

# ON THE DEFINITION OF KEY SECTORS AND THE STABILITY OF NET VERSUS GROSS MULTIPLIERS

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## Abstract

Industries often promote their interests by arguing that they have a large impact on the rest of the economy. The same line of reasoning is used when so-called key sectors for economic development are searched for. In both cases, a one-sided view of the dependence of the rest of the economy on the sector at hand is used, and sectors with large forward and backward linkages are selected as being strategically important to the region or nation at hand. This one-sided approach, however, disregards that the sectors selected may be heavily dependent on the rest of the economy, and may therefore in fact not be able to generate the growth impulses that their larger linkages are assumed to pass on to the rest of the economy. To avoid double-counting impacts and to reckon with the two-sided nature of the dependency between a sector and the economy at large, the *net multiplier* concept is shown to provide an adequate solution. However, both the standard (gross) multiplier and the new net multiplier are essentially static concepts. When the search is for strategic sectors for future development, the question of the stability of both measures unavoidably arises. Besides the stability of the input-output coefficients, the stability of net multipliers is also based on the stability of its additional “exogenous demand/total endogenous output” ratios, which are unstable by nature. We argue that this property should not be seen as a vice, but as an additional virtue of the net multiplier concept, as it forces the analyst to explicitly consider this inherent instability instead of assuming the problem away as is usually done when gross multipliers are used.

## Keywords

Key sectors, input-output analysis, net multipliers, import substitution, export substitution.

## 1 INTRODUCTION

Arguments for state aid and state intervention in favor of certain sectors of industry are mostly based on their assumed economic importance for the region or nation at hand. Quite often, the arguments are not primarily based on the own size and the direct impact of the sector or project at hand, but on its assumed *indirect* importance for the regional or national economy (cf. Oosterhaven, Eding and Stelder, 2001). To substantiate such claims consultants, but also academics, traditionally multiply direct employment or some other kind of size indicator with a sector- or project-specific employment or value added multiplier. The result is then presented as an estimate of the *total* economic impact of that sector, i.e. of the sum of the direct plus indirect plus induced plus whatever other impacts one can think of (see e.g. the case of plant closedowns in Cole, 1988, discussed in Jackson, Madden & Bowman, 1997, and Oosterhaven, 2000).

The main problem with this traditional approach is the claim of each and every sector to be economically more important than its own share in total employment or value added indicates. Naturally, this can not be true. When the claims of all sectors in an economy are added, an (implicit) estimate of the total size of the economy will result that is many times larger than its actual size. To avoid double-counting impacts and to solve the mix-up of endogenous and exogenous variables involved, the *net multiplier* concept was introduced (Oosterhaven & Stelder, 2002a, see Oosterhaven & Stelder, 2002b, Oosterhaven, van der Knijff & Eding, 2003, for empirical applications).

In this contribution we will link this new concept with the old concept of key sectors (Perroux, 1955, Hirschman, 1958), as that literature discusses the closely related issue of the strategic significance of individual sector of industry for the development of regions and nations. As to this type of search, the concept of net multipliers will be shown to add a new dimension to the selection of key sectors, because net multipliers not only consider the dependence of the rest of the economy on the sector at hand, but also consider the dependence of the sectors concerned on the rest of the economy. In fact, net multipliers look at two-way dependencies instead of one-way dependencies.

However, both the standard (gross) multiplier and the new net multiplier are essentially static concepts. When applied in a dynamic setting, the question of stability rises. The stability of the gross multipliers in the standard input-output model is based on the assumed stability of the input coefficients. The stability of net multipliers also needs to be based on the stability of its additional “exogenous demand/total output” ratios, which are unstable by nature. As opposed to De Mesnard (2002) we will argue that this property should not be seen as a vice but as an additional virtue of the net multiplier concept.

Section 2 reviews the basic cause of exaggerating sectoral impacts, i.e. the mix-up of exogenous and endogenous variables. This mix-up is closely related to the usual neglect in key sector analysis of *also* considering the ability of sectors to create exogenous growth impulses, next to the size of their forward and backward linkages. Section 3 formulates the properties of the solution to these related problems, the *net multiplier*. It is argued that the net multiplier thus provides an aid to the problem of selecting key industries for regional or national economic development. When applied to all sectors, the estimated total of all impacts will aggregate to the correct size of the whole economy. The net multiplier thus gives a measure of the *relative* importance of a certain sector for the economy at hand. Section 4 evaluates the ‘stable’ alternative proposed by De Mesnard (2002). Besides, it further deepens the understanding and the implications of the inherent instability of the net multiplier. These implications include the need to estimate future import and export substitution in order to measure the future relative (i.e. net) economic significance of a sector for the regional or national economy. Section 5 concludes.

