting ALOs. Let $\mathbf{X} = (X_1, \ldots, X_n)$ be the series of residuals of size n obtained after fitting a GARCH(1,1) model with errors following a standard normal distribution.

Step 1 Apply the DWT to the series of residuals $\mathbf{X}$ to obtain the first level wavelet coefficients $\mathbf{A}_1 = (a_1, \ldots, a_{n/2})$ and $\mathbf{D}_1 = (d_1, \ldots, d_{n/2})$.

Step 2 Set the threshold $k_1^{0.05}$ equal to the 95th-percentile of the distribution of the maximum of the first level detail coefficients (in absolute value) resulting from the DWT of n iid random variables following a standard normal distribution.

Step 3 Find $d_{max} = \max_{1 \leq j \leq n/2}\{|d_j| > k_1^{0.05}\}$, and let s be the position of d_{max} in the vector $\mathbf{D}$.

Step 4 Set $d_{max} = 0$ and construct $\widetilde{\mathbf{D}}_1$ as the vector equal to $\mathbf{D}_1$, but with a 0 in the s position; that is, $\widetilde{\mathbf{D}}_1 = (d_1, \ldots, d_{s-1}, 0, d_{s+1}, \ldots, d_{n/2})$.

Step 5 Recompose the series of residuals applying the inverse discrete wavelet transform (IDWT) to $\mathbf{A}_1$ and $\widetilde{\mathbf{D}}_1$.

Step 6 Repeat steps 1 to 5 until all the elements in the vector of the detail coefficients are lower (in absolute value) than the threshold $k_1^{0.05}$. Let $S = \{s_1, \ldots, s_\ell\}$ be the ordered set of indices containing the positions of the d_{max}'s.

Step 7 Use S to locate the exact positions of the outliers in the series of residuals $\mathbf{X}$. Let s be a generic element in S. Compute the sample mean of $\mathbf{X}$ without observations at locations $2s$ and $2s-1$:

$$\overline{x}_{n-2} = \frac{1}{n-2} \sum_{i \neq 2s, 2s-1} X_i$$

and set the position of the outlier equal to $2s$ if $|X_{2s} - \overline{x}_{n-2}| > |X_{2s-1} - \overline{x}_{n-2}|$, or equal to $2s-1$, otherwise.

Once the outlier positions in the series have been determined (using the series of residuals), we propose to correct those observations from the series of returns by hard-thresholding (HT) in the following way:

HT : Let $\{\mathbf{D}_1, \mathbf{A}_1\}$ be the vectors of wavelet coefficients from the first level decomposition of the return series. Assign zero to those elements in $\mathbf{D}_1$ whose indices belong to the set S and denote by $\widetilde{\mathbf{D}}_1$ the corrected first level detail coefficients. To reconstruct the series of returns, apply the inverse wavelet transform to $\mathbf{A}_1$ and $\widetilde{\mathbf{D}}_1$.

or either by soft-thresholding (ST):

ST : Let $\{\mathbf{D}_1, \mathbf{A}_1\}$ be the vectors of wavelet coefficients from the first level decomposition of the return series. Substitute those elements in $\mathbf{D}_1$ whose indices belong to the set S by $d_{s_i} - \text{sign}(d_{s_i})\, k_1^{0.05}$, for all $s_i \in S$, and denote by $\widetilde{\mathbf{D}}_1$ the corrected first level detail coefficients. To reconstruct the series of returns, apply the inverse wavelet transform to $\mathbf{A}_1$ and $\widetilde{\mathbf{D}}_1$.

In the original paper, we only considered to filter by hard-thresholding, but since soft-thresholding has become popular in the context of wavelet estimation and there is some evidence that for some particular situations it turns out to be superior to hard-thresholding (see Droge, 2006, for more details), we have included it for comparison.

3.2 Franses and Ghijsels (1999)'s proposal

Franses and Ghijsels (1999) exploited the analogy of the GARCH(1,1) model with an ARMA(1,1) model to adapt the method of Chen and Liu (1993b) to detect and correct additive level outliers in GARCH(1,1) models. In particular, the conditional variance equation (1) of the GARCH(1,1) model can be rewritten as an ARMA(1,1) for ε_t^2:

$$\varepsilon_t^2 = \alpha_0 + (\alpha_1 + \beta_1)\varepsilon_{t-1}^2 + v_t - \beta_1 v_{t-1}, \tag{2}$$

where $v_t = \varepsilon_t^2 - \sigma_t^2$. From the previous equation, v_t can be written as $v_t = \frac{-\alpha_0}{1-\beta_1 L} + \pi(L)\varepsilon_t^2$ with $\pi(L) = \frac{1-(\alpha_1+\beta_1)L}{1-\beta_1 L}$ and L is the lag operator. Now suppose that instead of the true series ε_t, we observe e_t defined by

$$e_t^2 = \varepsilon_t^2 + \omega_{AO}\, I_T(t),$$

where, as before, ω_{AO} represents the magnitude (or size) of the additive level outlier and $I_T(t) = 1$ for $t \in T$ and 0 otherwise, representing the presence of the outlier at a set of times T. Given this information, ϵ_t is equal to

$$\epsilon_t = v_t + \pi(L)\omega_{AO}\, I_T(t). \tag{3}$$

Equation (3) can be seen as a regression model of ϵ_t on x_t

$$\epsilon_t = \omega_{AO} x_t + v_t,$$

where

$$x_t = \begin{cases} 0, & \text{if } t < \tau, \\ 1, & \text{if } t = \tau, \\ -\pi_k, & \text{if } t > \tau + k \text{ and } k > 0. \end{cases}$$

We are assuming that there is an outlier of size ω_{AO} at time $t = \tau$. According to Franses and Ghijsels (1999) the detection of an ALO is based on the following test statistic:

$$\hat{\tau}(\tau) = \frac{\sum_{t=\tau}^{n} x_t\, \epsilon_t}{\hat{\sigma}_v \sqrt{\sum_{t=\tau}^{n} x_t^2}},$$

where $\hat{\sigma}_v$ is the estimated standard deviation of the residuals. The algorithm for the detection of an ALO outlier is then given by (see also Charles and Darné, 2005):

Step 1 Estimate a GARCH(1,1) model for e_t and obtain the estimates of the conditional variance $\hat{\sigma}_t^2$ and $\hat{\epsilon}_t = e_t^2 - \hat{\sigma}_t^2$.

