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OPTIMAL MARKET DESIGN

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Abstract

This paper introduces three methodological advances to study the optimal design of static and dynamic markets. First, we apply a mechanism design approach to characterize all incentive-compatible market equilibria. Second, we conduct a normative analysis, i.e. we evaluate alternative competition and innovation policies from a welfare perspective. Third, we introduce a reliable way to measure competition in dynamic markets with non-linear pricing. We illustrate the usefulness of our approach in several ways. We reproduce the empirical finding that innovation levels are higher in markets with lower price-cost margins, yet such markets are not necessarily more competitive. Indeed, we prove the Schumpeterian conjecture that more dynamic markets characterized by higher levels of innovation should be less competitive. Furthermore, we demonstrate how our approach can be used to determine the optimal combination of market regulation and innovation policies such as R&D subsidies or a weakening of the patent system. Finally, we show that optimal markets are characterized by strictly positive price-cost margins.

keywords: competition policy, dynamic markets, competition measures, Schumpeter, mechanism design

J.E.L. codes: K21, L40, O31

1. Introduction

Recent antitrust legislation, including the Microsoft and Intel cases and the Glaxo/Wellcome merger, has rekindled the controversy whether dynamic sectors, where firms invest in R&D to improve products and production processes, should be treated differently from static sectors, where products and technologies are more or less given. Specifically, the question is whether mergers and anti-competitive conduct by firms, e.g. the use of exclusive contracting, tying, and bundling, should be evaluated more leniently for dynamic sectors. This question dates back to Schumpeter (1942) – the main reason no consensus has emerged is the lack of a tractable framework in which (i) the welfare effects of competition and innovation can be evaluated and (ii) competition can be measured in a reliable way.

Regarding (i), the current literature that studies the effects of competition on innovation is positive, not normative, in nature: if an industry becomes more competitive, does innovation increase? Schumpeter (1942) argued that monopoly power is needed to foster innovation. An opposing viewpoint was first expressed by Arrow (1962), who formalized the idea that competition stimulates innovation. Many subsequent theoretical papers surveyed by Vives (2008) and Schmutzler (2010) indeed find a positive relation between competition and innovation (although exceptions exist). This suggests that competition policy should be more aggressive for dynamic sectors: as in static sectors, more intense competition leads to lower prices and, hence, less deadweight loss, and, in addition, competition stimulates innovation. However, this inference is logically incorrect – a normative conclusion cannot follow from a positive observation.

The correct normative question is whether the welfare-maximizing competition intensity is higher in dynamic sectors. To underline that normative and positive implications need not coincide, below we introduce a model where more intense competition, in the sense of lower price-cost margins, leads to more innovation.[1] Yet, the welfare-maximizing competition intensity is lower in dynamic sectors.

Regarding (ii), how should competition intensity be measured in dynamic sectors? Evans & Schmalensee (2001), Gilbert & Tom (2001), Hahn (2001), Katz & Shelanski (2005), and Katz & Shelanski (2007) provide arguments why standard competition measures, including concentration, profits, and price-cost margins, are misleading in dynamic sectors. That is, high concentration, high profits, and a high price-cost margin do not necessarily imply a lack of

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[1]Although the empirical evidence is also somewhat mixed, recent papers that document a positive effect of competition on innovation/productivity include Aghion, Bloom, Blundell, Griffith & Howitt (2005), Bassanini & Ernst (2002), Galdon-Sanchez & Schmitz (2002), Nickell (1996), Symeonidis (2002), and Porter (1990).

competition. In order to measure competition in a meaningful and reliable way, we introduce an alternative index that applies to both static and dynamic industries. The intuitive idea behind our competition index is that more competitive industries exhibit more payoff inequality, i.e. profit differences across firms of varying efficiency levels grow when competition increases.

The main contribution of this paper is to introduce a framework in which the welfare effects of competition can be analyzed, taking into account the effects of competition on innovation. To allow for a general yet tractable analysis, we use a mechanism design approach to describe the set of market equilibria. We prove Schumpeter's conjecture that dynamic markets should be less competitive, even when more intense competition leads to more innovation. We further illustrate the usefulness of our approach by considering the interaction between competition regulation and innovation policies.[2] Finally, we show that optimal markets are characterized by strictly positive price-cost margins.

Our paper adds to a large and growing theoretical literature that considers the relation between competition and innovation, see, for instance, the surveys by Vives (2008) and Schmutzler (2010) and references therein. Compared to previous literature, our approach has several distinguishing features that we detail below: (i) we apply a mechanism design approach to characterize market equilibria, (ii) we evaluate alternative competition and innovation policies from a welfare perspective, and (iii) we introduce a novel way to measure competition. Importantly, we allow for the possibility of non-linear pricing since this is common practice in many dynamic sectors, e.g. the telecom, pharmaceutical, semi-conductor, software, PC, cable, and petroleum-refining industries. Also many static sectors, including the airline and cereal industries, frequently employ non-linear pricing. In contrast to prior literature, we therefore do not assume that firms are restricted to use linear prices.

This paper is organized as follows. Section 2 details the methodological advances of our approach. In Section 3 we reproduce the positive finding that innovation levels are higher in markets with lower price-cost margins. We define the planner's welfare-maximization problem and derive the optimal market structure. In Section 4 we consider implications for regulatory design and prove Schumpeter's conjecture that more dynamic markets should be less competitive. For ease of exposition we make a number of simplifying assumptions to establish these results, which are relaxed in Section 5. Section 6 concludes. All proofs are in the Appendix.

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[2]Segal & Whinston (2007) analyze how innovation is affected by antitrust policies that restrict incumbent behavior, e.g. the use of exclusive contracts or predatory behavior. The basic tradeoff is that the protection of entrants raises their profits initially but lowers their profits once they become incumbents. In general, the effect of entrant protection on their (discounted) profits and innovation levels is not clear cut but cases can be identified where it results in higher levels of innovation.

2. Methodological Advances

In this section we introduce three methodological innovations to study the design of static and dynamic markets. First, we conduct a normative analysis whereas most policy conclusions are currently derived from positive results, e.g. whether or not competition stimulates innovation. Second, to study the impact of competition on innovation and welfare in a general yet tractable way, we employ a mechanism design approach to characterize all market equilibria. Third, we introduce a robust measure of competition. Many papers that study the correlation between competition and innovation use the price-cost margin as a measure of competition. As we argue below, when firms can reduce their production costs and use non-linear pricing, the price-cost margin is not a reliably measure. We end this section with a simple example to illustrate why the informational rents needed to ensure incentive compatibility preclude perfectly-competitive market outcomes.

2.1. Normative Approach

The discussion whether competition policy should be less or more strict in dynamic sectors typically focuses on the positive question whether more intense competition stimulates innovation. For example, the "Schumpeterian school," e.g. Katz & Shelanski (2005) and Ahlborn, Denicolo, Geradin & Padilla (2006), argues that competition policy should be more permissive in dynamic sectors because market power fosters innovation. However, a normative analysis is needed to justify this conclusion, i.e. competition policy should be more permissive only if the welfare-maximizing level of competition is lower in dynamic sectors. To underline the importance of a normative approach, below we introduce a model that replicates the finding of most theoretical papers that more intense competition (in the sense of lower price-cost margins) leads to more innovation. Yet, we show that the welfare-maximizing competition intensity should be lower in dynamic sectors compared to static ones, and, hence, competition policy should be less (not more) strict.

2.2. Mechanism Design

The effect of competition on innovation is typically analyzed for specific market environments. For instance, a common approach is to assume Cournot or Betrand competition and determine the effects of increased product substitutability, see, e.g., Vives (2008), and references therein. Since a change in the substitution elasticity between products affects utilities, a welfare analysis

is not meaningful in this context. More importantly, even if we change the degree of competition without affecting utilities, e.g. by changing conduct in the market, the analysis typically becomes intractable without providing general insights.

To facilitate a more general analysis, we employ a mechanism design approach to characterize the set of market equilibria. Myerson's (1981) revelation principle implies that for each market form there is an equivalent direct mechanism in which it is optimal for a firm to reveal its true type θ, e.g. the firm's marginal cost, to receive equilibrium revenue, $R(\theta)$, and produce equilibrium output, $q(\theta)$. By specifying a welfare function that takes into account the costs and benefits of innovation, the mechanism design approach allows us to derive the optimal revenue and output schedules $\{R(\cdot), q(\cdot)\}$. Admittedly, a competition authority or regulator typically cannot dictate arbitrary output and revenue schedules. However, instruments that implement incremental changes are possible and the mechanism design approach allows the regulator to choose such adjustments in an optimal manner. This would not be possible by analyzing only two (or more) specific market forms such as Bertrand and Cournot competition.[3]

Indeed, as a consequence of comparing only two specific market forms, the current literature comes with ambiguous predictions regarding the effects of competition on innovation. For example, in a recent survey, Schmutzler (2010) stresses that some theoretical papers find that more competition leads to more innovation while others conclude the opposite. Because the models in the survey differ in a number of aspects it is hard to grasp what drives these opposite conclusions. In contrast, as we show below, the mechanism design approach employed in this paper provides clear-cut predictions about how the optimal level of competition varies in response to changes in underlying market parameters. Furthermore, it allows us to study the interaction between market regulation and innovation policies such as R&D subsidies.

