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An Algebraic Analysis using Matrix Padé Approximation to Improve the Choice of Certain Parameter in Scalar Component Models

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Abstract

This paper presents an algebraic analysis using Matrix Padé Aproximation to improve the identification stage of the proposal in [6] on Scalar Component Models, specifically as it refers to the choice of a parameter they denote h. The original methodology in [6] is based on the construction and interpretation of a table whose elements are related to the singular value zero of certain relevant matrices in the process. We propose the alternative use of what we call a Ranks Table and the sure overall orders concept instead of the so-called overall orders. Ranks Table information allows for the improved interpretation and implication of the results and of potential computational and statistical properties.

Keywords: VARMA models, Scalar Component Models (SCM), Identification Stage, Sure overall orders, Corank and Rank Tables, Matrix Padé Approximation

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1. Introduction

Vector Autoregressive Moving Average (VARMA) models are used in multivariate time series analysis. Specifically, [6] proposed their original Scalar Component Model (SCM) methodology, the usefulness of which is widely acknowledged. Recently, [1] propose an extension to this methodology. They leave some matters unresolved. Mainly, we take an algebraic approach to the initial step in their procedure to resolve the uncertainty in the choice of a certain parameter h (which controls the dimension of relevant matrices and the choice of overall orders). Moreover we consider their tables, we study their algebraic properties and implications in depth so as to obtain further information to improve the initial step of [6] methodology.

2. Definitions and Notations

This section summarizes some of the results from [6] on which our analysis is based. More precisely, it consists of a full description of the table which they propose [5, pp.166-171] for identifying a VARMA pair of overall orders.

Consider a k-dimensional process $z_t=(z_{1t}, z_{2t}, \dots, z_{kt})'$ following the VARMA(p,q) model

$$\phi(B)z_t = \theta(B)a_t \tag{1}$$

where $\phi(B)=I-\phi_1B-\dots-\phi_pB^p$, $\theta(B)=I-\theta_1B-\dots-\theta_qB^q$, ϕs and θs are kxk matrices, B is the usual backshift operator and a_k is a sequence of independent k-variate random vectors with mean zero and definite covariance matrix Σ. *Exchangeable models* are special features of vector time series that do not occur in the univariate case. Two VARMA models are exchangeable if they are of finite order and give the same probability distribution of z_t. Since they have the same probability distribution, they have the same covariance structure and provide the same inference. The possibility of multiple-model representations with the same pair of minimum orders (p,q) gives rise to *the problem of identifiability* of a VARMA model. This identifiability problem has been discussed extensively in the literature without being fully resolved. Given a pair of minimum orders (p,q), knowing for certain that the model proposed is identifiable and that its parameters $\phi_1, \phi_2, ..., \phi_p, \theta_1, \theta_2, ..., \theta_q$, can be estimated would be a considerable contribution.

Definition 1 ([6]).- Given the VARMA(p,q) model (1), we say that a non-zero linear combination $y_{it}=v_0' z_t$, where v_0 is a k-vector, follows a *Scalar Component Model* with orders (p_i,q_i), $y_{it}\sim$ SCM(p_i,q_i), if v_0 has the properties: $v_0'\phi_{p_i} \neq 0'$ where $0\le p_i\le p$; $v_0'\phi_j = 0'$ for $j=p_i+1,...,p$; $v_0'\theta_{q_i} \neq 0'$ where $0\le q_i\le q$ and $v_0'\theta_j = 0'$ for $j=q_i+1,...,q$. Since $v_0'\phi(B)z_t = v_0'\theta(B)a_t$ the structure of y_{it} can be written as

$$y_{it} + \sum_{j=1}^{p_i} v_j' z_{t-j} = v_0' a_t + \sum_{j=1}^{q_i} h_j' a_{t-j} \text{ where } v_j' = -v_0'\phi_j \text{ and } h_j' = -v_0'\theta_j .$$

The notion of SCM is of enormous benefit because the effect is a reduction in the number of parameters in the VARMA representation. Since the choice of components, their orders and their SCM structures are not unique, Tiao and Tsay's (1989) goal is to obtain components which have the following minimal order property.

