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Using HP Filtered Data for Econometric Analysis: Some Evidence from Monte Carlo Simulations *

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Abstract

The Hodrick-Prescott (HP) filter has become a widely used tool for detrending integrated time series in applied econometric analysis. Even though the theoretical time series literature sums up an extensive catalogue of severe criticism against an econometric analysis of HP filtered data, the original Hodrick and Prescott (1980, 1997) suggestion to measure the strength of association between (macro-)economic variables by a regression analysis of corresponding HP filtered time series still appears to be popular. A contradictory situation which might be justified only if HP induced distortions were quantitatively negligible in empirical applications. However, this hypothesis can hardly be maintained as the simulation results presented within this paper indicate that HP filtered series give seriously rise to spurious regression results.

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1 Introduction

Two decades after its introduction in a working paper by Hodrick and Prescott (1980, 1997) the so-called Hodrick-Prescott (HP) filter plays a prominent role in econometric time series analysis.[1] The method is easily applied to extract a stochastic trend moving smoothly over time just as one would draw it with a free hand and generates stationary series for data generating processes (DGPs) being integrated up to order four. But even though the widespread usage of HP filtered data in empirical studies might seem well motivated at a first glance, the procedure has been and still is objected to severe criticism.[2]
On the one hand, from a theoretical point of view, its underlying assumption of an independent secular-cyclical decomposition of economic time series does not have to be fulfilled naturally.[3] On the other hand, from a methodological point of view, every filtering technique[4] has to distort the characteristics of the original data set as it is impossible to construct an ideal filter with a finite number of observations.[5] Thus, the practioner is confronted with a fundamental question: Does it make sense to use (HP) filtered data for further econometric analysis?

Despite all criticisms, the HP filter is still widely applied to actual and artificial time series. First of all, the influential paper by Hodrick and Prescott (1980, 1997) inspired many real business cycle advocates.[6] In this tradition, for instance, Fiorito and Kollintzas (1994) study output fluctuations of G7 economies whereas Christodoulakis, Dimelis and Kollintzas (1995) compare business cycle features of EC-members. Further exemplary applications of the HP technique can be found on the field of monetary policy analysis: Razzak (1997), e.g. estimates a non-linear expectations-augmented Phillips curve. Orphanides and van Norden (2002) com-

[1] See, e.g. Pedersen (2001): "The most widely used filter is the Hodrick-Prescott filter ...".

[2] The most popular reviewers of the HP filter are Cogley and Nason (1995), Ehlgen (1998), Guay and St-Amant (1997), Harvey and Jaeger (1993), King and Rebelo (1993) and Park (1996).

[3] As an example, see the following remark of King and Rebelo (1993): "The dividing lines virtually disappear in models of endogenous economic growth, in which transient displacements to the dynamic system have permanent consequences for the paths of economic quantities".

[4] Statistical theory offers a large set of alternative filtering methods like deterministic time trends, first-order differences, the Beveridge and Nelson procedure, unobserved components models or frequency domain techniques. As we do not intend to provide a complete survey of the variety of possible filtering procedures in this paper, the reader is invited to look up Canova (1994, 1998) or Pedersen (2001) for complementary presentations and comparisons of the most popular detrending methods.

[5] For an illustrative empirical study of this topic see Canova (1998) who points out "that both quantitatively and qualitatively 'stylized facts' of U.S. business cycles vary widely across detrending methods".
Studies based on prefiltered data therefore should be complemented by a comparison of the results for different detrending methods. See Bjørnland (2000) for an example.

[6] We do not replicate the complete list of standard references at this place as the interested reader finds a comprehensive summary of prominent HP applications in Ravn and Uhlig (2002).

pare HP filter performance with alternative detrending methods in estimating the output gap whereas Grant (2002) employs the HP technique in an empirical study of Okun's law. Recently, Bouakez, Cardia and Ruge-Murcia (2003) used linearly detrended as well as HP filtered data for a study of the effects of monetary shocks on output.

This established practice lately received new support by Pedersen (2001), who compares the distortionary effects of ten different linear filters and concludes that the HP filter is the less distorting one for a class of $AR(1)$-processes. Due to this lasting prominence,[7] the following pages examine the effects of HP filter-distortions in a regression context. After a formal description of the method and a closer discussion of its properties in section 2, section 3 presents the results of a Monte Carlo study which indicate a serious risk of spurious regressions. Section 4 summarizes the main findings and provides our conclusions.

2 The HP Filter

The decomposition procedure of the HP filter assumes that a given time series y_t was generated as sum of a cyclical component c_t and a stochastic trend τ_t being uncorrelated with this cycle:

$$y_t = \tau_t + c_t \; . \tag{1}$$

Being interested in smoothly varying trend components, Hodrick and Prescott (1980, 1997) operationalize this idea by a penalty function which balances the trade-off between "goodness-of-fit" and "degree of smoothness" of the trend estimate in the following way.

$$\min_{\tau_t} \sum_{t=1}^{T} (y_t - \tau_t)^2 + \lambda \sum_{t=2}^{T-1} \left[(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1}) \right]^2 \; . \tag{2}$$

Obviously, any empirical application of equation (2) forces the researcher to choose a numerical value for the smoothing parameter λ.[8] Hodrick and Prescott suggested to set $\lambda = 1600$ for quarterly data and, indeed, almost every quarterly based HP application did rely on this proposal whereas for other data frequencies no common practice seems to have been established until today.[9]

[7] See, e.g. Ravn and Uhlig (2002): "... [the HP filter] has withstood the test of time and the fire of discussion remarkably well. Thus ... it is likely that the HP filter will remain one of the standard methods for detrending."

[8] $0 \leq \lambda \leq \infty$. With λ increasing, the trend variability decreases. In the limiting case $\lambda \to \infty$ the trend becomes perfectly linear.

[9] By setting $\lambda = 1600$ Hodrick and Prescott aimed at fulfilling the restriction $\lambda = \frac{\sigma_\tau^2}{\sigma_c^2}$ (σ_τ and σ_c indicating the standard deviations of the trend and the cyclical component, respectively). Statistical derivations of optimal smoothing parameters for varying sampling frequencies are provided by Ravn and Uhlig (2002) and Pedersen (2001).

Early studies of the properties of the HP filter have been presented by Singleton (1988) and King and Rebelo (1993). For stationary series, it operates as a symmetric linear filter which induces no phase shifts, serves as close approximation to an ideal high-pass filter eliminating frequencies of 32 quarters or greater (see figure 1 which visualizes the power transfer functions of an ideal linear high-pass filter for frequencies of 32 quarters or greater and the one of the HP filter with $\lambda = 1269$)[10] and, as the results in Pedersen (2001) suggest, is less distorting than other approximate high-pass filters.

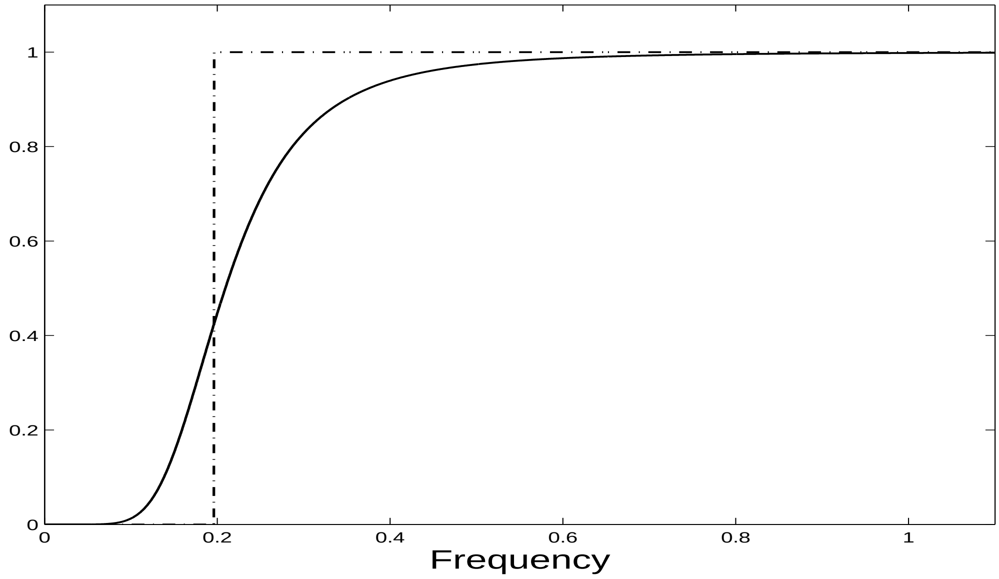

Figure 1: Power transfer functions: Ideal high-pass and HP filter (solid)

However, the HP filter is typically applied to non-stationary series. Even though King and Rebelo (1993) showed that it generates stationarity for data sets being integrated up to order four together with the conditions for the HP filter being optimal in the sense of Wiener (1949), heavy criticism rose against this common practice.