Step 2 Estimate $\hat{\tau}(\tau)$ for all possible $\tau = 1, ..., n$ and calculate $\hat{\tau}_{max} = max_{1 \leq \tau \leq n} |\hat{\tau}(\tau)|$. If the value of the test statistic is greater than the critical value C, an outlier is detected at time $t = \tau$ for which $\hat{\tau}(\tau)$ is maximized.

Step 3 Replace e_t^2 by $e_t^{*2} = e_t^2 - \hat{\omega}_{AO}$ and correct the series in the following way

$$e_t^* = \begin{cases} e_t, & \text{if } t \neq \tau, \\ \text{sign}(e_t)\sqrt{e_t^{*2}}, & \text{if } t = \tau. \end{cases}$$

Step 4 Repeat steps 1 to 3 for the corrected series until no other outlier is found.

The critical value C is crucial for the good performance of this proposal. Charles (2008) chose $C = 10$ based on the simulation results by Verhoeven and McAleer (2000) and Franses and van Dijk (2002).

4 Outlier's impact on the MCRRs

In the first part of this Section we study the effect of outliers on the MCRR estimates and in the second part, we compare four different proposals for reducing the MCRR estimation biases. We start simulating return series of different sample sizes ($n = 500, 1000, 5000$) from a GARCH(1,1) model with parameters $\{\alpha_0 = 0.0126, \alpha_1 = 0.0757, \beta = 0.9122\}$, that are chosen by fitting the model to a real return series. The frequency of the simulations is daily. The outliers are placed randomly in the series and each scenario (given n and ω_{AO}) involves 1000 Monte Carlo samples. Next we describe the considered situations:

[s1] One isolated outlier of three different magnitudes ($\omega_{AO} = 5\sigma_y$, $\omega_{AO} = 10\sigma_y$, $\omega_{AO} = 15\sigma_y$) in simulated series from a Gaussian GARCH(1,1) model. For each magnitude, the sample sizes considered are $n = 500, 1000, 5000$.

[s2] Two isolated outliers of size $\omega_{AO} = 15\sigma_y$ in simulated series from a Gaussian GARCH(1,1) model of sample sizes of $n = 500, 1000, 5000$.

[s3] Simulated series from a Gaussian GARCH(1,1) model of sample sizes $n = 500, 1000, 5000$, without outliers.

Capital risk requirements, given by the percentage of the initial value of the position for 95% coverage, are estimated for 1 day investment horizon for the simulated data. Therefore, we calculate the MCRRs for each Monte Carlo sample in each scenario, in the following way (hereafter, we call this procedure **M0** method): We start by fitting a Gaussian GARCH(1,1) model to the simulated series, using the G@RCH 4.2 package by Laurent and Peters (2006), and we generate 20000 paths of future values of the price series with the help of the parameter estimates, the disturbances obtained by sampling with replacement from the iid residuals (iid bootstrap), and the one-day ahead volatility forecasts. The maximum loss over a given holding period supposing there is only one futures contract is then obtained by computing $Q = (P_0 - P_1)$, where P_0 is the initial value of the position and P_1 is the lowest simulated price (for a long position) or the highest simulated price (for a short position) over the period. We assume that the position is opened on the final day of the sample (see Brooks et al., 2000; Brooks, 2002). [1] Without loss of generality, we can write $\frac{Q}{P_0} = \left(1 - \frac{P_1}{P_0}\right)$ for a long position, and $\frac{Q}{P_0} = \left(\frac{P_1}{P_0} - 1\right)$ for a short position. Remind that since P_0 is constant, the distribution of Q only depends on the distribution of P_1. We proceed as in Hsieh (1993) and Grané and Veiga (2008) assuming that simulated prices are lognormal distributed, a frequent hypothesis in the finance literature. Consequently, the maximum loss for a long position over the simulated days is given by $Q/P_0 = 1 - \exp(c_\alpha\, s + m)$, where c_α is the $\alpha \times 100$-th percentile of the standard normal distribution and s and m are the standard deviation and mean of the $\ln(P_1/P_0)$, respectively. The analogous for a short position is given by $Q/P_0 = \exp(c_{1-\alpha}\, s + m) - 1$, where $c_{1-\alpha}$ is the $(1-\alpha) \times 100$-th percentile of the standard normal distribution (see Brooks, 2002). In Tables 1–5 we present the MCRR estimates obtained as the mean of 1000 estimated MCRR values, computed as described above, and the standard deviation. Table 1 reports the results of the estimation of the MCRRs in the situations **s1**–**s3**, where we observe that the presence of outliers biases the estimates of MCRRs and that biases increase with the outlier size. We also detect that the MCRR estimates decrease with the sample size in the presence of outliers.

Table 1: **M0** method. Monte Carlo MCRRs for 95% coverage probability as a percent of the initial value of the simulated series (standard deviation) and e_r stands for the relative error.

