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News and knowledge capital

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Abstract

We explore the ability of a model with knowledge capital to generate business cycles driven by expectations of future movement in total factor productivity (TFP). These cycles are characterized by a boom in which consumption, investment, output and hours-worked all rise in advance of any actual movement in TFP. We model knowledge capital as an input into production which is endogenously produced through a learning-by-doing process. When firms receive news of an impending productivity increase, the value of knowledge capital rises, inducing the firm to hire more hours to "invest" in knowledge capital. The rise in the value of knowledge capital immediately raises the value of the firm, causing an appreciation in share prices, a feature that has empirical support. The increase in output of the firm allows both consumption and investment to rise despite the absence of any contemporaneous productivity shock. If the expected increase in productivity fails to materialize, the model generates a recession as well as a crash in the stock market.

KEYWORDS: expectations-driven business cycle, Pigou cycle, news shock, learning-by-doing, asset pricing
JEL CLASSIFICATION: E3

1 Introduction

A number of recent studies have attempted to develop models capable of generating expectations-driven business cycles. A key aspect of these cycles is that a boom is created in anticipation of future increases in productivity as opposed to the typical real business cycle model where the boom is driven by an unanticipated contemporaneous rise in productivity. Vector autoregression (VAR) evidence in favour of these cycles is provided in Beaudry and Portier ([5]). More recently, Beaudry and Lucke ([4]) and Schmitt-Grohe and Uribe ([30]) estimate the contribution of anticipated total factor productivity (TFP) shocks along with several other shocks typically used in the business cycle literature and find that anticipated TFP shocks account for a large fraction of the total variation in aggregate series. Despite the possibility that these "news" based shocks play a large role in modern business cycles, there are few models capable of effectively capturing even the most basic empirical features of these cycles. As discussed by Beaudry and Portier ([5]), Jaimovich and Rebelo ([19]) and others, the typical business cycle model is unable to deliver booms in which consumption, investment and hours all rise along with output in the periods after the news arrives but before the shock to productivity actually occurs[1].

Beyond co-movement between aggregate quantities, a robust feature of business cycles is co-movement with asset prices. For example, the S&P500 real price index leads GDP by about two quarters: at business cycle frequencies the contemporaneous correlation between the two measures is 0.42 while it is 0.56 two quarters ahead. Moreover, the idea that stock prices respond in advance of the increase in TFP was highlighted in the work of Beaudry and Portier ([6]). Intuitively this makes sense: news of impending productivity-increases and the ensuing flow of higher profits should induce an immediate increase in share prices.

The goal of our paper is to deliver co-movement in both aggregate real quantities and asset prices in response to news. This latter feature is an even higher hurdle for most models to cross. In order to get agents to increase investment expenditure on physical capital in advance of the actual rise in TFP, many studies utilize an adjustment cost specification which penalizes changes in the level of investment. An implication of this is that a rise in investment today lowers the value of installed capital. To the extent that the value of firms depends on the installed capital, this effect puts downward pressure on share prices.

[1]This is closely related to the analysis of Barro and King ([3]) which showed that consumption and hours-worked will negatively co-move for shocks other than contemporaneous productivity shocks.

In this paper we offer a simple variant of a standard business cycle model that generates the aforementioned co-movement through an intuitive mechanism. The modification is an environment in which agents' actions endogenously create productivity-increasing knowledge through a learning-by-doing (LBD) process. The idea is simple: the news that TFP will rise at a faster than normal pace in the future immediately increases the value of knowledge capital, which we model as an input into the production technology along with labour and physical capital. The rise in the value of knowledge capital creates an incentive for agents to increase the use of labour to accumulate more knowledge capital. The increase in knowledge capital and labour in turn induce investment expenditure on physical capital by raising it's productivity, and the ensuing expansion of output allows both consumption and investment to increase. Since we model firms as storehouses of knowledge capital, the rise in the value of knowledge capital leads to an immediate rise in the value of the firm and therefore its share price. Moreover, since the mechanism in our model resides on the production side of the economy, the model is able to generate an expectational-boom in real quantities and asset prices over a range of preference specifications. In addition, factor prices are procyclical because a rise in knowledge capital raises the marginal productivity of both labour and capital. The absence of adjustment costs prevents unrealistic spikes in interest rates, which are often features of models built to generate co-movement. We think that the introduction of one mechanism that simultaneously delivers all these key features of the data is a strength of the model over others which require one or more modification to the standard growth model to deliver each feature. In light of recent discussions in the literature, it is also interesting to be able to generate expectational-booms in a model without any frictions whatsoever.

The introduction of knowledge capital into the standard one good growth model is, in our opinion, a useful way to try to capture some of the real world complexity surrounding technical change while retaining the simplicity of the original model. While the literature on "news shocks" has focused a lot on the role of wealth effects in explaining why co-movement is hard to achieve, we think part of the problem is that when a productivity shock arrives, it immediately leads to an increase in output without requiring any change in actions on the part of agents. In practice however, the arrival of a new technology itself does nothing to increase output. Considerable resources have to be utilized to re-organize production in the economy including the acquisition of new skills and machines as well as the use of new processes and material and the production of new goods. In a one-good world (as measured by GDP) in which workers perform one identical task (measured by total hours), all these changes are hidden and all that we observe is that aggregate activity goes up in advance of total factor productivity. In other words, the economy needs to reinvent itself to take advantage of the productivity-increases enabled by the new technology, but all we see is a ramping-up of activity. We think that this idea of ramping-up to make the economy conform to the new technology can be captured simply and effectively in terms of investments in knowledge capital. In the model, in response to news about future productivity increases, firms "invest" in building knowledge capital by hiring workers beyond the level dictated by the current marginal product of labour. In doing so, firms sacrifice current profits for anticipated productivity-increases and higher profits in the future. This is similar to the economy devoting resources to re-organizing production activities in order to prepare for a new technology. The by-product nature of the learning-by-doing process also fits well into the one-good, one-task world view of the model. While the mix of tasks performed by workers and the mix of goods produced by firms may change during the expansion phase in order to enable the new technology, all that is recorded at the aggregate level is an increase in output and hours. Similarly in the model, the increase in hours leads simultaneously to more production today and to more knowledge capital which unleashes future increases in productivity.

Thus far, we have described predictions of the model when expected increases in future productivity are realized. The model also has very intriguing implications for situations when agents are disappointed. If the expected productivity shocks fail to materialize, agents find themselves with less wealth than expected and too much physical and knowledge capital relative to the actual state of TFP. This leads to a sharp drop in share prices, induced by a fall in the value of knowledge capital. This "bear market" is accompanied by a recession in which output, investment and hours all fall.

Our work builds on recent business cycle models by Chang et al ([9]) and Cooper and Johri ([11]) that incorporate various forms of learning-by-doing into dynamic general equilibrium models and show that they can be an effective propagation mechanism for shocks. While details differ, the two models share the feature that knowledge capital accumulation is a by-product of production activity. Analysis in both papers suggest that models with learning-by-doing improve the ability of the growth model to explain the response of the economy to productivity shocks. Johri ([21]) shows similar results for an economy with shocks to monetary policy. Both papers offer aggregate evidence in favour of learning-by-doing and build on an extensive empirical literature which documents the existence of learning effects in all sectors of the economy. Recent studies include Bahk and Gort ([2]), Irwin and Klenow ([18]), Jarmin ([20]), Benkard ([8]) and Thornton & Thompson ([31]). In this paper we adopt a specification based on that of Chang et al ([9])

where learning occurs as a by-product of past hours-worked. While many other specifications are possible, this one has the advantage of simplicity while still delivering the result [2].

There is a small but growing literature on expectations-driven business cycles. Beaudry and Portier ([5]) consider a model with a durable and non-durable good that are produced in two distinct sectors. A complementarity between the two allows both consumption and investment to rise in response to news about a productivity increase in the non-durable goods sector. Jaimovich and Rebelo ([19]) propose preferences that reduce or eliminate the strong wealth effect on leisure of an expected future increase in TFP or investment specific technical change. They demonstrate that when combined with capital utilization and adjustment costs to changes in investment, the model produces a strong expectational-boom. Christiano et al ([13]) show that the combination of the same specification of investment adjustment costs with habit formation in consumption produces an expectational-boom, however, they find that the model requires an implausible rise in the real interest rate and produces a counterfactual counter-cyclical asset price.[3] They then present a monetary version of the model with nominal wage rigidities and an inflation-targeting monetary authority that creates an expectational boom in real quantities and asset prices without as large a rise in the real interest rate. Den Haan and Kaltenbrunner ([14] present a matching model whereby matching frictions induce firms to post more vacancies in response to news, leading to an increase in employment which allows aggregate consumption and employment to co-move. Duper and Mehkari ([15]) show that a strictly convex frontier between consumption and investment and a high intertemporal elasticity of substitution can also deliver the result. Schmitt-Grohe and Uribe ([30]) investigate the role of news shocks in generating economic fluctuations by performing a structural Bayesian estimation on a model featuring habit-formation in consumption and leisure, a flow-specification of investment adjustment costs, and capacity utilization. By allowing for both anticipated (news) and unanticipated components for various shocks, they are able to perform a variance decomposition to determine the relative contribution of anticipated versus unanticipated shocks, and find that anticipated shocks to the permanent and temporary components of TFP account for more than two-thirds of aggregate fluctuations in U.S. postwar quarterly data.

In the remainder of the paper we proceed as follows. In Section 2 we discuss an example economy based on Chang et al ([9]). The purpose of

[2] Our work is also related to the ideas of human capital and organizational capital which have been explored in several studies, too numerous to cite.

[3] Since we do not use adjustment costs, our model does not suffer from an extreme jump in interest rates.

this example is to show the simplicity and strength of the mechanism built into the knowledge capital economy since it can generate co-movement in consumption, output, hours and investment without any other modification to the one sector growth model. It also illustrates the flexibility of the concept of knowledge capital. Like Chang et al, this section treats knowledge capital as being symmetric with human capital which is accumulated by the worker while Section 3 presents a model in which knowledge capital is accumulated by firms. In the former section, payments for knowledge capital go to the worker while in the latter section they lead to operating profits for firms. This feature is crucial for the model to display procyclical stock prices that rise before any changes in TFP. Since we parameterize the model to be consistent with US data and impose constant returns in the production technology, this implies a relatively small contribution of knowledge capital to firm output. As a result, we augment the model with variable capital utilization which magnifies the expectational-boom. We also discuss the impact on our results of changing preferences. The final section concludes.

2 An example

We begin with a simple example economy based on Chang et al ([9]) that makes clear how the learning-by-doing mechanism allows co-movement of hours, investment and consumption in response to news about a future rise in exogenous total factor productivity. Since this economy is taken more or less directly from Chang et al, we offer very little discussion of the modeling assumptions.[4]

The economy is populated by an infinitely-lived representative household whose preferences are defined over sequences of consumption C_t and leisure L_t with expected lifetime utility defined as

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \{\ln C_t + \chi L_t\}, \tag{1}$$

where β is the representative household's subjective discount factor and χ parameterizes the household's relative preference for leisure over consumption.

