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ON THE FISCAL TREATMENT OF LIFE EXPECTANCY RELATED CHOICES

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Abstract. In an overlapping generations economy setup we show that, if individuals can improve their life expectancy by exerting some effort, costly in terms of either resources or utility, the competitive equilibrium steady state differs from the first best steady state. This is due to the fact that under perfect competition individuals fail to anticipate the impact of their longevity-enhancing effort on the return of their annuitized savings. We identify the policy instruments required to implement the first-best into a competitive equilibrium and show that they are specific to the form, whether utility or resources, that the effort takes.

Keywords: life expectancy, health expenditures, taxation
JEL classification: H21, D91

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1. Introduction

In the last century, an unprecedented rise in life expectancy has been a pervasive phenomenon in both developed and developing countries. This has surely been due mostly to a host of causes affecting whole societies at large like, for instance, progress in medicine, improvements in agriculture, and better sanitary conditions, among others. Although there may also be among these causes a component that is related to individual behaviors or choices, its contribution to this dramatic increase in life expectancy is likely to have been small compared to those mentioned above. Nevertheless, an immediate consequence, among many others, of the increase in life expectancy is the pressure it puts on, for instance, the provision of health care, on pay-as-you-go pensions systems, on housing, etc.[1] Thus, as the constraints on these and other resources become tighter, the relative importance of the individual-specific causes of the increase in life expectancy may increase as well, and the question then arises about whether the decentralized choices made by the individuals about their efforts to have an ever increasing life expectancy are the right ones from an efficiency viewpoint.

Individuals can privately influence their life expectancy in various ways choosing to undertake actions and behaviors that tend to increase it, or to avoid those that may decrease it. Nevertheless, these choices typically imply a cost for them, either in terms of a disutility incurred or in terms of additional spending in, say, healthcare, and hence of forgone consumption. In effect, while the most obvious way to increase life expectancy is to increase medical treatment —which requires the actual spending of income— individuals can also make behavioral choices to that end (e.g. exercising, abstaining from smoking, eating a healthy diet, driving safely) that do not necessarily require an additional spending, but may inflict nonetheless some disutility on the individual.[2]

Despite the undisputable positive aspect of having a higher longevity, this overall increase has had also some detrimental external effects on, for example, pension systems, publicly provided healthcare, urban development, and the environment.

[1]Of course, a longer life, specifically a healthier one, increases also the labor force available for production at any time, which works in the opposite direction, but for the sake of simplicity we are going to make abstraction of this fact.

[2]On the impact of health expenditures on life expectancy, see Poikolainen (1986). Several studies have also shown the impact of factors such as physical activity (Kaplan et al.,1987 and Okamoto, 2006), overweight (see Solomon and Manson, 1997 and Bender et al. 1998) and smoking (Doll and Hill, 1950).

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The specific point this paper addresses is that, besides these well-known detrimental external effects, there exists another negative externality due to a higher life expectancy simply related to the impact that the individual's choice of *quantity* of life has on his *quality* of life, through the private resources he is left with for his extended life, if savings are annuitized. Becker and Philipson (1998) emphasized already how a rise in the quantity of life can affect its quality by showing that individuals investing in their longevity do not take into account that, by doing so, they influence the return of their annuitized savings. The result is too much investment in longevity compared to what would be optimal. Becker and Philipson (1998) thus suggests that one way to ensure a high return of savings should be to tax health expenditures (and thus, implicitly longevity). Some papers give recommendations in this direction. For example, Leroux (2008) showed that in the case of non-contractible effort to increase longevity, the social planner should tax second-period consumptions in order to reduce incentives for the individual to invest in longevity. Leroux et al. (2008a,b) studied the taxation of longevity-enhancing health expenditures and showed that three factors play a role in the choice of the adequate tax rate: (i) the possible misperception by the agents of their true survival probability; (ii) the Becker-Philipson effect, as described above; and, in case of asymmetric information, (iii) incentive constraints. Nevertheless, in Leroux (2008) and Leroux et al. (2008a,b) the framework was essentially static, with a 2-period-lived agent that solves a one-shot problem at the begining of the first period.

In this paper, on the contrary, we study the problem in a truly dynamic general equilibrium framework. Adressing the issue in a dynamic setup is the natural next step to undertake, since similar instances of inefficiencies due to an overlooked (by competitive agents) impact of individual saving decisions on the saving returns arise naturally in overlapping generations models as well (see Dávila (2008)). Thus we consider an overlapping generations economy in which individuals are identical except for the date they are born in. The representative agent is sure to live at least one period and at most two, conditional on a survival probability. He supplies inelastically labor when young and consumes from his labor income when young, and from his annuitized capital and monetary savings when old (if alive). We assume that the representative agent can influence his survival probability exerting some effort. We will distinguish between the case in which this effort entails a direct disutility but no additional spending (the disutility-effort case), and the case in which it requires some additional spending but has no direct impact on the agent's utility (the expenditure-effort case).[3] Thus, an expenditure-effort can be thought of

[3]We could as well assume that the individual exerts the two different types of efforts at the same time, but for the sake of simplicity, we consider them separately.

simply as health expenditures that enters the individual budget constraint and as resources unavailable for consumption or saving. A disutility-effort implies instead a cost in terms of utility only, entering negatively the utility function but not the budget constraint. It can be thought of generally as leading a "healthy" way of life (exercising, eating healthily, abstaining from smoking and other instantly gratificating pleasures, etc.), that might be unappealing to the individual at the time he exerts the effort, but that improves also his or her life expectancy and hence the prospects of enjoying utility from consumption in the second period of life.

Under the setup defined above, we show that, both in the disutility-effort and the expenditure-effort cases, the laissez-faire competitive equilibrium steady state level of individual effort is higher than the first-best steady state, and hence inefficient. For instance, in the expenditure-effort case the individuals do not take into account, as in Becker and Philipson (1998), that by investing in their longevity they also decrease the return of their annuitized savings —very much as in they do in Dávila (2008) by saving too much capital— and in that way they reduce their consumption possibilities in the second period. A similar effect is observed in the disutility-effort case. As a consequence, there is, as in the static case, room for a public intervention aiming at making the competitive equilibrium steady state with an annuity market for savings coincide with the first-best steady state. However, in the dynamic setup the policy instruments needed are different from those needed in the static case, and differ as well depending on whether the effort takes the form of a disutility or of an expenditure. In the disutility-effort case, we show to be optimal to announce a second-period lump-sum tax that depends on the second-period consumption of the previous generation and on the rate of growth of the population (net of the mortality rate between periods). Interestingly enough, at the competitive equilibrium steady state, the amount actually raised by the tax is zero in every period, so that the implementation of the first-best steady state allocation is achieved by the mere announcement of the policy. If, on the contrary, the effort is an actual expenditure (e.g. health expenditure), it simply requires to tax that expenditure at the young age and to make a lump-sum transfer of the same amount to the contemporary old. At the steady state, redistribution actually takes place, whenever there is demographic growth.

