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Roberto Leombruni

Abstract

This paper presents some artificial stylised facts emerging in a simulated contestable market where firms interact with each other in taking their stay or go decision. I use nearly zero-intelligence firms: no optimisation is considered, and all the firms sell at a fixed price an equal quantity of the good. The entry of new firms is triggered by the overall profitability of the market, measured by the spread between the average rate of profit and the interest rate. The exit decision is modelled *via* a mean field effect, to take into account in the decision process both the performance of the individual firm, and the information about the profitability of the market that can be abduced looking at the stay or go decision of the other firms. Financial requirements of production are considered, with a spread between creditor and debtor interest rates. The model is simulated with an ACE approach, using the Swarm libraries released by the Santa Fe Institute.

1 Introduction

Empirical literature in industrial dynamics has underlined since a long time many stylised facts relating firms distribution characteristics — such as the different dynamic path followed by large and small firms, a right skewed size distribution, the presence of an equilibrium positive firms turnover — that cannot be adequately tackled within the Representative Agent (RA) framework. As it seems, real markets are characterised by a ”tremendous within-industry heterogeneity” that is not cancelled out by the ’selection of the fittest’, and that requires a specific attention (Haltiwanger 1997).

The weaknesses of the RA framework have been pointed out also from a methodological point of view, and the effects that heterogeneity and interaction among individuals can have on macrodynamics — particularly in presence of market imperfections and strategic complementarities — have become a growing field of investigation (Kirman 1992 , Gallegati and Kirman 1999 , Delli Gatti et al. 2000).

A natural and sometimes ”radical” way of coping with heterogeneity is by means of agent-based simulations. We may see AB simulations as just a different path to simplification. Getting away from a 1:1 map of the world, the mainstream way is to model markets with an over simplified structure, and populate them with few categories of super-rational perfectly informed agents — some times capable of solving even uncomputable tasks. The opposite path is followed by AB practitioners, who usually endow their agents with just limited information and bounded rationality, and spend their degrees of freedom allowing for heterogeneity, learning, interaction and so on. The most radical way of doing this is to adopt ”zero-intelligence” (ZI) agents, and look whether their interaction in a well defined market microstructure give rise to macrobehaviours that tend to replicate those predicted by models with rational and informed agents (Gode and Sunder 1993 and 1997, Terna 1998, Mirowski and Somefun 2000). The spirit is well synthesised by Epstein: ”The issue is not how much rationality there is (at the micro level), but how little is enough to generate the macroequilibrium” (Epstein 1999).

I adopted this kind of modelling to show how ZI-agents can be useful in the investigation of industrial dynamics; namely, in the study of the relations between entry-exit of firms in/out of a contestable market, their financial position, and business fluctuations. The adoption of the ZI hypothesis allowed me to show how the free entry-exit of firms, together with heterogeneity in their financial position, are sufficient hypotheses to generate rather interesting aggregate dynamics, and to reproduce some stylised facts pointed out by the empirical literature on this topic. Among the artificial stylised facts produced running simulations of the model, there is a right skewed distribution for the equity base, attributable mainly to a composition effect; a long run positive firms turnover, due to a positive probability of exiting the market also during expansions; business cycles, due to the interaction component of the decision taken by the firms.

In next two sections, I briefly review the literature of interest. In the following, I present the algebra of the model, and a qualitative analysis of its dynamics with no heterogeneity among firms. Some other results on the dynamics emerging are given in section four, where I simulate the same base model. The effects of heterogeneity are then studied, putting idiosyncratic shocks on the price at which firms are selling their good. Some concluding remarks will follow.

2 Main empirical findings

The issue of firms' demography came to the attention of the scholars at the beginning of the Thirties, with the seminal work of Robert Gibrat (1931). His goal was to explain the skew distributions that could be observed in many contexts, among which in firm size in manufacturing industries. Gibrat firstly observed that his data fit well with a lognormal distribution. To ”generate” such form, he assumed a linkage between a firm's current size and his rate of growth: Namely, he proposed that a firm's absolute growth were a normally distributed random variable, whose mean was *proportional* to his actual size. In other words, that a firm *rate* of growth were a normally distributed random variable with mean independent of the firm's current size; the so called ”Law of proportionate effect” (see the survey in Schmalensee 1989).

After about two decades of but little research on this topic, starting from the late Fifties many empirical studies essentially confirmed Gibrat's law: At least on average, there seemed to be no relations between firms' size and their proportional rate of growth

(Hart and Prais 1956; Simon and Bonini 1958; Hymer and Pashigian 1962; Ijiri and Simon, 1977). The evidence on the lognormal shape of the distribution, however, was less sound; the ”main” fact appeared to be a right skewed distribution, whichever the underlying functional form.

These early investigations on the Gibrat's law had a severe limit in the data sets available at the time. In the last decades, researchers in a number of countries gained access to longitudinal data sets on business units — mainly of administrative source. It has been possible, then, to trace individual data on entries, exits, and life trajectories of firms, and a great deal of stylised facts has been produced with a greater detail. John Sutton (1997) and Richard Caves (1998) survey the many researches that have been produced on the ”Gibrats legacy” and the most recent findings about firms' demography. As regards Italy, an up to date investigation can be found in Lotti and Santarelli (2001).

Many of them point to a greater heterogeneity of behaviours. As regards the relations between growth rates and firms' size, the main refinement came from the analysis of small firms life trajectories. Beyond the average similarities between small and large firms' proportional growth, the small ones revealed a higher probability of going bankrupt; for those surviving, on the other hand, we usually observe higher and more variable growth rates than large firms (Evans 1987; Dunn and Samuelson 1988; early evidence in this direction in Mansfield, 1962).