		Long Position		Short Position	
	n	MCRR	e_r	MCRR	e_r
1 outlier	500	1.337 (0.645)	-0.134	1.361 (0.677)	-0.134
of size	1000	1.140 (0.557)	-0.261	1.105 (0.553)	-0.298
$\omega_{AO} = 5\sigma_y$	5000	1.056 (0.539)	-0.321	1.079 (0.551)	-0.319
1 outlier	500	1.283 (0.734)	-0.169	1.306 (0.779)	-0.169
of size	1000	1.135 (0.576)	-0.264	1.100 (0.573)	-0.301
$\omega_{AO} = 10\sigma_y$	5000	1.060 (0.557)	-0.319	1.084 (0.572)	-0.316
1 outlier	500	1.197 (0.841)	-0.223	1.220 (0.909)	-0.223
of size	1000	1.114 (0.647)	-0.278	1.080 (0.645)	-0.313
$\omega_{AO} = 15\sigma_y$	5000	1.066 (0.589)	-0.315	1.090 (0.611)	-0.312
2 outliers	500	1.751 (1.350)	0.134	1.713 (1.399)	0.090
of size	1000	1.172 (0.860)	-0.240	1.163 (0.909)	-0.261
$\omega_{AO} = 15\sigma_y$	5000	1.114 (0.608)	-0.284	1.141 (0.627)	-0.280
no outliers	500	1.544 (0.585)		1.571 (0.628)	
	1000	1.543 (0.534)		1.573 (0.555)	
	5000	1.556 (0.508)		1.584 (0.526)	

[1] "A futures contract is a standardized contract to buy (long position) or sell (short position) a specified commodity of standardized quality at a certain date in the future and at a market-determined price. Many times, the underlying asset to a futures contract may not be traditional "commodities" at all, but stock market indices or interest rates, etc... " (http://en.wikipedia.org/wiki/Futures_contract).

Given the evidence of the previous simulation study, the second part of this Section consists in the comparison of four proposals for reducing the MCRR estimation biases. In all these methods, the calculation of the MCRRs is analogous to **M0** method, but instead of starting by fitting a Gaussian GARCH(1,1) model directly to the simulated series, we start by:

[M1] fitting a Gaussian GARCH(1,1) model to the outlier filtered simulated data using the procedure of Grané and Veiga (2010) with hard-thresholding,

[M2] fitting a Gaussian GARCH(1,1) model to the outlier filtered simulated data using the procedure of Grané and Veiga (2010) with soft-thresholding,

[M3] fitting a Gaussian GARCH(1,1) model to the outlier filtered simulated data using the procedure of Franses and Ghijsels (1999),[2]

[M4] fitting a t-distributed GARCH(1,1) model, with endogenous degrees of freedom, directly to the simulated data.

As a general comment to these methods, we can say that the first three consist in detecting and correcting outliers before estimating the MCRRs, whereas the fourth method is an alternative way of dealing with outliers, consisting in fitting a model that can better accommodate these observations instead of correcting for them. In fact, **M4** method uses the t-distributed GARCH, that is the simplest model of this kind that can be fitted to the simulated data (see Carnero et al. (2008) for more sophisticated robust techniques to forecast volatility using GARCH models and Park (2002) for a proposal of an outlier robust GARCH). The performances of all these methods in situations **s1**–**s3** are reported in Tables 2–5.
From Tables 2 and 3 we first observe that applying **M1** or **M2** method leads intimately to the elimination of the relative error in the computation of the MCRR estimates. We also notice that the hard-thresholding correction performs slightly better than the soft- procedure for $n = 5000$.
From Table 4 we see that **M3** method underperforms comparatively to the wavelet procedures (**M1** and **M2** methods) for all sample sizes and outlier magnitudes, except for long position, $n = 1000$ and $\omega_{AO} = 15\sigma_y$, where the relative error is slightly small than those obtained with the wavelet procedures. Finally, from Table 5 we observe that, when applying **M4** method, the biases decrease with the sample size although they are pretty large in small samples when the outlier magnitudes are moderate or large.
Overall, the simulation study seems to emphasize the importance of outlier detection and correction in the context of risk measures opening room for further research.

5 Empirical Applications

In this Section we evaluate the performance of the methods presented in Section 4, whenever it is possible, for calculating the MCRRs on series of real data. Note that,

[2]The simulation study was conducted only for $n = 500, 1000$ because we have not found critical values for $n = 5000$ in the literature.

Table 2: **M1** method. Monte Carlo MCRRs for 95% coverage probability as a percent of the initial value of the simulated series corrected for outliers (standard deviation) and e_r stands for the relative error.

		Long Position		Short Position	
	n	MCRR	e_r	MCRR	e_r
1 outlier	500	1.541 (0.837)	-0.002	1.574 (0.908)	0.002
of size	1000	1.555 (0.535)	0.008	1.585 (0.557)	0.008
$\omega_{AO} = 5\sigma_y$	5000	1.559 (0.508)	0.002	1.587 (0.528)	0.002
1 outlier	500	1.558 (0.680)	0.009	1.586 (0.729)	0.010
of size	1000	1.567 (0.546)	0.016	1.597 (0.569)	0.015
$\omega_{AO} = 10\sigma_y$	5000	1.570 (0.526)	0.009	1.599 (0.549)	0.009
1 outlier	500	1.543 (0.836)	-0.001	1.575 (0.907)	0.003
of size	1000	1.582 (0.591)	0.025	1.614 (0.621)	0.026
$\omega_{AO} = 15\sigma_y$	5000	1.570 (0.526)	0.009	1.599 (0.549)	0.009
2 outliers	500	1.692 (0.760)	0.096	1.725 (0.812)	0.098
of size	1000	1.634 (0.599)	0.059	1.688 (0.635)	0.073
$\omega_{AO} = 15\sigma_y$	5000	1.588 (0.519)	0.021	1.618 (0.540)	0.021
no outliers	500	1.544 (0.585)		1.571 (0.628)	
	1000	1.543 (0.534)		1.573 (0.555)	
	5000	1.556 (0.508)		1.584 (0.526)	

Table 3: **M2** method. Monte Carlo MCRRs for 95% coverage probability as a percent of the initial value of the simulated series corrected for outliers (standard deviation) and e_r stands for the relative error.