The representative household operates a production technology that produces output Y_t according to the technology

$$Y_t = A_t \tilde{N}_t^{\alpha} K_t^{1-\alpha}, \tag{2}$$

[4]While Chang et al present a decentralized model, we focus on the associated planner's problem.

where A_t is the level of an exogenous stationary technology process, $\tilde{N}_t$ is effective labour hours, and K_t is physical capital which accumulates according to

$$K_{t+1} = (1-\delta)K_t + I_t. \tag{3}$$

Effective labour is defined as

$$\tilde{N}_t = H_t N_t, \tag{4}$$

where N_t is hours-worked and H_t is the stock of knowledge capital which accumulates according to

$$H_{t+1} = \Psi(H_t, N_t) = H_t^{\gamma} N_t^{1-\gamma}. \tag{5}$$

The idea here is that the actual contribution of labour to production is a combination of raw labour hours and knowledge capital which captures information about how best to use the labour input given the state of technology. As discussed in the introduction, the households acquires knowledge capital as a by-product of engaging in production.

Combining (2) and (4) we get

$$Y_t = A_t F(N_t, K_t, H_t) = A_t (H_t N_t)^{\alpha} K_t^{1-\alpha}. \tag{6}$$

The common exogenous total factor productivity process A_t evolves in logs according to the stationary AR(1) process

$$\ln A_t = \rho_A \ln A_{t-1} + \theta_{A,t}, \tag{7}$$

where $\rho_a < 1$ and $\theta_{A,t}$ is an exogenous period t innovation which we will define further below.

Each period, the household is endowed with one unit of time that can be allocated between leisure and hours-worked N_t according to

$$N_t + L_t = 1. \tag{8}$$

Finally, the economy's resource constraint is given by

$$C_t + I_t = Y_t. \tag{9}$$

The planner chooses contingent infinite sequences of C_t, N_t, K_{t+1} and H_{t+1} to maximize (1) subject to (3), (5), (6), (8) and (9). Making the appropriate substitutions and letting λ_t and Υ_t be the period t Lagrange multipliers on (9) and (5) respectively, the planner's first-order conditions are as follows:

$$u_C(C_t, L_t) = \lambda_t \tag{10}$$

$$u_L(C_t, L_t) = \lambda_t A_t F_{Nt} + \Upsilon_t \Psi_{Nt} \tag{11}$$

$$\lambda_t = \beta E_t \left\{ \lambda_{t+1} \left[A_{t+1} F_{Kt+1} + 1 - \delta \right] \right\} \tag{12}$$

$$\Upsilon_t = \beta E_t \left\{ \lambda_{t+1} A_{t+1} F_{Ht+1} + \Upsilon_{t+1} \Psi_{Ht+1} \right\}. \tag{13}$$

where $F_{Nt} = A_t \frac{\partial F(N_t, K_t, H_t)}{\partial N_t}$, $\Psi_{Nt} = \frac{\partial \Psi(H_t, N_t)}{\partial N_t}$ etc. These first-order conditions differ from those of the standard RBC model only by the addition of an additional term in the hours first-order condition (11) and an Euler equation for knowledge capital (13).

To interpret these two equations, first define $q_{ht} = \frac{\Upsilon_t}{\lambda_t}$ as the value of new knowledge capital in terms of consumption. Applying this definition and substituting out λ_t, we can re-write the knowledge capital and hours first-order conditions as

$$q_{ht} = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[A_{t+1} F_{Ht+1} + q_{ht+1} \Psi_{Ht+1} \right] \right\} \tag{14}$$

$$\frac{u_L(C_t, L_t)}{u_C(C_t, L_t)} = A_t F_{Nt} + q_{ht} \Psi_{Nt}. \tag{15}$$

Equation (14) shows that the value of the marginal unit of knowledge capital in terms of consumption is the stochastically-discounted future lifetime stream of the additional output generated from the additional knowledge capital. Note that the two terms on the right hand side of the equation suggest that additional knowledge not only contributes to output but also raises the marginal effectiveness of each hour in the learning process. Recognizing the connection between hours-worked and the creation of future knowledge capital, the planner does not merely equate the household's marginal rate of substitution of consumption for leisure to the marginal product of labour as would occur in the standard model. Instead, in this model, we see from equation (15) that the planner equates the household's marginal rate of substitution to the sum of the marginal product of labour and the value of the additional stock of knowledge generated by an increase in hours-worked today. From the perspective of explaining how the model can generate an increase in hours in response to news about a future productivity shock, it is helpful to note that a change in the value of knowledge capital, q_{ht}, will act as a shift factor for the labour supply curve mapped out in hours-productivity space. This shift factor is missing in standard models in the absence of a contemporaneous productivity shock.

While it may appear at first that the by-product nature of the learning process means that knowledge capital (and hence future productivity) can be acquired costlessly, this is not entirely correct. The planner will make considerable "unobserved investments" in knowledge capital. To see this, we can re-write (15) as $\frac{u_L(C_t,L_t)}{u_C(C_t,L_t)} - A_t F_{Nt} = q_{ht}\Psi_{Nt}$. The left hand side of this equation shows that the marginal rate of substitution is larger than the marginal product of labour. In other words, the planner is using more labour than is justified by the current payoff in terms of additional output. This additional use of labour is an "investment in the future". The per-hour cost of the investment is on the left hand side of the equation while the per-hour value of the investment is given by the right hand side. We can then define the total unmeasured investment in knowledge capital Λ_t as

$$\Lambda_t = \left\{ \frac{u_L(C_t, L_t)}{u_C(C_t, L_t)} - A_t F_{Nt} \right\} N_t. \tag{16}$$

2.1 The impact of news shocks

In this section we explore how news of an impending rise in total factor productivity is received by the economy described above. We contrast this with the response of a similar economy without knowledge capital. Our representation of news shocks is standard and follows Christiano et al ([13]). We provide for news about A_t by defining the innovation $\theta_{A,t}$ in equation (7) as

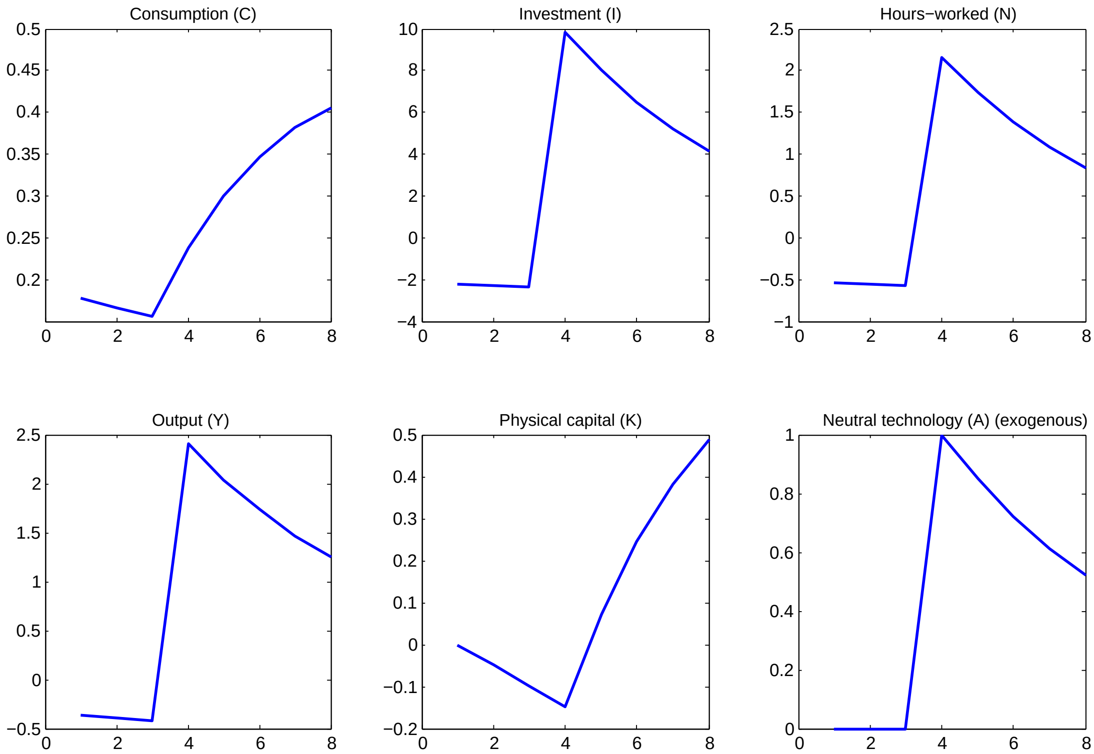
$$\theta_{A,t} = \epsilon^p_{A,t-p} + \varepsilon_{A,t}, \tag{17}$$

where $\epsilon^p_{A,t}$ is a news shock that agents receive in period t about the innovation $\theta_{A,t+p}$, and $\varepsilon_{A,t}$ is an unanticipated contemporaneous shock to $\theta_{A,t}$. The news shock $\epsilon^p_{A,t}$ has properties $E\epsilon^p_{A,t} = 0$ and standard deviation $\sigma_{\epsilon^p_A}$, and the contemporaneous shock $\varepsilon_{A,t}$ has properties $E\varepsilon_{A,t} = 0$ and standard deviation σ_{ε_x}. The shocks $\epsilon^p_{A,t}$ and $\varepsilon_{A,t}$ are uncorrelated over time and with each other.

Figure 1 shows the response of our benchmark standard RBC model to news in period 1 that a temporary but persistent increase in productivity will occur in period 4, represented by $\varepsilon_{At} = 0$ so that $\theta_{At} = \epsilon^p_{A,t-p}$. This and the next figure for the LBD model are based on the news turning out to be correct, ie., productivity actually does rise in period 4.[5] The figure clearly illustrates the difficulties in generating co-movement in response to news shocks. In period 1 consumption rises but investment and hours-worked decrease below steady state. Thereafter, these three variables slope downwards

[5]The equivalent baseline RBC model consists of setting $\tilde{N}_t = N_t$ and omitting the constraint (5). The common parameterization for both the RBC and LBD model behind these impulse responses is relatively standard: $\beta = 0.99$, $\alpha = 0.67$, $\delta = 0.022$, $\rho_A = 0.85$ and $p = 3$. In the LBD model, we set $\gamma = 0.8$, close to the estimate in Chang et al. A fully calibrated model will be presented in the next section.