For the purpose of the background knowledge to next section, the main stylised facts worth reminding about the different behaviour of small and large firms are the following (see again Caves 1998):

- the dependence of firms' growth rates on firm age and size;
- the negative relation between exit probabilities and firm size;
- the relative small size of firms' entering;
- the heteroskedasticity of growth rate variance with firm size;

To these, it should be added somewhat a weakening of the same findings, namely, the importance of idiosyncratic factors in explaining the overall firm performance (Haltiwanger 1997; Contini-Revelli 1992).

Another clear cut evidence emerged about the role of inflows and outflows of firms in business fluctuations. The first datum to cite relates with the co-movements of net business formation (entries minus exits) with gross national product. Chatterjee and Cooper (1993) reported for the United States a correlation between quarterly net business formation and gnp variations in 1955:1–1983:4 of 54%. For Italy, Novarese (2001) reports a correlation between yearly net business formation in the private sector and gnp growth in the years 1984–1998 of about 80%.

Secondly, we observe a positive turnover of firms also in ”equilibrium” conditions; that is, in every market, even when the stock of firms is roughly constant, we usually observe important inflows and outflows of firms. For Italy, in the years 1984–1998, firms' turnover (entry rates plus exit rates) as computed with the data of Business Registry was on average 15%; this figure rise to 19% using the Social Security data (Novarese 2001). As for as the impact of this flows on growth, Rajan and Zingales (1998) find in a sample of 42 countries that one-third of the growth in industries over the 1980s come from the creation of new firms.

Related to these evidence are the findings on the role of entry and exit on job creation and destruction. The Oecd estimates shares of job creation and destruction attributable to openings and closures of firms ranging from about 25% for Canada, to 67% in the United States[1]. In Italy, in the years 1984–1998, the same share hovered on 33%, that is, about one out of three new jobs created is attributable to the entry of new firms (Social Security data, Malpede and Cornaglia, 2001).

In the survey studies I cited, Sutton and Caves summarise the many efforts that have been directed in relating these dynamics to many micro- and macroeconomic covariates, among which R&D investments, entry barriers, industry concentration and so on. They devote less attention to the relations between financial variables and firms' demography. Various phenomena in corporate finance seem related to firm size and growth; a reference to some survey studies can be found in Kumar et al. (1999). To my purposes, the most sensible link is that going from firms' financial fragility to business fluctuations. The pathbreaking works on this topic are due to Hyman Minsky (1963, 1982), where a wide-ranging discussion about economic policy is strictly entangled with the analysis of the financial factors that led to the '29 crisis. The microeconometric research relating the ideas there put forth and industry dyamics is at his beginnings, partly because of the lack of data; theoretical investigation produced many contribution, to which I will point to in next section.

3 Modelling strategies on firm demography and business fluctuations

The "classic" view on the relations between firm size, the number of firms populating a market, their entry and their exit in a perfect competition framework can be traced back to Viner (1931). The key assumption is that of a representative firm with a strictly concave long run average cost function. This way, the optimal size of a firm is determined by the efficiency conditions of production, and the equilibrium number of firms is determined comparing this optimal size with the dimension of an exogenous demand. If q^e is the production that minimise variable costs at the level vc^e, the market will then be in equilibrium with a number of firms such that:

$$N^e = D(vc^e)/q^e \tag{1}$$

where $D(p)$ is demand at price p. Firms inflows and outflows, in this framework, are adjustment to the equilibrium, and will be typically driven by the demand side of the model: An expansion of market demand will cause the entry of new firms, a contraction the exit of some incumbent firms. Sheshinski and Dreze (1976) show that with these

[1] Oecd 1994, years from the mid-Eighties to 1991. The high figure for the United States stems (also) from the different data used, relative to plants and not firms.

assumptions another industry-structure implication of demand fluctuations can be the lowering of the average output of the representative firm. As for as dynamics, however, this framework is particularly poor, and its implications stay at odds with the empirical evidence.

The efforts of the literature to reconcile this raw portrait with empirical evidence have been mainly directed in two directions: A greater realism of the theoretical assumptions; a greater accordance of the models' results with the stylised facts on firms' demography. As to the first point, a crucial assumption that has been relaxed is the strict concavity of the cost function, in accordance with the observation that there is usually a wide production range for which variable costs are constant (Simon and Bonini 1958). The relevance of this hypothesis is straightforward: If the production function implies a flat bottom in the variable costs curve the simple mechanism sketched above breaks down, since a change of market demand can be absorbed (at least in part) by the active firms; a conclusion that seems in accordance with empirical research (Rajan and Zingales 1998). In this case, therefore, we can no more determine an equilibrium number of firms (at best an equilibrium range can be derived), and the qualitative assessment of market dynamics and firm flows becomes even looser than in Viner's case.

Keeping apart the realism of assumptions, the most challenging issue were the construction of models with aggregate implications more in accordance with the stylised facts that the empirical literature produced in the years. The contrast between the skew size distribution of firms in real markets and the standard model with N equal firms was firstly tackled by means of stochastic models, and various hypotheses about the random process guiding firm growth were tested (a survey in Steindl 1965).

This family of models has been criticised for its 'purely statistical' nature. This notwithstanding, they are in principle reconcilable with a flat-cost curve framework. If the microfoundations of incumbent firms' growth can be considered unsatisfactory, their equilibrium outcomes can be justified by models like Simon and Bonini's, and they generate aggregate results in accordance with the early stylised facts produced on firms' demography.

