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THE TAXATION OF SAVINGS IN OVERLAPPING GENERATIONS ECONOMIES WITH UNBACKED RISKY ASSETS

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Abstract. This paper establishes, in the context of the Diamond (1965) overlapping generations economy with production, that the risk that savings in unbacked assets (like fiat money or public debt) become worthless implies that, not only the first-best steady state, but even the best steady state attainable with those saving instruments fails to be a competitive equilibrium outcome under laissez-faire. It is nonetheless shown as well that this *best monetary steady state* can be implemented as a competitive equilibrium with the adequate policy of taxes on returns to capital, subsidies to returns to monetary savings, and lump-sum transfers. Interestingly enough, this policy requires no redistribution of income among agents, unlike the implementation of the first-best steady state. The policy is balanced every period at the steady state and, since no public spending exists in the model, it serves the only purpose of implementing a steady state that provides all agents with a higher utility than the laissez-faire competitive equilibrium steady state. The results thus provide a rationale for an active fiscal policy that has nothing to do with redistributive goals or the need to fund any kind of public sending.

Keywords: taxation of savings, overlapping generations, asset bubble.

JEL codes: E62, E21, E22, H21

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1. Introduction

The first-best steady state allocation of resources of the overlapping generations economy with production in Diamond (1965) is typically not a laissez-faire competitive equilibrium allocation unless the agents can save in terms of a fiat money (or another effectively unbacked asset like public debt) on top of physical capital. But even in the presence of a fiat money, for it to allow to attain the first-best steady state as a laissez-faire competitive equilibrium, money has to be absolutely riskless. In effect, as soon this asset risks becoming worthless (because of not being accepted by the next generation)[1] —even if this happens only with the slightest probability, which is arguably never zero in reality—, then there is no hope of attaining the first-best steady state in a decentralized way as a laissez-faire competitive equilibrium outcome.[2] But for anyone expecting that the market does anyway its best given the circumstances (i.e. given the unavoidable riskiness, even if small, of unbacked assets) news get even worse: not even the best steady state *implementable with the unbacked risky asset* is a laissez-faire competitive equilibrium outcome.[3] The good news is nonetheless that, if for some reason the intergenerational transfers required to implement the first-best are not possible and unbacked assets like money are unavoidably risky to some extent, at least the best steady state implementable by a risky money (the *best monetary steady state* henceforth) is a competitive equilibrium outcome under the right fiscal policy. This paper tells what this policy is.

More specifically, in Diamond (1965) the agents' only endowment is their ability to work when young. Output can be reproduced each period using the labor the agents supply and the amount of previously produced output that has not been consumed or used up in the production process yet (thought of as the aggregate level of capital). In such a set-up the best possible steady state —i.e. the steady state that maximizes the utility of the representative agent— requires, first, that the aggregate level of capital be such that the output net of capital replacement is maximized at each period; and, second, that this net production is split between young and old agents in such a way that the marginal rate of substitution between consumptions

[1] Or because of being reneged in the case of debt.

[2] That is to say, barring the possibility of resorting to outright redistributions across generations as a pay-as-you-go pension scheme does.

[3] In other words, a competitive equilibrium steady state fails to provide the representative agent with the highest utility not only *among feasible steady states* (regardless of whether it is dynamically efficient or not), but even among only those that are implementable through saving in the risky money.

when young and old equals the rate at which consumption can be redistributed from young to old at any given period. These two conditions amount to make both the marginal rate of substitution between present and future consumption and the marginal productivity of capital equal to the factor by which the population grows each period. Typically, this requires *not* to remunerate the factors of production by their marginal productivities or, alternatively, to make intergenerational redistributions of income, should the factors be remunerated this way. Any of these two ways to implement the first-best steady state is clearly at odds with what characterizes a laissez-faire competitive equilibrium, since the latter both remunerates the factors by their marginal productivities and does not allow for redistributions of income among agents. Nonetheless, if the agents can save part of their labor income in terms of an unbacked and hence intrinsically worthless asset, e.g. a fiat money (an asset bubble in Tirole (1985) terms),[4] then there is a specific portfolio of money and capital that, *if chosen* by the agents for their savings,[5] implements the first-best steady state as a competitive equilibrium outcome. Nevertheless, it is crucial for this result to hold true that every agent believes that money will not have, for sure, a zero exchange value next period.

In effect, at a competitive equilibrium steady state of the Diamond (1965) economy, the agents, in the absence of money or any other mechanism allowing to implement in a decentralized way intergenerational transfers of resources, may end up dumping with their saving decisions too much capital into the production process, compared to the level that maximizes net output. In order to convince them to withdraw part of these saving from the production process[6] and to devote them instead to increase the consumption of their parents, they need to be reassured that they will be treated in the same way by the next generation. That is to say, they must believe that the mechanism in place today allowing to make intergenerational transfers will still be there tomorrow when their turn comes to receive from it, instead of contributing to it. Whether this mechanism is fiat money, rolled-over public debt or a pay-as-you-go pensions system, the fact is that it amounts to just promises, and thus the essential element to make any such social contrivance to work is trust. Now, trusted promises risk not being honored. As a matter of fact, although fortunately these are rare events, it is nonetheless a fact of life that every now and then states do dissolve, wars are waged, revolutions topple governments, and as a result public debts of

[4]A fiat money in Samuelson (1958), or public debt that is rolled-over every period in Diamond (1965).

[5]It turns out that this *if* may be a too big if anyway (see the concluding section for a discussion of this issue).

[6]Which, incidentally, increases the marginal productivity of capital and hence the return to their own savings to an extent that offsets their lower level of savings.

previous governments are repudiated, money issued by former regimes becomes worthless, and pension claims are not honoured.[7] Financial crises in which banking and credit institutions disappear do happen and claimants lose their savings as a result. And, nevertheless, some *trust is put recurrently on similar promises*, institutions or social compacts almost immediately after such crises take place. Thus it seems to be inherent to intergenerational financial arrangements based on trust that there is some probability, no matter how small, that they collapse, only to be restarted little after. Weil (1989) established conditions for the existence of competitive equilibria in a Diamond (1965) economy with a money that risks losing value completely at any time, and the result was that existence obtained as long as this risk was small enough. In Weil (1989) the economy was supposed to revert to a non-monetary equilibrium once the bubble bursts, which happens in finite time with probability 1. Unfortunately, this is a counterfactual feature of the stochastic asset bubbles considered there, since asset bubbles are clearly recurrent, and money in particular, as a bubble, is immediately replaced by another money should it lose value completely. Thus, I consider instead a steady state in which a new money is issued (a new bubble starts) right after the dismissal of the current one, in case that event happens.