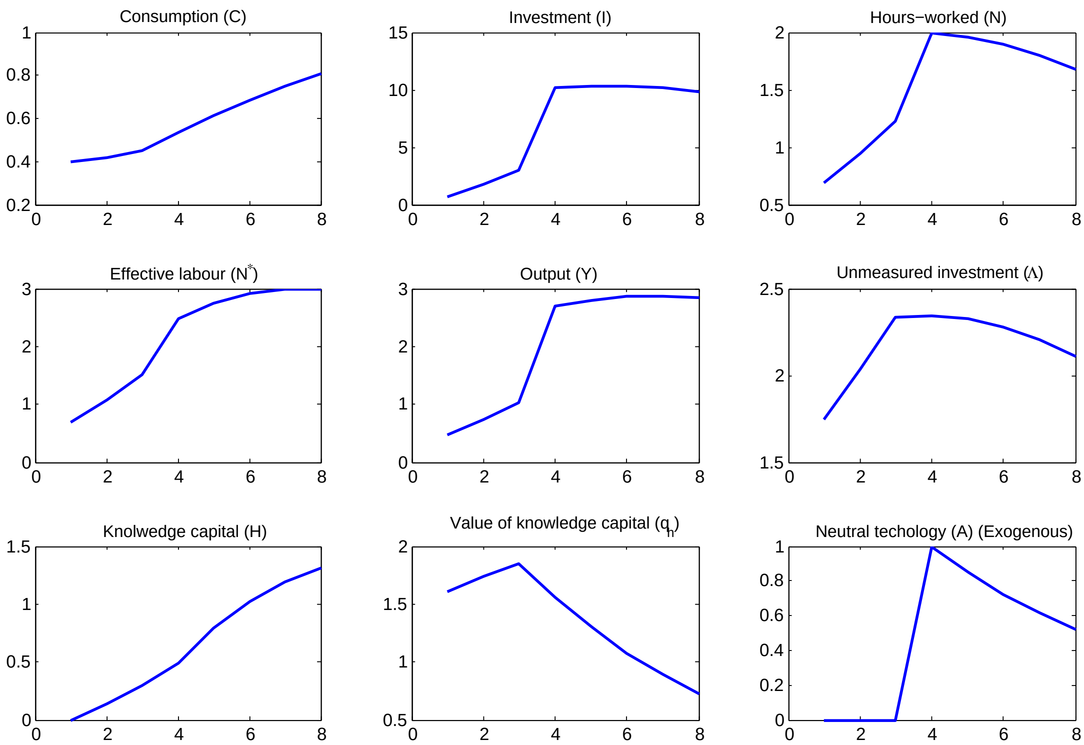
Figure 1: **Standard RBC** - News shock in period 1 about neutral tech. shock, tech. shock *fully realized* in period 4



slightly before reacting positively in the usual manner to the contemporaneous productivity shock in period 4. In response to the news, the wealth effect of the expected increase in future productivity causes households to increase consumption in the initial period, driving down the marginal utility of consumption, and producing a corresponding wealth effect on leisure through the hours first-order condition, causing households to reduce hours-worked. With the capital stock fixed in the initial period, the reduction in hours-worked reduces production, and therefore households "fund" the increase in consumption through a decrease in investment.

Figure 2 shows the response of the LBD model to the same news shock. In sharp contrast to the previous figure, hours-worked, investment, consumption and output all increase and slope upwards in response to the news shock. Note in particular the sharp increase in the value of knowledge capital, q_{ht}, in response to the news about future productivity increases. This increase in the value of knowledge capital, on its own, induces the agent to increase hours-

Figure 2: **Household knowledge** - News shock in period 1 about neutral tech. shock, tech. shock *fully realized* in period 4



worked. The aforementioned wealth effect on leisure is still in operation but is trumped by the desire to learn in preparation for the technological change. This can be seen in the plot for unmeasured investment which rises as soon as the news arrives, and peaks in the period before the technology shock actually hits the economy. The increase in hours-worked leads to a rise in output in the current period as well as an increase in knowledge capital in the subsequent period. Anticipating this, agents realize that the productivity of capital will rise in the next period also and therefore are induced to increase investment. The increase in output allows agents to simultaneously satisfy their desire for more consumption and investment.

Why does q_h rise? Firstly, recall that we defined q_h as the ratio of the shadow value of knowledge capital to that of goods, which in this model is also the shadow value of physical capital. The news that TFP will rise in the future causes the shadow value of physical capital to fall but the shadow value of knowledge capital to rise. This discrepancy between the behaviour of the two capital stocks may seem puzzling at first, therefore we discuss them in some detail. It is most convenient to think about the response of these prices in terms of demand and supply of the resources used to create the two capital stocks. On the demand side, the situation is symmetric. The planner realizes that the marginal products of physical and knowledge capital will both rise with TFP in period four, thus more of each input will be desired at that point. The supply side is, however, dramatically different in period four because of the technological environment of the model. The productivity shock implies that there will be additional goods available for use in consumption and for the creation of physical capital, even if the planner makes no changes at all. Thus there is a large increase in the supply of goods which drives its price down. In contrast, the technology does not expand the supply of knowledge capital in the economy, nor the primary input into knowledge capital, hours-worked. Rather, due to the wealth effect, the demand for leisure will increase, squeezing the availability of market hours needed for the creation of knowledge capital. This increase in the cost of creating knowledge capital leads to a rise in its shadow price. All together, this implies that the shadow value of goods falls while that of knowledge capital rises, both of which lead to a rise in q, the consumption value of knowledge capital. The form of the accumulation technology for knowledge capital determines how far forward in time the initial rise in q_{ht} occurs. The curvature in the functional form and the presence of constant returns in labour and knowledge capital all play a quantitative role in this regard. Diminishing returns to hours in the creation of knowledge encourages the planner to spread out the "investment" period. Furthermore, the presence of knowledge capital on the righthand side of the equation implies additional knowledge raises the marginal return to each hour in terms of knowledge created.

While the results of this section illustrate clearly the manner in which learning-by-doing is able to generate expectations driven business cycles, we think important characteristics of expectational booms cannot be explained by it. The most important of these is the co-movement of firm equity share prices. Often discussion of booms in the media do not distinguish between increases in the value of financial assets and in real quantities that macroeconomists tend to focus on. To an extent this could be because both tend to rise together in these boom periods. Beaudry and Portier ([6]) show that for the US, "news", as captured by innovations in their VAR results, lead to immediate increases in stock market values which are subsequently followed by increases in TFP. It would appear that stocks rise in anticipation of future increases in profits due to the increase in TFP. Our knowledge capital model has similar features. News about impending increases in TFP leads to an increase in the value of knowledge capital. If knowledge capital were accumulated by firms, this rise in value would also raise the value of the firms themselves. Share prices would rise in anticipation of the extra profits to be generated in the future. Interestingly, this suggests that the learning mechanism can simultaneously explain not only the co-movement in real quantities like hours-worked and investment, but also the increase in asset values.

This concept of firm value as a function of firm-specific knowledge is consistent with the idea of firm value in the organizational capital literature where organizational capital is typically viewed as an unobserved input into production. For example, Prescott and Visscher ([27]) refer to information accumulation within the firm as an explanation for the firm's existence. This information affects its production possibilities set, and thus acts as an asset for the firm which gives it value. Our interpretation of this value is similar: knowledge capital is productivity-enhancing, allowing firms to produce additional output for given levels of labour and capital without having to pay out additional rents in the future, creating a stream of profits which provide value to the firm. It differs from many models of organizational capital such as Atkeson and Kehoe ([1]) where the evolution of organizational capital is exogenous and not controlled by the firm. Moreover, our interpretation of knowledge capital as an asset owned by the firm is consistent with that idea of Rosen ([29]), who in reference to this type of knowledge writes that "specific knowledge is vested 'in the firm'. Then the asset is transferable by selling the firm, whose price, net of physical capital value, is in fact the market value of its specific capital" ([29]).

3 An economy with firm-specific capital

We now present our full model where knowledge is accumulated by firms as opposed to by workers as in the example economy. This will imply that firms will increase labour demand in response to the news, as opposed to workers increasing labour supply [6].

The economy consists of a continuum of identical infinitely-lived households on a unit measure, and a single competitive firm. Since we will impose constant returns to scale in production, it is convenient to assume that production occurs at a single representative firm that nonetheless behaves competitively and takes factor prices as given. This assumption has the advantage of suppressing notation associated with shares belonging to different firms. In general, we use lowercase variables to represent individual household quantities and economy-wide prices, and uppercase variables to

[6] It is entirely likely that both mechanisms are present in the data, but we explore only the former for clarity.

represent firm quantities. For notational simplicity, we assume that households own the stock of physical capital and sell capital services to the firm. In addition to markets for labour services, capital services and goods, we assume the existence of a stock market where households can buy and trade equity shares in the firm that represent claims to the firm's future profits.

3.1 Household

The household side of the model is relatively standard so we discuss it briefly. An individual household has preferences defined over sequences of consumption c_t and leisure l_t with expected lifetime utility defined as

$$\mathcal{U} = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t), \tag{18}$$

where β is the household's subjective discount factor.

Each period, the household supplies hours-worked n_t for wage w_t and capital services $\tilde{k}_t$ for price r_t. In addition, it receives dividend income d_t for each unit of its outstanding holdings of firm equity z_t. For convenience, we normalize the firm's outstanding number of shares to unity, and thus the household trades fractions of the firm's single equity share. The household allocates its earnings between consumption, investment in physical capital and equity shares. The household's period t budget constraint is given by

$$c_t + i_t + v_t z_{t+1} = w_t n_t + r_t \tilde{k}_t + [v_t + d_t]\, z_t, \tag{19}$$

where c_t is consumption, i_t investment in physical capital and v_t the price of equity. Capital services are defined as

$$\tilde{k}_t = u_t k_t, \tag{20}$$

where k_t is the household's stock of physical capital and u_t is the utilization rate of that capital. The household's physical capital evolves according to

$$k_{t+1} = [1 - \delta(u_t)]k_t + i_t \tag{21}$$

where the depreciation function $\delta(\cdot)$ satisfies the conditions $\delta'(\cdot) > 0$, $\delta''(\cdot) \geq 0$.

The household's problem is to choose sequences c_t, n_t, u_t, k_{t+1} and z_{t+1} to maximize (18) subject to (19), (20) and (21), yielding the standard first-order conditions

$$u_c(c_t, l_t) = \lambda_t \tag{22}$$

$$u_l(c_t, l_t) = \lambda_t w_t \tag{23}$$

$$\delta'(u_t) = r_t \tag{24}$$

$$\lambda_t = \beta E_t \left\{ \lambda_{t+1} \left[r_{t+1} u_{t+1} + 1 - \delta(u_{t+1}) \right] \right\} \tag{25}$$

$$v_t = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[v_{t+1} + d_{t+1} \right] \right\} = \sum_{s=1}^{\infty} E_t \left\{ \beta^s \frac{\lambda_{t+s}}{\lambda_t} d_{t+s} \right\}. \tag{26}$$

where it is clear from (26) that as usual the price of the firm's share v_t will equal the stochastically-discounted lifetime stream of the firm's dividends beginning in period $t+1$.

3.2 Firm

The firm produces output according to

$$Y_t = A_t F(N_t, K_t, H_t) = A_t N_t^{\alpha} \tilde{K}_t^{\theta} H_t^{\varepsilon}, \tag{27}$$

where A_t is aggregate exogenous neutral productivity defined as in (7) and H_t is the firm's stock of firm-specific knowledge capital, and where we restrict $\alpha + \theta + \varepsilon = 1$ to impose constant returns to labour, capital services and knowledge capital in production.

The firm's knowledge capital evolves as in our example economy as

$$H_{t+1} = \Psi(H_t, N_t) = H_t^{\gamma} N_t^{1-\gamma}. \tag{28}$$

Each period, the firm pays out a dividend D_t to shareholders defined as

$$D_t = Y_t - w_t N_t - r_t \tilde{K}_t. \tag{29}$$

Since the firm accumulates firm-specific knowledge capital through an internal learning-by-doing process, it faces the dynamic problem of choosing sequences of N_t, $\tilde{K}_t$ and H_{t+1} to maximize current and expected future lifetime dividends

$$V_t = D_t + E_t \sum_{s=1}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} \left\{ D_{t+s} \right\} = D_t + \bar{V}_t \tag{30}$$

subject to (27), (28), and (29), where the term $\beta^s \frac{\lambda_{t+s}}{\lambda_t}$ is the household's stochastic discount rate for period $t+s$, and where we have defined $\bar{V}_t = E_t \sum_{s=1}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} \left\{ D_{t+s} \right\}$ as the end-of-period discounted value of the firm's future lifetime stream of profits. Letting q_t be the Lagrange multiplier associated with (28), and making the appropriate substitutions, the firm's first-order conditions are then

$$w_t - A_t F_{Nt} = q_t \Psi_{Nt} \tag{31}$$

$$r_t = A_t F_{\tilde{K}t} \tag{32}$$

$$q_t = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[A_{t+1} F_{Ht+1} + q_{t+1} \Psi_{Ht+1} \right] \right\}. \tag{33}$$

Analogous to our example economy, the firm's knowledge capital first order condition (33) shows that the value, in terms of profits, of an additional unit of firm-specific knowledge is the stochastically discounted future lifetime stream of additional output created by that additional knowledge. As such, the firm's hours first-order condition now shows that in determining its optimal use of labour, the firm considers both the direct marginal productivity of that labour in current production plus the value in terms of profits of the additional future lifetime output brought about by increasing its stock of firm-specific knowledge from hiring more labour hours today.