2.3. Competition Measure

There are several reasons why the familiar price-cost margin is not suitable as a competition measure in dynamic markets. First, it may dramatically underestimate market power when firms can use non-linear pricing. To illustrate, consider a monopolist who uses a two-part tariff with marginal price equal to marginal cost and a fixed fee to appropriate consumer surplus. To conclude there is no market power simply because the price-cost margin is zero is incorrect.[4]

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[3]Suppose for certain parameters, Bertrand competition leads to higher welfare than Cournot competition. If the current level of competition is somewhere in between Bertrand and Cournot competition, it does not follow that a small step towards Bertrand competition will increase welfare.

[4]Calculating the price-cost margin with the fixed fee included does not resolve the problem – it would result in a positive price-cost margin suggesting there is a deadweight loss. However, depending on the model, there

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Second, in dynamic markets, firms can innovate to reduce their costs. Typically, conditional on cost, a high price-cost margin is interpreted as evidence of market power. However, conditional on price, a high price-cost margin signals efficiency. In other words, in dynamic markets, the price-cost margin is an ambiguous measure that cannot differentiate between market power and efficiency differences.

Third, standard competition measures can be misleading in dynamic industries where competition is often for the market rather than on the market.[5] That is, firms compete fiercely in terms of R&D and the firm with the best product is likely to capture most of the market. Hence, in terms of market shares, the product market is (very) concentrated. But this does not imply a lack of competition. Further, as mentioned above, in many dynamic industries (such as the software and pharmaceutical industries), marginal costs are close to zero resulting in price-cost margins close to 1. Finally, because R&D involves high risks, the return on a successful product will be high. Hence, profits in dynamic markets tend to be high, but, again, this does not imply a lack of competition since the ex ante expected return on R&D investments may be modest. Thus traditional competition measures such as concentration, price-cost margins, and profits can be misleading in dynamic sectors.

We therefore employ an alternative measure of competition, which is based on the idea that competition raises payoff inequality across different types of firms. Our competition measure is not restricted to markets and can also be applied to other environments, e.g. contests, auctions, tax schemes, etc. Let $F(\theta)$ denote the distribution of firms' types, with support $[\underline{\theta}, \bar{\theta}]$ and associated density $f(\theta)$, and let $\Pi_A(\theta)$ and $\Pi_B(\theta)$ denote the profit of a firm of type θ in environments A and B respectively. Throughout we use the convention that a higher θ reflects a more efficient type, e.g. a firm with lower production costs, so that $\Pi'(\theta) > 0$.

Definition 1. *Environment B is more competitive than environment A when $\Pi_B(\theta)$ is a convex transformation of $\Pi_A(\theta)$, or, equivalently, when $\mathcal{I}_B(\theta) \geq \mathcal{I}_A(\theta)$ for all θ, where*[6]

$$\mathcal{I}(\theta) \equiv \frac{\Pi''(\theta)}{\Pi'(\theta)} \tag{2.1}$$

Note that $\mathcal{I}(\theta)$ is unaffected when a constant is added to all payoffs, i.e. lump sum taxes have

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need not be such inefficiency.

[5]See, for instance, Ahlborn et al. (2006), Evans & Schmalensee (2001), Gilbert & Tom (2001), Katz & Shelanski (2005), and Katz & Shelanski (2007).

[6]To see that these are equivalent definitions, let $\phi : \mathbb{R}_+ \to \mathbb{R}_+$ denote an increasing function and let $\Pi_B(\theta) = \phi(\Pi_A(\theta))$. Then $\mathcal{I}_B = \mathcal{I}_A + (\phi''/\phi')\Pi'$, so $\mathcal{I}_B \geq \mathcal{I}_A$ if and only if ϕ is convex.

no effect. Likewise, $\mathcal{I}(\theta)$ is unaffected when all payoffs are multiplied by a constant, i.e. it is invariant with respect to changes in the unit of measurement.

To see how Definition 1 relates to payoff inequality, consider the familiar Lorenz curve

$$L(\theta) = \frac{\int_{\underline{\theta}}^{\theta} \Pi(\theta) dF(\theta)}{\int_{\underline{\theta}}^{\bar{\theta}} \Pi(\theta) dF(\theta)} \tag{2.2}$$

which is increasing in θ with $L(\underline{\theta}) = 0$ and $L(\bar{\theta}) = 1$. The basic idea behind our notion of more competition is that it results in an increase in payoff inequality, which means that the Lorentz curve shifts down.[7]

Proposition 1. *A more competitive environment results in more payoff inequality, i.e. $L_B(\theta) \leq L_A(\theta)$ for all θ.*

The next two examples demonstrate how $\mathcal{I}(\theta)$ can be used to define an increase in competition. The first example shows that, unlike the price-cost margin, our measure of competition can be applied to non-market contexts.[8]

Example 1 (progressive taxes). Consider an education-signaling model à la Spence, where workers make take-it-or-leave-it offers to firms consisting of an education level, e, and a wage, w. If the offer is accepted, the worker's payoff is $\Pi_w = (1 - \tau w)w - e/(2\theta)$ and the payoff of the firm is $\Pi_f = \theta - w$, otherwise the worker's payoff is $\Pi_w = -e/(2\theta)$ and the firm's payoff is $\Pi_f = 0$. Here $\theta \in [0,1]$ is the workers type or "ability," which is uniformly distributed and privately known, and $\tau < 3/4$ plays the role of a progressive tax. It is straightforward to derive the separating equilibrium for this setup: $e(\theta) = \theta^2(1 - \frac{4}{3}\tau\theta)$ and $w(\theta) = \theta$, resulting in worker profits of $\Pi_w = \theta(\frac{1}{2} - \frac{1}{3}\tau\theta)$. Hence, our competition measure is

$$\mathcal{I}(\theta) = \frac{1}{\theta - \frac{3}{4\tau}}$$

which is decreasing in τ, i.e. more progressive taxation corresponds to less intense competition between workers.

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[7]It should be stressed that a similar argument does not apply to the firms' profits directly. Indeed, an increase in competition does not preclude higher profits for (some) firms. Consider a homogeneous good duopoly with linear demand $p = 1 - q_1 - q_2$, where firm $i = 1, 2$ has constant marginal costs c_i. It is routine to verify that for c_1 substantially below c_2, firm 1 has higher profits under Bertrand competition than under Cournot competition, even though Bertrand is typically seen as more competitive.

[8]Boone (2008, p. 1250) defines the index $I(n) = \ln(C_n(q,n))$ where C denotes production costs, q output and n a firm's efficiency level. For the case of a market context, this index is related to the one of Definition 1.

Example 2 (Bertrand competition). Consider a model of Bertrand competition with $n \geq 1$ firms. A firm's constant marginal cost is given by $c = 1 - \theta$ where $\theta \in [0, 1]$ is uniformly distributed and privately known. Demand is $D(p) = (1 - p)^\alpha$ where $\alpha \geq 0$ reflects the demand elasticity ($\alpha = 0$ corresponds to the standard example of completely inelastic demand for 1 unit). It is straightforward to show that equilibrium prices are given by[9]

$$p(\theta) = 1 - \frac{n + \alpha - 1}{n + \alpha} \theta$$

with associated profits

$$\Pi(\theta) = \frac{(n + \alpha - 1)^\alpha}{(n + \alpha)^{\alpha+1}} \theta^{n+\alpha}$$

The competition index is readily calculated as

$$\mathcal{I}(\theta) = \frac{n + \alpha - 1}{\theta}$$

i.e. an increase in the number of firms (n) or in the elasticity of demand (α) raises competition.

2.4. Informational Rents

In the mechanism design approach of this paper, an important role is played by firms' informational rents needed to ensure incentive compatibility. In this section we illustrate with a simple example why these informational rents preclude perfectly-competitive outcomes.