## **2 ONE-SIDED KEY SECTOR MEASURES AND EXAGGERATING IMPACTS**

The *direct* economic importance of a certain sector can, seemingly easily, be measured by some kind of size indicator, preferably by its direct contribution to the gross regional or gross national product (GRP or GNP), or else by its direct contribution to total regional or national employment. The line of reasoning, and the

determination of the *indirect* economic importance of a sector normally starts with making an inventory of that sector's relations with other actors in the economy. However, a sector may have large backward and forward linkages, but that does not tell us whether that sector is (passively) receiving impulses from other sectors or (actively) sending impulses to other sectors. Consequently, without further information, the standard definitions of key sectors <sup>1</sup> do not indicate whether the sector at hand can really be considered a strategically important sector for regional or national development.<sup>2</sup>

To be labeled a *key sector*, in our opinion, besides (1) having *large linkages* to pass on growth impulses (Hirschman, 1958), a sector also needs (2) to be able *generate growth impulses* (Perroux, 1955). Perroux, and Krugman (1991) for that matter, emphasizes the importance of sectoral economies of scale, while Oosterhaven (1981, ch. 5) emphasizes the relative size of exogenous demand. Inter alia because of its measurability, we follow the last emphasis in our proposal to use the net multiplier concept also as a tool to select key sectors for regional and national economic development. The basic idea behind the net multiplier is to correct having large multipliers for being able to autonomously generate growth impulses. The specific way in which this is done of course depends on the type of model from which the net multipliers are derived.

If we look at the standard Leontief model, final demand for sectoral outputs  $\mathbf{f}$  is exogenous, and the causality runs as follows. Any change in final demand as well as in total demand for sectoral outputs is matched, without supply constraints, by endogenous sectoral production  $\mathbf{x}$ . Endogenous sectoral production, in turn, determines endogenous intermediate demand for sectoral outputs  $\mathbf{Ax}$  as well as endogenous demand for primary inputs, such as sectoral value added  $\mathbf{v}$  and sectoral

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<sup>1</sup> See Rasmussen (1957) for the standard definition of key sectors, based on backward and forward linkages, and Beyers (1976) for an improved definition of forward linkages. See Cuello, Mansouri & Hewings (1992) for a further discussion and other, non-related improvements in key sector definitions.

<sup>2</sup> Note that a comparable argument may be made as regards the hypothetical extraction approach to selecting key sectors (cf. Strassert, 1968-69, Schultz, 1977, Dietzenbacher & van der Linden, 1997, for discussion and applications).

employment  $\mathbf{e}$ . The model solution for *total* regional or national value added (the scalar  $v$ ) is the following:

$$v = \mathbf{i}' \mathbf{v} = \mathbf{v}_c' \mathbf{x} = \mathbf{v}_c' (\mathbf{I} - \mathbf{A})^{-1} \mathbf{f} = \mathbf{v}_c' \mathbf{L} \mathbf{f} \quad (1)$$

where  $\mathbf{v}_c'$  represents a row with value added coefficients,  $\mathbf{A}$  the matrix with intermediate input coefficients, and  $\mathbf{L}$  the Leontief-inverse (see Oosterhaven, 1981, ch. 2, or Miller & Blair, 1985, ch. 2, for details).

From (1) it is evident that the value added multipliers  $\mathbf{v}_c' \mathbf{L}$  may only be multiplied with exogenous final demand  $\mathbf{f}$  and *not* with endogenous total output  $\mathbf{x}$ . When the latter is done, this unavoidably leads to the over-estimation of the importance of the sector at hand. The reason is that (1) assumes that the intermediate part of total output ( $\mathbf{Ax}$ ) is endogenously determined by the size of (mainly the other sectors') total output. Multiplying the total of  $\mathbf{x}$  with the value added multipliers  $\mathbf{v}_c' \mathbf{L}$  results in double counting the endogenous part ( $\mathbf{x} - \mathbf{f}$ ).<sup>3</sup>

Things get even more wrong when ad hoc estimates of causal forward effects are added to the so-called direct effect, which total is then multiplied with standard employment or value added multipliers in order to estimate the so-called backward impacts of a certain industrial complex or a certain project. This procedure easily leads to triple counting of effects. Besides the above-mentioned double counting of part of the direct effect with part of the backward effects, it will also lead to double counting of part of both the direct and the backward effects with the forward effects.<sup>4</sup>

The principal reason for these over-estimations is simple: multipliers are used outside the context of the model from which they are derived, i.e. the fact that each

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<sup>3</sup> When calculating the importance of Schiphol airport for Dutch employment, for instance, BCI/NEI (1997) forget that part of the backward employment effect actually occurs within the aviation industry that was already included in the platform-tied employment at Schiphol. This platform-tied employment was assumed exogenous, but is not, at least not entirely. As a consequence, part of backward employment is double counted.