As the empirical findings gained details, however, this modelling strategy revealed unsatisfactory. Tackling with an equilibrium positive firm turnover, for instance, requires a more radical relaxing of the representative agent hypothesis: If all firms were characterised by an equal minimum of the long run average cost curve, a shock moving a market out of the equilibrium would imply just one-way flows, and when the equilibrium is established we should observe no entries and exits. The same can be said about the different growth paths followed by young and old firms.

The efforts to better fit the data, then, involved putting heterogeneity in some firm level characteristics, as in Lucas (1978), that considered different managerial abilities; or in the studies that considered differences in R&D and productivity levels (a most comprehensive investigation on this topics can be found in Sutton 1998).

The first attempt to give a comprehensive theoretical foundation of all aspects of firm's mobility in a competitive market structure were given in a pathbreaking paper by Boyan Jovanovic (1982). He proposed an evolutionary model of "noisy" selection, where firms are assumed heterogeneous with respect to their productivity, but uncover their true efficiencies through a Bayesian learning process. This give rise to patterns of entry, growth, and exit, that accounted for many of the departures from the proportional growth law, among which an higher level and variability in the growth rates of younger firms, a positive relation between firm age and size, and a positive relation between market profitability and concentration.

Individual productivity, in Jovanovic' model, does not change over time, and the selection ends up in an equilibrium in which there are no inflows and outflows of firms. Hopenhayn (1992), extends his results allowing for a stochastic evolution in the productivity level. The framework is the same: A population of firms with perfect foresight on prices and demand level, performing an intertemporal profit maximisation. He derives a limit distribution that adds to the main results of Jovanovic a positive equilibrium firms turnover. The dynamic behaviour of the model, however, is limited to a comparative statics analysis.

In the last decades, however, the representative agent framework has been questioned not only in its ”strongest” version, i.e. when the assumption is that realistic macrobehaviour can be obtained ignoring the heterogeneity of agents, but also when the heterogeneity, although considered, is managed via a massive set of unrealistic assumptions imposed to derive an exact microfoundation to the aggregate behaviour; in some way, cancelling out most of the potential effects that heterogeneity can have on the dynamics (Martel 1996; Kirman 1992). The relaxing of this (apparently) ”weak” RA hypothesis characterises a wide literature on the effects that heterogeneity and interaction among individuals can have on the macrodynamics in presence of market imperfections and strategic complementarities (Gallegati and Kirman 1999).

The relevance of a sounder modelling of heterogeneity and interaction for business dynamics has been pointed out to the industrial organisation literature in many contributions focusing on firms' financial fragility. Building on the seminal works of Hyman Minsky, many authors developed these ideas focusing on how firms' financial fragility and the presence of bankruptcy costs can shape macroeconomic behaviour (see for instance Greenwald and Stiglitz (1988, 1993), Bernanke and Gertler (1989, 1990), Kyiotaki and Moore (1997)). A feature common to these contribute is the adoption of a Dixit-Stiglitz model of product differentiation, and the assumption of heterogeneity in firms' equity base level. Delli Gatti and others (2000), built an explicit link between the business fluctuations emerging in these family of models, the evolution in the distribution, and the firms' inflows and outflows.

A convenient way of tackling with H&I is by means of a class of models developed within statistical mechanics to study the aggregate dynamics of particle systems. The earliest example of the use of statistical mechanics in the economic domain dates back to Föllmer (1974), and has been given a major impulse by the the seminal works of Brock and Durlauff on social interaction and binary choice models (see for instance Brock and Durlauff 2000).

The adoption of such tools in the field of industrial dynamics is rather at its beginning. An and Kiefer (1995), Cowan and Cowan (1998) and Dalle (1997) analyse processes of technology adoption in presence of local and global externalities. Ozman (2000) investigate the clustering in R&D activity.

4 The model

The model is an extension of a previous work (see Leombruni et al. 2001), where we introduced nearly zero-intelligence (ZI) firms to study industry dynamics. The use of ZI-agents, allowed us to derive a sort of benchmark for the behaviour of a market with free entry exit, linear production costs, and equity rationing.

There, the entry exit decision were essentially external to the firms: given a positive (negative) profitability of the market - measured by the spread between the interest rate of a secure asset and the average profit rate - a certain number of firms were driven into (out of) the market.

Here, we give back to the firms a bit of intelligence, to model the presence of local and dispersed knowledge on the market profitability.

On one side, potential entrants have access just to price signals, so that entries are still driven by the presence of more than normal profits.

On the other side, incumbents firms have at their disposal local knowledge on the market profitability. This information, however, is dispersed, and has to be extracted looking at the behaviour of the other competitors. The stay or go decision, then, is modelled assuming non-price interaction among firms: each of them will make its choice considering both their own performance, an the signals on the market profitability that they have looking at the stay or go decision taken by its competitors.

4.1 Firms and households

The demand side of the model consists of a constant and exogenous income Y, which is entirely spent by households to buy equal quantities of the goods produced by the incumbent firms. The price too is given, and is normalised to one plus an idiosyncratic shock identically and indipendently distributed across time and firms.

The N incumbent firms face fixed unitary (production) costs α. The ith firm's production is financed by means of its equity base A_i, and the eventual negative slack between equity and the financial requirements yields a cost of r times the slack. This is equivalent to assuming a spread between creditor and debtor r, the former being set to zero.