Having thus introduced some probability for the money bubble bursting, one can consider (as when money was assumed to be valued for sure) which is the best steady state that can be implemented *saving in such a "risky" money*, on top of in terms of capital. Of course, this will depend on the specific probability of money losing value, and as a first approach (admittedly unsatisfactory) I will consider that probability to be exogenously determined, as in Weil (1989). Thus I characterize below the best steady state that a risky money can buy, i.e. the best monetary steady state, for a given probability of money losing value completely. The best monetary steady state turns out not to be, unfortunately, a competitive outcome *under laissez-faire*. In other words, free markets are unable to reach the best steady state allocation of resources that is implementable with any given intergenerational transfers mechanism whenever (quite realistically) the latter may collapse at some point, no matter how small this risk is. That is bad news. The good news is that the best monetary steady state is nonetheless a competitive equilibrium outcome *under a well-defined policy of taxes and transfers* not requiring any intergenerational

[7]Although not a completely unbacked asset, the bonds issued by the Confederate States of America to finance the war effort during the American civil war were pledged not to be honored by the Union, and the paper money issued by the confederacy was just paper at the end of the conflict (although it had already almost zero exchange value by then, because of both massive printing by the confederacy and deliberate flooding of the south with counterfeited CSA dollars by the Union).

redistribution. More specifically, in the case in which the best monetary steady state over-accumulates capital with respect to the first-best steady state, this fiscal policy consists of (i) taxing linearly the returns to capital, (ii) subsidizing linearly monetary savings returns, and (iii) making second period lump-sum transfers (that at the steady state equilibrium will be equal to the taxes net of subsidies raised from the same generation).

In case it seems awkward that the implementation of the best monetary steady state may require the taxation of productive savings (in capital) and the subsidizing of unproductive ones (in money), one should recall that the dynamic inefficiency (besides the inefficiency generated by the risk that prevents the money to implement the first-best steady state) comes from the agents dumping of too much capital into the productive process, and hence the need to disincentive such savings. At the same time, unproductive (in a direct sense only) monetary savings[8] work instead in the direction of unclogging the production process in this case, from which the need to not to disincentive them follows. This result may challenge the widespread view that values only directly productive investments above supposedly unproductive or ”speculative” ones, and hence may provide some food for thought about what is the real role of each kind of investments.

The rest of the paper is organized as follows. Section 2 provides (mainly to fix notation and for the sake of completeness) the well-known characterization of the unique first-best steady state of the Diamond (1965) overlapping generations economy with production. In Section 3 I allow for the probability of money losing value completely to be positive at any time, and I characterize the laissez-faire competitive equilibrium steady state in that case. In Section 4 I show that, as a consequence of money being risky, the laissez-faire competitive steady state is not the best monetary steady state. Section 5 establishes that the best monetary steady state can nonetheless be made into a competitive outcome with the adequate policy of taxes and transfers, which I characterize there. A concluding Section 6 closes the paper.

2. The first-best steady state of the Diamond (1965) OG economy

In the Diamond (1965) overlapping generations economy with production, each of the 2-period-lived identical members of a population of overlapping generations

[8] That is to say, intergenerational transfers of resources diverted towards consumption, instead of towards production, actually.

(growing at a rate $n > -1$) is endowed with, say, 1 unit of labor when young and nothing when old. Consumption good can be produced out of their labor and of previously produced and not consumed good by means of a constant returns technology.[9] Utility from the consumption profile (c_0, c_1) is given by $u(c_0) + v(c_1)$, with u and v being as usual differentiable, strictly increasing, strictly concave on non-negative consumptions, and satisfying $\lim_{c_0 \to 0^+} u(c_0) = +\infty = \lim_{c_1 \to 0^+} v(c_1)$. Without loss of generality, and for the sake of notational simplicity, capital is assumed to depreciate completely every period.

At a steady state feasible allocation all agents consume the same profile and get, since they have identical preferences, the same utility. Steady states providing the highest possible utility to all agents are characterized thus by being solution to the problem

$$\begin{aligned} &\max_{0 \le c_0, c_1, k} u(c_0) + v(c_1) \\ &c_0 + \frac{c_1}{1+n} + k \le F\Big(\frac{k}{1+n}, 1\Big) \end{aligned} \tag{1}$$

where k is the output saved per worker each period (and used as capital the next period) and the feasibility constraint is hence written in per worker terms.[10] Under the assumptions made on u and v, a solution to (1) is completely characterized by the equations

$$\begin{aligned} \frac{u'(c_0)}{v'(c_1)} &= 1+n = F_K\Big(\frac{k}{1+n}, 1\Big) \\ c_0 + \frac{c_1}{1+n} &+ k = F\Big(\frac{k}{1+n}, 1\Big). \end{aligned} \tag{2}$$

In effect, such a level k of per worker capital savings maximizes net output at any period, while the latter is distributed between the young and old alive then so that the marginal rate of substitution between consumption when young c_0 and consumption when old c_1 equals always the rate at which they can be transformed into each other, i.e. the growth factor of the population $1 + n$.

[9]Only in the final section I will further assume, in order to establish a sign for the tax rate on capital returns, that the technology is described by a constant returns to scale Cobb-Douglas production function or a CES production function with elasticity of substitution smaller than 1. For all the other results constant returns to scale suffice.

[10]Although the choice of notation is always debatable, I will choose to write the model in terms of the *choice variables* of the agents and hence keep k for the per worker savings in terms of capital, instead of (as it is traditional) the level of capital per old agent. In the same vein I'll prefer explicit marginal productivities to so-called "intensive form" expressions that may obscure relations that are otherwise pretty clear (this is, at any rate, particularly true for the arguments and proofs presented below). Needless to say, the two choices are equivalent.

It easily follows from the assumptions on u and v that there is only one solution (c_0^*, c_1^*, k^*) to problem (1) and hence there exists a unique first-best steady state.

Proposition 1. *The Diamond* (1965) *overlapping generations economy with production has a unique first-best steady state, i.e. a unique feasible allocation such that*

(1) *it provides the same consumption profile to all generations*
(2) *at no other allocation providing the same consumption profile to all generations do the agents get a higher utility.*

Proof. Assume both (c_0, c_1, k) and (c_0', c_1', k') solve (1). Then necessarily

$$F_K\Big(\frac{k}{1+n}, 1\Big) = 1 + n = F_K\Big(\frac{k'}{1+n}, 1\Big) \tag{3}$$

so that $k = \bar{k} = k'$ for some $\bar{k}$, and since

$$\begin{aligned} &\max_{0 \le c_0, c_1} \; u(c_0) + v(c_1) \\ &c_0 + \frac{c_1}{1+n} \le F\Big(\frac{\bar{k}}{1+n}, 1\Big) - \bar{k} \end{aligned} \tag{4}$$

has a unique interior solution because of v and u being strictly concave and because of their behaviour at the boundary $\lim_{c\to 0^+} u(c) = +\infty = \lim_{c\to 0^+} v(c)$, then $(c_0, c_1) = (c_0', c_1')$ as well. Q.E.D.

