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Is inequality harmful for the environment in a growing economy!?

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Revised version.

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Abstract

In this paper we investigate the relationship between inequality and the environment in a growing economy from a political economy perspective. We consider an endogenous growth economy, where growth generates pollution and a deterioration of the environment. Public expenditures may either be devoted to supporting growth or abating pollution. The decision over the public programs is done in a direct democracy, with simple majority rule. We prove that the median voter is decisive and show that inequality is harmful for the environment: the poorer the median voter relative to the average individual, the less she will tax and devote resources to the environment, preferring to support growth.

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1 Introduction

The distributive issue associated to the protection of the environment has been widely studied in the literature, mostly in relation with the so-called "Environmental Kuznets Curve".[1] Empirically, Magnani (2000), using cross-section data, shows that there exists a negative relationship between inequality and public expenditures related to the protection of the environment. However, this issue has been studied by means of static models of the economy. But environmental problems are recognized to raise intertemporal trade-offs and the concern over environmental protection policies must focus on the dynamic and long-term consequences of any measure. On the other hand, the relationship between growth and inequality has also been thoroughly studied (see the survey by Aghion, Caroli and Garcia-Penalosa, 1999)[2] but these studies have neglected to include the interplay with environment issues.

The aim of this paper is to tackle the issue between inequality and environment protection in the context of a growing economy .[3] We do so adopting a political-economic perspective.[4] Basically, we develop the following view: both the protection of the environment and the growth process, while potentially conflicting, are major public concerns in modern polities. In particular, environment protection depends on how much public resources to devote to it, rather than to other policies aimed at sustaining growth and consumption. As far as individuals are differenciated with respect to their capital endowment and more generally wealth, they hold conflicting views on the necessity to protect the environment, relative to the goal of sustaining growth. In other words, the inequality schedule (that is the initial distribution of capital among agents) shapes the distribution of opinions on the trade-off between growth and environment protection. Through voting, these opinions are aggregated and lead to a political decision on the allocation of resources to environment protection. Therefore, inequality impacts on the environment, even when agents do not differ in their preferences over physical consumption and the environment.

In the economy which we consider, it impacts negatively. Indeed, inequality is harmful

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[1] See among recent references, Andreoni and Levinson (2001), Magnani (2000) and Torras and Boyce (1998). For a general survey on environment and growth, see Smulders (1999).

[2] Empirical studies on this issue lead to conflicting views: Perotti (1996) concluded that cross-country studies lead to a negative relationship. Later on, panel estimations lead to a positive relationship.

[3] Magnani (2000) refers to growth as the factor behind different levels of aggregate output but she does not model the growth process and therefore does not tackle the intertemporal trade-off raised by environmental policy, that is its impact on the saving decision. Marsiliani and Renström (2000) address this issue using an overlapping generations model, but they skip difficult questions related to the time-consistency of the political decisions. Jones and Manuelli (2001) also use an overlapping generations model to tackle the relationship between pollution and growth in a political setting, but they do not refer to inequality as the source of conflicts of interest among voters; they instead assume that the old generation has the power to choose.

[4] That political institutions matter in the understanding of the relationship between environment and growth is well illustrated by Earnhart (1997), comparing environment protection policies under communism and democracy.

to the environment: the more unequal a society (in a sense which will be made more precise below), the more resources will be used to sustain growth despite its negative impact on the state of the environment.[5] As a result, there is an inverse relationship between the concern for the environment and the growth rate chosen by the polity. We consider a growing economy with two central features. First, public expenditures contribute to growth, as in a AK endogenous growth model with productive public good. Second, aggregate production pollutes and deteriorates the state of the environment in a way which is detrimental to any one's utility since the environment is a pure public good in this economy. However, an active public policy is able to improve the environment: by devoting public resources to the protection of the environment, the government can fight off the adverse effects of growth on the environment. This obviously raises a public dilemma: how much resources to devote to the adverse goals of growth and environment protection?

The solution to this dilemma depends on how much the polity values the quality of its environment relative to its material well-being linked to consumption of physical goods. Assuming identical utility functions the arguments of which are the state of the environment and physical consumption, each agent in this economy views this trade-off according to her own wealth.[6] We find that the wealthier (relative to the mean initial capital endowment) an agent, the more she is in favor of taxation, in particular for the sake of depollution activities. This suggests that inequality matters a lot for the solution to this trade-off. In other words, there are conflicting views among individuals and the trade-off facing the entire polity can only be solved by means of a political decision.[7]

The quality of the environment is a pure public good. This may justify the use of public spending for environment improvement. On the other hand, when public resources are used to productive purposes, it uplifts the productivity of capital and labor, hence their marginal remunerations, in any period and thus, it increases the rate of growth. The marginal benefits of growth-enhancing public spending are larger, the poorer an agent is. A poorer agent faces a steeper trade-off between a marginal improvement of the environment and a marginal reduction in consumption. On the whole, standard economic and environmental issues are related and linked to inequality as agents with different endowments perceive differently this trade-off.

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[5] Admittedly, the relationship between pollution and growth is not simple to assess. There are many types of pollutants and many sources of growth, including R&D which may lead to the use of more environment-friendly technologies. However what we have in mind is the debate on global warming. There is much evidence that the man-made industrialization process leads to a significant increase in CO_2 emissions, adversely affecting world climate and welfare. See for example Holtz-Eakin and Selden (1995).

[6] That is, contrary to Magnani (2000), we do not assume that wealthier agents are characterized by a larger weight given to the state of the environment.