Each firm then will be characterised by the following profit equation:

$$\Pi_i = \frac{(1+\varepsilon_i-\alpha)Y}{N} - r\left(\frac{\alpha Y}{N} - A_i\right) \qquad \text{if } \frac{\alpha Y}{N} - A_i > 0 \tag{2a}$$

$$\Pi_i = \frac{(1+\varepsilon_i-\alpha)Y}{N} \qquad \text{otherwise} \tag{2b}$$

Firms remunerate their shareholders at the rate r, while the excess (shortfall) of the profit over the dividend increases (decreases) the equity base. Therefore, the motion equation of the equity of ith firm will be the following:

$$\begin{aligned} \Delta A_i &= \Pi_i - rA_i \\ &= A_i(\pi_i - r) \end{aligned} \tag{3}$$

with $\pi_i \equiv \Pi_i/A_i$.

To study the overall behaviour of our market, we will neglect for a while the idiosyncratic shocks on prices, and consider separately the dynamics of A and N. For the sake of simplicity, we'll also treat N as continuous.

4.2 Equity base equilibrium

Holding N constant, the equilibrium condition $\Delta A_i = 0$ is reached when

$$\pi_i = r \tag{4}$$

It can be shown that, if the condition $\alpha(1+r) < 1$ holds[2], equation (4) implies also no liability for the firm, so we can substitute (2b) into it to obtain the following equilibrium value of A:

$$A = \frac{Y(1-\alpha)}{rN} \tag{5}$$

If we draw equation (5) against N, we have the hyperbolic "Normal profits curve" (NPC) that identifies all the infinite couples (A, N) in which $\Delta A = 0$ (cp. figure 2). When any firm has an equity base lower than that of the NPC, the excess profit will accumulate until the equilibrium value is reached, and similarly in the opposite case. As a consequence, in equilibrium any initial heterogeneity in the equity wipes out.

4.3 Firms' turnover equilibrium

Now let's hold A constant and equal for all firms, and let us specify separately the entry and the exit mechanisms.

Given the partial equilibrium nature of the model, we assumed the existence out of our market of an unbounded set of potential entrants. We assumed also no entry barriers (pure contestable market hypothesis), so that the inflow of new firms is triggered simply by the difference between the average rate of profit and the interest rate.

Entries, then, will "happen" only when the market is not too crowded for the given aggregated demand, that is when actual N is lower than the value that satisfies $\pi = r$.

The condition is the same than (4), but the passage to (the inverse of) equation (5), necessary to compute an N^* of equilibrium, is less direct. In fact, here we are considering only the entries in the market, so that the N^* that we can compute from (5) is just a *benchmark* with which to compare the actual number of firms: it is the maximum number of firms compatible, given A, with at least normal profits. Whether or not that value will be a feasible equilibrium depends also on the exit fluxes; and as we will show, the answer is no.

As a consequence, the equilibrium in N given A does not imply no liability for the firms, and to solve $\pi = r$ we must consider both profit equations (2a-2b).

[2] The condition states that the unitary production costs, augmented for the costs of their financing, must be lower than the price. It is a sort of minimum requirement for the market to exist, and we'll assume it always verified.

The switch between the two profit equations is the "No liability line" (NLL) we drawn on figure 2, defined by the condition $\alpha Y/N = A$. In the region lying over it, firms have no liability, the "active" profit equation will be (2b), and the N^* satisfying $\pi = r$ can again be found on the NPC.

In the region below it, the "active" profit equation will be (2a). It is easy to see that if $\alpha(1+r) < 1$ is verified, with any couple $(A, N$ falling in this area firms are earning more than normal profits. Hence, new firms will continue to entry until N goes over the NLL.

In conclusion, given A, the unique N compatible with the condition $\pi = r$ is that read on the NPC. To determine the entity of the inflow, then, we calculate an equilibrium N^* *via* equation (5), and let the number of entrant I be equal to the difference - if positive - between actual N and N^*, times an adjustment coefficient γ. Writing it in relative terms, we'll have

$$i \equiv \frac{I}{N} = \max\left\{\gamma\left[\frac{Y(1-a)}{rAN} - 1\right], 0\right\} \tag{6}$$

Turning to the incumbent firms, the decision they have to take is whether to stay in the market or to leave it. As said, the information they have to base on to take their choice is twofold: their own performance, and the profitability of the market.

To evaluate the latter, they try to extract some information looking at the *behaviour* of their competitors. When they observe that some of them are leaving, they interpret this fact as a signal that the market profitability is getting worse. *Via* this interaction effect, then, a firm has a positive probability of exiting even if it is making more than normal profits.

Formally, this is equivalent to the presence of strategic complementarities: firms will judge convenient to make a stay or go choice of the same sign of that of their competitors.

A convenient way of tackling this social interaction effect, is to slightly modify a mean field effect model as those proposed by Brock and Durlauf (see for instance), to let the external field be determined endogenously.

Calling ω_i the choice of the ith firm, and $\omega_{\sim i}$ the average choice of its competitors, we write the expected benefit of ith firm as:

$$V(\omega_i) = h_i\omega_i + J\omega_i E_i(\omega_{\sim i}) + \eta(\omega_i) \tag{7}$$

where:

$\omega_. = -1$ stands for the "go" choice, and $\omega_. = 1$ for the "stay" choice;

h_i, the external field, is a measure of the firm's own profitability;

J is a parameter measuring the strength of the interaction;

$E_i(.)$ is the operator that gives the expectations of firm i;

$\eta(\omega_i)$ is a random term whose realisation depends on the decision taken, independent across individuals.