Note that once the firm has created the extra unit of knowledge capital, its contribution to additional output each period $t + s$ thereafter, as given by $A_{t+s} F_{Ht+s}$ in (33), represents a stream of profits for the firm over the life of the knowledge capital. This occurs because once it is created, it is held costlessly by the firm. To see this, combine (29), (31), (32) and (33) along with the specific functional forms of $F(\cdot)$ and $\Psi(\cdot)$ to give

$$q_t = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[\frac{D_{t+1}}{H_{t+1}} + q_{t+1} \frac{H_{t+2}}{H_{t+1}} \right] \right\}, \tag{34}$$

which shows that the marginal value of an additional unit of knowledge capital is the additional profit created by the extra unit (which happens to also equal the average profit per unit) plus the value of future units of knowledge capital made possible.

Recall from the previous section that the right hand side of (31) may be thought of as unmeasured investment by the firm in knowledge capital per hour. The extent to which this term influences the firm's labour decision will depend on the current value of knowledge capital, q_t. When $q_t > 0$, the firm will wish to use labour at a level in excess of that of a standard neoclassical firm. From the lens of the standard firm's problem (where firms hire labour up to the point where the marginal product of labour equals the wage rate), it appears as if the firm wishes to hire "too much" labour because the marginal product is below the current wage rate. In fact, the firm is investing in knowledge capital by trading off lower current profit for higher future profit. This investment in knowledge capital responds to q_t, the value of knowledge, and shows up as "unmeasured investment" since the investment is embedded in wage payments. Like the example economy, if news about future changes in TFP increases q_t, the firm will respond by attempting to hire more labour. This will, in turn, raise the value of the firm and it's shares. It is easy to check that changes in the value of the firm are in fact equal to the total value of unmeasured investment. We show in the Appendix that the end-of-period t value of the firm and therefore price of equity can be expressed as

$$\bar{V}_t = q_t H_{t+1}, \tag{35}$$

which shows that the value of the firm is determined by the total value of its existing stock of knowledge, obtained as a product of the marginal value of firm-specific knowledge and the stock of firm-specific knowledge.[7]

3.3 Equilibrium

Equilibrium in this economy is defined by contingent infinite sequences of c_t, n_t, $\tilde{k}_t$, u_t, k_{t+1}, z_{t+1} for each household, N_t, $\tilde{K}_t$, H_{t+1}, Y_t for the firm, aggregate states $\int_0^1 H_t di = \mathbb{H}_t$, $\int_0^1 k_t di = \mathbb{K}_t$, and prices $w_t = w(\mathbb{H}_t, \mathbb{K}_t, A_t)$, $r_t = r(\mathbb{H}_t, \mathbb{K}_t, A_t)$ and $v_t = v(\mathbb{H}_t, \mathbb{K}_t, A_t)$ that satisfy the following conditions: (i) the allocations solve each household's problem taking prices as given; (ii) the allocations solve the firm's problem taking prices as given; (iii) the equity market clears, $\int_0^1 z_t di = 1$; (iv) the labour market clears, $\int_0^1 n_t di = N_t$; (v) the capital services market clears, $\int_0^1 \tilde{k}_t di = \tilde{K}_t$, and; (vi) the aggregate resource constraint holds, $C_t + I_t = Y_t$, where $\int_0^1 c_t di = C_t$ and $\int_0^1 i_t di = I_t$. Finally, we note that (26) and (30) imply that $v_t = V_t$.

3.4 Solution method and parameterization

In order to solve the model it is convenient to work with the associated central planner's version given in the Appendix. We solve the model by linearizing the model equations around the steady-state and then use the singular linear difference system reduction method of King and Watson [25]. We assign values to the parameters of the model using typical values established in the

[7] By assuming that the value of the firm derives solely from its knowledge capital we do not mean to suggest that physical capital plays no role. However, it is convenient to let households accumulate physical capital and focus the analysis of the firm's problem on the novel mechanism. In any case, the role played by capital and variation in the price of capital on firm values is well documented and understood.

literature, and later provide sensitivity analysis to discuss the dependence of the results on these values.

First, we set the share of time allocated to the market in steady-state N_{SS} to 0.2, and the household's subjective discount factor β to 0.99. For the knowledge capital parameters, we start by choosing $\varepsilon = 0.15$, which is approximately the midpoint of the range of 0.08-0.26 estimated in Cooper and Johri ([11]). This value is equivalent to a learning rate of around 10%, which is half of that typically estimated in the learning literature. This is also the value of the contribution of organizational capital used in Atkeson and Kehoe ([1]). For the knowledge capital accumulation equation we pick $\gamma = 0.8$, which is close to the value estimated by Chang et al ([9]). As we will show later, we find that the model results are quite robust to variations in γ over the range of values estimated in the literature such as Cooper and Johri ([11]) and Johri and Letendre ([22]).

Next, we choose the remaining parameters in the production technology and the capital depreciation rate. We require that the model deliver a steady-state labour share, S_N, of approximately 0.67, which in the model is given by

$$S_n = \alpha + \left(\frac{(1-\gamma)}{\xi + (1-\gamma)} \right) \varepsilon, \tag{36}$$

where $\xi = 1/\beta - 1$.

This yields a value of 0.53 for α, and, with constant returns to N_t, $\tilde{K}_t$ and H_t in the production function, a value of 0.32 for θ. Next, we determine the capital depreciation rate such that the model delivers a capital-output ratio of 10, yielding a value of 0.022 for δ.

The parameterization of the learning-by-doing technology has implications for steady state profit. We show in the Appendix that with constant returns in both $F(\cdot)$ and $\Psi(\cdot)$, the share of profit is very small but positive and is given by

$$\frac{D}{Y} = \left(\frac{\xi}{\xi + (1-\gamma)} \right) \varepsilon, \tag{37}$$

where $\xi = 1/\beta - 1$ is the household's subjective discount rate, and γ is the parameter in the accumulation equation for knowledge capital.

We note that the above expression (37) for $\frac{D}{Y}$ assumes that the household accumulates physical capital, and thus this profit share represents the steady state contribution of knowledge capital to profit net of physical capital. For the above parameterization, $\left(\frac{\xi}{\xi+(1-\gamma)} \right) \approx 0.048$ yielding $\frac{d}{y} \approx 0.007$.

We set the elasticity of the marginal capital depreciation function $\epsilon_u = \frac{\delta''(u)}{\delta'(u)} u$ to 0.15, which is within the range of values considered by King and Rebelo ([24]) and the same value as that used by Jaimovich and Rebelo ([19]).

For the exogenous technology shock process that includes news shocks, we set the persistence to $\rho_A = 0.85$, which is in the middle of the values of 0.83 and 0.89 estimated by Christiano et al ([13]) and Schmitt-Grohe and Uribe ([30]) respectively. Following the literature, we set $p = 3$, implying that in each period agents receive news about total factor productivity 3 periods in the future.

Since the learning-by-doing mechanism in this model is primarily a production-side mechanism, we explore the impact of three different forms of preferences on our results. These are:

1. Standard indivisible labour preferences separable in consumption and leisure with specification

$$u(c_t, l_t) = \frac{s_t^{1-\sigma}}{1-\sigma} + \chi \frac{(l_t)^{1-\nu}}{1-\nu}, \tag{38}$$

   with $\sigma = 1$ and $\nu = 0$, therefore implying log consumption and linear leisure per Hansen's ([17]) indivisible labour model.

2. Indivisible labour preferences not separable in consumption and leisure of the form used by King and Rebelo ([24]) in their application of Rogerson and Wright's ([28]) generalization of indivisible labour to nonseparable preferences. These preferences still fall within the general class of "KPR preferences" described in King, Plosser and Rebelo ([23]). With these preferences, the stand-in representative agent has the preference specification

$$u(c_t, l_t) = \frac{1}{1-\sigma} \left\{ c_t^{1-\sigma} \upsilon^*(l_t)^{1-\sigma} - 1 \right\} \tag{39}$$

   where $\upsilon^*(l) = \left[\left(\frac{1-l_t}{H}\right) \upsilon_1^{\frac{1-\sigma}{\sigma}} + \left(1 - \frac{1-l_t}{H}\right) \upsilon_2^{\frac{1-\sigma}{\sigma}} \right]^{\frac{\sigma}{1-\sigma}}$, and where H is the fixed shift length, and υ_1 and υ_2 are constants representing the leisure component of utility of the underlying employed group (who work H hours) and unemployed group (who work zero hours) respectively. We set $\sigma = 2$ in this case. [8]

3. "JR preferences" of the form proposed by Jaimovich and Rebelo ([19]),

[8] We note that for $\sigma = 1$, the linearized form of these nonseparable indivisible labour preferences is equivalent to the linearized form of the standard separable indivisible labour preferences that we consider in the first preference case.

Table 1: Firm knowledge capital interpretation - calibration

σ	ν	ζ	β	NSS	α	θ	ε	$\delta(u)$	ϵ_u	ρ_A	p	γ	η
KPR separable indivisible labour preferences													
1	0	n/a	0.99	0.2	0.53	0.32	0.15	0.022	0.15	0.85	3	0.8	0.2
KPR nonseparable indivisible labour preferences													
2	n/a	n/a	0.99	0.2	0.53	0.32	0.15	0.022	0.15	0.85	3	0.8	0.2
JR preferences													
1	1.16	0.01	0.99	0.2	0.53	0.32	0.15	0.022	0.15	0.85	3	0.8	0.2

with specification [9]

$$u(c_t, l_t, x_t) = \frac{(c_t - \psi(1-l_t)^{\nu} x_t)^{1-\sigma} - 1}{1-\sigma}, \tag{40}$$

$$x_t = c_t^{\zeta} x_{t-1}^{1-\zeta} \tag{41}$$

and where we set $\sigma = 1$, $\nu = 1.16$ and $\zeta = 0.01$ based on Schmitt-Grohe and Uribe's ([30]) estimation of these preferences. As detailed in Jaimovich and Rebelo ([19]), the ζ parameter has the effect of parameterizing the wealth effect to leisure and nests both "GHH preferences" ($\zeta = 0$) proposed by Greenwood, Hercowitz and Huffman ([16]) and "KPR preferences", with lower ζ implying a lower wealth effect to leisure.

Table 1 summarizes our parameterization of the model.

