**Evaluation of Nonlinear time-series models for real-time
business cycle analysis of the Euro area**

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Evaluation of Nonlinear time-series models for real-time business cycle analysis of the Euro area[*]

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Abstract

In this paper, we aim at assessing Markov-switching and threshold models in their ability to identify turning points of economic cycles. By using vintage data that are updated on a monthly basis, we compare their ability to detect ex-post the occurrence of turning points of the classical business cycle, we evaluate the stability over time of the signal emitted by the models and assess their ability to detect in real-time recession signals. In this respect, we have built an historical vintage database for the Euro area going back to 1970 for two monthly macroeconomic variables of major importance for short-term economic outlook, namely the Industrial Production Index and the Unemployment Rate.

1 Introduction

Recently, we witnessed the development of new modern tools in business cycle analysis, mainly based on nonlinear parametric modelling. Nonlinear models have the great advantage of being flexible enough to take into account certain stylized facts of the economic business cycle, such as asymmetries in the phases of the cycle. In this respect, emphasis has been placed on the class of nonlinear dynamic models that accommodate the possibility of regime changes.

Especially, Markov-switching (MS) models popularized by Hamilton (1989) have been extensively used in business cycle analysis in order to describe economic fluctuations. The first application of a switching-regime model to the business cycle analysis is Hamilton (1989)'s seminal paper: the starting idea is that the mean growth rate of the quarterly

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GDP in the USA is not constant in time and, in the model, this fact is taken into account by imposing the mean growth rate to depend on a two-state latent discrete variable which follows a first-order Markov chain. A positive growth rate and a negative one are found for the two regimes, providing thus strong evidence that the change in regime is strictly connected with the business cycle reference chronology of turning points defined by the NBER.

Since this seminal application, Markov-switching models became popular in business cycle analysis and have been widely used and extended allowing for more than two regimes, time varying probabilities, a vector of endogenous and dynamic factors. We refer, for example, among others to Clements and Krolzig (1998), Krolzig (1997), Krolzig and Sensier (2000), Krolzig and Toro (1999), Diebold and Rudebusch (1996, 1999), Kim and Nelson (1998, 1999), Chauvet (1998), Kaufmann (2000), Layton (1996), Layton and Smith (2007), Ferrara (2003), Anas et al. (2007b) or Billio et al. (2007).
The multivariate extension of Markov-switching models allows one to take into account the co-movements among countries and sectors which is crucial when dealing with the business cycle (Burns and Mitchell, 1946), Diebold and Rudebusch (1996, 1999), Kim and Nelson (1999). In this framework, MS-VAR processes have been introduced by Krolzig (1997), where the conditional stochastic process is a Gaussian classical VAR(p) and the regime generating process is again a Markov chain. These multivariate models allow for a multi-countries and multi-sectors simultaneous analysis in the search of a common business cycle (which is represented by the latent variable) for a given area. It is possible to quote among others Artis, Krolzig and Toro (2004), Clements and Krolzig (1998), Krolzig (2003), who developed a Markov-switching VAR model in order to identify the European business cycle, and Ahrens (2002), who evaluated the informational content of the term structure as a predictor of recessions in eight OECD countries, as multi-countries examples.

Besides the well-known Markov-switching approach, another parametric model allows for different regimes in business cycle analysis, the threshold autoregressive (TAR) model, first proposed by Tong and Lim (1980). This model is able to produce limit cycle, time irreversibility and asymmetry behavior of a time-series. TAR models have been used to describe the asymmetry observed in the quarterly US real GDP by various authors, such as Tiao and Tsay (1994), Potter (1995) and Proietti (1998) for instance, or to model US Unemployment Rate (Hansen, 1997). With the TAR model the transition variable is observed: it may be either an exogenous variable, such as a leading index for example, or a linear combination of lagged values of the series. In this latter case, the model is referred to as a self-exciting threshold autoregressive (SETAR) model. This is the main difference with the Markov-switching model whose parameters of the autoregressive data generating process vary according to the states of the latent Markov chain. For example, Montgomery et al. (1998) proposed a SETAR model for the US Unemployment Rate, while Ferrara and Guégan (2006) used a SETAR model to detect in real-time, through a dynamic simulation approach, the dates of peaks and troughs in the industrial business cycle.

We resort to these two different classes of models because they underpin a complementary

rather than an alternative approach, as the notion captured by them is not exactly the same. Thus, here, we propose to compare their capability to date and detect in real-time turning points on two different economic datasets: Industrial Production Index and Unemployment Rate. These two series are of major importance for short-term economic outlook, are usually considered to date economic cycles and are available for the Euro area on a monthly basis. To accomplish this task, we first preselect a wide set of economically meaningful possible specifications (i.e. that according to the previous literature have been proved to adhere to the stylized facts of economic cycles) and then specify either on the Industrial Production Index and the Unemployment Rate time-series the best SETAR and MS models according to some goodness of fit measures. After this specification phase, we increase successively the original data sets by moving through all the available vintage monthly releases, and we specify again and re-estimate the models at each step in order to detect in real-time occurrence of turning points which can characterize changes in the economic phases. Since the best specified model can change through the different vintages, given the wider set of possible models for the MS approach, we prefer to define the relevant MS models using a grid-search-like method that selects the model that on average is better able to timely detect turning points of a given reference chronology.

The paper is organized as follows: in Section 2 we describe the two data sets and the way to build the vintage data. Section 3 briefly presents the two classes of models used in this study and the estimation procedures. Section 4 is devoted to the specification phase of SETAR ad MS models, for both of the two series. Section 5 examines the capability of the two models, fitted either to the Industrial Production Index and Unemployment Rate time-series, to detect turning points in real-time. Finally, Section 6 concludes.

2   Data sets

In this section we describe the data sets for the Industrial Production Index and the Unemployment Rate (hereafter referred to as IPI and UR, respectively) considered in this paper: both of them refer to the Euro area (12 members) aggregate; we specifically focus on the description of the vintage database and the method followed to obtain the historical part.

For the IPI series, the available data are all the 69 monthly releases[1] issued by Eurostat from June 2003 to February 2009, and contained in the recently disclosed HistoDB database (last access in March 2009). Moreover, a previously backcalculated series by Eurostat of Euro 12 IPI is also available from January 1970 to December 2006. Since the aim of the paper is to compare the two types of models both from a datation and detection point of view, we built an historical vintage data base by backcalculating all the available releases.

We evaluated alternative possibilities to reconstruct a longer database. Given the drawbacks of the ARIMA approach[2] in the context of economic time-series (lack of a straight

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[1]Industrial Production Index, total industry, excluding construction, seasonally and working days adjusted, base year 2000.

[2]Two solutions are available: estimate an ARIMA on the current series, reverse the model and use it to produce some forecasts; reverse the series, fit an ARIMA model and use it to produce forecasts. Both

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economic interpretation of the reverted time-series, impossibility of resorting on temporal reversion test due to the presence of a trend, asymmetric seasonal components and asymmetric cycles) and the limited number of periods that it is possible to reach into the past (otherwise, either only a tendency will be estimated, or the estimates will converge to the long run level, or they will explode), we preferred to discard this possibility. On the contrary, since the time-series we want to reconstruct and the related time-series share a sufficiently wide overlapping period, following the approach developed in Caporin and Sartore (2006), we chose to estimate the IPI within a regression framework.

In particular, we specified for the backcalculation a linear regression model over the 1st order differenced series, extended with ARMA terms in the residuals. Denoting by Y_t^1 the release of the Industrial Production Index to be historically reconstructed and by X_t^1 the so-called related time-series in the Caporin and Sartore's approach, which is the aforementioned backcalculated IPI time-series which starts in January 1970, the general model estimated is the one specified below:

$$\Delta Y_t^1 = \beta \Delta X_t^1 + \delta D_t + \gamma T_t + \Phi^{-1}(L)\Theta(L)\epsilon_t,$$

where Δ is the first order difference operator, D_t is a (12×1)-vector containing a set of monthly dummies, T_t is the time-trend, ϵ_t is an innovation process and $\Phi^{-1}(L)\Theta(L)$ are the ARMA polynomials.

As a matter of example, in Figure 1 we depict the pattern of the IPI time-series released in September 2008, the IPI time-series used to carry out the retropolation exercise and the resulting backcalculated vintage IPI.
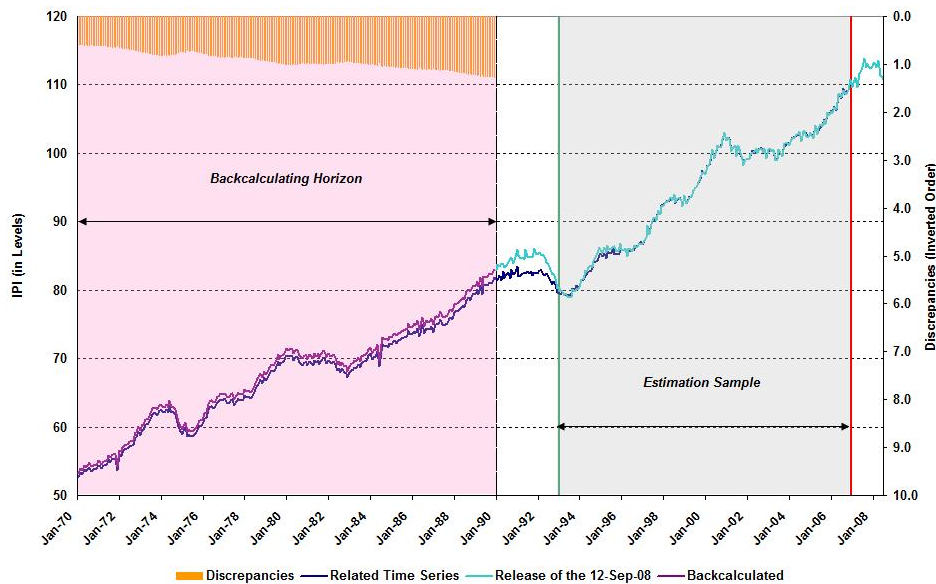

Figure 1: September 2008 IPI release historically reconstructed working on the 1st order differences.