First, due to its imposed restrictions, a careless application of HP filtered data can be criticized from an economical perspective. Singleton (1988), e.g. points out that the fundamental hypothesis of independent trend and cyclical components was already rejected by Burns (1934) whereas Smant (1998) stresses the perfect-foresight character of HP detrended series.[11] Furthermore, equation (1) neglects any influences due to additional time series components like seasonal variations

[10] From the definition of the fundamental frequency $\omega = 2\pi/T$ (T representing period length in the time domain), it immediately follows that an ideal high-pass filter for $T = 32$ should give zero weights to frequencies less than $\pi/16 \approx 0.196$ whereas the higher frequencies should be given a weight equal to one.

[11] A critique that can be raised against every filter calculating actual estimates on the basis of future observations.

or noise terms. Empirical applications of the HP technique are therefore often performed to seasonal adjusted series. Nevertheless, as any seasonal adjustment procedure also creates distortions, this common practice cannot be justified easily by statistical theory.
Second, from a methodological perspective, King and Rebelo (1993) showed that HP filtering can only be optimal when the original series were integrated of order two. But as many macroeconomic time series are well approximated by first difference stationary DGPs, the warning of Cogley and Nason (1995) against applying the HP filter to first-order integrated or near unit root processes must be taken seriously. Similar criticism is raised by Harvey and Jaeger (1993), Guay and St-Amant (1997) and Park (1996). They show that an application of the HP filter to integrated series creates artificial business cycles. Furthermore, this problem deepens with the degree of integration. In addition to that, Ehlgen (1998) shows that even if the optimality conditions for the HP filter are met its application always alters the autocorrelations and standard deviations of original series.

3 Simulation

This section summarizes the findings of two Monte Carlo studies considering the main characteristics of HP filtered series in a regression setup. Within subsection 3.1 two independent autoregressive processes are simulated and regressed on each other. Our findings indicate serious pitfalls for a regression analysis of HP detrended series: For near unit root DGPs, OLS-test statistics of prefiltered series do suffer from significant size distortions. Furthermore, the additional simulation exercises of subsection 3.2 point out that HP filering might seriously detach the underlying dynamics of a multivariate DGP.

3.1 MC-Study of Independent AR(1)-Processes

Within this subsection, we are going to present the results of $AR(1)$-simulation exercises which use the findings of Pedersen (2001) that HP detrending with $\lambda = 1269$ is the least distorting filtration method[12] for a stationary autoregressive process of the form

$$y_t = 0.9 \cdot y_{t-1} + u_t \ . \tag{3}$$

[12] Note that Pedersen derives this result by a comparison of ten different filters including the seemingly preferable Baxter and King (1999) band-pass filter. Additionally, Pedersen also shows that the extra-distortions induced by the suboptimal standard $\lambda = 1600$ assumption seem to be small overall.

For each simulation run we generated a pair of independent time series

$$y_t^1 = \phi^1 y_{t-1}^1 + \epsilon_t \quad (4)$$
$$y_t^2 = \phi^2 y_{t-1}^2 + \psi_t \,, \quad (5)$$

where ϵ_t and ψ_t are *iid*-distributed white noise processes with standard deviations $\sigma_\epsilon = 0.0375$ and $\sigma_\psi = 0.15$, respectively. As the HP filter assumes the absence of high frequency signals like, e.g. seasonal dummies, this setup – labelled setup A in the sequel – seems preferable to us.[13] Subsequently, the artificial series passed the HP filter with $\lambda = 1269$, yielding artificial trend and cyclical components ${y_t}^i_{HP}$ and ${y_t}^i_{cycl}$. Then, the following OLS-regressions were estimated:

$$y_t^1 = \alpha_1 y_{t-1}^1 + \alpha_2 y_{t-1}^2 + u_t \quad (6)$$
$$y_{t\,cycl}^1 = \beta_1 y_{t-1\,cycl}^1 + \beta_2 y_{t-1\,cycl}^2 + v_t \quad (7)$$
$$y_{t\,HP}^1 = \gamma_1 y_{t-1\,HP}^1 + \gamma_2 y_{t-1\,HP}^2 + w_t \quad (8)$$

This procedure was replicated a thousand times in order to record the events of 5%-significant t-statistics of the estimated (structural non-explanatory) parameters $\hat{\alpha}_2$, $\hat{\beta}_2$ and $\hat{\gamma}_2$.[14]
Finally, these simulations were performed for 81 different parameter-constellations fulfilling $0 \leq \phi^i \leq 0.99$.[15] For illustrative purposes figure 2 visualizes a stochastic realization of both processes (upper panel) together with their estimated HP trends (lower panel) for the near unit root case $\phi_1 = \phi_2 = 0.99$.[16]

3.1.1 Results for the Unfiltered Series

Running 1000 regressions with a chosen significance-level of 5% should yield about 50 (erroneously) significant $\hat{\alpha}_2$-estimates for the original time series realizations. An examination of table 1 shows that this assumption seems to be violated only in the near unit root case $\phi^1 = \phi^2 = 0.99$. A successful filter, however, might at least slightly decrease the number of significant estimation results for $\phi^i \in \{0.90, 0.95\}$.

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[13] Nevertheless, additional simulation exercises based on seasonal DGPs have also been carried out. See appendix A for an account of these complementary findings.

[14] Each simulation run was based on 480 observations generated by equations (4) and (5) with starting values y_1^1, y_1^2 set equal to zero. Yet, to avoid distortions due to initializing-effects, the first 80 observations were dropped, so the effective estimation samples were based on 400 artificial observations. Replications of this simulation exercise with 50, 100 and 200 observations per sample did not seem to affect the qualitative findings.

[15] This interval was chosen, because Pedersens technique for the derivation of optimal smoothing parameters is restricted to the class of stationary DGPs.

[16] The corresponding HP cyclical components are plotted in figure 3.

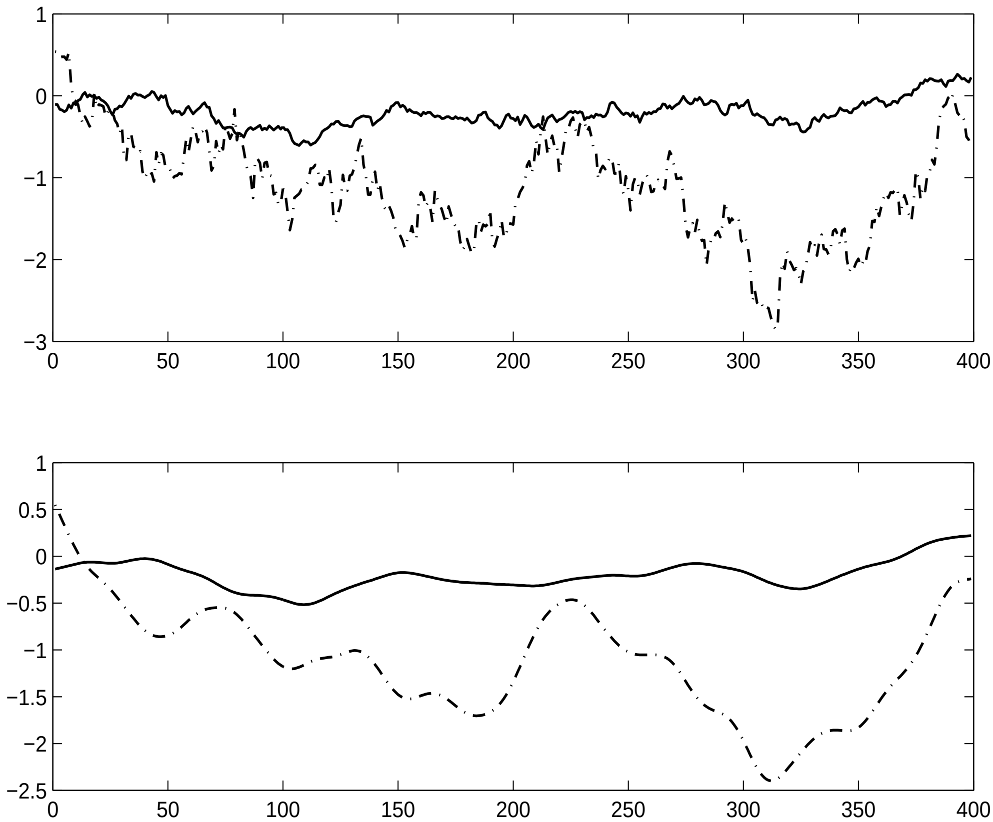

Figure 2: DGP-Realizations (upper panel) and estimated HP trends

Table 1: Setup A: Regression results for equation (6)

