CentER

Discussion Paper

No. 2010–43

WHEN CAN INSURERS OFFER PRODUCTS THAT DOMINATE DELAYED OLD-AGE PENSION BENEFIT CLAIMING?

By Lisanne Sanders, Anja De Wagenaere, Theo E. Nijman

April 2010

ISSN 0924-7815

TILBURG UNIVERSITY

When can Insurers Offer Products that Dominate Delayed Old-Age Pension Benefit Claiming?

Lisanne Sanders,[*] Anja De Waegenaere,[†] and Theo E. Nijman[‡]

April 15, 2010

Abstract

It is common practice for public pension schemes to offer individuals the option to delay benefit claiming until after the normal retirement age and adjust the annual benefit level as a result. This adjustment is often not actuarially neutral with respect to the age at which benefits are claimed. The degree of actuarial nonequivalence varies by interest rates as well as individual characteristics such as gender and age. In this paper we show that actuarial nonequivalence can imply that deferring benefit claiming is suboptimal, irrespective of the preferences of the individual. Specifically, we derive preference-free conditions under which delaying benefit claiming is dominated by claiming benefits early, and using them to buy super-replicating annuity products from an insurance company. We find that the degree of actuarial nonequivalence in public pension schemes is such that such dominating strategies can exist even when the purchase of annuities would be significantly more costly than what is currently observed. If individuals choose to strategically exploit these dominating strategies, this will affect benefit claiming behavior, which in turn affects long run program costs.

Keywords: Pension Benefit Claiming; Delay Options; Actuarial Nonequivalence; Preference-free Dominance

JEL classification: H55, D14, G22

[*] Department of Econometrics and OR, Tilburg University, CentER for Economic Research, Netspar, PO Box 90153, 5000LE Tilburg, The Netherlands.

[†] Corresponding author: Tel.: +31-134662913; fax: +31-134663280; E-mail address: a.m.b.dewaegenaere@TilburgUniversity.nl, Department of Econometrics and OR, Tilburg University, CentER for Economic Research, Netspar, PO Box 90153, 5000LE Tilburg, The Netherlands. This author gratefully acknowledges financial support from NWO.

[‡] Department of Finance, Tilburg University, CentER for Economic Research, Netspar, PO Box 90153, 5000LE Tilburg, The Netherlands.

1 Introduction

In many countries, individuals can decide either to claim their Social Security old-age pension benefits once the minimal retirement age has been reached, or to delay benefit claiming. In case of delay, the individual is offered the same choice next period and so on, until either the maximum age at which benefits can be claimed has been reached or benefits have been claimed. When an individual defers pension receipts, the benefit level is subjected to an actuarial adjustment for each year that benefit claiming is delayed.[1] In many cases, the adjustment is a constant fraction of the benefit level at the normal retirement age, irrespective of age, gender, and other individual characteristics. In the U.S. and the U.K., the benefit levels increase by respectively 8% and 10.4% for each year benefit claiming is delayed (see Diamond, 2005; Queisser and Whitehouse, 2006). In the Netherlands a proposal has been put forward to increase the benefit level by only 5% for each year of delay. As argued by, e.g., Horneff et al. (2008), governments seem to want simple and standardized rules for annuitization applied to a large heterogeneous group of retirees, which may be the reason for choosing a fixed instead of an age-dependent accrual.

The adjustment of the benefit level in case of delayed benefit claiming is typically not actuarially neutral in the sense that the expected present value of the missed benefits in case benefit claiming is delayed is typically not equal to the expected present value of the additional benefits received once benefits are claimed (see, e.g., Coile et al., 2002; Duggan and Soares, 2002; Brown, 2003; Desmet and Jousten, 2003; Sun and Webb, 2009). This lack of actuarial neutrality occurs for several reasons. First, the expected present value of the missed and additional benefits in case of delayed benefit claiming depends on the term structure of interest rates. Higher short-term interest rates typically decrease the expected present value of the missed benefits relative to the expected present value of the additional benefits. The opposite holds for high long-term interest rates. The adjustment of the benefit level in case of delayed benefit claiming, however, is typically fixed for a number of years and therefore not adjusted for changes in the term structure of interest rates. Second, an age-independent accrual leads to actuarial unfairness because, as age increases, the number of years over which the increased benefit level should be paid out decreases, and the missed benefits due to deferral of one more year increase. Finally, the expected present value of the missed and additional benefits in case of delayed benefit claiming depend on survival rates, which depend on individual characteristics such as gender and socio-economic status. Thus, heterogeneity among participants leads to actuarial nonequivalence at the individual level (see Brown, 2003; Desmet and Jousten, 2003).[2]

[1] Such possibilities exist in Social Security pension systems in, e.g., the U.S., the U.K., the Netherlands, Japan, Germany, France, Australia. (see Queisser and Whitehouse, 2006).

[2] The actuarial nonequivalence is well-documented in the literature. For example, Duggan and Soares (2002) calculate actuarially fair adjustment factors when benefits are claimed at ages 62 to 70, and find that results depend strongly on both gender and discount rate. They also find that the annual accrual for delayed benefit claiming of 8%, given in the U.S., is too low in most cases. Desmet and Jousten (2003) show that there is a high degree of heterogeneity among participants of a large public pension system, so that benefit adjustments that are based on the ”average” participant can lead to large degrees of actuarial unfairness at the individual level .

As argued by Duggan and Soares (2002) actuarially nonequivalent benefit adjustments may have unintended consequences in the sense that they affect claiming behavior. Coile et al. (2002) and Sun and Webb (2009) consider optimal claiming of Social Security benefits in the U.S., and argue that even when the adjustment of the benefit level is lower than actuarially fair, delaying benefit claiming can be attractive to risk averse individuals. This occurs because a risk-averse individual attaches more value to the increased longevity insurance due to the higher benefit level.[3] Coile et al. (2002) find that delaying Social Security annuitization for a period of time after the minimal retirement age is optimal in a wide variety of cases under expected utility maximization. Sun and Webb (2009) find that, for plausible preference parameters, the optimal age to claim Social Security benefits for single individuals is between 67 and 70.

Our goal in this paper is to show that the actuarial unfairness inherent in many public pension systems implies that an individual who wishes to defer the receipt of pension benefits may be better off by claiming Social Security benefits immediately and using them to buy annuity products. Consider, for example, a man aged 66 who would like to receive pension benefits as of age 67. He can do so by deferring benefit claiming with one year, which implies that his benefit level will be increased. Suppose now that the level of the accrual is actuarially unfair for this particular man in the sense that the expected present value of the missed benefits at age 66 is higher than the expected present value of the additional benefits received as of age 67. If the difference is sufficiently large, insurers may be able to offer a deferred annuity that starts to pay out as of age 67, with a benefit level that is higher than the accrual offered by the pension provider, and for a periodic premium that is lower than the benefits received in case they are claimed at age 66. If this is the case, the individual is better off by claiming benefits at age 66, and using these benefits to buy the deferred annuity.

In this paper we characterize conditions under which insurers can offer *super-replicating* annuity products. The annuity product is super-replicating if it satisfies two conditions. First, it can be bought for a periodic premium that is at most equal to the benefit level obtained in case Social Security benefits are claimed immediately. Second, upon annuitization it yields a benefit level that is at least equal to the benefit level received in case Social Security benefits would have been claimed at that age. If these two conditions are satisfied, deferred benefit claiming is dominated because the individual is better off by claiming benefits immediately and using them to buy the annuity product. An important aspect of this approach is that because the annuity product is super-replicating, there is preference-free dominance of immediate benefit claiming. All that is required for the individual to prefer claiming benefits immediately and using them to buy the annuity product is that more is preferred to less. To characterize such preference-free dominance conditions, we consider two cases. First we consider the case where an individual at a given age decides as of which age he would like to receive

[3]There is an extensive literature that characterizes individuals' optimal behavior with regard to the timing and level of annuitization of their wealth (see, e.g., Yaari, 1965; Brugiavini, 1993; Brown, 2001; Milevsky, 2001; Brown, 2003; Davidoff et al., 2005; Gupta and Li, 2007; Horneff et al., 2006; Milevsky and Young, 2007a,b; Gerrard et al., 2010, to name just a few). Our focus is on claiming behavior in Social Security systems with delay options.

his pension benefits, and derive conditions under which insurers can offer deferred annuities that the individual prefers above deferring benefit claiming. Next, we determine conditions under which insurers can offer super-replicating annuity options for those individuals who want to defer receipt of pension benefits until an unspecified age. The individual who buys the annuity option can, year by year, decide whether he wants to annuitize, or defer annuitization for at least one more year.

Whether insurers will be able to offer super-replicating annuity products depends on the degree of actuarial unfairness in the Social Security system, as well as on how insurers price annuity products. Two factors are important. First, annuities offered by insurance companies are typically also not actuarially fair in the sense that the premium includes a load to cover costs. Second, in contrast to Social Security providers, insurers can adjust premium conditions to the prevailing term structure of interest rates. Moreover, they can to some extent differentiate premiums based on individual characteristics that affect survival rates. We first consider the case where insurers can differentiate premiums on the basis of age and gender only, and characterize conditions on the level of the premium load and the term structure of interest rates under which they can offer super-replicating annuity products to men and women, respectively. We find that there is ample room for insurers to profitably offer annuity products that men prefer above deferring benefit claiming. For women it is less likely that dominating strategies exist. We then consider the case where insurers can also differentiate premiums based on factors that are correlated with educational level. This additional flexibility increases the room for insurers to offer super-replicating annuity products, in particular to individuals with lower educational levels. This occurs because individuals with lower educational levels have lower life expectancy, and therefore the accruals offered by the social security system are more unfair for them.

Our results potentially have important implications because the existence of super-replicating annuity products can alter claiming incentives and may thereby distort benefit acceptance decisions. Specifically, it can imply that individuals may decide not to defer benefit claiming, even though they do wish to defer annuitization. This can affect the long-run program costs of public pensions (see Hurd et al., 2004). Benefit claiming decisions are not only important for public pensions but also for defined benefit (DB) pensions. It is not uncommon that participants in a (DB) pension plan can, at least to some extent, choose at which age they claim benefits. The annual benefit level is then adjusted to the age at which benefits are first claimed. When the adjustments are not actuarially neutral with respect to the age at which benefits are claimed, participants may choose to strategically exploit outside options offered by insurance companies. This may affect claiming behavior, which in turn affects the plan's liabilities.

The remainder of this paper is organized as follows. Section 2 discusses factors that generate actuarial unfairness in Social Security pension systems with delay options. In Sections 3 and 4 we consider the case where insurers differentiate premiums based on gender only, and characterize conditions under which they can offer super-replicating annuity products for men and women, respectively. We also quantify the potential gains for both individuals and insurers. Section 3 considers individuals who wish to defer the receipt of pension benefits to a specific age. Section 4 extends the analysis to cases where the individual wishes to defer the receipt of pension benefits to an unspecified age. In Section 5 we illustrate the potential gains when insurers can, in addition to gender, also differentiate premiums on the basis of factors correlated with educational level. We end with the conclusions in Section 6.