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approaches present some problems.

Now we move on to consider the second variable, namely the Euro area total Unemployment Rate. From the HistoDB database issued by Eurostat (last access in June 2009 for this variable) we selected 97 monthly releases, from May 2001 to May 2009, of the seasonally adjusted UR. The delay between the issuance date of the time-series and the underlying month is of two months, hence the release issued in month t contains observations of the UR until month $t-2$. For example, the last available vintage data released in May 2009 refers to March 2009. As previously explained for the case of the Industrial Production Index, also the vintage releases of the UR have been historically reconstructed following the approach proposed by Caporin and Sartore (2006); in this case, the previously backcalculated time-series dates back to January 1971 instead of January 1970. In this case we considered a linear regression model over the 12th order difference series, extended with ARMA terms in the residuals, as described in the following equation:

$$\Delta^{12}Y_t^2 = \beta\Delta^{12}X_t^2 + \delta D_t + \gamma T_t + \Phi^{-1}(L)\Theta(L)\epsilon_t,$$

where Y_t^2 is the release of the total Unemployment Rate to be historically reconstructed and X_t^2 is the related time-series. For instance, taking into consideration the release issued in August 2008, the related retropolated time-series is depicted in Figure 2.
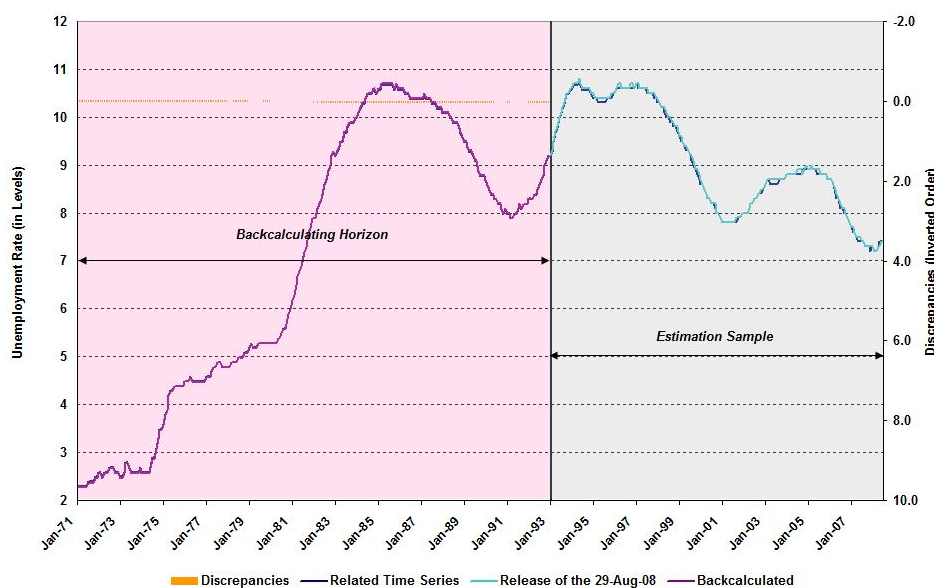

Figure 2: August 2008 Unemployment Rate release historically reconstructed working on the 12th order differences.

3   Models and Methodology

In this section, we specify the two classes of models which permit to take into account the existence of various regimes or states in real data. For sake of simplicity, we describe the models by only taking into consideration the simple specification that allows for two

regimes, but they can be easily generalised to more regimes. For a review concerning these kinds of processes, we refer to Tong (1990), Krolzig (1997) and Franses and van Dijk (2000).

3.1 Threshold processes

The covariance-stationary process $(Y_t)_t$ follows a two-regime threshold autoregressive process, denoted TAR$(2,p_1,p_2)$, if it verifies the following equation:

$$Y_t = (1 - I(Z_{t-d} > c))(\phi_{1,0} + \sum_{i=1}^{p_1} \phi_{1,i} Y_{t-i} + \sigma_1 \varepsilon_t)$$

$$+ I(Z_{t-d} > c)(\phi_{2,0} + \sum_{i=1}^{p_2} \phi_{2,i} Y_{t-i} + \sigma_2 \varepsilon_t),$$

where c is the threshold, $d > 0$ the delay, $(\varepsilon_t)_t$ a standardised white noise process and $(Z_t)_t$ the transition variable. Here, $I(\bullet)$ is the indicator function such that $I(Z_{t-d} > c) = 1$ if $Z_{t-d} > c$ and zero otherwise. If, $\forall t$, $Z_t = Y_t$, the process is referred to as self-exciting TAR process (SETAR). For a given threshold c and the position of the random variable Z_{t-d} with respect to this threshold c, the process $(Y_t)_t$ follows here a particular AR(p) model. The model parameters are $\phi_{j,i}$, for $i = 0, \ldots, p_j$ and $j = 1, 2$, the standard variance errors σ_1 and σ_2, the threshold c and the delay d. This model has been first introduced by Tong and Lim (1980).

Using some algebraic notations, the model (1), with $p_1 = p_2 = p$, can be rewritten as a regression model. Denote $I_d(c) \equiv I(Z_{t-d} > c)$, $\Phi_1 = [\phi_{1,0}, \cdots, \phi_{1,p}]'$, $\Phi_2 = [\phi_{2,0}, \cdots, \phi_{2,p}]'$ and $\mathbf{Y'_{t-1}} = [1, Y_{t-1}, \cdots, Y_{t-p}]$, then, we get the following alternative representation from (1):

$$Y_t = (1 - I_d(c))\mathbf{Y'_{t-1}}\Phi_1 + I_d(c)\mathbf{Y'_{t-1}}\Phi_2 + ((1 - I_d(c))\sigma_1 + I_d(c)\sigma_2)\varepsilon_t. \tag{1}$$

We assume now that we observe $(Y_1, \cdots, Y_n)$ data stemming from model (1). The equation (1) is a regression equation (albeit nonlinear in parameters) and an appropriate estimation method for the parameters is the least squares (LS) method. Note that under the assumption that $(\varepsilon_t)_t$ is a Gaussian strong white noise, LS is equivalent to the maximum likelihood estimation. Since the regression equation (1) is nonlinear and discontinuous, the easiest method to obtain the LS estimates is to use sequential conditional LS.

For a given value of c, using the notation $\mathbf{Y'_t}(c) = [\mathbf{Y'_t}(1 - I_d(c)), \mathbf{Y'_t}(I_d(c))]$ and $\Phi = (\Phi_1, \Phi_2)$, the LS estimates for the parameters Φ are:

$$\hat{\Phi}(c) = (\sum_{i=1}^{n} \mathbf{Y'_t}(c)\mathbf{Y_t}(c))^{-1}(\sum_{i=1}^{n} \mathbf{Y'_t}(c)Y_t), \tag{2}$$

with residuals $\hat{\varepsilon}_t(c) = Y_t - \mathbf{Y'_t}(c)\hat{\Phi}(c)$, and the residual variance is:

$$\hat{\sigma}_n^2(c) = \frac{1}{n} \sum_{t=1}^{n} \hat{\varepsilon}_t(c)^2. \tag{3}$$

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The LS estimate of c is the value that minimizes equation (3):

$$\hat{c} = argmin_{c \in C} \hat{\sigma}_n^2(c), \tag{4}$$

where $C = [C_1, C_2]$, C_1 and C_2 are real numbers.

In practice, we first need to determine the parameters c, d, p_1 and p_2 in order to estimate all the parameters of model (1). We can proceed in the following way:

- The threshold parameter c is chosen by grid-search procedure. The grid points are obtained using the quantiles of the sample under investigation. One generally uses equally spaced quantiles from the 10 (percent) quantiles and ending at the 90 (percent) quantiles.

- Now, for each fixed pair (d, c_i), $0 < d < D$, $i = 1, \ldots m$, the appropriate TAR model has to be identified.

- The AIC criterion is used for selection of the orders p_1 and p_2.

In this context, the used AIC criterion is given by:

$$AIC(p_1, p_2, d, c) = \ln\left(\frac{1}{n}\sum_{t=1}^{n} \hat{\epsilon}_t^2\right) + 2\frac{p_1 + p_2 + 2}{n}, \tag{5}$$

where $\hat{\epsilon}_t$ denotes the residuals.

Finally, the model with the parameters p_1^*, p_2^*, d^* and c^* that minimize the AIC criterion can be retained. Since for different d there are different numbers of values that can be used for estimation, the following adjustment should be done, with $n_d = \max(d, P)$, $P = \max(p_1^*, p_2^*)$:

$$AIC(p_1^*, p_2^*, d^*, c^*) = \min_{p_1, p_2, d, c} \frac{1}{n - n_d} AIC(p_1, p_2, d, c). \tag{6}$$

On each state, it is possible to propose more complex stationary models like ARMA(p,q) processes.