[7] Empirical studies support this view. Kahn and Matsusaka (1997) show that the environment is found to be a normal good for most people, except those with very high incomes. They stress that income and price can explain most of the variation in voting; there is little need to introduce "preference" variables such as political ideology.

Applying the majority decision making rule to the political resolution of the trade-off we have just mentioned, readily leads us to our conclusion. The poorer the median voter, the more she cares about material well-being, the more she is willing to channel public resources to the sustaining of growth and the less she will devote resources to the restoration of the environment altered by economic growth.[8] The more unequal a society, for a given amount of initial wealth, the more resources will be devoted to the upholding of growth and therefore the more degraded will be the environment.[9]

In the next section, we develop the model of a growing economy with environment. In section 3, we address the political decision to be taken over the protection of the environment. The last section concludes.

2   The model

In this section, we set up the model of an economy where growth has an adverse influence on the environment. Pollution is a direct consequence of production[10] and public resources can either sustain growth or improve the environment. Infinitely lived agents care both about their consumption profile and the state of the environment over time.

Individuals differ in their initial endowment of capital. The political decision about taxation and spending is taken at the beginning of time according to a simple majority rule. The public budget is balanced at each period. Hence, the polity has to make a joint decision about the tax rate and the split of public receipts into growth-enhancing and environment-linked spending. This decision is irrevocable, made before any capital accumulation decision and will be applied at each period. When this decision is reached, each individual acts as an intertemporal utility-maximizer, taking the public decision as given and decides about her intertemporal saving schedule.

2.1   Production

The production function is similar to that of Alesina and Rodrik (1994), which is adapted from Barro (1990). A public good G_Y is produced by government and contributes to production in addition to capital and labor. The aggregate production function is

$$Y = AK^{\alpha}(G_Y)^{1-\alpha}L^{1-\alpha} \tag{1}$$

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[8] In this paper, when we refer to "poorer" or "wealthier" agents, it is based on their initial endowment, and relative to the average initial endowment. That is why we stress inequality rather than (absolute) poverty.

[9] So this paper is related to the literature on the political economy of redistribution (See Roberts, 1977, and Meltzer and Richards, 1981). Some papers have cast the issue of redistribution within an endogenous growth model (see Alesina and Rodrik, 1994, Fiaschi, 1999, Kempf and Rossignol, 2005). Here we use a similar framework to address the trade-off between growth and environment protection.

[10] Alternatively, pollution could be linked to consumption expenditures. In our balanced-growth model with no transition to the steady-state, this does not entail any major difference.

where K represents aggregate capital and L aggregate labor. The factors are remunerated at their marginal productivity:

$$r = \frac{\partial Y}{\partial K} = \frac{\alpha Y}{K} \text{ and } \widehat{w} = \frac{\partial Y}{\partial L} = \frac{(1-\alpha)Y}{L} \tag{2}$$

then $rK + \widehat{w}L = \alpha Y + (1-\alpha)Y = Y$.

There are N agents in the economy (N odd). At time 0, each agent is endowed with a given quantity of initial capital $k_i(0)$, and $k_i(0) \neq k_j(0)$, for any $i \neq j$. There are no two identical endowments. Without loss of generality, we rank individuals according to their endowments: $k_i(0) < k_j(0)$, for any $i < j$. An agent is characterized by a median initial capital endowment, denoted by k_m.

The total public spending is $G = G_Y + G_E$, where G_Y represents the public spending contributing to production, and G_E the amount of public spending against pollution. The product is taxed to finance public spending: $G = \tau Y$.[11] We denote by τ_Y the part of the tax which finances G_Y, and τ_E the part of the tax which finances G_E. It means that $G_Y = \tau_Y Y$, and $G_E = \tau_E Y$. We denote by $\tau = \tau_E + \tau_Y$ the overall tax rate. We restrict $\tau \in (0,1)$ as the overall tax rate τ cannot be bigger than 1. According to (1), the public good G_Y cannot be negative, hence $\tau_Y > 0$. τ_E can be either positive or negative, which implies that $\tau_E > 0$ corresponds to the case of a policy aiming at protecting the environment, $\tau_E \leq 0$ corresponds to the case where the environment is depleted for the sake of growth. Here, we focus on the former case. In brief, the public decision amounts to choosing a pair of tax rates (τ_E, τ_Y). This pair will be applied at any period.[12]

Inserting the relation $G_Y = \tau_Y Y$ in equation (1), we get:

$$Y = AK^\alpha(\tau_Y Y)^{1-\alpha}L^{1-\alpha} = AK^\alpha(\tau_Y L)^{1-\alpha}Y^{1-\alpha} \tag{3}$$

which is equivalent to:

$$Y = A^{1/\alpha}K(\tau_Y L)^{(1-\alpha)/\alpha} \tag{4}$$

We normalize labor ($L = 1$), thus:

$$Y = A^{1/\alpha}\tau_Y^{(1-\alpha)/\alpha}K \tag{5}$$

Using (2), we get:

$$r = \alpha A^{1/\alpha}\tau_Y^{(1-\alpha)/\alpha} \text{ and } \widehat{w} = (1-\alpha)A^{1/\alpha}\tau_Y^{(1-\alpha)/\alpha}K. \tag{6}$$

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[11] Here we differ from Alesina-Rodrik who consider a tax based on capital.

[12] We shall discuss this assumption in section 3.