<sup>4</sup> When evaluating the economic impact of a rail freight line from the port of Rotterdam to the Ruhr-area, for instance, Knight Wendling (1992) added the backward effects of the Rotterdam port industries on inland freight transport to the forward effects of the freight line on the Rotterdam port industries.

sector is partly dependent on growth impulses of other sectors is not reckoned with. The simple remedy "don't do it" is too naive. Practitioners will continue to need simple devices like multipliers, which they will unavoidably want to simply multiply with total direct employment or value added. Moreover, an alternative, two-sided dependency measure is needed to replace the standard one-sided backward and forward linkages measures to select key sectors for regional or national development. Our proposal is to solve both problems in one go.

### 3 THE REMEDY: USE NET MULTIPLIERS

The label *net multiplier* is used to indicate any multiplier that may rightfully be multiplied with (total) sectoral output, value added or employment without resulting in an overestimation of that sector's economic importance. More precisely, in the case of total sectoral output, the *Type I net total output multipliers* are defined as:  $\mathbf{i}' (\mathbf{I} - \mathbf{A})^{-1} \langle \mathbf{f}_c \rangle$ . Where  $\mathbf{i}' (\mathbf{I} - \mathbf{A})^{-1}$  are the standard total output multipliers and  $\langle \mathbf{f}_c \rangle$  is a diagonal matrix with the fractions of total sectoral output that may rightfully be considered exogenous (i.e.  $f_j / x_j$ ). Adding these latter fractions corrects for a sector's dependence on other sectors' growth impulses, i.e. it corrects for the endogeneity of total output.

In the case of value added and employment multipliers, the corresponding net multipliers need of course to be multiplied with total sectoral value added or employment, and not with total output. This means that the ordinary value added and employment multipliers first need to be standardized (see Oosterhaven, 1981, ch. 4; Miller and Blair, 1985, ch. 4) before the corresponding net multiplier can be formulated. This leads to the following definition of the *Type I net value added multipliers*:

$$\boldsymbol{\mu}_I' = \mathbf{v}_c' (\mathbf{I} - \mathbf{A})^{-1} \langle \mathbf{v}_c \rangle^{-1} \langle \mathbf{f}_c \rangle = \mathbf{v}_c' \mathbf{L} \langle \mathbf{v}_c \rangle^{-1} \langle \mathbf{f}_c \rangle \quad (2)$$

In (2),  $\langle \mathbf{f}_c \rangle$  is again the diagonal matrix with the sectoral final output ratios that secures the net character of the multipliers, while  $\langle \mathbf{v}_c \rangle^{-1}$  represents the diagonal

inverse of the sectoral value added ratios that secures the standardization with respect to sectoral value added. Thus, (2) indicates the economy-wide value added impact of one unit of value added of a specific sector, corrected for the partly endogenous character of that one unit.

When the standard Leontief model is extended with endogenous household consumption expenditures (see Oosterhaven, 1981, ch. 6; Batey, 1985), the *Type II net value added multipliers* are defined as:<sup>5</sup>

$$\mu_{II}' = \mathbf{v}_c' (\mathbf{I} - \mathbf{A} - \mathbf{Q})^{-1} \langle \mathbf{v}_c \rangle^{-1} \langle \mathbf{f}_c^* \rangle = \mathbf{v}_c' \mathbf{L}^* \langle \mathbf{v}_c \rangle^{-1} \langle \mathbf{f}_c^* \rangle \quad (3)$$

In (3),  $q_{ij}$  from  $\mathbf{Q}$  indicates the endogenous consumption expenditures on products from sector  $i$  paid for from incomes earned in sector  $j$  per unit of output in sector  $j$ , while  $\langle \mathbf{f}_c^* \rangle$  now represents what may be called the Type II final output ratios, which are defined as:  $\mathbf{f}^* = \mathbf{f} - \mathbf{Q}\mathbf{x}$ . Since consumption is now also endogenous, only  $\mathbf{f} - \mathbf{Q}\mathbf{x}$  remains as *exogenous* final demand.