Assuming the difference $\eta(1) - \eta(-1)$ to be logistically distributed, we'll have that the probability that ith firm will stay in the market is proportional to:

$$Pr[\omega_i = 1] = Pr[V(1) > V(-1)] \propto \exp\{\beta[h_i + JE_i(\omega_{\sim i})]\} \tag{8}$$

where β is the inverse of the scale parameter of the logistic distribution. Note that the introduction of a random term in the expected benefit V implies a positive exit probability not only with h positive, but also when the firm observes none of its competitors exiting.

Using (8), we can compute the mathematical expectations of the individual behaviour $E(\omega_i)$, $i = 1 \dots N$, and aggregate them to compute the expected average choice ω in the population. Imposing a coherence between this latter and the expectations of the individuals, i.e. letting $E(\omega) = E_i(\omega_{\sim i}) \forall i$, and remembering that the equity base, the market share, and hence the firms' profitability situations as expressed by h_i are the same for all firms, we can write the condition for the average choice ω^* to be an expectational equilibrium:

$$\omega^* = \tanh\{\beta[h + J\omega^*]\} \tag{9}$$

Since $\tanh(.)$ is continuous, and is a contraction of $[-1;1]$ into itself, there is at least one solution to (9); when $J\beta < 1$, it can be shown that the solution is unique. Assuming this condition verified, we can re-scale ω^* to obtain a unique value for the rate o of exiters which gives a self-consistent equilibrium in the expectations:

$$o = \frac{1 - \omega^*}{2} \tag{10}$$

Now that we have defined the entry exit rules, we can put them together to write the equilibrium condition $\Delta N = 0 \Leftrightarrow i = o$. We can proceed as follows.

Firstly, we can observe that for h to be a measure of the firm's profitability it must be increasing in π, which in turn is a continuous decreasing function of N, defined in $(0, \infty)$ and there differentiable almost everywhere.

If we let also $h(N)$ be continuous, and observe that equations (9-10) define implicitly o as a decreasing function of h, with values in (0; 1), we'll have that

$$o(N) : \Re^+ \mapsto (0;1) \tag{11}$$

is a continuous and increasing function of N.

By a simple analysis of the two curves $i = i(N) - o = o(N)$, descends that there is a value N strictly lower than $\frac{Y(1-\alpha)}{rA}$ in which entries are equal to exits; that is, firms turnover will be in equilibrium in a point lying strictly under the NPC.

A convenient way to better qualify this equilibrium is to consider both i and o as functions of the product AN. In figure 1 we plotted them with values of the parameters satisfying the hypotheses adopted till now, and specifying h as:

$$h = \frac{\pi - r}{r}$$

If we read this figure "forgetting" for a while the A on the X-axes, we can see the N of equilibrium for the turnover derived above.

The key observation, is that the function $o(.)$ is separable in AN. If we assume the same separability for the function $i(.)$, we'll have that the equilibrium condition for the

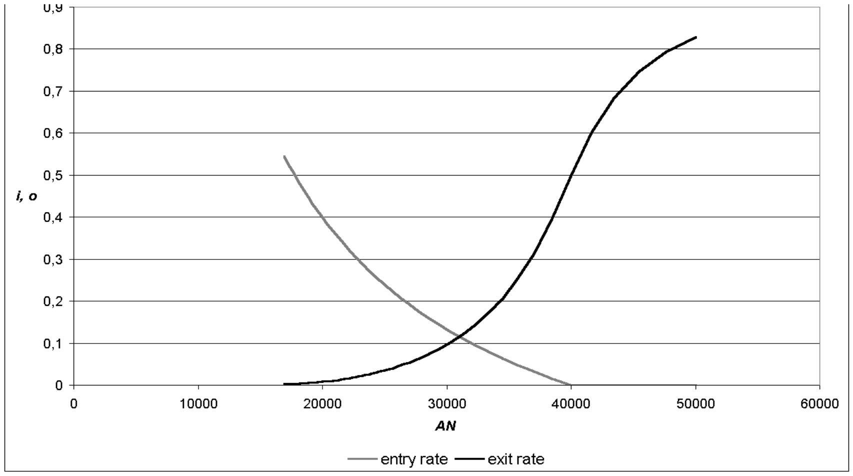

Figure 1: Firms' turnover equilibrium.

firms' turnover will depend only on the *product* of A and N. In the figure, a change $\Delta N = 1/\Delta A$ will not change the position of the entry/exit rates curves, and hence the position of the equilibrium.

If we call k the value of the product AN at which the two curves intersect, we can write the map that defines the turnover equilibrium in the space (N, A) as:

$$N = \frac{k(.)}{A} \tag{12}$$

where $k(.)$ doesn't depends on A, but depends on all the other parameters of the model.

This turnover equilibrium (TE) curve acts as an attractor for N (cp. figure 2). Given A, for values of N lower than that identified by equation (12), we'll be at the left of the equilibrium of figure 1: entries will be higher then exits, and N will increase. Similarly in the opposite case.

4.4 Market dynamics and heterogeneity

In the last two paragraphs, we derived the equilibrium condition for the equity base as a function of N, and the equilibrium condition for the firms' turnover as a function of A.

In figure 2 we draw the two equilibrium maps, together with the no liability line. The direction of the trajectories around these maps is also reported: according to what stated in paragraph 4.2, in the area above the NPC the equity will decrease; while in paragraph 4.3 we saw that in the area above the TE curve there will be an excess of exits on entries, and N will decrease - similarly in the opposite cases.