Definition 2.- Let y_{it} follow the SCM(p_i,q_i) structure and write $o_i=p_i+q_i$. Let $OR(y_t)=\{o_{(1)}\le o_{(2)}\le ... \le o_{(k)}\}$ be the set of orders o_is. We say that a vector of k linearly independent (l.i.) scalar components y_t is of *minimal order* if there exists no other vector of k l.i. components y_t^* with $OR(y_t^*) = \{o_{(1)}^* \le \ldots \le o_{(k)}^*\}$ such that $o_{(i)}^* \le o_{(i)}$ for $1\le i\le k$ and a strict inequality holds for some i.

Definition 3.- Each set of k l.i. SCMs with orders (p_i,q_i), i=1,2,...,k, represents the process and *gives rise* to the pair of *overall orders* (p,q) where $p=\max\{p_i\}$, $q=\max\{q_i\}$.

2.1. The Difference in Corank

[6] use properties of autocovariance matrices of z_t as a basic tool for finding SCMs. The rank properties and the eigenstructure of the sample covariance matrices are the mathematical tools they use to justify their procedure.

Let A be an rxs real matrix and x be an s-dimensional vector (in [6] r≥s but in this paper it is possible that r<s). We say that x is a *right singular vector for the singular value of A* if Ax=0. Rank(A)=s-v where v is the corank of A (we have not taken the terminology from [6] for x (a right vector corresponding to a zero of A) and v (the number of zeros associated with l.i. right vectors of A), but used instead an accepted well-known alternative from linear algebra). Next, for h≥0, m≥0, j≥0, let

$$\Gamma(m,h,j)=\begin{pmatrix} \Gamma_{j+1} & \Gamma_j & \cdots & \Gamma_{j+1-m} \\ \Gamma_{j+2} & \Gamma_{j+1} & \cdots & \Gamma_{j+2-m} \\ \vdots & \vdots & \cdots & \vdots \\ \Gamma_{j+1+h} & \Gamma_{j+h} & \cdots & \Gamma_{j+1+h-m} \end{pmatrix}$$

be the k(h+1)xk(m+1)-dimensional matrix where $\Gamma_i=E(z_{t-i}\, z_i')$ is the lag i autocovariance matrix of z_t. Given h, [6] define D(m,h,j) to be: a) the corank of Γ(m,h,j) for m=0, j>0 or m≥0, j=0; b) the diagonal difference in corank of Γ(m,h,j) and Γ(m-1,h,j-1) for m≥1 and j≥1. Given h, they arrange D(m,h,j) in a two-way table according to (m,j), m≥0, j≥0. We call it the *Incremental Corank Table*. If the process can be represented by a set of k l.i. SCM(p_i,q_i), i=1,2,...,k, having the minimal order property, p=max{p_i}, q=max{q_i} and it does not have any other *non-nested* exchangeable representations (the orders (p,q) and (s,r) of two VARMA exchangeable representations are said to be *non-nested* if either (p<s, q>r) or (p>s, q<r)). [6] claim that for any h≥m, D(m,h,j) $\begin{cases} =k & \text{if } m \ge p, j \ge q \\ <k & \text{otherwise} \end{cases}$. If the process has another VARMA(s,r) exchangeable representation whose SCMs also have the minimal order property, (p,q) and (s,r) being *non-nested* orders, then

$$D(m,h,j) = \begin{cases} =k & \text{if}\,(m \ge p, j \ge q) \cup (m \ge s, j \ge r) \\ <k & \text{otherwise} \end{cases} \tag{2}$$

This can be generalized to the case in which z_t has more than two minimal exchangeable representations. Thus, they affirm that this table makes it possible to identify a VARMA overall order of z_t by searching for a lower right rectangular pattern of k in each place (m,j) for m≥p, j≥q.

3. Algebraic and Conceptual Analysis

Two examples will illustrate the need for a more in-depth analysis of the different types of SCM representations. To calculate Γ_h we have taken into account that, *if z_t is stationary,* z_t=W(B)a_t , *where* $W(B) \equiv \phi^{-1}(B)\theta(B) \equiv \sum_{j=0}^{\infty} W_j B^j$, $W_0 = I$, W_j *being a* $k \times k$ *matrix for* $j=0,1...$

and $\Gamma_h = \sum_{j=0}^{\infty} W_j \Sigma W_{j-h}'$ for any integer h , where $W_j=0$ if $j<0$, [5]. Note that, in Example 1, $W_j=0$ if $j>4$ and, in Example 2, $W_j=0$ if $j>5$.