Standard input-output analysis tells us that the standard Type II multipliers with  $\mathbf{L}^*$  are larger than the standard Type I multipliers with  $\mathbf{L}$  (provided of course that  $\mathbf{Q} > \mathbf{0}$ ). In case of the net multipliers, however, no such systematic relation can be found. This observation implies that commercially motivated extensions of the input-output model that aim at getting larger multipliers do not produce the required enlargement when net multipliers are used. This follows from the following property.

**Theorem I.** *The correctly weighted average of all sectoral net multipliers equals unity.*<sup>6</sup>

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<sup>5</sup> Type I and Type II net *employment* multipliers are defined analogously by replacing  $\mathbf{v}_c$  with  $\mathbf{e}_c$ , containing the sectoral employment/output ratios. Using this transformation, all statements on value added multipliers also apply to employment multipliers, and so on.

<sup>6</sup> Note that Theorem I in Oosterhaven & Stelder (2002a) does not speak of 'correctly weighted' but of 'output weighted', which is wrong. We thank Ron Miller for pointing this out.

‘Correctly’ means that employment multipliers should be weighted with employment shares, and so on. With  $v = \mathbf{i}' \mathbf{v}$ , the economy-wide total value added, Theorem I for Type I net value added multipliers implies that  $\boldsymbol{\mu}_I' (\mathbf{v} v^{-1}) = I$ .

Proof:  $\boldsymbol{\mu}_I' (\mathbf{v} v^{-1}) = \mathbf{v}_c' \mathbf{L} \langle \mathbf{v}_c \rangle^{-1} \langle \mathbf{f}_c \rangle \mathbf{v} v^{-1} = \mathbf{v}_c' \mathbf{L} \langle \mathbf{f} \rangle \langle \mathbf{v} \rangle^{-1} \mathbf{v} v^{-1} = \mathbf{v}' \langle \mathbf{v} \rangle^{-1} \mathbf{v} v^{-1} = I$

The proof for the Type II net multipliers  $\boldsymbol{\mu}_{II}'$  runs analogous with  $\mathbf{L}$  and  $\mathbf{f}$  replaced by  $\mathbf{L}^*$  and  $\mathbf{f}^*$ . The proof for the net output multipliers also runs analogous, but without the  $\mathbf{v}_c$  matrices and vectors, and with  $(\mathbf{v} v^{-1})$  replaced by  $(\mathbf{x} x^{-1})$ .

The above theorem precisely represents the first reason for developing the concept of the net multiplier. As a consequence, the net multipliers avoid the double counting of impacts, as also follows from the next property.

**Theorem II.** *Multiplied with the correct sectoral totals and summed over all sectors, net multipliers reproduce the exact total for the whole economy.*

‘Correct’ again means that employment multipliers should be multiplied with employment, and so on. For Type I net value added multipliers Theorem II implies  $\boldsymbol{\mu}_I' \mathbf{v} = v$ .

Proof:<sup>7</sup>  $\boldsymbol{\mu}_I' \mathbf{v} = \mathbf{v}_c' \mathbf{L} \langle \mathbf{v}_c \rangle^{-1} \langle \mathbf{f}_c \rangle \mathbf{v} = \mathbf{v}_c' \mathbf{L} \langle \mathbf{v} \rangle^{-1} \langle \mathbf{f} \rangle \mathbf{v} = \mathbf{v}_c' \mathbf{L} \langle \mathbf{f} \rangle \mathbf{i} = \mathbf{v}' \mathbf{i} = v$

The proofs for the other multipliers run analogous, as indicated above.

Economically, Theorem I and II underscore that sectors with multipliers smaller than one will be more dependent on other sectors, than those other sectors are dependent upon them. The most extreme case being, of course, a zero net multiplier. Given the above, the interpretation of this extreme is simple. Such sectors have a zero

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<sup>7</sup> Note that the order of reduction here is different from that in Oosterhaven and Stelder (2002a), which was not correct. We again thank Ron Miller for pointing this out.



(exogenous) final output, which indicates that they are *not* able to generate exogenous growth impulses themselves. This does not imply that these sectors are not important. It only implies that their growth is entirely dependent upon the impulses they receive from and through other sectors. For that reason, we do not want to label them as key sectors for regional or national economic development. In our definition, having large forward or backward linkages is not enough. A key sector also needs the ability to function as an ‘industrie motrice’ (Perroux, 1955).