$T = 400$	ϕ_1								
ϕ_2	0.00	0.05	0.10	0.25	0.50	0.75	0.90	0.95	0.99
0.00	66	42	72	57	45	49	46	49	40
0.05	52	60	42	41	53	52	55	62	43
0.10	51	51	66	55	44	57	40	57	57
0.25	47	40	51	59	69	53	52	43	56
0.50	49	52	40	50	60	53	51	49	56
0.75	37	55	55	55	53	52	54	45	50
0.90	47	57	58	49	53	53	64	64	55
0.95	60	44	65	47	59	54	58	64	71
0.99	48	51	51	53	57	49	55	61	94

Entries indicate number of significant t-statistics
for $\hat{\alpha}_2$ (at 5%-level) in 1.000 replications

3.1.2 Results for the HP Cyclical Components

The estimated HP cyclical components $y^1_{t\,cycl}$, $y^2_{t\,cycl}$ shown in figure 3 correspond to the exemplary pair of series introduced by figure 2.

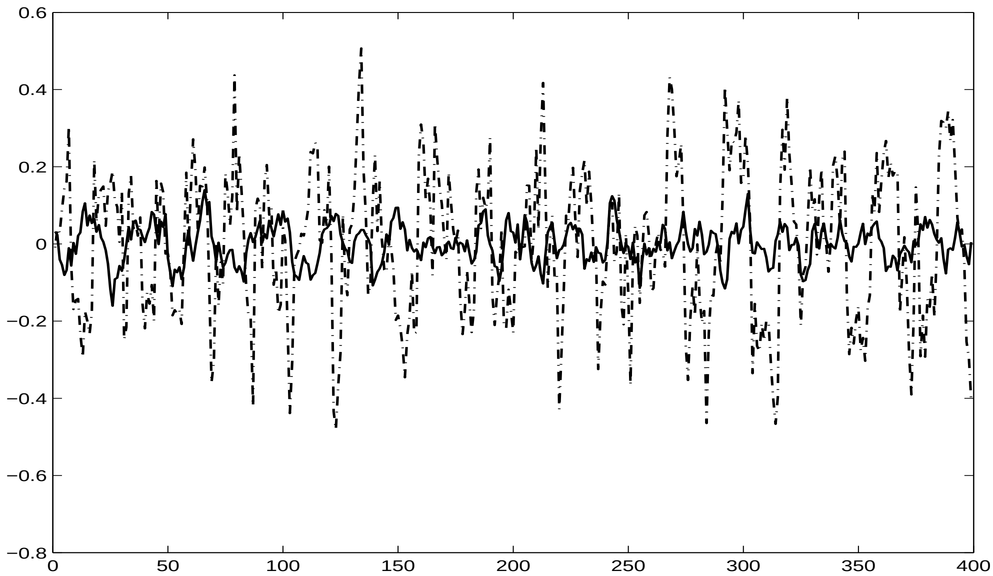

Figure 3: HP cyclical components

Simulating 1000 pairs of artificial HP cyclical components with varying constellations of the autoregressive parameters the regression setup according to equation (7) yields the results summarized by table 2. Comparing these results with the ones for the unfiltered series, some quite amazing observations have to be pointed out.

Table 2: Setup A: Regression results for equation (7)

$T = 400$	ϕ_1								
ϕ_2	0.00	0.05	0.10	0.25	0.50	0.75	0.90	0.95	0.99
0.00	72	52	76	64	53	64	62	40	56
0.05	66	66	52	52	63	62	58	45	47
0.10	71	47	76	63	53	62	64	52	70
0.25	57	49	61	61	78	62	41	43	54
0.50	48	59	55	68	83	81	58	63	69
0.75	56	60	58	62	74	70	77	78	64
0.90	55	65	52	64	69	98	93	91	99
0.95	55	51	66	58	72	92	96	97	117
0.99	54	52	47	63	79	84	94	90	92

Entries indicate number of significant t-statistics
for $\hat{\beta}_2$ (at 5%-level) in 1.000 replications

Whereas for the unfiltered series the evidence of distorted significance statistics appeared limited to the $\phi^1 = \phi^2 = 0.99$ case, for the estimated cyclical components the whole interval $0.75 \leq \phi^i \leq 0.99$, at least, must be suspected of generating spurious regression results. Furthermore, for the one and only really problematic case of table 1 (the nearly integrated case with both autoregressive parameters close to one) the qualitative improvements gained by estimating filtered series instead of nearly integrated level data seem to be rather poor.[17]

Overall, the HP application obviously worsened things. For the unfiltered data, only 1 parameter-regime did yield more than 75 significant estimates whereas for the HP cyclical components the number of these events increased up to 20. To put it another way: In almost 25% of our simulation exercises the estimated HP cyclical components of two independent DGPs suffered evidently from spurious regression results.

3.1.3 Results for the Artificial HP Trends

Our study restricts itself to an analysis of stationary DGPs, so the estimated HP trends themselves also have to be stationary. An exemplary time series plot of two HP estimates ${y^1}_{HP}, {y^2}_{HP}$ for the most interesting parameter constellation $\phi^1 = \phi^2 = 0.99$ has already been introduced by the lower panel of figure 2. As expected, almost all variability of the original series (upper panel of figure 2) has been smoothed out by the HP filter. Note however that the filtration process obviously induced spurious cyclical patterns for series ${y^2}_{HP}$. Note further that, whereas series ${y^1}_{HP}$ seems to run more or less vertically at a first glance, a preliminary visual inspection cannot exclude the potential pitfall of both filter-outcomes following similar filtration induced structures.[18]
We therefore turn over to an inspection of the regression results for equation (8), i.e. regressing HP trend components on each other, which have been summarized in table 3. The findings of table 3 are apparently striking. There do exist severe pitfalls in an econometric interpretation of HP trends:
For more than 82% of the parameter combinations in our simulation study the number of significant estimates $\hat{\gamma}_2$ exceeds the number of the insignificant ones. Even for the "best performing" simulation ($\phi_1 = 0.99, \phi_2 = 0.50$), more than every fourth regression wrongly indicates a significant correlation.[19]

[17] Yet, as can be checked by the tables of appendix A, this conclusion is slightly softened for the seasonal DGPs.

[18] As the ordinates of figure 2 have been scaled to match the relative high standard deviation of y^2 it might be impossible to recognize similar patterns for the less volatile y^1 series.

[19] Furthermore, this problem seriously deepens in the presence of additional seasonal signals. See appendix A.

Table 3: Setup A: Regression results for equation (8)

$T = 400$	ϕ_1								
ϕ_2	0.00	0.05	0.10	0.25	0.50	0.75	0.90	0.95	0.99
0.00	574	537	566	567	604	570	537	484	311
0.05	592	581	573	564	555	603	522	479	301
0.10	595	579	589	573	576	561	532	459	302
0.25	574	573	549	604	553	578	544	491	282
0.50	587	585	568	575	585	570	536	483	269
0.75	581	580	591	617	585	565	539	488	318
0.90	618	622	618	609	635	577	592	564	380
0.95	638	633	635	646	643	636	644	624	459
0.99	658	679	683	663	660	662	660	661	625

Entries indicate number of significant t-statistics
for $\hat{\gamma}_2$ (at 5%-level) in 1.000 replications

Overall, there seems to exist a positive correlation between HP distortions and the variance of the original DGPs.[20] However, with regard to table 3 we can hardly expect suitable situations for any regression analysis of HP trend estimates.

[20] The events of spurious significance tend to decrease with rising ϕ_1 and to increase with rising ϕ_2.

3.2 MC-Study of an Estimated VAR-Process

This section presents simulation results for an exemplary empirical HP application.[21] After a short description of the example, subsection 3.2.2 studies the impact of HP filtering on simulated series with well defined causality structure. Subsection 3.2.3 completes this analysis with a study of the impact of the HP filer on two independently generated series.

