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Subgame-perfect free trade networks in a four-country model

Masaki Iimura
Graduate School of Humanities and Social Sciences, University of Tsukuba

Tatsuhiro Shichijo
Department of Economics, Osaka Prefecture University

Toru Hokari
Faculty of Economics, Keio University

Abstract

Goyal and Joshi (2006, Int Econ Review) apply the notion of ``pairwise stable networks'' introduced by Jackson and Wolinsky (1996, J Econ Theory) to a model of free trade network formation, and show that (i) every pairwise stable network is either complete or almost complete (with all countries except one forming direct links), and (ii) the complete network maximizes global welfare. In this note, we use essentially the same model as their model with four countries, and investigate which network is more likely to be realized than others by considering a dynamic process introduced by Jackson and Watts (2002, J Econ Theory).

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1 Introduction

Goyal and Joshi (2006) apply the notion of *pairwise stable networks* introduced by Jackson and Wolinsky (1996) to a model of free trade network formation, and show that (i) every pairwise stable network is either complete or almost complete (with all countries except one forming direct links), and (ii) the complete network maximizes global welfare. In this note, we use essentially the same model as their model with four countries, and investigate which network is more likely to be realized than others by considering a dynamic process introduced by Jackson and Watts (2002).[1]

2 The model

There are four countries, each of which has one firm producing a homogeneous good. Each firm sells in the domestic market as well as in the foreign markets. Let $p = a - q$ be the inverse demand function in each market, where p is a price and q is a market supply. For simplicity, we assume that the production cost is zero. If two countries are connected by a direct link, the transportation cost is zero. If they are not directly connected, the per-unit transportation cost (= tariff) is τ. Given a configuration of such a free trade network, the firms compete in each market in a Cournot fashion. Each country's welfare is the sum of the consumer's surplus in the domestic market, the tariff revenue, and the total profit of the domestic firm. There are eleven patterns of networks. Assuming that $a > 4\tau$, the welfare of each country in each network configuration is shown in Figures 1.

We assume that the formation of a link requires the consent of both parties involved, but severance can be done unilaterally. Assuming that each country is myopic, a network is **pairwise stable** (Jackson and Wolinsky 1996) if (i) no pair of countries want to form a new link between them, and (ii) no country wants to sever any single direct link. Whether a particular network is pairwise stable and/or Pareto optimal depends on the relative values of a and τ as shown Table 1.

Let $N \equiv \{1, 2, 3, 4\}$ be the set of countries. For each pair $i, j \in N$, let ij denote the link between them. We do not distinguish ij and ji. A **network** on N is a set of links. Let $\mathcal{G}_N$ denote the set of all

[1] Iimua, Murakoshi, and Hokari (2007) conduct a similar exercise for a model of market sharing agreements (Belleflamme and Bloch 2004) with three firms.

Conditions on a and τ	pairwise stable	Pareto optimal
$\tau < \frac{2a}{19}$	(6)	(3b), (3c), (4b), (5), (6)
$\frac{2a}{19} < \tau < \frac{2a}{9}$	(6)	(2b), (3b), (3c), (4b), (5), (6)
$\frac{2a}{9} < \tau < \frac{10a}{47}$	(6)	(2b), (3a), (3b), (3c), (4b), (5), (6)
$\frac{10a}{47} < \tau < \frac{6a}{25}$	(3b), (6)	(2b), (3a), (3b), (3c), (4b), (5), (6)
$\frac{6a}{25} < \tau < \frac{a}{4}$	(3b), (6)	(2b), (3a), (3c), (4b), (5), (6)

Table 1: Pairwise stable and Pareto optimal networks.

networks on N. For each $g \in \mathcal{G}_N$ and each $i \in N$, let $u_i(g)$ denote the welfare of country i in network g, as described in Figure 1. For each $g \in \mathcal{G}_N$ and each $ij \in g$, let $g - ij$ denote the network generated by deleting ij from g. For each $g \in \mathcal{G}_N$ and each $ij \notin g$, let $g + ij$ denote the network generated by adding ij to g.