2 Actuarial unfairness

Existing literature shows that the option to delay Social Security benefit claiming is often actuarially unfair in the sense that the expected present value of the additional benefits in case of deferred benefit claiming is strictly lower than the expected present value of the missed benefits (see, e.g., Coile et al., 2002; Duggan and Soares, 2002; Brown, 2003; Desmet and Jousten, 2003; Sun and Webb, 2009). This unfairness implies that individuals who wish to defer the receipt of pension benefits may be better off by claiming benefits immediately, and using them to buy annuity products at the market. Our goal is to characterize under which conditions insurers can offer annuity products that individuals prefer above deferring benefit claiming.

We focus on cases in which an individual wishes to delay the receipt of pension benefits beyond the so-called full retirement age, which we denote by $\underline{x}$.[4] Each year, the individual decides either to claim old-age pension benefits immediately, or to delay benefit claiming for a period of at least one year.[5] In case of delay, the individual is offered the same choice next year and so on, until either the maximum age at which benefits can be claimed has been reached or benefits have been claimed. We denote the maximum age at which benefits can be claimed by $\overline{x}$. When the individual claims benefits, he receives them in the form of a *whole life annuity* that periodically pays a fixed amount as long as he is alive. Without loss of generality, we normalize the annual benefit level in case benefits are claimed at the full retirement age to 1. For each year of delay, the benefit level increases by a fixed amount a, for some $a > 0$. Therefore, in case benefit claiming is deferred until age $y > \underline{x}$, the annual benefit level is equal to $1 + (y - \underline{x}) \cdot a$.

Whether insurers will be able to offer more attractive delay options clearly depends on the degree of actuarial unfairness in the Social Security system. This degree of unfairness depends not only on the accrual a, but also on the term structure of real interest rates and individual characteristics that affect survival probabilities (see, e.g., Duggan and Soares, 2002). First, higher long-term interest rates lead to less expensive annuities, which

[4] In many countries (including, e.g., the U.S.), individuals can also claim pension benefits at an earlier age than the full retirement age, in which case the benefit level is adjusted downwards. Our focus is on delayed benefit claiming.

[5] It is not uncommon that individuals can decide on a monthly basis to claim benefits or delay benefit claiming. For expositional convenience, we assume that the decision is made annually.

Term Structures of Real Interest Rates

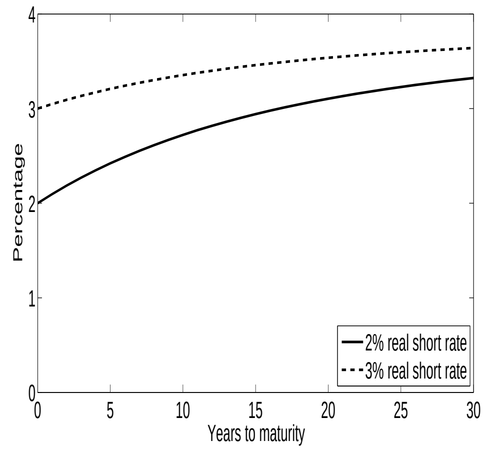


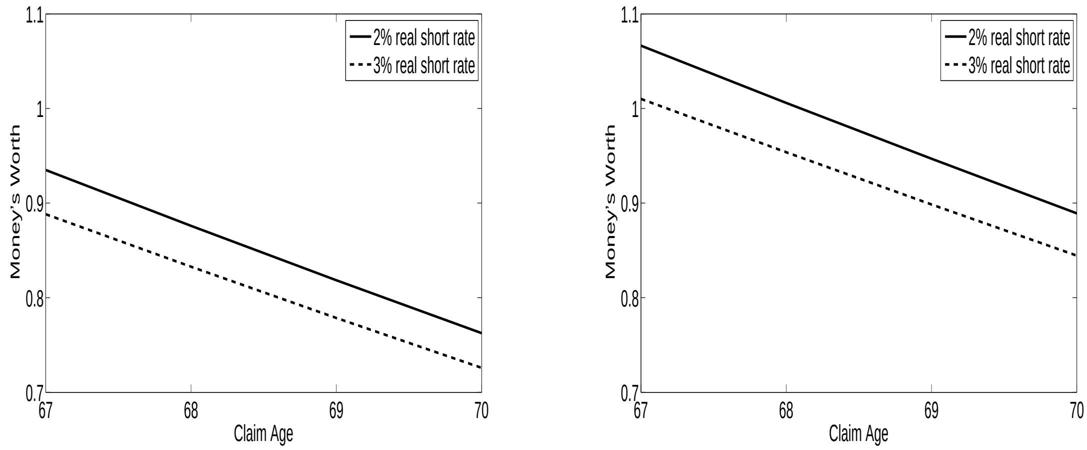
Figure 1: Term structures of real interest rates (in percentages), generated by a one-factor Vasicek model with parameters given in Table 5 in Appendix B.

may result in an opportunity for insurance companies to outperform the Social Security provider. Second, the delayed retirement credit does not differ with individual characteristics (such as, e.g., gender) even though survival probabilities do differ with these characteristics. This leads to actuarial nonequivalence at the individual level. Thus, even if the system would be fair for the "average" individual, it would be unfair to certain groups of individuals (see, e.g., Brown, 2003; Desmet and Jousten, 2003). Insurers can, at least to some extent, differentiate premiums and may therefore be able to offer more attractive delay options to those individuals for which the Social Security system is actuarially unfair.

To illustrate that the degree of actuarial unfairness can be significant, and that it depends strongly on both the term structure of real interest rates and individual characteristics, we determine the *money's worth* of deferring the receipt of pension benefits. The money's worth of the option to delay benefit claiming is defined as the ratio of the expected present value of the missed benefits over the expected present value of the additional benefits received as of claiming age (see, e.g., Sun and Webb, 2009). Let us denote $R^{(\tau)}$ for the τ-years real interest rate, and ${}_\tau p_x$ for the probability that an individual with age x survives at least the first τ years. Now consider an individual aged x who wants to defer the receipt of pension benefits to age y. Because the missed benefit equals $1 + a(x - \underline{x})$ at ages $x, \cdots, y - 1$, and the additional benefit equals $a \cdot (y - x)$ annually as of age y, the money's worth of deferring benefit claiming from age x to age y, denoted by $MW(y, x)$, is given by:

$$MW(y,x) = \frac{a \cdot (y-x) \cdot \left(\sum_{\tau=y-x}^{\infty} \frac{{}_\tau p_x}{\left(1+R^{(\tau)}\right)^\tau} \right)}{(1 + a(x - \underline{x})) \cdot \left(\sum_{\tau=0}^{y-x-1} \frac{{}_\tau p_x}{(1+R^\tau)^\tau} \right)}.$$

The money's worth of deferring Social Security benefit claiming



(a) Men

(b) Women

Figure 2: The money's worth of deferring Social Security benefit claiming from age 66 to age y (i.e., $MW_{y,66}$) as a function of y, for men (left panel) and women (right panel), and for two term structures of interest rates, generated by a one-factor Vasicek model with parameters given in Table 5 in Appendix B. The solid lines (dashed lines) correspond to a real short rate of 2% (3%). The annual accrual is $a = 8\%$, and the full retirement age is set at $\underline{x} = 66$. The survival probabilities are those of U.S. males (females) for the period 2000-2004.

Figure 2 displays the money's worth of delaying benefit claiming from age 66 to age y, for $y = 67, \cdots, 70$, for men and women, and for the two term structures of real interest rates displayed in Figure 1.[6] The solid (dashed) lines correspond to the lower (upper) term structure. We consider the U.S. setting in which the annual accrual offered by the Social Security system equals 8% (i.e., $a = 0.08$), and the full retirement age equals 66 (i.e., $\underline{x} = 66)$. Survival rates are those of U.S. males (females) for the years 2000 up to and including 2004, as reported in the Human Mortality Database.[7]

The option to defer benefit claiming to age $y > 66$ is actuarially unfair if the corresponding money's worth is below one, because this indicates that the expected present value of the additional annuity received as of age y in return for delaying benefit claiming is strictly lower than the present value of the missed benefits at ages $66, \cdots y - 1$. Figure 2 shows that the degree of actuarial unfairness can be substantial, and that it depends strongly on the term structure of real interest rates as well as on individual characteristics such as gender and the preferred deferral period. First, comparing the solid and the dashed lines shows that the deferral option is more unfair when interest rates are high. When interest rates are higher (dashed lines), the money's worth shifts downwards for both men and women, and for all deferral periods. Higher long term interest rates decrease the

[6]The term structures are generated by a one-Vasicek model with parameters as displayed in Table 5 in Appendix B, and with a short rate of 2% (solid lines) and 3% (dashed lines), respectively.

[7]Human Mortality Database. University of California, Berkeley (USA), and Max Planck Institute for Demographic Research (Germany). Available at www.mortality.org or www.humanmortality.de (data downloaded on 05-01-2009). The survival rates are displayed in Figure 9 in Appendix A.

value of the additional benefits relative to the value of the missed benefits, and therefore make deferral more actuarially unfair. The figure also shows that the system is more unfair for men than for women, and more unfair for those who wish to defer for a longer period. Because women have higher life expectancy than men, they are expected to receive the increased benefit for a longer period of time. Therefore, the money's worth of deferring benefit claiming is significantly lower for men than for woman. Consider, for example, the case where the real interest rate is upward sloping from 2% for the real short rate to just above 3.3% for a maturity of 30 years (Figure 1, solid line).[8] The money's worth for men is below one for all deferral periods. For women, the money's worth is above one for deferral of at most two years, but strictly below one for longer deferral periods. Finally, for both men and women and for both term structures, the money's worth of deferring benefit claiming is decreasing in the length of the deferral period. Stated differently, the system is more unfair for those who would like to delay benefit claiming more than for those who would like to delay benefit claiming just a couples of years.

The above results suggest that the degree of actuarial unfairness in the Social Security system is substantial, in particular for those who wish to defer benefit claiming for a longer period. In the next sections we show that this unfairness implies that individuals who wish to defer the receipt of pension benefits may be better off by claiming benefits immediately, and using them to buy annuity products at the market.

3 Dominating strategies using deferred annuities

In this section we characterize conditions under which the market can offer annuity products that are preferred by individuals above deferring pension benefit claiming. The annuity products must be attractive for both insurers and participants, implying that insurers should be able to offer them on profitable terms and individuals should achieve a higher benefit level by buying these products than by deferring benefit claiming. Conditions will be determined under which this holds. When these conditions are satisfied, claiming benefits early and using them to buy a deferred annuity dominates deferring benefit claiming in the sense that the former strategy is preferred to the latter, irrespective of the individual's preference relation.[9] An example of such a preference-free choice is given below.

Suppose that a man with current age 66 would like to receive pension benefits as of age 67.

[8]The results in this case are similar to those reported in Sun and Webb (2009) using survival rates of the Social Security administration, and a flat term structure of 3%.