#### 4 ON THE STABILITY OF NET VERSUS GROSS MULTIPLIERS

For policy purposes, one would of course primarily be interested in the future values of net multipliers and not in their past values. The same holds of course for the standard (gross) measures of backward linkages, which are used in the standard definition of key sectors. However, as noted by De Mesnard (2002), the ratios of exogenous demand to total output  $f_c$  are endogenous to the Leontief model, and this makes the net multiplier inherently unstable. De Mesnard considers this instability undesirable and proposes a ‘stable’ alternative. First, we will discuss this alternative and then consider whether the instability of the net multiplier really is a vice or whether it may be a virtue.

De Mesnard’s alternative net multipliers  $\mathbf{m}'$  (for total output) are attractively simple. They equal the indirect part of the corresponding standard (gross) multipliers and standard (gross) backward linkages, which means that they equal the standard output multiplier  $\mathbf{i}'\mathbf{L}$  minus the direct effect  $\mathbf{i}'$ :

$$\mathbf{m}' = \mathbf{i}' (\mathbf{A} + \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \dots) = \mathbf{i}' \mathbf{A} \mathbf{L} = \mathbf{i}' (\mathbf{L} - \mathbf{I}) = \mathbf{i}' \mathbf{L} - \mathbf{i}' \quad (4)$$

Obviously, whenever  $\mathbf{A}$  is stable  $\mathbf{m}'$  will be stable. However, as a solution to the twin problem discussed in section 2, this alternative has three disadvantages.

As already noted by De Mesnard,  $\mathbf{m}'$  has the same rank order and, one may add, the same absolute mutual sectoral differences as the standard (gross) output multiplier  $\mathbf{i}'\mathbf{L}$ . Thus, this alternative does not have any informational value above the

standard multiplier. The reason is, that it does not add any information a sector's dependency on *other* sectors. It only gives an alternative measure of the other sectors' dependency on the sector at hand, just like the standard gross multiplier and the standard definition of backward linkages derived there from.

Second, the above Theorems I and II are not valid for  $\mathbf{m}'$ . Multiplied with total output,  $\mathbf{m}'$  does not generate the actual size of the economy's aggregate output, as follows from: <sup>8</sup>

$$\mathbf{m}' \mathbf{x} = \mathbf{i}' (\mathbf{L} - \mathbf{I}) \mathbf{x} = \mathbf{i}' \mathbf{L} (\mathbf{x} - \mathbf{f}) = \mathbf{i}' \mathbf{L} \mathbf{A} \mathbf{x} \neq \mathbf{i}' \mathbf{L} \mathbf{f} = x \quad (5)$$

The result may be either larger or smaller, primarily depending on whether total intermediate output is larger or smaller than total final output. Consequently, the output weighted average of the alternative net multipliers  $\mathbf{m}' (\mathbf{x} \mathbf{x}^{-1})$  does not equal unity either. The latter will only be true in the rare case in which the output weighted gross multiplier precisely equals 2.

The economic weakness is that  $\mathbf{m}'$  does not adequately correct for the partial endogeneity of total output, because it does not really reckon with the two-sided dependency of sectors. Instead it simply subtracts the direct impact from the total impact, which does not solve the exogenous/endogenous variables mix-up nor the conceptual double counting of endogenous intermediate output.

Hence,  $\mathbf{m}'$  does not provide a real alternative for the twin problem tackled by the net multipliers defined in (2) and (3).

Nevertheless, De Mesnard (2002) rightfully draws attention to the inherent instability of our definition of a net multiplier. Indeed, any impulse to the exogenous demand of a certain sector  $j$  will increase the exogenous final output ratio of that sector ( $f_j / x_j$ ) and thus its net multiplier. Next, the output of all other sectors will increase indirectly, and this will decrease their final output ratios and thus their net multipliers. In the new equilibrium, however, both Theorems still hold true.

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<sup>8</sup> The same conclusion holds when  $\mathbf{m}'$  is applied to changes in total output as in the original definition (De Mesnard, 2002). This may easily be verified by replacing  $\mathbf{x}$  with  $\Delta \mathbf{x}$ .