We also saw that the latter lies strictly under the former, so that between the two we have an attractor-basin where the system will fall with certainty.

The dynamics inside the basin is that depicted: with agents all equal, the market will go towards higher equity and lower N.

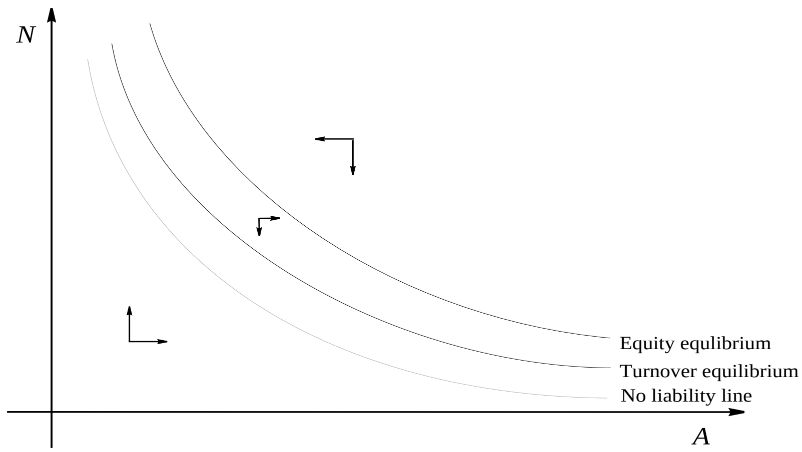

Figure 2: (A,N) phase diagram - no heterogeneity.

We must consider two more factors to better assess the behaviour of our market: the role of liabilities, and that of heterogeneity.

In the figure, we draw the no liability line under the two other curves, but that is not always the case. The position of the NLL and of the NPC depends only on the parameters r and α, and on the level of the aggregated demand Y, while the TE curve will depend also on the parameters β and J, with which we modelled the strategic interaction among firms, and on γ. For some configurations of these three latter parameters, then, the TE curve can go under the NLL. This will change slightly the trajectories around the equilibrium maps, since both the equity and the turnover will evolve according to a different profit equation.

Actually, the TE curve itself will have a different formulation, since the implicit function $k(.)$ of the model parameters will change. Without getting into details, it can be shown that it will lie under the TE curve as calculated ignoring financial costs. With no heterogeneity, anyhow, this will not change the qualitative dynamics depicted so far.

When we allow for heterogeneity in the firms' equity base, the portrait can change sensibly.

On one side, the equilibrium condition we derived for the firms turnover is again no more directly applicable. While the entries are driven only by the average measure of the market profitability, the exit rule has a nonlinearity that makes the aggregation sensible to the dispersion of the distribution. Experimentally, as we'll see, the heterogeneity implies a downward shift in the TE.

In addition, if the equity dispersion goes over a threshold, a not-empty subset of firms will lie under the NLL. This will change both the benefit function on which they base their stay or go choice on, and the motion equation of their equity. In other words, we can have two non empty subset of firms following two different dynamic regimes.

On the other side, the direction followed inside the basin attractor can change its sign, provided that the new firms entering the market have an average equity lower than the incumbents'. This composition effect, in fact, countervails the tendency in the rise of the average *A* when we are under the NPC.

Both these effects are easily handled building an agent based simulation of our model, with which we can also have some hints on the dynamics around these maps.

5 The simulations

In this section we first report some technical details concerning the simulations conducted. We then proceed to show the artificial time series generated, first reproducing the results above derived assuming no heterogeneity among firms and a unique equilibrium in the stay or go choice, then relaxing both of them.

5.1 Technical details

The simulations have been conducted in Swarm, a set of software libraries developed at the Santa Fe Institute in New Mexico - starting from 1995 -, to help simulating complex systems. The underlying programming languages of Swarm are Objective-C and Java, which object-oriented architecture is particularly suited to run agent-based simulations[3]. In a nutshell, a simulation in Swarm is built up putting together a population of artificial agents - independent "pieces" of software containing the agents' behavioural algorithms and their vector of state variables, and a schedule with the ordered list and the timing of all the actions each agent will make.

The core of our simulation has been built up with three kinds of agents: firms and households (whose state variables and behaviour are a straightforward translation of the model characteristics described above), and an environment, whose role is to collect and distribute statistics, to co-ordinate entries and exits, and to behave as an interface between the other agents.

For every time-step of the simulation, the main items of the schedule are the following:

1. Households: do shopping;
2. Environment: check for bankruptcies;
3. Firms: take the stay or go decision;
4. Environment: create new firms.

During the first step, a representative household spends entirely its yearly income. It divides it equally among the incumbent firms, except for a random multiplicative shock on the price payed to each seller. The shock is extracted from a uniform distribution with mean one, whose support is a simulation parameter modifiable at start

[3]For economic simulations in Swarm see Luna and Stefansson (2001) and Luna and Perrone (2002 . Technical details on the Swarm Toolkit can be found on the web, at http://www.swarm.org.

up. The eventual slack between yearly income and actual spending is added to the following period income.

In the second step of the schedule the environment checks if there have been "hard" bankruptcies, i.e. if any firm's equity base has fallen under a threshold - that we set to zero. Afterwards, it asks to all firms to take their stay or go decision, providing them with the macro variables values they need in order to take it.