2.2 Quality of environment

We assume that the quality E of the environment (with $E \geq 0$) is a decreasing function of the production Y (because of negative externalities of the production),[13] and an increasing function of the public spending against pollution G_E:

$$E = E(Y, G_E) \tag{7}$$

G_E and Y are instantaneous values, that means for example that a quick increase of G_E improves immediately E. In other words, we consider the pollution as a flow, and not as a stock (see Marrewijk et al., 1993).

We assume that E is a homogeneous function of Y and G_E. More precisely, E is homogeneous of degree 0, which means that E is a function of the ratio $\frac{G_E}{Y} = \tau_E$. This homogeneity assumption means that growth is not the "ideal" solution of the environmental problems, neither ineluctably harmful for the environment.[14] Hence we can write:

$$E = E(\tau_E). \tag{8}$$

2.3 Individual utility function

The instantaneous utility function of agent i depends on her level of consumption c_i and on the quality of the environment E (which does not depend on the agent i), $E \geq 0$. We assume that this total utility is separable in the physical individual consumption c_i and in the quality of the environment E, since there is no *a priori* interaction between these two aspects. It means in particular that it is not a product but a sum:

$$U(c_i, E) = \ln(c_i) + \widetilde{V}(E). \tag{9}$$

Because of (8), we get:

$$U(c_i, E) = \ln(c_i) + V(\tau_E) \tag{10}$$

and we assume that the V function is increasing and concave: $V' > 0$ and $V'' < 0$. An improvement in the environment improves utility at a decreasing pace. Despite the separability of this function, we will show that c_i depends on τ_E (cf. (17)).

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[13] For this reason, the environment appears as specific compared to more standard public goods, like national defense or education, which are not considered as directly adversely affected by the production process. E equal to zero or negative amounts to an unsufferable environment.

[14] If E is homogeneous of degree β, then (7) leads to

$$\forall \mu > 0, \ E(\mu Y, \mu G_E) = \mu^\beta E(Y, G_E)$$

1. If $\beta > 0$, when we multiply Y and G_E by a factor $\mu > 1$, then the quality of the environment is increased by a factor μ^β. In particular growth will automatically increase E. We will tend to ecological heavens.

2. If $\beta < 0$, when we multiply Y and G_E by a factor $\mu > 1$, then the quality of the environment decreases. In particular the growth will automatically decrease E. We will tend to ecological hell.

To sum up, if $\beta > 0$, any action is useless since the economic growth will solve the ecological problems, and if $\beta < 0$, the growth must be stopped because an indefinite growth would ineluctably harm the environment.

6

2.4 Accumulation decisions and growth

When deciding about saving, an individual takes as given and constant over time the pair of tax rates. Given (6), the net income of agent i is:

$$y_i = (rk_i + \widehat{w}l_i)(1 - \tau) \tag{11}$$

where l_i is the inelastic labor supply of agent i, equal to $L/N = 1/N$.

The agent i maximizes her intertemporal utility under budget constraint:

$$\max_{c_i} W^i = \int_0^{+\infty} e^{-\rho t} \left[\ln(c_i(t)) + V(\tau_E)\right] dt \tag{12}$$
$$\text{such that } \dot{k}_i = (rk_i + \widehat{w}l_i)(1 - \tau) - c_i.$$

Since the tax rate τ_E and the state of the environment E are beyond the reach of agent i, the program of agent i becomes:

$$\max_{c_i} W^i = \int_0^{+\infty} e^{-\rho t} \ln(c_i(t)) dt \tag{13}$$
$$\text{such that } \dot{k}_i = (rk_i + \widehat{w}l_i)(1 - \tau) - c_i.$$

The Hamiltonian of (13) is:

$$\mathcal{H} = \ln(c_i(t)) + \lambda \left[(rk_i + \widehat{w}l_i)(1 - \tau) - c_i\right] \tag{14}$$

with $\frac{\partial \mathcal{H}}{\partial c_i} = \frac{1}{c_i} - \lambda$ and $\frac{\partial \mathcal{H}}{\partial k_i} = \lambda r(1 - \tau)$.

The solution to (14) is:

$$\frac{\partial \mathcal{H}}{\partial c_i} = 0$$
$$\text{and } \dot{\lambda} = \rho\lambda - \frac{\partial \mathcal{H}}{\partial k_i}$$

which leads to $\lambda = \frac{1}{c_i}$ and $\dot{\lambda} = \rho\lambda - \lambda r(1 - \tau)$. Since $\frac{\dot{\lambda}}{\lambda} = -\frac{\dot{c}_i}{c_i}$, finally $\frac{\dot{c}_i}{c_i} = r(1 - \tau) - \rho$ obtains.

We assume that we are on a balanced growth path, i.e. $\frac{\dot{c}_i}{c_i} = \frac{\dot{k}_i}{k_i}$. The growth rate η is then given by:

$$\eta = \frac{\dot{c}_i}{c_i} = \frac{\dot{k}_i}{k_i} = r(1 - \tau) - \rho. \tag{15}$$

Introducing this value in the budget constraint, we get:

$$\dot{k}_i = (rk_i + \widehat{w}l_i)(1 - \tau) - c_i$$

and:

$$\frac{\dot{k}_i}{k_i} = \frac{1}{k_i}(rk_i + \widehat{w}l_i)(1 - \tau) - \frac{c_i}{k_i}.$$

Then:

$$r(1 - \tau) - \rho = \left(r + \widehat{w}\frac{l_i}{k_i}\right)(1 - \tau) - \frac{c_i}{k_i}. \tag{16}$$