Consider a homogeneous good industry with two firms, where each firm is equally likely to have an efficient cost function $c_l(q)$ or an inefficient cost function $c_h(q)$, where $c_h(q) \geq c_l(q)$ and $c'_h(q) > c'_l(q)$ for all possible output levels, q. Hence, there are three market constellations, $(c_l, c_l), (c_l, c_h), (c_h, c_h)$, which occur with probability 1/4, 1/2, and 1/4 respectively. In each case, the market equilibrium specifies the total level of output, $Q_{ll} > Q_{lh} > Q_{hh}$, which is split evenly between symmetric firms while in the asymmetric case the efficient firm has a larger market share $x > 1/2$.

Suppose the social planner wants to implement equilibrium outcomes that maximize consumer welfare. Standard intuition suggests this can be accomplished by imposing the following

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[9]The profit of a firm of type θ who acts as if of type θ' is given by

$$\Pi(\theta', \theta) = (1 - p(\theta'))^\alpha (p(\theta') - (1 - \theta))\theta'^{n-1}$$

Using the first order condition for θ' evaluated at $\theta' = \theta$, yields the optimal prices given in Example 2.

perfect-competition properties: (i) marginal costs are equalized between firms and (ii) price, or marginal utility, is set equal to marginal cost. However, this intuition is incorrect because it overlooks the informational rents needed to implement these properties in equilibrium.

Let t_l (t_h) denote the expected transfers to an efficient (inefficient) firm, where the expectation is taken over the competitor's type. The relevant constraints are the individual-rationality constraint for the inefficient firm

$$t_h - \tfrac{1}{2}c_h((1-x)Q_{lh}) - \tfrac{1}{2}c_h(\tfrac{1}{2}Q_{hh}) \geq 0$$

and the incentive-compatibility constraint for the efficient firm

$$t_l - \tfrac{1}{2}c_l(\tfrac{1}{2}Q_{ll}) - \tfrac{1}{2}c_l(xQ_{lh}) \geq t_h - \tfrac{1}{2}c_l((1-x)Q_{lh}) - \tfrac{1}{2}c_l(\tfrac{1}{2}Q_{hh})$$

The planner's problem can be written as

$$\max_{Q_{ll},Q_{lh},Q_{hh},x} \left\{ \tfrac{1}{4}U(Q_{ll}) + \tfrac{1}{2}U(Q_{lh}) + \tfrac{1}{4}U(Q_{hh}) - t_h - t_l \right\}$$

subject to the above two constraints. To maximize consumer surplus, transfers are minimized and chosen such that the above constraints hold with equality. Solving for the transfers in this manner, the first-order conditions for $Q_{ll}, Q_{lh}, Q_{hh}, x$ can be written as

$$U'(Q_{ll}) = c_l'(\tfrac{1}{2}Q_{ll})$$
$$U'(Q_{lh}) = xc_l'(xQ_{lh}) + (1-x)c_h'((1-x)Q_{lh}) + (1-x)\left\{c_h'((1-x)Q_{lh}) - c_l'((1-x)Q_{lh})\right\}$$
$$U'(Q_{hh}) = c_h'(\tfrac{1}{2}Q_{hh}) + \left\{c_h'(\tfrac{1}{2}Q_{hh}) - c_l'(\tfrac{1}{2}Q_{hh})\right\}$$
$$c_l'(xQ_{lh}) = c_h'((1-x)Q_{lh}) + \left\{c_h'((1-x)Q_{lh}) - c_l'((1-x)Q_{lh})\right\}$$

Note that the terms between the curly brackets are strictly positive. Hence, unless both firms are efficient, price does not equal marginal cost. Moreover, when there is an efficient and inefficient firm, the market is not split efficiently, i.e. marginal costs are not equalized. In other words, equilibrium outcomes with the aforementioned perfect-competition properties are not optimal, i.e. they do not maximize consumer surplus. The reason is that implementing these outcomes requires high transfers, or informational rents, which lowers consumer surplus.

Generalizing the example to more than two firms and more than two efficiency levels quickly becomes intractable. This is not the case for the model discussed below, which assumes a continuum of types.

3. A Mechanism Design Approach

We use a two-stage model of R&D and market competition. In the first stage, firms invest in R&D, which determines their efficiency levels, or production costs, for the second stage in which there is market competition. In particular, the cost to a firm of type θ to achieve efficiency level n is given by $\Psi(n - \theta)$, where $\Psi : \mathbb{R}_+ \to \mathbb{R}_+$ is increasing so that, ceteris paribus, a higher type corresponds to a lower R&D cost. In the market-competition stage, firms choose outputs given their efficiency levels. In particular, when a firm with efficiency level n produces an output q, its production costs are $C(q - n)$, where $C : \mathbb{R}_+ \to \mathbb{R}_+$ is increasing so that, ceteris paribus, a higher efficiency level corresponds to a lower production cost.

Let $q(\theta)$ denote the amount of output that consumers purchase from a firm with type θ. Then consumers' (gross) utility is given by $\int U(q(\theta)) dF(\theta)$, with $U(0) = 0$, $U'(q) > 0$, and $U''(q) \leq 0$. If $U''(q) < 0$ then products are differentiated and consumers value variety, while the linear case $U''(q) = 0$ means products are perfect substitutes.

Assumption 1. *Marginal production costs, $C'(\cdot)$, and marginal R&D costs, $\Psi'(\cdot)$, are increasing and convex. Or, equivalently, the "supply functions" $\psi(\cdot) \equiv \Psi'^{(-1)}(\cdot)$ and $c(\cdot) \equiv C'^{(-1)}(\cdot)$ are increasing and concave.*

Note that, roughly speaking, the requirement is that R&D and production costs are quadratic or more convex.

Assumption 2. *The hazard rate $f(\theta)/(1 - F(\theta))$ is non-decreasing.*

A non-decreasing hazard-rate is equivalent to log-concavity of the density $f(\cdot)$, and, hence, the distribution function, $F(\cdot)$. The class of log-concave densities includes many well-known and commonly-used densities, e.g. the normal, uniform, and exponential densities.

Assumptions 1 and 2 can be relaxed considerably, see the discussion in Section 5. They are imposed here to streamline the presentation, in particular, to ensure uniqueness and regularity of the optimal market structure.

The output a firm chooses depends not only on its efficiency level but also on the market structure the regulator offers, as characterized by the revenue-output menu, $\{R(\cdot), q(\cdot)\}$. So when a firm chooses a production level q its revenue is $R(q)$, where $R(\cdot)$ can be non-linear, as is the case, for instance, with two-part or multi-part pricing. Since firms' types, their R&D investments, and their efficiency levels are unobservable, the regulator needs to take into

account firms' incentive constraints at both the innovation and market-competition stages when choosing the market structure $\{R(\cdot), q(\cdot)\}$.[10]

To analyze firms' incentive constraints, we invoke the revelation principle and consider the direct mechanism where instead of choosing an efficiency level and output, a firm of type θ simply reports θ_1 (θ_2) in the first (second) stage to maximize:

$$\Pi(\theta) \;=\; \max_{\theta_1, \theta_2} \big\{ R(q(\theta_2)) - C(q(\theta_2) - n(\theta_1)) - \Psi(n(\theta_1) - \theta) \big\} \tag{3.1}$$

This yields the first-order conditions

$$0 \;=\; R'(q(\theta)) - C'(q(\theta) - n(\theta)) \tag{3.2}$$

$$0 \;=\; C'(q(\theta) - n(\theta)) - \Psi'(n(\theta) - \theta) \tag{3.3}$$

The first-order condition (3.2) can be simplified by differentiating a firm's net profit

$$\Pi(\theta) \;=\; R(q(\theta)) - C(q(\theta) - n(\theta)) - \Psi(n(\theta) - \theta) \tag{3.4}$$

with respect to θ, which together with the first-order condition (3.3) yields $\Pi'(\theta) = \Psi'(n(\theta) - \theta)$. Note that this also follows more directly from an Envelope Theorem argument applied to $\Pi(\theta)$ in (3.1). Firms' incentive constraints (3.2) and (3.3) can thus be neatly summarized as:

$$\Pi'(\theta) \;=\; \Psi'(n(\theta) - \theta) \;=\; C'(q(\theta) - n(\theta)) \tag{3.5}$$

These incentive constraints can easily be inverted to derive the equilibrium R&D and output levels. To simplify notation we define the marginal profit schedule $\pi(\theta) \equiv \Pi'(\theta)$.

Proposition 2. *Incentive-compatible R&D and market outputs are characterized by*

$$n(\theta) \;=\; \theta + \psi(\pi(\theta)) \tag{3.6}$$

$$q(\theta) \;=\; \theta + c(\pi(\theta)) + \psi(\pi(\theta)) \tag{3.7}$$

and are non-decreasing in θ.