3.2.1 The Empirical Example

The upper panel of figure 4 shows a plot of logged real GDP series for the United States and Germany over the period 1975:1-2001:4.[22] In addition to the level data, a time series plot of the corresponding HP estimated cyclical components[23] is given by the lower panel of figure 4.

The filtration process obviously succeeded in separating two stationary[24] series which seem to be governed by similar dynamic patterns over the sample period. In applied econometric research one would usually tend to interpret these similarities as international business cycle linkages which might be analyzed by the means of correlation-analysis or error-response-estimates (see, e.g. International Monetary Fund, 2001, Sachverständigenrat zur Beurteilung der gesamtwirtschaftlichen Entwicklung, 2001 or Weyerstraß, 2002).[25] For our simulation study, we follow the approach employed by the German council of economic advisors[26], i.e. considering error-response-estimates for the HP estimated cyclical components. However, even if we assume the existence of a cointegration relation between the level series due to theoretical considerations, the results of table 2 indicate that any apparent correlation between HP filtered series might be an artefact generated by the filtration process itself.

[21] This approach corresponds to the idea of "data based Monte Carlo methods" introduced by Ho and Sørensen (1996).

[22] Data taken from the OECD Main Economic Indicators Database. The German series is based on observations for unified Germany. However, given that data for unified Germany are not available prior to 1990, we follow the approach chosen by the German council of economic advisors (Sachverständigenrat zur Beurteilung der gesamtwirtschaftlichen Entwicklung, 2001): The official data series for unified Germany have been recursively backcasted from 1993 on by historical West-German growth rates.

[23] According to the common $\lambda = 1600$ assumption of empirical research.

[24] For convenience, the results of augmented Dickey-Fuller-Tests for the series of figure 4 are given by table 14 in the appendix.

[25] An alternative approach would consist in a test for cointegration of the level data using, e.g. the Engle and Granger (1987) procedure. Let y^{GER} and y^{US} denote the logged level-GDP series for Germany and the U.S., which have to be treated as $I(1)$-variables (see the corresponding ADF-test results of table 14 in the appendix). Regressing y^{GER} on y^{US} and a constant (see table 15 in the appendix), the null of no cointegration between both series cannot be rejected at a 10%-level (see table 16 in the appendix). Consequently, a regression analysis of these level series might be subject to the spurious regression fallacy.

[26] Sachverständigenrat zur Beurteilung der gesamtwirtschaftlichen Entwicklung (2001).

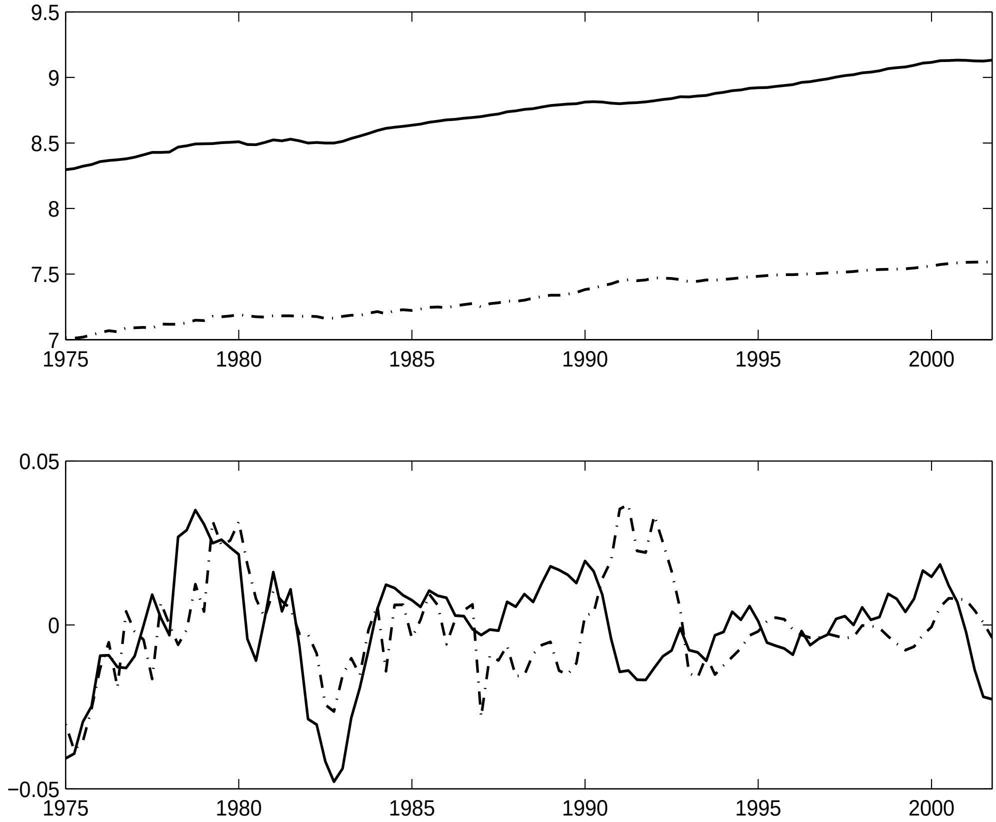

Figure 4: German (dashed) and U.S. (solid) logged GDP, HP cyclical components

We try to assess the risk of a spurious regression fallacy for this example by means of Monte Carlo simulations based on data generating processes which mimic the empirical data under two assumptions. In subsection 3.2.2 the case of a cointegrated bivariate DGP is analyzed, while in 3.2.3, the simulated DGP does not allow for any business cycle spill-overs between both series.

3.2.2 Simulation Results for Interdependent Processes

Within this subsection we are going to simulate a cointegrated bivariate DGP. Aiming at the specification of a well defined structural VAR, we start by estimating an unrestricted VAR-model for the logged level data. Considering standard information criteria we choose a lag order of $p = 2$, so we estimate the following model:[27]

$$\boldsymbol{Y}_t = \boldsymbol{C} + \boldsymbol{\Phi}_1 \boldsymbol{Y}_{t-1} + \boldsymbol{\Phi}_2 \boldsymbol{Y}_{t-2} + \boldsymbol{\epsilon}_t \,, \qquad (9)$$

[27] See table 17 for a detailed account of estimation results.

where

$$\boldsymbol{Y}_t = \begin{pmatrix} y_t^{US} \\ y_t^{GER} \end{pmatrix}, \qquad \boldsymbol{C} = \begin{pmatrix} c^{US} \\ c^{GER} \end{pmatrix}, \qquad \boldsymbol{\epsilon}_t = \begin{pmatrix} \epsilon_t^{US} \\ \epsilon_t^{GER} \end{pmatrix},$$

$$\boldsymbol{\epsilon}_t \sim N(\boldsymbol{0}, \boldsymbol{\Sigma}), \text{ and } \qquad E(\boldsymbol{\epsilon}_t \boldsymbol{\epsilon}'_{t-h}) = \begin{cases} \boldsymbol{\Sigma} & \text{for } h = 0 \\ \boldsymbol{0} & \text{for } h \neq 0 \end{cases}.$$

Economic theory suggests a strong impact of y^{US} on y^{GER} but only modest feedback effects from y^{GER} to y^{US}.[28] And, indeed, the OLS-estimation of a VEC-specification of equation (9):

$$\boldsymbol{\Delta Y}_t = \boldsymbol{C_{VEC}} + \boldsymbol{\alpha} \cdot EC + \boldsymbol{\Phi_{VEC}} \boldsymbol{\Delta Y}_{t-1} + \boldsymbol{\epsilon}_t, \tag{10}$$

with

$$EC = 1.9724 + y_t^{US} \underset{[-15.2266]}{-1.4614} \cdot y_t^{GER}, \tag{11}$$

results in a significant loading coefficient in the German equation whereas the U.S. series does not seem to adjust to the long-run German development.[29]

We therefore decide to simulate a DGP according to the following parameter restrictions:

$$\boldsymbol{\Phi_{VEC}} = \begin{pmatrix} \phi_{11}^{VEC} & 0 \\ \phi_{21}^{VEC} & \phi_{22}^{VEC} \end{pmatrix}, \qquad \boldsymbol{\alpha} = \begin{pmatrix} 0 \\ \alpha_2 \end{pmatrix}. \tag{12}$$

Under these restrictions, the reorganized results of a SUR-estimation of equation 10 yield the following parametrization:[30]

$$\begin{aligned} y_t^{US} &= 0.0052 + 1.3280 \cdot y_{t-1}^{US} - 0.3280 \cdot y_{t-2}^{US} + \epsilon_t^{US} \\ y_t^{GER} &= 0.1116 + 0.1247 \cdot y_{t-1}^{US} - 0.0707 \cdot y_{t-2}^{US} \\ &\quad + 0.8580 \cdot y_{t-1}^{GER} + 0.0630 \cdot y_{t-2}^{GER} + \epsilon_t^{GER}, \\ \hat{\Sigma} &= \begin{pmatrix} 6.11E-05 & 7.14E-06 \\ 7.14E-06 & 8.57E-05 \end{pmatrix}. \end{aligned} \tag{13}$$

The simulation setup considered within this subsection can now be summarized as follows: Taking the historical observations for 1974:3 and 1974:4 as starting values a stochastic simulation of model (13) generates a pair of artificial time series y^{US} and y^{GER}, each consisting of 400 observations.