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We can note that $\widehat{w} = \omega K$, setting $\omega = (1-\alpha)A^{1/\alpha}\tau_Y^{(1-\alpha)/\alpha}$. Equation (16) becomes:

$$r(1-\tau) - \rho = \left(r + \omega\frac{Kl_i}{k_i}\right)(1-\tau) - \frac{c_i}{k_i}$$

which implies:

$$
\begin{aligned}
c_i &= \left[\left(r + \omega\frac{Kl_i}{k_i}\right)(1-\tau) - r(1-\tau) + \rho\right]k_i \\
&= \left[\rho + \omega\frac{Kl_i}{k_i}(1-\tau)\right]k_i.
\end{aligned}
$$

We denote by $\sigma_i = \frac{Kl_i}{k_i} = \frac{\overline{k}}{k_i}$, the ratio of mean capital to the capital owned by agent i. The set of these ratios characterizes the inequality schedule of this economy. It is independent of the time since l_i is constant, and K and k_i grow at the same rate η. This is an important property of this type of model: when tax rates remain constant over time, there is no modification of relative inequality between agents over time, even though each of them is getting richer.

The higher σ_i, the poorer agent i relative to the average capital endowment. We can rewrite the previous equation as follows:

$$c_i = \rho k_i + \omega(1-\tau)\overline{k} = \left[\rho + \omega\sigma_i(1-\tau)\right]k_i. \tag{17}$$

This equation gives the optimal level of consumption of agent i, taking τ as given. c_i is of course an increasing function of k_i : the richer agent i, the higher her consumption. Also quite intuitively, the higher the total tax rate that she bears, the lower her consumption. Remark that the consumption/capital ratio is an increasing function of σ_i : for a given overall tax rate τ, the poorer an individual, the more she consumes relative to her endowment, that is the less she saves. This is in line with the result obtained by Alesina and Rodrik in their simpler model.

3 The political economy of taxes and the environment

As in Alesina and Rodrik, we impose that tax rates be constant over time and the political decision on the tax rates (τ_Y, τ_E) be taken before the accumulation process starts. It is taken according to majority rule. The timing of the moves is as follows:

1. The endowment schedule $\{k_i(0)\}$ is determined;

2. The political decision is made, according to majority rule, simultaneously on τ_Y^i and τ_E^i;

3. Agents choose their saving plans.

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It is important to note that in the context of this model, these assumptions imply that the relative endowment of agents do not change and therefore the initial median voter retains this property forever. This ensures the time-consistency of the political decision.[15] Despite the multidimensionality of the decision, we show that the generalized median voter theorem applies in this context. Hence, we first investigate the preferred policy for any individual, and then address the issue of the political decision itself.

3.1 The preferred tax policy of agent i.

We now search for the preferred tax policy (τ_Y^i, τ_E^i) of agent i, when she takes into account her reaction function as a private intertemporal maximizer.

This is obtained by solving the following program:

$$
\max_{\tau_Y^i, \tau_E^i} W^i = \int_0^{+\infty} e^{-\rho t} \left[\ln(c_i(t)) + V(\tau_E)\right] dt
$$
$$
\text{such that} \quad : \quad c_i = \left[\rho + \omega\sigma_i(1-\tau)\right] k_i.
$$

Since k_i grows at the rate η, we know that:

$$
c_i = \left[\rho + \omega\sigma_i(1-\tau)\right] k_i(0)e^{\eta t}.
$$

and the program becomes:

$$
\max_{\tau_Y, \tau_E} W^i = \int_0^{+\infty} e^{-\rho t} \left[\ln\left(\left[\rho + \omega\sigma_i(1-\tau)\right] k_i(0)e^{\eta t}\right) + V(\tau_E)\right] dt. \tag{18}
$$

Remark that:

$$
W^i = \int_0^{+\infty} e^{-\rho t} \left[\ln\left(\rho + \omega\sigma_i(1-\tau)\right) + \ln(k_i(0)) + \eta t + V(\tau_E)\right] dt \tag{19}
$$

$$
= \frac{1}{\rho}\ln\left(\rho + \omega\sigma_i(1-\tau)\right) + \frac{1}{\rho}\ln(k_i(0)) + \frac{1}{\rho^2}\eta + \frac{1}{\rho}V(\tau_E) \tag{20}
$$

with $\eta = r(1-\tau) - \rho$. Thus:

$$
\rho W^i = \ln\left(\rho + \omega\sigma_i(1-\tau)\right) + \ln(k_i(0)) + \frac{r(1-\tau)}{\rho} - 1 + V(\tau_E) \tag{21}
$$

where $\tau = \tau_E + \tau_Y$, $r = \alpha A^{1/\alpha}\tau_Y^{(1-\alpha)/\alpha}$, $\omega = (1-\alpha)A^{1/\alpha}\tau_Y^{(1-\alpha)/\alpha} = (\frac{1-\alpha}{\alpha})r$.

We can prove that there exists an identical linear relationship between the tax rates preferred by any agent i:

Lemma 1 *For any agent i, her preferred tax rate pair (τ_E^i, τ_Y^i) satisfies a unique linear relationship:*

$$
\tau_Y^i + (1-\alpha)\tau_E^i. = (1-\alpha). \tag{22}
$$

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[15]Although not its optimality, compared to the solution obtained if the (identical) median voter were allowed to vote sequentially at each period. On this point, see Krusell et al. (1997).