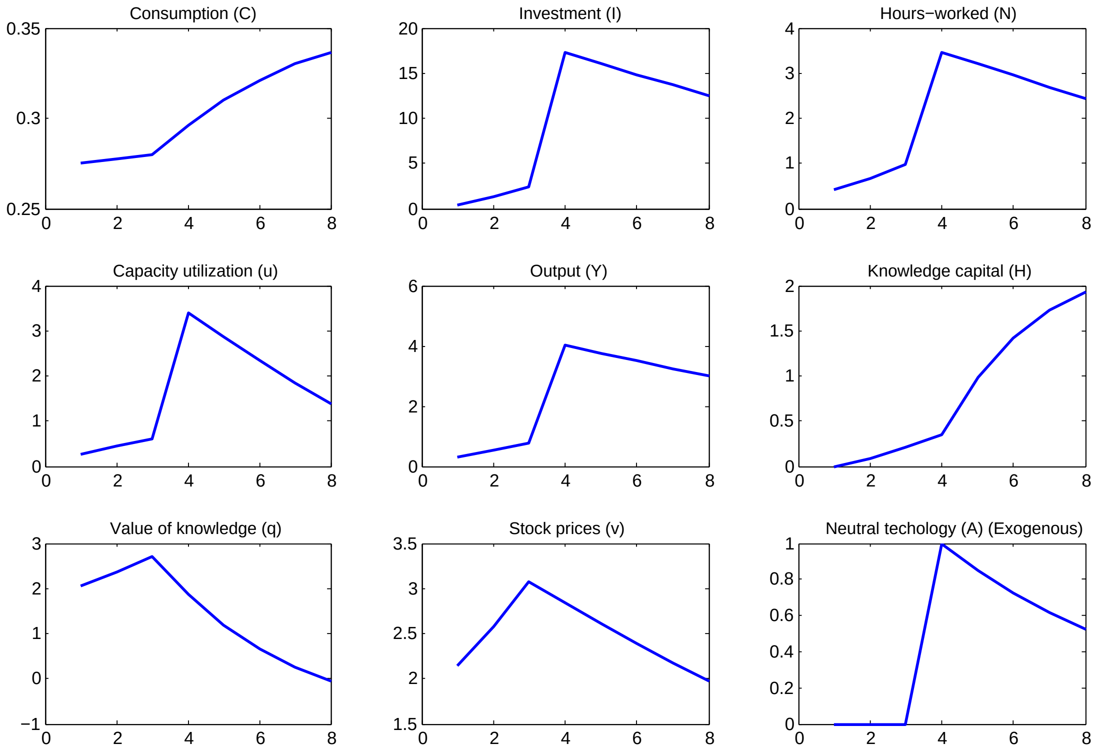
4 Results

We begin this section with a discussion of how the economy described above reacts to news of a 1% increase in TFP in period 4, and then an eventual realization of that shock in period 4. As discussed above, we present results for three different preferences.

Figure 3 shows the response of the economy using separable preferences. As expected from the results of Section 2, consumption, investment, hours-worked and output all rise above steady state levels immediately. Upon

[9] The inclusion of the term X_t introduces another state variable into our system (X_{t-1}) and another first-order condition for X_t to the household problem. See Jaimovich and Rebelo ([19]) for a complete discussion of these preferences.

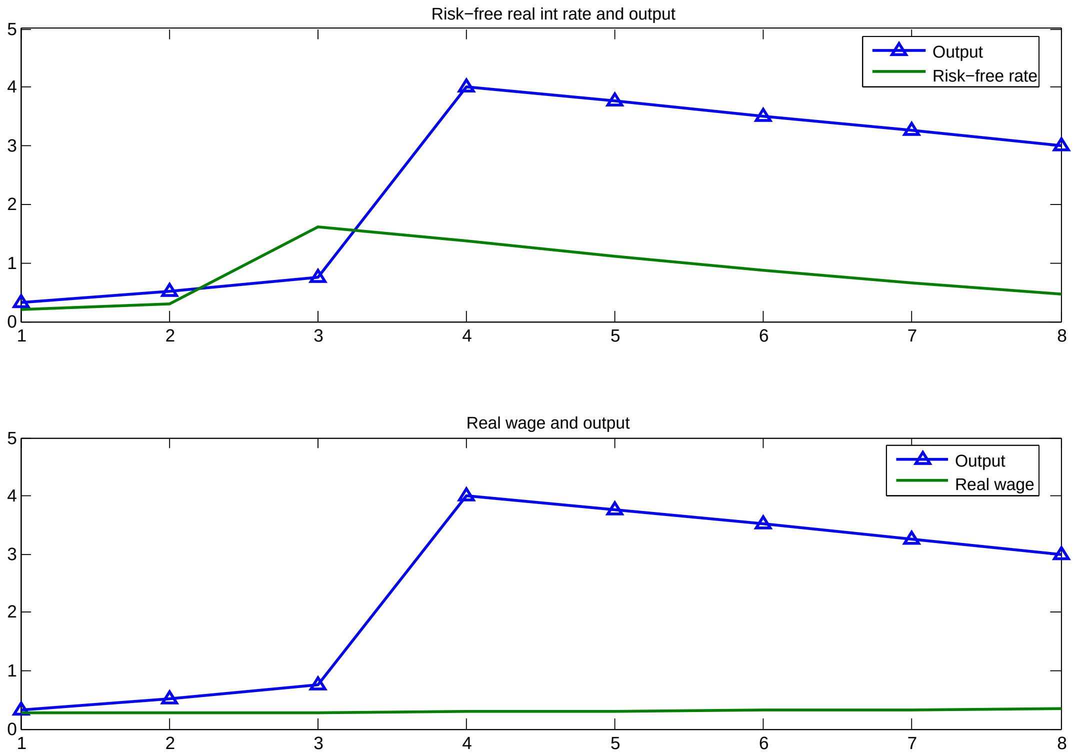
Figure 3: **Firm-specific knowledge: separable indivis lbr pref's** - News shock in period 1 about neutral tech. shock in period 4, tech. shock *fully realized* in period 4



receipt of the news, the rise in the value of knowledge capital q shifts out the firm's labour demand as the firm realizes that it needs to invest in accumulating more knowledge capital by increasing hours, creating a corresponding increase in demand for capital services, and raising the overall level of production. More interestingly, stock prices jump up by over 2 percent upon arrival of the news, and then continue to rise, peaking in the period before the technology shock actually raises TFP. [10] This pattern of share prices reacting well in advance of any movements in productivity is reminiscent of the discussions in Beaudry and Portier ([6]). The rise in the value of the firm results both from a rise in the value of knowledge capital, q, which rises two percent and stays above steady state for several periods, and the increase in knowledge capital itself, which begins to rise in the period after the news

[10] The model is also capable of generating expectational-booms in response to investment specific technology shocks. Plots for these are available from the authors.

Figure 4: **Factor prices: separable indivis lbr pref's** - News shock in period 1 about neutral tech. shock in period 4, tech. shock *fully realized* in period 4



arrives.

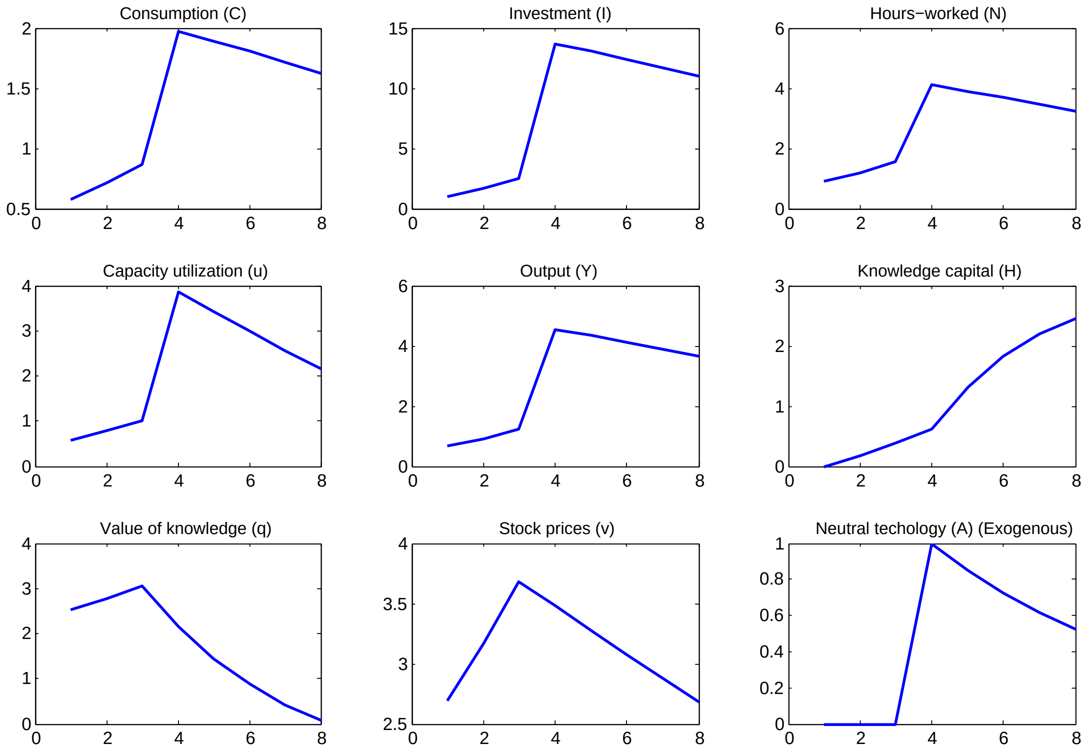
Figure 4 shows the response of factor prices. Both wages and interest rates rise along with output but their response is more muted than that of output.

4.1 The effect of varying preferences

We can influence the response of consumption and investment by altering preferences. Below we consider two possibilities. In Figure 5 we show the case of indivisible labour with nonseparability in consumption and leisure. Figure 6 shows the response of preferences based on Jaimovich and Rebelo ([19]). As both figures show, consumption now rises more than in the case of separable preferences, both in response to arrival of the news, and when the TFP shock actually hits. In both cases investment responds less aggressively.

For the nonseparable indivisible labour preferences, with $\sigma > 1$, the

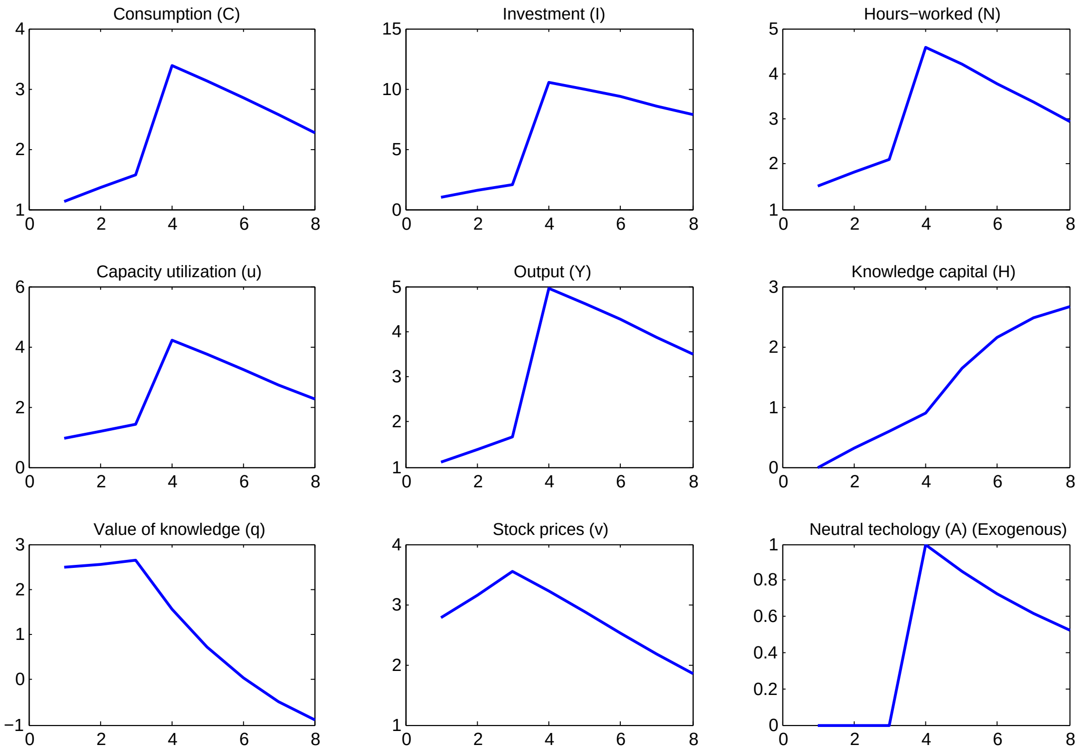
Figure 5: **Firm-specific knowledge: nonseparable indivis lbr pref's** - News shock in period 1 about neutral tech. shock in period 4, tech. shock *fully realized* in period 4



marginal utility of consumption is increasing in hours-worked, making consumption track closer to hours-worked. As discussed by King and Rebelo([24]), when $\sigma > 1$, the combination of nonseparability of consumption and leisure and indivisibility in hours-worked imply that the consumption of the employed group will exceed that of the unemployed group. An increase in total hours, which occurs along the extensive employment margin, represents an increase in the number of individuals moving from unemployment to employment. Since the employed enjoy higher consumption levels, total consumption responds more than in the separable case. Figure 7 shows the impact of changing sigma on the response of consumption, investment and hours as σ varies from 1 to 3.[11] The sharp increase in the response of both variables as σ increases is clearly visible.