Our paper can be related to the growing literature dealing with endogenous longevity in overlapping generations setups. Some papers have already emphasized the role of endogenous longevity in shaping growth and savings patterns (see, for example, Chakraborty, 2004) as well as the environment (Jouvet et al., 2007). Other papers have studied how the golden rule is modified by the introduction of endoge-

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nous longevity, inducing the under-accumulation of capital when longevity depends on public health expenditures (De la Croix and Ponthière, 2008). These papers differ however from ours in several respects. First, all of them consider health expenditures as a publicly-provided good, so that individuals have no direct control over their life expectancy. Second, they consider, *for a given public policy*, either the competitive equilibrium steady state when the consumption-saving choice has longevity consequences, as in Chakraborty (2004) or the first-best steady state (as in De la Croix and Ponthière, 2008), but none of them shows that the laissez-faire competitive equilibrium steady state with annuitized savings typically differs from the first-best steady state. In particular, to the best of our knowledge, no paper has yet established that the combination of private health expenditures and of an annuity market requires an active fiscal policy if the first-best steady state is to be implemented as a competitive equilibrium. Moreover, we identify the different policies required for the implementation of the first-best depending on the specific form that the life expectancy-increasing effort can take.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 shows for the disutility-effort case that the competitive equilibrium steady state typically differs from the first-best steady state, and shows how to restore the first-best. Section 4, does the same but for the expenditure-effort case. Section 5 concludes.

2. The model

Time is discrete, and at every date t, a generation of identical agents is born. The size of the generations increases in time at a rate n. Agents live at least one period and at most two, conditional to a survival with probability $\pi(e^t)$ that they can influence by the choice of some effort level e^t. A period-t agent supplies inelastically when young his labor (normalized to 1) for a real wage rate w_t that he can split as he wishes between first period consumption c_0^t and saving, which he can hold in either capital or intrinsically worthless money. His capital savings k^t earn a return r_{t+1}at $t+1$, while monetary holdings M^t bought at a real price $\frac{1}{p_t}$ at t are worth $\frac{1}{p_{t+1}}M^t$ at $t+1$. Savings (augmented of their return) are used for second period consumption c_1^t. Note that the probability of survival $\pi(e^t)$ represents also the proportion of individuals born at t who survive to the next period. Finally, effort can be costly to agents either in terms of utility (Section 3) or in terms of forgone income for consumption (Section 4). The first case tries to capture the influence on life expectancy of individual behavioral choices that are unrelated to income

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but undesirable per se, while the second case can be simply thought of as standard health expenditures.

Consider first the utility-effort case. The probability of survival $\pi(e^t)$ depends on an effort level e^t —with $\pi'(e^t) > 0$ and $\pi''(e^t) < 0$— that creates a linear disutility[4] γe^t (γ represents thus the intensity of the effort disutility, assumed to be identical across individuals).[5] The utility from consumption when young and old is given by the differentiably increasing and concave functions $u(c)$ and $v(c)$ respectively with $\lim_{c_0 \to 0} u'(c_0) = +\infty = \lim_{c_1 \to 0} v'(c_1)$. The lifetime utility of the representative agent born at time t is then

$$U(c_0^t, c_1^t, e^t) = u(c_0^t) + \pi(e^t)v(c_1^t) - \gamma e^t. \tag{1}$$

Since effort has no impact at all on the agent's income, his budget constraints at periods t and $t+1$ are respectively

$$\begin{aligned} c_0^t + k^t + \frac{1}{p_t} M^t &= w_t \\ c_1^t = r_{t+1} k^t + \frac{1}{p_{t+1}} M^t. \end{aligned} \tag{2}$$

Consider now the income-effort case. We assume that the individual spends an amount e^t of his income in health care, which influences his survival probability, equal to $\pi(e^t)$ (as before $\pi'(e^t) > 0$ and $\pi''(e^t) < 0$). In this case, in the utility function above $\gamma = 0$ so that the agent's utility is now

$$U(c_0^t, c_1^t, e^t) = u(c_0^t) + \pi(e^t)v(c_1^t) \tag{3}$$

but the agent bears a cost in terms of resources, e^t which reduces the first-period income available for consumption and saving:

$$\begin{aligned} c_0^t + k^t + \frac{1}{p_t} M^t + e^t = w_t \\ c_1^t = r_{t+1} k^t + \frac{1}{p_{t+1}} M^t. \end{aligned} \tag{4}$$

[4]Note that we obtain the same results by assuming convex disutility of effort. For simplicity of exposure, we stick to the linear case.
[5]For the case where the effort disutility differs across individuals in a static setup, see Leroux (2008).

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Note that, as opposed to other endogenous longevity models (e.g. Chakraborty (2004) and De la Croix and Ponthiere (2008)), in the two cases above the level of effort e^t is chosen by the individual himself.

Production is standard: at every period, firms produce, out of capital and labor, a single good that can be either consumed (possibly as health expenditure) or saved to be used as capital for production the next period. The production function $F(K, L)$ exhibits constant returns to scale and good and factors markets are perfectly competitive, so that the wage rate equals the marginal productivity of labor and the annuitized marginal productivity of capital remunerates the latter. Hence, at equilibrium

$$\begin{aligned} w_t &= F_L(\frac{k^{t-1}}{1+n}, 1) \\ r_{t+1} &= F_K(\frac{k^t}{1+n}, 1)\frac{1}{\pi(e^t)} \end{aligned} \tag{5}$$

given that, at every period t, aggregate capital K_t equals at equilibrium the previous period aggregate savings in terms of capital $(1+n)^{t-1}k^{t-1}$ (for the sake of simplicity capital is assumed to depreciate completely in one period), and aggregate labour L_t equals $(1+n)^t$. Note that, according to the equations above, capital savings are assumed to be invested into a fund that lends to firms and gets therefore the marginal productivity of capital. Since the return to capital savings is annuitized, it depends on the survival probability $\pi(e^t)$, and hence on effort e^t. Indeed, the return to the aggregate savings invested in the fund is augmented by the fact that a proportion $1 - \pi(e^t)$ individuals of each generation does not survive and therefore profits are to be distributed among the proportion $\pi(e^t)$ of survivors only. This is a crucial feature of our model.

3. Case in which increasing life expectancy is costly in terms of utility

In this section, we assume that the longevity-enhancing effort has a cost in terms of utility only, such as eating a healthy diet, not smoking, exercising, etc.

3.1 First-best steady state.

Firstly, we characterize the first-best steady state, i.e. the steady state that maximizes the utility of the representative agent solving the problem

$$\begin{aligned} &\max_{c_0,c_1,k,e} u(c_0)+\pi(e)v(c_1)-\gamma e \\ &\text{s.t.} \quad c_0+\frac{\pi(e)}{1+n}c_1+k=F(\frac{k}{1+n},1) \end{aligned} \tag{6}$$

where k is the steady state per capita savings in terms of capital. The constraint in the optimization problem above is the resource constraint requiring that the output per worker allows at any time to satisfy the consumption of the young and old agents alive that period, the latter being only a proportion $\frac{1}{1+n}$ of the former (of which, moreover, only a fraction $\pi(e)$ would have survived) because of the population growth. The first-order conditions characterizing the solution to the problem above are

$$\begin{pmatrix} u'(c_0) \\ \pi(e)v'(c_1) \\ 0 \\ \pi'(e)v(c_1)-\gamma \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ \frac{\pi(e)}{1+n} \\ 1-F_K(\frac{k}{1+n},1)\frac{1}{1+n} \\ \frac{\pi'(e)}{1+n}c_1 \end{pmatrix} \tag{7}$$

for some $\lambda \neq 0$, given the monotonicity of u, along with the resource constraint in the optimization problem above. Equivalently, the first-best steady state is a profile c_0^*, c_1^*, e^*, k^* satisfying the equations:

$$\begin{aligned} \frac{u'(c_0)}{v'(c_1)} = 1+n =& F_K(\frac{k}{1+n},1) \\ c_0+\frac{\pi(e)}{1+n}c_1+k =& F(\frac{k}{1+n},1) \\ \pi'(e)v(c_1) = \gamma +& \pi'(e)v'(c_1)c_1. \end{aligned} \tag{8}$$