Proof. see Appendix. ■

According to this lemma, any individual is facing a dilemma between physical goods and the environment. She wishes to save and invest, so as to incur higher future consumption. By the same reasoning, she wants her accumulation effort to be well remunerated. This can be obtained by channeling some public funds obtained from taxation into the production sector and not just in depollution activities. On the whole, this accumulation of capital and this production of goods will deteriorate the environment, relative to its state in a no production, no public policy economy. Hence, the inverse relationship between both taxation ratios reflects the trade-off between future increased consumption and environment quality.

A key feature of this lemma is that there is unanimity on the linear relation (22) between τ_Y^i and τ_E^i as this relation does not depend on the capital ratio for agent i, σ_i, and therefore is identical for any agent i. Given (22), the political problem becomes unidimensional and the generalized median voter theorem applies.[16] Even though the choice is made simultaneously on the joint pair of tax rates, it is as if the two tax rates were chosen sequentially with τ_E^i being chosen first and then τ_Y^i chosen according to (22).[17] For a given state of the environment, that is a given τ_E^i, everybody wants to set τ_Y^i so as to maximize the growth rate, which leads to (22). There is no disagreement on this choice. The only debate is therefore on the choice of τ_E. This is decided according to the median voter theorem.

Hence, given (22), we can replace τ_Y as a function of τ_E, and concentrate on the choice of τ_E.[18] Then:

$$r = \alpha A^{1/\alpha} \tau_Y^{(1-\alpha)/\alpha} = \alpha A^{1/\alpha} \left[(1-\alpha)(1-\tau_E)\right]^{(1-\alpha)/\alpha}. \tag{23}$$

Given (22) and the definition of $\tau = \tau_Y + \tau_E$, we immediately get:

$$1 - \tau = \alpha(1 - \tau_E). \tag{24}$$

Thus, using (23) and (24), we get:

$$r(1 - \tau) = C(1 - \tau_E)^{1/\alpha}$$

where $C \equiv \alpha^2 A^{1/\alpha}(1-\alpha)^{(1-\alpha)/\alpha} > 0$. Here C is treated as a constant insofar as it does not depend on the policy instruments.

(21) becomes:

$$\rho W^i = \ln\left(\rho + \left(\frac{1-\alpha}{\alpha}\right)\sigma_i C(1-\tau_E)^{1/\alpha}\right) + \ln(k_i(0)) + \frac{C(1-\tau_E)^{1/\alpha}}{\rho} - 1 + V(\tau_E). \tag{25}$$

---

[16] A similar reasoning has been applied by Fiaschi (1999) and Kempf and Rossignol (2005).

[17] We could present the sequential moves in reverse order, and reason on the choice of τ_Y^i first and then, the choice of τ_E^i.

[18] We could equivalently express τ_E^i as a function of τ_Y^i and concentrate on the choice of the latter rate. The same results would obtain.

The environmental policy τ_E^i preferred by agent i maximizes ρW^i given by (25). To keep the model analytically tractable, we use the following specification for $V(\tau_E)$:

$$V(\tau_E) = b\frac{(\tau_E)^{1-\lambda}}{1-\lambda}, \ \lambda \neq 1$$
$$V(\tau_E) = b\ln(\tau_E), \ \lambda = 1$$

where $b > 0, \lambda > 0$. As $V(\cdot)$ is increasing and concave, an increase in the tax rate τ_E incrases the quality of the environment and therefore welfare, at a decreasing pace. The coefficient b corresponds to the relative weight given to the environment. The higher b, the more an agent values the environment, relative to consumption. Given this specification, τ_E has to be positive: we focus on the case where public policy aims at protecting the environment.[19]

The following lemma helps us to better understand the trade-off faced by an agent, and will be useful later:

Lemma 2 *There exists $\widehat{b} > 0$ such that:*

i/ if $b < \widehat{b}$, for any σ_i, the environmental tax τ_E^i preferred by agent i is unique, belongs to $(0, \widehat{\tau}_E]$ with $\widehat{\tau}_E = \frac{\lambda\alpha}{\lambda\alpha+1-\alpha} < 1$ and is decreasing in σ_i;

ii/ if $b > \widehat{b}$, there exists an endowment ratio $\sigma^*(b)$ such that any agent i characterized by $\sigma_i < \sigma^*(b)$ prefers $\tau_E^i = 1$.

Proof. See Appendix. ∎

The case $\tau_E = 1$ is a corner solution, meaning that the best solution is to stop production. It implies $\tau_Y = 0$, $r = \omega = 0$, $G_E = \tau_E Y = Y = 0$, $c_i = \rho k_i$. In this case, each agent lives in isolation and consumes over time her own initial endowment. According to the first part of the lemma, there exists a threshold value $\widehat{b}$ for the environment weight, such that for any $b < \widehat{b}$, even the richest individuals want to save and invest for future production at the expense of the environment. When the relative weight given to the environment is low enough, it leads any individual to a compromise between future production and environment quality. Given the properties of the utility function with $b < \widehat{b}$, and a given pair (τ_Y, τ_E), the less endowed an agent, the higher her marginal utility of physical production. On the other hand, the marginal utility coming from the environment is the same for any agent. Hence the poorer an agent, the less she wants to devote public resources to depollution activities. On the other hand, if $b > \widehat{b}$, at least some agents can be so rich