From the statement of problem (1) above it is clear that its only constraint, the feasibility constraint, allows to distribute freely the output of each period among the contemporaneous young and old agents in order to maximize the representative agent's utility. As a consequence, the agents need not receive at the first-best steady state (c_0^*, c_1^*, k^*) the marginal productivity of the factors they contribute to the production of output in case this was a private property economy in which young agents only have their labor endowment and old agents only the return to their saved labor income, i.e. the return to capital. In effect, this would only be the case if it happened to be the case that

$$\begin{aligned} c_0^* + k^* &= F_L\Big(\frac{k^*}{1+n}, 1\Big) \\ c_1^* &= F_K\Big(\frac{k^*}{1+n}, 1\Big) \end{aligned} \tag{5}$$

which is not guaranteed by the conditions (2) characterizing the first-best steady state (c_0^*, c_1^*, k^*). In other words, the first-best steady state needs not be (and will typically not be) a competitive equilibrium outcome in the absence of some mechanism allowing to implement intergenerational transfers. Nonetheless, if the first-best steady state (c_0^*, c_1^*, k^*) solution to (2) is such that

$$\frac{c_1^*}{1+n} - k^* \geq 0 \tag{6}$$

then, as it is well known, it can be attained as a competitive equilibrium of such a private property economy introducing an unbacked (and hence intrinsically worthless) asset like fiat money that the agents can trade for the good, and in terms of which they can therefore save as well,[11] *conditional to the probability of this money being accepted next period being* 1. In effect, given the solution (c_0^*, c_1^*, k^*) satisfying (2) there exists an $m^* = \frac{c_1^*}{1+n} - k^* \geq 0$ (and hence m^* equals $F_L(\frac{k^*}{1+n}) - k^* - c_0^*$ as well from the feasibility condition in (2) above and the homogeneity of degree 1 of F) such that

$$\begin{aligned} c_0^* + k^* + m^* &= F_L\Big(\frac{k^*}{1+n}, 1\Big) \\ c_1^* &= F_K\Big(\frac{k^*}{1+n}, 1\Big)k^* + (1+n)m^* \end{aligned} \tag{7}$$

so that if *every period* the young agents buy from the old agents the fiat money in exchange for an amount m^* of the good, getting thus a return $1+n$ on it next period, then the first-best steady state (c_0^*, c_1^*, k^*) obtains as a competitive equilibrium steady state.[12]

The condition stated above —namely, that the probability money is not accepted next period is zero— is notwithstanding crucial for the decentralization of the first-best as a competitive outcome this way.[13] In effect, in the next section I show the

[11] Quite another thing is whether they would choose to do so in the adequate amount. On this issue, see the discussion in the concluding section.

[12] In the case in which $\frac{c_1^*}{1+n} - k^* < 0$, if every period the young agents receive a transfer from the old agents of an amount m^* of the good, then the first-best steady state (c_0^*, c_1^*, k^*) obtains as well, but for this transfers to be implemented at a competitive equilibrium there needs to be an additional infinitely lived agent, a bank, willing to buy from the young agents an IOU bearing an interest of $1+n$ in exchange of a credit in the bank with which agent t buys an amount $-m^* > 0$ of the good from agent $t-1$, allowing thus the latter to pay back his own standing IOU to the bank. Note that it is implicitly assumed that the agents pay back their IOU's when old *with probability* 1. Note also that the intermediary bank would thus make no gains or losses.

[13] Similarly for the assumption that the agents repay with probability 1 their IOU's when old in the case $\frac{c_1^*}{1+n} - k^* < 0$.

consequences of a positive probability for money becoming worthless at any period —i.e. the consequences of money being "risky"[14]— and, as it will become clear there, the new unique[15] competitive equilibrium steady state supported by such a stochastic bubble asset turns out to be not only distinct from the unique first-best steady state, but even from the best monetary steady state.[16] The latter can however be implemented as a competitive outcome under the fiscal policy detailed in Section 5 further below.[17]

3. Laissez-faire competitive equilibria with "risky" money

Suppose that in the Diamond (1965) overlapping generations economy with production there is a stochastic asset bubble, i.e an unbacked and intrinsically worthless asset like fiat money that is traded at positive prices against the good and that is risky in the sense that with some probability $\pi \in (0, 1)$ the money accepted by generation t in exchange of goods will still be legal tender at $t + 1$, but with some positive probability $\tilde{\pi} = 1 - \pi$ it will not. In the second case, a financial disaster is supposed to have happened between the date at which agent t decides to accept intrinsically worthless money in exchange for goods as a means of saving, and the date at which he intends to spend his monetary savings in old age consumption. As a result of that event, part of his claims over second period resources, namely those held in money, have thus been wiped out. Note that it is the old agent's claims over these resources and not the resources themselves that disappear, so that at equilibrium these resources go to someone else, which in this set up can only be the contemporaneous young agent. An abrupt, sudden redistribution of wealth takes place when this happens, as it is the case when, for instance, bubbles burst, devaluations take place, debt is repudiated, or a currency issued by a toppled government is dismissed during troubled times like wars, revolutions, and other types of social

[14]As in Weil (1989), except for the fact that in that paper the "steady state" equilibrium switches to the moneyless steady state once money becomes worthless. Here, on the contrary, a new money replaces the dismissed one when that happens and, as a consequence, the equilibrium will be truly stationary.

[15]For a probability of money becoming worthless small enough (see Proposition 2).

[16]Unique as well for a probability of money becoming worthless small enough (see Proposition 3).

[17]In the absence of money or any other intergenerational transfers mechanism, the competitive equilibrium steady state differs typically also not only from the first-best steady state, but also from the best steady state that can be implemented through the existing markets for capital and labor. An adequate policy of taxes and transfers allows to implement nevertheless this constrained best steady state (see Dávila (2008)).

crises. Since in a stationary environment the previous generation $t-1$ faced the same risk, then generation t may find itself, when young, with their real wage being worth in a newly issued money enough to afford the resources that could have been claimed by the old had their money not become worthless. Effectively, the risk of loss of value of the monetary savings of the old is *as if* the young found themselves with the (real) monetary savings of the previous generation in their hands with some probability $\tilde{\pi}$ as well. Nevertheless, the representative agent chooses the amount and composition of his savings portfolio prior to this uncertainty being resolved. The representative agent's problem becomes therefore in this case

$$\begin{aligned} \max_{c_0^t, \tilde{c}_0^t, c_1^t, \tilde{c}_1^t, k^t, m^t} \pi u(c_0^t) + & \tilde{\pi} u(\tilde{c}_0^t) + \pi v(c_1^t) + \tilde{\pi} v(\tilde{c}_1^t) \\ c_0^t + k^t + m^t & \leq w_t \\ \tilde{c}_0^t + k^t + m^t & \leq w_t + \frac{\rho_t m^{t-1}}{1+n} \\ c_1^t & \leq r_{t+1} k^t + \rho_{t+1} m^t \\ \tilde{c}_1^t & \leq r_{t+1} k^t \\ 0 \leq & c_0^t, \tilde{c}_0^t, c_1^t, \tilde{c}_1^t, k^t \end{aligned} \tag{8}$$

where $\tilde{c}_i^t, c_i^t$ are agent t's consumption at $t+i$, for $i = 0, 1$, conditional to the money bubble bursting or not respectively, m^t is the real savings in risky money by agent t, and ρ_{t+1} is its return if still valued at $t+1$. Note that, given the monotonicity of preferences, the problem of agent t reduces to choosing k^t and m^t *before* the uncertainty about the exchange value of agent $t-1$'s money holdings (and a fortiori of agent t's as well) is resolved.[18]

Note again that, according to (8) above, the problem faced by the representative agent is *as if* there is every period a probability $\tilde{\pi}$ that the old agent transfers the returns to his monetary savings to the contemporaneous young agent. This is actually equivalent to the situation in which there is every period a probability $\tilde{\pi}$ that the monetary price of the good becomes infinity in the old money hold entirely by the old agents, while in the new money in which the young agents get paid their

[18] As in footnote 12, a negative amount $m^t < 0$ stands for the resources bought by agent t from agent $t-1$ with the credit obtained from issuing an IOU to the bank. There is the risk at every period that the IOU issued by agent $t-1$ is not repaid at t, reducing thus to zero the real value of credit obtained from the bank by agent t from the issuance of his IOU (decided prior to to this uncertainty being resolved) which is anyway still owed at $t+1$. Note that the constrained set is compact, and hence the agent's problem is well defined, as long as $r_{t+1} < \rho_{t+1}$, i.e. if the risky asset (the money or IOU) bears a higher return than the safe asset (capital).