		Long Position		Short Position	
	n	MCRR	e_r	MCRR	e_r
1 outlier	500	1.562 (0.591)	0.012	1.590 (0.635)	0.012
of size	1000	1.556 (0.532)	0.008	1.586 (0.555)	0.008
$\omega_{AO} = 5\sigma_y$	5000	1.559 (0.508)	0.002	1.587 (0.527)	0.002
1 outlier	500	1.550 (0.722)	0.004	1.580 (0.775)	0.006
of size	1000	1.575 (0.559)	0.021	1.606 (0.586)	0.021
$\omega_{AO} = 10\sigma_y$	5000	1.566 (0.510)	0.006	1.595 (0.531)	0.007
1 outlier	500	1.532 (0.931)	-0.008	1.566 (1.017)	-0.003
of size	1000	1.586 (0.640)	0.028	1.610 (0.676)	0.024
$\omega_{AO} = 15\sigma_y$	5000	1.583 (0.532)	0.017	1.612 (0.557)	0.018
2 outliers	500	1.615 (1.222)	0.046	1.653 (1.307)	0.052
of size	1000	1.566 (0.899)	0.015	1.605 (0.993)	0.020
$\omega_{AO} = 15\sigma_y$	5000	1.623 (0.574)	0.043	1.655 (0.601)	0.045
no outliers	500	1.544 (0.585)		1.571 (0.628)	
	1000	1.543 (0.534)		1.573 (0.555)	
	5000	1.556 (0.508)		1.584 (0.526)	

although these methods have been described for simulated data, we keep the same notation while working with real data.

We start by introducing the three stock market indexes: the Dow Jones index, the FTSE-100 index and the S&P 500 index. The data was collected from Yahoo Finance (`http://finance.yahoo.com/`) and spans the period of April 2, 1984-July 29, 2008. Figure 2 depicts the three return series, $y_t = (\log p_t - \log p_{t-1}) \cdot 100$, where p_t is

Table 4: **M3** method. Monte Carlo MCRRs for 95% coverage probability as a percent of the initial value of the simulated series (standard deviation) and e_r stands for the relative error.

		Long Position		Short Position	
	n	MCRR	e_r	MCRR	e_r
1 outlier of size	500	1.530 (0.575)	-0.009	1.547 (0.606)	-0.015
$\omega_{AO} = 5\sigma_y$	1000	1.531 (0.531)	-0.008	1.559 (0.552)	-0.009
1 outlier of size	500	1.422 (0.616)	-0.079	1.437 (0.634)	-0.085
$\omega_{AO} = 10\sigma_y$	1000	1.504 (0.556)	-0.025	1.538 (0.580)	-0.022
1 outlier of size	500	1.597 (0.749)	0.034	1.607 (0.791)	0.036
$\omega_{AO} = 15\sigma_y$	1000	1.578 (0.699)	0.023	1.612 (0.775)	0.025
2 outliers of size	500	1.915 (1.986)	0.240	2.002 (2.524)	0.274
$\omega_{AO} = 15\sigma_y$	1000	1.622 (1.263)	0.079	1.698 (1.960)	0.079
no outliers	500	1.544 (0.585)		1.571 (0.628)	
	1000	1.543 (0.534)		1.573 (0.555)	

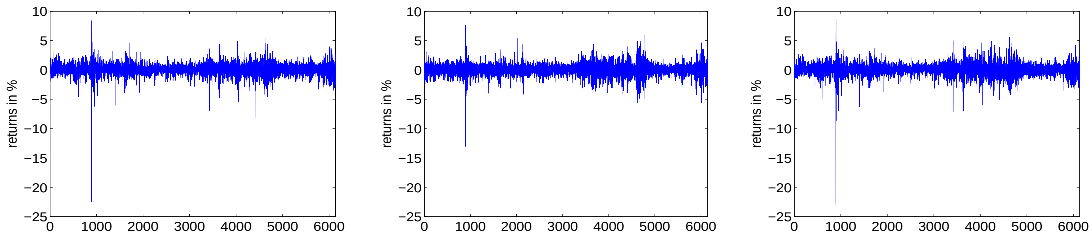
Table 5: **M4** method. Monte Carlo MCRRs for 95% coverage probability as a percent of the initial value of the simulated series (standard deviation) and e_r stands for the relative error.

		Long Position		Short Position	
	n	MCRR	e_r	MCRR	e_r
1 outlier	500	1.593 (0.611)	0.032	1.623 (0.661)	0.033
of size	1000	1.572 (0.537)	0.019	1.602 (0.560)	0.018
$\omega_{AO} = 5\sigma_y$	5000	1.596 (0.516)	0.026	1.619 (0.534)	0.022
1 outlier	500	1.714 (0.834)	0.110	1.751 (0.919)	0.115
of size	1000	1.651 (0.599)	0.070	1.684 (0.628)	0.071
$\omega_{AO} = 10\sigma_y$	5000	1.582 (0.539)	0.017	1.611 (0.565)	0.017
1 outlier	500	1.872 (1.224)	0.212	1.924 (1.419)	0.225
of size	1000	1.763 (0.739)	0.142	1.803 (0.784)	0.146
$\omega_{AO} = 15\sigma_y$	5000	1.596 (0.516)	0.026	1.619 (0.534)	0.022
2 outliers	500	2.126 (1.359)	0.377	2.188 (1.536)	0.393
of size	1000	1.932 (1.068)	0.252	1.989 (1.217)	0.264
$\omega_{AO} = 15\sigma_y$	5000	1.685 (0.645)	0.083	1.719 (0.682)	0.085
no outliers	500	1.544 (0.585)		1.571 (0.628)	
	1000	1.543 (0.534)		1.573 (0.555)	
	5000	1.556 (0.508)		1.584 (0.526)	

the value at time t of the corresponding index and Table 6 reports some descriptive statistics. From Table 6 we observe that the three return series are negatively skewed and have significant kurtosis, ranging from 10.370 for the FTSE-100 to 54.929 for the Dow Jones. High kurtosis could be caused by the presence of some outliers and some outlier detection methods, specially in the multivariate context, are based on this information (see for example Peña and Prieto, 2001; Galeano et al., 2006). Table 6 also contains the results of the Kiefer and Salmon (1983) test, which is a formal test of normality in the context of conditional heteroscedastic series.[3]