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[19]A negative τ_E could be considered using a different specification of the utility function such as

$$V(\tau_E) = b\frac{(a+\tau_E)^{1-\lambda}}{1-\lambda}, \ \lambda \neq 1$$
$$V(\tau_E) = b\ln(a+\tau_E), \ \lambda = 1,$$

where a is positive, with a chosen tax rate superior to $-a$. Results are unchanged when we use this formulation.

so as to prefer autarky and no production rather than contributing to the degradation of the environment, as a side effect of more physical production. Given her endowment, a relatively rich agent gives a high enough weight to the environment that she prefers to stop production so as to preserve the quality of the environment. Poorer agents prefer an intermediate positive value for τ_E lower than 1. Remark that when $b < \widehat{b}$, the richer an agent, the higher the overall tax rate she wishes. This comes from the inverse relationship between σ_i and τ_E^i, and Lemma 1.[20]

3.2 The political decision.

We can now address the issue of the choice of the tax policy when majority rule applies. We assume that no agent, whatever her relative endowment, prefers to stop production, that is $b < \widehat{b}$. This assumption is plausible given the inescapable evidence of productive activities through the world. Given the two previous lemmas, we can state the following:

Proposition 3 *i/ The median voter chooses a taxation policy such that $\tau_E^m \in (0, \widehat{\tau}_E\,]$ and $\tau_Y^m = (1-\alpha)(1-\tau_E^m)$.*

ii/ τ_Y^m is an increasing function of σ_m, whereas τ_E^m and τ^m are decreasing functions of σ_m, given that $\tau^m = \tau_Y^m + \tau_E^m$.

Proof. Part i/ directly comes from applying the generalized median voter theorem (Plott, 1967). As for part ii/ of this proposition, we can see τ_E^m is a decreasing function of σ_m since from Lemma 2, τ_E^i is decreasing in σ_i. The rest follows according to (22) and (24). ∎

Remember that $\sigma_m = \frac{\overline{k}}{k_m}$ denotes the capital ratio associated with the median voter. Part $i/$ of this proposition states that the median voter is able to decide over fiscal policy. This comes from the fact that there is an unanimous agreement on the linear relationship between the two tax rates and the generalized median theorem can apply. Hence the political decision depends on the ratio σ_m (see eq.(25)). The ratio σ_m can be seen as the politically relevant inequality index in this economy. The higher σ_m, the more unequal this economy in a political sense.[21] Empirically, the plausible case is $\sigma_m > 1$ (i.e. $k_m < \overline{k}$). Proposition 3 means that the higher the inequality, the higher τ_Y^m, and the lower τ_E^m and τ^m. In a very unequal society, the political decisionmaking process privileges the production (τ_Y high) but sacrifices the environment (τ_E low). Moreover, given (22), τ is equal to $(1-\alpha) + \alpha\tau_E$. Hence it depends positively on τ_E. This explains that τ^m is a decreasing function of σ_m. A poorer agent tends to give higher weight to her material well-being

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[20]In the case of no concern for the environment ($b = 0$), the solution simplifies to $\tau_Y^i = (1-\alpha)$ and $\tau_E^i = 0$, for any agent i. This is similar to the result obtained by Barro (1990).

[21]Of course, this index does not capture all dimensions of inequality, and ignore the various moments of the initial capital distribution. The point here is that the political decision hinges on this sole index.

over time, relative to the environment, than a richer one. Consequently, when confronted with the issue of taxing and allocating the proceeds of taxes to either a growing pie or an improved environment, she tends to support both lower taxes and a higher share of the public budget devoted to growth-enhancing expenditures than to toil at the environment. The poorer the median voter (that is, the higher σ_m), the lower τ and τ_E. Note that these rates do not depend on the average level of income $\overline{k}$, but only on the inequality ratio σ_m.

Turning to the consequences on growth of this political decision leads to the following:

Corollary 4 *The growth rate is an increasing function of σ_m.*

Proof.   Immediate since the growth rate η is equal to $C\left(1 - \tau_E^m\right)^{1/\alpha} - \rho$ ∎

This result directly comes from the property of the growth process. The growth rate is an increasing function of the share of aggregate product used to enhance the technological component τ_Y. Hence, the poorer the median voter relative to the average agent, the more she will channel public expenditures to the growth process. Altogether, this corollary claims that in this economy, there is an inverse relationship between the steady-state growth rate and the quality of environment, and that this inverse relationship has its roots in the inequality schedule.   This supports the view that it is impossible to disentangle environmental and productive issues because of inequality: agents with different endowments have a different appreciation of the trade-off between physical consumption and the deterioration of the environment over time.

4   Conclusion

In this paper, we address the important issue of the long-term impact of income distribution on the environment. We claim that income inequality is harmful for the environment in so far as a concern for a cleaner environment draws public resources away from growth-enhancing uses. This trade-off generates a conflict of interest: relatively poor people are more interested in fostering physical growth at the expense of a clean environment whereas relatively rich people are more concerned with the quality of the environment and are more willing to spend for depollution purposes, even if this means a less productive economy in the long run.

This conflict shapes the political debate and generates the main result of the paper: the poorer is the median voter, relatively to the average agent in the economy, the more deteriorated the environment will be, sacrificed to more physical production, that is, higher growth. Of course, this does not contradict the standard result also obtained from the model: a richer (aggregate) society takes better care of the environment than a poorer one as it devotes more aggregate resources (G_E) to environment protection. But here, the

relative weight given to this aim (G_E/Y) does not depend on the aggregate endowment level of a society but on its distribution.