Note that firms' innovation and output levels are uniquely determined by the marginal profit

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[10]In most regulation models, e.g. Laffont & Tirole (1993), it is assumed that the regulated firm reveals its type θ after which efficiency n, output q, and revenue R are determined. In a market context this assumption is less appealing. Below we therefore allow a firm with efficiency $n(\theta_1)$ to choose output $q(\theta_2)$.

schedule $\pi(\theta)$. Furthermore, setting the profit of the lowest type, $\underline{\theta}$, to zero,[11] the marginal schedule determines firms' net profits as $\Pi(\theta) = \int_{\underline{\theta}}^{\theta} \pi(t)dt$ and, hence, revenues as

$$R(\theta) = \int_{\underline{\theta}}^{\theta} \pi(t)dt + \Psi(\psi(\pi(\theta))) + C(c(\pi(\theta))) \tag{3.8}$$

To summarize, the market mechanism $\{R(\cdot), q(\cdot)\}$ is uniquely characterized by the marginal profit schedule. We show below how to choose $\pi(\cdot)$ to maximize welfare.

First, we demonstrate that our model reproduces the positive relation between competition and innovation typically reported in the empirical literature where competition is measured using the price-cost margin.

Proposition 3. *Innovation levels are higher in markets with lower price-cost margins.*

With linear pricing, a lower price-cost margin implies more competition and higher consumer surplus. With non-linear pricing this is not necessarily the case, e.g. when a monopolist uses a two-part tariff both the price-cost margin and consumer surplus may be zero. The correct way to measure competition in this case is to use Definition 1. As we show in the next section this results in the opposite conclusion, i.e. innovation levels are higher with less competition.

3.1. A Normative Approach

Consider the regulator's problem of maximizing some welfare standard subject to the incentive constraints in (3.5). We assume the regulator puts less weight, $\beta \leq 1$, on producer surplus than on consumer surplus.[12] As a consequence, the regulator will optimally set the profit of the lowest-type firm to zero as assumed above. The planner's problem is to maximize welfare:

$$
\begin{aligned}
W \;=\; & \int_{\underline{\theta}}^{\overline{\theta}} \Big\{ U(q(\theta)) - R(q(\theta)) + \beta(R(q(\theta)) - C(q(\theta) - n(\theta)) - \Psi(n(\theta) - \theta)) \Big\} dF(\theta) \\
& + \int_{\underline{\theta}}^{\overline{\theta}} \lambda(\theta)\big(R'(q(\theta)) - C'(q(\theta) - n(\theta))\big) d\theta \\
& + \int_{\underline{\theta}}^{\overline{\theta}} \mu(\theta)\big(\Psi'(n(\theta) - \theta) - C'(q(\theta) - n(\theta))\big) d\theta \\
& + \eta\, \Pi(\underline{\theta})
\end{aligned}
\tag{3.9}
$$

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[11]We show below that it is welfare maximizing to do so.

[12]To illustrate, the DOJ and FTC explicitly state that their "fundamental goals" are "enhancing consumer welfare and promoting innovation," see DOJ & FTC (2007).

with respect to $\{R(\cdot), q(\cdot)\}$. The multipliers $\lambda(\cdot), \mu(\cdot)$ implement the firms' incentive constraints and η implements the zero-profit condition for the lowest type, $\underline{\theta}$.

In the proof of Proposition 4, see the Appendix, we derive the necessary first-order conditions by optimizing (3.9). Here we follow a simpler approach by using the incentive-compatible outcomes of Proposition 2 and the expression for revenue in (3.8), which enables us to write welfare as

$$W = \int_{\underline{\theta}}^{\overline{\theta}} W(\theta, \pi(\theta)) dF(\theta)$$

where

$$W(\theta, \pi) = U(\theta + c(\pi) + \psi(\pi)) - C(c(\pi)) - \Psi(\psi(\pi)) - (1 - \beta) \frac{1 - F(\theta)}{f(\theta)} \pi \qquad (3.10)$$

Note that $W(\theta, 0) > 0$,[13] i.e. there is a social gain even when a firm does not innovate, $n(\theta) = \theta$. Here we focus on the case where firms of all types are R&D active,[14] i.e. $n(\theta) > \theta$, and discuss extensions in Section 5.

Assumption 3. *Firms of all types are R&D active.*

The optimal marginal profit schedule follows by maximizing $W(\theta, \pi)$ with respect to π, taking into account the incentive-compatibility constraints that R&D and market outputs be increasing, see Proposition 2. Note that outputs are increasing if the marginal profit schedule is, a condition that is readily established when goods are substitutes.

Assumption 4. *Products are substitutes, $U(q) = q$.*

As we show in the proof of Proposition 4, with substitutes, the necessary first-order condition for welfare maximization implies that the marginal profit is increasing.

Proposition 4. *Under Assumptions 1-4, the optimal market mechanism is characterized by the marginal profit schedule, $\pi(\theta)$, that is the unique solution to*

$$\left(c'(\pi(\theta)) + \psi'(\pi(\theta))\right)\left(1 - \pi(\theta)\right) = (1 - \beta) \frac{1 - F(\theta)}{f(\theta)} \qquad (3.11)$$

Furthermore, $\pi(\theta)$ is positive and increasing.

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[13]Here we assume that $C(0) = C'(0) = 0$ and $\Psi(0) = \Psi'(0) = 0$. In particular, there are no fixed costs.

[14]This condition is met, for instance, when also $C''(0) = \Psi''(0) = 0$.

It is important to point out that the "standard" Assumptions 1-4 (convexity of costs, non-decreasing hazard rate, all firms are R&D active, and substitute products) have been imposed to simplify the analysis, not because they are necessary conditions for any of the results that follow. In Section 5 we discuss extensions that relax Assumptions 1-4.

One easy corollary to Proposition 4 is that due to firms' information rents needed to guarantee incentive compatibility, optimal markets are characterized by positive price-cost margins.

Proposition 5. *Optimal markets have strictly positive price-cost margins.*

To understand this result recall that the price-cost margin is $(U'(q) - C'(q - n))/U'(q)$, which, using Assumption 4 and the incentive-compatible outputs of Proposition 2, can be written as $1 - \pi(\theta)$. The necessary condition (3.11) then implies a strictly positive price-cost margin for all $\theta < \bar{\theta}$.[15]

Furthermore, (3.11) implies that price-cost margins are lower in dynamic than in static markets for which $\psi(\pi(\theta)) \equiv 0$. However, as we show next this implies that optimal dynamic markets are *less* (not more) competitive, i.e. the price-cost margin provides the wrong information about the degree of competition in a market.

4. Implications for Market Design

In this section, we explore several comparative statics properties of the optimal market structure. We first note a useful relation between the competition index of Definition 1 and the price-cost margin.

Proposition 6. *The marginal profit schedule is decreasing in the competition index*

$$\pi(\theta) = \exp\left(-\int_\theta^{\bar{\theta}} \mathcal{I}(t)dt\right) \tag{4.1}$$

where $\mathcal{I}(\theta) \geq 0$ for all $\underline{\theta} \leq \theta \leq \bar{\theta}$, and it is decreasing in the price-cost margin

$$\pi(\theta) = 1 - PCM(\theta) \tag{4.2}$$

Hence, an increase (decrease) in the optimal marginal profit schedule, which reflects a decrease (increase) in competition, results in a lower (higher) price-cost margin.

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[15]Boone (2009) makes this point for static industries (possibly with a small number of firms) and shows how the optimal price-cost margin varies with entry costs.

In other words, the competition index of Definition 1 results in *opposite* predictions regarding the degree of competitiveness in the market than the price-cost margin. The reason is that with non-linear pricing, the price-cost margin overstates the positive effects of higher outputs and understates the negative effects of higher firm profits. We illustrate this point in the next section.

4.1. Consumer versus Producer Surplus

Intuitively, if the regulator puts more weight on producer surplus then firms will be better off and consumers will be worse off. This comparative static exercise can be used to demonstrate the shortcomings of the price-cost margin in markets with non-linear pricing. From (3.11), an increase in β implies that the marginal profit schedule, $\pi(\theta)$, rises and the price-cost margin therefore falls. In other words, when more weight is placed on producer surplus, the price-cost margin erroneously indicates that the market becomes more competitive. In contrast, the competition measure of Definition 1 predicts a decrease in competition.

To glean some intuition, it is useful to draw an analogy with the situation where a monopolist sets price equal to marginal cost in a world with two-part tariffs. In our model, as β rises, innovation levels and output levels go up since $n(\cdot)$ and $q(\cdot)$ are increasing in π. However, firms' profits go up as well and consumer surplus *falls*, just as in the example where a monopolist uses a two-part tariff.