[3]The Kiefer and Salmon (1983) test is given by $KS_N = (KS_S)^2 + (KS_K)^2$, where $KS_S = \sqrt{\frac{n}{6}}\left[\frac{1}{n}\sum_{t=1}^{n} y_t^{*3} - \frac{3}{n}\sum_{t=1}^{n} y_t^{*}\right]$, $KS_K = \sqrt{\frac{n}{24}}\left[\frac{1}{n}\sum_{t=1}^{n} y_t^{*4} - \frac{6}{n}\sum_{t=1}^{n} y_t^{*2} + 3\right]$ and y_t^* are the stan-

Figure 2: Returns in percentage for (a) Dow Jones index, (b) FTSE-100 index and (c) S&P 500 index.



(a) (b) (c)

Table 6: Descriptive statistics for the daily stock index returns.

Stock index returns	Dow Jones	FTSE-100	S&P 500
Mean	0.038	0.026	0.040
Variance	1.043	1.076	1.194
Skewness	-2.413^*	-0.511^*	-2.035^*
Kurtosis	54.929^*	10.370^*	46.107^*
KS_S	-67.963	-18.050	-59.646
KS_K	744.493	135.842	653.651

5.1 Detection of outliers using wavelets

In order to check if there are some outliers in the data, we apply Grané and Veiga (2010)'s procedure to the residual series of the estimated GARCH(1,1). The proposal by Franses and Ghijsels (1999) is not used for checking the presence of outliers due to its poor performance in the simulation study and the unavailability of critical values and studies that report the method's performance (under these critical values) for sample sizes comparable to those considered in this study.

Table 7: Observations identified as possible additive level outliers for $\alpha = 0.05$ in the three series of stock market indexes.

Dow Jones	FTSE-100	S&P 500
896	3980	896
1399	4785	1399
1928		3431
3431		3643
4407		5777

Threshold value: $k_1^{0.05} = 4.3042$.

dardized returns. If the distribution of y_t^* is conditional $N(0,1)$, then KS_S and KS_K are asymptotically $N(0,1)$ and KS_N is asymptotically $\chi^2(2)$.

Table 7 contains the results of the outlier detection, using a threshold value of $k_1^{0.05} = 4.3042$ computed from 20000 Monte Carlo samples of size $n = 6100$. The observation positions presented in Table 7 are already those corresponding to real data. We observe that observations 896, 1399 and 3431 are considered outliers for the S&P 500 series. The first observation corresponds to October 19, 1987, the day that subsequently became known as "Black Monday". Both the Dow Jones and the S&P 500 lost more than twenty percent of their total value on that day. The second outlier detected corresponds to October 13, 1989, the day on which another crash occurred that was apparently caused by a reaction to a news story of a $6.75 billion leveraged buyout deal for UAL Corporation, the parent company of United Airlines, which eventually fell through. Finally, observation 3431 corresponds to October 27, 1997, when a mini crash caused by an economic crisis in Asia occurred. These same observations are also detected as outliers in the Dow Jones series. Other observations identified as outliers for the Dow Jones are, for instance, observation 4407 (September 17, 2001), which corresponds to the first day that the New York Stock Exchange opened for trading after the terrorist attack on the USA on the 11th of September, 2001. Concerning to the S&P 500, there is another observation (observation 5777) that is detected as an outlier, which corresponds to February 27, 2007, the day of the big decline in Chinese stocks and the news of the weakness in some key readings on the U.S. economy. Regarding to FTSE-100, observation 4785 corresponds to March 19, 2003, the day when the reaction of share prices in London to the expected onset of hostilities in the Gulf took place. Observation 3980 (December 31, 1999) is also considered an outlier and it corresponds to a market correction, since on December 30, 1999, the FTSE-100 reached its highest value to the date. Our procedure is quite effective in capturing the most important crashes in three important international stock markets, the New York Stock Exchange, NASDAQ and the London Stock Exchange.

5.2 Estimating the MCRRs on real data

Tables 8–10 contain the results of the calculation of MCRRs, for the three series of stock market indexes, using **M0**, **M1**, **M2** and **M4** methods. All series show larger MCRRs for short positions than for long positions and the differences increase with the investment horizon h. As an example, for the **M0** method applied to Dow Jones, approximately 2.3%, 5.1% and 6.8% of the value of a long position (as a percentage of the initial value of the position) will be enough to cover 95% of the expected losses if the position is held for 1, 5 and 10 days, respectively. The MCRRs for the same horizons but short positions are approximately 2.4%, 5.3% and 7.5%, respectively. This finding could be explained by the existence of a positive drift in the returns over the sample period, indicating that the series are not symmetric about zero. Indeed, the means of all return series are positive over the sample period.

When applying **M1** or **M4** method, we observe differences on the MCRR estimates, specially for longer investment periods. In both cases the estimates are larger than those obtained with **M0** method, but we still register differences between them. The same phenomenon was observed in the simulated study of Section 4, where we detected that the MCRR biases obtained using **M4** method were still pretty large in samples of size $n = 500, 1000$ and moderate in samples of size $n = 5000$, although

Table 8: Minimum capital risk requirements for 95% coverage probability as a percent of the initial value of the Dow Jones Index.