More importantly, the fact that the net multiplier of the primarily affected sector increases, whereas all others will decrease, is precisely what the net multiplier is intended to pick up. The growth of the final output ratio of the sector at hand measures its larger ability to generate its own growth impulses, while the smaller final output ratios of the other sectors measure that they have become more dependent on the sector at hand. Thus, the net multiplier of the sector at hand should increase, while the others should decrease in the way indicated.

Finally, the fact that the standard (gross) multipliers are stable whenever  $\mathbf{A}$  and  $\mathbf{Q}$  are stable, precisely indicates that they are not the proper tool to use when measuring the *key character* of a sector for the regional or national development, neither before nor after a final demand impulse.

To study the implications of the instability deeper, first, consider a *closed* economy. In such an economy, assuming fixed input price ratios, the stability of the input coefficients, and thus that of the standard (gross) multipliers, is a technological feature. In a growing *open* regional or national economy, multiplier stability also implies the absence of import substitution, i.e. the absence of increased self-reliance through the increased size and diversity of the regional economy. Formulated more precisely: the stability of the standard multiplier also implies the stability of the import coefficients. This follows for an open economy  $r$  from the following definitional relationships:

$$a_{ij}^{rr} = m_{ij}^{rr} a_{ij}^{\bullet r} \quad \forall r, i, j \quad \text{or in matrix notation:} \quad \mathbf{A} = \mathbf{M} \otimes \mathbf{A}^t \quad (6)$$

with  $\otimes$  indicating the (element-by-element) Hadamard product, and:

$a_{ij}^{\bullet r}$  the regional technical coefficient, with  $\bullet$  indicating a summation over all regions of origin, and

$m_{ij}^{rr}$  the intra-regional trade or self-sufficiency coefficient, equaling one minus the regional import coefficient.

Stability of  $\mathbf{M}$ , however, is economically unlikely whenever the growth of exogenous demand is substantial, since larger economies, as a rule, are relatively more self-sufficient than smaller economies. The same holds for the export ratios that constitute the main part of the exogenous final output ratios  $\mathbf{f}_c$  in the net multiplier formulas (2) and (3). Whenever an economy grows the export part of those ratios will tend to decline because of increasing self-sufficiency.

One way to further analyze the implication of instability in input coefficients is using the Field of Influence approach of Sonis & Hewings (1992). This approach relates to the standard (gross) multipliers, but could easily be adapted for an application to our net multipliers. The approach is also useful because it can directly be tied in to the discussion on the definition of key sectors (Sonis, Hewings & Guo, 2000). We prefer to use a variant of the more traditional decomposition of economic growth, because it uses the economically constituent parts of the net multiplier from (6).

In the case of a Type I input-output model, we thus further analyze the instability of net multipliers by inspecting the average of the *polar decompositions* of output growth (see Oosterhaven & van der Linden, 1997, for the first application, and Dietzenbacher & Los, 1998, for a further discussion):<sup>9</sup>

$$\Delta x = \mathbf{i}' \mathbf{L}_1 \mathbf{F}_{c1} \mathbf{x}_1 - \mathbf{i}' \mathbf{L}_0 \mathbf{F}_{c0} \mathbf{x}_0 =$$

$$\frac{1}{4} \mathbf{i}' \Delta \mathbf{L} (\mathbf{F}_{c0} + \mathbf{F}_{c1}) (\mathbf{x}_0 + \mathbf{x}_1) + \frac{1}{4} \mathbf{i}' (\mathbf{L}_0 + \mathbf{L}_1) \Delta \mathbf{F}_c (\mathbf{x}_0 + \mathbf{x}_1) + \frac{1}{4} \mathbf{i}' (\mathbf{L}_0 + \mathbf{L}_1) (\mathbf{F}_{c0} + \mathbf{F}_{c1}) \Delta \mathbf{x} \quad (7a)$$

$$\text{with: } \Delta \mathbf{L} = \frac{1}{2} \mathbf{L}_1 \Delta \mathbf{M} \otimes (\mathbf{A}_0^t + \mathbf{A}_1^t) \mathbf{L}_0 + \frac{1}{2} \mathbf{L}_1 (\mathbf{M}_0 + \mathbf{M}_1) \otimes \Delta \mathbf{A}^t \mathbf{L}_0 \quad (7b)$$

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<sup>9</sup> The decomposition formula for a Type II input-output model or any other linear demographic model (cf. Batey, 1985) runs essentially analogous, but with added terms with  $\Delta \mathbf{Q}^t$  (changes in technical consumption coefficients) and  $\Delta \mathbf{M}^q$  (changes in intra-regional consumption trade coefficients).

where  $\mathbf{F}_c$  is the diagonal matrix with the “exogenous demand / total output” ratios,  $\mathbf{i}'\mathbf{L}_0\mathbf{F}_{c0}$  is the net multiplier at  $t = 0$ , and  $\mathbf{i}'\mathbf{L}_1\mathbf{F}_{c1}$  is the net multiplier at  $t = 1$ .