To show that consumer surplus falls with β in our model, recall that consumer surplus is given by

$$CS = W - \beta \int_{\underline{\theta}}^{\bar{\theta}} (1 - F(\theta))\pi(\theta)d\theta$$

Differentiating with respect to β yields

$$\frac{dCS}{d\beta} = -\beta \int_{\underline{\theta}}^{\bar{\theta}} (1 - F(\theta))\frac{d\pi(\theta)}{d\beta}d\theta$$

which is negative since $\pi(\theta)$ is increasing in β. The intuition is that while outputs rise, and, hence, so does $U(q(\theta))$, incentive compatibility requires that firms' revenues $R(q(\theta))$ rise even faster, and overall consumer surplus falls.

The lower consumer surplus is the sign of a *less* competitive market, in contrast with the prediction of the price-cost margin. The measure of competition in Definition 1 supports this conclusion: an increase in β raises $\pi(\theta)$, which implies a lower index $\mathcal{I}(\theta)$, see (4.1).
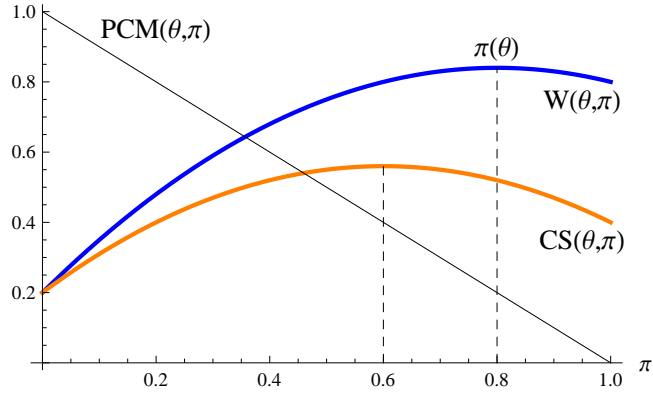
Figure 1: Total welfare, $W(\theta, \pi)$, consumer welfare, $CS(\theta, \pi)$, and the price-cost margin, $PCM(\theta, \pi)$, for fixed $\theta = 1/5$ as a function of the marginal profit, π. The welfare-maximizing marginal profit is denoted $\pi(\theta)$.

Example 3 (quadratic costs). Suppose R&D and production costs are quadratic, $\Psi(z) = C(z) = \frac{1}{2} z^2$, and firms' types are uniform on $[0, 1]$. The optimal marginal profit schedule is readily calculated as

$$\pi(\theta) = 1 - \frac{1}{2}(1 - \beta)(1 - \theta)$$

The conditions of Proposition 2 simplify to:

$$q(\theta) = \theta + 2 - (1 - \beta)(1 - \theta)$$

$$n(\theta) = \theta + 1 - \frac{1}{2}(1 - \beta)(1 - \theta)$$

and the price-cost margin is $PCM(\theta) = \frac{1}{2}(1 - \beta)(1 - \theta)$. Note that an increase in β lowers the price-cost margin and raises outputs. At the same time, an increase in β raises firms' profits and *lowers* consumer welfare. The correct measure of competition is

$$\mathcal{I}(\theta) = \frac{1}{\frac{1+\beta}{1-\beta} + \theta}$$

which is *decreasing* in β, i.e. a higher weight on producer surplus results in less competition.

The non-monotonic relationship between the price-cost margin and (consumer) welfare is illustrated in Figure 1, which is based on the parameters of Example 3. For low values of π, the price-cost margin falls with π and consumer welfare rises. However, for high enough values of π, both the price-cost margin and consumer welfare fall with π. The intuition is that firms' informational rents, which have to be paid for by the consumers, grow with π. In other words,

while further increases in π result in more output, increasingly higher revenues are required to keep these high outputs incentive compatible. The welfare-maximizing level of the marginal profit is denoted by $\pi(\theta)$ in Figure 1. Note that at this level, the price-cost margin and consumer welfare move in the same direction, i.e. a further decrease in the price-cost margin corresponds to *lower* consumer welfare.

4.2. Hazard-Rate Dominance

As a second comparative static, suppose the type distribution $F(\cdot)$ *hazard-rate dominates* $G(\cdot)$, i.e. $f(\theta)/(1 - F(\theta)) \leq g(\theta)/(1 - G(\theta))$ for all θ. Recall that hazard-rate dominance implies first-order stochastic dominance,[16] so there is a shift towards higher types under $F(\cdot)$ compared to $G(\cdot)$. In other words, the type distribution $F(\cdot)$ describes a more competitive situation.

Note that when the hazard rate falls, the right side of (3.11) increases and, hence, the optimal marginal profit schedule falls. The price-cost margin $PCM(\theta)$ therefore predicts a *decrease* in competition, while $\mathcal{I}(\theta)$ correctly predicts an *increase* in competition, see (4.1).

4.3. Schumpeter's Conjecture for Dynamic Markets

We say an industry is more dynamic when marginal R&D costs are lower. One tractable way to parameterize this is to scale the R&D cost function $\Psi_\nu(z) = \nu\, \Psi(z/\nu)$ where $\nu > 0$, so that $\Psi'_\nu(z) = \Psi'(z/\nu)$, and an increase in ν lowers the marginal R&D costs (as Ψ is convex). In other words, a higher ν corresponds to a more dynamic market and the limit $\nu = 0$ corresponds to a static market in which innovation is prohibitively costly. The necessary condition (3.11) becomes:

$$\big(\nu\, \psi'(\pi_\nu(\theta)) + c'(\pi_\nu(\theta))\big)\big(1 - \pi_\nu(\theta)\big) \;=\; (1 - \beta)\frac{1 - F(\theta)}{f(\theta)} \tag{4.3}$$

where we used that $\psi_\nu(\cdot) = \nu\, \psi(\cdot)$.

Proposition 7 (Schumpeter). *More dynamic markets should be less competitive.*

The logic is that for a more dynamic market with a higher ν, the marginal profit schedule, $\pi_\nu(\theta)$, has to rise in order to maintain the equality in (4.3) since the left side is decreasing in $\pi_\nu(\theta)$. Using (4.1) this implies that the competition index $\mathcal{I}_\nu(\theta)$ has to fall with ν.

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[16]Since $\int_{\underline{\theta}}^\theta f(x)/(1 - F(x))dx = \log(1/(1 - F(\theta))) \leq \log(1/(1 - G(\theta)))$ so $F(\theta) \leq G(\theta)$ for all θ.

Example 3 (continued). It is straightforward to solve for the optimal marginal profit schedule

$$\pi_\nu(\theta) = \frac{\beta + \nu + (1 - \beta)\theta}{1 + \nu}$$

which is increasing in ν, resulting in a competition index that is decreasing in ν

$$\mathcal{I}_\nu(\theta) = \frac{1}{\frac{\nu + \beta}{1 - \beta} + \theta}$$

4.4. Spillovers

Above we defined dynamic markets in terms of lower marginal R&D costs resulting in higher levels of innovation. Another characteristic feature of dynamic markets is that spillovers are important: knowledge or efficiency gains realized by one firm benefit others and have a positive effect for the sector as a whole. Of course, spillovers may reduce a firm's incentive to innovate since others can free ride on its efforts. Another possible interpretation of knowledge spillovers is therefore one of weak patents. The question we want to address is whether an industry with more knowledge spillovers should be more (or less) competitive. Or, equivalently, should an industry with weaker patent protection be more (or less) competitive?[17]

Let $N \equiv \int_{\underline{\theta}}^{\overline{\theta}} n(\theta)dF(\theta)$ denote the average knowledge generated by the industry. Then given a firm's efficiency level and the average knowledge in the sector, production costs are:

$$C\big(q - (1 - \alpha)n - \alpha N\big)$$

Hence spillovers are increasing in α and the limit $\alpha = 0$ corresponds to the case of no spillovers, e.g. when patents offer perfect protection from imitation. The planner's problem is to maximize

$$
\begin{aligned}
W = {}& \int_{\underline{\theta}}^{\overline{\theta}} \big\{ U(q(\theta)) - (1 - \beta)\Pi(\theta) - C(q(\theta) - (1 - \alpha)n(\theta) - \alpha N) - \Psi(n(\theta) - \theta)) \big\} dF(\theta) \\
& + \int_{\underline{\theta}}^{\overline{\theta}} \lambda(\theta)\big(\pi(\theta) - \Psi'(n(\theta) - \theta)\big)d\theta \\
& + \int_{\underline{\theta}}^{\overline{\theta}} \mu(\theta)\big(\Psi'(n(\theta) - \theta) - (1 - \alpha)C'(q(\theta) - (1 - \alpha)n(\theta) - \alpha N)\big)d\theta \\
& + \xi\Big(N - \int_{\underline{\theta}}^{\overline{\theta}} n(\theta)dF(\theta)\Big)
\end{aligned}
$$

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[17] See DOJ & FTC (2007) for a discussion about the interaction between competition policy and intellectual property rights.