Example 1: $z_t + \begin{pmatrix} 0 & 0 \\ -1/2 & 0 \end{pmatrix} z_{t-1} = a_t + \begin{pmatrix} 0 & 0 \\ -1/2 & 0 \end{pmatrix} a_{t-1} + \begin{pmatrix} 1/2 & 1/4 \\ 0 & 0 \end{pmatrix} a_{t-2}$

Example 2: $z_t + \begin{pmatrix} -1 & -1 \\ 1 & 1 \end{pmatrix} z_{t-4} = a_t$

If the table is built up with the values D(m,m,j), i.e. h=m, for m≥0, j≥0, the results will be, respectively:

m \ j	0	1	2	3	4	5
0	1	1	1	2	2	2
1	0	0	1	2	2	2
2	0	1	2	2	2	2
3	0	1	2	2	2	2
4	0	1	2	2	2	2
5	0	1	2	2	2	2

m \ j	0	1	2	3	4	5
0	2	2	2	1	2	2
1	2	2	1	0	2	2
2	2	1	0	0	2	2
3	1	0	0	0	2	2
4	2	2	2	2	2	2
5	2	2	2	2	2	2

Considering the first table (Example 1), note that z_{1t}~SCM(0,2) with $v_0=(1\ 0)$, z_{2t}~SCM(1,1) with $v_0=(0\ 1)$, z_t is of minimal order and has no non-nested exchangeable representation, p=1 and q=2. However, it can be seen that (2) does not hold because D(1,1,2)=1≠k.

Considering the second table (Example 2), note that, for instance, if (m,m,j)∈{(0,0,0), (0,0,1), (0,0,2), (1,1,0), (1,1,1), (2,2,0)}, then D(m,m,j)=k. This shows that, in this example, (2) does not hold.

[6] assume that h≥m, and they suggest that a high value of h is preferred. In practice, h controls the dimension of Γ(m,h,j), the effective sample size in the estimation, etc. Thus, a low value of h would reduce computation. As a compromise, [6] suggest using h=m in this first stage of the analysis to reduce computation. However, they note that this choice of h risks underspecifying MA models in the presence of skipping MA lags, e.g. seasonal MA models. Other values of h may be used when skipping lag is likely to occur. In searching for minimal order SCMs, it is

reasonable to let h be guided by the specified overall order (p,q). In general, [6] use h=m+q-j at the (m,j) position. We will see that this is not the best approach.

In [6], several experts on the subject comment at length on the topics covered in the article. We note in particular, as it is central to our study: the uncertainty in selecting the value of h. For instance, Priestley comments in [6]: "Of further interest is the question about how robust the identification procedure is with regard to the choice of m and h. (...) Is varying the value of m and/or h likely to result in substantially different orders for the component models?". The first step needed to analyze these aspects in depth is to detect and outline the definitions which are crucial to this analysis. As previously noted, the concept of overall order, as it is understood in [6], is "unsure" in the sense that sometimes it can be smaller than required and non optimum in the sense that sometimes it is larger than necessary. It seems logical to propose the following definition. Its improvements, though subtle, are important both in theory and in practice.

Definition 4 ([4]).- We say that (s,r) is a pair of *sure overall orders* if and only if

$$\text{rank}\,\Gamma(s-1,s-1,r-1)=\text{rank}\,\Gamma(s+u,s+u,r+u) \quad \forall\ u\geq 0$$

Definition 5.- We say that the k(p+1)-dimensional vector $v'=(v_0'...v_p')$, *gives rise to* a SCM(p,q) for z_t if $v_0\neq 0$ and $\exists$ h_1, h_2,...,h_q such that $y_t=v_0'z_t$ satisfies: y_t +

$$\sum_{j=1}^{p} v_j' z_{t-j} = v_0' a_t + \sum_{j=1}^{q} h_j' a_{t-j}\ .$$

Proposition 1.- If (s,r) is a pair of *sure overall orders*, then:

a) $\exists$ $v' = (v_0' \ldots v_s')$, with $v_0\neq 0$, such that $\Gamma(s,h,r)v=0$ $\forall h\geq s-1$. That is, $\exists$ $v' = (v_0' \ldots v_s')$, with $v_0\neq 0$, such that v is a right singular vector for the singular value 0 of $\Gamma(s,h,r)$, $\forall h\geq s-1$.