[11]Recall that for $\sigma = 1$, the linearized form of the generalized nonseparable indivisible labour preferences is equivalent to the linearized form of the standard separable indivisible labour preferences with log consumption that we consider in the first preference case.

Figure 6: **Firm-specific knowledge: JR pref's** - News shock in period 1 about neutral tech. shock in period 4, tech. shock *fully realized* in period 4



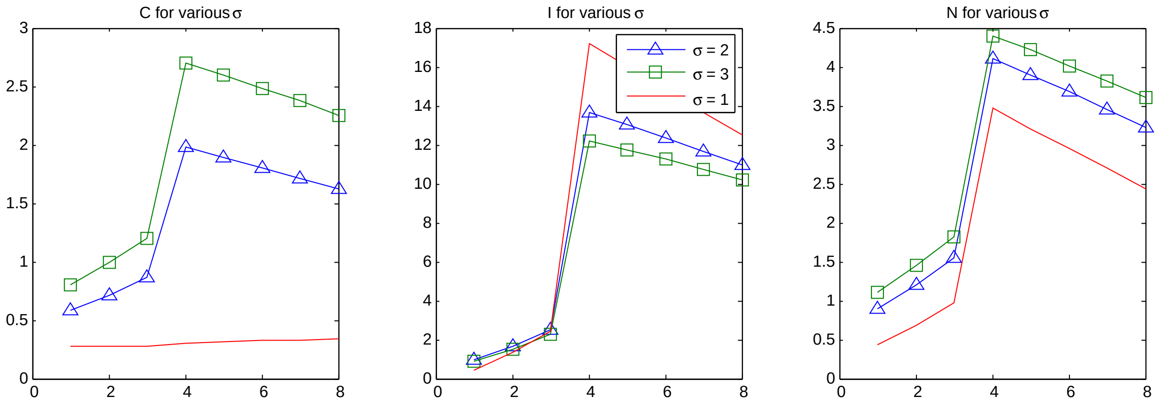
The nonseparablility built into JR preferences also boosts the response of consumption by making the marginal utility of consumption depend on labour. These preferences, however, also offer the benefit of being able to parameterize the wealth effect on leisure. Given the choice of $\zeta = 0.01$, the wealth effect on leisure is so small that it does not counteract the shift in labour demand caused by the jump in q, and thus the response of hours is greater than in the previous cases.

Having discussed the impact of changing preferences on the results, for the remainder of our results we will focus on the nonseparable case where $\sigma = 2$.

4.2 Effect of varying γ

Next we turn to the impact of varying γ. In Section 3.4 we stated that our chosen value of $\gamma = 0.8$ represented a typical value from the literature. Figure 8 shows the effects on consumption, investment and hours of varying

Figure 7: **Effect of varying σ: nonseparable indivis lbr pref's** - News shock in period 1 about neutral tech. shock in period 4, tech. shock *fully realized* in period 4



γ. Clearly the model continues to display co-movement over this range of γ, however, the increased curvature from using a lower γ strengthens the response of hours and therefore the other variables over this range.

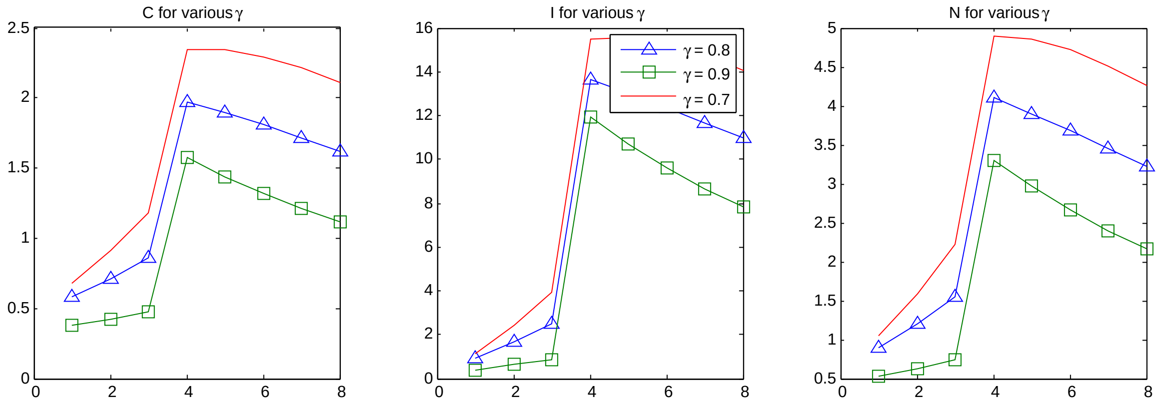
4.3 The role of capital utilization

Due to the assumption of constant returns to all factors in production, the contribution of knowledge capital to output is much smaller than was the case in the human capital example economy discussed earlier. As a result, with K and H fixed in the initial period, without capital utilization, the increase in hours-worked alone cannot raise output sufficiently to finance both an increase in consumption and investment in period 1. Adding capital utilization to the model allows capital services to expand along with labour and therefore increase the responsiveness of output. The optimal determination of utilization can be seen by combining the household's first-order condition for utilization (24) with the firm's first-order condition for capital services (32), and imposing equilibrium to give

$$\delta'(u_t)K_t = A_t F_{ut}. \tag{42}$$

Note that unlike models that include both utilization and intertemporal adjustment costs to capital or investment, here there is no direct intertemporal link to the optimal level of utilization (such as through changes in the relative price of investment or capital which would alter the cost of adjusting

Figure 8: **Effect of varying γ: nonseparable indivis lbr pref's** - News shock in period 1 about neutral tech. shock in period 4, tech. shock *fully realized* in period 4



utilization). Utilization simply responds to changes in its marginal product through the variation of the other factors of production, or changes to the stock of capital. Thus the role of capacity utilization in the model is to simply amplify the boom.

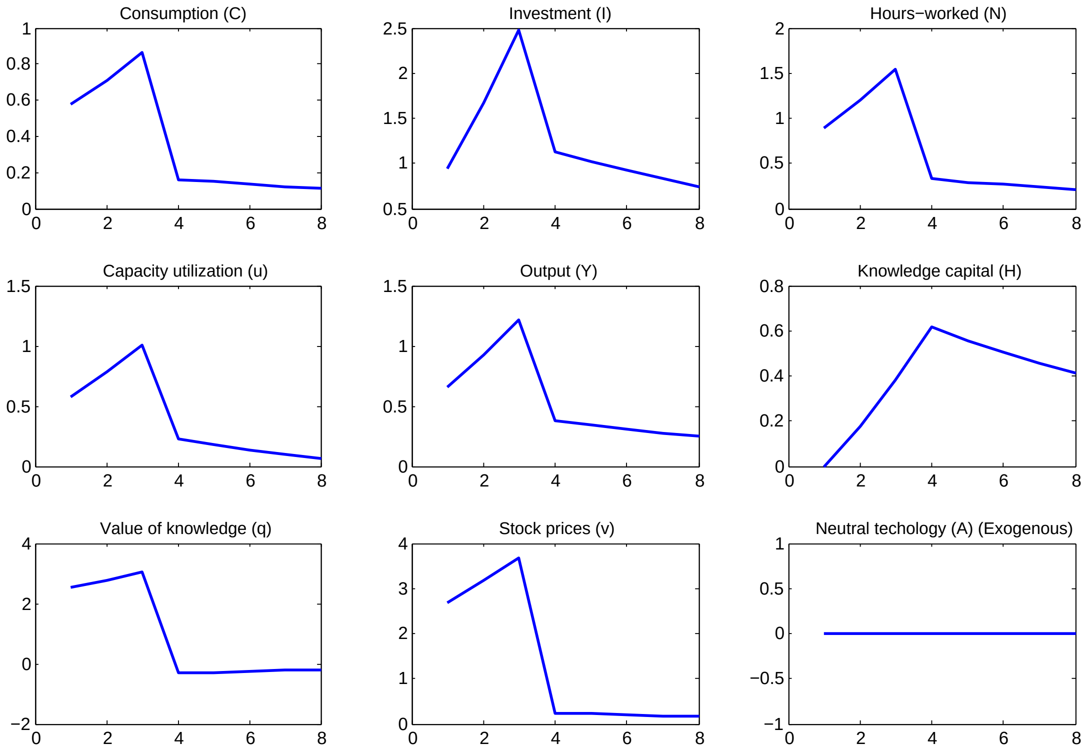
However, while capacity utilization acts as a magnification device for the boom, it cannot deliver one in the absence of knowledge capital. Absent knowledge capital, hours would fall upon receipt of the news which would reduce the marginal productivity of varying capital utilization and this would, in turn (42) induce a reduction in utilization which would further magnify the contraction of output [12].

4.4 A Boom-Bust Episode

Thus far, we have considered the artificial situation in which the expected increases in productivity are fully realized. In reality, agents' forecasts about future fundamentals will be imperfect, and expectations will be continuously revised. A question investigated at length by Beaudry and Portier ([5]) and Christiano et al ([13]) asks whether "boom-bust" behaviour in aggregate quantities and asset prices could result when a news shock turns out be to ex-post "too optimistic" and expectations are as a consequence revised downwards. In this section we briefly consider the extreme situation in this regard where agents receive news of an expected future increase in TFP that

[12] Impulse response plots for this case are available from the authors.

Figure 9: **A boom-bust episode: nonseparable indivis lbr pref's** - News shock in period 1 about neutral tech. shock in period 4, tech. shock ***not*** *realized* in period 4



turns out be to fully unrealized.