3.2   Markov-switching processes

The covariance-stationary Markov-switching (MS) model has been first introduced by Quandt (1958), then reconsidered by Neftçi (1982, 1984) and popularized in economics by Hamilton in 1989. The process $(Y_t)_t$ follows a two-regime Markov-switching model, denoted MS(2) - AR(p), if it verifies the following equation:

$$Y_t - \mu(S_t) = \sum_{i=1}^{p} \phi_i(S_t)(Y_{t-i} - \mu(S_{t-i})) + \sigma(S_t)\varepsilon_t, \tag{7}$$

where the non-observed process $(S_t)_t$ is an ergodic Markov chain and where $(\varepsilon_t)_t$ is a standardized white noise process. The parameters $\mu(S_t)$, $\phi_1(S_t)$, $\ldots$, $\phi_p(S_t)$ and $\sigma(S_t)$

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describe the dependence of the process $(Y_t)_t$ to the current regime S_t. The associated transition matrix of the Markov chain $(S_t)_t$ is defined by:

$$P[S_t = j | S_{t-1} = i] = p_{ij}, \tag{8}$$

with $0 < p_{ij} < 1$ for $i, j = 1, 2$ and $\sum_{j=1}^{2} p_{ij} = 1$. In the two-regime case, the unconditional probabilities associated to the process $(Y_t)_t$ are equal to:

$$P[S_t = 1] = \frac{1 - p_{22}}{1 - p_{11} + 1 - p_{22}} = \pi,$$

and

$$P[S_t = 2] = 1 - \pi.$$

Various extensions of Markov-switching processes have been proposed in the literature, ranging from time-varying transition probabilities (see Filardo and Gordon, 1994, Layton and Smith, 2007) to multivariate models.

Regarding the parameter estimation issue, from an observed trajectory $(y_1, \ldots, y_T)$, the maximum likelihood method is used in connection with the Expectation Maximization (EM) algorithm. The EM algorithm is an iterative technique for maximizing the likelihood function in case of models with missing observations or models where the observed time-series depends on some unobservable latent stochastic variables. Hamilton (1990) reports that the EM algorithm is, in general, relatively robust with respect to poorly chosen starting values of the parameters, quickly moving to a reasonable region of the likelihood surface. Furthermore, as shown in Hamilton (1990), the EM algorithm can be used in conjunction with the filtering and smoothing algorithm to draw inference over the state allocation under the simplifying assumption of knowing the parameter of the Markov-switching model.

4   Specification phase

For both approaches the analysis starts with a specification process in order to come up with the model that is best suited to adhere to business cycle features; we thus present for both SETAR and MS models, the specifications that have been retained and report the estimates obtained by fitting the selected models to the first available vintage release of each of the two time-series, namely IPI and UR. As discussed in Section 1, for the MS approach a further step has been considered to select the best specification that also consider a goodness of fit measure with respect to a reference chronology. In the following, we thus briefly describe both the specification phase of SETAR models and the selection procedure of MS models.

4.1   Nonlinear modelling for the Industrial Production Index

First, we focus on the IPI time-series, more in detail, on its growth rate over three months, which is denoted by $(y_t^1)_t$. The first available vintage series was released in mid-June 2003, then it ends in April 2003 and has been historically reconstructed back in time to January 1970, as described in Section 2. For both SETAR and MS models we describe the specification for $(y_t^1)_t$.

4.1.1 SETAR specification

In order to implement a SETAR modelling approach, we needed to determine the order p of the AR models on each state, the delay d and the threshold c. For the autoregressive order p, we proceeded by using a descendent stepwise approach by considering first $p = 12$. To determine the delay d, we applied different tests using several values of p. In particular we considered the Tsay (1989) test with $p = 6$, which accepted the null of linearity for $d = 1$ but rejected the null for $d = 2$ at the usual risk $\alpha = 0.05$, implying thus the presence of two regimes. The Tsay test also rejected the null for $d = 3$ with a risk $\alpha = 0.10$. We also considered the Hansen (1997) test with bootstrapped p-values (500 replications are used). For the usual type I risk $\alpha = 0.05$, the Hansen test rejected the null for $d = 2$ and $d = 4$, and accepted the null for all the other values. Thus, according to the testing results, we decided to take as transition variable y_{t-2}^1. To determine the threshold c, we used a grid-search procedure over the various thresholds lying between the 10% and 90% quantiles, and identified that $\hat{c} = 0.2138$ returned the lowest Akaike Information Criterion (AIC).

The model that we finally retain for the centered series is the following:

$$
\begin{aligned}
y_t^1 = & (0.2954 y_{t-1}^1 + 0.2116 y_{t-2}^1)(1 - I_{[y_{t-2}^1 > 0.2138]}) + \hat{\sigma}_\varepsilon^1 \varepsilon_t \\
& + (0.4249 y_{t-1}^1 + 0.1088 y_{t-2}^1 - 0.1889 y_{t-3}^1 + 0.2180 y_{t-4}^1) I_{[y_{t-2}^1 > 0.2138]} + \hat{\sigma}_\varepsilon^2 \varepsilon_t.
\end{aligned}
\tag{9}
$$

and Table 1 provides the estimated parameters and the respective standard errors.

Table 1: Parameter estimates and standard errors for a 2-regime SETAR model fitted to the 1st vintage IPI series (January 1970 – April 2003).

	Low regime $[y_{t-2} \leq 0.2138]$	High regime $[y_{t-2} > 0.2138]$
$\hat{\mu}$	-0.1962 (-1.14)	0.8415 (7.29)
$\hat{\phi}_1$	0.2954 (3.61)	0.4249 (6.73)
$\hat{\phi}_2$	0.2116 (2.59)	0.1088 (1.60)
$\hat{\phi}_3$		-0.1888 (-2.79)
$\hat{\phi}_3$		0.2180 (3.46)
$\hat{\sigma}_\varepsilon$	1.0299	0.7869

This specification phase has been repeated for all the 69 vintages. The specified models showed to be quite stable and thus we have preferred to fix and use this first model for the next phase of real-time detection. It is noteworthy that an alternative approach for model selection could have been to minimize a goodness of fit criterion that measures also the ability to replicate the cyclical phases, as for example the QPS criterion described below in equation (10); indeed, as it will be explained in the next paragraph, this further step has been considered for Markov-switching models.

4.1.2   Markov-switching specification

A wide range of Markov-switching models have been taken into account and according to the specification strategy suggested by Krolzig (1997), that considers the best ARMA representation defined according to the AIC criterion, we specified the model for the first release and repeated this step for each one of the 69 releases of the IPI time-series. Since the MS specification was less stable than the SETAR one and we were interested in considering also multivariate and time-varying transition probabilities models, we introduced a further specification step. This means that each pre-specified model has been successfully fitted to each one of the 69 releases of the IPI time-series. We have then computed the filtered probabilities of being in recession and computed for each release and each model some goodness of fit measure with respect to a reference chronology. Finally, for each model we have averaged these measures over all the releases and then selected the model with the best fit.

As for the statistics computed to assess the ability of Markov-switching models to locate turning points, we considered the classical Quadratic Probabilistic Score (QPS) of Brier (1950) and the Concordance Index for regular periodic behavior in the business cycles proposed by Harding and Pagan (2002). Moreover, we adopted as reference chronology the turning points dating provided in Anas et al. (2007a) for the industrial business cycle. More in detail, the classical QPS is defined as follows:

$$QPS = \frac{1}{T}\sum_{t=1}^{T}\left(P_t - RC_t\right)^2,\tag{10}$$

where, for $t \in \{1,\ldots,T\}$, P_t is the filtered probability of being in recession in month t as it stems from the estimation of the Markov-switching model and RC_t is a binary random variable that is equal to 1 during recessionary phases of the business cycle and 0 otherwise, according to the reference chronology.

The second statistic computed as goodness of fit measure to the reference chronology is the Concordance Index, which is defined as follows:

$$CI = \frac{1}{T}\left[\sum_{t=1}^{T}I_t \times RC_t + \sum_{t=1}^{T}(1 - I_t)\times(1 - RC_t)\right],\tag{11}$$

where RC_t is the same variable already employed in the QPS, which represents the turning points of the reference chronology, while I_t is a binary random variable that assumes value 1 if the latent state variable S_t is in the recessionary phase of the business cycle and 0 otherwise. In this respect it is fundamental the criterion adopted to discriminate when the unobserved state-variable S_t takes on value 1 or 0. Following the approach proposed by Hamilton (1989), we chose the natural threshold of 0.5 to transform the filtered probability into a binary variable.

The final specification step thus requires to select the model that returns the lowest QPS and the highest Concordance Index.

Among all the considered Markov-switching models, specifications that do not allow for autoregressive dynamics in the observable variable present the better goodness of fit mea-

sure and Table 2 reports the QPS and Concordance Index statistics for some selected models that share an autoregressive polynomial of order 0.

Table 2: Mean QPS and Concordance Index of some selected Markov-switching models fitted to the 69 releases of the IPI.

Model	QPS	Concordance Index
MSI(2)-AR(0)	0.080	0.898
MSIH(2)-AR(0)	0.113	0.844
MSI(3)-AR(0)	0.097	0.890
MSIH(3)-AR(0)	0.084	0.894
MSI(4)-AR(0)	0.127	0.858
MSIH(4)-AR(0)	0.087	0.900

According to the figures reported in Table 2, we were not able to find a Markov-switching model that returned at the same time the lowest QPS and the highest Concordance Index; indeed, these statistics are often too close. Therefore, we supported the quantitative analysis by graphical inspection and singled out two models out of the six reported in Table 2, namely[3] MSI(2)-AR(0) and MSIH(3)-AR(0). We retained these two specifications as they return the best performances and also because they differ in two respects: the number of regimes of the state-variable and the possibility of allowing for regime-dependent heteroskedasticity.