b) Given $v' = (v_0' \ldots v_s')$, with $v_0\neq 0$, v is a right singular vector for the singular value 0 of $\Gamma(s,s-1,r)$ $\Leftrightarrow$ v *gives rise* to a SCM(s,r).

c) Given V_0 / $|V_0|\neq 0$, $\exists$(p,q) / (p,q)$\leq$(s,r) such that $y_t=V_0z_t$ can be represented as k l.i. SCMs with overall orders (p,q).

Proof: a) (s,r) is a pair of *sure overall orders* $\Rightarrow$ rank Γ(s-1,s-1,r-1)=rank Γ(s+u,s+u,r+u), $\forall$ u≥0 $\Rightarrow$ the last k(u+1) columns of Γ(s+u,s+u,r+u) are linearly dependent (l.d.) on its first ks columns and the last k(u+1) rows of Γ(s+u,s+u,r+u) are l.d. on its first ks rows $\Rightarrow$ rank Γ(s-1,s-1,r-1)= rank Γ(s,s-1,r) = rank Γ(s,s+u,r), $\forall$u≥0.

Since rank Γ(s-1,s-1,r-1) = rank Γ(s,s-1,r), according to the Rouché Frobenius Theorem, the system

$$\Gamma_{i-1}\phi_1 + ... + \Gamma_{i-s}\phi_s = \Gamma_i \qquad i=r+1,r+2,\ldots,r+s \tag{3}$$

has a solution. Besides, since rank Γ(s-1,s-1,r-1) = rank Γ(s,s+u,r), $\forall$u≥0, any solution of (3) is also a solution of $\Gamma_{i-1}\phi_1 + ... + \Gamma_{i-s}\phi_s = \Gamma_i$, $\forall$i≥r+1. Considering, for instance, v_0=(-1 0 ... 0)' and v_j the first column of ϕ_j for j=1,...,s, we have at least one v / Γ(s,h,r)v=0, $\forall$h≥s-1.

b) Given (s,r), a pair of *sure overall orders*, v, with $v_0 \neq 0$, is a right singular vector for the singular value 0 of Γ(s,s-1,r) $\Leftrightarrow$ Γ(s,s-1,r)v=0 $\Leftrightarrow$ Γ(s,h,r)v=0, $\forall$h≥s-1 $\Leftrightarrow$ $\Gamma_{i-1}v_1 + ... + \Gamma_{i-s}v_s = -\Gamma_i v_0$, $\forall$i≥r+1 $\Leftrightarrow$ v *gives rise* to a SCM(s,r) for z_t.

c) Taking into account the proof of a), (s,r) is a pair of *sure overall orders* $\Leftrightarrow$ any solution of (3) is also a solution of $\Gamma_{i-1}\phi_1 + ... + \Gamma_{i-s}\phi_s = \Gamma_i$, $\forall$i≥r+1 $\Leftrightarrow$ $\exists\theta_1,...,\theta_q$ such that z_t has a VARMA(s,r) representation, $z_t - \phi_1 z_{t-1} - ... - \phi_s z_{t-s} = a_t - \theta_1 a_{t-1} - ... - \theta_r a_{t-r} \Leftrightarrow \forall V_0 \,/\, |V_0| \neq 0$, $V_0 z_t - V_0\phi_1 z_{t-1} - ... - V_0\phi_s z_{t-s} = V_0 a_t - V_0\theta_1 a_{t-1} - ... - V_0\theta_r a_{t-r}$.

Given that a V_0 / $V_0\phi_j=0$ for j=p+1,...,s and $V_0\theta_i=0$ for i=q+1,...,r, could exist, we can affirm that $\exists$(p,q) / (p,q)≤(s,r) such that $y_t=V_0 z_t$ can be represented as k l.i. SCMs with overall orders (p,q).
□

Corollary 1: If (s,r) is a pair of sure overall r then the optimum value of h is s-1.

Proof: From Proposition 1, b), all the possible SCMs with overall orders (s,r) can be obtained from the system Γ(s,s-1,r)v=0. □

The following result tries to account for a process where $\exists$ k l.i. SCMs with overall orders (p,q) and (p,q) is not a pair of *sure overall orders*. This situation arises, for instance, in Example 1 considering (p,q)=(1,2).