The first equation in the first line equates the marginal rate of substitution (actually $\frac{u'(c_0)}{\pi(e)v'(c_1)}$) between first and second period consumptions to the rate at which resources can be transferred from the first to the second period of life (namely $\frac{1+n}{\pi(e)}$); it determines thus the optimal level of savings. The second equation in the first line, on the other hand, pins down the optimal level of individual capital savings. The second line is the feasibility constraint, while the last line determines the optimal level of effort. This last condition is specific to the endogenous life expectancy setup we are considering, and it states that the optimal level of effort should be such that the marginal cost of effort (the right-hand side) should equate its marginal benefit (the left-hand side). While the marginal benefit is simply given by the marginal increase of the survival probability times the utility of second period consumption,

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the marginal cost of increasing survival consists of the sum of a direct marginal utility cost of increasing effort (namely γ) and an indirect cost in terms of the additional pressure on resources (i.e. $\lambda\frac{\pi'(e)}{1+n}c_1 = \pi'(e)v'(c_1)c_1$ from the second first-order condition in (7)). This latter effect follows from the fact that an increase in everyone's survival chances creates an additional demand for the existing resources. As it will be seen in the next section, this additional cost of an increased life expectancy is not taken into account by the individuals when choosing their effort level in a competitive equilibrium under laissez-faire.

3.2 Laissez-faire competitive equilibrium steady state with money.

We turn now to characterizing the competitive equilibrium steady state allocation under laissez-faire. The representative agent's problem amounts (i) to choose how much to save and how to allocate his savings between capital and money, and (ii) to choose how much effort to make to increase the chances of surviving into the second period, given his preferences and his budget constraints:

$$\begin{aligned} \max_{c_0^t, c_1^t, k^t, e^t, M^t} & u(c_0^t) + \pi(e^t)v(c_1^t) - \gamma e^t \\ c_0^t + k^t + \frac{1}{p_t}M^t &= w_t \\ c_1^t &= r_{t+1}k^t + \frac{1}{p_{t+1}}M^t. \end{aligned} \tag{9}$$

The first-order conditions characterizing the solution to this problem are

$$\begin{pmatrix} u'(c_0^t) \\ \pi(e^t)v'(c_1^t) \\ 0 \\ 0 \\ \pi'(e^t)v(c_1^t) - \gamma \end{pmatrix} = \lambda^t \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} + \mu^t \begin{pmatrix} 0 \\ 1 \\ -r_{t+1} \\ -\frac{p_t}{p_{t+1}} \\ 0 \end{pmatrix} \tag{10}$$

for some λ^t and μ^t, along with the budget constraints of the optimization problem above, or equivalently

$$\begin{aligned} \frac{u'(c_0^t)}{v'(c_1^t)} = \pi(e^t)\frac{p_t}{p_{t+1}} &= \pi(e^t)r_{t+1} \\ c_0^t + k^t + \frac{1}{p_t}M^t &= w_t \\ c_1^t = r_{t+1}k^t + \frac{1}{p_{t+1}}M^t \\ \pi'(e^t)v(c_1^t) = \gamma. \end{aligned} \tag{11}$$

At the competitive equilibrium, the two conditions in (5) equating at every period, the wage rate to the marginal productivity of labor and the rental rate of capital to its annuitized marginal productivity, must be satisfied as well. Thus at any time t adding up the budget constraints of the young and old alive, it must hold

$$c_0^t + \frac{\pi(e^{t-1})}{1+n}c_1^{t-1} + k^t + \frac{1}{p_t}M^t = F_L(\frac{k^{t-1}}{1+n},1) + F_K(\frac{k^{t-1}}{1+n},1)\frac{k^{t-1}}{1+n} + \frac{\pi(e^{t-1})}{p_t}\frac{M^{t-1}}{1+n} \tag{12}$$

where (because of the feasibility of the allocation of resources and the constant returns to scale of the technology) the first three terms of the left-hand side cancel out with the first two of the right-hand side at equilibrium, so that at any t it must hold

$$\frac{M^t}{M^{t+1}} = \frac{1+n}{\pi(e^t)}. \tag{13}$$

Thus, at equilibrium, the individual monetary holdings must always decrease at a slower pace than in the standard 2-period lifetime case with certainty (where they decrease every period by a constant factor $\frac{1}{1+n}$). This accounts for the fact that some individuals die in the end of the first period.

At a competitive equilibrium steady state the monetary savings held by the agents must be constant in real terms, i.e. $\frac{M^t}{p^t} = \frac{M^{t+1}}{p^{t+1}}$ always, and therefore prices must decrease at the same rate, so that it holds

$$\frac{p_t}{p_{t+1}} = \frac{1+n}{\pi(e)} \tag{14}$$

where e is the steady state individual level of effort. Therefore, the competitive equilibrium steady state under laissez-faire consists of a profile $\bar{c}_0, \bar{c}_1, \bar{e}, \bar{k}, \bar{m}$ satisfying

$$\begin{aligned} \frac{u'(c_0)}{v'(c_1)} &= 1+n = F_K(\frac{k}{1+n},1) \\ c_0 + k + m &= F_L(\frac{k}{1+n},1) \\ \frac{\pi(e)}{1+n}c_1 &= F_K(\frac{k}{1+n},1)\frac{k}{1+n} + m \\ \pi'(e)v(c_1) &= \gamma. \end{aligned} \tag{15}$$

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These equations would be equivalent to those characterizing the first-best steady state[6] if it were not for the term $\pi'(e)v'(c_1)c_1$ appearing in the last equation on the first-best conditions (8), but missing in the competitive equilibrium steady state conditions (15). As a consequence, the laissez-faire competitive equilibrium steady state is not the first-best steady state, as the next proposition establishes.

Proposition 1. *In the standard Diamond* (1965) *overlapping generations economy with production and money, the laissez-faire competitive equilibrium steady state is inefficient when the agents can choose the disutility they are willing to incur in order to increase their life expectancy.*

Proof. Let (c_0^*, c_1^*, k^*, e^*) be the first-best steady state solution to (8), and $(\bar{c}_0, \bar{c}_1, \bar{k}, \bar{m}, \bar{e})$ be the laissez-faire competitive equilibrium steady state solution to (15). It follows trivially from the last equation in each of the systems (8) and (15) that, should the two steady states coincide, then since

$$\pi'(\bar{e})v(\bar{c}_1) = \gamma = \pi'(e^*)[v(c_1^*) - v'(c_1^*)c_1^*] \tag{16}$$

it would hold also

$$\pi'(e^*)v'(c_1^*)c_1^* = 0 \tag{17}$$

which cannot hold for an interior steady state guaranteed by the good behavior at the boundary of the representative agent's utility. Q.E.D.

As noted above, the term $\pi'(e)v'(c_1)c_1$ (which from the first-best first order conditions (8) is equivalent to $\lambda\frac{\pi'(e)}{1+n}c_1$) measures the indirect cost of an increase in life expectancy implied by the additional pressure put on resources by a bigger fraction of survivors. This cost is not taken into account by the individuals in a competitive equilibrium. In effect, price-taking individuals disregard the impact of their effort —through a higher life expectancy— on the return to their own savings. More specifically, they take as given the return to capital r_{t+1} while it happens to be at equilibrium a function $F_K(\frac{k^t}{1+n}, l)/\pi(e^t)$ of their own effort e^t. The same remark holds for the return to his monetary savings which, with perfect foresight, he takes as given to be p_t/p_{t+1}, while it turns out to depend at equilibrium on his effort, according to $(1+n)/\pi(e^t)$. As a consequence, the agents overinvest in their life expectancy with respect to the efficient level, living in expectation longer lives while saving in terms of capital the same amount, which leads them to enjoy lower levels of consumption in both periods, as the following proposition shows.