[9]When the insured claims benefits, they generally are taxed. However, in case the income is used as a premium for annuities, they are in many cases received taxfree and then taxed when the annuity pays out. In the U.S. there are some qualified retirement accounts in which individuals can invest taxfree. The wealth invested can then be used to finance annuities, where the payments of the annuities are taxed (see Brown et al., 2001). We assume a tax system were both premiums and returns on the premiums for annuities are exempted from taxation, and only the annuity payments are taxed.

Furthermore, assume that the benefit level of his pension when he claims benefits immediately equals 100 and that when the man delays benefit claiming by one year, his future benefit level will be increased by 8%. Thus, when he defers pension benefit claiming from age 66 to age 67, he will receive an annual benefit of 108 as of age 67. Now suppose that the man is able to buy a deferred annuity at the market which gives an annual benefit of 9 as of age 67 for a price of 100. When he claims benefits immediately and uses the benefits to finance this deferred annuity, he will receive an annual benefit level of 109 as of age 67. We will therefore argue that, independent of the individual's preferences, claiming benefits at age 67 is dominated by claiming benefits at age 66 and using the benefits as a premium for a deferred annuity that starts to pay out at age 67. The different strategies are displayed in Table 1.

Strategy	Annual Cash flow at age					
	66	67	68	69	70	....
Claim 66	100	100	100	100	100	...
Claim 67	-	108	108	108	108	...
Claim 66, buy deferred annuity	-	109	109	109	109	...

Table 1: The annual payments in a stylized example for a man with age 66, for an accrual a of 8%, and for different strategies.

From Table 1 it is clear that claiming benefits at age 67 is dominated by claiming benefits at age 66 and using the benefits received that year to buy a deferred annuity that starts to pay out at age 67. Of course this is just a stylized example and we still have to analyze the conditions under which insurers can indeed offer a higher benefit level. In the next subsection we determine sufficient conditions under which the market can outperform the option to delay as offered by the Social Security Administration. Unless mentioned otherwise, we consider the U.S. setting in which the full retirement age is 66 (i.e., $\underline{x} = 66$), the maximum retirement age is 70 (i.e., $\overline{x} = 70$), and the annual accrual offered by the Social Security system is 8%.[10]

3.1 Characterizing conditions for dominance

In this section we consider an individual who, at a given age x (e.g., the full retirement age), decides as of which age he would like to receive his pension benefits, and derive preference-free conditions under which insurers can offer deferred annuities that the individual prefers above deferring benefit claiming.

For an individual aged x, deferring benefit claiming to age y can be considered as buying a deferred real annuity. The premium equals the missed benefits at ages $x, \cdots, y-1$. In return for this premium, a deferred

[10] Because there is an earnings test for claiming benefits before the full retirement age (i.e., between the age of 62 and 65) (see e.g., Song and Manchester, 2007), we focus on individuals who wish to delay benefit claiming beyond the full retirement age of 66. However, the analysis can be easily extended to individuals who want to claim before the full retirement age.

annuity with a benefit level of $(y-x)\cdot a$ as of age y is received. For example, in case benefit claiming is deferred to age $\underline{x}+1$, a premium of 1 (i.e., the benefit level in case the individual would have claimed at age $\underline{x}$) is used to finance a deferred annuity with start age $\underline{x}+1$, and benefit level a. If the expected present value of the additional benefits is lower than the premium paid (i.e., when the money's worth of this deferred annuity is less than one), the deferred annuity offered by the pension provider is actuarially unfair, and so the market may be able to outperform the pension provider by offering a more attractive deferred annuity.

Suppose that an individual with age x would like to receive pension benefits as of age y, with $y > x$. He could do so by deferring benefit claiming until age y, in which case the benefit level will equal $1+(y-\underline{x})\cdot a$. Alternatively, however, the individual could claim benefits at age x, and (conditional on being alive) use the benefits received up to age y as periodic premiums to finance a deferred annuity that starts to pay out at age y.[11] Let $b_{y,x}$ denote the benefit level offered by the insurer. Then the aggregate benefit level received as of age y equals the sum of the Social Security benefits that were claimed at age x, $1+(x-\underline{x})\cdot a$, and the payoff from the deferred annuity, $b_{y,x}$, i.e.,

$$B_{y,x} := 1+(x-\underline{x})\cdot a + b_{y,x}. \tag{1}$$

This strategy is preferred if insurers can offer a deferred annuity with a benefit level $b_{y,x}$ that is strictly higher than the accrual offered by the Social Security system, i.e., if

$$b_{y,x} > (y-x)\cdot a, \tag{2}$$

Indeed, (1) and (2) imply that the aggregate benefit level is strictly higher than the benefit level received in case Social Security benefit claiming is deferred to age y, i.e., $B_{y,x} > 1+(y-\underline{x})\cdot a$.

Whether insurers will be able to offer deferred annuities that individuals prefer above deferring benefit claiming clearly depends on the prices charged for deferred annuities. The annuity insurers offer is in general not actuarially fair because insurers impose a premium load. The load may include costs for administration and adverse selection, but also a risk premium, and is typically expressed as a percentage l of the premium (see, e.g., Mitchell et al., 1999). Now consider an individual who claims benefits at age x, and uses the benefits received at ages $x,\cdots,y-1$, as periodic premiums to finance a deferred annuity that starts to pay out at age y. Then, the benefit level $b_{y,x}$ that insurers would offer follows from setting the expected present value of the

[11] Alternatively, the individual could use only part of the claimed benefits to buy a deferred annuity. It can be verified that assuming that the claimed benefits are fully used is without loss of generality. Deferring benefit claiming is dominated by claiming immediately if and only if this is the case when the claimed benefits are fully used.

premium net of cost loading equal to the expected present value of the payments of the deferred annuity, i.e.,

$$(1-l)\cdot(1+(x-\underline{x})\cdot a)\cdot\left(\sum_{\tau=0}^{y-x-1}\frac{{}_{\tau}p_x}{\left(1+R^{(\tau)}\right)^{\tau}}\right)=b_{y,x}\cdot\left(\sum_{\tau=y-x}^{\infty}\frac{{}_{\tau}p_x}{\left(1+R^{(\tau)}\right)^{\tau}}\right). \tag{3}$$

Combined with (2), this implies that claiming benefits immediately and using them to buy a deferred annuity dominates deferring benefit claiming if

$$b_{y,x}:=\frac{(1-l)\cdot(1+(x-\underline{x})\cdot a)\cdot\left(\sum_{\tau=0}^{y-x-1}\frac{{}_{\tau}p_x}{\left(1+R^{(\tau)}\right)^{\tau}}\right)}{\left(\sum_{\tau=y-x}^{\infty}\frac{{}_{\tau}p_x}{\left(1+R^{(\tau)}\right)^{\tau}}\right)}>(y-x)\cdot a. \tag{4}$$

Whether this condition can be satisfied depends on the term structure of real interest rates as well as on the premium load l. In the next subsection, we investigate the effect of the term structure of real interest rates and the premium load on the existence of dominating strategies.

3.2 Effect of term structure and premium load

In this subsection conditions are characterized under which insurers can profitably offer deferred annuities that individuals prefer above deferring benefit claiming. To do so, we compare the benefit levels individuals can obtain by either delaying benefit claiming or by claiming immediately and using the benefits to buy a deferred annuity at the market. We first consider a base case in which the term structure of real interest rates is as displayed in Figure 1, solid line. It is upward sloping from 2% for the real short rate to just above 3.3% for a maturity of 30 years. The premium load equals 7.3%, i.e., $l = 0.073$.[12] We then investigate the sensitivity of the results with respect to changes in the term structure of real interest rates or in the premium load.

Table 2 displays the benefit levels for the base case.

For any given age $y = 66, \cdots, 70$, the diagonal displays the benefit level received as of age y when Social Security benefits are claimed at age y, and the off-diagonal elements (i.e., for $x < y$) yield the benefit level the individual receives as of age y when he claims Social Security benefits at an earlier age x, and uses them to finance a deferred annuity that starts to pay out at age y. If the latter exceeds the former (bold entries), deferring benefit claiming is suboptimal. For example, in case a man aged 66 would like to receive pension benefits as of age 68, the dominating strategy he can follow is claiming benefits immediately and using these benefits to buy a deferred annuity which starts paying off at age 68. Men with age 67 or higher and women with age 68

[12] For most maturities the interest rate is lower than the 3% real interest rate as assumed in for instance Sun and Webb (2009), and Coile et al. (2002). The load is taken from the 1999 annuity value per premium dollar computed on an after tax basis in Mitchell et al. (1999).

Annuity	Claim age (x) Men					Claim age (x) Women				
Age (y)	66	67	68	69	70	66	67	68	69	70
66	1.00					1.00				
67	1.08	1.08				1.07	1.08			
68	**1.17**	**1.17**	1.16			1.15	1.16	1.16		
69	**1.27**	**1.27**	**1.26**	1.24		1.23	**1.24**	**1.25**	1.24	
70	**1.39**	**1.38**	**1.37**	**1.35**	1.32	**1.33**	**1.34**	**1.34**	**1.33**	1.32

Table 2: The aggregate benefit level received as of age y for an individual aged x, when Social Security benefits are claimed at age x and used to finance a deferred annuity that starts to pay out at age y ($B_{y,x}$, off-diagonal elements), and when claiming Social Security benefits is deferred to age y (diagonal elements). The left (right) panel corresponds to men (women). The bold entries represent dominating strategies. The annual accrual a equals 8% and the load l equals 7.3%. The survival probabilities are those of U.S. males (females) for the period 2000-2004. The term structure of real interest rates is as displayed in Figure 1, solid line.

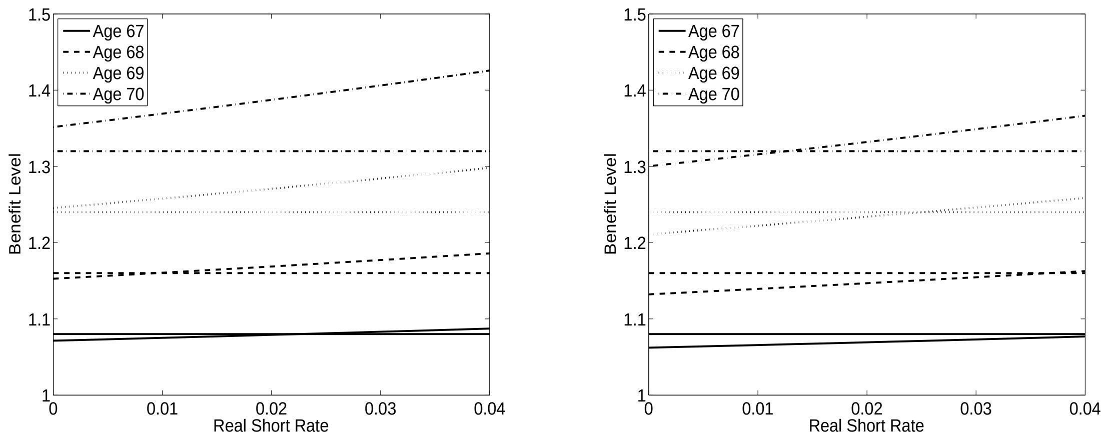
or higher are better off by claiming benefits immediately and using them to buy a deferred annuity than by delaying benefit claiming, regardless of how long they wish to defer the receipt of their pension benefits.