Figure 9 shows the response of our model economy to news of a 1% increase in TFP in period 4 and then no eventual realization of that shock in period 4. Since agents proceed as if the shock will be realized, the response of the model in periods 1 to 3 is identical to the previous case: firms and households ramp-up investment in physical and knowledge capital, hours, output, factor prices and stock prices rise along with consumption. When the shock fails to materialize in period 4 however, agents revise their expectations downward and a recession ensues. Agents realize that the excess capacity of the economy needs to be worked off and the value of knowledge capital plummets. This leads to a sharp revision in the value of firms and a "correction in the stock market". While stock prices fall quickly, the response of other variables is more gradual. The last panel of the figure shows that this boom-bust in asset prices and quantities occurs in the absence of any exogenous technical change. More importantly, consistent with discussions

Table 2: Robustness

Preferences	ε [a]	ϵ_u	γ [b]	ν	ζ
KPR separable	0.12-0.54	0.01-0.18	0.57-0.87	-0.67-0.54	n/a
KPR nonseparable	0.06-0.34	0.01-0.31	0.29-0.94	n/a	n/a
JR	0.01-0.25	0.01-0.71	0-0.98 [c]	0.83-1.59	0-0.15

[a] Due to CRS in $F(\cdot)$, a change to ε implies a change to α and/or θ. For this exercise we keep θ constant (through constant δ) and vary α. This alters the labour share slightly but by no more than 0.02- for all parameter ranges in this column.

[b] A change in γ affects the labour share, but the change is less than 0.02 unless otherwise indicated.

[c] Labour share varies 0.68-0.63 over this range

in Beaudry and Portier ([5]), the model generates both a crash in stock prices and overall recession without any true technological regress or variation in monetary policy.

4.5 Robustness

In this section we explore the model's sensitivity to key parameter values with regards to its ability to generate an expectations-driven business cycle. We maintain a strict definition of this type of cycle that in response to news, C, I and N must be at or above steady-state in period 1, and slope upwards beginning either in period 1 or thereafter. Table 2 shows the results of our robustness check for the three different preference specifications. Following the approach of Jaimovich and Rebelo ([19]), we vary only 1 parameter from each baseline paramaterization and report the range of the parameter over which the model can still exhibit an expectations-driven business cycle.

It is clear that the parameter ranges are most limited for the KPR separable preferences, especially for the parameters ϵ_u and ν that provide for the high-substitution response. Allowing for nonseparability significantly expands the parameter range, especially by lowering the required ε. We note that without adjustment costs to investment or capital, a typical parameterization for the first two KPR preference specifications that "fails" to exhibit an expectations-drive business cycle under this definition is often characterized by C and N that are above steady state and sloping upwards in accordance with our definition, but an I that begins below steady state - in some cases less than 0.01% below steady state - yet still sloping upwards and often above steady-state in periods 2 or 3, and giving the overall impression of a "boom".

While we could likely remedy this behaviour and expand our robustness set by adding real frictions to the model, for the purposes of illustrating our primary mechanism we abstain from additional features and disqualify any parameterization that causes either C, I or N to move even slightly below steady state.

Allowing for JR preferences expands the parameter range even further, especially for ε, ϵ_u and ν because the absence of a strong wealth effect to leisure eliminates the need for as elastic a response of output. Even though JR preferences allow for very low learning rates as captured by $\varepsilon < 0.05$, this leads to small increases in variables above their steady state values. For the parameter ζ, we note that above 0.15, while C, I and N rise above steady state, N and/or I begin to slope downwards following their initial rise and thus we exclude values above 0.15 given our definition.

Across all the preference specifications it is clear that our model requires a high labour supply elasticity. Even with JR preferences with no wealth effect, our range of $\nu = 0.83 - 1.59$ is low compared to the range of robustness for the equivalent parameter found by Jaimovich and Rebelo ([19]) in their model. This is not surprising, however, given that the critical mechanism in our model requires a substantial increase in labour supply in response to the shift in labour demand induced by the news.

5 Conclusion

In this paper we highlight the role of knowledge capital in enabling the existence of expectations driven cycles. We present a model in which firms accumulate knowledge capital as a function of hours-worked at the firm. Since the learning process is internalized by firms, their demand for labour exceeds that implied by equating the wage rate to the marginal product of labour. This occurs because firms take into account not only the current increase in output but also the value of the additional knowledge capital generated by the marginal hour of work hired. This latter effect operates as a "shift" factor for a labour demand curve drawn in wage and hours space and is key to enabling an expectations driven cycle. When news of future increases in technology arrive, the value of the firm's knowledge capital rises. This induces the firm to hire more labour at any given wage rate and results in increased production. The subsequent increase in knowledge capital also induces the accumulation of physical capital in anticipation of higher productivity next period. Meanwhile households wish to consume more in anticipation of higher income when the new technology eventually arrives. The increase in hours allows output to rise enough for both consumption and investment to co-move. When expectations about future productivity increases are not realized, the model generates a complete boom-bust cycle.

We note that unlike most models of expectations driven cycles, the rise in investment occurs in the absence of any investment adjustment costs or any other frictions. The ability of the model to generate these cycles in the absence of adjustment costs on changes in investment, a feature that is often built into business cycle models, is worth emphasizing. These costs often have unpleasant implications for factor prices as well as for firm values. As discussed by Christiano et al ([13]) and Jaimovich and Rebelo ([19]) they imply that the price of capital will fall and interest rates spike upwards in an unrealistic way. Since the model does not have adjustment costs, it does not rely on changes in the price of capital to raise firm values. Rather, the key mechanism is the rise in the value of knowledge capital and its accumulation that raises the value of the firm. This leads to an appreciation in the price of equity shares. Evidence suggests that the boom in stock prices leads increases in total factor productivity. We show our model is consistent with this lead-lag relationship. Moreover the very mechanism that generates the expectational-boom also leads to a rise in asset values.

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A Appendix

A.1 Central Planner's problem

The representative household has preferences defined over sequences of consumption C_t and leisure L_t with expected lifetime utility defined as

$$\mathcal{U} = E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t), \tag{43}$$

where β is the representative household's subjective discount factor and the period utility $u(C_t, L_t)$ function falls within the standard general class of preferences detailed in King, Plosser and Rebelo ([23]).

The representative household operates a production technology that produces output Y_t according to the technology

$$Y_t = A_t F(N_t, K_t, H_t) = A_t N_t^{\alpha} \tilde{K}_t^{\theta} H_t^{\varepsilon}, \tag{44}$$

where A_t is the level of an exogenous stationary technology process, N_t is hours-worked, $\tilde{K}_t$ is capital services and H_t is the stock of knowledge capital.

Capital services are defined as

$$\tilde{K}_t = u_t K_t, \tag{45}$$

where K_t is the stock of physical capital, and u_t is the utilization rate of that capital. Physical capital evolves according to

$$K_{t+1} = [1 - \delta(u_t)]K_t + I_t \tag{46}$$

where I_t is investment, and where the depreciation function $\delta(\cdot)$ satisfies the conditions $\delta'(\cdot) > 0$, $\delta''(\cdot) \geq 0$.

The common exogenous total factor productivity process A_t evolves in logs according to the stationary AR(1) process

$$\ln A_t = \rho_A \ln A_{t-1} + \epsilon^p_{A,t-p} + \varepsilon_{A,t}, \tag{47}$$

where $\rho_a < 1$, ϵ^p_{At} is a news shock that agents receive in period t about the innovation θ_{At+p}, and $\varepsilon_{A,t}$ is an unanticipated contemporaneous shock.

The stock of knowledge capital H_t evolves according to

$$H_{t+1} = \Psi(H_t, N_t) = H_t^\gamma N_t^{1-\gamma}. \tag{48}$$

Each period, the representative household is endowed with one unit of time that can be allocated between leisure and hours-worked N_t according to

$$N_t + L_t = 1. \tag{49}$$

Finally, the economy's resource constraint is given by

$$C_t + I_t = Y_t. \tag{50}$$

Combining the above equations, the planner's consolidated resource constraint is

$$C_t + K_{t+1} - [1 - \delta(u_t)]K_t = A_t N_t^\alpha (u_t K_t)^\theta H_t^\varepsilon. \tag{51}$$

The central planner chooses contingent infinite sequences of C_t, N_t, u_t, K_{t+1} and H_{t+1} to maximize (43) subject to equations (48) and (51).

Letting Υ_t and λ_t be the period Lagrange multipliers on (48) and (51)) respectively, the planner's first-order conditions are as follows:

$$u_C(C_t, L_t) = \lambda_t \tag{52}$$

$$U_L(C_t, L_t) = \lambda_t A_t F_{Nt} + \Upsilon_t \Psi_{Nt} \tag{53}$$

$$\delta'(u_t)K_t = A_t F_{ut} \tag{54}$$

$$\lambda_t = \beta E_t \left\{\lambda_{t+1} \left[A_{t+1} F_{Kt+1} + 1 - \delta(u_{t+1})\right]\right\} \tag{55}$$

$$\Upsilon_t = \beta E_t \left\{\lambda_{t+1} A_{t+1} F_{Ht+1} + \Upsilon_{t+1} \Psi_{Ht+1}\right\} \tag{56}$$

Define $q_{ht} = \frac{\Upsilon_t}{\lambda_t}$ as the value of new knowledge capital in terms of consumption. Applying this definition and substituting out λ_t, we can re-write the knowledge capital and hours first-order conditions as

$$q_{ht} = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[A_{t+1} F_{Ht+1} + q_{ht+1} \Psi_{Ht+1}\right] \right\} \tag{57}$$

$$\frac{u_L(C_t, L_t)}{u_C(C_t, L_t)} = A_t F_{Nt} + q_{ht} \Psi_{Nt} \tag{58}$$

A.2 The value of the firm

In this section we investigate steady-state firm profits and the time t value of the firm. As in the main body of the paper, lower case variables indicate individual agent quantities or economy-wide prices, and upper case variables indicate aggregate quantities.

A.2.1 Steady-state firm profits

First, we re-write the firm's two technologies $F(\cdot)$ and $\Psi(\cdot)$ without imposing any particular returns to scale as

$$Y_t = A_t F(N_t, \tilde{K}_t, H_t) = N_t^{\alpha}(\tilde{K}_t)^{\theta} H_t^{\varepsilon} \tag{59}$$

and

$$\Psi(H_t, N_t, \tilde{K}_t) = H_t^{\gamma} N_t^{\eta}. \tag{60}$$

Since $H_{t+1} = H_t^{\gamma} N_t^{\eta}$, in steady state $H = N^{\frac{\eta}{1-\gamma}}$ and thus

$$Y = AN^{\alpha+\frac{\eta\varepsilon}{1-\gamma}}\tilde{K}^{\theta} = A\tilde{F}(N, \tilde{K}), \tag{61}$$

so that in steady state the firm's production function can be expressed as a function of just labour and capital services. From (61) we can see that for a given α, θ and γ, we can impose constant returns to labour and capital services in steady-state such that $\alpha + \theta + \frac{\eta\varepsilon}{1-\gamma} = 1$ by either: (i) $\eta < 1 - \gamma$ (DRS in $\Psi(\cdot)$) and $\varepsilon > 1 - \alpha - \theta$ (IRS in $F(\cdot)$), or (ii) $\eta > 1 - \gamma$ (IRS in $\Psi(\cdot)$) and $\varepsilon < 1 - \alpha - \theta$ (DRS in $F(\cdot)$), or (iii) $\eta = 1 - \gamma$ (CRS in $\Psi(\cdot)$) and $\varepsilon = 1 - \alpha - \theta$ (CRS in $F(\cdot)$. In this paper we impose case (iii), and as such in steady-state,

$$Y = AN^{1-\theta}\tilde{K}^{\theta}., \tag{62}$$

where we explicitly note that despite the presence of knowledge capital as an input into production, in steady state, the production of Y displays CRS to just labour and physical capital services. We can see how this relates to firm profits in steady-state by expressing $D = Y - wN - r\tilde{K}$ as a share of output as

$$\frac{D}{Y} = 1 - \frac{wN}{Y} - \frac{r\tilde{K}}{Y} = 1 - S_N - S_{\tilde{K}}, \tag{63}$$

where S_N is the steady-state labour share and $S_{\tilde{K}}$ is the steady-state capital services share. Applying (59) and (60) to the firm's first-order conditions gives h and

$$S_{\tilde{K}} = \theta, \tag{64}$$

so that

$$\frac{D}{Y} = 1 - \alpha - \theta - \left(\frac{\eta}{1/\beta - \gamma}\right)\varepsilon = 1 - [\alpha + \left(\frac{\eta}{1/\beta - \gamma}\right)] - \theta, \tag{65}$$

or in our case with $\eta = 1 - \gamma$ and $\varepsilon = 1 - \alpha - \theta$

$$\frac{D}{Y} = \left(\frac{1/\beta - 1}{1/\beta - \gamma}\right)\varepsilon = \left(\frac{\xi}{\xi + (1 - \gamma}\right)[1 - \alpha - \theta], \tag{66}$$

where $\xi = 1/\beta - 1$ is the household-owner's subjective discount rate. From (65) and (66) it is evident that the steady-state profit share will be affected not only by the steady-state returns to scale to N and $\tilde{K}$ as implied in (61), but also the household-owner's subjective rate of time discount β. With $\beta < 1$, the share of profits is slightly positive, even though $F(\cdot)$ exhibits constant returns to N and $\tilde{K}$.

