ARE POLISH FIRMS RISK-AVERTING OR RISK-LOVING? EVIDENCE ON DEMAND UNCERTAINTY AND THE CAPITAL-LABOUR RATIO IN A TRANSITION ECONOMY[♣]

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Som-theme E: Financial Markets and Institutions

Abstract

This paper investigates the effect of demand uncertainty on the capital-labour ratio of non-financial firms in Poland in order to infer the firms' risk behaviour. A generic model is used to characterise a utility maximising firm in a transition economy with demand uncertainty and imperfect competition. It is assumed that labour is completely variable and capital is quasi-fixed. The demand for capital, and hence the capital-labour ratio, derives from the optimisation of expected costs and the firm's pricing and output decisions, and crucially depends on the sign of the covariance term *i.e.* the firm's risk behaviour. The main proposition of the model is that if firms are risk-loving, an increase in demand uncertainty increases the capital-labour ratio, whereas the capital-labour ratio would decrease when a firm is risk-averting. The model is estimated using data from a cross-section of 148 non-financial firms in Poland. The results unambiguously show that there exists a significant positive relationship between demand uncertainty and the capital-labour ratio. This finding suggests that Polish firms are risk-lovers. The evidence may have important implications for the needed set of regulations and corporate governance in Poland as part of the necessary economic reform.

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1. Introduction

A key policy issue in the emerging market economies of Eastern and Central Europe is to engender the transition to the price mechanism for the allocation of resources. For policy makers, an important part of the transition process involves the design of the needed set of regulations and corporate governance in the face of extreme forms of uncertainty, especially characterised by unstable market conditions in which firms as well as financial institutions operate. For example, firms are concerned about conditions of demand uncertainty as well as input choices. In this context, government regulations seem to be necessary to facilitate a stable business environment, promote business confidence and optimism and thus generate a stable growth path for the transition economies.

The main literature that may be invoked to shed light on the above economic problems, at the firm level, relates to models of firm behaviour under uncertainty, and shows that demand uncertainty affects firms's input choices and cost functions (see, for example, Halthausen, 1976; and Ghosal, 1991, 1995). A common element of these models is that they put forward some plausible conditions under which stochastic demand conditions may determine the capital-labour ratios. For example, when capital is chosen *ex ante* but labour is determined after the level of demand is observed, firms may respond to demand uncertainty by using a smaller amount of capital and thus operate with a lower capital-labour ratio in a sub-optimal manner. However, the special line of inquiry taken by Holthausen (1976) argues that the way in which firms react to stochastic demand conditions depends on their risk behaviour. This suggests that more information about firm's risk behaviour may be an important input in the development of an efficient system of government regulations. In the context of emerging market economies, effective regulation may minimise the risk exposure of firms. In contrast, Ghosal (1991) introduces the argument of firm size into the analysis, arguing that an increase in firm size counteracts the negative relationship between demand uncertainty and the capital-labour ratio.

This paper investigates the effect of demand uncertainty on the capital-labour ratio of firms in Poland. The motivation of the paper is that, under given conditions characterizing the economic transition process in Poland, the impact of demand uncertainty on the capital-labour ratio sheds light on the firms' risk behaviour. It is argued that if firms are risk-loving, an increase in demand uncertainty increases the capital-labour ratio, whereas the capital-labour ratio would decrease when a firm is risk-averting. Hence, the main proposition put forward in the paper is that the elasticity of the capital-labour ratio with respect to demand uncertainty serves as an indirect indicator of the risk behavior of Polish firms. The empirical results of the paper shows that there is a significant positive relationship between demand uncertainty and the capital-labour ratio of Polish firms. This finding suggests that Polish firms are risk-lovers. We conclude that the evidence may have important consequences for the needed set of regulations and corporate governance in Poland.