Let us consider the following discrete-time dynamic process introduced by Jackson and Watts (2002). At each period $t \in \{1, 2, \ldots, T\}$, two countries are chosen randomly. If they are already linked directly, they can decide whether to keep the link or sever it. If they are not linked directly, they can decide whether to form a new link between them.[2]

Assuming that each country is myopic, one can compute the transition probabilities in each period (Figure 2). Also, assuming that the process starts with the empty network, one can compute the probability that each network is realized in the beginning of each period. For each $t \in \{2, 3, \ldots, T\}$ and each $g \in \mathcal{G}_N$, let $p^t(g)$ denote the probability that network g is realized in the beginning of period t. Then

$$\begin{aligned} p^t(g) \quad = \quad & \frac{1}{6} \sum_{ij \in g} \left[p^{t-1}(g - ij) + p^{t-1}(g)\right] \times \texttt{AND}\left(u_i(g) \geq u_i(g - ij), u_j(g) \geq u_j(g - ij)\right) \\ & + \frac{1}{6} \sum_{ij \notin g} \left[p^{t-1}(g + ij) + p^{t-1}(g)\right] \times \texttt{OR}\left(u_i(g) > u_i(g + ij), u_j(g) > u_j(g + ij)\right), \end{aligned}$$

where AND and OR are functions such that for each pair of condi-

[2]Dutta, Ghosal, and Ray (2005) study an infinite-horizon dynamic process similar to that of Jackson and Watts (2002) assuming that each player is farsighted. In their setting, when a pair of players is selected, each of them has an additional option of severing existing links with *other players* unilaterally. For simlicity, we do not incorporate such a feature into our model.

	(3b)	(6)
$T=10$	0.389106	0.112912
$T=20$	0.429438	0.472647
$T=30$	0.429629	0.554233
$T=40$	0.429630	0.567758
$T=50$	0.429630	0.569948

Table 2: Probabilities with which networks (3b) and (6) are realized in the final period when each country is myopic and $\frac{10a}{47} < \tau < \frac{a}{4}$.

tions A and B,

$$\begin{aligned} \texttt{AND}(A,B) &\equiv \begin{cases} 1 & \text{if both } A \text{ and } B \text{ are true,} \\ 0 & \text{otherwise,} \end{cases} \\ \texttt{OR}(A,B) &\equiv \begin{cases} 1 & \text{if either } A \text{ or } B \text{ is true,} \\ 0 & \text{otherwise.} \end{cases} \end{aligned}$$

Let us assume that $\frac{10a}{47} < \tau < \frac{a}{4}$. Then (3b) and (6) are pairwise stable. We can use the above dynamic process to see which one is more likely to be realized. Table 2 summarizes the result of computation. We can see that (6) is more likely to be realized than (3b), but the probability with which (3b) is realized is not negligible.

3 Subgame-perfect networks

Next, we assume that each country is farsighted in the sense that each country maximizes the expected value of the sum of discounted payoffs, with a common discounting factor $\beta \in (0,1]$. Then the above dynamic process defines an extensive form game. Assuming that T is finite, we use backword induction to find a subgame-perfect equilibrium of this game. Although there are many subgame-perfect equilibria, we are interested in the one in which two countries form a new link whenever it is profitable for both to do so.

For each $t \leq T$, each $g \in \mathcal{G}_N$, and each $i \in N$, let $V_i^t(g)$ denote a subgame-perfect equilibrium payoff to i in the subgame starting from period t with network g. Note that for each $g \in \mathcal{G}_N$ and each $i \in N$, $V_i^T(g) = u_i(g)$. For each $t \leq T-1$, each $g \in \mathcal{G}_N$, and each $i \in N$, the Bellman equation can be written as