To facilitate a comparison with the outcomes of the specification phase performed for SETAR models, Tables 3 and 4 report the estimates of the parameters obtained either by fitting a MSI(2)-AR(0) and MSIH(3)-AR(0) model to the 3-month growth rate of the IPI, $(y_t)_t$, in the period January 1970 – April 2003, that is the time-horizon covered by the first vintage release of the endogenous variable.

Table 3: Parameter estimates and standard errors for a MSI(2)-AR(0) model fitted to the 1st vintage IPI series (January 1970 – April 2003).

	Regime 1	Regime 2
$\hat{\mu}(S_t)$	-1.0186 (0.1656)	0.7307 (0.0568)
$\hat{\sigma}_\varepsilon$	0.8965	
Transition Matrix		
	Regime 1	Regime 2
Regime 1	0.8730	0.1270
Regime 2	0.0236	0.9764

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[3]We consider Krolzig's (2007) classification and thus I means regime dependent intercept, while H regime dependent heteroskedasticity.

Table 4: Parameter estimates and standard errors for a MSIH(3)-AR(0) model fitted to the 1st vintage IPI series (January 1970 – April 2003).

	Regime 1	Regime 2	Regime 3
$\hat{\mu}\,(S_t)$	-1.0622 (0.2487)	0.3854 (0.0668)	1.4312 (0.1098)
$\hat{\sigma}_\varepsilon$	1.0769	0.63452	0.76243
Transition Matrix			
	Regime 1	Regime 2	Regime 3
Regime 1	0.8235	0.1053	0.07117
Regime 2	0.04979	0.9228	0.02741
Regime 3	0.0001916	0.09610	0.9037

The interpretation in terms of business cycle phases of the MSI(2)-AR(0) is straightforward, as the first regime locates recession phases and the second pinpoints expansion phases. As for the economic meaning given to the MSIH(3)-AR(0) model, the first regime is assumed to locate recession phases, while the second regime could be interpreted as identifying slowdown phases that not necessarily lead to a recession and the third regime can be labelled as expansion phase.

4.2 Nonlinear modelling for the Unemployment Rate

We focus now on the Unemployment Rate for the Euro area; more in detail, on its inverted differences over 3 months, which is denoted by $(y_t^2)_t$. The first vintage was released in May 2001, then it ends in March 2001 and has been historically reconstructed back in time to January 1971. Either for SETAR and MS models, we provide the specification analysis for this series.

4.2.1 SETAR specification

The same procedures as for IPI are applied to determine the order p, the delay d and the threshold c. The Tsay (1989) test with $p = 10$ rejected the null of linearity for $d = 1, 2, 5$ at the usual risk $\alpha = 0.05$. The null hypothesis was accepted for other values of d. We also considered the Hansen (1997) test with bootstrapped p-values (500 replications). For the type I risk $\alpha = 0.10$, the Hansen test rejected the null for $d = 2, 3, 4$ and accepted the null for all the other values. Thus, we decided to take as transition variable y_{t-2}^2. Now, applying a 3-regime SETAR model to the series (y_t^2), the AIC corresponding to a set of thresholds $\hat{c}$ have been computed and the minimum was reached for $\hat{c}_1 = -0.1401$ and $\hat{c}_2 = 0.0442$. *In fine*, the estimated model we have retained is given in Table 5.

As for the IPI series, we repeated the specification phase for several vintages. Since the specified models showed to be quite stable, we used this specification for the real-time detection analysis.

Table 5: Parameter estimates and standard errors for a 3-regime SETAR model applied to the 1st vintage series of Unemployment Rate (January 1971 − March 2001).

	Low regime $[y^1_{t-2} < -0.1401]$	Intermediate regime $[-0.1401 \leq y^1_{t-2} \leq 0.0442]$	High regime $[y^1_{t-2} > 0.0442]$
$\hat{\mu}$	-0.2868	-0.0369	0.1667
$\hat{\phi}_1$	0.6501		0.3138
$\hat{\phi}_2$	0.3682		
$\hat{\phi}_3$	-0.6635		
$\hat{\phi}_4$	0.2399		0.2373
$\hat{\phi}_5$	0.3664	0.2629	
$\hat{\phi}_6$	-0.2153		
$\hat{\sigma}_\varepsilon$	0.1227	0.1228	0.0981
n	114	108	115

4.2.2 Markov-switching specification

The same procedure described for IPI series has been applied to the Unemployment Rate; once again, the reference business cycle chronology is taken from Anas et al. (2007a), however in this case we adopted the quarterly dating referred to the classical business cycle and transformed it on a monthly basis by considering as turning point the month in the middle of each quarter in which a turning point has occurred.

In the same way as for the IPI, also for the UR we mainly considered Markov-switching models that do not allow for autoregressive dynamics in the observable variable. These models and their respective QPS and Concordance Index statistics are listed in Table 6. Contrary to the previous case of the IPI, MSI(4)-AR(0) model turned out to be the unique selected specification as it is characterized by the lowest QPS and the highest Concordance Index among all the considered models.

Table 6: Mean QPS and Concordance Index of some selected Markov-switching models fitted to the 97 releases of the Unemployment Rate.

Model	QPS	Concordance Index
MSI(2)-AR(0)	0.180	0.798
MSIH(2)-AR(0)	0.324	0.637
MSI(3)-AR(0)	0.133	0.854
MSIH(3)-AR(0)	0.148	0.840
MSI(4)-AR(0)	0.123	0.870
MSIH(4)-AR(0)	0.128	0.865

Table 7 reports the parameter estimates of the MSI(4)-AR(0) model; as for the first and second regimes of the state variable, we maintain the same economic interpretation given for the MSIH(3)-AR(0) model fitted to the IPI, namely they locate recession and

slowdown phases, respectively; as for the third and fourth regimes, the former pinpoints low-expansion phases and the latter high-expansion phases.

Table 7: Parameter estimates and standard errors for a MSI(4)-AR(0) model fitted to the 1st vintage Unemployment Rate series (January 1971 − March 2001).

	Regime 1	Regime 2	Regime 3	Regime 4
$\hat{\mu}\left(S_t\right)$	-0.4371 (0.0152)	-0.1487 (0.0186)	0.0326 (0.0144)	0.2209 (0.0125)
$\hat{\sigma}_\varepsilon$	0.097129			
Transition Matrix				
	Regime 1	Regime 2	Regime 3	Regime 4
Regime 1	0.9258	0.07383	0.0003755	3.422e-007
Regime 2	0.03756	0.8370	0.1190	0.006463
Regime 3	4.612e-008	0.1010	0.8790	0.01996
Regime 4	2.350e-009	8.480e-005	0.03405	0.9659

5   Real-time turning point detection

In this section we compare the previously selected SETAR and MS models in terms of their ability to detect in real-time turning points of the classical business cycle, either when they are fitted to the IPI and UR series. However, it is important to underline that the reference dating chronologies (see Anas et al., 2007a), which could be used as benchmark to compute goodness of fit statistics (QPS and Concordance Index) for the selected models, do not cover and only partially cover, respectively, the sample horizons through which span the IPI and UR vintages. Indeed, both the reference dating chronologies end in December 2002, while the real-time exercise for the IPI series starts only in April 2003 and for the UR series in March 2001; therefore, in the former case they are not overlapping at all and only slightly overlapping in the latter case. As a result, real-time analysis is mainly tackled by graphical inspection and by taking into account our a-priori beliefs about the evolution of the business cycle outside the reference chronology.

The lack of quantitative measures for the months outside the reference chronology are partially overcome by taking into exam the historical ability, of both SETAR and MS models, to ex-post date the occurrence of turning points. Indeed, this ex-post exercise is the starting point for the analysis of each of the two time-series considered in this paper; once we have measured to what extent either SETAR and MS models are able to locate the turning points of the respective reference chronology, we are allowed to tackled the real-time analysis.

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5.1 Real-time detection based on the Industrial Production Index

As discussed above, first we graphically analysed the issue of ex-post dating of turning points of the Industrial Production Index by looking at Figure 3, which is divided in three panels:

- the lower panel shows the industrial recessionary phases derived by applying the 0.5 "natural rule" to regime 1 filtered probabilities obtained by fitting the MSI(2)-AR(0) model to the first vintage of the IPI;

- the mid panel depicts the recessionary periods as stated in the reference chronology;

- the upper panel replicates the same exercise of the lower panel for the case of the 2-regime SETAR model.
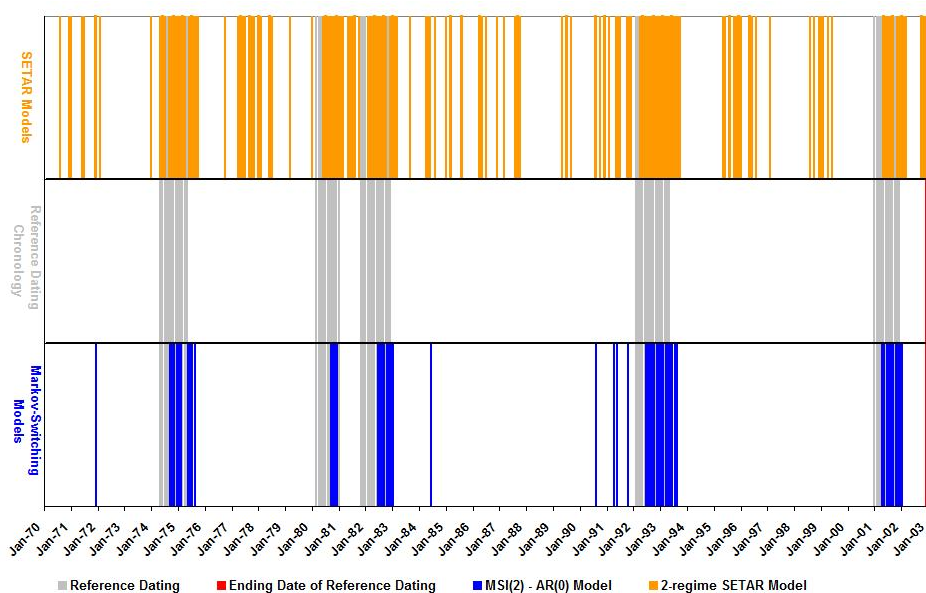
Figure 3: Classical business cycle recession dating (mid panel) is compared to recession dates obtained either by fitting MSI(2)-AR(0) model (lower panel) and 2-regime SETAR model (upper panel) to the 1st vintage release, issued in June 2003, of the IPI series.