The above results correspond to the term structure as displayed in Figure 1, solid line. Higher long-term interest rates make deferred annuities cheaper, and so it becomes more likely that insurers will be able to offer deferred annuities that individuals prefer above deferring benefit claiming. The opposite holds for lower long-term interest rates. To investigate the sensitivity of our results with respect to changes in the term structure of real interest rates, we use a one-factor Vasicek model (Vasicek, 1977). In this one-factor model, the term structure is fully determined by the short rate, and so the sensitivity of the results with respect to the term structure of real interest rates can be investigated by varying the short rate. Details on the one-factor Vasicek model can be found in Appendix B.

Figure 3 displays the benefit level that an individual aged 66 can obtain as of age y, for $y = 67, \cdots, 70$, as a function of the real short rate, and for two strategies: claiming benefits immediately and using them to finance a deferred annuity that starts to pay out at age y (upward sloping lines), and deferring benefit claiming until age y (horizontal lines).

The figure shows that for each annuity age y, there exists a critical value of the real short rate at which the individual is indifferent between these two strategies. Whenever the short rate is higher than this critical value, annuities are relatively cheap, and insurers can profitably offer annuities that yield higher benefit levels than the accrual offered by the Social Security (upward sloping line higher than horizontal line). Thus, deferring benefit claiming is dominated by claiming benefits immediately and using them to buy a deferred annuity. Below the critical real short rate, deferring pension benefit claiming is preferred above buying additional annuities at the market. Second, the figure shows that dominating strategies are more likely to exist for men than for women. For a man aged 66 who would like to receive pension benefits as of age 67 (solid lines), claiming benefits early to finance a deferred annuity dominates delayed benefit claiming in case the real short rate is

The aggregate benefit level as a function of the real short rate



(a) Men (b) Women

Figure 3: The aggregate benefit level received as of age y, as a function of the real short rate at age 66, when Social Security benefits are claimed at age 66 and used to finance a deferred annuity that starts to pay out at age y ($B_{y,66}$, upward sloping lines), and when claiming Social Security benefits is deferred to age y (horizontal lines). The left (right) panel corresponds to men (women). The annual accrual a equals 8% and the load l equals 7.3%. The survival probabilities are those of U.S. males (females) for the period 2000-2004. The term structure of real interest rates corresponding to a specific real short rate is generated with a one-factor Vasicek model, with parameters given in Table 5 in Appendix B.

above 2.25%. For a woman, the critical real short rate for deferral of one year equals 4.7%, which is quite high. As a result, dominating strategies are not likely to exist in this case. Finally, the figure shows that for both men and women, the critical real short rate decreases when the age as of which they would like to receive pension benefits increases. For men (women), it decreases to -1.8% (1.2%) for deferral to age 70 (dashed-dotted lines). This occurs because the system is more unfair for those who would like to delay benefit claiming more than for those who would like to delay benefit claiming just a couples of years (recall that the money's worth of deferring benefit claiming decreases when the deferral period increases, see Figure 2). Consequently, there is more room for dominance for individuals who wish to delay the receipt of pension benefits for a longer period.

The above results correspond to settings where the premium load equals 7.3%. It is immediately clear from (4) that a higher premium load reduces the benefit level that insurers can offer for a given premium, and therefore makes it less likely that insurers are able to offer deferred annuities that individuals prefer above deferring benefit claiming. In order to investigate the sensitivity of our results to the level of the premium load, we determine the load such that the individual is indifferent between deferring benefit claiming, and claiming immediately and buying a deferred annuity. Consider an individual aged x would like to receive pension benefits as of age y, with $y > x$. The individual is indifferent between the two strategies if they yield the same benefit level, i.e., if

$$b_{y,x} = (y - x) \cdot a.$$

Therefore, it follows from (4) that the indifference load $l_{\max}$ is given by:

$$l_{\max} = 1 - \frac{(y-x) \cdot a \cdot \left(\sum_{\tau=y-x}^{\infty} \frac{{}_\tau p_x}{\left(1+R^{(\tau)}\right)^\tau} \right)}{(1 + (x - \underline{x}) \cdot a) \cdot \left(\sum_{\tau=0}^{y-x-1} \frac{{}_\tau p_x}{\left(1+R^{(\tau)}\right)^\tau} \right)} = 1 - MW(y, x).$$

As long as the premium load that is strictly lower than $l_{\max}$, the market offers deferred annuities that (combined with the Social Security benefits claimed at age x) give a higher benefit level than the benefit level offered by the Social Security provider in case benefit claiming is delayed until age y. Thus, deferring benefit claiming is dominated by claiming immediately.

Table 3 displays the maximum load under which claiming Social Security benefits and using them to buy a deferred annuity dominates deferring benefit claiming, for all possible combinations of the claim age x and the annuity age $y > x$.

Annuity	Claim Age (x) Men				Claim Age (x) Women			
Age (y)	66	67	68	69	66	67	68	69
67	6.51				-6.64			
68	12.41	16.39			-0.06	4.29		
69	18.14	21.82	24.90		5.30	9.86	13.72	
70	23.72	27.10	29.93	32.10	11.06	15.31	18.91	21.97

Table 3: The maximum load $l_{\max}$ (in percentages) under which, at age x, deferring benefit claiming to age $y > x$ is dominated by claiming Social Security benefits at age x and using them to buy a deferred annuity that starts to pay out at age y. The left (right) panel corresponds to men (women). The accrual is set at $a = 8\%$. The survival probabilities are those of U.S. males (females) for the period 2000-2004. The term structure of real interest rates is as displayed in Figure 1, solid line.

For men aged 66 who would like to receive pension benefits as of age 67, the load insurance companies can impose should be below 6.5%. However, for men who wish to defer the receipt of pension benefits until at least age 68, loads can be imposed that are significantly higher than the benchmark level of 7.3%. For women aged 66 who would like to receive pension benefits as of age 67 or 68, dominating strategies will not exist because a negative load is needed. This occurs because for them the option to defer benefit claiming in the Social Security system is more than actuarially fair (i.e., the money's worth is higher than one; see Figure 2, solid line). For women aged 68 or women who would like to defer benefit claiming for a longer period, the loads are also significantly higher than the benchmark level.

4 Dominating strategies using annuity options

In the previous section we characterized conditions under which it is optimal for the individual to claim pension benefits at an earlier age than the age as of which he wants to receive annuity benefits, and use the pension benefits to buy a deferred annuity. We considered the case where an individual at a given age decides as of which age he would like to receive pension benefits, so that a deferred annuity with the corresponding deferral period can be bought. This section considers an individual who wishes to defer the receipt of pension benefits until an unspecified age. We develop an annuity product, called an *annuity option*, in which the individual can, year by year, decide whether he wants to annuitize or defer annuitization for at least one more year. We characterize conditions under which insurers can offer annuity options that super-replicate those offered by the Social Security provider.

4.1 Super-replicating annuity options

In this subsection we design an *annuity option* that super-replicates the option to delay benefit claiming in the Social Security system. The individual who buys this option pays a periodic premium (in case he is still alive) until the time he decides to annuitize, and from there on receives annuity payments from the insurer. The level of the periodic premium depends on the age at which the product is bought. The level of the annuity payment depends on the age at which the option to annuitize is exercised, as well as on the age at which the option is bought. Let us denote:

- x for the age at which the insured buys the annuity option;
- $Y \in [x+1, \overline{x}]$ for the age at which the insured annuitizes. Y is unknown until it is reached, we denote y for any given realization of Y;
- $\pi(x)$ for the premium paid at ages $z \in [x, Y-1]$, conditional on being alive, and given that the annuity option was bought at age x;
- $b_{y,x}$ for the benefit level of the annuity, conditional on annuitizing at age y, and given that the annuity option was bought at age x. We assume that:

$$0 = b_{x,x} \leq b_{x+1,x} \leq \cdots \leq b_{\overline{x},x}.$$

At each age $z \in [x+1, \overline{x}-1]$, the individual decides either to pay a premium of $\pi(x)$ and defer annuitization for at least another year, or to stop paying premium and annuitize. When he annuitizes, he receives an immediate annuity from the insurance company with a benefit level $b_{y,x}$ that depends on his current age y, and the age x at which he bought the annuity option. The benefit levels are determined at the moment the annuity option is bought.

This annuity option (weakly) dominates the option to delay benefit claiming in the Social Security system if the periodic premium is at most equal to the benefit level obtained in case benefits are claimed at age x, and, *for each possible annuity age* y, the level of the annuity payment is at least equal to the accrual offered by the Social Security system in case benefit claiming would have been delayed to that age, i.e.,

$$\pi(x) \leq 1 + (x - \underline{x}) \cdot a, \tag{5}$$

$$b_{y,x} \geq (y - x) \cdot a, \text{ for all } y = x + 1, \cdots, \overline{x}. \tag{6}$$

If these two conditions are satisfied with at least one strict inequality, then for an individual aged x who did not yet claim pension benefits, further deferring benefit claiming is dominated by claiming benefits (of $1+(x-\underline{x})\cdot a$) and using (part of) these benefits to pay the periodic premiums for the annuity option. Indeed, (5) implies that the benefits are sufficient to pay the periodic premium, and (6) implies that, for any given annuity age y, the aggregate benefit level (from Social Security benefits claimed at age x and from the annuity option),

$$B_{y,x} := 1 + (x - \underline{x}) \cdot a + b_{y,x},$$

is weakly higher than the benefit level received in case Social Security benefit claiming is deferred to age y.

Whether insurers will be able to offer super-replicating annuity options clearly depends on how they are priced. Because the risk associated with uncertainty in the age at which the individual will exercise the option to annuitize cannot be hedged, the payoffs of the annuity option cannot be replicated by payoffs from existing assets. In the following subsection we determine conditions under which there exists a selffinancing strategy that super-replicates the payoffs of the annuity option. The strategy is selffinancing if any new assets or annuity payments can be financed from revenues from previously bought assets combined with the premium received from the individual. If these conditions are satisfied, insurers can offer annuity options that satisfy the dominance conditions (5) and (6), while making nonnegative profits in each future year.