$$\begin{aligned}
V_i^t(g) &= u_i(g) \\
&+\frac{\beta}{6}\sum_{jk\in g} V_i^{t+1}(g) \times \texttt{AND}\left(V_j^{t+1}(g-jk) \geq V_j^{t+1}(g), V_k^{t+1}(g-jk) \geq V_k^{t+1}(g)\right) \\
&+\frac{\beta}{6}\sum_{jk\in g} V_i^{t+1}(g-jk) \times \texttt{OR}\left(V_j^{t+1}(g-jk) > V_j^{t+1}(g), V_k^{t+1}(g-jk) > V_k^{t+1}(g)\right) \\
&+\frac{\beta}{6}\sum_{jk\notin g} V_i^{t+1}(g+jk) \times \texttt{AND}\left(V_j^{t+1}(g+jk) \geq V_j^{t+1}(g), V_k^{t+1}(g+jk) \geq V_k^{t+1}(g)\right) \\
&+\frac{\beta}{6}\sum_{jk\notin g} V_i^{t+1}(g) \times \texttt{OR}\left(V_j^{t+1}(g+jk) < V_j^{t+1}(g), V_k^{t+1}(g+jk) < V_k^{t+1}(g)\right).
\end{aligned}$$

After solving these equations backwardly, assuming that the game starts with the empty network, one can compute the probability that each network is realized in the beginning of each period. For each $t \in \{2, 3, \ldots, T\}$ and each $g \in \mathcal{G}_N$, let $\pi^t(g)$ denote the probability that network g is realized in the beginning of period t. Then

$$\begin{aligned}
\pi^t(g) &= \frac{1}{6}\sum_{ij\in g}\left[\pi^{t-1}(g-ij) + \pi^{t-1}(g)\right] \times \texttt{AND}\left(V_i^t(g) \geq V_i^t(g-ij), V_j^t(g) \geq V_j^t(g-ij)\right) \\
&+\frac{1}{6}\sum_{ij\notin g}\left[\pi^{t-1}(g+ij) + \pi^{t-1}(g)\right] \times \texttt{OR}\left(V_i^t(g) > V_i^t(g+ij), V_j^t(g) > V_j^t(g+ij)\right).
\end{aligned}$$

Let us assume that $a = 100$, $\tau = 23$, and $\beta = 0.9$. Then we have $\frac{10a}{47} < \tau < \frac{a}{4}$ so that (3b) and (6) are pairwise stable. As we have seen in the previous section, if each country is myopic, although (6) is more likely to be realized than (3b), the probability with which (3b) is realized is not negligible. We would like to know what happens if each country is farsighted. Table 3 summarizes the result. We can see from the table that the probability with which the complete network is realized becomes very close to 1.[3]

[3]Excel files that are used to solve the Bellman equations are available from the authors on request. These files and additional figures can be downloadable at `http://www.eco.osakafu-u.ac.jp/~shichijo/profile/network/network.html`

	(3b)	(6)
$T = 10$	0.062717	0.189043
$T = 20$	0.001338	0.818923
$T = 30$	2.34×10^{-5}	0.969786
$T = 40$	4.07×10^{-7}	0.995103
$T = 50$	7.06×10^{-9}	0.999209

Table 3: Probabilities with which networks (3b) and (6) are realized in the final period when each country is farsighted and $(a, \tau, \beta) = (100, 23, 0.9)$.

Since the number of networks is $2^6 = 64$, the number of the Bellman equations in each period is $4 \times 64 = 256$. However, since the model is anonymous, there is a way to reduce the number of "states" in each period to 20 by using the same argument as in Iimura, Murakoshi, and Hokari (2007).[4]

[4]A list of the Bellman equations in this alternative approach is provided in the appendix, which is downloadable at the webpage mentioned in footnote 3.