[28] Beck and Winker (2004) analyze these feedback mechanisms in a structural model of the German economy complemented by VEC models for the bilateral trade flows.

[29] See table 18 in the appendix.

[30] See table 19 in the appendix for a comprehensive overview of the estimation results.

These simulated level series then have to pass the HP filter (yielding a pair of artificial cyclical components $\{y^{US}_{cycl}, y^{GER}_{cycl}\}$)[31] as well as the first difference filter (yielding a pair of artificial growth rates series $\{y^{US}_{\Delta}, y^{GER}_{\Delta}\}$).
Next, for each pair of simulated series ($\{y^{US}, y^{GER}\}, \{y^{US}_{cycl}, y^{GER}_{cycl}\}, \{y^{US}_{\Delta}, y^{GER}_{\Delta}\}$)[32] a bivariate VAR($p$)-model is estimated according to the following specification-algorithm:

1. Choosing the Initial Lag Length $\hat{p}_{SC}$

   As the Schwarz-Criterion is known to be strongly consistent (see, e.g. Lütkepohl, 1993, p. 132), the initial lag length $\hat{p}_{SC}$ is estimated by minimizing the Schwarz-Criterion for all $p \in \{1, 2, \ldots, 9\}$.[33]

2. Hypothesis Testing

   (a) Residual Autocorrelation
   The presence of residual autocorrelation is tested up to order 9 by means of a multivariate LM-test (see, e.g. Johansen, 1995, p. 22, for details). In case of rejection of the null at a 5%-level, the initial lag order is increased by one and the VAR is being reestimated with the new specification. This procedure is repeated until the hypothesis of no serial correlation cannot be rejected. However, the maximum lag order is limited to $\hat{p}_{MAX} = 9$. Therefore, even in case of any remaining autocorrelations, this testing procedure will be abandoned for lag orders greater than nine.

   (b) Residual Heteroskedasticity
   As far as the autocorrelation test was not terminated with $\hat{p}_{MAX}$ the residual diagnostics continue with a system analogue of White's (1980) LM-test for heteroskedasticity (see Doornik, 1996, for computational details). Again, a 5%-significant system LM-statistic causes model-reestimation with increased lag order. This procedure is repeated until the null of no heteroskedasticity cannot be further rejected or $\hat{p}_{MAX}$ is reached, respectively.

[31] Note that the underlying DGP rests on $AR(2)$-processes. Considering this class of autoregressive processes, Pedersen (2001) indicates that HP filtering with a conventional smoothing parameter of $\lambda = 1600$ appears to induce worse distortions than, inter alia, HP detrending based on a numerical value of $\lambda = 1007$. We therefore decided on a dual simulation setup: The estimation procedures described within this chapter have been simulated for the common $\lambda = 1600$ approach as well as for Pedersens $\lambda = 1007$ suggestion. However, our findings appear to be very robust to changes in the smoothing parameter.

[32] See figure 5 for exemplary realizations of these series.

[33] Obviously, the "cut off" lag had to be chosen ad hoc. However, considering the implied loss of degrees of freedom we doubt that any macroeconometrician would consider more than nine lags in a quarterly based VAR-analysis whereas, e.g. Canova and Marrinan (1998) estimate a VAR with nine lags and a constant on detrended output series.

(c) Normality Test
For the multivariate normality test we employ a residual factorization originally suggested by Doornik and Hansen (n.d.). The resulting test statistic is invariant to ordering and scale of the estimated VAR and follows a $\chi^2(4)$-distribution under the null (see Doornik and Hansen, n.d., for details). Rejection of the normality assumption again forces a VAR-reestimation with increased lag length as far as the maximum lag order is not exceeded.

The algorithm therefore results with three separately estimated VARs (for the simulated level series, their first differences and the HP estimated trend-deviations, respectively), each of them following the structure given by equation (14).

$$\boldsymbol{Y}_t^n = \hat{\boldsymbol{C}}^n + \hat{\boldsymbol{\Phi}}_1^n \boldsymbol{Y}_{t-1}^n + \ldots + \hat{\boldsymbol{\Phi}}_{\hat{p}^n}^n \boldsymbol{Y}_{t-\hat{p}^n}^n + \hat{\boldsymbol{\epsilon}}_t^n \,, \tag{14}$$

$$\hat{\boldsymbol{\epsilon}}_t \sim N(\boldsymbol{0}, \hat{\boldsymbol{\Sigma}}) \,, \quad \text{and} \qquad E(\hat{\boldsymbol{\epsilon}}_t \hat{\boldsymbol{\epsilon}}'_{t-h}) = \begin{cases} \hat{\boldsymbol{\Sigma}} & \text{for } h = 0 \\ \boldsymbol{0} & \text{for } h \neq 0 \end{cases} ,$$

with $\boldsymbol{Y}^n$ denoting the corresponding pair of series, i.e.

$$\boldsymbol{Y}^n = \begin{pmatrix} y^{US} \\ y^{GER} \end{pmatrix} , \quad \text{or} \begin{pmatrix} y^{US}_{cycl} \\ y^{GER}_{cycl} \end{pmatrix} , \quad \text{or} \begin{pmatrix} y^{US}_{\Delta} \\ y^{GER}_{\Delta} \end{pmatrix} , \text{ respectively} \,,$$

and $0 < \hat{p}^n < 10$.

Next, we test within each of the estimated VARs whether the lagged values of y^{GER} (y^{GER}_{cycl}, y^{GER}_{Δ}, respectively) could be excluded from the regressor list for the corresponding U.S. series, i.e. we test:

$$H_0 : \hat{\boldsymbol{\Phi}}_i^n = \begin{pmatrix} \hat{\phi}^i_{11} & 0 \\ \hat{\phi}^i_{21} & \hat{\phi}^i_{22} \end{pmatrix} \tag{14.1}$$

versus

$$H_1 : \hat{\boldsymbol{\Phi}}_i^n = \begin{pmatrix} \hat{\phi}^i_{11} & \hat{\phi}^i_{12} \\ \hat{\phi}^i_{21} & \hat{\phi}^i_{22} \end{pmatrix} , \qquad i = 1, \ldots, \hat{p}^n \,.$$

Additionally, we also test whether the lagged U.S. observations might be excluded from the regressor list for the corresponding German series:

$$H_0 : \hat{\boldsymbol{\Phi}}_i^n = \begin{pmatrix} \hat{\phi}^i_{11} & \hat{\phi}^i_{12} \\ 0 & \hat{\phi}^i_{22} \end{pmatrix} \tag{14.2}$$

versus

$$H_1 : \hat{\boldsymbol{\Phi}}_i^n = \begin{pmatrix} \hat{\phi}^i_{11} & \hat{\phi}^i_{12} \\ \hat{\phi}^i_{21} & \hat{\phi}^i_{22} \end{pmatrix} , \qquad i = 1, \ldots, \hat{p}^n \,.$$