labor income that price adjusts to clear markets by allowing the holders of the new money to be able to claim with their labor income the resources $\rho_t m^{t-1}/(1+n)$ that the old agents cannot claim anymore.[19]

Under the standard assumptions made on u and v, the unique solution to problem (8) is characterized by the first order conditions

$$\begin{aligned}\frac{\pi u'(c_0^t)+\tilde{\pi}u'(\tilde{c}_0^t)}{\pi v'(c_1^t)+\tilde{\pi}v'(\tilde{c}_1^t)}&=r_{t+1}\\ \frac{\pi u'(c_0^t)+\tilde{\pi}u'(\tilde{c}_0^t)}{\pi v'(c_1^t)}&=\rho_{t+1}\end{aligned} \tag{9}$$

along with the budget constraints in (8) above.[20]

From the constant returns to scale of the production function, at equilibrium capital and labor are remunerated by their marginal productivities so that

$$\begin{aligned}r_{t+1}&=F_K\Big(\frac{k^t}{1+n},1\Big)\\ w_t&=F_L\Big(\frac{k^{t-1}}{1+n},1\Big)\end{aligned} \tag{10}$$

[19]In the case $m_t<0$, the price of the good in terms of the *credit* obtained by agent t from the issuance of his IOU to the bank becomes infinity (since agent $t-1$ does not sell any amount of the good to cancel a debt that he is not paying back anymore), while terms of the young agents' labor income it adjusts to clear markets given that the old will not put the resources $\rho_t m^{t-1}/(1+n)$ on sale in the market anymore.

[20]Note that the agent's optimal choice determines now not only the overall level of savings k^t+m^t chosen by agent, given w_t, r_{t+1}, and ρ_{t+1}, but also the very composition of the savings portfolio, i.e. k^t and m^t (the system of equilibrium equations (9) along with the budget constraints in (8) reduces in fact to a system of two equations in k^t and m^t). This is in sharp contrast with what happens in the absence of risk, when only the market clearing condition for capital pins down the individual's savings portfolio. In effect, for both assets to be held simultaneously they must earn the same return, making the agent indifferent to the composition of his portfolio. Interestingly, it follows from the conditions (9) that, at any competitive equilibrium, the return to money (unproductive savings) has to be necessarily larger than the return to capital (productive savings), that is to say $r_{t+1}<\rho_{t+1}$.The higher real return for monetary savings is clearly a consequence of the fact that money is a riskier asset than capital in this setup, so that it requires to bear a higher return for the agents to be willing to accept it at equilibrium. It may seem surprising at first, since the only productive investments here are, at least directly, those made in terms of capital. It is worth stressing, at any rate, that money (or for the same token public debt, pay-as-you-go pension systems, or any other intergenerational transfers mechanism) is an unproductive investment only in a strictly direct technological and physical sense, since by allowing to support higher levels of net output at equilibrium, it cannot be deemed socially unproductive, if only because it allows to implement a better steady state. Social arrangements or institutions thus certainly matter.

must hold at every period t as well. Moreover, since the population grows at a rate $n > -1$, from the agents budget constraints follows that at equilibrium, whether the money bubble bursts or not at any given period t, it holds[21]

$$c_0^t + \frac{c_1^{t-1}}{1+n} + k^t + m^t = F_L\Big(\frac{k^{t-1}}{1+n}, 1\Big) + F_K\Big(\frac{k^{t-1}}{1+n}, 1\Big)\frac{k^{t-1}}{1+n} + \frac{\rho_t m^{t-1}}{1+n} \tag{11}$$

from where the feasibility condition is equivalent to

$$\rho_t \frac{m^t}{m^{t+1}} = 1+n. \tag{12}$$

At a competitive equilibrium steady state it then necessarily holds that

$$\rho_t = 1+n \tag{13}$$

and letting the per worker steady state demand for real balances be m, the profile of contingent consumptions and monetary and capital savings of a competitive equilibrium steady state $(c_0^e, c_1^e, \tilde{c}_0^e, \tilde{c}_1^e, k^e, m^e)$ is characterized by satisfying the equations

$$\begin{aligned}
&\frac{\pi u'(c_0) + \tilde{\pi} u'(\tilde{c}_0)}{\pi v'(c_1) + \tilde{\pi} v'(\tilde{c}_1)} = F_K\Big(\frac{k}{1+n}, 1\Big) \\
&\frac{\pi u'(c_0) + \tilde{\pi} u'(\tilde{c}_0)}{\pi v'(c_1)} = 1+n \\
&c_0 + k + m = F_L\Big(\frac{k}{1+n}, 1\Big) \\
&\tilde{c}_0 + k = F_L\Big(\frac{k}{1+n}, 1\Big) \\
&c_1 = F_K\Big(\frac{k}{1+n}, 1\Big)k + (1+n)m \\
&\tilde{c}_1 = F_K\Big(\frac{k}{1+n}, 1\Big)k
\end{aligned} \tag{14}$$

It follows from the existence of a unique competitive equilibrium steady state with sure money (i.e. with $\tilde{\pi} = 0$), namely the first-best steady state (c_0^*, c_1^*, k^*), that there exists a unique monetary competitive equilibrium steady state $(c_0^e, c_1^e, \tilde{c}_0^e, \tilde{c}_1^e, k^e, m^e)$ when money risks becoming worthless as long as the probability $\tilde{\pi}$ of this event is small enough, as the next proposition establishes.

[21] In the event the bubble bursts the condition (11) holds for $\tilde{c}_0^t$ and $\tilde{c}_1^{t-1}$.

Proposition 2. *The Diamond* (1965) *overlapping generations economy with production has a unique competitive equilibrium steady state if the probability* $\tilde{\pi} \in [0,1)$ *of money becoming worthless is small enough.*

Proof. The system of equations characterizing a competitive equilibrium steady state, for any $\tilde{\pi}$, is

$$\begin{aligned} \pi\big[u'(c_0) - F_K\Big(\frac{k}{1+n},1\Big)v'(c_1)\big] + \tilde{\pi}\big[u'(\tilde{c}_0) - F_K\Big(\frac{k}{1+n},1\Big)v'(\tilde{c}_1)\big] &= 0 \\ \pi\big[u'(c_0) - (1+n)v'(c_1)\big] + \tilde{\pi}u'(\tilde{c}_0) &= 0 \\ c_0 + k + m - F_L\Big(\frac{k}{1+n},1\Big) &= 0 \\ \tilde{c}_0 + k - F_L\Big(\frac{k}{1+n},1\Big) &= 0 \\ c_1 - F_K\Big(\frac{k}{1+n},1\Big)k - (1+n)m &= 0 \\ \tilde{c}_1 - F_K\Big(\frac{k}{1+n},1\Big)k &= 0 \end{aligned} \tag{15}$$

When $\tilde{\pi} = 0$ the system (15) has the unique first-best steady state (c_0^*, c_1^*, k^*) solution to (2) as solution, and in fact by Proposition 1 as the only solution, m being the level of monetary savings $m^* = \frac{c_1^*}{1+n} - k^*$ implementing the first-best steady state as a competitive equilibrium when $\frac{c_1^*}{1+n} - k^* \geq 0$,[22] and $\tilde{c}_0$ and $\tilde{c}_1$ being variables determined by k^* simultaneously but irrelevant in this case.[23]

The Jacobian of the system in (15) is, with obvious notation,[24]

[22]In the case $\frac{c_1^*}{1+n} - k^* < 0$, this is the amount young agents borrow from the infinitely lived intermediary in the credit market, to be repaid with an interest n when old.