By graphical inspection, it came out a marked difference between selected SETAR and MS models; on one hand, MSI(2)-AR(0) model lags behind the 2-regime SETAR model in detecting four out of the five recessions located by the reference chronology and, on the other hand, the former model emits many more short-lived recession signals than the latter one. Due to the short-lived duration of these events, we though interesting to gauge the effect of introducing a censoring rule so as to get rid of the signals that do not actually represent a recession. This was achieved by introducing a restriction according to which a phase of the cycle must last at least three months to be recognized as a recession or expansion. It is worth noting that, although this rule reduces risk of false signals, it has the drawback of delaying turning points detection by three months. In Figure 4 we propose the previous graph when the aforementioned censoring rule is applied.
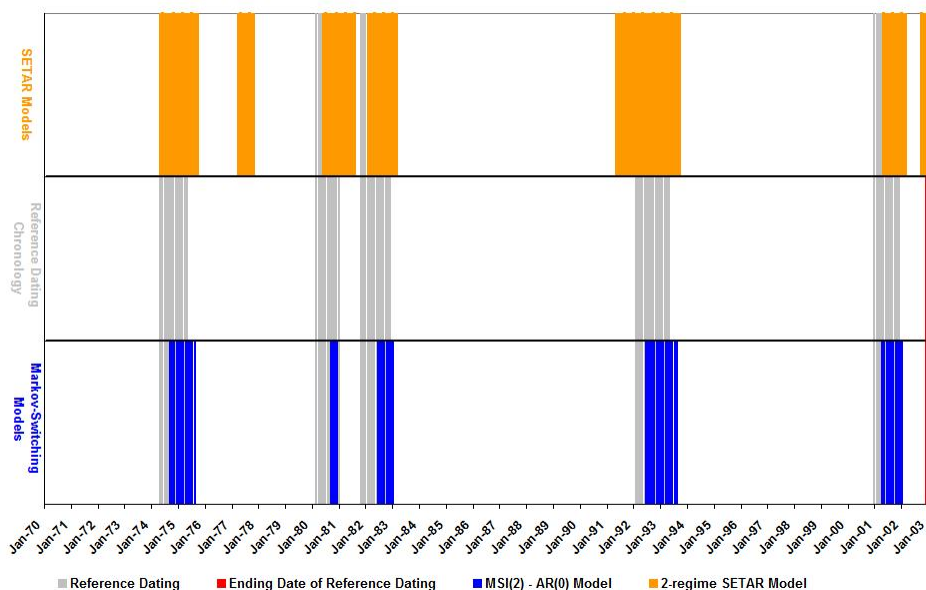
Figure 4: Classical business cycle recession dating (mid panel) is compared to recession dates obtained either by fitting MSI(2)-AR(0) model (lower panel) and 2-regime SETAR model (upper panel) to the 1st vintage release, issued in June 2003, of the IPI series; recession signals must satisfy a censoring rule stating that a recession must last at least 3 months.

By applying the censoring rule, we have discarded the noise that characterized the raw output of the 2-regime SETAR model, so as to be facilitated in analysing either the missed recessionary episodes and the extra cycles detected by the two models. On one hand, the MSI(2)-AR(0) model encounters difficulties in indentifying the double-dip recession of the early '80s, but no extra-cycles is detected; on the other hand, the 2-regime SETAR model does not miss any recession episodes of the reference dating, but a false signal of recession is emitted between April and November 1977[4], furthermore it anticipates the starting date and postpones the ending date of the recession between 1992 and 1993.

The following Figures 5 and 6 replicate the above analysis when the MSI(2)-AR(0) is replaced with the MSIH(3)-AR(0) model. Since the result in terms of detected turning points almost matches the one already described in the case of the MSI(2)-AR(0) model, we refer the reader to the previous discussion.

As previously stated, we prefer to ground on a quantitative basis the insights gained by graphical inspection. Due to the nature of the outcome returned by SETAR models (binary variable rather than probabilities), we can compare them with Markov-switching models only on the basis of the Concordance Index. Table 8 reports the Concordance Index computed on selected SETAR and MS models estimated on the first available release

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[4] A further recession is detected starting in October 2002, but since it is located on the border of the reference chronology, we discarded it as it could be subject to edge effects.
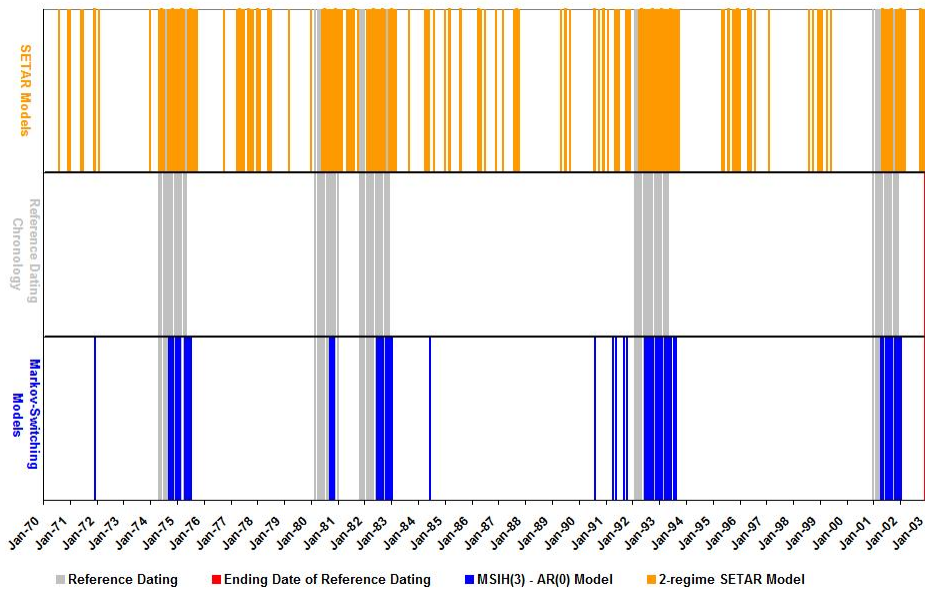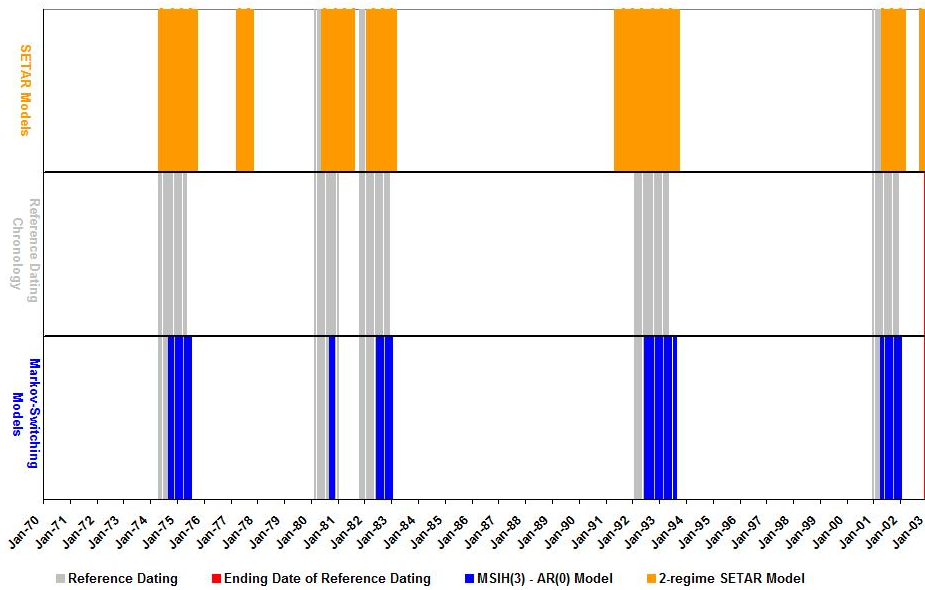
Figure 5: Classical business cycle recession dating (mid panel) is compared to recession dates obtained either by fitting MSIH(3)-AR(0) model (lower panel) and 2-regime SETAR model (upper panel) to the 1st vintage release, issued in June 2003, of the IPI series.

Figure 6: Classical business cycle recession dating (mid panel) is compared to recession dates obtained either by fitting MSIH(3)-AR(0) model (lower panel) and 2-regime SETAR model (upper panel) to the 1st vintage release, issued in June 2003, of the IPI series; recession signals must satisfy a censoring rule stating that a recession must last at least 3 months.

(June 2003), with and without censoring rule and on different time horizons. According to the definition of the Concordance Index, the higher its value, the closer the related dating to the reference chronology is.