The paper makes at least three major contributions. First, it introduces an important variation to recent research on firm behaviour under demand uncertainty (e.g. Ghosal, 1991, 1995) by focusing on the risk behaviour (specifically, risk-averting versus risk-loving) of firms under conditions which are characteristic of transition economies such as Poland. Second, while the literature on input choices and the stochastic nature of demand has generated a number of theoretical models, the empirical dimension is less well developed; in this context, this paper contributes to recent efforts to provide econometric evidence that may facilitate the evaluation of these models. Third, the paper presents evidence which, in some respects, calls for a re-examination of the findings of previous work on demand uncertainty and factor input choices.

The remainder of this paper is structured as follows. Section 2 sets out the theoretical model. The dataset and empirical procedures, including the method for estimating demand uncertainty, are described in Section 3. Section 4 presents the estimation results. Section 5 concludes.

2. The theoretical model

The model we use is based on Holthausen (1976). The main difference of our model from the one by Holthausen (1976) is that, for reasons of convenience, we specify the main equations which capture the business environment in emerging market economies. The purpose of the model is to show that risk-loving firms faced with demand uncertainty prefer to have a high capital-labour ratio, whereas risk-averting firms prefer low capital-labour ratios.

The model assumes that the firm is an imperfect competitor who behaves as a price setter. The firm operates in a world where demand is uncertain. Its demand function is therefore specified as:

$$q = u - \varphi p$$

where q is total demand, u is a random term and p is the price level. The firm produces goods with a standard neoclassical Cobb-Douglas production function with two inputs, capital (k) and labour (l):

$$q = k^{\alpha} l^{(1-\alpha)}$$

A crucial assumption is that labour is completely variable, whereas capital is quasi-fixed. This is taken into account by assuming that capital input is chosen before actual demand is observed, whereas the demand for labour takes place after demand is observed. If the capital stock is set, and actual demand is observed, labour input will always be chosen so that any level of demand is satisfied. To simplify the analysis, labour demand is not considered as a decision variable, but is taken into account implicitly by a labour-requirements function, which is specified as:

$$l = (q/k^{\alpha})^{(1/1-\alpha)}$$

The firm optimizes the expected utility of profit (Π):

$Max_{(p,k)}\ \ EU(\Pi)$

where U is a von Neumann-Morgenstern utility function, and E is an expectations operator. The decision variables are the price level and the capital stock. Profit is defined as:

$\Pi = \ p(u\text{-}\varphi p) - w((u\text{-}\varphi p)/k^{\alpha})^{(1/1-\alpha)}$ - $ic(k)$

where w is the wage rate; c is the price of capital; and i is the cost of capital. The demand function and the labour requirement functions are taken into account. It is assumed that $w,c,\ i$ are given for the firm. The first-order condition with respect to k is:

$$E\left\{U^{'}(\Pi)\left(w\frac{\alpha}{1-\alpha}\frac{l}{k}-ic\right)\right\}=0$$

For two random variables, $E(X,Y) = E(X)E(Y) + cov(X,Y)$, so that the above equation can be rewritten as:

$$EU^{'}(\Pi)E\left(w\frac{\alpha}{1-\alpha}\frac{l}{k}-ic\right)+w\frac{\alpha}{1-\alpha}\operatorname{cov}\left(\frac{l}{k},U^{'}(\Pi)\right)=0$$

so that:

$$E\left(\frac{l}{k}\right)=\frac{1-\alpha}{\alpha}\frac{ic}{w}-\frac{\frac{1}{k}\operatorname{cov}\left(l,U^{'}(\Pi)\right)}{EU^{'}(\Pi)}$$

It follows that, on the one hand, the demand for capital is bigger than the amount of capital the firm uses if expected costs are minimized for a given level of output in the case where the left-hand side of the above expression is smaller than ic/w. Since, $U^{'}(\Pi) > 0$, a firm will demand more capital than the cost minimizing amount when the covariance term is positive. On the other hand, in the case where the left-hand side of the expression in the above given equation is bigger than ic/w, the demand for capital is smaller than the amount of capital the firm uses if expected costs are minimized for a given level of output. Since, $U^{'}(\Pi) > 0$, a firm will demand less capital than the cost minimizing amount when the covariance term is negative. The crucial indicator, therefore, is the sign of the covariance term.