[23]When $\tilde{\pi} = 1$ the system has as solution for $(\tilde{c}_0, \tilde{c}_1, k)$ the competitive equilibrium steady state in the absence of money $(\bar{c}_0, \bar{c}_1, \bar{k})$ solution to the first, fourth, and sixth equations in (14), or equivalently with money known to be worthless next period and hence worthless today (with c_0, c_1, m required to satify the third and fifth equations in (15) at indeterminate but irrelevant levels).

[24]That is to say, $u'' = u''(c_0)$, $\tilde{u}'' = u''(\tilde{c}_0)$, $v'' = v''(c_1)$, $\tilde{v}'' = v''(\tilde{c}_1)$, $F_K = F_K\big(\frac{k}{1+n},1\big)$ $F_{KK} = F_{KK}\big(\frac{k}{1+n},1\big)$ $F_{LK} = F_{LK}\big(\frac{k}{1+n},1\big)$ at a solution $c_0, \tilde{c}_0, c_1, \tilde{c}_1, k, m$ to the system.

$$\begin{pmatrix} \pi u'' & \tilde{\pi}\tilde{u}'' & -\pi F_K v'' & -\tilde{\pi} F_K \tilde{v}'' & 0 & -[\pi v' + \tilde{\pi}\tilde{v}']F_{KK}\frac{1}{1+n} \\ \pi u'' & \tilde{\pi}\tilde{u}'' & -\pi(1+n)v'' & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 - F_{LK}\frac{1}{1+n} \\ 0 & 1 & 0 & 0 & 0 & 1 - F_{LK}\frac{1}{1+n} \\ 0 & 0 & 1 & 0 & -(1+n) & -F_K - F_{KK}\frac{1}{1+n} \\ 0 & 0 & 0 & 1 & 0 & -F_K - F_{KK}\frac{1}{1+n} \end{pmatrix} \quad (16)$$

and it is regular for $\tilde{\pi} = 0$. In effect, the last four rows are clearly linearly independent, and the first two rows can be combined linearly with the last four rows in order to turn them into (1) a block of zeros in their first four columns, and (2) in their last two columns the block, for $\tilde{\pi} = 0$,

$$\begin{pmatrix} -u'' - F_K(1+n)v'' & -(F_K + F_{KK}\frac{1}{1+n})(F_K v'' + u''\frac{1}{1+n}) - v' F_{KK}\frac{1}{1+n} \\ -u'' - (1+n)^2 v'' & -(F_K + F_{KK}\frac{1}{1+n})((1+n)v'' + u''\frac{1}{1+n}) \end{pmatrix}. \quad (17)$$

Since for $\tilde{\pi} = 0$, k is the only k^* such that $F_K\left(\frac{k}{1+n}, 1\right) = 1 + n$, then this block is regular, and therefore so is the entire Jacobian. By the continuity of the determinant of the Jacobian in (16)[25] with respect to $\tilde{\pi}$, it is still regular for any $\tilde{\pi} < \varepsilon$ and some $\varepsilon > 0$ small enough. As a consequence, the existence and uniqueness of the solution to the system with $\tilde{\pi} = 0$ (i.e. the existence and uniqueness of the first-best steady state) implies, by the Implicit Function Theorem, the existence and local uniqueness of the competitive equilibrium steady state for all $\tilde{\pi} \in [0, \varepsilon)$. Moreover, the local uniqueness is global for all $\tilde{\pi} \in [0, \varepsilon)$ since otherwise either the correspondence from $\tilde{\pi}$ to the set solutions to (15) is not locally a function at $\tilde{\pi} = 0$ (which we just proved it is), or it is not upper hemicontinuous, which it is as well.[26] Q.E.D.

[25]Which is clearly equal to the determinant of the block (17), i.e.

$$-\left[(1+n)^2 v'' + u''\right] v' F_{KK}\frac{1}{1+n} < 0.$$

[26]In effect, since at every point of the graph of the correspondence from $\tilde{\pi}$ to the set solutions to (15) the correspondence is locally a C^1 function of $\tilde{\pi}$ for every $\tilde{\pi} \in [0, \varepsilon)$, then for every sequence $\{\tilde{\pi}_n, (c_0^n, \tilde{c}_0^n, c_1^n, \tilde{c}_1^n, k^n, m^n)\}_{n\in\mathbb{N}}$ within the graph of that correspondence such that $\{\tilde{\pi}_n\}_{n\in\mathbb{N}}$ converges to 0 there exists a convergent subsequence. Since moreover all the left-hand sides in (15) are continuous with respect to all $c_0, \tilde{c}_0, c_1, \tilde{c}_1, k, m$ and $\tilde{\pi}$, then (15) still holds true in the limit of the subsequence when $\tilde{\pi} \to 0^+$, which establishes the upper hemicontinuity at $\tilde{\pi} = 0$.
Incidentally, it can be easily checked that for for $\tilde{\pi} = 1$ the Jacobian is singular. When money is

It is worth noting that the existence result provided in Proposition 2 is a general property that does not depend on the uniqueness of the moneyless steady state $(\bar{c}_0, \bar{c}_1, \bar{k})$ solution to to the first, fourth and sixth equations in (14) with $\tilde{\pi} = 1$, as opposed to the condition for existence provided in Proposition 3 in Weil (1989). In effect, it is established in Weil (1989) that, *if there is a unique steady state for the moneyless economy*, then there exists a competitive equilibrium steady state with a stochastic asset bubble[27] if, and only if,

$$\pi > \frac{F_K(\frac{\bar{k}}{1+n}, l)}{1+n} \tag{18}$$

that is to say, if, and only if, the probability of money losing completely its value is low enough. As a consequence, *if there is a unique steady state for the moneyless economy*, there cannot be a competitive equilibrium steady state with a stochastic asset bubble if

$$F_K(\tfrac{\bar{k}}{1+n}, l) \geq 1 + n \tag{19}$$

that is to say, in the case the *moneyless* competitive equilibrium steady state is dynamically efficient. It turns out this leaves open the question of whether there exist stochastic asset bubbles when there are several steady states (possibly dynamically efficient) of the moneyless economy.

4. The best steady state that "risky" money can buy

Let us consider now the best monetary steady state —i.e. the steady state maximizing the utility of the representative agent under the constraints of using the risky money for intergenerational transfers and remunerating factors by their marginal

worthless, the system (15) has a trivial indeterminacy in c_0, c_1, m.