Firms' incentive constraints

$$\pi(\theta) = \Psi'(n(\theta) - \theta) = (1-\alpha)C'(q(\theta) - (1-\alpha)n(\theta) - \alpha N)$$

can be inverted to derive the incentive-compatible market outputs (as in Proposition 2)

$$n(\theta) = \theta + \psi(\pi(\theta))$$

$$q(\theta) = (1-\alpha)(\theta + \psi(\pi(\theta))) + c\left(\frac{\pi(\theta)}{1-\alpha}\right) + \alpha \int_{\underline{\theta}}^{\bar{\theta}} (t + \psi(\pi(t)))dF(t)$$

Using these, welfare can be simplified as $W = \int_{\underline{\theta}}^{\bar{\theta}} W(\theta, \pi(\theta))dF(\theta)$ with

$$W(\theta, \pi) = \theta + c\left(\frac{\pi}{1-\alpha}\right) + \psi(\pi) - C\left(c\left(\frac{\pi}{1-\alpha}\right)\right) - \Psi(\psi(\pi)) - (1-\beta)\frac{1-F(\theta)}{f(\theta)}\pi$$

where we used the substitutes assumption $U(q) = q$.

Proposition 8. *With spillovers, the optimal market mechanism is characterized by the unique solution to*

$$\psi'(\pi_\alpha(\theta))\left(1 - \pi_\alpha(\theta)\right) + \frac{1}{1-\alpha}c'\left(\frac{\pi_\alpha(\theta)}{1-\alpha}\right)\left(1 - \frac{\pi_\alpha(\theta)}{1-\alpha}\right) = (1-\beta)\frac{1-F(\theta)}{f(\theta)} \qquad (4.4)$$

with $\pi_\alpha(\theta)$ positive and increasing. Furthermore, weakening perfect ($\alpha = 0$) patent protection is socially beneficial.

The effect of increased spillovers (or weaker patents) on the optimal market structure is complicated by the fact that the marginal profit is not fixed to 1 for the highest possible type, $\bar{\theta}$. From (4.4) it is easy to see that $1 - \alpha < \pi(\bar{\theta}) < 1$, so the relation between the marginal profit schedule and the competition index in (4.1) is no longer valid. For the specific setup of Example 3, however, it is straightforward to determine the optimal combination of innovation policy and market design.

Example 3 (continued). The optimal marginal profit schedule

$$\pi_\alpha(\theta) = \frac{(1 + \beta(1-\alpha))(1-\alpha)}{1 + (1-\alpha)^2} + \frac{(1-\beta)(1-\alpha)^2}{1 + (1-\alpha)^2}\theta$$

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is increasing in α, resulting in a competition index that is decreasing in α

$$\mathcal{I}_\alpha(\theta) \;=\; \frac{1}{\frac{\frac{1}{1-\alpha}+\beta}{1-\beta}+\theta}$$

In other words, weaker patents should be accompanied with less aggressive market competition.

4.5. R&D Subsidies

Besides competition policy there are a number of other instruments the regulator can use to stimulate innovation, e.g. R&D subsidies in the form of tax-breaks to firms that are paid for by the consumers. Subsidizing a fraction σ of firms' R&D costs changes firms' incentive constraints to $\pi(\theta) = (1-\sigma)\Psi'(n-\theta)$, and requires taxation on consumers:

$$W = \int_{\underline{\theta}}^{\overline{\theta}} \left\{ U(q(\theta)) - (1-\beta)\Pi(\theta) - C(q(\theta) - n(\theta)) - \Psi(n(\theta) - \theta) \right\} dF(\theta)$$

$$+ \int_{\underline{\theta}}^{\overline{\theta}} \lambda(\theta)\big(\pi(\theta) - (1-\sigma)\Psi'(n(\theta) - \theta)\big) d\theta$$

$$+ \int_{\underline{\theta}}^{\overline{\theta}} \mu(\theta)\big((1-\sigma)\Psi'(n(\theta) - \theta) - C'(q(\theta) - n(\theta))\big) d\theta$$

Firms' incentive constraints

$$\pi(\theta) \;=\; (1-\sigma)\Psi'(n(\theta) - \theta) \;=\; C'(q(\theta) - n(\theta))$$

can be inverted to simplify the welfare function to

$$W(\theta, \pi) \;=\; \theta + \psi\big(\frac{\pi}{1-\sigma}\big) + c(\pi) - C(c(\pi)) - \Psi\big(\psi\big(\frac{\pi}{1-\sigma}\big)\big) - (1-\beta)\frac{1-F(\theta)}{f(\theta)}\pi$$

where we used the substitutes condition as before.

Proposition 9. *With subsidies, the optimal market mechanism is characterized by the unique solution to*

$$\frac{1}{1-\sigma}\psi'\big(\frac{\pi_\sigma(\theta)}{1-\sigma}\big)\big(1 - \frac{\pi_\sigma(\theta)}{1-\sigma}\big) + c'(\pi_\sigma(\theta))\big(1 - \pi_\sigma(\theta)\big) \;=\; (1-\beta)\frac{1-F(\theta)}{f(\theta)} \qquad (4.5)$$

with $\pi_\sigma(\theta)$ positive and increasing. Furthermore, small subsidies are socially beneficial.

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Example 3 (continued). The optimal marginal profit schedule is given by

$$\pi_\sigma(\theta) = \frac{(1 + \beta(1 - \sigma))(1 - \sigma)}{1 + (1 - \sigma)^2} + \frac{(1 - \beta)(1 - \sigma)^2}{1 + (1 - \sigma)^2} \theta$$

which is everywhere decreasing in σ also for $\theta = \bar{\theta}$. Indeed, $\partial_\sigma \partial_\theta \pi_\sigma(\theta) < 0$, i.e. the decrease in the marginal profit due to an increase in σ is *largest* for $\theta = \bar{\theta}$. This implies that $\mathcal{I}_\sigma(\theta)$ must fall with σ, indicating that the market with subsidies should be less competitive. A direct computation shows

$$\mathcal{I}_\sigma(\theta) = \frac{1}{\frac{\frac{1}{1-\sigma} + \beta}{1 - \beta} + \theta}$$

which is decreasing in σ, i.e. also subsidies should be accompanied by less aggressive market competition.

5. Extensions

In this section we demonstrate via illustrative examples how our approach can be applied when Assumptions 1-4 are not met. Importantly, the relaxation of any of the four assumptions requires only minor changes in the derivation of the optimal market mechanism.

5.1. R&D Inactivity

Recall that a firm of type θ is R&D inactive when $n(\theta) = \theta$, which is socially optimal if the cost of being active, i.e. the informational rents necessary to maintain incentive compatibility, exceeds the benefit to consumers.[18]

Example 4. Suppose costs are quadratic as in Example 3 and the type density is $f(\theta) = 2\theta$ for $0 \leq \theta \leq 1$. Welfare is given by

$$W(\theta, \pi) = \theta + 2\pi - \pi^2 - (1 - \beta)\frac{1 - \theta^2}{2\theta}\pi$$

and the informational rent term dominates for small θ. Hence, low-type firms are R&D inactive. Let $0 < \theta_0 < 1$ be the threshold type for which $\partial_\pi W(\theta_0, \pi)|_{\pi=0} = 0$. Then the optimal market

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[18]Another reason for firms to be inactive is when there are fixed costs, i.e. when $C(0) > 0$ or $\Psi(0) > 0$. In this case, it can be optimal to have an interval of types, $[\underline{\theta}, \theta_0]$, that do not enter the industry.

mechanism $\{R(\cdot), q(\cdot)\}$ is given by

$$q(\theta) = \begin{cases} \theta & \text{if} \quad \theta \leq \theta_0 \\ \theta + 2\pi(\theta) & \text{if} \quad \theta > \theta_0 \end{cases}$$

and

$$R(\theta) = \begin{cases} 0 & \text{if} \quad \theta \leq \theta_0 \\ \int_{\theta_0}^{\theta} \pi(t)dt + \pi(\theta)^2 & \text{if} \quad \theta > \theta_0 \end{cases}$$

where $\pi(\theta) = 1 - \frac{1}{4}(1-\beta)(1/\theta - \theta)$ is positive and increasing for $\theta_0 < \theta \leq 1$.