Long Position					Short Position				
h	**M0**	**M1**	**M2**	**M4**	h	**M0**	**M1**	**M2**	**M4**
1	2.30	2.17	2.23	2.16	1	2.40	2.27	2.33	2.25
5	5.06	4.69	4.83	4.79	5	5.32	5.11	5.18	5.03
10	6.85	6.42	6.59	6.52	10	7.53	7.34	7.39	7.15
30	10.66	10.29	10.33	10.41	30	12.77	12.91	12.69	12.45
90	13.56	13.98	13.37	14.00	90	19.97	21.70	20.30	20.61
180	13.71	14.67	13.62	14.53	180	25.78	29.00	26.36	27.39

they tended to decrease with n.

Table 9: Minimum capital risk requirements for 95% coverage probability as a percent of the initial value of the FTSE-100 Index.

Long Position					Short Position				
h	**M0**	**M1**	**M2**	**M4**	h	**M0**	**M1**	**M2**	**M4**
1	2.11	2.12	2.12	2.15	1	2.20	2.21	2.20	2.23
5	4.64	4.65	4.63	4.74	5	4.85	4.88	4.86	4.92
10	6.42	6.44	6.41	6.59	10	7.01	7.04	7.01	7.12
30	10.07	10.12	10.05	10.47	30	12.13	12.23	12.12	12.40
90	13.40	13.50	13.33	14.27	90	18.94	19.25	18.93	19.77
180	13.95	14.08	13.86	15.07	180	23.81	24.37	23.84	25.38

Table 10: Minimum capital risk requirements for 95% coverage probability as a percent of the initial value of the S&P 500 Index.

Long Position					Short Position				
h	**M0**	**M1**	**M2**	**M4**	h	**M0**	**M1**	**M2**	**M4**
1	2.16	2.07	2.11	2.13	1	2.26	2.19	2.22	2.22
5	4.73	4.53	4.62	4.67	5	4.95	4.92	4.91	4.88
10	6.56	6.29	6.44	6.51	10	7.13	7.17	7.15	7.06
30	10.73	10.44	10.62	10.92	30	12.67	13.08	12.92	12.80
90	15.46	15.80	16.02	16.72	90	22.10	24.32	23.57	23.55
180	17.22	18.88	18.90	20.07	180	29.63	35.00	33.09	33.52

5.3 Out-of-sample performance

For a full evaluation of the results, we perform out-of-sample conditional tests on the MCRRs calculated using **M0**, **M1**, **M2** and **M4** methods.
By definition, the failure rate of a model is the number of times the estimated MCRRs are inferior to the returns (in absolute value). If the MCRR model is correctly

specified, the failure rate should be equal to the pre-specified MCRR level (in our case, 5%).[4] Therefore, we calculate the MCRRs for one day horizon for both long and short positions and then check if these MCRRs are exceeded by price movements in day $t+1$. We roll this process forward and we calculate the MCRRs for 504 days. We test the proposals' performance on the most volatile period by using the first 3431 (Dow Jones and S&P 500) and 3981 (FTSE-100) observations for the estimation of the models, leaving the following 504 observations for the performance evaluation.

Table 11: Estimates of the failure rate (proportions of exceedances) obtained one day ahead. The MCRR's are computed to cover 95% of expected losses.

	Dow Jones		FTSE-100		S&P 500	
	L. Position	S. Position	L. Position	S. Position	L. Position	S. Position
M0	6.7%	4.6%	8.7%	4.2%	6.7%	5.4%
M1	6.2%	5.0%	8.7%	4.8%	5.8%	5.4%
M2	6.7%	4.8%	8.7%	4.8%	6.3%	6.0%
M4	6.5%	4.6%	8.5%	4.4%	6.0%	5.2%

In Table 11 we present the number of violations of the MCRR estimates. Both for the Dow Jones and FTSE-100, the short position MCRR number of violations (in percentage) obtained from **M0** and **M4** methods almost never exceeds the 5% nominal value. This indicates that these models generate "slight" excessive MCRRs for these series and this position when we do not correct for outliers. On the contrary, for all series and long positions these models tend to over reject. The hard-thresholding (**M1** method) presents the closest failure rate to the 5% for two out of three results, while the other procedures register only one (**M2** and **M4**) or zero favorable results (**M0**).

Since the calculation of the empirical failure rate defines a sequence of ones (MCRR violation) and zeros (no MCRR violation), we can test if the theoretical failure rate, f, is equal to 5%, i.e., $H_0 : f = 5\%$ vs. $H_1 : f \neq 5\%$. Standard evaluation of the failure rate proceeds by simply comparing the percentage of exceedances to the true failure rate. But, as was pointed out in the works by West (1996) and McCracken (2000) when parameters are estimated, parameter uncertainty can play a role in out-of-sample inference. According to Christoffersen (1998), testing for conditional coverage is important in the presence of higher order dynamics and he proposed a procedure that is composed of three tests.[5] The first tests for the unconditional coverage (denoted LR_{uc}), the second for the independence part of the conditional

[4]For a long position the failure rate is obtained as the percentage of negative returns smaller than the one day ahead MCRRs calculated for long positions. Analogously, for a short position the failure rate is estimated as the percentage of positive returns larger than the one day ahead MCRRs calculated for short positions (see Giot and Laurent, 2003, 2004).