[27]That reverts to the moneyless steady state once it bursts. The equilibrium conditions in Proposition 3 in Weil (1989) are nonetheless equivalent to those characterizing the recurrent stochastic asset bubbles being considered here, i.e. a profile $(c_0^e, c_1^e, \tilde{c}_0^e, \tilde{c}_1^e, k^e, m^e)$ solution to (14).

productivities. It would be characterized as a solution to

$$
\begin{aligned}
\max_{c_0,c_1,\tilde{c}_0,\tilde{c}_1,k,m} \; & \pi u(c_0) + \tilde{\pi} u(\tilde{c}_0) + \pi v(c_1) + \tilde{\pi} v(\tilde{c}_1) \\
& c_0 + k + m \leq F_L\Big(\frac{k}{1+n}, 1\Big) \\
& \tilde{c}_0 + k \leq F_L\Big(\frac{k}{1+n}, 1\Big) \\
& c_1 \leq F_K\Big(\frac{k}{1+n}, 1\Big) k + (1+n)m \\
& \tilde{c}_1 \leq F_K\Big(\frac{k}{1+n}, 1\Big) k \\
& 0 \leq c_0, c_1, \tilde{c}_0, \tilde{c}_1, k
\end{aligned}
\tag{20}
$$

Thus, the best monetary steady state is a profile $(c_0, c_1, \tilde{c}_0, \tilde{c}_1, k, m)$ satisfying the set of first order conditions and budget constraints

$$
\begin{aligned}
\frac{\pi u'(c_0) + \tilde{\pi} u'(\tilde{c}_0)}{\pi v'(c_1) + \tilde{\pi} v'(\tilde{c}_1)} &= \frac{F_K\Big(\frac{k}{1+n}, 1\Big) + F_{KK}(\frac{k}{1+n}, 1)\frac{k}{1+n}}{1 - F_{LK}(\frac{k}{1+n}, 1)\frac{1}{1+n}} \\
\frac{u'(c_0)}{v'(c_1)} &= 1+n \\
c_0 + k + m &= F_L\Big(\frac{k}{1+n}, 1\Big) \\
\tilde{c}_0 + k &= F_L\Big(\frac{k}{1+n}, 1\Big) \\
c_1 &= F_K\Big(\frac{k}{1+n}, 1\Big) k + (1+n)m \\
\tilde{c}_1 &= F_K\Big(\frac{k}{1+n}, 1\Big) k
\end{aligned}
\tag{21}
$$

Note that, as opposed to what happens at the competitive equilibrium steady state, the impact of savings in terms of capital on the real wage and the return to capital is now taken into account in the first equation in (21) through the changes in the marginal productivities of capital and labor that an increase in saving in capital induces.

It follows from the existence of a unique first-best steady state, that there exists a unique steady state implementable with the risky money that maximizes the representative agent's utility as long as the probability $\tilde{\pi}$ of money becoming worthless is small enough, as the next proposition establishes.

Proposition 3. *The Diamond* (1965) *overlapping generations economy with production has a unique best monetary steady state*[28] *if the probability* $\tilde{\pi} \in (0,1)$ *of money becoming worthless is small enough.*

Proof. The system of equations characterizing the steady state that maximizes the representative agent's utility while remunerating factors by their marginal productivities and using monetary savings to implement intergenerational transfers is (using notation previously introduced)[29]

$$\begin{aligned}\left(1+n+F_{KK}\frac{k}{1+n}\right)\left[\pi u'+\tilde{\pi}\tilde{u}'\right]-\left(F_K+F_{KK}\frac{k}{1+n}\right)\left[\pi v'+\tilde{\pi}\tilde{v}'\right](1+n)=0\\ u'(c_0)-(1+n)v'(c_1)=0\\ c_0+k+m-F_L\left(\frac{k}{1+n},1\right)=0\\ \tilde{c}_0+k-F_L\left(\frac{k}{1+n},1\right)=0\\ c_1-F_K\left(\frac{k}{1+n},1\right)k-(1+n)m=0\\ \tilde{c}_1-F_K\left(\frac{k}{1+n},1\right)k=0\end{aligned} \tag{22}$$

When $\tilde{\pi}=0$ the system has as its only solution the unique first-best steady state (c_0^*, c_1^*, k^*) (m being again the level of per worker monetary savings[30] implementing the first-best steady state as a competitive equilibrium, and $\tilde{c}_0$ and $\tilde{c}_1$ being variables once more determined simultaneously but irrelevant in this case).[31]

[28] That is to say, a unique steady state that maximizes the representative agent's utility *among those implementable through monetary savings* (on top of savings in capital).
[29] That is to say, $u''=u''(c_0)$, $\tilde{u}''=u''(\tilde{c}_0)$, $v''=v''(c_1)$, $\tilde{v}''=v''(\tilde{c}_1)$, $F_K=F_K\left(\frac{k}{1+n},1\right)$ $F_{KK}=F_{KK}\left(\frac{k}{1+n},1\right)$ $F_{LK}=F_{LK}\left(\frac{k}{1+n},1\right)$ at a solution $c_0, \tilde{c}_0, c_1, \tilde{c}_1, k, m$ to the system.
[30] Or IOU's, if negative.
[31] When $\tilde{\pi}=1$ the system determines the choice $(\tilde{c}_0, \tilde{c}_1, k)$ maximizing the representative agent's utility with no intergenerational transfers but remunerating factors by their productivities (with c_0, c_1, m required to satify the second, third and fifth equations in (20) given k).

The Jacobian of the system is

$$\begin{pmatrix} \pi A u'' & \tilde{\pi} A \tilde{u}'' & -\pi B v'' & -\tilde{\pi} B \tilde{v}'' & 0 & C \\ u'' & 0 & -(1+n)v'' & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 - F_{LK}\frac{1}{1+n} \\ 0 & 1 & 0 & 0 & 0 & 1 - F_{LK}\frac{1}{1+n} \\ 0 & 0 & 1 & 0 & -(1+n) & -F_K - F_{KK}\frac{1}{1+n} \\ 0 & 0 & 0 & 1 & 0 & -F_K - F_{KK}\frac{1}{1+n} \end{pmatrix} \tag{23}$$

with the notation introduced previously, and letting $A = 1 + n + F_{KK}\frac{k}{1+n}$, $B = \Big(F_K + F_{KK}\frac{k}{1+n}\Big)(1+n)$, and

$$\begin{aligned} C = \big(F_{KKK}\frac{k}{(1+n)^2} &+ F_{KK}\frac{1}{1+n}\big)\big[\pi u' + \tilde{\pi}\tilde{u}'\big] \\ &- \big(F_{KKK}\frac{k}{(1+n)^2} + F_{KK}\frac{2}{1+n}\big)\big[\pi v' + \tilde{\pi}\tilde{v}'\big](1+n) \end{aligned} \tag{24}$$

This Jacobian is regular for $\tilde{\pi} = 0$. In effect, the last four rows are clearly linearly independent, and the first two rows can be combined linearly with the last four rows to turn them into (1) a block of zeros in their first four columns, and (2) in their last two columns the block, when $\tilde{\pi} = 0$,

$$\begin{pmatrix} -Au'' - (1+n)Bv'' & C - (\frac{1}{1+n}Au'' + Bv'')(F_K + F_{KK}\frac{1}{1+n}) \\ -u'' - (1+n)^2 v'' & -(\frac{1}{1+n}u'' + (1+n)v'')(F_K + F_{KK}\frac{1}{1+n}) \end{pmatrix} \tag{25}$$

since for $\tilde{\pi} = 0$, $F_K = 1 + n$. But since $u' = (1+n)v'$ and, for $\tilde{\pi} = 0$, then $(1+n)A = B$ and hence

$$\begin{aligned} C = \Big(F_{KKK}\frac{k}{(1+n)^2} + F_{KK}\frac{1}{1+n}\Big)\big(u' - (1+n)v'\big) - F_{KK}\frac{1}{1+n}v'(1+n) \\ = -F_{KK}v' > 0 \end{aligned} \tag{26}$$

so that the block (25) is regular, and therefore so is the entire Jacobian. By the continuity of the determinant of the Jacobian in (23)[32] with respect to $\tilde{\pi}$, it is still