To summarize, in the optimal market mechanism, firms with types $0 \leq \theta \leq \theta_0$ are R&D inactive and earn zero net profits. For higher-type firms, the optimal marginal profit schedule is determined as before.

5.2. Non-Convex Marginal Costs

The assumption of convex marginal R&D and production costs guarantees that the optimal marginal profit schedule is unique. Consider instead the case where R&D and production costs are convex but marginal costs are not.

Example 5. Suppose firms' types are uniform on $[0, 1]$ and costs are $\Psi(x) = C(x) = \frac{4}{3}x\sqrt{x}$. The necessary first-order condition for welfare maximization (3.11) becomes

$$\pi(\theta)(1 - \pi(\theta)) = (1 - \beta)(1 - \theta)$$

First, note that if $\beta < 3/4$ there is no solution for types $\theta \leq 1 - 1/(4(1-\beta))$, i.e. low-type firms are R&D inactive. Second, whenever a solution exists, there are actually two solutions: either $\pi(\theta) \leq 1/2$ or $\pi(\theta) \geq 1/2$. The second-order condition for welfare maximization is that $1 - 2\pi(\theta) \leq 0$, so only the solution where $\pi(\theta) \geq 1/2$ corresponds to a maximum. For example, for $\beta = 3/4$, the optimal market mechanism $\{R(\cdot), q(\cdot)\}$ is given by

$$q(\theta) = \theta + \frac{\pi(\theta)^2}{2}$$

and

$$R(\theta) = \int_0^{\theta} \pi(t)dt + \frac{\pi(\theta)^3}{3}$$

where $\pi(\theta) \geq 1/2$ is the larger solution to $\pi(\theta)(1 - \pi(\theta)) = (1 - \theta)/4$.

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To summarize, when marginal costs are not convex, there may exist multiple solutions to the necessary first-order condition (3.11). The second-order condition for welfare maximization selects the solution for which $\pi'(\theta) \geq 0$, similar to the regular case of Section 3.[19]

5.3. Non-Monotone Hazard Ratio

When the hazard rate is not everywhere decreasing, the marginal profit schedule that solves the necessary first-order condition (3.11) may not be everywhere non-decreasing. As a result, R&D and production outputs may be decreasing, which conflicts with incentive compatibility. The solution in this case is to "iron out" the marginal profit schedule, a well-known procedure in the study of optimal auctions, see Myerson (1981).

Example 6. Suppose costs are quadratic and the type density is $f(\theta) = \frac{1}{12} + 11(\theta - \frac{1}{2})^2$ for $0 \leq \theta \leq 1$. The solution to the first-order condition (3.11) is given by

$$\tilde{\pi}(\theta) = 1 - \frac{1}{2}(1 - \beta)\frac{1 - F(\theta)}{f(\theta)}$$

For $\beta = 3/4$, the marginal profit schedule and the associated outputs and revenues are shown by the dashed lines in the left, middle, and right panels of Figure 2 respectively. Note that output $\tilde{q}(\theta) = \theta + 2\tilde{\pi}(\theta)$ is decreasing for some types and, hence, violates incentive compatibility (see Proposition 2).

The correct solution for the optimal marginal profit schedule is shown by the solid line in the left panel of Figure 2. To derive this solution, note that welfare is quadratic in $\pi(\theta)$ so it is maximized by choosing $\pi(\theta)$ to minimize $\int_0^1 (\pi(\theta) - \tilde{\pi}(\theta))^2 dF(\theta)$ under the constraint $\pi'(\theta) \geq -\frac{1}{2}$. This is equivalent to choosing $q(\theta) = \theta + 2\pi(\theta)$ to minimize $\int_0^1 (q(\theta) - \tilde{q}(\theta))^2 dF(\theta)$ under the restriction that $q'(\theta) \geq 0$. The solid line in the middle panel of Figure 2 solves this minimization problem since the weighted area between this line and the dashed line adds up to zero. Finally, the optimal revenue schedule shown by the solid line in the right panel follows from (3.8). Note that the optimal revenue schedule is flat when the optimal output is, since (3.8) implies

$$R'(\theta) = \pi(\theta)\, q'(\theta)$$

i.e. $R' = 0$ when $q' = 0$, so low-type firms that produce the same output all receive the same revenue.

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[19]Let $SOC_\pi < 0$ denote the second-order condition for welfare maximization. Then differentiating the first order condition for $\pi(\theta)$ yields $SOC_\pi \pi'(\theta) = (1 - \beta)((1 - F(\theta)/f(\theta))'$. This implies $\pi'(\theta) \geq 0$ since $SOC_\pi < 0$ and the inverse hazard rate is non-increasing.
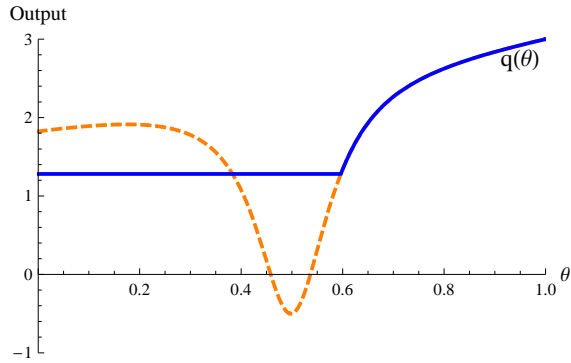
Figure 2: Ironing out the marginal profit schedule (left), output (middle), and revenue (right).

To summarize, when the hazard rate of the type distribution is non-monotone, the optimal marginal profit schedule follows by applying standard "ironing" techniques to the solution of the first-order condition for welfare maximization.

5.4. Taste for Variety

When consumers have a taste for variety, the utility function, $U(q)$, is concave. The first-order condition for welfare maximization

$$\big(c'(\pi(\theta)) + \psi'(\pi(\theta))\big)\big(U'(\theta + c(\pi(\theta)) + \psi(\pi(\theta))) - \pi(\theta)\big) \;=\; (1-\beta)\frac{1-F(\theta)}{f(\theta)}$$

no longer implies that $\pi(\theta)$ is non-decreasing. Hence, the associated output $q(\theta)$ is not necessarily non-decreasing, violating incentive compatibility (Proposition 2). As in the previous example, the solution is to "iron out" the marginal profit schedule.

6. Conclusions

Every micro-economics text book lists the conditions under which perfect competition guarantees welfare-maximizing outcomes. However, in many industries, perfect competition is not the relevant benchmark. In oligopolistic settings, firms have market power and price-taking behavior cannot realistically be assumed. For a competition authority in charge of regulating such a sector the relevant question is: should the competition intensity in this industry be increased? Or, more generally, what industry characteristics determine whether the welfare-maximizing competition intensity should be low or high? In this paper, we have introduced a general and tractable framework to analyze this question.

As an application of our approach, we have confirmed Schumpeter's conjecture that the welfare-maximizing level of competition is lower in dynamic sectors compared to static ones. In other words, it is socially desirable that firms in innovative sectors where R&D is important enjoy more market power than firms in static industries. Our approach also enables us to determine how market regulation interacts with policies aimed at stimulating innovation. In many dynamic markets, firms generate knowledge that cannot be fully appropriated using patents. Or governments use subsidy schemes to stimulate R&D investments. In an example we showed that both a weakening of the patent system and the introduction of R&D subsidies should be accompanied by less aggressive market regulation.

Importantly, our framework can be applied to analyze other sectors. Two examples that currently have a high policy priority are the finance and health industry. Some people have argued that the crisis in the banking sector was caused by (too) intense market competition, which led banks to accept more and more risk in order to stay ahead of rivals. Could more market power in the finance sector have mitigated the crisis by reducing incentives to take risks? Since less intense competition reduces profit variability, would bankruptcies have been less likely? The normative mechanism-design approach put forth in this paper allows us to study whether optimal competition levels are lower in industries where moral-hazard problems play an important role.

Another timely application concerns the study of markets with adverse selection such as the health industry, where perfect competition does not necessarily result in Pareto-efficient outcomes. Does this imply that less intense competition raises welfare? We intend to extend our approach to this and other applications in future work.