Table 8: Concordance Index for selected MS and SETAR models fitted to the 1st vintage release of the IPI series (January 1970 − April 2003).

Horizon	Model	Concordance Index	
		No Censoring Rule	*3-month Censoring Rule*
1970 − 2002	**MSI(2)-AR(0)**	0.893	0.914
	MSIH(3)-AR(0)	0.893	0.914
	2-regime SETAR	0.716	0.862
1970 − 1986	**MSI(2)-AR(0)**	0.865	0.885
	MSIH(3)-AR(0)	0.870	0.885
	2-regime SETAR	0.708	0.849
1986 − 2002	**MSI(2)-AR(0)**	0.926	0.946
	MSIH(3)-AR(0)	0.922	0.946
	2-regime SETAR	0.721	0.882

By looking at Table 8, and in particular to the results referring to the whole time-horizon covered by the reference chronology (1970 − 2002), both Markov-switching models return Concordance Index values that are higher than the one of the SETAR model; that is, the former models are better able than the latter one to date turning points defined by the reference chronology. This can be explained by the huge amount (although mainly short-lived) of false signals emitted by the 2-regime SETAR model when we did not consider any censoring rule. In fact, when we imposed the aforementioned 3-month censoring rule, all the short-lived false signals have been discarded and the Concordance Index of the 2-regime SETAR model rose considerably from 0.716 to 0.862, whereas the figure for both MS models improved only from 0.893 to 0.914.

Finally, we joined graphical inspection and quantitative measures to draw some conclusions. The former approach suggests that both chosen MS models outperform 2-regime SETAR model as they did not locate any false recession cycles, while the latter did. Furthermore, we notice that both MS models have improved more than the 2-regime SETAR model in their respective ability to date turning points of the industrial cycle along the sample horizon; as a matter of fact, although we cannot rely to any statistical significance test, by comparing the concordance indexes computed in the equally-wide periods 1970 − 1986 and 1986 − 2002, it follows that both the MS models enhanced their capability of detecting turning points, while the figures for the 2-regime SETAR model were almost unchanged. Therefore, as we are interested in using SETAR and MS models in a real-time analysis to detect the occurrence of turning points, we should be aware of the different performances over time of these models.

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The previous analysis was based on the outcomes derived by fitting either SETAR and MS models to the first available release of the IPI time-series. In what follows, we leave this static perspective and move on to a real-time analysis. In this respect, we considered all the 69 vintage releases of the IPI, issued from June 2003 to February 2009, which have been sequentially modelled as a MSI(2)-AR(0) or MSIH(3)-AR(0) model and as a 2-regime SETAR model. Due to the dynamic nature of this exercise, the information set used to draw inference about the parameters changes according to the evolution of the releases, in the sense that it includes an increasing number of observations and it also reflects the data revision effect. Then, for each release, a binary variable was returned for the last month in the sample, which assumed value 1 if a recession signal was detected and 0 otherwise. Stated differently, these are the same inferential and detection issues that we would have faced if we had actually performed the study in real-time. All these binary variables have been collected up and graphically represented in the following Figures 7, 8, 9 and 10.

Real-time analysis confirmed one of the main findings stemming from the ex-post dation exercise: unlike Markov-switching models, the 2-regime SETAR model returns many short-lived recession signals. Although we were prevented from discriminating right from wrong signals because of the lack of a reference chronology in the real-time analysis, we filtered out recession signals by constraining them to last at least three months. The resulting recession periods are depicted in Figures 8 and 10.
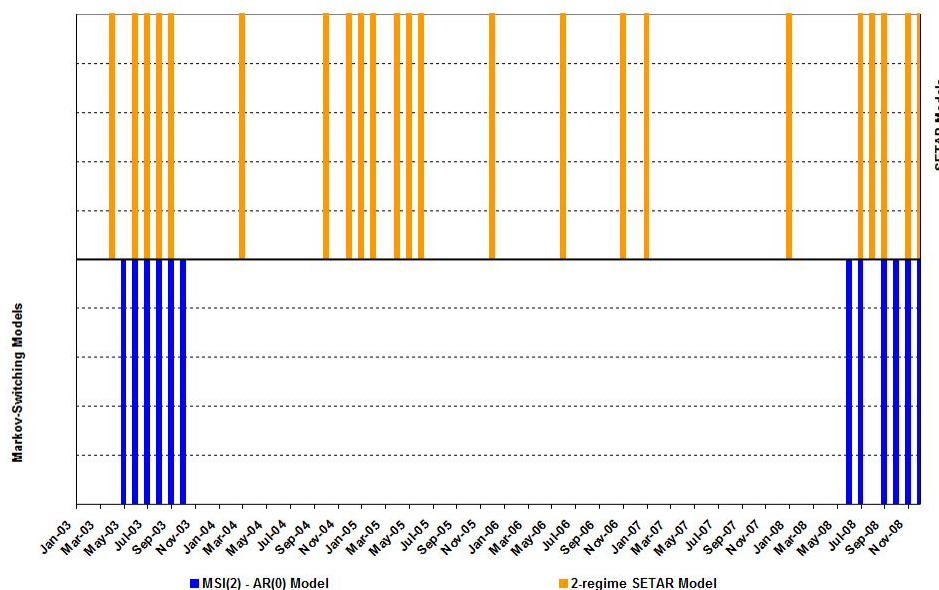

Figure 7: Real-time recession signals obtained either by fitting MSI(2)-AR(0) model (lower panel) and 2-regime SETAR model (upper panel) to all the 69 vintage releases, issued from June 2003 to February 2009, of the IPI series.

From April 2003 to December 2008, the 2-regime SETAR model locates three recession phases; two of them are identified, although with a different timing between them and also with respect to the SETAR model, by both MS models. On the contrary, MS models
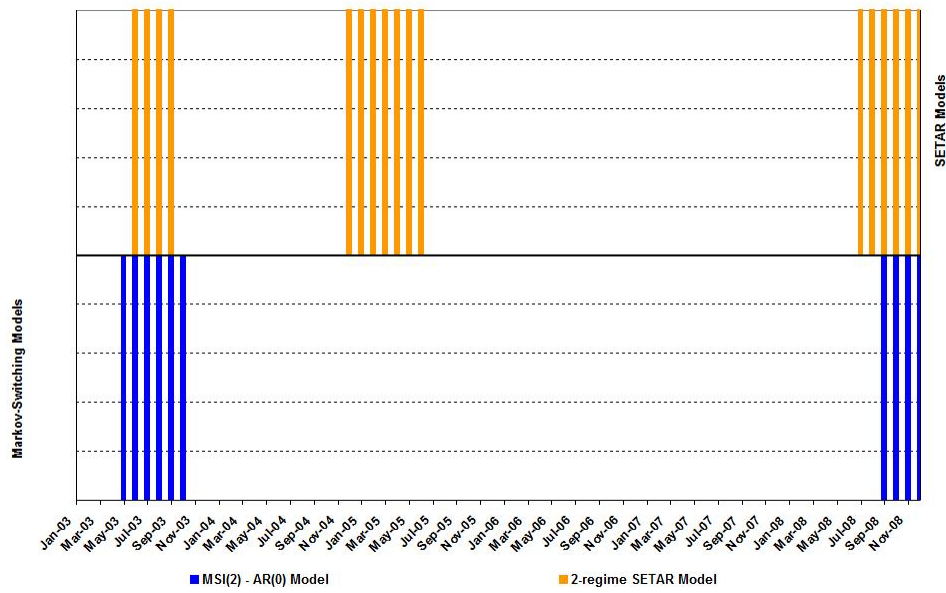
Figure 8: Real-time recession signals obtained either by fitting MSI(2)-AR(0) model (lower panel) and 2-regime SETAR model (upper panel) to all the 69 vintage releases, issued from June 2003 to February 2009, of the IPI series; recession signals must satisfy a censoring rule stating that a recession must last at least 3 months.
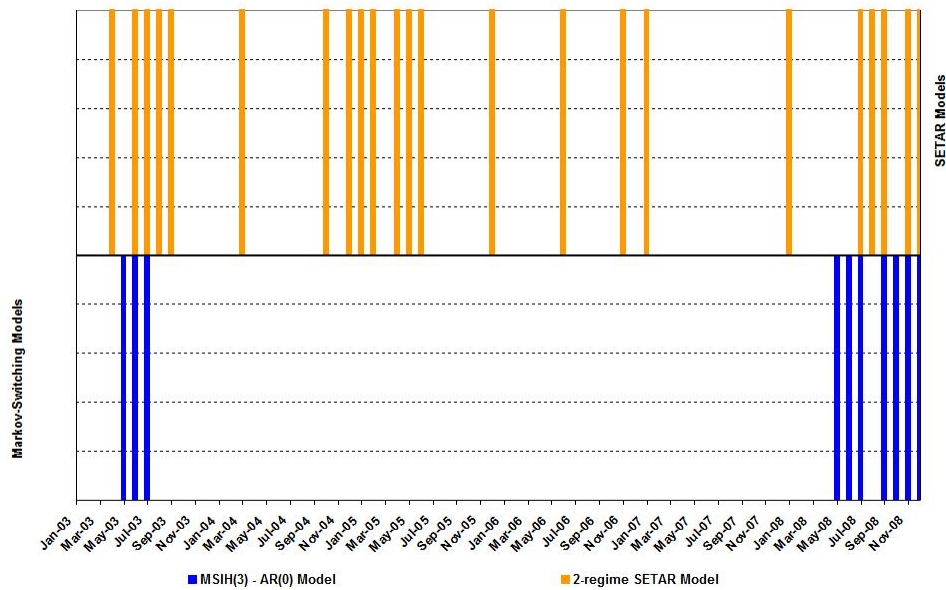

Figure 9: Real-time recession signals obtained either by fitting MSIH(3)-AR(0) model (lower panel) and 2-regime SETAR model (upper panel) to all the 69 vintage releases, issued from June 2003 to February 2009, of the IPI series.