[32] Which is clearly equal to the determinant of the block (25), i.e.

$$-\big[(1+n)^2 v'' + u''\big]F_{KK}v' < 0.$$

regular for any $\tilde{\pi} < \varepsilon$ and some $\varepsilon > 0$ small enough. As a consequence, the existence and uniqueness of the solution to the system with $\tilde{\pi} = 0$ (i.e. the existence and uniqueness of the first-best steady state) implies, by the Implicit Function Theorem, the existence and local uniqueness of the best monetary steady state for all $\pi \in [0, \varepsilon)$. Moreover, the local uniqueness is global for all $\tilde{\pi} \in [0, \varepsilon)$ since otherwise either the correspondence from $\tilde{\pi}$ to the set solutions to (22) is not locally a function at $\tilde{\pi} = 0$ (which we just proved it is), or it is not upper hemicontinuous, which it is as well.[33] Q.E.D.

Comparing equations (14) characterizing the laissez-faire competitive equilibrium steady state with risky money, to equations (21) characterizing the best steady state with risky money, it becomes apparent that the two steady states do not coincide, as the next proposition establishes.

Proposition 4. *In the Diamond* (1965) *overlaping generations economy with production, the best monetary steady state is not a competitive equilibrium outcome under laissez-faire if the probability $\tilde{\pi}$ of money becoming worthless is positive.*

Proof. In effect, should the best monetary steady state $(c_0, \tilde{c}_0, c_1, \tilde{c}_1, k, m)$ solution to (21) coincide with a competitive equilibrium steady state $(c_0^e, \tilde{c}_0^e, c_1^e, \tilde{c}_1^e, k^e, m^e)$ solution to (14), then from the second equation in both (14) and (21) it would hold

$$\frac{\pi u'(c_0) + \tilde{\pi} u'(\tilde{c}_0)}{\pi v'(c_1)} = 1 + n = \frac{u'(c_0)}{v'(c_1)} \tag{27}$$

from which $\frac{\tilde{\pi} u'(\tilde{c}_0)}{\pi v'(c_1)} = 0$ would follow, which under the assumptions made on u and v cannot be. Q.E.D.

As a consequence of Proposition 4, the laissez-faire competitive equilibrium steady state is not the best steady state in which the economy can be even when the risky money is the only in asset, besides capital, in which the agents can save. There is nonetheless an active fiscal policy allowing to decentralize the best monetary steady state when money is risky as a competitive equilibrium, as the next section establishes.

[33]For the same reasons as in Proposition 2. Incidentally, It can be easily checked that for for $\tilde{\pi} = 1$ the Jacobian is singular.

5. Implementing the best monetary steady state through taxes and transfers

Assume the government announces at each period t that it will

(1) tax linearly the capital returns at $t+1$ at a rate

$$\tau_t = 1 - \frac{1+n}{F_K\left(\frac{k^{t-1}}{1+n},1\right)} \cdot \frac{F_K\left(\frac{k^{t-1}}{1+n},1\right) + F_{KK}(\frac{k^{t-1}}{1+n},1)\frac{k^{t-1}}{1+n}}{(1+n) + F_{KK}(\frac{k^{t-1}}{1+n},1)\frac{k^{t-1}}{1+n}} \tag{28}$$

(2) subsidize returns from monetary savings at $t+1$ at a rate

$$\mu_t = \frac{\tilde{\pi} u'(F_L\left(\frac{k^{t-1}}{1+n},1\right) - k^{t-1})}{\pi v'(F_K\left(\frac{k^{t-1}}{1+n},1\right)k^{t-1} + (1+n)m^{t-1})}\frac{1}{1+n} \tag{29}$$

(3) transfer to agent t at $t+1$ the following lump sum, depending on whether agent t's monetary savings have lost or not completely their value respectively

$$\begin{aligned}\tilde{T}_t &= \tau_t F_K\Big(\frac{k^{t-1}}{1+n},1\Big)k^{t-1} \\ T_t &= \tau_t F_K\Big(\frac{k^{t-1}}{1+n},1\Big)k^{t-1} - \mu_t(1+n)m^{t-1}\end{aligned} \tag{30}$$

Then the representative agent's problem becomes

$$\begin{aligned}\max_{c_0^t,\tilde{c}_0^t,c_1^t,\tilde{c}_1^t,k^t,m^t} \quad & \pi u(c_0^t) + \tilde{\pi} u(\tilde{c}_0^t) + \pi v(c_1^t) + \tilde{\pi} v(\tilde{c}_1^t) \\ & c_0^t + k^t + m^t \leq w_t \\ & \tilde{c}_0^t + k^t + m^t \leq w_t + \frac{\rho_t m^{t-1}}{1+n} \\ & c_1^t \leq (1-\tau_t)r_{t+1}k^t + (1+\mu_t)\rho_{t+1}m^t + T_t \\ & \tilde{c}_1^t \leq (1-\tau_t)r_{t+1}k^t + \tilde{T}_t \\ & 0 \leq c_0^t, \tilde{c}_0^t, c_1^t, \tilde{c}_1^t, k^t\end{aligned} \tag{31}$$

and the new equilibrium conditions are the first-order conditions

$$\begin{aligned}\frac{\pi u'(c_0^t) + \tilde{\pi} u'(\tilde{c}_0^t)}{\pi v'(c_1^t) + \tilde{\pi} v'(\tilde{c}_1^t)} &= (1-\tau_t)r_{t+1} \\ \frac{\pi u'(c_0^t) + \tilde{\pi} u'(\tilde{c}_0^t)}{\pi v'(c_1^t)} &= (1+\mu_t)\rho_{t+1}\end{aligned} \tag{32}$$

along with the budget constraints in (31) and the remunerations to factors by their marginal productivities. Thus the competitive equilibrium steady state is now characterized by

$$
\begin{aligned}
\frac{\pi u'(c_0)+\tilde{\pi}u'(\tilde{c}_0)}{\pi v'(c_1)+\tilde{\pi}v'(\tilde{c}_1)} &= (1-\tau)F_K\Big(\frac{k}{1+n},1\Big)\\
\frac{\pi u'(c_0)+\tilde{\pi}u'(\tilde{c}_0)}{\pi v'(c_1)} &= (1+\mu)(1+n)\\
c_0+k+m = F_L\Big(\frac{k}{1+n},1\Big)&\\
\tilde{c}_0+k = F_L\Big(\frac{k}{1+n},1\Big)&\\
c_1 = (1-\tau)F_K\Big(\frac{k}{1+n},1\Big)k+(1+\mu)(1+n)m+T&\\
\tilde{c}_1 = (1-\tau)F_K\Big(\frac{k}{1+n},1\Big)k+\tilde{T}.&
\end{aligned}
\tag{33}
$$

Since it follows from the policy above that, at a steady state,

$$
(1-\tau)F_K\Big(\frac{k}{1+n},1\Big) = (1+n)\frac{F_K\Big(\frac{k}{1+n},1\Big)+F_{KK}(\frac{k}{1+n},1)\frac{k}{1+n}}{(1+n)+F_{KK}(\frac{k}{1+n},1)\frac{k}{1+n}} \tag{34}
$$

$$
(1+\mu)(1+n) = \frac{\tilde{\pi}u'(\tilde{c}_0)}{\pi v'(c_1)}+(1+n) \tag{35}
$$

and

$$
\begin{aligned}
\tilde{T} &= \tau F_K\Big(\frac{k}{1+n},1\Big)k\\
T &= \tau F_K\Big(\frac{k}{1+n},1\Big)k-\mu(1+n)m
\end{aligned}
\tag{36}
$$

then it is straightforward to check that the systems (33) and (21) are the same one, and therefore the competitive equilibrium steady state under this policy provides the representative agent the same contingent consumptions profile, savings, and portfolio as the best steady state implementable through the risky money. This result is summarized in the following proposition.