4.2 The financing strategy of the insurer

We first design a strategy such that at every possible exercise date, the insurer holds a portfolio of zero-coupon bonds with a market price equal to the price of the annuity in case the insured annuitizes at that date. If the insured does not annuitize, the payoff of the bond portfolio is used to finance a new bond portfolio. If the insured annuitizes, the bond portfolio is sold to finance the immediate annuity. Formally, suppose that an individual aged x buys an annuity option at time $t = 0$, and consider the following strategy:

- At age x, the insurer knows that the benefit level of the annuity will be at least $b_{x+1,x}$. He buys a portfolio of zero-coupon bonds which cash flow matches the expected payments (plus cost loading) of the annuity in case the insured annuitizes at age $x + 1$. Because survival probabilities beyond the age of 110 are negligibly small, he buys a bond portfolio that pays off

$$\left(\frac{b_{x+1,x}}{1-l}\right) \cdot \ {}_s p_x, \text{ in years } s = 1, ..., 110 - x.$$

- At age $x < z < Y$, the insured does not yet annuitize, and the insurer knows that the benefit level upon annuitization will be at least $b_{z+1,x}$, i.e., the benefit level increases by at least

$$\widetilde{b}_{z,x} = b_{z+1,x} - b_{z,x}.$$

  Therefore, the insurer buys a portfolio of zero-coupon bonds with cash flows $\left(\frac{\widetilde{b}_{z,x}}{1-l}\right) \cdot \ {}_s p_x$, in years $s = z - x + 1, \cdots, 110 - x$. Combined with bonds bought at ages $x, \cdots, z - 1$, this implies that he holds a portfolio of zero-coupon bonds with cash flows

$$\left(\sum_{\tau=x}^{z} \frac{\widetilde{b}_{x,\tau}}{1-l}\right) \cdot \ {}_s p_x = \left(\frac{b_{z+1,x}}{1-l}\right) \cdot \ {}_s p_x, \text{ in years } s = z - x + 1, ..., 110 - x.$$

  He receives a cash flow of $\left(\frac{b_{z,x}}{1-l}\right) \cdot \ {}_{z-x} p_x$ from previously bought bonds, as well as a premium payment equal to $\pi(x)$ from every insured that survived. Combined, the expected cash inflow equals

$$\left(\pi(x) + \frac{b_{z,x}}{1-l}\right) \cdot \ {}_{z-x} p_x.$$

- At age $z = Y$, the insured annuitizes. The insurer holds a portfolio of bonds, bought at ages $x, \cdots, Y - 1$, with aggregate payoff

$$\left(\frac{b_{Y,x}}{1-l}\right) \cdot \ {}_s p_x, \text{ in years } s = Y - x, ..., 110 - x.$$

  The market price of this bond portfolio equals the price of the annuity that pays off $b_{Y,x}$ in every future year that the insured is alive.

This strategy yields the desired payoff as of age Y. For it to be selffinancing, however, revenue at each age before annuitization needs to be sufficient to finance the new bond portfolio. In every year in which the insured has not yet exercised the annuity option, the insurer receives revenue which consists of the premium paid by the insured and the cash flow of previously bought bonds which mature. From this revenue, he needs to finance a bond portfolio. The strategy therefore involves losses when the price of the bond portfolio exceeds the revenue. Moreover, for ages $z > x$, the price of the bond portfolio that needs to be bought at age z depends on the term structure of real interest rates in year $t = z - x > 0$. To eliminate this interest rate risk, the insurer can, for each age $z = x + 1, \cdots, \overline{x} - 1$, buy a call option with maturity date $t = z - x$ on the corresponding bond portfolio. To minimize the price of the call options while still guaranteeing that revenue is sufficient to buy the bond portfolio, we set the strike price $K(z, x)$ of the call option on the bond portfolio that needs to be bought at age z equal to revenue received at that age, i.e.,

$$K(z,x) = \left(\pi(x) + \frac{b_{z,x}}{1-l}\right) {}_{z-x}p_x. \tag{7}$$

In the following table we summarize the insurer's revenue and expenses at each age, with and without call options. We denote $P_{Calls}(x)$ for the date $t = 0$ price of the portfolio of call options. Moreover, to avoid overloaded notation, we denote $P_{Bonds}(z, x)$ for the date $t = z - x$ price of the bond portfolio that needs to be bought at age z.

		Age $z = x$	Age $z \in [x+1, Y-1]$
Revenue		$\pi(x)$	$\left(\pi(x) + \frac{b_{z,x}}{1-l}\right) \cdot {}_{z-x}p_x$
Expenses	without options	$P_{Bonds}(x,x)$	$P_{Bonds}(z,x)$
	with options	$P_{Calls}(x) + P_{Bonds}(x,x)$	$\min\{P_{Bonds}(z,x), K(z,x)\}$
Profit	without options	$+/-$	$+/-$
	with options	$+/-$	$+$

Table 4: The insurer's revenue and expenses at age z (i.e., in year $t = z - x$), for $z = x, \cdots, Y - 1$, for an insured who buys the annuity option at age x and exercises it at age Y, and for two financing strategies: the case where the insurer buys call options and the case where he does not buy call options. The last two rows display the sign of the corresponding profit (revenue minus expenses).

With call options, expenses at age x increase, but expenses at ages $z \in [x+1, Y-1]$ (weakly) decrease because the required bond portfolio can be bought at the minimum of the market price and the strike price of the call option. Moreover, because the strike price of the call option on the bond portfolio that needs to be bought at a given age is set equal to the revenue at that age, the revenue always weakly exceeds the expenses at any age $z > x$. Thus, the insurer can offer the annuity option at a nonnegative profit in every year if and only if revenue exceeds expenses at age x, i.e., if and only if

$$P_{Calls}(x) + P_{Bonds}(x, x) \leq \pi(x). \tag{8}$$

Our goal is to characterize conditions on the premium load l, and the term structure of real interest rates under which the dominance conditions (5) and (6), and the profit condition (8) are satisfied. When these conditions are satisfied, insurers can profitably offer annuity options such that individuals who wish to defer the receipt of pension benefits until an unspecified age are better off by claiming benefits and using them to buy the annuity option. This approach is conservative in the sense that it assumes that the insurer wishes to eliminate all interest rate risk. If insurers are willing to bear some risk, the conditions under which they can offer super-replicating annuity options will become less strict.

4.3 When can the insurer profitably offer a super-replicating annuity option?

In this section we first determine conditions on the term structure of real interest rates under which insurers can profitably offer super-replicating annuity options. We then investigate the sensitivity of these results to the level of the premium load charged by insurers. Finally, we quantify the potential gains for insurers from offering super-replicating annuity options.

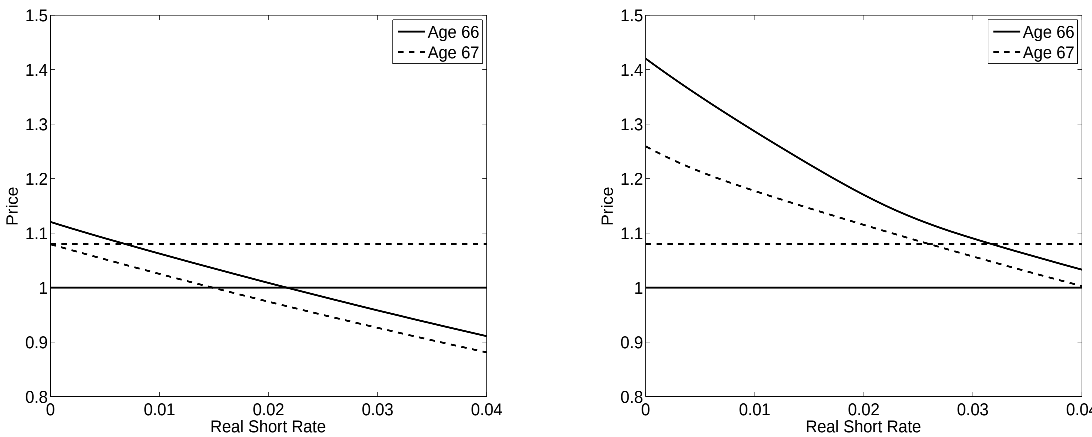
To characterize conditions on the term structure of real interest rates and the profit loading under which insurers can profitably offer super-replicating annuity options, we consider the annuity option that *replicates* the option to defer benefit claiming in the Social Security system, i.e., we set

$$\pi(x) = 1 + (x - \underline{x}) \cdot a, \tag{9}$$

$$b_{y,x} = (y - x) \cdot a, \text{ for all } y = x + 1, \cdots, 70. \tag{10}$$

and investigate under which conditions an insurer who uses the selffinancing strategy defined in Subsection 4.2 makes a strictly positive profit in the year in which the annuity option is sold (i.e., $P_{Calls}(x) + P_{Bonds}(z, x) < \pi(x)$). If this is the case, the insurer can profitably offer the replicating annuity option because, as can be seen from Table 4, the revenue weakly exceeds expenses in all future years. Moreover, since the profit in the first year is strictly positive, either the annual premium $\pi(x)$ could be decreased or the benefit level for at least one annuity age y could be increased, so that a super-replicating annuity option can be offered while still making a positive profit. An individual who wishes to defer annuitization until an unspecified age is then better off by claiming benefits and using them to buy that annuity option than by further delaying benefit claiming. Indeed, either the individual has strictly more wealth before annuitization (if $\pi(x) < 1 + (x - \underline{x}) \cdot a$), or the benefit level as of annuitization is strictly higher for at least one annuity age (if $b_{y,x} > (y - x) \cdot a$).

Expenses and revenue of the insurer



(a) Men (b) Women

Figure 4: The insurer's revenue ($\pi(x)$, horizontal lines) and expenses ($P_{Call}(x) + P_{Bonds}(x, x)$, downward sloping lines) in the year in which the annuity option is sold, as a function of the real short rate at that time. The solid (dashed) lines correspond to an individual who buys the annuity option at age $x = 66$ ($x = 67$). The accrual a offered by the Social Security system is set at 8%, and the profit load l equals 7.3%. The left (right) panel corresponds to men (women). The survival probabilities are those of U.S. males (females) for the period 2000-2004. The term structure of real interest rates corresponding to a specific real short rate is generated with a one-factor Vasicek model, with parameters given in Table 5 in Appendix B.

Figure 4 displays the insurer's revenue (horizontal lines) and expenses (downward sloping lines) in the year in which the annuity option is sold, as a function of the real short rate at that time.[13] The revenue equals the premium paid by the individual. The expenses are equal to the price of the portfolio of call options and bonds that needs to be bought at the time the contract is sold (for details, see Table 4). The figure considers two cases. The solid lines correspond to an annuity option sold to an individual aged 66, for a periodic premium of 1 (i.e., the Social Security benefit level if benefits are claimed at age 66). The dashed lines correspond to an annuity option sold to an individual aged 67, for a periodic premium of 1.08 (the Social Security benefit level if benefits are claimed at age 67).

First consider men who buy the annuity option at age $x = 66$ (left panel, solid lines). The figure shows that there exists a critical value of the real short rate at which the insurer's expenses in the first year are equal to the revenue (the premium received from the insured). When the real short rate is above the critical value of 2.25%, the portfolio of call options and bonds becomes less expensive, i.e., the expenses decrease. This implies

[13] Recall that in the one-factor Vasicek model, the term structure is fully determined by the short rate, and so the sensitivity of the results with respect to the term structure of real interest rates can be investigated by varying the short rate. Details on the Vasicek model as well as on how the price of the portfolio of call options and bonds is determined can be found in Appendices B and C.