The covariance term can be signed by considering the effects of an increase in the random term. Hence:

$$\frac{\partial U'(\Pi)}{\partial u} = \frac{\partial \Pi}{\partial u} U''(\Pi) = \left[p - w \frac{\partial L(q,k)}{\partial q} \right] \left[\frac{\partial q(p,u)}{\partial u} \right] U''(\Pi)$$

The first term in brackets on the right-hand side of the above equation is always positive under the assumption that the price is higher than the marginal cost of production. Since q_u >0, the sign of $\delta U'(\Pi)/\delta u$ depends on the sign of $U''(\Pi)$. And it follows that:

$$\frac{\partial l}{\partial u} = \frac{\partial l}{\partial q} \frac{\partial q}{\partial u} = \frac{1}{1-\alpha} \left(\frac{q}{k^{\alpha}} \right)^{\frac{\alpha}{1-\alpha}} k^{-\alpha} \geq 0$$

Hence, the covariance depends on $U''(\Pi)$. If $U''(\Pi) < 0$, which corresponds to a concave function and risk-averting behaviour, the covariance term is negative. In that case less capital is used than the cost minimizing outcome. If $U''(\Pi) > 0$, and hence the firm is risk-loving, the covariance term is positive, and accordingly the capital labour ratio is above the cost minimizing level. If $U''(\Pi) = 0$, $U'(\Pi) = constant$, implying that utility is linear, the firm is risk-neutral; in that case the covariance term equals zero, so that the capital-labour ratio equals the cost minimizing level.[1]

In summary, therefore, the model characterises a representative utility maximising firm in a transition economy with uncertain demand conditions; the firm is a price setter under imperfect competition. It is assumed that labour is completely variable and capital is quasi-fixed, and the firm chooses capital input before actual demand is observed. Labour demand is not considered as a decision variable in the sense that after actual demand is observed, the capital stock is set and labour input is chosen to satisfy the level of demand. The demand for capital, and hence the capital-labour ratio, derives from the optimisation of expected costs and the firm's pricing and output decisions, and crucially depends on the sign of the covariance term *i.e.* the risk behaviour of the firm. There are three possibilities. One, if the optimisation process yields a negative covariance term, the result corresponds to a concave function and risk-averting behaviour, indicating that less capital is used than the cost minimizing outcome. Two, if the optimisation process yields a positive covariance term, the result corresponds to a convex function and risk-loving behaviour of the firm, indicating that the capital-labour ratio is above the cost minimizing level. Three, if the optimisation process yields a zero covariance term, the capital-labour ratio equals the cost minimizing level, implying that utility is linear and the firm is risk-neutral.

The testable proposition of the model, therefore, is that risk-loving firms faced with demand uncertainty prefer to have high capital-labour ratios, whereas risk-averting firms

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[1] Note that strictly speaking the above theoretical analysis gives the *overall* impact, rather than the *marginal* impact, of uncertainty on the capital-labour ratio. However, the empirical equation and the regressions in the empirical part of the paper reflect the *marginal* (partial) effects of demand uncertainty on the capita-labour ratio.

prefer low capital-labour ratios. To represent this proposition, a corresponding empirical equation can be specified as follows:

$$(K/L)_i = \beta_0 \pm \beta_1 UNC_i + \beta_2 W_i + \varepsilon_i$$

where *UNC* denotes a measure of demand uncertainty; *W* is the cost per employee and serves as a control variable in estimation and testing; ε is an error term; and $i = 1, 2, 3,.....,n$ firms. The sign of β_1 is theoretically dichotomous (±) and captures the competing hypotheses on the risk behaviour of firms: if the firms are risk-loving, the sign is positive; a negative sign indicates risk-averting firms.[2]

3. The dataset and empirical procedures

We use the dataset AMADEUS which contains firm level financial data for more than 200,000 non-financial firms in Europe, including Poland. The data are provided by local bureaus and AMADEUS compiles the dataset by only including companies which satisfy one of the following size criteria:
(i) a turnover greater than 10 million Euro;
(ii) a number of employees greater than 150;
(iii) a total asset sum greater than 10 million Euro.