[5]The unconditional coverage test is a standard likelihood ratio test given by

$$LR_{uc} = -2log\left[L(p; I_1, I_2, ..., I_n)/L(\hat{\pi}; I_1, I_2, ..., I_n)\right] \overset{asy}{\sim} \chi^2(1),$$

where $\{I_t\}_{t=1}^n$ is the indicator sequence, p is the theoretical coverage, $\hat{\pi} = n_1/(n_0 + n_1)$ is the maximum likelihood estimate of the alternative failure rate π, n_0 is the number of zeros and n_1 is the number of ones in the sequence $\{I_t\}_{t=1}^n$.

coverage hypothesis (denoted LR_{ind}) and the third is a joint test of coverage and independence (denoted LR_{cc}). With this complete procedure it is possible to check if the dynamics or the error distribution is misspecified or both.
Table 12 reports the results of the likelihood ratio tests for conditional coverage. Concerning to the short position the four methods pass the three tests. Regarding to the long position none of them pass the joint test of coverage and independence for the FTSE-100. Overall, **M1** method performs the best, followed by **M4** and **M2** and the poorest performance is registered by **M0**.
Hence, given the simulation and the out-of-sample results we may conclude that the detection and correction of outliers using the proposal by Grané and Veiga (2010) and hard-thresholding generates more accurate estimates of MCRRs than the proposal of fitting a model that better accommodates the outliers. By far, the worst is doing nothing.

Table 12: p-values for the null hypotheses $f = \alpha$, with α =5%. LR_{uc}, LR_{ind}, LR_{cc}, stand for the LR test of unconditional coverage, the LR test of independence and the joint test of coverage and independence, respectively.

Long Position

	Dow Jones			FTSE-100			S&P 500		
	LR_{uc}	LR_{ind}	LR_{cc}	LR_{uc}	LR_{ind}	LR_{cc}	LR_{uc}	LR_{ind}	LR_{cc}
M0	0.087	0.328	0.010	0.000	0.894	0.000	0.087	0.029	0.001
M1	0.252	0.471	0.024	0.000	0.894	0.000	0.448	0.554	0.593
M2	0.087	0.328	0.010	0.000	0.894	0.000	0.181	0.040	0.003
M4	0.127	0.941	0.020	0.001	0.816	0.000	0.340	0.055	0.006

Short Position

	Dow Jones			FTSE-100			S&P 500		
	LR_{uc}	LR_{ind}	LR_{cc}	LR_{uc}	LR_{ind}	LR_{cc}	LR_{uc}	LR_{ind}	LR_{cc}
M0	0.648	0.958	0.859	0.377	0.176	0.260	0.716	0.678	0.813
M1	0.967	0.813	0.923	0.805	0.121	0.277	0.716	0.080	0.191
M2	0.805	0.884	0.914	0.805	0.121	0.277	0.340	0.498	0.474
M4	0.648	0.958	0.859	0.504	0.156	0.280	0.871	0.092	0.227

The likelihood ratio test of independence is

$$LR_{ind} = -2log\left[L(\hat{\Pi}_2; I_1, I_2, ..., I_n)/L(\hat{\Pi}_1; I_1, I_2, ..., I_n)\right] \overset{asy}{\sim} \chi^2(1),$$

where $\hat{\Pi}_1 = \begin{bmatrix} n_{00}/(n_{00}+n_{01}) & n_{01}/(n_{00}+n_{01}) \\ n_{10}/(n_{10}+n_{11}) & n_{11}/(n_{10}+n_{11}) \end{bmatrix}$, $\hat{\Pi}_2 = \begin{bmatrix} 1-\hat{\pi}_2 & \hat{\pi}_2 \\ 1-\hat{\pi}_2 & \hat{\pi}_2 \end{bmatrix}$, n_{ij} is the number of observations with value i followed by j and $\hat{\pi}_2 = (n_{01}+n_{11})/(n_{00}+n_{10}+n_{01}+n_{11})$.

The joint test of coverage and independence is given by

$$LR_{cc} = -2log\left[L(p; I_1, I_2, ..., I_n)/L(\hat{\Pi}_1; I_1, I_2, ..., I_n)\right] \overset{asy}{\sim} \chi^2(1).$$

6 Conclusion

This paper evidences the impact of outliers on the estimation of the MCRRs using a Gaussian GARCH(1,1) model and compares four different approaches that attenuate these effects. The first three proceed by detecting and correcting outliers before estimating these risk measures with the GARCH(1,1) model while the fourth procedure fits a t-distributed GARCH(1,1) model directly to the data. The former group includes the proposals by Franses and Ghijsels (1999) and Grané and Veiga (2010) with hard- and soft-thresholding. The simulation results gather that detecting and filtering outliers decrease intimately the MCRR estimates' biases, specially when the detection and filtering is done with Grané and Veiga (2010)'s method (either with hard- or soft- thresholding). The poor performance of the proposal of Franses and Ghijsels (1999) may be due to the fact that it is based on an iterative outlier detection and filter method and throughout the iterative process the estimates of the parameters may be affected by the presence of remaining outliers. The t-distributed GARCH(1,1) model generates MCRR estimates' biases pretty large in small samples when the outlier magnitudes are moderate or large. Although these biases tend to decrease with the sample size, even for a sample size of $n = 5000$, they are larger than those obtained with the proposals of Franses and Ghijsels (1999) and Grané and Veiga (2010). The empirical application and the out-of-sample results favor the wavelet detection procedure with hard-thresholding filtering since it overperforms the other methods even in pretty volatile periods.

References

Baillie, R. and T. Bollerslev (1989). The message in daily exchange rates: a conditional variance tale. *Journal of Business and Economic Statistics 7*, 297–309.

Bilen, C. and S. Huzurbazar (2002). Wavelet-based detection of outliers in time series. *Journal of Computational and Graphical Statistics 11*, 311–327.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics 31*, 307–327.

Bollerslev, T. (1987). A conditionally heteroskedastic time series model for speculative prices and rates of return. *Review of Economic and Statistics 69*, 542–547.

Brooks, C. (2002). *Introductory Econometrics for Finance.* Cambridge, University Press.

Brooks, C., A. Clare, and G. Persand (2000). A word of caution on calculating market-based minimum capital risk requirements. *Journal of Banking and Finance 24*, 1557–1574.