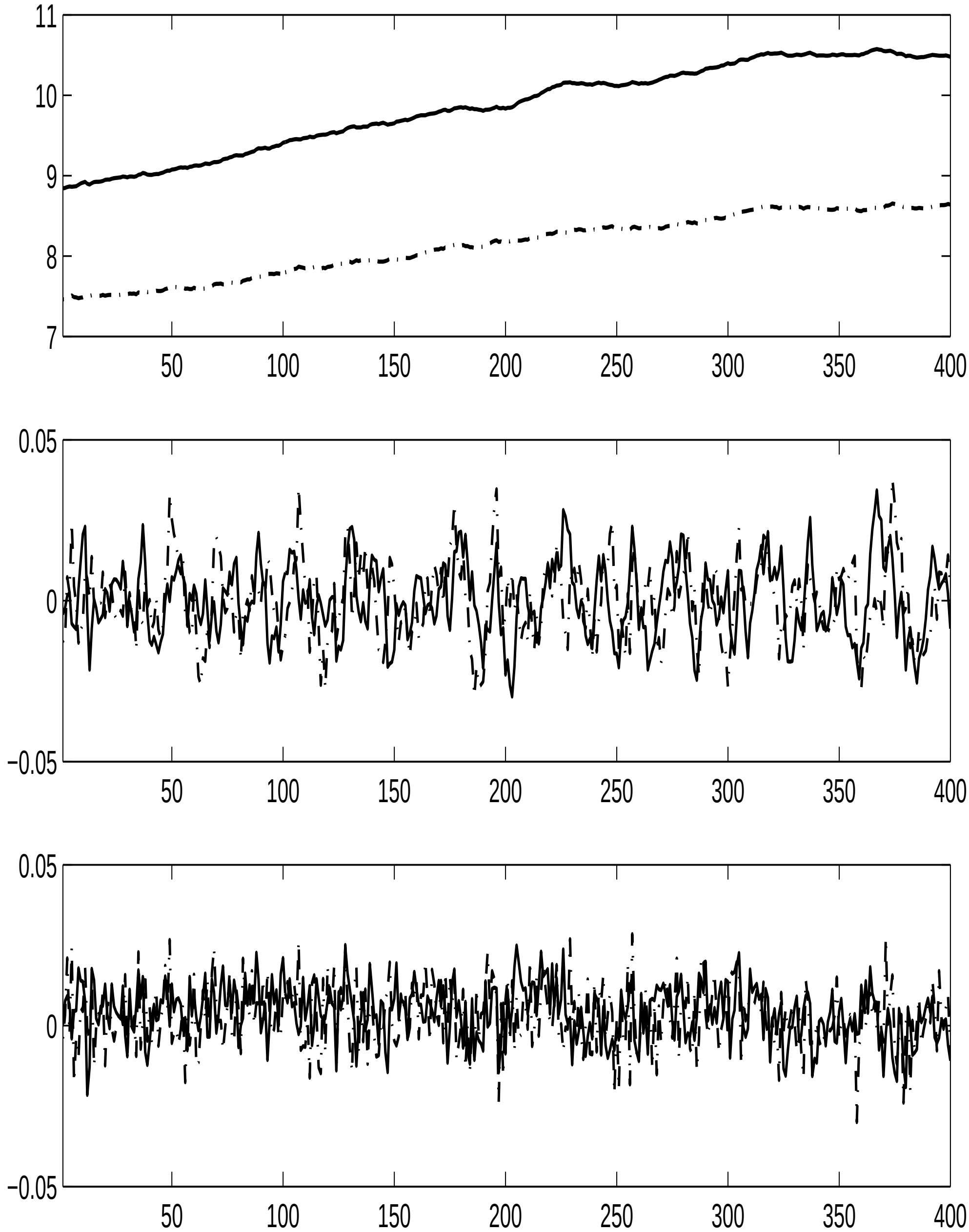

Figure 5: Simulated level series (upper panel), corresponding HP estimated cyclical components (mid panel) and simulated growth rates (lower panel)

Both tests are carried out as likelihood ratio tests by multiplying the difference between the log-determinants of the restricted and the unrestricted models by the number of degrees of freedom.[34]

[34] The resulting statistic λ_{LR} is known to have an asymptotical χ^2-distribution. However,

Note that the estimated LR-statistics for the simulated level data should not suffer from any size distortions as the underlying DGP does establish a cointegration relation between both series. Note further that the DGP parametrization given by equation (13) restricts the order of integration of both series to one. Consequently, we should not expect any problems arising from a spurious regression fallacy. These theoretical considerations can be checked easily by counting the number of significant test results in sufficient replications of the simulation setup.

With the given set of historical starting values, the simulation exercise was performed a thousand times. Then, a second set of 1000 simulations was computed, based on historical observations for 1974:4 and 1975:1, followed by a third loop of 1000 simulations (starting in 1975:1 and 1975:2) and so on.Finally, after 72 replications of the simulation tasks, our resulting findings can be summarized as shown by table 4.[35]

Table 4: Interdependent simulation results

λ		Quantities of significant LR-test statistics					
	Regressors	$L(y^{GER})$	$L(y^{US})$	$L(y^{GER}_{cycl})$	$L(y^{US}_{cycl})$	$L(y^{GER}_{\Delta})$	$L(y^{US}_{\Delta})$
1600	*Mean*	64.2	995.3	128.6	492.1	54.6	772,6
	Std.Dev.	6.6	2.1	14.7	44.8	7.1	31.0
1007	*Mean*	62.5	994.9	126.8	464.6	55.3	773.9
	Std.Dev.	7.2	2.1	16.8	41.3	7.5	32.7

Mean indicates the mean number of significant LR-statistics (at 5%-level) over 72 simulations each consisting of 1.000 tested sets of hypotheses.
Std.Dev. stands for the sample standard deviation of the mean.

At a 5% level, successive replications of 1000 tests for the non-binding hypothesis of the German series Granger-causing the U.S. series should end up with a mean of approximately 50 accidental significant statistics. For the level data (column 3) as well as for the first difference filter (column 7) this assumption seems to be reasonably fulfilled. Yet, the test statistics for the HP detrended series (column 5) obviously suffers from size distortions: The empirical size of a likelihood ratio test based on estimated HP trend deviations approximately equals the empirical size of the level data multiplied by a factor of two.

taking estimation errors and the limited length of economic time series into account, one would usually prefer to divide λ_{LR} by the number of tested restrictions as this results with an approximately F-distributed test statistic. Therefore, all reported results for the tests (14.1), (14.2) and (15.1) actually rest on a comparison of the computed values for $\frac{\lambda_{LR}}{\hat{p}^n}$ with the tabulated 5% critical values of a F-distribution with $(\hat{p}^n)$ numerator and $(400 - 2 \cdot \hat{p}^n - 1)$ denominator degrees of freedom. (See, e.g. Lütkepohl, 1993, p. 93f, for a methodological discussion of this topic.)

[35] As indicated by its first column, table 4 summarizes the results of two independent simulation exercises. Overall, a change in the HP smoothing parameter does not appear to have a significant impact on our findings.

Furthermore, for the HP filtered series the true characteristics of the original DGP can only be tested significantly in about 50% of our simulation runs (column 6), whereas for the first difference filter this figure after all exceeds 75%. However, due to the higher convergence rate of integrated series, causality-tests based on the original level data impressively prove their superior power in this simulation setup (column 4).

3.2.3 Independent Simulation Results

This section completes our study with a simulation setup based on two independent univariate DGPs. The following pair of autoregressive equations seems to provide a reasonable description of the logged level data:

$$\begin{aligned} y_t^{US} &= 1.3057 \cdot y_{t-1}^{US} - 0.3050 \cdot y_{t-2}^{US} + \epsilon_t^{US} \\ y_t^{GER} &= 0.9093 \cdot y_{t-1}^{GER} + 0.1273 \cdot y_{t-2}^{GER} + 0.1073 \cdot y_{t-3}^{GER} \\ &\quad + 0.0913 \cdot y_{t-4}^{GER} - 0.2348 \cdot y_{t-5}^{GER} + \epsilon_t^{GER}, \\ \hat{\Sigma} &= \begin{pmatrix} 6.13E-05 & 1.28E-05 \\ 1.28E-05 & 8.75E-05 \end{pmatrix} . \end{aligned} \tag{15}$$

According to the foregoing simulation setup, 1000 pairs of independent time series y^{US}, y^{GER}, each spanning over 400 observations, were generated by stochastic simulations of model (15). These series again had to pass the HP filter as well as the first difference filter, yielding output gap estimates as well as growth rates estimates. Then, following the algorithm described in section 3.2.2, VAR-models were specified and estimated for each couple of simulated series.
As the true DGP given by model (15) is characterized by a zero skew diagonal for all parameter matrices $\mathbf{\Phi}_i$ $(i = 1, 2, \ldots, 5)$, we are now especially interested in the empirical size of a likelihood ratio test for the following restrictions:

$$H_0 : \hat{\mathbf{\Phi}}_i^n = \begin{pmatrix} \hat{\phi}_{11}^i & 0 \\ 0 & \hat{\phi}_{22}^i \end{pmatrix} \tag{15.1}$$

versus

$$H_1 : \hat{\mathbf{\Phi}}_i^n = \begin{pmatrix} \hat{\phi}_{11}^i & \hat{\phi}_{12}^i \\ \hat{\phi}_{21}^i & \hat{\phi}_{22}^i \end{pmatrix} , \qquad i = 1, \ldots, \hat{p}^n .$$

Replicating the testing procedures a thousand times for 72 different starting values, the resulting findings can be summarized as shown by table 5: The averaged results for the differenced series range around their nominal level of 5% whereas more than 20% of the level-based test procedures wrongly indicate the tested restrictions as binding. However, for the HP detrended data the amount of spurious regression results appears to be important, too. The empirical size of the HP based likelihood ratio tests approximately equals the nominal size multiplied by a factor of two (see column 4 of table 5).