Proposition 2.- If ∃ k l.i. SCMs with overall orders (p,q) and (p,q) is not a pair of *sure overall orders*, then:

a) ∃ (s,r) with $s \geq p$, $r \geq q$ / (s,r) is a pair of *sure overall orders* and moreover

b) v, with $v_0 \neq 0$, is a right singular vector for the singular value 0 of $\Gamma(p,r+s-q-1,q)$ ⇔ v gives rise to a SCM(p,q).

Proof: From Theorem 2 and Proposition 2 in [3] -given in the field of Matrix Padé Approximation-, if z_t follows a VARMA model then there exists at least one pair of *sure overall orders* for which a) holds –in practice take into account Property 1 of Ranks Table in the next section-. Indeed, we can choose a pair (s,r) with minimum s+r. From the proof of Proposition 1, (s,r) is a pair of *sure overall orders* if and only if the system $\Gamma(s,s-1,r)v=0$ is equivalent to the system $\Gamma(s,s+u,r)v=0$, $\forall u \geq 0$. Any $v'=(v_0' \ldots v_p')$, with $v_0 \neq 0$, that *gives rise* to a SCM(p,q) satisfies

$$\Gamma_i v_0 + \Gamma_{i-1} v_1 + ... + \Gamma_{i-p} v_p = 0 \qquad \forall i \geq q+1 \tag{4}$$

therefore $v^{*\prime}=(v_0' \ldots v_s' \ 0')$ -where 0 is a column with k(p-s) zeros- is a solution of

$$\Gamma_i v_0 + \Gamma_{i-1} v_1 + ... + \Gamma_{i-s} v_s = 0 \qquad \forall i \geq r+1.$$

Consequently, if v is a solution of $\Gamma(p,r+s-q-1,q)v=0$ then v* is a solution of (4). Therefore, v, with $v_0 \neq 0$, is a right singular vector for the singular value 0 of $\Gamma(p,r+s-q-1,q)$ ⇔ v gives rise to a SCM(p,q). □

Corollary 2: If (p,q) is not a pair of sure overall orders and we choose a pair of sure overall order (s,r) with s+r minimum, $s \geq p$, $r \geq q$, then the optimum value of h is r+s-q-1.

Proof: Indeed, all the possible SCMs with overall orders (p,q) can be obtained by solving the system $\Gamma_{i-1}\phi_1 + ... + \Gamma_{i-s}\phi_s = \Gamma_i$, for i=q+1, q+2,…,r+s without loss of generality. We have considered $V_0=I$-. □

Note that with Proposition 1 and 2 and their Corollaries we have certainty in the choice of h.

4. The Pattern of the Ranks Table

Taking into account the above propositions and [3], it is useful to define a table, *Ranks Table*, arranging rank $\Gamma(i-1,i-1,j-1)$ in each cell (i,j) for $i\geq 0$, $j\geq 0$. By convention, rank $\Gamma(-1,-1,j-1)=0$, $\forall j\geq 0$. Note that in each cell the dimension of the matrix involved is lower than the dimension of the corresponding matrix in the *Incremental Corank Table*. We denote by $T(i,j)$ the value of cell (i,j) of *Ranks Table* and by R the set of *sure overall orders*, that is, $R=\{(i,j) \,/\, T(i,j)=T(i+u,j+u), \forall u\geq 0\}$.

Ranks Table Properties.- The following properties are obtained keeping in mind the main results previously expounded, as well as the properties and propositions in [3].

Property 1.- $(s,r)\in R \Leftrightarrow$ (s,r) is a pair of *sure overall orders* $\Leftrightarrow$ any (u,v) / $u\geq s$, $v\geq r$ is a pair of *sure overall orders* $\Leftrightarrow$ if $u\geq s$ and $v\geq r$ then $(u,v)\in R$

As a consequence of Property 1 we can mark a rectangle whose upper left corner is the cell (s,r) / $T(i,j)=T(i+u,j+u)$, $\forall u\geq 0$, $\forall (i,j)\geq(s,r)$.