Uniformity is achieved by standardization of accounting information. A big advantage of the dataset is that it comprises listed as well as unlisted firms. For Poland, where there is only a small amount of firms listed on the stock market, this implies that much more firms can be taken into account in the analysis. However, the fact that the dataset focuses on unlisted firms is for some types of analysis also a drawback. For example, the dataset does not contain market values; this limitation precludes analyses for which Tobin's Q is required. Another drawback of this dataset is that, for most countries including Poland, it only contains yearly data for the period 1992-1996. Obviously, this is a main shortcoming. But it should be taken into account that for most Eastern European countries, given the structural break associated with the collapse of the communist system, it would be questionable whether data for the pre-1990 period, if they were available, could be useful at all.

Due to the limited time period for which the data are available, our analysis is based on averages for the 1992-1996 period. By using averages, cyclical influences are avoided.[3] We average data for all original variables prior to any other transformation. All

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[2] In some recent studies, the theoretical dichotomy is not addressed. For example, Ghosal (1991) postulates only a negative relationship between demand uncertainty and the capital-labour ratio. However, unlike this paper, the model by Ghosal (1991) is specified as $(K/L) = f\,(UNC, SIZE)$ where *SIZE* is a measure of firm size, and serves to counteract the negative relationship between demand uncertainty and the capital-labour ratio.

[3] By avoiding cyclical fluctuations, we are able to focus on the underlying nature of the relationship between demand uncertainty and the capital-labour ratio. Moreover, the period of 1992-1996, for which the data are available, is not characterised by any important cyclical downturns in Poland or the

data are denominated in millions of current US dollars. Since US inflation has been moderate in the sample period, this transformation helps to avoid inflation problems.

The calculation of one important variable in the analysis – demand uncertainty - requires time series data. In order to be able to make a reasonable estimate of demand uncertainty, we only consider firms for which at least four years of the relevant data are available. Moreover, we balance the data set so that in all regressions the same amount of firms are taken into account. Finally, to account for extreme outliers, we delete from all variables used in the estimates the five lowest and the five highest values. The final dataset contains 148 Polish firms.

The approach we use to construct the demand uncertainty variable consists of two steps. In the first step, we formulate and estimate a forecasting equation which determines the expected part of the variable under consideration. In line with many other empirical studies (see, among others, Aizenman and Marion (1993), Ghosal (1995) and Ghosal and Loungani (1996)), we use an autoregressive process as the forecasting equation. Due to the limited time series data available per firm, we use a first-order autoregressive process. We consider two alternative specifications, with and without a constant.

$$ln\ SAL_t = a_1\ ln\ SAL_{t-1} + e_t$$

$$ln\ SAL_t = a_2 + a_3\ ln\ SAL_{t-1} + \mu_t$$

where *SAL* denotes sales of firms; a_1 and a_3 are the autoregressive parameters; a_2 is a constant; *ln* denotes natural logarithm; and e_t and μ_t are error terms.[4] We estimate the above given equations for all firms in our dataset.

In the second step of our procedure, the proxies for demand uncertainty are obtained by calculating the standard deviation of the unexpected part of *SAL*, *i.e.* the residuals from the forecasting equations given above. Hence, *UNC1* refers to the proxy obtained from the equation with a constant, while *UNC* refers to the proxy obtained from the equation without a constant.[5]

In all, therefore, we initially extract the following set of variables from the dataset: *K* = tangible assets; *L* = the number of employees; *CEMPL* = the total cost of employees; and *SAL* = sales. Basing on these variables, we construct the variables used in the estimates, namely *CLRAT* = the capital-labour ratio, calculated as *K/L*; *W* = cost per employee, calculated as *CEMPL/L* and used as a control variable in the estimates; and *UNC*

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rest of the world economy.