[6] To be more precise they would rather imply the first-best conditions, but under conditions guaranteeing the uniqueness of a first-best steady state that amounts to the same thing.

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Proposition 2. *In the standard Diamond* (1965) *overlapping generations economy with production and money, if the agents can choose the disutility to incur in order to increase their life expectancy, then at the laissez-faire competitive equilibrium steady state profile of consumptions, savings, and life-expectancy effort* $(\bar{c}_0, \bar{c}_1, \bar{k}, \bar{m}, \bar{e})$ *satisfying* (15)*, the agents' first and second period consumptions are lower and the effort devoted to increase their life expectancy* $\bar{e}$ *bigger than at the first-best profile* (c_0^*, c_1^*, k^*, e^*) *satisfying* (8)*, i.e.*

$$\begin{aligned} c_1^* &> \bar{c}_1 \\ c_0^* &> \bar{c}_0 \\ k^* &= \bar{k} \\ e^* &< \bar{e}. \end{aligned} \tag{18}$$

Proof. Firstly, $\bar{k} = k^*$ follows trivially from the equalization of the marginal productivity of capital to the rate of growth of the population in both the laissez-faire competitive equilibrium steady state and the first-best steady state.

As for the level of effort e, let us see first that necessarily $e^* \leq \bar{e}$.

(1) Assume $e^* > \bar{e}$, and assume also that $c_1^* \geq \bar{c}_1$. Then

$$\frac{\pi(e^*)}{1+n} c_1^* > \frac{\pi(\bar{e})}{1+n} \bar{c}_1 \tag{19}$$

and hence $c_0^* < \bar{c}_0$ from the equation

$$c_0^* + \frac{\pi(e^*)}{1+n} c_1^* = F(\frac{k^*}{1+n}, 1) - k^* = F(\frac{\bar{k}}{1+n}, 1) - \bar{k} = \bar{c}_0 + \frac{\pi(\bar{e})}{1+n} \bar{c}_1 \tag{20}$$

so that

$$u'(c_0^*) > u'(\bar{c}_0). \tag{21}$$

Moreover, since $c_1^* \geq \bar{c}_1$, then

$$\frac{1}{v'(c_1^*)} \geq \frac{1}{v'(\bar{c}_1)}. \tag{22}$$

Therefore,

$$\frac{u'(c_0^*)}{v'(c_1^*)} \geq \frac{u'(c_0^*)}{v'(\bar{c}_1)} > \frac{u'(\bar{c}_0)}{v'(\bar{c}_1)} \tag{23}$$

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which cannot be since both at the competitive equilibrium steady state and the first-best steady state these marginal rates of substitution are equal to the rate of growth of the population $1+n$.

(2) Assume otherwise that $e^* > \bar{e}$ and $c_1^* < \bar{c}_1$. Then $\pi'(e^*) < \pi'(\bar{e})$ since π is concave, and $v(c_1^*) < v(\bar{c}_1)$, so that

$$\pi'(e^*)v(c_1^*) < \pi'(\bar{e})v(\bar{c}_1) \tag{24}$$

but then for the last equations in conditions (8) and (15) to hold that would require

$$\pi'(e^*)v'(c_1^*)c_1^* < 0 \tag{25}$$

which cannot be either.

Therefore, necessarily $e^* \leq \bar{e}$.

Let us see now that $e^* < \bar{e}$ indeed.

(1) Assume that $e^* = \bar{e}$ and that $c_1^* > (<)\bar{c}_1$. Then

$$\frac{\pi(e^*)}{1+n}c_1^* > (<)\frac{\pi(\bar{e})}{1+n}\bar{c}_1 \tag{26}$$

and hence $c_0^* < (>)\bar{c}_0$ by (20), from which

$$u'(c_0^*) > (<)u'(\bar{c}_0). \tag{27}$$

Moreover, since $c_1^* > (<)\bar{c}_1$, then

$$\frac{1}{v'(c_1^*)} > (<)\frac{1}{v'(\bar{c}_1)}. \tag{28}$$

Therefore,

$$\frac{u'(c_0^*)}{v'(c_1^*)} > (<)\frac{u'(c_0^*)}{v'(\bar{c}_1)} > (<)\frac{u'(\bar{c}_0)}{v'(\bar{c}_1)} \tag{29}$$

which again cannot be since both at the competitive equilibrium steady state and the first best steady state these marginal rates of substitution are equal to the growth factor of the population $1+n$.[7]

(2) Assume that $e^* = \bar{e}$ and assume moreover that $c_1^* = \bar{c}_1$. Then

$$\frac{\pi(e^*)}{1+n}c_1^* = \frac{\pi(\bar{e})}{1+n}\bar{c}_1 \tag{30}$$

and hence $c_0^* = \bar{c}_0$, i.e. $(c_0^*, c_1^*, e^*) = (\bar{c}_0, \bar{c}_1, \bar{e})$ which cannot be by Proposition 1.

[7] Note that although admittedly repetitive, the argument cannot be collapsed into a single step.

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Therefore, necessarily $e^* < \bar{e}$.

Finally, assume $c_1^* \leq \bar{c}_1$. Then, as previously,

$$\frac{\pi(e^*)}{1+n} c_1^* < \frac{\pi(\bar{e})}{1+n} \bar{c}_1 \tag{31}$$

and hence $c_0^* > \bar{c}_0$ by (20), from which

$$u'(c_0^*) < u'(\bar{c}_0). \tag{32}$$

Moreover, since $c_1^* \leq \bar{c}_1$, then

$$\frac{1}{v'(c_1^*)} \leq \frac{1}{v'(\bar{c}_1)}. \tag{33}$$

Therefore,

$$\frac{u'(c_0^*)}{v'(c_1^*)} \leq \frac{u'(c_0^*)}{v'(\bar{c}_1)} < \frac{u'(\bar{c}_0)}{v'(\bar{c}_1)} \tag{34}$$

which cannot be since both at the competitive equilibrium steady state and the first best steady state these marginal rates of substitution are equal to the growth factor of the population $1 + n$.[8]

Therefore, necessarily $c_1^* > \bar{c}_1$.

As a consquence, since both at the first-best steady state and the laissez-faire competitive steady state, it holds

$$\frac{u'(c_0^*)}{v'(c_1^*)} = 1 + n = \frac{u'(\bar{c}_0)}{v'(\bar{c}_1)} \tag{35}$$

$c_1^* > \bar{c}_1$ implies $c_0^* > \bar{c}_0$ as well.

Q.E.D.

In the following section, we show how to decentralize the first-best steady state as a competitive equilibrium.

[8] The same remark as in footnote 6 applies here.

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3.3 Implementation of the First-Best Steady State as a competitive equilibrium steady state.

Note that many instances of unhealthy behaviors with a direct link with life expectancy that do not have an impact on the agent's budget constraints (like not exercising or taking prolonged sunbaths) go, for that same reason, untaxed.[9] Moreover, in many cases, it is not possible to tax them indirectly either, by taxing, for example, saving returns (held in terms of either capital or money). In effect, on the one hand, taxing savings may disincentive the prospect of a high life expectancy and, thus, it could discourage a healthy behavior. But, on the other hand, taxing savings distorts the consumption-saving decision, modifying the condition equating the intertemporal marginal rate of substitution of consumption to the return to savings in (15), which would make it impossible to coincide with the first-best steady state.[10]

Therefore, consider instead the following policy. Announce at each period t to the newborn generation that in the second period a lump-sum tax/subsidy of an amount $c_1^{t-1} \ln \frac{\pi(e^t)}{\pi(e^{t-1})}$ will be raised/transferred. Note that although $\pi(e^t)$, the survival rate of generation t, is not known at the time t of the announcement (everything else is), it will crucially be nonetheless known at the time the policy will have to be implemented in $t+1$. As a matter of fact, the individuals are given the opportunity —by their knowledge of the exact form the lump-sum tax/subsidy will take— to manipulate the tax or subsidy. As a consequence, they will change their behavior. Interestingly enough, it turns out that, since they modified their behavior, it is actually *them* who are being manipulated by the policy maker in order to implement the first-best steady state.