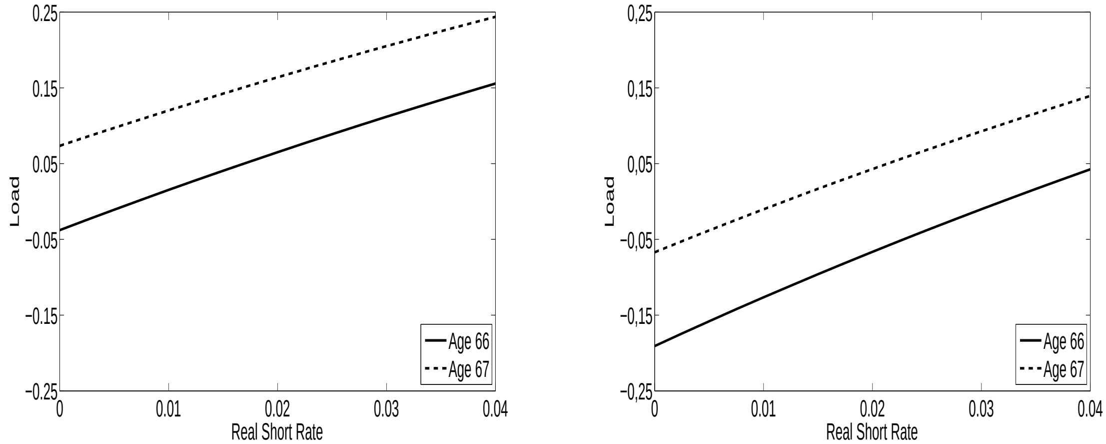
that the insurer can profitably offer a super-replicating annuity option. Men aged 66 who wish to defer benefit claiming until an unspecified age are then better off by claiming benefits immediately and using them to buy that annuity option. Indeed, that strategy yields a higher benefit level, regardless of when they will decide to annuitize. When the annuity option is bought at age of 67 instead of age 66 (left panel, dashed lines), the conditions for dominance are even more likely to be fulfilled. The reason is that the maximum premium the insurer can ask (the benefit level in case Social Security benefits are claimed at age 67) increases from 1 to 1.08, and the minimum benefit level that he needs to offer when the individual annuitizes at age y (the accrual offered by the Social Security benefits when benefit claiming is deferred to age y) decreases from $(y-66)\cdot a$ to $(y-67)\cdot a$. Therefore, the insurer's revenue increases (the horizontal line shifts upwards), and the expenses decrease (the downward sloping line shifts downwards). For men aged 67 (left panel, dashed lines), a positive real short rate is enough for them to prefer buying an annuity option above deferring benefit claiming. For women (right panels), dominating strategies are less likely to exist. Because they have higher life expectancy, the option offered by the Social Security provider is less unfair for them. For women aged 66, the real short rate would need to be well above 4%, which is unlikely to be the case. For women aged 67, insurers can offer annuity options that they prefer above deferring benefit claiming if the real short rate is above 2.75%.

The above results correspond to a premium load of 7.3%. In order to investigate the sensitivity of our results with respect to the level of the premium load, we determine the maximum value of the premium load under which an insurer who follows the selffinancing strategy described in Subsection 4.2 can profitably offer the replicating annuity option. Specifically, we determine the load such that the insurer's expenses in the first year equal the premium received from the insured in that year, i.e., $P_{Calls}(x) + P_{Bonds}(z,x) = \pi(x)$. Whenever the load charged by insurers is strictly lower than this maximum load, the market can offer super-replicating annuity options that individuals strictly prefer above deferring Social Security benefit claiming.

Figure 5 displays the maximum feasible load as a function of the real short rate. The solid (dashed) lines correspond to a super-replicating annuity option sold to an individual aged 66 (67). The left panel corresponds to men; the right panel corresponds to women. Because higher values of the real short rate make annuities less expensive, the maximum feasible load is increasing in the real short rate. There is more room for insurers to offer annuity products that individuals prefer above deferring benefit claiming when interest rates are high. For men who buy the annuity option at age 66, the feasible load is above 7% whenever the short rate is at least 2%. For women, the maximum load is negative for most realistic values of the real short rate, indicating that dominating strategies are not likely to exist. However, when the product is bought at age 67, the maximum feasible load increases significantly for both men and women.

The above results were determined for the case where the insurer uses a conservative financing strategy in which all interest rate risk is eliminated. Insurers, however, may be willing to take some risk, which implies

Maximum feasible load as a function of the real short rate



(a) Men (b) Women

Figure 5: The maximum load $l_{\max}$ (in percentages) under which insurers can offer a super-replicating annuity option to men (left panel) and to women (right panel) aged 66 (solid lines) and aged 67 (dashed lines), as a function of the real short rate. The accrual a is set at 8%. The survival probabilities are those of U.S. males (females) for the period 2000-2004. The term structure of real interest rates corresponding to a specific real short rate is generated with a one-factor Vasicek model, with parameters given in Table 5 in Appendix B.

that there may be more room to offer super-replicating annuity options. To conclude this section, we therefore quantify the potential gains for insurers from offering super-replicating annuity options under the two financing strategies described in Subsection 4.2: eliminating all interest rate risk by buying a portfolio of call options, and accepting some interest rate risk. In both cases, the financing strategy is such that upon annuitization, the insurer holds a portfolio of bonds with a market price equal to the market price of the annuity. Therefore, the insurer's profit consists of profit made in all years prior to annuitization.

As an illustration, we consider a super-replicating annuity option sold to an individual aged 66 for a periodic premium equal to 1 (the Social Security benefits claimed at age 66), with benefit levels given by:

$$
\begin{aligned}
b_{y,x} \quad &= 0.08, &&\text{for } y = 67,\\
&= 0.08 + (y-67)\cdot 0.09, &&\text{for } y = 68, \cdots, 70.
\end{aligned}
\tag{11}
$$

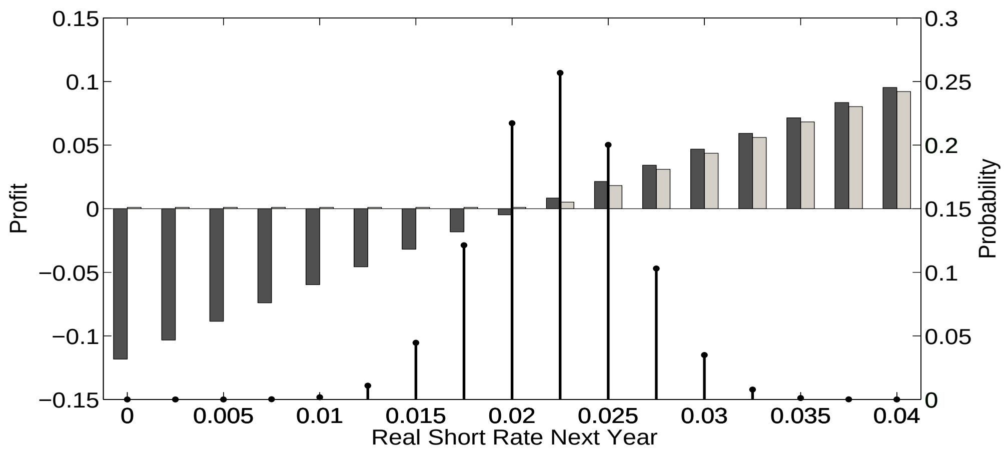
Thus, the benefit level received upon annuitization is strictly higher than the accrual offered by the Social Security system as soon as annuitization is delayed until at least age 68. We determine the probability distribution of the present value of the insurer's profits in all years prior to annuitization, in case the individual exercises the option to annuitize at age 68. The profit in the first year depends on the short rate at the time the annuity option is sold (i.e., when the individual turns 66); the profit made in the year in which the insured turns 67 depends on the short rate next year (see Table 4 for details on these profits). The former is known when the contract is offered, but the latter is stochastic.

Figure 6 displays the probability distribution of the insurer's profit for two values of the short rate at the time the contract is sold. The upper (lower) panel corresponds to the case where the real short rate at the time the contract is sold equals 2.25% (3%). In each case, the figure displays the present value of the insurer's profit as a function of the real short rate next year (bars), as well as the probability that the real short rate next year falls into the corresponding bracket (stems). It considers two financing strategies: buying call options (light grey bars) and not buying call options (dark grey bars). The premium load is set equal to 7.3%. Profit values are displayed on the left y-axis, probability values are displayed on the right y-axis.

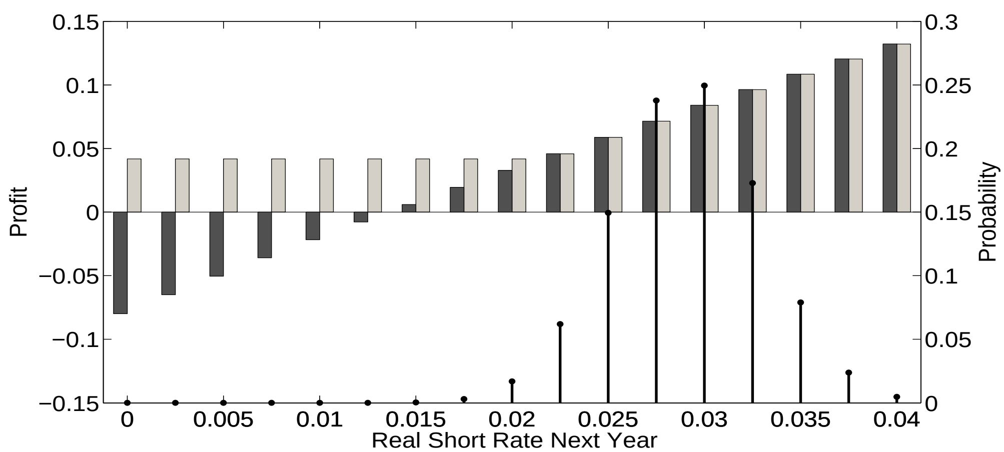
The figure shows that for both financing strategies and for both values of the short rate at the time the contract is sold, the insurer's profit is (weakly) increasing in the real short rate next year. This occurs because and higher short rate next year makes the bond portfolio that needs to be bought at age 67 less expensive. Comparing the upper and the lower panel shows that profits are also increasing in the current real short rate. A higher real short rate at the time the contract is bought (lower panel) makes the portfolio that needs to be bought in the first year less expensive, and, in addition, makes it more likely that the short rate in the second year is also higher, so that the bond portfolio that needs to be bought at age 67 also becomes less expensive. When the short rate at the time the contract is sold equals 3%, the insurer's profit is almost surely positive even when interest rate risk is not hedged.