Carnero, M., D. Peña, and E. Ruiz (2007). Effects of outliers on the identification and estimation of GARCH models. *Journal of Time Series Analysis 28*, 471–497.

Carnero, M., D. Peña, and E. Ruiz (2008). Estimating and forecasting GARCH volatility in the presence of outliers. *Working Paper AD-13, Ivie.*

Charles, A. (2008). Forecasting volatility with outliers in GARCH models. *Journal of Forecasting 27*, 551–565.

Charles, A. and O. Darné (2005). Outliers and GARCH models in financial data. *Economic Letters 86*, 347–352.

Chen, C. and L. Liu (1993a). Forecasting time series with outliers. *Journal of Forecasting 12*, 13–35.

Chen, C. and L. Liu (1993b). Joint estimation of model parameters and outlier effects. *Journal of American Statistical Association 88*, 284–297.

Christoffersen, P. (1998). Evaluating interval forecasts. *International Economic Review 39*, 841–862.

Doornik, J. and M. Ooms (2005). Outlier detection in GARCH models. *Tinbergen Institute, Discussion Paper*.

Droge, B. (2006). Minimax regret comparison of hard and soft thresholding for estimating a bounded mean. *Statistics & Probability Letters 76*, 83–92.

Engle, R. (1982). Autoregressive conditional heteroskedasticity with estimates of the variance of U.K. inflation. *Econometrica 50*, 987–1008.

Fox, A. (1972). Outliers in time series. *Journal of Royal Statistical Society B 34*, 350–363.

Franses, P. and H. Ghijsels (1999). Additive outliers, GARCH and forecasting volatility. *International Journal of Forecasting 15*, 1–9.

Franses, P. and D. van Dijk (2002). *Nonlinear Time Series Models in Empirical Finance*. Cambridge University Press: Cambridge, UK.

Galeano, P., D. Peña, and R. Tsay (2006). Outlier detection in multivariate time series by projection pursuit. *Journal of the American Statistical Association 101*, 654–669.

Giot, P. and S. Laurent (2003). Value-at-risk for long and short trading positions. *Journal of Applied Econometrics 18*, 641–663.

Giot, P. and S. Laurent (2004). Modelling daily value-at-risk using realized volatility and ARCH type models. *Journal of Empirical Finance 11*, 379–398.

Glosten, L., R. Jagannathan, and D. Runkle (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. *Journal of Finance 48*, 1779–1801.

Grané, A. and H. Veiga (2008). Accurate minimum capital risk requirements: A comparison of several approaches. *Journal of Banking and Finance 32*, 2482–2492.

Grané, A. and H. Veiga (2010). Wavelet-based detection of outliers in financial time series. *Computational Statistics and Data Analysis, doi:10.1016/j.csda.2009.12.010*.

Grossi, L. and F. Laurini (2009). A robust forward weighted lagrange multiplier test for conditional heteroscedasticity. *Computational Statistics and Data Analysis 53*, 2251–2263.

Hotta, L. and R. Tsay (1998). Outliers in GARCH processes. *Manuscript. Graduate School of Business, University of Chicago*.

Hsieh, D. (1993). Implications of nonlinear dynamics for financial risk management. *The Journal of Financial and Quantitative Analysis 28*, 41–64.

Kiefer, N. and M. Salmon (1983). Testing normality in econometric models. *Economics Letters 11*, 123–127.

Laurent, S. and J. Peters (2006). *G@RCH 4.2, Estimating and Forecasting ARCH Models*. Timberlake Consultants.

Ledolter, J. (1989). The effect of additive outliers on the forecasts from ARIMA models. *International Journal of Forecasting 5*, 231–240.

Lunde, A. and P. Hansen (2005). A forecast comparison of volatility models: Does anything beat a GARCH(1,1). *Journal of Applied Econometrics 20(7)*, 873–889.

McCracken, M. (2000). Robust out-of-sample inference. *Journal of Econometrics 99*, 195–223.

Park, B.-J. (2002). An outlier robust GARCH model and forecasting volatility of exchange rate returns. *Journal of Forecasting 21*, 381–393.

Peña, D. (2001). Outliers, influential observations and missing data. In D. Peña, G. Tiao, and R. Tsay (Eds.), *A Course in Time Series*, New York, pp. 136–170. Wiley.

Peña, D. and F. Prieto (2001). Multivariate outlier detection and robust covariance matrix estimation. *Technometrics 43*, 286–310.

Percival, D. and A. Walden (2000). *Wavelet Methods for Time Series Analysis*. New York: Cambridge University Press.

Sakata, S. and H. White (1998). High breakdown point conditional dispersion estimation with aplication to S&P500 daily returns volatility. *Econometrica 66*, 529–567.

Taylor, S. (1986). *Modelling Financial Time Series*. Wiley, New York.

Teräsvirta, T. (1996). Two stylized facts and the GARCH(1,1) model. *Working Paper 96, Stockholm School of Economics*.

Teräsvirta, T. (2006). An introduction to univariate GARCH models. *SSE/EFI Working Papers in Economics and Finance* (646).

Tsay, R. (2005). *Analysis of Financial Time Series*. New Jersey: John Wiley & Sons, Inc.

Van Dijk, D., P. Franses, and A. Lucas (1999). Testing for ARCH in the presence of additive outliers. *Journal of Applied Econometrics 14*, 539–562.

Verhoeven, P. and M. McAleer (2000). Modelling outliers and extreme observations for ARMA-GARCH processes. Working paper, University of Western Australia.

West, K. (1996). Asymptotic inference about predictive ability. *Econometrica 64*, 1067–1084.

Zhang, X. (2004). Assessment of local influence in GARCH processes. *Journal of Time Series Analysis 25*(2), 301–313.