In effect, the representative agent's problem becomes now (with the second period

[9]Others (like smoking and drinking alcohol) do. And others still that could be taxed (like eating junk food) are not, yet. Nevertheless, harmful behaviors, to one-self or to others, are taxed indeed, through fines (e.g. for speeding and other instances of dangerous driving).

[10]For instance, in Leroux (2008), it is shown that, in a static partial equilibrium framework, the first-best allocation can be restored through a tax on savings or, equivalently, on second period consumption. In this case, the individual has less incentives to invest in a higher life expectancy as his second period consumption is distorted downward.

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lump-sum tax/subsidy)

$$\max_{c_0^t, c_1^t, k^t, e^t, M^t} u(c_0^t) + \pi(e^t)v(c_1^t) - \gamma e^t$$
$$c_0^t + k^t + \frac{1}{p_t}M^t = w_t \tag{36}$$
$$c_1^t = r_{t+1}k^t + \frac{1}{p_{t+1}}M^t - c_1^{t-1}\ln\frac{\pi(e^t)}{\pi(e^{t-1})}$$

the solution of which is characterized by the first-order conditions

$$\begin{pmatrix} u'(c_0^t) \\ \pi(e^t)v'(c_1^t) \\ 0 \\ 0 \\ \pi'(e^t)v(c_1^t) - \gamma \end{pmatrix} = \lambda^t \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} + \mu^t \begin{pmatrix} 0 \\ 1 \\ -r_{t+1} \\ -\frac{p_t}{p_{t+1}} \\ c_1^{t-1}\frac{\pi'(e^t)}{\pi(e^t)} \end{pmatrix} \tag{37}$$

along with the budget constraints of the optimization problem above or, equivalently, by the system of equations

$$\frac{u'(c_0^t)}{v'(c_1^t)} = \pi(e^t)\frac{p_t}{p_{t+1}} = \pi(e^t)r_{t+1}$$
$$c_0^t + k^t + \frac{1}{p_t}M^t = w_t \tag{38}$$
$$c_1^t = r_{t+1}k^t + \frac{1}{p_{t+1}}M^t - c_1^{t-1}\ln\frac{\pi(e^t)}{\pi(e^{t-1})}$$
$$\pi'(e^t)v(c_1^t) = \gamma + \pi'(e^t)v'(c_1^t)c_1^{t-1}.$$

As before, at equilibrium the two conditions (5) determining the wage and rental rates are still satisfied. As for the feasibility condition, adding up the budget constraints of the agents living at any given period t one gets

$$c_0^t + \frac{\pi(e^{t-1})}{1+n}c_1^{t-1} + k^t + \frac{1}{p_t}M^t =$$
$$F_L(\frac{k^{t-1}}{1+n}, 1) + F_K(\frac{k^{t-1}}{1+n}, 1)\frac{k^{t-1}}{1+n} + \frac{\pi(e^{t-1})}{1+n}\frac{1}{p_t}M^{t-1} \tag{39}$$
$$- \frac{\pi(e^{t-1})}{1+n}c_1^{t-2}\ln\frac{\pi(e^{t-1})}{\pi(e^{t-2})}.$$

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Note again that (because of the feasibility and constant returns to scale) the first three terms of the left-hand side cancel out with the first two of the right-handside, so that (39) it is equivalent to

$$\frac{1}{p_t}M^t = \frac{\pi(e^{t-1})}{1+n}\frac{1}{p_t}M^{t-1} - \frac{\pi(e^{t-1})}{1+n}c_1^{t-2}\ln\frac{\pi(e^{t-1})}{\pi(e^{t-2})} \tag{40}$$

which, at the steady state, implies again

$$\frac{p_t}{p_{t+1}} = \frac{1+n}{\pi(e)} \tag{41}$$

as the last term in (40) vanishes. Therefore, the competitive equilibrium steady state is now a profile $\bar{c}_0, \bar{c}_1, \bar{e}, \bar{k}, \bar{m}$ satisfying

$$\begin{aligned} \frac{u'(c_0)}{v'(c_1)} &= 1+n = F_K(\frac{k}{1+n},1) \\ c_0+k+m &= F_L(\frac{k}{1+n},1) \\ c_1 &= F_K(\frac{k}{1+n},1)\frac{1}{\pi(e)}k + \frac{1+n}{\pi(e)}m \\ \pi'(e)v(c_1) &= \gamma + \pi'(e)v'(c_1)c_1. \end{aligned} \tag{42}$$

The solution to this system coincides with the solution to equations (8) above.[11] Note that the tax/susbidy $c_1^{t-1}\ln\frac{\pi(e^t)}{\pi(e^{t-1})}$ is zero at the steady state, so that no tax or subsidy is actually raised or handed out in that case, keeping the government budget trivially balanced. As a matter of fact, the mere announcement of the policy makes the agents modify their choices in such a way that the first-best steady state is attained in a decentralized way when this was not possible under laissez-faire. This result is summarized in the next proposition.

Proposition 3. *In the standard Diamond* (1965) *overlapping generations economy with production and money, if the agents can choose the disutility to incur in order to increase their life expectancy, the first-best profile* (c_0^*, c_1^*, k^*, e^*) *satisfying* (8) *is*

[11] To be precise, every solution to this system is also a solution to equations (8). Therefore, under conditions guaranteeing the uniqueness of the first-best steady state, the two systems of equations are equivalent.

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a competitive equilibrium outcome if a second period a lump-sum tax (subsidy) of an amount $c_1^{t-1} \ln \frac{\pi(e^t)}{\pi(e^{t-1})}$ *is raised (transferred) from (to) each generation* t.

This policy restores the first-best steady state for two reasons. First, adjusting their effort, the individuals directly reduce the tax they face (or increase the subsidy they receive) when old and, second, they adjust their probability of survival according to the prospect of facing a tax which reduces their future consumption or a subsidy that increases it. By imposing a lump-sum subsidy or tax on consumption when old, the planner makes more or less attractive the prospect of survival and thus provides incentives to the individual to choose the right level of effort.

4. Case in which increasing life expectancy is costly in terms of resources

Assume now that the individual can increase his life expectancy at some cost in terms of resources, so that the individual can divert part of his first period income away from consumption and saving, in order to increase his chances of survival. Thus, this effort appears directly in the individual's first period budget constraint instead of directly in the individual's utility. As in the previous case, we will characterize first the first-best steady state, then the competitive equilibrium steady state under laissez-faire, and finally the policy that implements the first-best steady state as a competitive equilibrium outcome.