We now discuss the effect of the financing strategy. When call options are bought, the first year profit is strictly lower, but the second year profit is weakly higher because the bond portfolio that needs to be bought at age 67 can then be bought at the minimum of the market price and the strike price of the call option. Because the market price of the bond portfolio is decreasing in the short rate, there exists a critical value of the short rate in the second year such that the present value of profits with call options is lower (higher) when the short rate is below (above) the critical value. Specifically, when the short rate in the second year is above 2.125%, the market price of the bond portfolio is lower than the strike price of the call option. Therefore, the profit made in the second year is the same for the two financing strategies, and so the present value of profits is lower when call options are bought. The difference (the price of the call options) is about 0.5% of the annual premium when the current short rate is 2.5% (upper panel, dark grey bars), and negligibly small in case the current short rate is 3% (lower panel). When the short rate in the second year falls below the critical level of 2.125%, the price of the bond portfolio is strictly higher than the strike price. Therefore, the second year's profit is zero in case the insurer bought call options, but strictly negative in case he did not. So, without call options the present value of profits can be negative, but the size and likelihood of such losses depend strongly on the current real short rate. When the current short rate is 2.25% (upper panel), a loss is made whenever the short rate falls below the critical level of 2.125%. In contrast, when the current short rate is 3% (lower panel), the profit made

The present value of the insurer's profit



(a) real short rate of 2.25% at age 66



(b) real short rate of 3% at age 66

Figure 6: The present value of the insurer's profit for a man who buys the annuity option at age 66 and exercises it at age 68. The bars represent the present value of the insurer's profit as a function of the real short rate next year, for two financing strategies: buying call options (light grey bars) and not buying call options (dark grey bars). The stems represent the probability that the real short rate next year falls into the corresponding bracket.Profit values are displayed on the left y-axis, probability values are displayed on the right y-axis. The upper (lower) panel corresponds to the case where the real short rate at age 66 equals 2.25% (3%). The benefit levels of the annuity option are as given in (11). The accrual offered by the Social Security system is set at 8%. The premium load is set equal to 7.3%. The survival probabilities are those of U.S. males (females) for the period 2000-2004. The term structure of real interest rates corresponding to a specific real short rate is generated with a one-factor Vasicek model, with parameters given in Table 5 in Appendix B.

in the first year is significantly higher, and high enough to compensate for the loss made in the second year as long as the short rate is in the second year is not below 1.375%. The probability that the short rate falls below this level is negligibly small, so that the insurer almost surely makes no losses, even when interest rate risk is not hedged.

4.4 How much can individuals gain from buying annuity options?

The previous subsection shows that, depending on the real short rate and the premium load, insurers can make significant profits from offering a replicating annuity option. This suggests that they may also be able to offer annuity options with benefit levels that are significantly higher than those offered by the Social Security system, while still making a nonnegative profit. In this subsection we quantify the potential gains for individuals from such super-replicating annuity options.

Recall that in case of delayed Social Security benefit claiming, the accrual received for an additional year of delay equals a for every year of delay. Such a fixed accrual implies that the deferral option is more unfair for those who wish to defer for a longer period (recall that the money's worth is decreasing in the length of the deferral period, see Figure 2). This occurs because the expected number of years over which the additional benefit payment should be made decreases when benefit claiming is delayed further. Consequently, insurers might be able to offer annuity options in which the accrual received for an additional year of delay increases each year. To illustrate the potential gains for insureds, we consider the case where the insurer offers an annuity option with the following conditions:

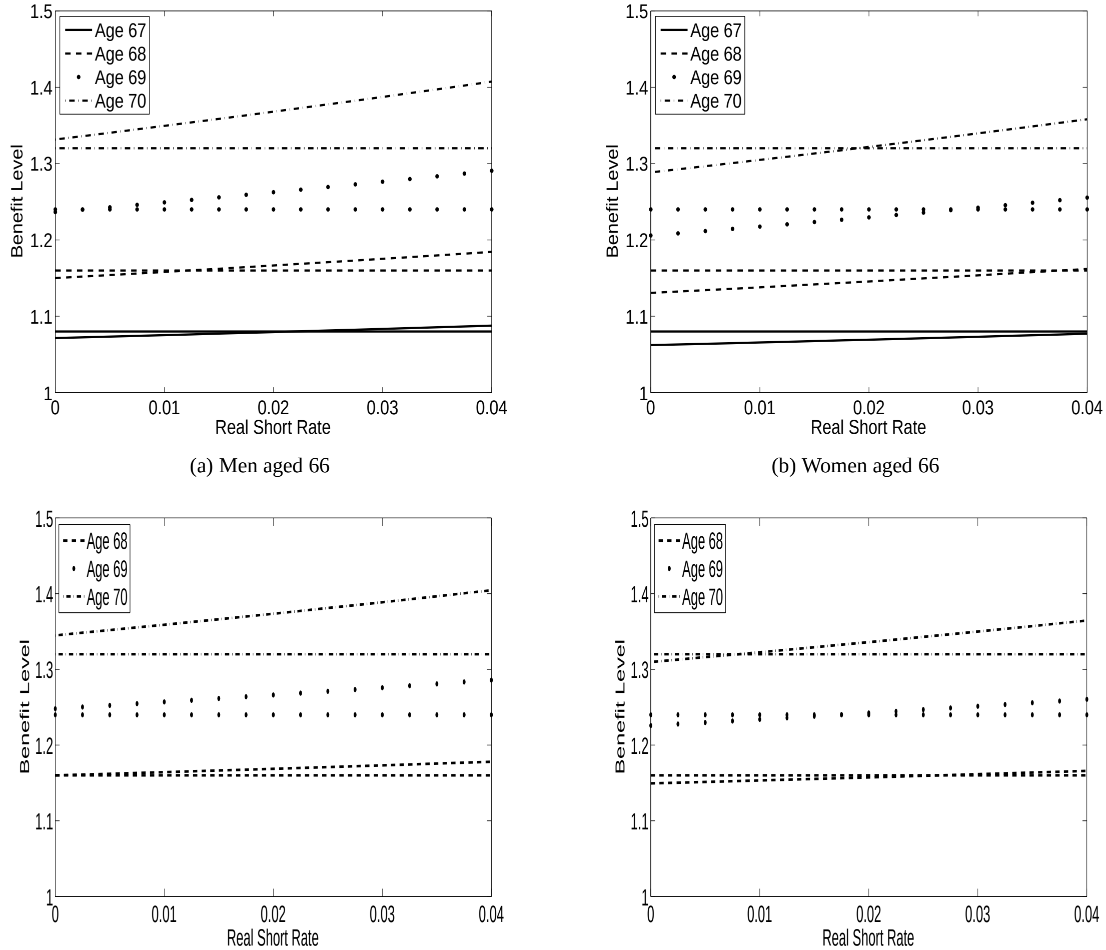
$$\pi(x) = 1 + (x - \underline{x}) \cdot a, \tag{12}$$

$$b_{y,x} = \sum_{\tau=0}^{y-x-1} (1+c)^{\tau} \cdot b, \tag{13}$$

for some $b, c \in (0, 1]$. Thus, the annual premium for the annuity option is equal to the Social Security benefits received in case they are claimed at age x, and the accrual received for an additional year of delay increases by c% each year. Consider, for example, an individual aged $x = \underline{x} = 66$ who would like to defer the receipt of pension benefits. If he claims Social Security benefits immediately and uses them to buy the annuity option, he will receive an annual benefit level as of age 69 of $1 + \left[1 + (1+c) + (1+c)^2\right] \cdot b$. In contrast, if he delays Social Security benefit claiming until age 69, he receives $1 + 3 \cdot a$.

Figure 7 displays the total benefit levels received as of age $y = 67, \cdots, 70$, as a function of the real short rate, for two strategies: the case where the individual claims benefits at age x and uses them to buy the annuity

Benefit level as a function of the real short rate



(a) Men aged 66

(b) Women aged 66

(c) Men aged 67

(d) Women aged 67

Figure 7: The aggregate benefit level received as of age y, as a function of the real short rate at age x, when Social Security benefits are claimed at age x and used to buy the annuity option ($B_{y,x}$, upward sloping lines), and when claiming Social Security benefits is deferred to age y (horizontal lines). The upper (lower) panel corresponds to $x = 66$ ($x = 67$). The left (right) panel corresponds to men (women). The annual accrual a equals 8% and the load l equals 7.3%. The benefit levels of the annuity option are as defined in (13) with $c = 0.1$. The survival probabilities are those of U.S. males (females) for the period 2000-2004. The term structure of real interest rates corresponding to a specific real short rate is generated with a one-factor Vasicek model, with parameters given in Table 5 in Appendix B.

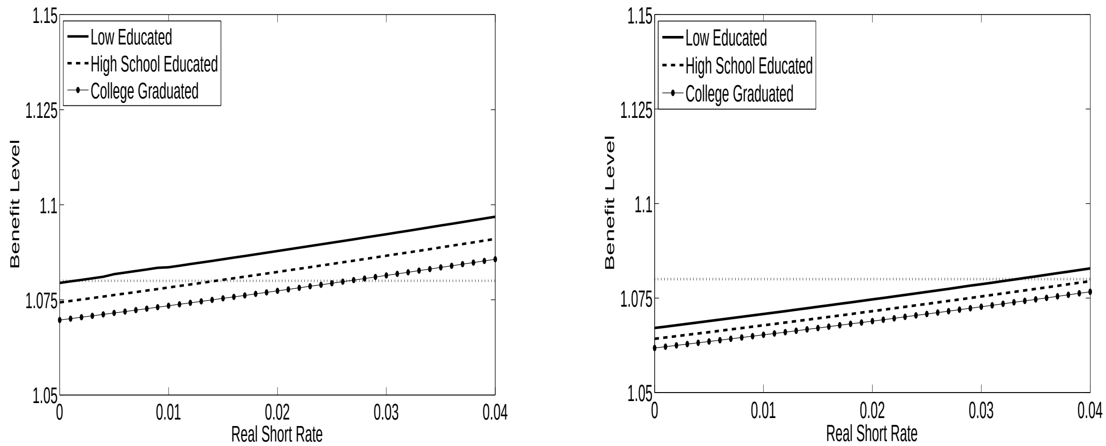
option (upward sloping lines), and the case where he defers benefit claiming until age y (horizontal lines). The upper (lower) panels correspond to $x = 66$ ($x = 67$). The benefit levels offered in the annuity option are as defined in (13). To illustrate the potential gains for individuals, we choose $c = 10\%$, and let b be the level that insurance companies can offer in a competitive market in which excess profits are driven to zero (i.e., condition (8) is satisfied in equality). The accrual offered in the Social Security system is set at $a = 8\%$ and the premium load is set at $l = 7.3\%$.

The figure shows that insurance companies are able to offer an attractive alternative to the option to defer pension benefit claiming as offered by the Social Security provider when the real short rate is sufficiently high. Strict dominance occurs when for every given annuitization age y, the benefit level received in case the annuity option is bought is higher than when benefit claiming is deferred (i.e., the upward sloping line is above the horizontal line for all annuity ages y). In order to have strict dominance a real short rate of 2.25% is needed for men and of 4% for women. However, some individuals may know for sure that they do not wish to annuitize before a certain age. In such cases, insurers are able to offer attractive annuity options even when the short rate is lower. Suppose, for example, that an individual with age 66 knows that he would like to defer annuitization until at least age 68. Then, dominating annuity options exist already when the real short rate is above 1% for men and 3.5% for women. When the individual knows he would like to defer until at least 69, the critical values of the real short rate decrease to 0.25% for men, and 2.75% for women. There is even more room for insurers to offer attractive annuity options when the option is bought at age 67 (lower panels). An individual can, for example, defer social security benefit claiming until age 67, and then use the claimed benefits to buy an annuity option. In this case, insurers can offer a product that dominates further delay of pension benefit claiming irrespective of the real short rate for men. The reason is that a higher annual premium is paid (1.08 instead of 1) and that the minimum required benefit level upon annuitization (the accrual offered by the Social Security benefits when benefit claiming is further deferred to age y) decreases.