A.2.2 Period t dynamic value of firm

Having established the firm's steady-state profits, we now obtain an expression for the dynamic period t value of the firm. We follow an approach similar to that used by Jaimovich and Rebelo ([19]) to value the firm using a recursive formulation of the firm's problem.

First, we note that the stochastic process (47) can be represented by the first-order system

$$\ln A_t = \rho_A \ln A_{t-1} + \Omega_t^p + \varepsilon_{A,t} \tag{67}$$

$$\Omega_t^p = \Omega_{t-1}^{p-1} \tag{68}$$

$$\Omega_t^{p-1} = \Omega_{t-1}^{p-2} \tag{69}$$

$$\vdots \tag{70}$$

$$\Omega_t^{p-p+1} = \Omega_t^{p-p} = \Omega_{t-1}^0 \tag{71}$$

$$\Omega_t^0 = \epsilon_t. \tag{72}$$

Defining the $p \times 1$ vector $\Omega_t = \begin{bmatrix} \Omega_t^P \\ \Omega_t^{P-1} \\ \vdots \\ \Omega_t^0 \end{bmatrix}$, the system can then be represented compactly using matrix notation by

$$\ln A_t = \rho_A \ln A_{t-1} + \Omega_t^p + \varepsilon_{A,t} \tag{73}$$

$$\Omega_t = B\Omega_{t-1} + G\epsilon_t \tag{74}$$

where B is a $p \times p$ matrix with all zeroes except for ones on the first diagonal above the main diagonal, and G is a $p \times 1$ vector with all zeroes except for a one in the final row. Using this notation, the agent's exogenous states are then A_t and Ω_t, the former representing the current state of technology and the latter the agent's current information set of news shocks relevant for forecasting the future state of technology.

We can then re-formulate the firm's problem (30) recursively as

$$V(H, A, \Omega) = \max_{N, \tilde{K}', H'} \left\{ \lambda[AF(N, \tilde{K}, H) - wN - r\tilde{K}] + \beta EV(H', A', \Omega') \right\} \tag{75}$$

subject to

$$H' = \Psi(H, N) \tag{76}$$

and where the firm takes as given from the aggregate

$$\mathbb{K}' = \mathbb{K}(\mathbb{H}, \mathbb{K}, A, \Omega) \tag{77}$$

$$\mathbb{H}' = \mathbb{H}(\mathbb{H}, \mathbb{K}, A, \Omega) \tag{78}$$

$$\ln A_t = \rho_A \ln A_{t-1} + \Omega_t^p + \varepsilon_{A,t} \tag{79}$$

$$\Omega_t = B\Omega_{t-1} + G\epsilon_t \tag{80}$$

$$\lambda' = \lambda(A, H, K, \Omega) \tag{81}$$

$$w = w(\mathbb{H}, \mathbb{K}, A, \Omega) \tag{82}$$

$$r = r(\mathbb{H}, \mathbb{K}, A, \Omega) \tag{83}$$

Note that we have defined $V(H, A, \Omega)$ in terms of the household-owners' utility as given by λ, so that in terms of the notation used in (30) in the main text, $V_t = \frac{V(H,A,\Omega)}{\lambda}$. Letting Υ be the Lagrange multiplier on (76), we write (75) as

$$V(H, A, \Omega) = \max_{N, \tilde{K}', H'} \left\{ \lambda[AF(N, \tilde{K}, H) - wN - r\tilde{K}] + \Upsilon[\Psi(H, N) - H'] + \beta EV(H', A', \Omega') \right\}. \tag{84}$$

Solving the maximization on the right-hand side gives

$$w = AF_N(N, \tilde{K}, H) + \frac{\Upsilon}{\lambda}\Psi_n(h, n) \tag{85}$$

$$r = AF_{\tilde{K}}(N, \tilde{K}, H) \tag{86}$$

$$\Upsilon = \beta EV_1(H', A', \Omega'). \tag{87}$$

Now define $\bar{V}(H, A, \Omega) = \beta EV(H', A', \Omega')$ as the end-of-period value of the firm, which is related to (30) in the main text by $\bar{V}_t = \frac{\bar{V}(H,A,\Omega)}{\lambda}$, and to the price v_t of a share of the firm's equity through the household's Euler equation for equity (26) as $v_t = \bar{V}_t = \frac{\bar{V}(H,A,\Omega)}{\lambda}$.

Since it can be shown that $\bar{V}(H, A, \Omega)$ is homogenous of degree 1 in H (which we prove below in A.2.3 below), we can write

$$\bar{V}(H, A, \Omega) = \beta E V_1(H', A, \Omega)H'. \tag{88}$$

Substituting the firm's H' first-order condition (??) into (88) then gives

$$\bar{V}(A, H) = \Upsilon h' \tag{89}$$

as the end-of-period value of the firm in terms of the household-owners' utility. Using the notation in the main text, this then gives

$$\bar{V}_t = v_t = \frac{\Upsilon_t}{\lambda_t} H_{t+1}. \tag{90}$$

Or, defining $q_t = \frac{\Upsilon_t}{\lambda_t}$,

$$\bar{V}_t = v_t = q_t H_{t+1}, \tag{91}$$

so that the period t price v_t of the firm's equity share is the product of the value of knowledge capital in terms of consumption today and next period's stock of knowledge capital.

It only remains to establish that $\bar{V}(H, A, \Omega)$ is homogenous of degree 1 in H, which we do in the following section.

A.2.3 Degree 1 homogeneity of $\bar{V}(H, A, \Omega)$

The firm's recursive problem (75) in our model has the unique property that both the return-function $\lambda[AF(N, \tilde{K}, H) - wN - r\tilde{K}]$ and the constraint $H' = \Psi(H, N)$ are homogeneous of degree 1 (hod 1) in N, $\tilde{K}$ and H. In what follows we will show that these properties then imply that $V(H, A, \Omega)$ and thus $\beta EV(H', A', \Omega')$ are hod 1 in H.

First, defining any contingent sequence $\{H_t\}_{t=1}^{\infty}$ as a plan, let

$$\Pi(H_0, A_0, \Omega_0) = \{\{H_t\}_{t=1}^{\infty} : H_{t+1} = \Psi(H_t, N^*(H_t, A_t, \Omega_t)), t = 0, 1, ...\} \tag{92}$$

be the set of plans that are feasible from (H_0, A_0, Ω_0) in that they satisfy the initial conditions H_0, A_0, and Ω_0 and the constraint $H_{t+1} = \Psi(H_t, N^*(H_t, A_t, \Omega_t))$, where $N_t = N^*(N_t, A_t, \Omega_t)$ is the policy function for N_t, and $\underline{N}$ a typical feasible plan in $\Pi(N_0, A_0, \Omega_0)$. Then let

$$u(\underline{H}) = \sum_{t=0}^{\infty} \left\{ \lambda[AF(N^*(H_t, A_t, \Omega_t), \tilde{K}^*(H_t, A_t, \Omega_t), H_t) - wN^*(H_t, A_t, \Omega_t) - r\tilde{K}^*(H_t, A_t, \Omega_t)] \right\} \tag{93}$$

be the discounted sum of values of the return function over some feasible plan $\underline{H}$, where $\tilde{K}_t = \tilde{K}^*(H_t, A_t, \Omega_t)$ is the policy function for $\tilde{K}_t$. The maximum value function is then defined as $V^*(H_0, A_0, \Omega_0) = \max_{\underline{H} \in \Pi(H_0, A_0, \Omega_0)} u(\underline{H})$. By the hod 1 of our return function,

$$\begin{aligned} &\lambda[AF(N^*(\theta N_t, A_t, \Omega_t), \tilde{K}^*(\theta H_t, A_t, \Omega_t), \theta H_t) - wN^*(\theta H_t, A_t, \Omega_t) - r\tilde{K}^*(\theta H_t, A_t, \Omega_t)] \quad (94) \\ &\quad = \theta\lambda[AF(N^*(H_t, A_t, \Omega_t), \tilde{K}^*(H_t, A_t, \Omega_t), H_t) - wN^*(H_t, A_t, \Omega_t) - r\tilde{K}^*(H_t, A_t, \Omega_t)] \end{aligned}$$

for some value θ, and therefore

$$\begin{aligned} u(\theta\underline{H}) &= \theta \sum_{t=0}^{\infty} \left\{ \lambda[AF(N^*(H_t, A_t, \Omega_t), \tilde{K}^*(H_t, A_t, \Omega_t), H_t) - wN^*(H_t, A_t, \Omega_t) - r\tilde{K}^*(H_t, A_t, \Omega_t)] \right\} (\\ &= \theta u(\underline{H}). \end{aligned}$$

Similarly, by the hod 1 or our constraint, $\Psi(\theta H_t, N^*(\theta H_t, A_t, \Omega_t)) = \theta\Psi(H_t, N^*(H_t, A_t, \Omega_t))$, and therefore

$$\underline{H} \in \Pi(H_0, A_0, \Omega_0) \Leftrightarrow \theta\underline{H} \in \Pi(\theta H_0, A_0, \Omega_0). \quad (96)$$

As a result,

$$\begin{aligned} V^*(\theta H_0, A_0, \Omega_0) &= \max_{\theta\underline{H} \in \Pi(\theta H_0, A_0, \Omega_0)} u(\theta\underline{H}) \quad (97) \\ &= \max_{\underline{H} \in \Pi(H_0, A_0, \Omega_0)} \theta u(\underline{H}) \\ &= \theta V^*(H_0, A_0, \Omega_0), \end{aligned}$$

which implies that for any given state (H, A, Ω) , $V(\theta H, A, \Omega) = \theta V(H, A, \Omega)$, and therefore $\beta EV(\theta H', A', \Omega') = \theta\beta EV(H', A', \Omega')$, and thus $\bar{V}(H, A\Omega)$ is homogenous of degree 1.