[4] Given the short span of the data for each firm (5 annual observations), we did not experiment with more complicated specifications of the autoregressive process e.g. fitting a linear trend or specifying a quadratic function to capture the fluctuations of sales around the trend.

[5] As observed by Aizenman and Marion (1993) the main shortcoming of this method is that future changes in the environment that are fully anticipated are still treated as uncertain.

and *UNC1* which denote the proxies for demand uncertainty, calculated by using *SAL*, as explained above.

[Table 1 about here]

In Table 1 we give some statistical moments of the main variables used in estimating the model. The first and second moments (mean and standard deviation) suggest fairly good dispersion characteristics for a cross-section sample of 148 Polish firms. For example, the capital-labour ratio ranges from a minimum of 0.0012 to a maximum of 0.064 with a mean value of 0.013 and a standard deviation of 0.0107; demand uncertainty measured without a constant ranges from 0.85 to a maximum of 78.62 with a mean value of 9.64 and a standard deviation of 12.19; the measure of demand uncertainty that includes a constant range from 0.31 to 50.51 with a mean value of 5.96 and standard deviation of 8.23; wages range from 0.0025 to 0.0107 with a mean of 0.0044 and a standard deviation of 0.0014. The results thus show a wide diversity in the indicators, suggesting extreme outliers in the data are unlikely.

4. Estimation results

Although the economic theory discussed in Section 2 yields hypotheses on the effect of demand uncertainty and wages on the capital-labour ratio, the economic functional relationship is not completely specified. We therefore empirically explore the exact form of the estimating equation by experimenting with variants of the empirical equations in linear, quasi-logarithmic and logarithmic forms. In all cases, however, we did not find it plausible to include in the model an interaction between demand uncertainty and wages as an additional argument in the equation i.e. $\beta_3 \, (UNC \times W)_i$. The estimation results are presented in Table 2.

[Table 2 about here]

The results show that the fit of the equations is generally good in terms of the F-statistic; the adjusted R^2 is low, which is typical of cross-section regressions of this type. Given that heteroskedasticity could be important across firms, the t-values were computed from White's heteroskedasticity-consistent standard errors (White, 1980); the t-statistics show that all the coefficients are significant.

The evidence is unambiguous. Irrespective of whether the empirical model is specified in its linear, quasi-logarithmic, or logarithmic variants, the evidence shows that there exists a significant positive relationship between demand uncertainty and the capital-labour ratio. In this specific study, therefore, the evidence resolves the theoretical dichotomy in the empirical equation by showing that with respect to a sample of 148 firms in the transition economy of Poland, the sign of β_1 is empirically positive, with statistically significant coefficients. The evidence is therefore unequivocal that Polish firms are risk-loving; with an increase in demand uncertainty, the firms tend to increase their capital-labour ratio.

The logarithmic variants of the model offer additional information about the elasticities. For model variant (5) in Table 2, when demand uncertainty is measured on the basis of the equation without a constant in the first-order autoregressive process, it is shown that, on average, a one percent rise in demand uncertainty will result in a 0.22 percentage increase in the capital-labour ratio. The elasticity falls to 0.15 when demand uncertainty is measured on the basis of the equation without a constant in the first-order autoregressive process, as in model variant (6).[6] The F-test statistic indicates that model variant (5) outperforms the rest of the model variants considered in this study. As theoretically postulated in Sections 2 and 3, the labour costs variable is empirically positive for all model variants explored in this paper.

5. Summary and conclusion

This paper investigates the effect of demand uncertainty on the capital-labour ratio of non-financial firms in Poland. A generic model is used to characterise a representative firm operating under imperfect competition and uncertain demand conditions in a transition economy. The main testable proposition of the model is that if firms are risk-loving, an increase in demand uncertainty increases the capital-labour ratio, whereas the capital-labour ratio would decrease if firms are risk-averting.