Table 5: Independent simulation results

	Quantities of significant LR-Test statistics			
λ	Regressors	$L(y)$	$L(y_{cycl})$	$L(y_{\Delta})$
1600	*Mean*	209.1	107.9	54.2
	Std.Dev.	12.9	11.8	6.2
1007	*Mean*	207.2	107.5	53.5
	Std.Dev.	14.2	9.3	7.3

Mean: Mean number of significant LR-statistics (5%-level).
Std.Dev.: Sample standard deviation.

Summarizing the results of tables 4 and 5, there is obviously much to be said against an regression analysis of HP detrended time series. Yet, where do these deficiencies actually stem from? Subsection 3.2.4 will give a hint.

3.2.4 Further Simulation Results: Estimated Lag Orders

Let us turn back to subsection 3.2.2 for a short time before we come to our conclusions. As already introduced by equations (14) and (9), this setup dealt with the estimation of VAR($\hat{p}$)-processes for a simulated DGP which constitutes a cointegrated VAR(2) in levels. We know that the applied specification algorithm allowed for a maximum lag order of $\hat{p}_{MAX} = 9$, but which lag specifications have been overall preferred? Table 6 gives the answer and reveals some insightful structures.

Table 6: Interdependent simulation results

λ		Quantities of lag order estimates $\hat{p}$								
1600	Series	1	2	3	4	5	6	7	8	9
	Levels	22.3	680.1	24.9	9.9	4.4	2.1	1.0	0.4	254,9
	Gaps	13.3	544.4	42.8	19.1	11.7	7.3	4.8	3.2	353.6
	Diffs.	747.3	40.1	14.1	7.1	3.7	1.8	0.7	0.4	184,8
1007	*Levels*	21.8	676.0	25.3	9.9	4.6	2.1	1.3	0.4	258.7
	Gaps	12.1	561.8	50.0	18.7	12.4	6.8	4.4	3.0	330.8
	Diffs	746.5	41.4	14.5	6.9	3.5	1.7	0.7	0.3	184.3

Figures indicate the average number of selected $VAR(\hat{p})$ specifications after 72 repetitions of the simulation exercise described in subsection 3.2.2.

Averaged over 72 different simulation exercises our specification procedure detects the true lag order of the differenced series (symbolized by the $Diffs.$ entry) in three out of four cases.[36] For the simulated level series (*Levels*) at least two out of three specified VARs end up with the correct lag length. For the HP detrended series (*Gaps*), however, this amount decreases to approximately 55%.
Additionally, due to failed residual miss-specification tests, about every third HP based VAR-model is being estimated with the maximum lag length. This figure averages twice the amount of miss-specified models for the differenced series. Compared to the results for the simulated level series it still outnumbers the amount of miss-specified level models for more than a third. An objective evidence of the underlying causations of this effect remains for future research.

4 Conclusion

Hodrick and Prescott (1980, 1997) originally suggested to measure the strength of association between (macro-)economic variables by a regression analysis of corresponding HP filtered time series. With regards to our findings this proposal should not be accepted for the following reasons:

1. For cointegrated series, the power of traditional test statistics seems to be heavily weakened by pre-filtering whereas the test statistics of the unfiltered series are known to converge at a higher rate.

2. For not cointegrated series, the problem of spurious regression results might be extensively worsened by an application of the HP procedure whereas the first difference filter proves to provide reliable results.

Our experimental setup therefore confirms and strengthens the complementary methodological criticisms about the HP approach being subject to the Nelson and Kang (1981) critique *"...that inappropriate detrending of time series will tend to produce apparent evidence of periodicity which is not in any meaningful sense a property of the underlying system. ... The dynamics of econometric models estimated from such data may well be wholly or in part an artifact of the trend removal procedure."*

[36] A look back to equations (9) and (10) confirms that the first differences of the simulated DGP can be represented as a VAR(1)-process.

A Further Univariate Simulation Results

Within section 2 we already mentioned that HP outcomes might become distorted by neglected high frequency signals like, e.g. seasonal variations. Consequently, the previously presented simulation results considered “plain” autoregressive DGPs without any seasonal influences. However, as empirical time series usually are characterized by significant seasonal patterns, this appendix summarizes complementary results for seasonal DGPs.[37] By adding four seasonal dummies to equations (4) and (5) we constructed the following DGPs:

$$y_t^1 = \phi^1 y_{t-1}^1 + \theta_1^1 s_1^1 + \theta_2^1 s_2^1 + \theta_3^1 s_3^1 + \theta_4^1 s_4^1 + \epsilon_t \qquad (16)$$
$$y_t^2 = \phi^2 y_{t-1}^2 + \theta_1^2 s_1^2 + \theta_2^2 s_2^2 + \theta_3^2 s_3^2 + \theta_4^2 s_4^2 + \psi_t \qquad (17)$$

Again, 480 observations were generated, now according to processes (16) and (17) with starting values y_1^1, y_1^2 set equal to zero. Concerning the numerical parameter values, two different setups were simulated: Whereas setup B considers the case of centralized seasonals ($\sum_{i=1}^4 \theta_i^j = 0 \ \forall \ j \in \{1,2\}$), the complementary non-centralized ($\sum_{i=1}^4 \theta_i^2 \neq 0$) case is studied within setup C. Table (7) summarizes the individual parameter constellations.

Table 7: Simulation setups

Setup	DGP No.	σ	θ_1	θ_2	θ_3	θ_4
B	(1)	0.0375	-0.3	0.4	0.6	-0.7
B	(2)	0.15	-06	-0.2	0.1	0.7
C	(1)	0.0375	-0.3	0.4	0.6	-0.7
C	(2)	0.15	0.2	0.1	-0.7	0.1

Seasonal dummies were also added to the OLS-regressor-lists. The estimated equations therefore can be formally written as:

$$y_t^1 = \alpha_1 y_{t-1}^1 + \alpha_2 y_{t-1}^2 + \alpha_3 s_1 + \alpha_4 s_2 + \alpha_5 s_3 + \alpha_6 s_4 + u_t \qquad (18)$$
$$y_{t\,cycl}^1 = \beta_1 y_{t-1\,cycl}^1 + \beta_2 y_{t-1\,cycl}^2 + \beta_3 s_1 + \beta_4 s_2 + \beta_5 s_3 + \beta_6 s_4 + v_t \qquad (19)$$
$$y_{t\,HP}^1 = \gamma_1 y_{t-1\,HP}^1 + \gamma_2 y_{t-1\,HP}^2 + \gamma_3 s_1 + \gamma_4 s_2 + \gamma_5 s_3 + \gamma_6 s_4 + w_t \qquad (20)$$

Simulating and regressing thousand artificial series for varying autoregressive parameters and bookkeeping the events of 5%-significant estimates $\hat{\alpha}_2$, $\hat{\beta}_2$ and $\hat{\gamma}_2$, we derived the following tables of results:

[37] In additional simulation exercises deterministic time trends were also included in the regressor lists for y_t and $y_{t\,HP}^1$. This kind of modification did not seem to induce any systematic variation of the findings so the corresponding results will not be published, though, it goes without saying that they are available on request to the authors.