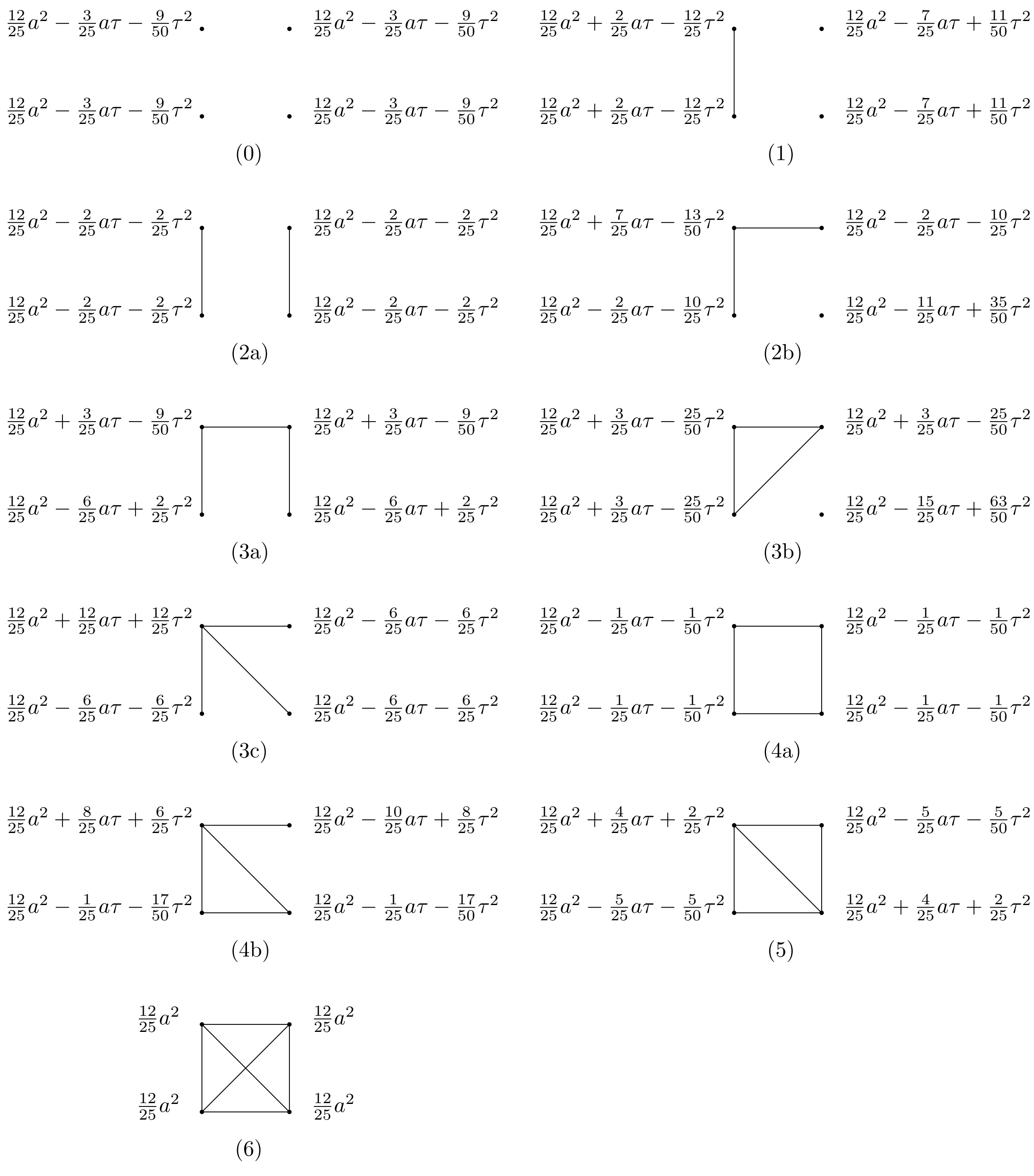

Figure 1: The welfare of each country in each network.

References

[1] Belleflamme, P. and F. Bloch (2004) "Market sharing agreements and collusive networks" *International Economic Review* **45**, 387–411.

[2] Dutta, B., S. Ghosal, and D. Ray (2005) "Farsighted network formation" *Journal of Economic Theory* **122**, 143–164.

[3] Goyal, S. and S. Joshi (2006) "Bilateralism and free trade" *International Economic Review* **47**, 749–778.

[4] Iimura, M., S. Murakoshi, and T. Hokari (2007) "Subgame-perfect market sharing agreements" *Economics Bulletin* **3**(7), 1–14.

[5] Jackson, M.O. and A. Watts (2002) "The evolution of social and economic networks" *Journal of Economic Theory* **106**, 265–295.

[6] Jackson, M.O. and A. Wolinsky (1996) "A strategic model of social and economic networks" *Journal of Economic Theory* **71**, 44–74.