The model used to obtain the environment is simple. It is based on an AK model of endogenous growth. Other theories of endogenous growth have been offered in the literature, based on human capital, on the growth of good variety, or on R&D competition. It would be worthwhile to incorporate in these theories some environmental features and see whether they sustain the growth vs environment trade-off we have been able to exploit here.

Appendix

A   Proof of Lemma 1.

We search for the pair (τ_E, τ_Y) maximizing W^i. This implies to solve the following problem: $\max_{\tau_E} \left(\max_{\tau_Y} \left(W^i \right) \right) = \max_{\tau_Y} \left(\max_{\tau_E} \left(W^i \right) \right)$. Hence, let's fix τ_E and search for the best τ_Y.[22] For τ_E given, to maximize W^i means to maximize $\frac{r(1-\tau)}{\rho} + \ln \left(\rho + (\frac{1-\alpha}{\alpha})r(1-\tau)\sigma_i \right)$. This is an increasing function of $r(1-\tau)$, thus it is equivalent to search for τ_Y maximizing $r(1-\tau) = r(1 - \tau_Y - \tau_E)$. We set

$$f(\tau_Y) = r(1 - \tau_Y - \tau_E) = \alpha A^{1/\alpha} \tau_Y^{(1-\alpha)/\alpha} (1 - \tau_Y - \tau_E). \tag{26}$$

Derivating this function, we get:

$$f'(\tau_Y) = \alpha A^{1/\alpha} \left[(\frac{1-\alpha}{\alpha}) \tau_Y^{\frac{1}{\alpha}-2}(1-\tau) - \tau_Y^{(1-\alpha)/\alpha} \right] \tag{27}$$

$$= \alpha A^{1/\alpha} \tau_Y^{\frac{1}{\alpha}-2} \left[(\frac{1-\alpha}{\alpha})(1-\tau) - \tau_Y \right] \tag{28}$$

Therefore:

$$f'(\tau_Y) \geq 0 \Leftrightarrow \tau_Y \leq (\frac{1-\alpha}{\alpha})(1 - \tau_Y - \tau_E) \tag{29}$$

$$\Leftrightarrow \alpha \tau_Y \leq (1-\alpha)(1 - \tau_Y - \tau_E) \tag{30}$$

$$\Leftrightarrow \tau_Y \leq (1-\alpha)(1 - \tau_E). \tag{31}$$

The maximum is attained at τ_Y such that $\tau_Y = (1-\alpha)(1 - \tau_E)$. We can note that for τ_E given, $\tau_Y = (1-\alpha)(1 - \tau_E)$ maximizes the growth rate $\eta = r(1-\tau) - \rho$ since it maximizes $r(1-\tau)$.

B   Proof of Lemma 2.

We are looking for τ_E^i which maximizes $\rho W^i(\tau_E)$, for a given agent i. We assume that $\lambda \neq 1$. The proof is similar when $\lambda = 1$.

According to (25), we have

$$\rho W^i = \ln \left(1 + D_i(1 - \tau_E)^{1/\alpha} \right) + \frac{C}{\rho}(1 - \tau_E)^{1/\alpha} + b\frac{\tau_E^{1-\lambda}}{1 - \lambda} + const$$

setting $D_i \equiv \left(\frac{1-\alpha}{\alpha} \right) \sigma_i \frac{C}{\rho}$.

We introduce the new variable $x = (1 - \tau_E)^{1/\alpha}$ where $\tau_E \in [0; 1]$ and $x \in [0; 1]$.

Let $U_i(x) \equiv \ln \left(1 + D_i x \right) + \frac{C}{\rho} x + b\frac{(1-x^\alpha)^{1-\lambda}}{1-\lambda}$.

It is clear that τ_E maximizes W^i on $[0; 1]$ if and only if x maximizes $U_i(x)$ on $[0; 1]$.

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[22] We could equivalently use the reverse order of maximization.

Let $U_0(x) \equiv \frac{C}{\rho}x + b\frac{(1-x^\alpha)^{1-\lambda}}{1-\lambda}$. Then $U_0(x) = U_i(x)$ for $D_i = 0$. This corresponds to the case of an infinitely rich agent i, when the number of agents is infinite. We deduce:

$U_0'(x) = \frac{C}{\rho} - b\alpha(1-x^\alpha)^{-\lambda}x^{\alpha-1}$

$U_0"(x) = -b\alpha x^{\alpha-2}(1-x^\alpha)^{-\lambda-1}\left[(\lambda\alpha+1-\alpha)x^\alpha - (1-\alpha)\right]$

and $U_0"(x) < 0 \Leftrightarrow x > \widehat{x}$ where $\widehat{x} = \left(\frac{1-\alpha}{\lambda\alpha+(1-\alpha)}\right)^{1/\alpha} \in (0;1)$. Then

$$U_0 \text{ is convex on } (0;\widehat{x}) \text{ and concave on } (\widehat{x};1). \tag{32}$$

Before proving Lemma 2, we will first prove that there exists $\widehat{b} \in (0;+\infty)$ such that for any $b > 0$:

$$b \quad < \quad \widehat{b} \Rightarrow \max_{x\in(0;1]} U_0(x) > U_0(0) \tag{33}$$

$$b \quad > \quad \widehat{b} \Rightarrow \max_{x\in(0;1]} U_0(x) < U_0(0). \tag{34}$$

Remark that

$$
\begin{aligned}
U_0(x) \quad &< \quad U_0(0) \\
&\Leftrightarrow \quad \frac{C}{\rho}x + b\frac{(1-x^\alpha)^{1-\lambda}}{1-\lambda} < \frac{b}{1-\lambda} \\
&\Leftrightarrow \quad \frac{\frac{C}{\rho}x\,(1-\lambda)}{1-(1-x^\alpha)^{1-\lambda}} < b.
\end{aligned}
$$

Setting $\widehat{b} \equiv \max_{x\in(0;1]} \frac{C}{\rho}x\left[\frac{1-\lambda}{1-(1-x^\alpha)^{1-\lambda}}\right] \in (0;+\infty)$, we obtain:

- if $b < \widehat{b}$, there exists $z \in (0;1]$ such that $b < \frac{\frac{C}{\rho}z(1-\lambda)}{1-(1-z^\alpha)^{1-\lambda}}$, i.e. $U_0(z) > U_0(0)$ and (33) is proved.