A. Appendix: Proofs

Proof of Proposition 1. Let $\phi : \mathbb{R}_+ \to \mathbb{R}_+$ denote a convex and increasing function and let $\Pi_B(\theta) = \phi(\Pi_A(\theta))$. Then

$$\frac{L_B''(\theta)}{L_B'(\theta)} - \frac{L_A''(\theta)}{L_A'(\theta)} = \left(\phi'(\Pi(\theta)) - \frac{\phi(\Pi(\theta))}{\Pi(\theta)}\right)\frac{\pi(\theta)}{\phi(\Pi(\theta))} \geq 0$$

i.e. $L_B(\theta)$ is more convex than $L_A(\theta)$. Since $L_A(\underline{\theta}) = L_B(\underline{\theta}) = 0$ and $L_A(\bar{\theta}) = L_B(\bar{\theta}) = 1$, a more convex Lorenz curve implies a downward shift, i.e. $L_B(\theta) \leq L_A(\theta)$ for all θ.   *Q.E.D.*

Proof of Proposition 2. Let $\theta_1 < \theta_2$, then incentive compatibility requires

$$R(q(\tilde{\theta}_1)) - C(q(\tilde{\theta}_1) - n(\theta_1)) - \Psi(n(\theta_1) - \theta_1) \geq R(q(\tilde{\theta}_2)) - C(q(\tilde{\theta}_2) - n(\theta_2)) - \Psi(n(\theta_2) - \theta_1)$$

$$R(q(\tilde{\theta}_2)) - C(q(\tilde{\theta}_2) - n(\theta_2)) - \Psi(n(\theta_2) - \theta_2) \geq R(q(\tilde{\theta}_1)) - C(q(\tilde{\theta}_1) - n(\theta_1)) - \Psi(n(\theta_1) - \theta_2)$$

for all $\tilde{\theta}_1, \tilde{\theta}_2$. Adding these inequalities yields

$$\Psi(n(\theta_2) - \theta_1) - \Psi(n(\theta_2) - \theta_2) \geq \Psi(n(\theta_1) - \theta_1) - \Psi(n(\theta_1) - \theta_2),$$

so the result that $n(\cdot)$ is increasing follows if and only if $\Psi(x - \theta_1) - \Psi(x - \theta_2)$ is increasing in x. Equivalently, $\Psi'(x - \theta_1) \geq \Psi'(x - \theta_2)$, which holds if and only if $\Psi'(\cdot)$ is increasing, i.e. $\Psi(\cdot)$ is convex. The proof that $q(\cdot)$ is increasing is similar.   *Q.E.D.*

Proof of Proposition 3. The price-cost margin is given by

$$PCM(\theta) \equiv \frac{U'(q(\theta)) - C'(q(\theta) - n(\theta))}{U'(q(\theta))} = 1 - \pi(\theta)/U'(\theta + c(\pi(\theta)) + \psi(\pi(\theta))) \qquad \text{(A.1)}$$

Since $U'' \leq 0$, the price-cost margin is decreasing in $\pi(\theta)$. Innovation levels, however, are increasing in $\pi(\theta)$, see (3.6).   *Q.E.D.*

Proof of Proposition 4. Using the expression for the firm's net profit (3.4) we can reformulate welfare as

$$W = \int_{\underline{\theta}}^{\bar{\theta}} \left\{U(q(\theta)) - (1 - \beta)\Pi(\theta) - C(q(\theta) - n(\theta)) - \Psi(n(\theta) - \theta))\right\}dF(\theta)$$

$$+ \int_{\underline{\theta}}^{\bar{\theta}} \lambda(\theta)\big(\pi(\theta) - \Psi'(n(\theta) - \theta)\big)d\theta$$

$$+ \int_{\underline{\theta}}^{\bar{\theta}} \mu(\theta)\big(\Psi'(n(\theta) - \theta) - C'(q(\theta) - n(\theta))\big)d\theta$$

$$+ \eta\,\Pi(\underline{\theta}) \qquad \text{(A.2)}$$

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which now has to be maximized with respect to $\{\Pi(\cdot), q(\cdot)\}$. The first-order condition with respect to $\Pi(\cdot)$, together with the transversality condition $\lambda(\bar{\theta}) = 0$, implies:

$$\lambda(\theta) = (1 - \beta)(1 - F(\theta)) \tag{A.3}$$

The first-order condition with respect to $q(\cdot)$ is

$$\big(U'(q(\theta)) - C'(q(\theta) - n(\theta))\big)f(\theta) - \mu(\theta)C''(q(\theta) - n(\theta)) = 0,$$

and the first-order condition for $n(\cdot)$ is

$$\lambda(\theta)\Psi''(n(\theta) - \theta) - \mu(\theta)\big(C''(q(\theta) - n(\theta)) + \Psi''(n(\theta) - \theta)\big) = 0.$$

So $\mu(\theta) = \lambda(\theta)\Psi''/(C'' + \Psi'')$ and the first-order condition for $q(\cdot)$ can be rewritten as

$$U'(q(\theta)) - C'(q(\theta) - n(\theta)) = (1 - \beta)\frac{1 - F(\theta)}{f(\theta)}\frac{C''(q(\theta) - n(\theta))\Psi''(n(\theta) - \theta)}{C''(q(\theta) - n(\theta)) + \Psi''(n(\theta) - \theta)} \tag{A.4}$$

Note that output is distorted compared to the first-best solution ($U' = C'$) except at the top, $\theta = \bar{\theta}$, where the hazard ratio vanishes. Assumption 3 implies $n(\theta) > \theta$ and thus $\pi(\theta) > 0$. Using the definitions of the supply functions $\psi(z) = \Psi'^{(-1)}(z)$ and $c(z) = C'^{(-1)}(z)$ and the incentive constraints in (3.5), the first-order condition (A.4) can be rewritten as

$$\big(c'(\pi(\theta)) + \psi'(\pi(\theta))\big)\big(U'(\theta + c(\pi(\theta)) + \psi(\pi(\theta))) - \pi(\theta)\big) = (1 - \beta)\frac{1 - F(\theta)}{f(\theta)}$$

The left side is strictly decreasing in $\pi(\theta)$ since $c(\cdot)$, $\psi(\cdot)$ and $U(\cdot)$ are concave, so the solution is unique. Moreover, $U'(q) = 1$ when products are substitutes, in which case there is no explicit θ dependence on the left side while the right side is decreasing in θ. Hence, $\pi'(\theta) \geq 0$. *Q.E.D.*

Proof of Proposition 5. From (A.1) and (A.4) we conclude that the price-cost margin is proportional to $(1 - \beta)(1 - F(\theta))/f(\theta)$ since $\Psi'' > 0$, $C'' > 0$, and $U' > 0$. The inverse hazard rate is positive for all $\theta < \bar{\theta}$. *Q.E.D.*

Proof of Proposition 6. Note from (3.11) that $\pi(\bar{\theta}) = 1$. Recall that $\mathcal{I}(\theta) = \pi'(\theta)/\pi(\theta) = \partial_\theta \log(\pi(\theta))$, which, upon integration and using the boundary condition $\pi(\bar{\theta}) = 1$, yields (4.1). Moreover, the ratio $\pi'(\theta)/\pi(\theta)$ is positive since the optimal marginal profit schedule is positive and increasing (see Proposition 4). *Q.E.D.*

Proof of Proposition 7. Let $\nu_1 > \nu_2$, and suppose, in contradiction, that $\mathcal{I}_{\nu_1}(\theta) > \mathcal{I}_{\nu_2}(\theta)$ for some θ. From (4.1) this implies that $\pi_{\nu_1}(\theta) < \pi_{\nu_2}(\theta)$ for some θ, which yields a contradiction since the left-side of (4.3) is decreasing in $\pi_\nu(\theta)$. *Q.E.D.*

Proof of Proposition 8. Note that the left side of (4.4) is strictly decreasing in $\pi_\alpha(\theta)$ since

$c(\cdot)$ and $\psi(\cdot)$ are concave, so the solution $\pi_\alpha(\theta)$ is unique. Furthermore, the left side of (4.4) has no explicit θ dependence while the right side is non-increasing in θ, so $\pi'_\alpha(\theta) \geq 0$. Using an envelope argument shows that

$$\frac{dW}{d\alpha}\bigg|_{\alpha=0} = \int_{\underline{\theta}}^{\overline{\theta}} (1 - \pi(\theta))\pi(\theta)c'(\pi(\theta))dF(\theta) > 0$$

so welfare rises with α when α is small. $\hfill Q.E.D.$

Proof of Proposition 9. The proof that $\pi_\sigma(\theta)$ is unique, positive, and increasing follows from similar arguments as in the proof of Proposition 8. An envelope argument shows that

$$\frac{dW}{d\sigma}\bigg|_{\sigma=0} = \int_{\underline{\theta}}^{\overline{\theta}} (1 - \pi(\theta))\pi(\theta)\psi'(\pi(\theta))dF(\theta) > 0$$

so welfare rises with σ when σ is small. $\hfill Q.E.D.$

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