4.1 First-best steady state.

The first-best steady state results in this case from solving the problem

$$\begin{gathered} \max_{c_0, c_1, k, e} u(c_0) + \pi(e)v(c_1) \\ c_0 + \frac{\pi(e)}{1+n}c_1 + k + e = F(\frac{k}{1+n}, 1) \end{gathered} \tag{43}$$

where e denotes the resources devoted to increase the individuals' life expectancy (through their probability of survival) as, say, health expenditures, and that enters directly the feasibility constraint. The solution to the optimization problem above is characterized by the first-order conditions

$$\begin{pmatrix} u'(c_0) \\ \pi(e)v'(c_1) \\ 0 \\ \pi'(e)v(c_1) \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ \frac{\pi(e)}{1+n} \\ 1 - F_K(\frac{k}{1+n}, 1)\frac{1}{1+n} \\ 1 + \frac{\pi'(e)}{1+n}c_1 \end{pmatrix} \tag{44}$$

along with the constraint of the problem above. Equivalently, a first-best steady state consists of a profile c_0^*, c_1^*, e^*, k^* satisfying

$$\begin{aligned} \frac{u'(c_0)}{v'(c_1)} &= 1+n = F_K(\frac{k}{1+n}, 1) \\ c_0 + \frac{\pi(e)}{1+n}c_1 + k + e &= F(\frac{k}{1+n}, 1) \\ \pi'(e)v(c_1) &= (1+n)v'(c_1) + \pi'(e)v'(c_1)c_1. \end{aligned} \tag{45}$$

Note that the first line is the same condition as the one obtained in the case where increasing life expectancy is costly in terms of utility in (8): first, the equality between the inter-temporal marginal rate of substitution and the rate at which consumption can be transferred between the two periods, and second, the maximization of output net of capital replacement. The feasibility condition in the second line includes now as an expenditure the resources e devoted to pin down the life expectancy of the individual, i.e. health expenditures. Thus output net of replacement of used up capital must be at any period equal to the consumption of young individuals, plus the consumption of the survivors of the preceding generation, *and the health expenditures*.

Finally, the last condition differs from the one obtained in the utility-effort case in (8). Indeed, the term $(1+n)v'(c_1)$ is now substituted to the term γ in the right-hand side. As before, this condition still requires that, at the first-best steady state, the marginal benefit of increasing the life expectancy, $\pi'(e)v(c_1)$, exactly matches its marginal cost which, in this case, consists of (i) the direct impact that an increase in health expenditures has on second period consumption —reducing it at a rate $\frac{1+n}{\pi(e)}$ and hence reducing second period utility at a rate $(1+n)v'(c_1)$ (first term on the right-hand side)— and of (ii) the indirect cost (common to both the utility-effort and the resources-effort cases) in terms of the additional pressure on resources following from bigger cohorts of survivors (the second term $\lambda\frac{\pi'(e)}{1+n}c_1 = \pi'(e)v'(c_1)c_1$ in the right-hand side).

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4.2 Competitive equilibrium steady state under laissez-faire.

The representative agent's problem under perfect competition is in this case

$$\max_{c_0^t, c_1^t, k^t, e^t, M^t} u(c_0^t) + \pi(e^t)v(c_1^t)$$

$$c_0^t + k^t + \frac{1}{p_t}M^t + e^t = w_t \tag{46}$$

$$c_1^t = r_{t+1}k^t + \frac{1}{p_{t+1}}M^t.$$

As in the utility-effort case, the individual has to decide how much to save as well as the composition of his savings portfolio in terms of capital and money. The difference now comes from the fact that, he must decide as well how much of his income to devote to health expenditures e^t in order to pin down the optimal (from his viewpoint) life expectancy. The solution to the agent's problem is characterized by the first-order conditions

$$\begin{pmatrix} u'(c_0^t) \\ \pi(e^t)v'(c_1^t) \\ 0 \\ 0 \\ \pi'(e^t)v(c_1^t) \end{pmatrix} = \lambda^t \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} + \mu^t \begin{pmatrix} 0 \\ 1 \\ -r_{t+1} \\ -\frac{p_t}{p_{t+1}} \\ 0 \end{pmatrix} \tag{47}$$

along with the budget constraints in the problem above. Equivalently, agent t's choice is the solution to

$$\begin{aligned} \frac{u'(c_0^t)}{v'(c_1^t)} &= \pi(e^t)\frac{p_t}{p_{t+1}} = \pi(e^t)r_{t+1} \\ c_0^t + k^t + \frac{1}{p_t}M^t + e^t &= w_t \\ c_1^t &= r_{t+1}k^t + \frac{1}{p_{t+1}}M^t \\ \pi'(e^t)v(c_1^t) &= \pi(e^t)v'(c_1^t)r_{t+1}. \end{aligned} \tag{48}$$

At the competitive equilibrium, the wage and rental rate are still determined by the conditions (5) determining the wage and rental rate of capital, so that the return to savings invested in capital by a generation depends on the survival rate of that same generation. Under competitive conditions, the individuals take these variables as given. Again, from the addition of the budget constraints of the agents alive at any given period t

$$c_0^t + \frac{\pi(e^{t-1})}{1+n}c_1^{t-1} + k^t + \frac{1}{p_t}M^t + e^t = \\ F_L(\frac{k^{t-1}}{1+n},1) + F_K(\frac{k^{t-1}}{1+n},1)\frac{k^{t-1}}{1+n} + \frac{\pi(e^{t-1})}{p_t}\frac{M^{t-1}}{1+n} \tag{49}$$

it follows that the feasibility of the allocation is equivalent to

$$\frac{M_t}{M_{t+1}} = \frac{1+n}{\pi(e^t)} \tag{50}$$

which at a steady state implies also

$$\frac{p_t}{p_{t+1}} = \frac{1+n}{\pi(e)}. \tag{51}$$

Therefore, a competitive equilibrium steady state under laissez-faire consists of a profile $\bar{c}_0, \bar{c}_1, \bar{e}, \bar{k}, \bar{m}$ such that

$$\begin{aligned} \frac{u'(c_0)}{v'(c_1)} &= 1+n = F_K(\frac{k}{1+n},1) \\ c_0 + k + m + e &= F_L(\frac{k}{1+n},1) \\ c_1 &= \frac{1}{\pi(e)}F_K(\frac{k}{1+n},1)k + \frac{1+n}{\pi(e)}m \\ \pi'(e)v(c_1) &= (1+n)v'(c_1). \end{aligned} \tag{52}$$

Only the last equation in the system above differs from the one in the first-best system of equations in (45). Indeed, compared to the first-best system (45), the term $\pi'(e)v'(c_1)c_1$ is missing in (52), which is simply due to the fact that the return to savings invested in capital, $r_{t+1} = \frac{1}{\pi(e^t)}F_K(\frac{k^t}{1+n},l)$ and in money, $p_t/p_{t+1} = (1+n)/\pi(e^t)$, are taken as given by the individual under perfect competition. He does not take into account that, by investing in his longevity, he is also going to modify the overall return of his savings and thus, his consumption possibilities when old. As a consequence, the suboptimality of the competitive equilibrium steady state follows, as the following proposition establishes.

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Proposition 4. *In the standard Diamond* (1965) *overlapping generations economy with production and money, the laissez-faire competitive equilibrium steady state is inefficient when the agents can choose the resources to invest into increasing their life expectancy.*

Proof. Letting (c_0^*, c_1^*, k^*, e^*) be the first-best steady state solution to (45), and $(\bar{c}_0, \bar{c}_1, \bar{k}, \bar{m}, \bar{e})$ be the laissez-faire competitive equilibrium steady state solution to (52), it follows trivially from the last equation in each of the systems (45) and (52) that should the two coincide, then since

$$\pi'(\bar{e})v(\bar{c}_1) = (1+n)v'(\bar{c}_1) = (1+n)v'(c_1^*) = \pi'(e^*)[v(c_1^*) - v'(c_1^*)c_1^*] \tag{53}$$

it would hold also

$$\pi'(e^*)v'(c_1^*)c_1^* = 0 \tag{54}$$

which cannot hold for an interior steady state guaranteed by the good behavior at the boundary of the agent's utility. Q.E.D.