Proposition 3. *In the Diamond* (1965) *overlapping generations economy with production, if money risks becoming worthless with a positive but small enough proba-*

bility π, then the best monetary steady state is the unique competitive equilibrium steady state under the following policy

(1) *tax capital returns at $t+1$ at a rate*

$$\tau_t = 1 - \frac{1+n}{F_K\left(\frac{k^{t-1}}{1+n}, 1\right)} \cdot \frac{F_K\left(\frac{k^{t-1}}{1+n}, 1\right) + F_{KK}(\frac{k^{t-1}}{1+n}, 1)\frac{k^{t-1}}{1+n}}{(1+n) + F_{KK}(\frac{k^{t-1}}{1+n}, 1)\frac{k^{t-1}}{1+n}} \tag{37}$$

(2) *subsidize monetary savings returns at $t+1$ at a rate*

$$\mu_t = \frac{\tilde{\pi}u'(F_L\left(\frac{k^{t-1}}{1+n}, 1\right) - k^{t-1})}{\pi v'(F_K\left(\frac{k^{t-1}}{1+n}, 1\right)k^{t-1} + (1+n)m^{t-1})} \frac{1}{1+n} \tag{38}$$

(3) *transfer to each agent of generation t at $t+1$ the lump sum $\tilde{T}_t$ or T_t defined as*

$$\begin{aligned} \tilde{T}_t &= \tau_t F_K\left(\frac{k^{t-1}}{1+n}, 1\right)k^{t-1} \\ T_t &= \tau_t F_K\left(\frac{k^{t-1}}{1+n}, 1\right)k^{t-1} - \mu_t(1+n)m^{t-1} \end{aligned} \tag{39}$$

depending on, respectively, whether agent t's monetary savings have lost or not completely their value at $t+1$.

A few remarks are in order at this point. First note that the tax and transfers policy announced at any period t is defined as a function of the capital savings decided by the generation born at $t-1$. Therefore, the policy is defined in terms of information that is both known at the time of its announcement and not manipulable by the agents to which it applies. Second, by construction the government does not incur in any deficit or surplus at the steady state, since the amount raised by the tax in a distortionary way is given back as a lump sum to the same agents in the same period.

Note finally that whether the returns to capital need to be taxed (as opposed to subsidized), i.e. whether $\tau > 0$, hinges on the following inequality holding true

$$\frac{F_K\left(\frac{k}{1+n}, 1\right)}{1+n} > \frac{F_K\left(\frac{k}{1+n}, 1\right) + F_{KK}(\frac{k}{1+n}, 1)\frac{k}{1+n}}{(1+n) + F_{KK}(\frac{k}{1+n}, 1)\frac{k}{1+n}} \tag{40}$$

at the best monetary steady state allocation $(c_0, \tilde{c}_0, c_1, \tilde{c}_1, k, m)$, which —given that $(1+n) + F_{KK}(\frac{k}{1+n}, 1)\frac{k}{1+n} > 0$ for any Cobb-Douglas or CES technology with an elasticity of substitution smaller than 1—[34] holds if, and only if,

$$1 > \frac{F_K\left(\frac{k}{1+n}, 1\right)}{1+n} \tag{41}$$

or equivalently iff $k^* < k$, where k^* is the first-best level of per worker capital savings satisfying $F_K(\frac{k^*}{1+n}, 1) = 1 + n$, and k is the level of capital at the best monetary steady state solving (21). In other words, if at the best monetary steady state the level of capital is higher than at the first-best steady state, then its implementation as a competitive equilibrium requires taxing the return of capital savings. Otherwise the returns to capital need to be subsidized as well. Note, however, that according to Proposition 3 in Weil (1989) this case can only happen if in the absence money the economy has more than one steady state.

6. Concluding remarks

To conclude, and for the sake of completeness, some comments on the usual argument about the implementability of the first-best steady state as a monetary equilibrium with riskless money are in order. Clearly, the implementation of the first-best steady state as a competitive equilibrium typically requires to hold strictly positive amounts of both money (or public debt)[35] and capital. Therefore, in the absence of uncertainty both assets must have the same return at the steady state, so that the agents are indifferent about the composition of their savings portfolio. In other words, the agents' (although not the planner's) choice of the *composition* of their savings portfolio is completely indetermined at the first-best steady state (even if the *level* is not). Thus, although there exists indeed a way to support the

[34]In effect, the right-hand side of the first equation must be positive in (21), i.e.

$$\frac{F_K\left(\frac{k}{1+n}, 1\right) + F_{KK}\left(\frac{k}{1+n}, 1\right)\frac{k}{1+n}}{(1+n) + F_{KK}\left(\frac{k}{1+n}, 1\right)\frac{k}{1+n}} > 0$$

but the numerator is positive for any constant returns to scale Cobb-Douglas technology $F(K, L) = AK^{\alpha}L^{1-\alpha}$ and any CES technolgy $F(K, L) = A[aK^r + (1-a)L^r]^{\frac{1}{r}}$ with $r < 0$ and hence elasticity of substitution $s = \frac{1}{1-r} \in [0, 1)$, so that the denominator is positive as well.

[35]Or the issuance of IOU's by the agents when young if they want to transfer income from their old age to their young age.

first-best steady state using money to place some of the agents' savings, nothing in the model explains why the agents would actually *choose* to place their savings precisely the way that allows to do so. As a matter of fact, nothing *within the model* leads the agents to choose the composition of the portfolio the implements the first-best.[36]

Note finally that this indeterminacy is not of the same nature than, say, that of the production plan at equilibrium of a firm with a constant returns to scale technology. In effect, in that case it is widely assumed that production just adjusts to a demand that is well determined by prices. Nevertheless, in the case of the choice savings portfolio at the first-best steady state, on the two sides of the money market sits the same representative agent, and *both* sides face then the same indeterminacy. As a result, there is no well-determined side of the market in this case that is able to anchor an indeterminate side. This points to the existence of an element, missing from the model, that would explain why the agents would choose to save exactly the right amounts of capital and money that allow to put the economy on the best possible steady state. Interestingly enough, the introduction of the risk of money losing completely value, even if this risk is minimal, helps to pin down completely both the level as well as the composition of the agents' savings portfolio at the steady state.

A number of issues clearly remain to be addressed in this setup as, for instance, the dynamics out of the steady state, the cost of moving to such a steady state, or the endogeneization of the probability of breakdown of the intergenerational transfers mechanism. These and other issues are left for further research in the future.

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