To implement the stay or go choice, we had to better specify equation (7). For the external field we used the following:

$$h_i = \frac{1}{b}\sum_{i=1}^{b} L^i(\pi_i - r),$$

where L is the lag operator, and b is the number of periods considered by the firms to evaluate their own profitability. For what concerns the expectations on the behaviour of their competitors, we assumed again an adaptive mechanism: they have been put equal to the lagged proportion of exiters re-scaled to the interval [-1, 1]:

$$E_i(\omega_{\sim i}) = 1 - 2L(o)$$

Finally, the entry of new firms is driven by the Environment object. If there's a positive slack between the overall rate of profit and the interest rate, the environment will create a number of new firms according to equation (6). In accordance with empirical findings, we let new entrants have an average equity base lower than the entrants. However, not to put a bias towards a predetermined equity level, we extracted it from its distribution among the first quantiles of the incumbents.

5.2 Simulation runs

To get started, we simulated the model with no shocks on prices, and a unique solution for the stay or go choice - the condition being $\beta J < 1$ [4].

As we can see in figure 3, the qualitative outcomes we obtain are in line with those predicted by the formal study of the model. Once the trajectory goes over the TE curve[5], the market starts moving towards low N and high A; the path seems quite close to the TE curve, showing faster adjustments in the number of firms.

Putting an idiosyncratic shock on prices, this portrait changes as follow.

First of all, the dynamic of the average equity base will be dumped by a composition effect, since new firms will be endowed on average of an equity base lower than the incumbents. As a consequence, the equilibrium condition $\Delta A = 0$ will be satisfied somewhere *under* the NPC.

The movement towards low N and high A, then, will be either slower, or even reversed.

As a first step towards this second case, we calibrated the simulation parameters so to obtain the almost coincidence of the two curves. In figures 4 and 5 we can see two

[4] Assuming, as we are doing, adaptive expectations, the equilibrium is also stable (cp. Brock and Durlauf 2000).

[5] The turnover equilibrium curve has been plotted solving numerically $k(.)$ in function of the parameters.

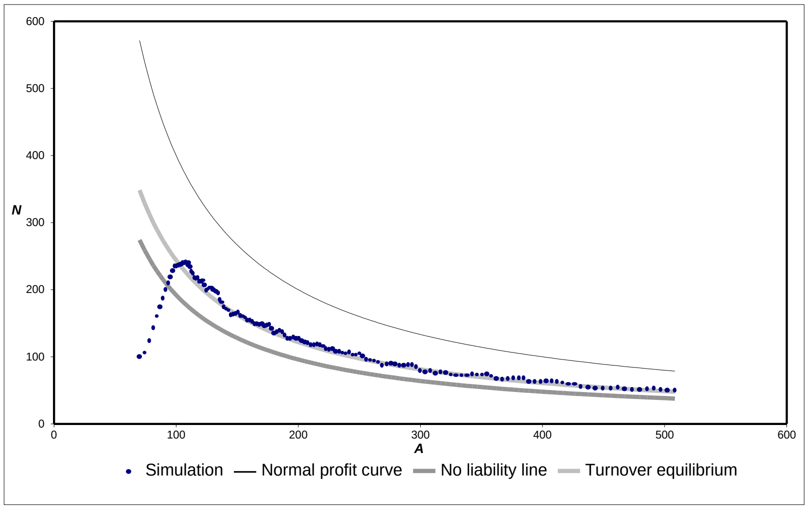

Figure 3:

recurrent stylised facts that characterise the dynamics also in this quasi-equilibrium case, namely: a right skewed distribution of the equity base, and irregular fluctuations of the number of firms' time series[6].

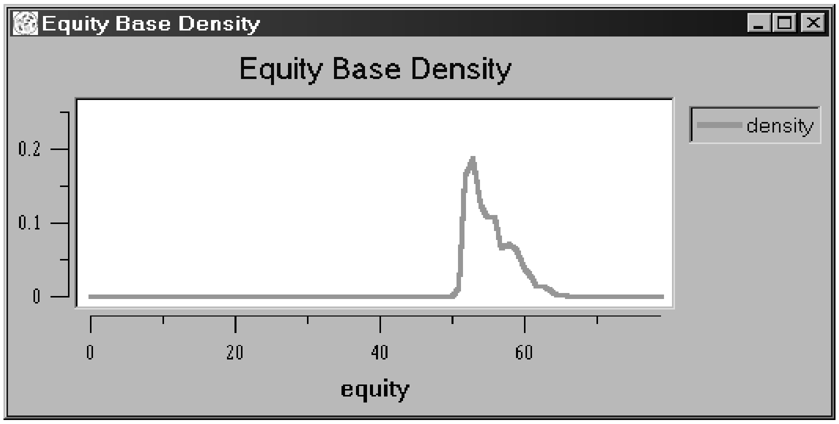

Figure 4:

[6]The (most "crucial") parameters values were the following: $r = 0.02$, $\alpha = 0.96$, $\beta = 2.2$, $J = 0.6$. In this case, so as in some following simulations, we have $\beta J > 1$, so that we can have multiple equilibria in the stay or go decision. Anyway, it can be shown that a jump from one equilibrium to another can only occur either with a very large exogenous shock, or with a downward shift in the h, towards negative values. The latter in our model cannot occur, since h is endogenous, and anneals rapidly to positive values, the former seemed to us out of the scope of our analysis.