As in the previous utility-effort case, the fact that the individuals do not take into account the stress that a higher life expectancy puts on the available resources leads them to invest too much resources into it compared to what would be the optimal amount, i.e. $\bar{e} > e^*$. The next proposition establishes this.

Proposition 5. *In the standard Diamond* (1965) *overlapping generations economy with production and money, if the agents can choose the resources to invest into increasing their life expectancy, then at the laissez-faire competitive equilibrium steady state profile of consumptions, savings, and life-expectancy effort* $(\bar{c}_0, \bar{c}_1, \bar{k}, \bar{m}, \bar{e})$ *satisfying* (52)*, the agent's second-period consumption is not bigger and the resources invested into increasing his life expectancy* $\bar{e}$ *are not smaller than at the first-best profile* (c_0^*, c_1^*, k^*, e^*) *satisfying* (45)*, while his capital savings are the same, i.e.*

$$\begin{aligned} c_1^* &\geq \bar{c}_1 \\ k^* &= \bar{k} \\ e^* &\leq \bar{e} \end{aligned} \tag{55}$$

Proof. [12] Firstly, $\bar{k} = k^*$ follows trivially from the equalization of the marginal productivity of capital to the rate of growth of the population in both the laissez-faire competitive equilibrium steady state and the first-best steady state.

[12]The proof parallels that of the utility-effort case, but maybe surprisingly has a few twists that

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As for the level of effort e and the second -period consumption c_1, let us see first that necessarily $e^* \leq \bar{e}$ and $c_1^* \geq \bar{c}_1$.

(1) Assume $e^* > \bar{e}$ and $c_1^* \geq \bar{c}_1$. Then

$$\frac{\pi(e^*)}{1+n}c_1^* + e^* > \frac{\pi(\bar{e})}{1+n}\bar{c}_1 + \bar{e} \tag{56}$$

and hence $c_0^* < \bar{c}_0$ from the equation

$$c_0^* + \frac{\pi(e)}{1+n}c_1^* + e^* = F(\frac{k^*}{1+n}, 1) - k^* = F(\frac{\bar{k}}{1+n}, 1) - \bar{k} = \bar{c}_0 + \frac{\pi(\bar{e})}{1+n}\bar{c}_1 + \bar{e} \tag{57}$$

so that

$$u'(c_0^*) > u'(\bar{c}_0). \tag{58}$$

Moreover, since $c_1^* \geq \bar{c}_1$, then

$$\frac{1}{v'(c_1^*)} \geq \frac{1}{v'(\bar{c}_1)}. \tag{59}$$

Therefore,

$$\frac{u'(c_0^*)}{v'(c_1^*)} \geq \frac{u'(c_0^*)}{v'(\bar{c}_1)} > \frac{u'(\bar{c}_0)}{v'(\bar{c}_1)} \tag{60}$$

which cannot be since both at the competitive equilibrium steady state and the first best steady state these marginal rates of substitution are equal to the growth factor of the population $1+n$.

As a consequence, either $e^* \leq \bar{e}$, or $c_1^* < \bar{c}_1$, or both hold.

(2) Assume that both $e^* \leq \bar{e}$ and $c_1^* < \bar{c}_1$ hold. Then, as previously,

$$\frac{\pi(e^*)}{1+n}c_1^* + e^* < \frac{\pi(\bar{e})}{1+n}\bar{c}_1 + \bar{e} \tag{61}$$

and hence $c_0^* > \bar{c}_0$ by (57), from which

$$u'(c_0^*) < u'(\bar{c}_0). \tag{62}$$

make it significantly different. Notably, a consequence of them is that no relation can be established between the first period consumptions $\bar{c}_0$ and c_0^*, as well as that neither $c_1^* > \bar{c}_1$ nor $e^* < \bar{e}$ are guaranteed anymore.

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Moreover, since $c_1^* \leq \bar{c}_1$, then

$$\frac{1}{v'(c_1^*)} \leq \frac{1}{v'(\bar{c}_1)}. \tag{63}$$

Therefore,

$$\frac{u'(c_0^*)}{v'(c_1^*)} \leq \frac{u'(c_0^*)}{v'(\bar{c}_1)} < \frac{u'(\bar{c}_0)}{v'(\bar{c}_1)} \tag{64}$$

which cannot be since both at the competitive equilibrium steady state and the first best steady state these marginal rates of substitution are equal to the growth factor of the population $1+n$.

Therefore, either $e^* \leq \bar{e}$ and $c_1^* \geq \bar{c}_1$, or $e^* > \bar{e}$ and $c_1^* < \bar{c}_1$.

(3) Assume $e^* > \bar{e}$ and $c_1^* < \bar{c}_1$. Then $v'(c_1^*) > v'(\bar{c}_1)$ holds, as well as $\pi'(e^*) < \pi'(\bar{e})$ and $v(c_1^*) < v(\bar{c}_1)$, and hence

$$\pi'(e^*)v(c_1^*) < \pi'(\bar{e})v(\bar{c}_1) \tag{65}$$

But since,

$$\begin{aligned} \pi'(e^*)v(c_1^*) &= (1+n)v'(c_1^*) + \pi(e^*)v'(c_1^*)c_1^* \\ \pi'(\bar{e})v(\bar{c}_1) &= (1+n)v'(\bar{c}_1) \end{aligned} \tag{66}$$

then necesarily $\pi(e^*)v'(c_1^*)c_1^* < 0$, which cannot be.

Therefore $e^* \leq \bar{e}$ and $c_1^* \geq \bar{c}_1$.

Q.E.D.

It is worth noting that, as opposed to what happened in the disutility-effort case, nothing can be said now about how do the first-period consumptions c_0^* and $\bar{c}_0$ compare. This is simply due to the fact that when e enters the budget constraint, it gives one additional degree of freedom to the problem, which leaves undetermined how c_0^* and $\bar{c}_0$ compare.

4.3 FBSS implementation as a competitive equilibrium steady state with money and taxes.

Contrarily to what happened in the utility-effort case, health expenditures can be taxed or subsidized directly. This simplifies considerably the implementation of the first-best steady state. For instance, assume that the government taxes health expenditures at a rate σ^t and hands at $t+1$ a lump-sum transfer T^t to agents born at time t. In this case, the representative agent's problem becomes

$$\max_{c_0^t, c_1^t, k^t, e^t, M^t} u(c_0^t) + \pi(e^t)v(c_1^t)$$

$$c_0^t + k^t + \frac{1}{p_t}M^t + (1+\sigma^t)e^t = w_t \tag{67}$$

$$c_1^t = r_{t+1}k^t + \frac{1}{p_{t+1}}M^t + T^t$$

The solution to this problem is characterized by the first-order conditions

$$\begin{pmatrix} u'(c_0^t) \\ \pi(e^t)v'(c_1^t) \\ 0 \\ 0 \\ \pi'(e^t)v(c_1^t) \end{pmatrix} = \lambda^t \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 1+\sigma^t \end{pmatrix} + \mu^t \begin{pmatrix} 0 \\ 1 \\ -r_{t+1} \\ -\frac{p_t}{p_{t+1}} \\ 0 \end{pmatrix} \tag{68}$$

and the budget constraints in the problem above, or, equivalently, by

$$\begin{aligned} \frac{u'(c_0^t)}{v'(c_1^t)} &= \pi(e^t)\frac{p_t}{p_{t+1}} = \pi(e^t)r_{t+1} \\ c_0^t + k^t + \frac{1}{p_t}M^t + (1+\sigma^t)e^t &= w_t \\ c_1^t &= r_{t+1}k^t + \frac{1}{p_{t+1}}M^t + T^t \\ \pi'(e^t)v(c_1^t) &= \pi(e^t)v'(c_1^t)r_{t+1}(1+\sigma^t). \end{aligned} \tag{69}$$