Using cross-section data for 148 non-financial Polish firms, we test variants of the model in order to estimate the elasticity of the capital-labour ratio with respect to demand uncertainty and thus establish the risk behavior of Polish firms. The econometric results resolve the theoretical dichotomy and show that an increase in demand uncertainty leads to an increase in the capital-labour ratio of Polish firms. This finding suggests that Polish firms are risk-lovers. In general, the results are robust to the functional form of model specification (linear, semi-logarithmic and logarithmic) as well as the measure of demand uncertainty.

The knowledge of how firms react to uncertain demand conditions, uncovered in this study, offers a number of policy implications for Poland and some other transition economies. First, the evidence may have important consequences for the needed set of regulations and corporate governance in Poland as part of the necessary economic management during the transition period. Regulatory policy may be applied to influence the level of demand uncertainty as well as factor demand and factor productivity. For example, an introduction of a sales tax is expected to operate directly on the sales volume of firms and hence on the fluctuation of actual sales, and indirectly on the unexpected component of the sales (or demand uncertainty). Since demand uncertainty directly affects the capital-labour ratio, the effect may work its way to demand and pricing of the firm's factor inputs such as capital and labour (e.g. employment by firms) and the productivity of these factors. Second, the results shed light on how risk-loving firms behave in the face of a downturn in business confidence. The measure of demand uncertainty used in this paper derives from the unpredictable component of sales by firms and can therefore serve as a useful indicator of business confidence and consumer optimism. An increase in demand uncertainty reduces consumer confidence but risk-loving firms tend to increase their capital-labour ratio. However, to be able to refine the regulatory and other implications of this study, further and more detailed econometric research is necessary.

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[6] In general, however, the results reported in Table 2 are reliably robust to the specific measure of demand uncertainty *i.e. UNC* or *UNC1*.

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Table 1: Statistical moments of the main variables used in model estimation

	Mean	Median	Stand Dev.	Max.	Min.
UNC	9.64	5.96	12.19	78.62	0.85
UNC1	5.96	3.55	8.23	50.51	0.31
CLRAT	0.013	0.010	0.0107	0.064	0.0012
W	0.0044	0.0039	0.0014	0.0107	0.0025

Note: *UNC* denotes the proxy for demand uncertainty obtained by estimating the sales (*SAL*) equation without a constant;

UNC1 refers to the proxy for demand uncertainty obtained by estimating the *SAL* equation with a constant;

CLRAT is the capital-labour ratio, calculated as *K/L*, where *K* is tangible assets and *L* is the number of employees;

W is cost per employee, calculated as *CEMPL/L*, where *CEMPL* is the total cost of employees.

Table 2: Estimation results for a cross-section sample of 148 Polish firms

	(1) *CLRAT*	(2) *CLRAT*	(3) *Ln(CLRAT)*	(4) *Ln(CLRAT)*	(5) *Ln(CLRAT)*	(6) *Ln(CLRAT)*
UNC	0.00020 (2.15)		0.0247 (3.12)			
UNC1		0.00026 (2.01)		0.0188 (3.06)		
Ln(UNC)					0.2288 (3.50)	
Ln(UNC1)						0.1578 (2.81)
W	1.137 (1.91)	1.198 (1.95)	94.11 (2.34)	98.49 (2.37)		
Ln(W)					0.531 (2.72)	0.614 (3.00)
Constant	0.0064 (2.40)	0.0066 (2.35)	-5.14 (-27.34)	-5.13 (-26.30)	-2.09 (-1.92)	-1.04 (-1.27)
F	6.37	5.24	7.59	6.40	11.26	8.39
Adj. R^2	0.07	0.06	0.08	0.07	0.12	0.09

Note: t-values are between parentheses, below the estimated coefficients; the values are based on White's heteroskedasticity consistent standard errors. Equations (1) - (6) in the table are variants of the following generic model: $CLRAT_{i,t} = \beta_0 \pm \beta_1 UNC_{i,t} + \beta_2 W_{i,t} + \varepsilon_{i,t}$, where *CLARET* is the capital - labour ratio; *UNC* is a measure of demand uncertainty; *W* is a labour cost variable; subscripts *i,t* denote firm *i* at time *t; and* ε is the error term.