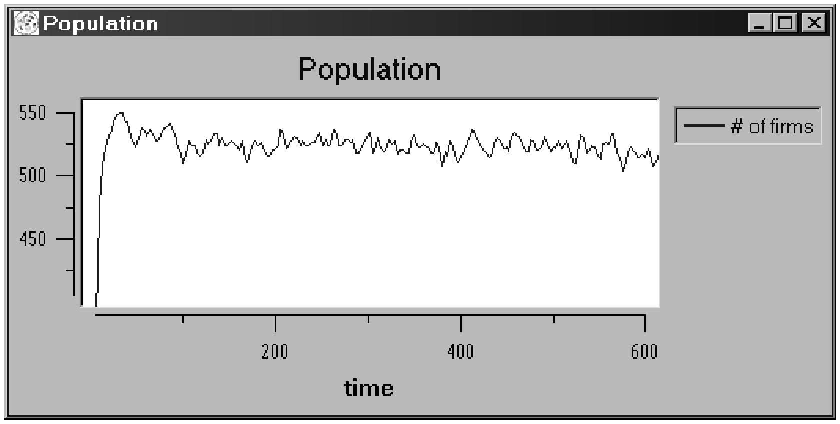

Figure 5:

The former is surely related to the composition effect due to the low equity level of the entrants; so as the latter are obviously fired by the stochastic components of the simulation.

Both of them, however, are strictly related also to the interaction term ruling the exits. As a matter of fact, the relation between the asymmetry and the parameters J and β is straightforward, since they influence directly the entity of the exit fluxes; and hence also the way in which the turnover changes the composition of the population. For some set of parameters, the skewness can also turn negative: when all the firms face no liability costs, ”little is better”: a lower A brings an higher rate of profit, which in turn is sufficient to countervail the signal received by the exit fluxes, strongly amplified by the high J. While the right tail of the distribution can be almost be cancelled out. With heterogeneity, and a non empty quota of firms facing financial constraints, this somewhat counterintuitive effect disappears.

The strenght of interaction has a quite direct relationalso with business fluctuations, *via* two effects: it can stuck an agent to a behaviour consistent with the others’ one, hence stabilising it; and it can amplify random shocks that wouldn’t elsewhere have great effect on the aggregate.

Letting the heterogeneity be higher - relatively to the distance between the NPC and the NLL - we can observe a change in the dynamic regime.

We already saw how the NPC, because of the composition effect, can be lower than in the base model. It happens something similar to the TE curve, that is, the curves that we draw as a function of the *average* value of the equity, overestimates the level at which there is equilibrium in the firms turnover. This is a common result of all the simulations conducted, and maybe it was predictable, studying more deeply the nonlinear specification of the entry-exit rules.

A little bit harder to predict, was the fact that the two curves can change their relative position. In figure 6, we report the outcomes of two simulations, conducted with all the structural parameters equal, but with a different variance of the random shock. As a benchmark, we draw only the NLL together with the series, since it is the only curve whose position is not changed by the heterogeneity.

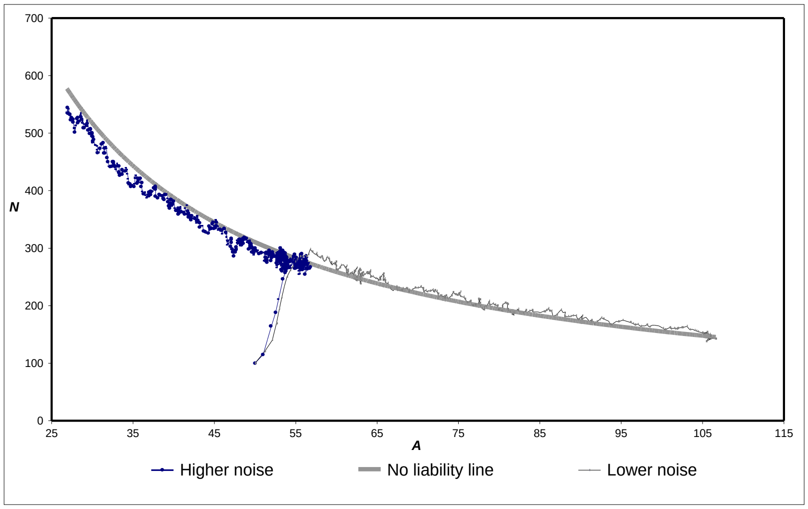

Figure 6:

Apparently, the higher dispersion in the equity base caused the ”empiric” firms’ turnover equilibrium curve to go above the line that stabilise the average *A*. This fact, changed the direction followed by the system inside the attractor basin, and caused the second experiment end towards higher values of *N*. In the figure, actually, we reported three hundred time steps for the series with the lower noise, and six hundreds for the higher variance one.

This latter, in fact, for the first three hundred time steps remains about the same levels of *N*. Afterwards, ”something happens”, and the series starts moving.

6 Conclusions

Perfect competition is usually associated with the idea that firms take for exogenous the behaviour of their competitors. I studied a case in which this idea is not connected with the assumption of no interaction at all between agents. Namely, I considered the non price interaction between incumbent firms, due to the process of knowledge extraction - about the market profitability - in which they are involved in.

This assumption has been modelled with a mean field effect, in which the external field - the firms’ own profitability - is endogenous, since it depends also on the entry fluxes and on the dynamics of their equity base.

Allowing for idiosyncratic shocks on prices, the simulations produced some artificial stylised facts of interest: a right skewed distribution for the equity base, attributable mainly to a composition effect; a long run positive firms turnover, which is related, among the others, to the dispersion of the equity distribution; business cycles, triggered by the stochastic components of the model and magnified by the interaction component of the decision taken by the firms.

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