At a competitive equilibrium, the conditions (5) determining the wage and rental rate of capital still hold. We require also that the government runs a balanced budget at every period, so that in every period t it must hold

$$e^t\sigma^t = T^{t-1}\frac{\pi(e^{t-1})}{(1+n)} \tag{70}$$

where the amount raised by taxes on health expenditures on the left-hand side matches at every period the amount handed out to the survivors of the previous generation, on the right-hand side. Finally, adding up the budget constraints of the agents alive at any given period

$$\begin{aligned} &c_0^t + \frac{\pi(e^{t-1})}{1+n}c_1^{t-1} + k^t + \frac{1}{p_t}M^t + (1+\sigma^t)e^t = \\ &F_L(\frac{k^{t-1}}{1+n}, 1) + F_K(\frac{k^{t-1}}{1+n}, 1)\frac{k^{t-1}}{1+n} + \frac{\pi(e^{t-1})}{p_t}\frac{M^{t-1}}{1+n} + T^{t-1}\frac{\pi(e^{t-1})}{(1+n)} \end{aligned} \tag{71}$$

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it follows that the feasibility condition is again equivalent to

$$\frac{M_t}{M_{t+1}} = \frac{1+n}{\pi(e^t)} \tag{72}$$

which at the steady state requires

$$\frac{p_t}{p_{t+1}} = \frac{1+n}{\pi(e)}. \tag{73}$$

Therefore, the competitive equilibrium steady state is characterized now by a profile $\bar{c}_0, \bar{c}_1, \bar{e}, \bar{k}, \bar{m}$ satisfying

$$\begin{aligned}
\frac{u'(c_0)}{v'(c_1)} = 1+n =& F_K(\frac{k}{1+n}, 1) \\
c_0 + k + m + (1+\sigma)e =& F_L(\frac{k}{1+n}, 1) \\
c_1 = \frac{1}{\pi(e)} F_K(&\frac{k}{1+n}, 1)k + \frac{1+n}{\pi(e)} m + T \\
\pi'(e)v(c_1) = v'(c_1)&(1+n)(1+\sigma) \\
e\sigma = T&\frac{\pi(e)}{1+n}.
\end{aligned} \tag{74}$$

Comparing conditions (74) with those characterizing the first-best steady state in (45), it is straightforward to check that they share the same solution if the tax rate is[13]

$$\sigma = \frac{\pi'(e)}{1+n} c_1 \tag{75}$$

Therefore, in order to implement the first-best steady state, the government just needs to announce at the beginning of each period t that (i) health expenditures are going to be taxed then at a rate

$$\sigma^t = \frac{\pi'(e^{t-1})}{1+n} c_1^{t-1} \tag{76}$$

[13]It can be easily verified that an equivalent expression for the optimal tax rate at the first-best steady state is

$$\sigma = \frac{v'(c_1)c_1}{v(c_1) - v'(c_1)c_1}$$

Note, that if $v(.)$ has constant elasticity of substitution, $v(x) = x^\epsilon$, this tax takes the form $\varepsilon/(1-\varepsilon)$ and depends thus only on the parameter ϵ and not on the particular value of the steady state second period consumption c_1.

(which depends only on known variables and cannot be manipulated by individuals born in period t) and (ii) a lump-sum transfer will be made to period-t agents at $t+1$ of an amount equal to[14]

$$T^t = e^{t-1}\sigma^t \frac{1+n}{\pi(e^{t-1})} = \frac{\pi'(e^{t-1})e^{t-1}}{\pi(e^{t-1})} c_1^{t-1} \tag{77}$$

The lump-sum transfer depends thus on the elasticity of the survival probability with respect to health expenditures and on the consumption when old of the previous generation. Replacing these two expressions into conditions in (74) characterizing the competitive equilibrium steady state with taxes, it is straightforward to check that at the steady state the conditions coincide with those of the first-best steady state in (45),[15] so that such tax-and-transfers scheme implements the first-best steady state. This result is summarized in the next proposition.

Proposition 6. *In the standard Diamond* (1965) *overlapping generations economy with production and money, if the agents can choose the resources to invest into increasing their life expectancy, the first-best profile* (c_0^*, c_1^*, k^*, e^*) *satisfying* (45) *is a competitive equilibrium outcome if such expenditure is taxed at a rate*

$$\sigma^t = \frac{\pi'(e^{t-1})}{1+n} c_1^{t-1} \tag{78}$$

for each generation t*, and a second period lump-sum transfer is made to each generation* t *of an amount*

$$T^t = \frac{\pi'(e^{t-1})e^{t-1}}{\pi(e^{t-1})} c_1^{t-1}. \tag{79}$$

Finally, consider the *expected* per capita net taxes paid by any given generation t, i.e. $\tau^t = \sigma^t e^t - \pi(e^t)T^t$ (note that the transfer T^t is conditional on the individual's survival, while the contribution is paid in first period, with certainty). Replacing for the expressions of σ^t and T^t, it amounts to

$$\tau^t = \pi'(e^{t-1})c_1^{t-1}[\frac{e^t}{1+n} - \frac{\pi(e^t)e^{t-1}}{\pi(e^{t-1})}] \tag{80}$$

[14]Note that the formulation of the transfer T^t is defined such that it depends only on variables which cannot be manipulated by the individuals born in period t. The consequence of such an assumption is that the budget balance condition, although satisfied at the steady state, will not be satisfied ex post, outside the steady state.

[15]Under assumtions guaranteeing the uniqueness of the latter.

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which, at the steady state, becomes

$$\tau = \pi'(e)c_1e[\frac{1}{1+n} - 1] < 0. \tag{81}$$

These expected net taxes are negative simply because of our assumption of positive demographic growth as (if $n = 0$, we would also have $\tau = 0$). This is not incompatible with budget balance at each period, which is guaranteed by (70).

5. Conclusion

In this paper, we address in a dynamic setup the externality created by expenses or individual behaviors that have an impact on the individual's life expectancy. Becker and Philipson (1998) first showed in a static setup how the individuals' attempts to increase the "quantity" of their life also affect the "quality" of it in a way that they do not perfectly anticipate, which typically leads to an inefficient outcome. More specifically, we show, in this paper, that in an overlapping generations economy with production à la Diamond (1965) the competitive equilibrium steady state still differs from the first-best steady state because of this external effect of longevity on the return to savings, both when individuals can affect their life expectancy by means of health expenditures, or when they can do it by just improving their habits in a way that is costly for them in terms of utility (but at no cost in terms of resources). The externality is created by the fact that individuals do not take into account that their life expectancy affects the return to their annuitized savings (held either in money or in capital) and, hence, their consumption possibilities when old. In this case, they are likely to invest too much in their longevity in comparison to what would be optimal. We show nonetheless that the first-best steady state can be decentralized as a competitive equilibrium in both cases if the government announces and implements the adequate policy of taxes and transfers, and we identify this policies.

Still our paper could be extended in several ways. First, we consider a type of effort which is costly in terms of utility and in terms of resources but we excluded the case where the effort requires time investment. This would certainly have implications on the labour supply. Moreover, we assume a perfect annuity market, which may be far from what is observed in reality; in a extension of this paper, we should relax this assumption. This is on our research agenda.

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