A.1 Setup B Regression Results

Table 8: Setup B: Regression results for equation (18)

$T = 400$	ϕ_1								
ϕ_2	0.00	0.05	0.10	0.25	0.50	0.75	0.90	0.95	0.99
0.00	62	44	51	54	40	49	55	48	44
0.05	52	60	55	41	42	42	39	48	47
0.10	54	52	44	48	52	44	57	48	30
0.25	55	51	51	47	40	50	50	51	54
0.50	51	52	40	41	57	46	61	49	52
0.75	53	51	55	57	44	60	54	51	54
0.90	55	53	47	46	53	50	61	60	75
0.95	56	55	49	47	56	53	64	64	93
0.99	57	55	47	62	66	58	70	83	114

Table 9: Setup B: Regression results for equation (19)

$T = 400$	ϕ_1								
ϕ_2	0.00	0.05	0.10	0.25	0.50	0.75	0.90	0.95	0.99
0.00	74	66	62	63	59	57	39	43	40
0.05	61	65	61	53	59	64	51	45	45
0.10	61	57	54	59	69	50	45	53	46
0.25	67	64	63	66	61	53	51	58	46
0.50	56	62	60	55	71	70	75	65	60
0.75	52	59	67	54	70	89	65	79	74
0.90	46	61	52	54	77	72	89	86	96
0.95	64	62	46	48	68	88	94	79	90
0.99	50	76	54	54	88	76	89	85	101

Table 10: Setup B: regression results for equation (20)

$T = 400$	ϕ_1								
ϕ_2	0.00	0.05	0.10	0.25	0.50	0.75	0.90	0.95	0.99
0.00	630	600	639	611	619	587	567	550	520
0.05	623	618	603	594	618	595	596	495	492
0.10	632	637	614	625	588	594	571	527	451
0.25	610	649	601	602	603	585	575	476	422
0.50	609	620	609	600	601	599	569	523	441
0.75	586	626	624	597	565	612	558	542	429
0.90	623	617	636	625	613	602	601	595	509
0.95	623	626	638	632	651	639	664	610	562
0.99	652	618	642	665	652	684	668	674	687

A.2 Setup C Regression Results

Table 11: Setup C: Regression results for equation (18)

$T = 400$	ϕ_1								
ϕ_2	0.00	0.05	0.10	0.25	0.50	0.75	0.90	0.95	0.99
0.00	65	46	49	46	49	63	48	45	54
0.05	46	48	52	50	49	58	54	55	49
0.10	42	53	51	42	57	48	55	50	41
0.25	58	39	49	59	54	50	52	41	48
0.50	45	52	40	47	42	51	49	42	52
0.75	61	59	57	48	46	53	61	57	41
0.90	41	46	44	55	57	46	57	61	59
0.95	40	55	43	43	35	57	54	64	88
0.99	52	56	53	52	54	57	64	76	141

Table 12: Setup C: Regression results for equation (19)

$T = 400$	ϕ_1								
ϕ_2	0.00	0.05	0.10	0.25	0.50	0.75	0.90	0.95	0.99
0.00	72	46	49	50	57	53	41	41	53
0.05	47	57	63	62	52	64	48	46	39
0.10	52	61	58	50	63	57	58	53	43
0.25	64	45	56	60	75	54	58	58	57
0.50	60	64	50	61	73	69	75	56	65
0.75	70	66	67	61	68	89	80	79	72
0.90	45	47	61	71	62	83	101	82	74
0.95	44	54	49	59	46	84	91	73	102
0.99	44	60	69	57	70	112	84	83	98

Table 13: Setup C: Regression results for equation (20)

$T = 400$	ϕ_1								
ϕ_2	0.00	0.05	0.10	0.25	0.50	0.75	0.90	0.95	0.99
0.00	623	610	635	615	608	581	570	545	508
0.05	592	621	589	597	587	589	547	524	460
0.10	612	573	586	626	599	556	549	531	435
0.25	580	582	579	583	591	567	562	497	467
0.50	590	570	595	593	613	591	584	499	517
0.75	594	596	597	616	615	592	583	515	465
0.90	621	628	624	602	619	625	617	591	490
0.95	636	637	644	629	651	668	601	619	545
0.99	674	665	660	628	690	678	666	692	663

B Further VAR-Simulation Results

In the regression tables, estimated standard deviations are shown in parenthesis whereas the t-statistics are be given in brackets.

Table 14: Results of ADF-Tests for individual series

Sample: 1975:1 2001:4		
Series	Lags	Statistic
y^{US}	12	-0.3205
y^{GER}	0	-0.9010
y^{US}_{cycl}	12	-3.8932**
y^{GER}_{cycl}	4	-4.6986**
y^{US}_{Δ}	11	-4.0987**
y^{GER}_{Δ}	4	-4.2403**

Table 15: Static regression results

	Dependent Variable
Regressor	y^{GER}_t
y^{US}_t	0.7063 (0.0122) [58.1084]
c	1.1573 (0.1062) [10.8957]
Regression statistics Sample 1975:1 2001:4	
R^2	0.9696
$\overline{R}^2$	0.9693
$\sum \hat{\epsilon}^2_t$	0.0956
$\hat{\sigma}$	0.0300

Table 16: Engle-Granger-Test results

	Sample: 1976:2 2001:4				
Series	Lags	Statistic	$t^{0.10}$	$t^{0.05}$	$t^{0.01}$
$\hat{u}_t$	4	-2.72	-3.04	-3.34	-3.90
$t^{0.01}$, $t^{0.05}$ and $t^{0.10}$ denote asymptotic critical values for cointegration tests at a 1%, 5% and 10% level, given by table 20.2 of Davidson and MacKinnon (1993). Lag order selection based on the AIC.					

Table 17: Unrestricted VAR-Estimation results

VAR-Regression Results Sample: 1975:1 2001:4		
	Dependent Variable	
Regressor	y_t^{US}	y_t^{GER}
y_{t-1}^{US}	1.3222 (0.0933) [14.1753]	0.1242 (0.1108) [1.1210]
y_{t-2}^{US}	-0.3169 (0.0967) [-3.2754]	-0.0687 (0.1149) [-0.5975]
y_{t-1}^{GER}	- 0.0039 (0.0820) [-0.0481]	0.8570 (0.0974) [8.7991]
y_{t-2}^{GER}	-0.0065 (0.0796) [-0.0821]	0.0622 (0.0946) [0.6574]
c	0.0355 (0.0419) [0.8471]	0.1123 (0.0497) [2.2574]
Regression statistics		
R^2	0.9989	0.9971
$\overline{R}^2$	0.9989	0.9969
$\sum \hat{\epsilon}_t^2$	0.0066	0.0093
$\hat{\sigma}$	0.0080	0.0095
$F : L(y^{GER}) = 0$	0.0770	414.2914
$p - value$	0.9259	0.0000
$F : L(y^{US}) = 0$	1520.7040	3.7920
$p - value$	0.0000	0.0258

Table 18: VECM-Estimation results

VECM Regression Results Sample: 1975:1 2001:4		
Cointegration-Relation: $EC = 1.9724 + y_t^{US} \underset{[-15.2266]}{-1.4614} \cdot y_t^{GER}$		
	Dependent Variable	
Regressor	Δy_t^{US}	Δy_t^{GER}
EC	0.0092 (0.0180) [0.5085]	0.0551 (0.0214) [2.5770]
Δy_{t-1}^{US}	0.3138 (0.0963) [3.2597]	0.0690 (0.1142) [0.6046]
Δy_{t-1}^{GER}	0.0071 (0.0793) [0.0890]	-0.0622 (0.0941) [-0.6612]
c	0.0052 (0.0011) [4.8289]	0.0051 (0.0013) [3.9811]
Regression statistics		
R^2	0.1123	0.0790
$\overline{R}^2$	0.0867	0.0525
$\sum \hat{\epsilon}_t^2$	0.0066	0.0093
$\hat{\sigma}$	0.0080	0.0095

Table 19: Bivariate DGP – SUR estimation results

DGP Regression Results Sample: 1975:1 2001:4		
Cointegration-Relation: $EC = 1.9724 + y_t^{US} \underset{[-15.2266]}{-1.4614} \cdot y_t^{GER}$		
	Dependent Variable	
Regressor	Δy_t^{US}	Δy_t^{GER}
EC	0.0000	0.0540 (0.0209) [2.5877]
Δy_{t-1}^{US}	0.3280 (0.0898) [3.6545]	0.0707 (0.1120) [0.6311]
Δy_{t-1}^{GER}	0.0000	-0.0630 (0.0919) [-0.6861]
c	0.0052 (0.0010) [5.1074]	0.0051 (0.0013) [4.0524]
Regression statistics		
R^2	0.1101	0.0790
$\overline{R}^2$	0.1017	0.0525
$\sum \hat{\epsilon}_t^2$	0.0066	0.0093
$\hat{\sigma}$	0.0079	0.0095

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