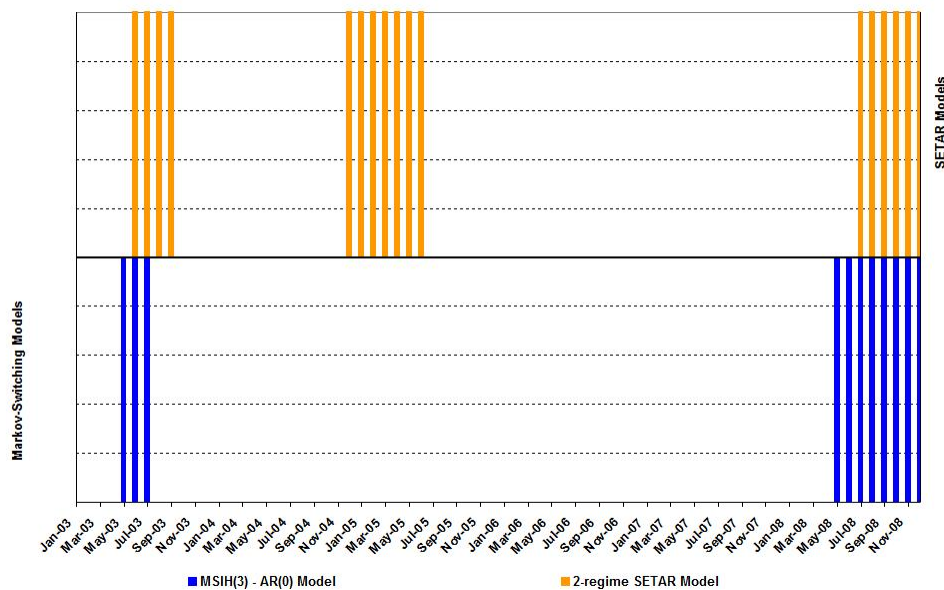do not detect at all a recession phase between December 2004 and June 2005.

Figure 10: Real-time recession signals obtained either by fitting MSIH(3)-AR(0) model (lower panel) and 2-regime SETAR model (upper panel) to all the 69 vintage releases, issued from June 2003 to February 2009, of the IPI series; recession signals must satisfy a censoring rule stating that a recession must last at least 3 months.

As for the most recent months in the sample, the MSIH(3)-AR(0) model is the one that detects earliest a peak of the industrial cycle in April 2008, that is, a recession starts in May 2008; the 2-regime SETAR model locates this peak to be occurring in June 2008 and the MSI(2)-AR(0) only in August 2008. In the light of our knowledge of the ongoing economic conjuncture, according to which a slowdown started in mid 2008, the MSIH(3)-AR(0) model deserved to be singled out among the models here considered.

To conclude, the fact that from an historical perspective the MSIH(3)-AR(0) model returned the highest Concordance Index reinforces the potential role that it could play in timely detecting forthcoming turning points.

5.2   Real-time detection based on the Unemployment Rate

Also in the case of the Unemployment Rate, we tackled the comparison between SETAR and MS models either by graphical inspection and also by providing a quantitative measure of their ability to ex-post date turning points of the reference chronology. Note that in this case, we considered as benchmark the global economic cycle, rather than the industrial cycle previously considered.

For the UR, only two models have been retained, namely a 3-regime SETAR model and a MSI(4)-AR(0) model; these models are graphically compared in Figure 11, which is organized as before in three panels:

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- the lower panel shows the classical business cycle recessionary phases obtained by applying the 0.5 "natural rule" to regime 1 filtered probabilities derived by fitting the MSI(4)-AR(0) model to the first vintage release of the Unemployment Rate;

- the mid panel depicts the recessionary periods as stated in the reference chronology;

- the upper panel replicates the same exercise of the lower panel for the case of the 3-regime SETAR model.

It came out that the 3-regime SETAR model has, although to a lesser extent, a feature in common to the previous 2-regime SETAR model fitted to the IPI series, namely the great number, compared to MS models, of false and short-lived signals of recession. For what concerns the reduction in the number of short-lived recession signals, it could be straightforwardly explained by the smoother behaviour of the Unemployment Rate time-series when compared to the IPI series.
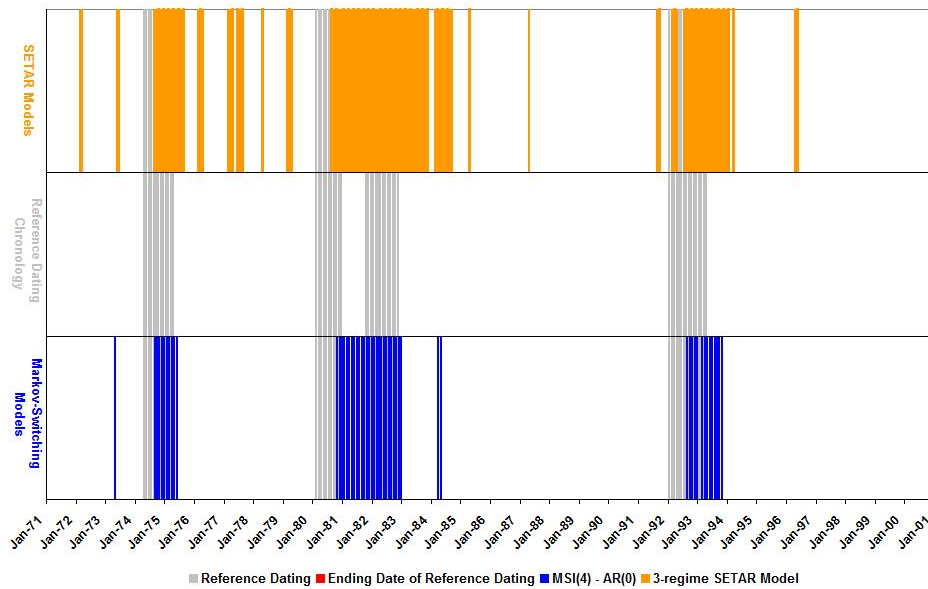

Figure 11: Classical business cycle recession dating (mid panel) is compared to recession dates obtained either by fitting MSI(4)-AR(0) model (lower panel) and 3-regime SETAR model (upper panel) to the 1st vintage released, issued in May 2001, of the TUR series.

Also in case of the UR series, we preliminary constrained the recession signals to last at least three months by applying a censoring rule. The resulting datings of the classical business cycle are depicted in Figure 12.
As expected, all the short-lived recession signals that affected both SETAR and MS models have been wiped out; however, three false recession cycles still persist in the dating obtained from the 3-regime SETAR model; furthermore, this model sensibly postpones the ending date of the double-dip recession until November 1984, while the reference chronology declared it to have come to end in December 1982; the same is true for what
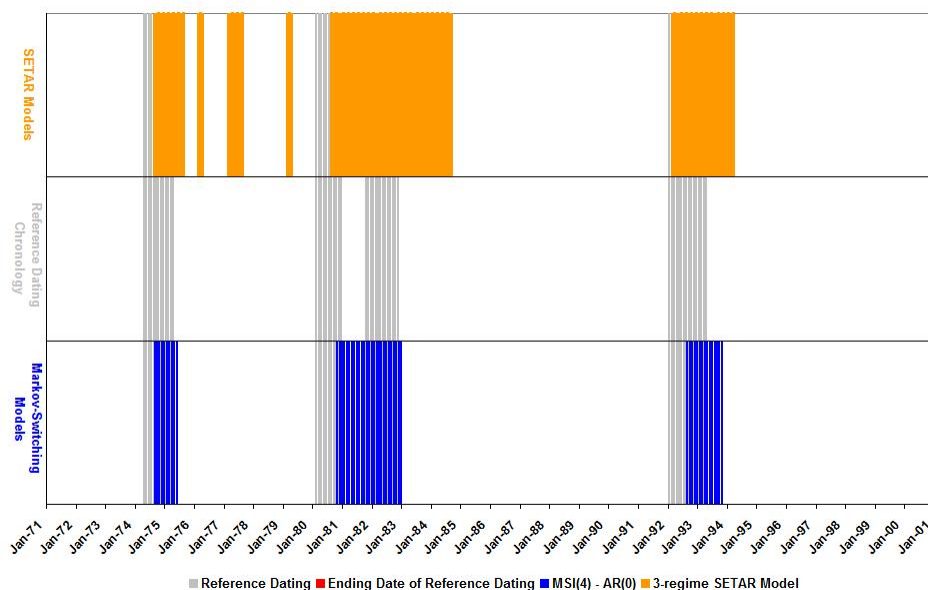
Figure 12: Classical business cycle recession dating (mid panel) is compared to recession dates obtained either by fitting MSI(4)-AR(0) model (lower panel) and 3-regime SETAR model (upper panel) to the 1st vintage released, issued in May 2001, of the TUR series; recession signals must satisfy a censoring rule stating that a recession must last at least 3 months.

concerns the $1992 - 1993$ recession, whose ending date is moved ahead in time from May 1993, according to the reference chronology, to April 1994. Contrary to SETAR model, the MSI(4)-AR(0) model does not locate any extra cycle. Moreover, on one hand, neither model misses a recession period of the reference chronology, but on the other hand, neither of them is able to distinguish as separate phases the two downturns of the double-dip recession in the early '80s.