Property 2.- There exist k l.i. SCMs with overall orders (0,q) $\Leftrightarrow$ (0,q) is a pair of *sure overall orders* and (0,q-1) is not $\Leftrightarrow$ $(0,q)\in R$ and $(0,q-1)\notin R$. Moreover these SCMs are identifiable.

Note that with Property 2, the *Ranks Table* allows us to affirm whether the model is a pure VMA or not, in contrast to the situation in [6].

Property 3.- There exist k l.i. SCMs with overall orders (p,0) $\Leftrightarrow$ (p,0) is a pair of *sure overall orders* and (p-1,0) is not $\Leftrightarrow$ $(p,0)\in R$ and $(p-1,0)\notin R$. Moreover these SCMs are identifiable.

Property 4.- If $T(p,q)=pk$ and $(p,q)\notin R$ then k l.i. SCMs with overall orders (p,q) do not exist.

Property 5.- If $T(p,q-1)<T(p,q)$ then k l.i. SCMs with overall orders (p,q-1) do not exist.

Property 6.- If $(s,r)\in R$, $(s-1,r-1)\notin R$ and $T(s,r)=sk$, then k l.i. SCMs with overall orders (s-1,r-1) do not exist.

Property 7.- If $(s,r)\in R$ and $T(s,r)=sk$ then $\forall V_0$ such that $|V_0|\neq 0$, the k l.i. SCMs $y_t=V_0z_t$ are identifiable considering that their overall orders are (s,r). (Note that y_t could have other exchangeable representations but with different pairs of overall orders).

Property 8.- If (s,r)∈*R*, ∃ k l.i. SCMs y_t with overall orders (p,q) / (p,q)≤(s,r), ∃ u / (p+u,q+u)∈*R* and *T*(p+u,q+u)=pk then the k SCMs y_t are identifiable considering that their overall orders are (p,q).

Property 9.- Suppose (p,q)∉*R*. If ∃ (s,r)∈*R* such that (p,q)≤(s,r), ∃ u / (p+u,q+u)∈*R*, then:

∃ k l.i. SCMs with overall orders (p,q) ⇔ rank Γ(p-1,r+s-q,q-1)=*T*(p+u,q+u).

Note: to apply this property it is preferable to choose (s,r) such that s+r is minimum, satisfying the hypothesis in Property 9.

The *Ranks Table* for Examples 1 and 2 are, respectively, the following:

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	2	1	1	1	0	0
2	4	4	3	2	1	0
3	6	6	5	3	2	1
4	8	8	7	5	3	2
5	10	10	9	7	5	3

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	2	0	0	0	1	0
2	4	2	0	1	2	1
3	6	4	3	2	3	2
4	8	7	6	5	4	3
5	10	8	7	6	5	4

Considering the first table (Example 1), from Property 2, there exist k l.i. SCMs with overall order (0,3), which are identifiable. However, we should like to know if there exist k l.i. SCMs with overall orders (1,2). From Property 9, considering (s,r)=(2,2), (p,q)=(1,2) and u=1, we calculate rank Γ(0,2,1)=2. Therefore there exist k l.i. SCMs with overall orders (1,2) because the value in position (2,3) is 2. From Property 8, they are identifiable.

Considering the second table (Example 2), from Property 2 there exist k l.i. SCMs with overall order (0,4), which are identifiable. From Property 3 there exist k l.i. SCMs with overall order (4,0), which are identifiable. From Property 5, considering (p,q)=(3,4), k l.i. SCMs with overall order (3,3) do not exist; therefore, for this process, Property 9 is not necessary.

5. Conclusions

This paper resolves the theoretical uncertainty about the choice of the parameter h in [6]. The definition of a pair of *sure overall orders* -instead of a pair of *overall orders*- and the construction of the *Ranks Table* –instead of the *Incremental Corank Table*- improve the interpretation of the results, add useful properties and reduce the computation required thanks to a reduction in the dimension of the matrices involved. As a consequence, statistical properties are also potentially improved, although estimated rank tables (Incremental Corank Table and Ranks Table) is not a straightforward procedure.

As an extension of this paper, there is a vast amount of literature connecting the type of structured matrices in this paper to rational approximation, matrix orthogonal polynomials, linear system theory, system identification, etc. which could shed more light on successfully completing the proposal in [6], similar to the approach we have taken in our work. See, for instance [2] and the references therein.

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