- if $b > \widehat{b}$, then $b > \frac{C}{\rho}x\left[\frac{1-\lambda}{1-(1-x^\alpha)^{1-\lambda}}\right]$ for any $x \in (0;1]$, i.e. $U_0(0) > U_0(x)$ for any $x \in (0;1]$ and (34) is proved.

According to (32), U_0' is increasing on $(0;\widehat{x})$ and decreasing on $(\widehat{x};1)$. Since $\lim_{x\to 0} U_0'(x) = -\infty$ and $\lim_{x\to 1} U_0'(x) = -\infty$ two cases are possible:

(I) if $U_0'(\widehat{x}) \le 0$ then $U_0'(x) \le 0$ for every $x \in (0;1)$;

(II) if $U_0'(\widehat{x}) > 0$ then there exist x_0 and y_0 such that:

$0 < y_0 < \widehat{x} < x_0 < 1$ and: $U_0'(x) > 0$ for $x \in (y_0;x_0)$ and $U_0'(x) < 0$ for $x \notin [y_0;x_0]$.

We will now prove Lemma 2.

A/ Suppose that $b < \widehat{b}$. According to (33), U_0' is not always negative on $(0;1)$, so (II) is true. Finally, we have found $x_0 \in (\widehat{x};1)$ such that $U_0(x_0) > U_0(x)$ for any $x \in [0;1]$ with $x \ne x_0$.

Similarly, we want to prove that for $D_i > 0$, there exists $x_i \in (\widehat{x};1)$ such that $U_i(x_i) > U_i(x)$ for any $x \in [0;1]$ with $x \ne x_i$. If $x \le \widehat{x} < x_0$, $U_i(x) = U_0(x) + \ln(1+D_ix) < U_0(x_0) + \ln(1+D_ix_0) = U_i(x_0)$ because $U_0(x) < U_0(x_0)$ and $\ln(1+D_ix) < \ln(1+D_ix_0)$ since $D_i > 0$. Hence, $\max_{x\in[0;\widehat{x}]} U_i(x) < \max_{x\in[0;1]} U_i(x)$

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Since $U_i''(x) = U_0''(x) - \frac{D_i^2}{(1+D_i x)^2}$ with U_0 concave on $(\widehat{x}; 1)$, then U_i is concave on this interval too, and finally the maximum of U_i on $[0; 1]$ is attained at a unique point x_i, and $x_i \in (\widehat{x}; 1)$.

Let us show that x_i is an increasing function of D_i. Assume that $0 < D_i < D_j$.

If $x < x_i$ then $U_j(x) = U_i(x) + \ln\left(\frac{1+D_j x}{1+D_i x}\right) < U_i(x_i) + \ln\left(\frac{1+D_j x_i}{1+D_i x_i}\right) = U_j(x_i)$ because $U_i(x) < U_i(x_i)$ and $\ln\left(\frac{1+D_j x}{1+D_i x}\right) \leq \ln\left(\frac{1+D_j x_i}{1+D_i x_i}\right)$. Hence, $\max_{x \in [0; x_i)} U_j(x) < \max_{x \in [0; 1]} U_j(x)$, which implies that $x_i \leq x_j$.

Remark that $U_j'(x) = \frac{D_j}{1+D_j x} - \frac{D_i}{1+D_i x} + U_i'(x)$, and $U_i'(x_i) = 0$ since U_i is maximum at x_i. Then, we have $U_j'(x_i) = \frac{D_j}{1+D_j x_i} - \frac{D_i}{1+D_i x_i} > 0$. We conclude that $x_i < x_j$.

We have thus proven that, if $b < \widehat{b}$, for any $D_i > 0$, there is a unique x_i which maximizes $U_i(x)$ on $[0; 1]$, and that $x_i \in (\widehat{x}; 1)$, x_i is an increasing function of D_i. According to our change of variables, it means that Lemma 2 (i) is proven, with $\tau_E^i = 1 - x_i^\alpha$, and $\widehat{\tau}_E = 1 - \widehat{x}^\alpha = 1 - \frac{1-\alpha}{\lambda\alpha+(1-\alpha)} = \frac{\lambda\alpha}{\lambda\alpha+(1-\alpha)}$

B/ Suppose that $b > \widehat{b}$. According to (34), $U_0(x) < U_0(0)$ for any $x \in (0; 1]$. For D_i small enough, we will have: $\forall x \in (0; 1]$, $U_i(x) = U_0(x) + \ln(1 + D_i x) < U_0(0) = U_i(0)$. It means that Lemma 2 (ii) is proven with $\tau_E^i = 1 - x_i^\alpha = 1$, since here $x_i = 0$, for D_i small enough.

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