The above features are confirmed by figures reported in the upper frame of Table 9.

Table 9: Concordance Index for selected MS and SETAR models fitted to the 1st vintage release of the TUR series (January 1971 $-$ March 2001).

Horizon	Model	Concordance Index	
		No Censoring Rule	*3-month Censoring Rule*
$1971 - 2001$	**MSI(4)-AR(0)**	0.881	0.892
	3-regime SETAR	0.772	0.797
$1971 - 1985$	**MSI(4)-AR(0)**	0.853	0.870
	3-regime SETAR	0.655	0.672
$1986 - 2001$	**MSI(4)-AR(0)**	0.907	0.913
	3-regime SETAR	0.885	0.918

When no censoring rule is applied, the Concordance Index computed for the selected MS model is 0.881, while it is 0.772 for the SETAR model. These values slightly increased when the 3-month censoring rule is taken into account. For the 3-regime SETAR model, there is only a small improvement in the Concordance Index, since only few of the false signals are short-lived, while the remaining do satisfy the aforementioned censoring rule; also for the MSI(4)-AR(0) model, the improvement in the Concordance Index is limited because only two short-lived extra cycles were detected by this model.

Differently from what seen with the IPI series, in the case of the UR series, both selected SETAR and MS models are characterized by higher values of the Concordance Index in the second part of the sample, namely 1986 − 2001.

In line with these results, the MSI(4)-AR(0) could be preferred to the 3-regime SETAR model, independently of any preferences on extra- and missing-cycles, because the former model dominates the latter one in terms of its ability to date turning points of the reference chronology.

Also for the Unemployment Rate, we conclude our discussion by performing a real-time analysis starting from May 2001 by fitting either SETAR and MS models to each monthly release of the UR series until May 2009.
Figure 13 shows the binary variable representing the recession signals obtained by fitting either the 3-regime SETAR and MSI(4)-AR(0) model to the sequence of all the 97 vintage releases of UR series; and Figure 14 is obtained by imposing the 3-month censoring rule.
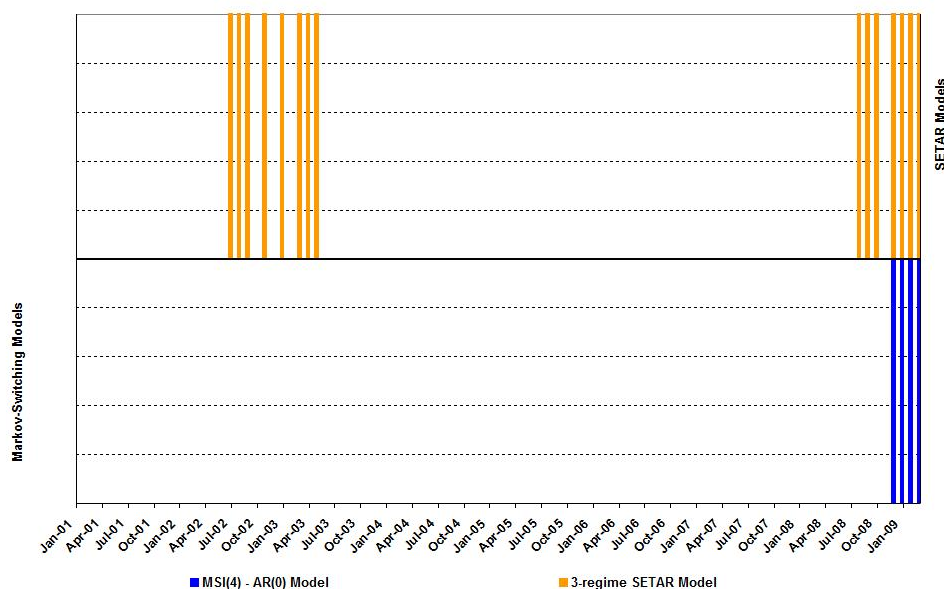

Figure 13: Real-time recession signals derived either by fitting MSI(4)-AR(0) model (lower panel) and 3-regime SETAR model (upper panel) to all 97 vintage releases, issued from May 2001 to May 2009, of the TUR series.
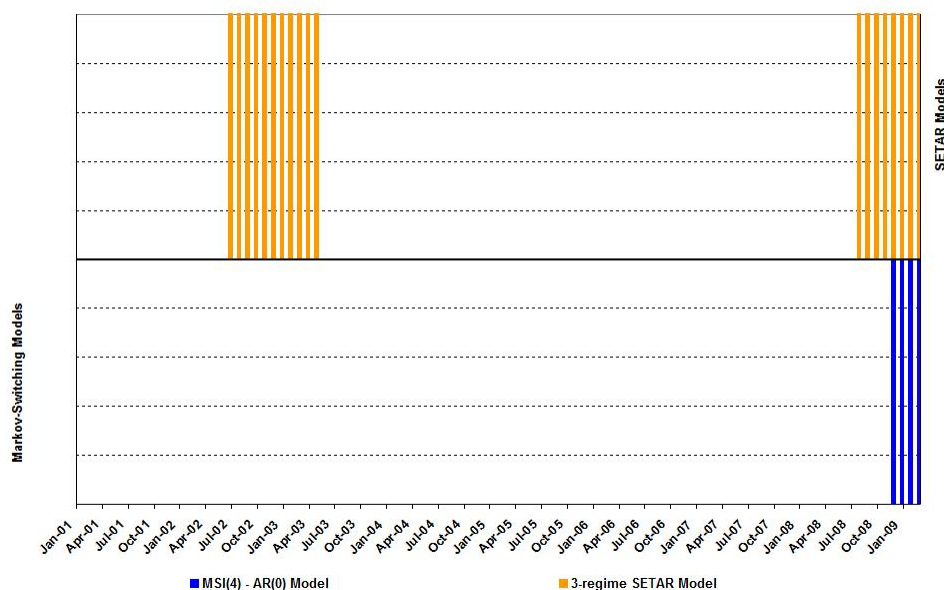
Figure 14: Real-time recession signals obtained either by fitting MSI(4)-AR(0) model (lower panel) and 3-regime SETAR model (upper panel) to all 97 vintage releases, issued from May 2001 to May 2009, of the TUR series; recession signals must satisfy a censoring rule stating that a recession must last at least 3 months.

By graphical inspection of Figures 13 and 14 we are able to indentify two main differences between the 3-regime SETAR and MSI(4)-AR(0) model; on one hand, the 3-regime SE-TAR model locates a recession between July 2002 and May 2003, whereas MSI(4)-AR(0) model does not; on the other hand, this latter model lags behind the former in signalling the starting date of the ongoing recession, namely December 2008 vs. August 2008.

6 Conclusions

In the paper we aim at assessing Markov-switching and SETAR models, either in their ability to detect the occurrence of turning points of economic cycles and the stability over time of the signals they emitted in a real-time analysis. We resort to these different classes of models because they underpin a complementary rather than an alternative approach, as the notion captured by them is not exactly the same. In the framework of SETAR models the transition variable is observed, in particular it is a linear combination of lagged values of the series; this is the main difference with MS models whose parameters of the autoregressive data generating process vary according to the states of a latent Markov chain.

These tasks haves been accomplished by fitting both classes of models to ad hoc histori-cally reconstructed databases, either of the Industrial Production Index and Unemploy-ment Rate of the Euro area, that are updated on a monthly basis so as to simulate a real-time exercise. These two series have been considered since they are commonly em-ployed to date economic cycles and are available for the Euro area on a monthly basis.

For what concerns Markov-switching models, a specification that allows for state-dependent heteroskedasticity seems to play an important role when modelling time-series that are quite noise, like the IPI time-series; this conclusion is drawn from the fact that MSIH(3)-AR(0) model is well-suited to deal with the IPI time-series, while in the case of the smoother series of the Unemployment Rate, an homoskedastic MSI(4)-AR(0) model returns an higher performance than heteroskedastic specifications. In this respect, it could be interesting to investigate Markov-switching models that further extends heteroskedasticity by specifying a Markov chain that drives the changes in the variance of the observable process.

On the contrary of Markov-switching models, we have experienced how SETAR models can return many short-lived recession signals and, although they could be easily driven out by applying some simple censoring rule, this can represent a drawback as far as real-time detection of turning points is concerned. In fact, resorting to a censoring rule usually ends up in delaying the signal of a turning point.

With regards to the ex-post dating, Markov-switching models have been proved to return higher values for the Concordance Index than the SETAR models. However, if on one hand, MS models emit a delayed signal with respect to SETAR models but without detecting extra-cycles; on the other hand, SETAR models do not miss any recession episodes of the reference chronology but emit some false signal of recession. The choice between them must then resort to the policy maker's preferences, according to the weights attached to extra- and missed-cycle, respectively.

Although we do not have performed a complete dynamic analysis in order to assess the evolution of ex-post detected turning points, as we move through all the vintages releases, we note that, for both IPI and Unemployment Rate, Markov-switching models improve their capability in correctly detecting turning points in the second half of the sample horizon compared to the performance recorded in the first half. The same is true also for the SETAR model when it is fitted on the Unemployment Rate. Moreover and more importantly, we can observe that both Markov-switching and SETAR models convey signals that are robust and stable over time, that is moving through the vintage releases considered, MS and SETAR models stem signals that do not vary sensibly.

To conclude, also if the Markov-switching models show some higher ability to date turning points, it is not possible to clearly state their superiority since if a missing a cycle is a worse error than emitting false signals, as it can be, then a SETAR model could be preferred to a MS one.

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