Clearly, (7) indicates that the change in aggregate output  $\Delta x$  can be attributed to:

- changes in the Leontief-inverse  $\Delta\mathbf{L}$ , which in turn depend on technological changes  $\Delta\mathbf{A}^t$  and changes in self-sufficiency ratios  $\Delta\mathbf{M}$  (i.e. import substitution),
- changes in the exogenous demand ratios  $\Delta\mathbf{F}_c$ , which in turn strongly depend on changes in the export ratios (i.e. export substitution), and
- changes in sectoral total output  $\Delta\mathbf{x}$  that are unjustly assumed exogenous, precisely for which misuse the net multiplier concept was developed.

When any of the above changes are assumed to be zero, the corresponding term disappears.

Many practitioners believe that the technical coefficients  $\mathbf{A}^t$  are stable, but that assumption only leads to the disappearance of the last term in (7b), and a simplification of its first term into  $\mathbf{L}_1 \Delta\mathbf{M} \otimes \mathbf{A}^t \mathbf{L}_0$ . However, this indicates that the standard (gross) multipliers  $\mathbf{i}'\mathbf{L}$  will still be unstable whenever the self-sufficiency ratios  $\mathbf{M}$  are changing, which will be the case whenever import prices or tariffs are changing or when an economy grows. The only difference with the net multipliers is that the exogenous demand ratios  $\mathbf{F}_c$  in the latter case may never be assumed to be stable, such because of the endogeneity of  $x_j$  in their denominators. The above decomposition, thus, emphasizes that the inherent instability of  $\mathbf{F}_c$  is only one of the sources of instability of the net multiplier. The other sources are the instability of the import ratios and the instability of the technical coefficients, which the net multiplier shares with the standard (gross) multiplier.

The only, but important difference and advantage is that the net multiplier concept forces the user to make assumptions about the interconnected changes in import and export ratios. In the case of the standard multipliers only the change in import ratios has to be reckoned with. But in that case, the practitioner too easily assumes those changes away, whereas he/she is forced to explicitly consider these whenever net multipliers are used.

## 5 CONCLUSION

Section 2 and 3 analyze why claims of economic importance based on standard (gross) multipliers are often misleadingly high, and why the key sector concept should be broadened to include not only the size of its forward and backward linkages, but also a sector's ability to generate autonomous growth. Using net multipliers is a remedy against this systematic upward bias and the neglect of the endogeneity of part of total output. About half of the net multipliers will be smaller than one and about the other half will be larger than one. This property gives a useful numerical expression to the notion that certain sectors may be more dependent on the rest of the economy than the rest of the economy is dependent on them. In this way, net multipliers serve as an alternative to the standard (one-sided dependency) way of finding key sectors for developing economies.

In section 4 we argue that the inherent instability of the net multiplier should be seen as a virtue and not as a vice. A strong exogenous growth of a certain sector, through either import substitution or export substitution, makes the rest of the economy more dependent on this sector. Thus, its own net multiplier ought to increase, while those of the other sectors ought to decrease. They should *not* remain stable. In fact, treating the standard multiplier as being stable may be seen as the real vice, since that practice assumes away the inherent instability of import ratios and even technical coefficients, whenever an economy is growing over time.

More generally, depending on the relative size of import versus export substitution, the net multiplier may either rise or fall, whereas the gross multiplier most probably only rises when the economy grows. This would imply that each individual economic activity, when evaluated in the standard (gross) way, over time becomes more important for the economy at large. This clearly does not make sense. Even though individual net multipliers change, their weighted average is constant and equal to unity. Thus, when the relative economic importance of individual economic activities is evaluated by means of the net multiplier no systematic upward or downward bias occurs, even though it changes over time.

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## REFERENCES

- Batey, Peter J 1985. "Input-Output Models for Regional Demographic-Economic Analysis: Some Structural Comparisons" *Environment and Planning A* 17: 77-93.
- BCI/NEI 1997. *Ruimtelijke-economische verkenning van de Toekomstige Nederlandse Luchtvaart Infrastructuur*. Nijmegen/Rotterdam: Buck Consultants International/Nederlands Economisch Instituut.
- Beyers, William B 1976. "Empirical identification of key sectors: some further evidence" *Environment and Planning A* 8/2: 231-6.
- Cole, Sam 1988. "The delayed impacts of plant closures in a reformulated Leontief model" *Papers of the Regional Science Association* 65: 135-149.
- Cuello, Frederico A, Fayçal Mansouri & Geoffrey J D Hewings 1992. "The Identification of Structure at the Sectoral level: a Reformulation of the Hirschman-Rasmussen Key Sector Indices" *Economic Systems Research* 4/4: 285-96.
- Dietzenbacher, Erik & Jan A van der Linden, 1997. "Sectoral and Spatial Linkages in the EC Production Structure" *Journal of Regional Science* 37: 235-57.
- Dietzenbacher, Erik & Bart Los, 1998. "Structural Decomposition Techniques: Sense and Sensitivity" *Economic Systems Research* 10/4: 307-23.
- Hirschman, A.O. 1958. *The Strategy of Economic Development*. New Haven; Yale University Press.
- Jackson, Randall W, Moss Madden & Harry A Bowman 1997. "Closure in Cole's Reformulated Leontief Model" *Papers in Regional Science* 76/1: 21-28.
- Knight Wendling Consulting 1992. *Macro-economische en maatschappelijke kosten-baten analyse van de Betuweroute*. Amsterdam: Rapport voor het Ministerie van V & W.
- Krugman, Paul 1991. *Geography and Trade*. London: MIT Press.
- Mesnard, Louis de 2002. "Note about the concept of "net multipliers" *Journal of Regional Science* 42/3: 545-8.

- Miller, Ronald E & Peter D Blair 1985. *Input-Output Analysis: Foundations and Extensions*. Englewood Cliffs: Prentice-Hall.
- Oosterhaven, Jan 1981. *Interregional Input-Output Analysis and Dutch Regional Policy Problems*, Aldershot, UK: Gower.
- Oosterhaven, Jan & Jan A van der Linden 1997. "European Technology, Trade and Income Changes for 1975-85: An Intercountry Input-Output Decomposition" *Economic Systems Research* 9/4: 393-411.
- Oosterhaven, Jan 2000. "Lessons from the debate on Cole's model closure" *Papers in Regional Science* 79/2: 233-42.
- Oosterhaven, Jan, Gerard J Eding & Dirk Stelder 2001. "Clusters, Linkages and Interregional Spillovers: Methodology and Policy Implications for the Two Dutch Mainports and the Rural North" *Regional Studies* 35/9: 809-22.
- Oosterhaven, Jan & Dirk Stelder 2002a. "Net Multipliers Avoid Exaggerating Impacts: With a bi-regional illustration for the Dutch transportation sector" *Journal of Regional Science* 3/42: 533-43.
- Oosterhaven, Jan & Dirk Stelder 2002b. "On the Economic Impact of the Transport Sector: A Critical Review with Dutch Bi-Regional Input-Output Data" in: G J D Hewings, M Sonis & D Boyce (eds) *Trade, Networks and Hierarchies: Modeling Regional and Interregional Economies*. Springer-Verlag, Berlin, 2002: 119-33
- Oosterhaven, Jan, Ed C van der Knijff & Gerard J Eding, 2003. "Estimating interregional economic impacts: an evaluation of nonsurvey, semisurvey, and fullsurvey methods" *Environment and Planning A* 35/1: 5-18.
- Perroux, Francois 1955. "Note sur la notion de pole de croissance" *Economique Appliquée* 1-2: 307-22.
- Rasmussen, P N 1957. *Studies in Intersectoral Relations*. Amsterdam: North-Holland.
- Schultz, Siegfried 1977. "Approaches to identifying key sectors empirically by means of input-output analysis" *Journal of Development Studies* 14/1: 77-94.
- Sonis, Michael & Geoffrey J D Hewings, 1992. "Coefficient change in input-output models: Theory and applications" *Economic Systems Research* 4/1: 143-57.
- Sonis, Michael, Geoffrey J D Hewings & Jiemin Guo, 2000. "A new image of classical key sector analysis: Minimum information decomposition of the Leontief inverse" *Economic Systems Research* 12/3: 401-23.
- Strassert, G 1968-69. "Zur Bestimmung Strategischer Sektoren mit Hilfe von Input-Output Modellen" *Jahrbücher für Nationalökonomie und Statistik* 182: 211-15.