5 Dominating Strategies using differentiated survival rates

In the previous sections we characterized conditions under which insurers can offer super-replicating annuity products, taking into account that they can differentiate premium and benefit levels on the basis of gender. There is strong empirical evidence, however, that mortality rates also depend substantially on individual characteristics such as, for example, educational level. This heterogeneity leads to actuarial nonequivalence at the individual level (see, e.g., Brown, 2003; Desmet and Jousten, 2003). In contrast to Social Security providers, insurers may, at least to some extent, be able to differentiate premiums on factors that affect survival probabilities. If this is the case, there is more room to offer super-replicating annuity products for those individuals for

Benefit level as a function of the real short rate



(a) Men (b) Women

Figure 8: The benefit level $(B_{67,66})$ as a function of the real short rate for different groups who buy an option to annuitize at age 66 and annuitize at age 67. The horizontal line denotes the benefit level when benefits are claimed at age 67. A factor c of 10% and a load l of 7.3% were assumed. The survival probabilities are those of U.S. males (females) for the period 2000-2004. The term structure of real interest rates corresponding to a specific real short rate is generated with a one-factor Vasicek model, with parameters given in Table 5 in Appendix B.

which the accruals offered in the Social Security system are more unfair. To illustrate the potential effects, we characterize conditions under which insurers are able to offer the super-replicating annuity option defined in (13) to groups of individuals who differ in educational level. Three educational levels are distinguished: less than high school, high school plus up to three years of college, and college graduates. We use relative mortality factors differentiated to age, gender, and educational level determined by Brown (2003) to calculate the differentiated survival probabilities (see Appendix D). As in the previous section we consider the case where excess profits are driven to zero, i.e., the benefit level b is such that the insurer's profit in the first year is zero.

Figure 8 displays the benefit level that an individual aged 66 can obtain as of age 67, as a function of the real short rate at age 66, and for two strategies: claiming benefits immediately and using them to buy the annuity option (upward sloping lines), and deferring benefit claiming until age 67 (horizontal lines). It distinguishes three educational levels: low education (solid lines), high school education (dashed lines), college graduate (dashed-dotted lines). The figure shows that the critical level of the real short rate above which insurers can offer super-replicating annuity options is increasing in the educational level. Because individuals with lower educational levels have lower life expectancy, they expect to receive the additional benefits offered by the Social Security system for a shorter period of time, which implies that the system is more unfair for them. The differences for men are large. For men with low education, the critical short rate is 0.2%. For college graduates, it increases to 2.7%.

6 Conclusions

In many countries accruals to annual pension benefits are offered to those who claim benefits later. Typically, these accruals are fixed for a number of years, and are independent of both interest rates and individual characteristics such as gender. In addition, the accrual received for an additional year of delay is typically a fixed percentage of the benefit level in case benefits are claimed at the full retirement age. The actuarially fair value of the additional deferred annuity that the individual receives in case he delays benefit claiming, however, depends nontrivially on the length of the deferral period, the term structure of real interest rates, and individual characteristics that affect survival rates. As a consequence, public pension systems with fixed accruals are not actuarially fair, and the degree of unfairness varies over time (as it depends on the term structure of real interest rates). Moreover, the degree of unfairness depends on individual characteristics.

We show that the actuarial unfairness implies that individuals who wish to defer the receipt of pension benefits may be better off by claiming benefits and using them to buy annuity products at the market. Conditions under which it is optimal for them to do so are investigated in a preference-free setting assuming only that more is preferred to less. We first quantify the degree of unfairness in the public pension system on the basis of the market term structure of real interest rates, generated by a Vasicek term structure model. We then characterize conditions under which insurers can offer attractive deferral options without taking any interest rate risk. Our results suggest that there is a broad range of settings (for market conditions, required premium loads, and individual characteristics) in which insurers can profitably offer deferral options that are more actuarially fair than those offered by the public pension provider. Individuals can exploit these options by claiming benefits early, and using them to buy annuity products from insurers. The potential gains for individuals and insurers increase when market conditions are more favorable (e.g., when interest rates are relatively high), and when insurers have more flexibility to differentiate premium and benefit levels on the basis of individual characteristics. If individuals choose to strategically exploit outside options offered by insurance companies, this will affect benefit claiming behavior, which in turn affects long run program costs.

Appendices

Appendix A Survival Probabilities

Throughout the paper, we use the one-year mortality probabilities differentiated to age and gender reported by the Human Mortality Database for U.S. males (females) for the year 2000 up to and including 2004.[14] Let q_x denote the probability that an individual with age x dies within one year. The probability that an individual is alive over τ years conditional on being alive at age x is given by:

$$_{\tau}p_x = \prod_{\upsilon=1}^{\tau}(1 - q_{x+\upsilon-1})$$

Figure 9 displays the cumulative survival probabilities, conditional on being alive at age 66, i.e., $_{\tau}p_{66}$, as a function of τ.

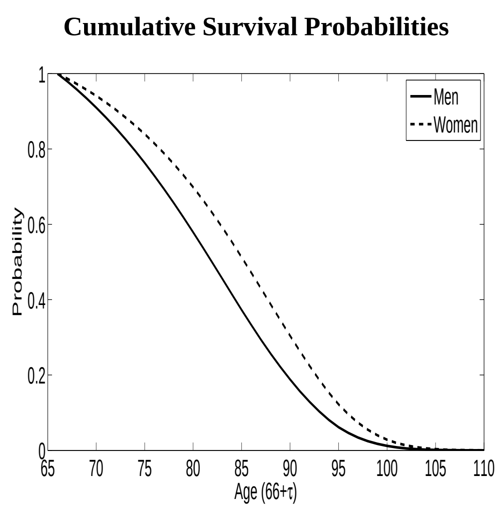


Figure 9: The cumulative survival probabilities ($_{\tau}p_{66}$), as a function of age ($66 + \tau$) for men (solid line) and women (dashed line) respectively with age 66.

Appendix B The One-factor Vasicek Model

The Vasicek model assumes that the instantaneous real short rate at time t, r_t, is generated by:

$$dr_t = \kappa[\theta - r_t]dt + \sigma dW_t, \;\; r(0) = r_0,$$

[14]Human Mortality Database. University of California, Berkeley (USA), and Max Planck Institute for Demographic Research (Germany). Available at www.mortality.org or www.humanmortality.de (data downloaded on 05-01-2009).

where W_t is a Wiener process, θ denotes the long-run mean, κ the parameter of mean reversion, and $\sigma$the volatility.

The time-t price of a zero-coupon bond which matures at time T, denoted by $P(r_t, t, T)$, is given by:

$$P(r_t, t, T) = \exp\{A(t, T)\} \cdot \exp\{-B(t, T) \cdot r_t\},$$

with

$$\begin{aligned} B(t, T) &= \frac{1 - \exp\{-\kappa \cdot (T - t)\}}{\kappa}, & (14) \\ A(t, T) &= [B(t, T) - (T - t)] \left(\frac{\kappa \cdot (\kappa\theta + \lambda\sigma) - \sigma^2/2}{\kappa^2} \right) - \frac{\sigma^2}{4\kappa} \cdot B(t, T)^2, & (15) \end{aligned}$$

and where λ denotes the market price of risk. Then, the time-t real interest rate for a maturity of $T - t$ years given that the short rate at time t equals r_t, is given by:

$$R(r_t, t, T) = \frac{-\log P(r_t, t, T)}{T - t}.$$

Throughout the paper we use the parameter values displayed in Table 5.

	Vasicek model
κ	0.1
θ	0.02
σ	0.004
λ	0.5

Table 5: The parameter values of the Vasicek model for interest rate.

The long-term average θ is set equal to 2%. Moreover, the market price of risk λ is set equal to 0.5. This reflects a setting in which the real interest rate for a maturity of six years is 0.5% higher than the short rate. The benchmark case displayed in Figure 1, solid line, corresponds to the case where the real short rate equals the long-term average θ.

Appendix C Pricing call options on bond portfolios

In this subsection we determine the price $P_{Calls}(x)$ of the portfolio of call options that the insurer buys in order to eliminate interest rate risk. Jamshidian (1989) has derived an exact formula to price options on (coupon-bearing) bonds, assuming that interest rates are generated by a one-factor Vasicek model. The pricing problem is further addressed in Hull (2003) and Brigo and Mercurio (2001). Let us denote $P(r,t,s)$ for the date-t price of a zero-coupon bond with maturity date s, given that the real short rate at time t equals r. The date-0 price of a call option with strike price K and maturity date t, on a zero-coupon bond with maturity s and principal L, is given by:

$$C(s,t,K,L) = L \cdot P(r_0,0,s)\Phi(h) - K \cdot P(r_0,0,t) \cdot \Phi(h-\sigma_P),$$

where $\Phi(\cdot)$ denotes the standard normal cumulative distribution function, r_0 denotes the real short rate at time 0, and h and σ_P are respectively given by:

$$h = \frac{1}{\sigma_P} \ln\{\frac{L \cdot P(r_0,0,s)}{P(r_0,0,t) \cdot K}\} + \frac{\sigma_P}{2}, \tag{16}$$

$$\sigma_P = \frac{\sigma}{\kappa}(1-\exp(-\kappa(s-t)))\sqrt{\frac{1-\exp(-2\kappa t)}{2\kappa}}, \tag{17}$$

respectively, where κ denotes the parameter of mean-reversion and σ denotes the volatility of real short rate.

Recall that, for each age $z = x+1, \cdots, \overline{x}-1$, the insurer needs to buy a call option with strike price $K(z,x)$ given by (7), on a portfolio of zero-coupon bonds with maturity dates $s = z-x+1, ..., 110-x$, and with corresponding principals $L_{z,x,s} = \left(\frac{\widetilde{b}_{z,x}}{1-l}\right) {}_sp_x$. The price of this call option is equal to the price of a *portfolio* of call options, one for each individual zero-coupon bond, where the strike prices $K(z,x,s)$ of the individual call options are such that $\sum K(z,x,s) = K(z,x)$, and they all have the same exercise region, i.e.,

$$\begin{aligned} & K(z,x,s) = L_{z,x,s} \cdot P(r^*, z-x, s) \\ \text{with } r^* \text{ such that:} \quad & \sum_{s=z-x+1}^{110-x} L_{z,x,s} \cdot P(r^*, z-x, s) = K(z,x). \end{aligned} \tag{18}$$

Given that a call option is needed for every age $z = x+1, \cdots, \overline{x}-1$, the price of the portfolio of call options equals:

$$P_{Calls}(x) = \sum_{z=x+1}^{\overline{x}-1} \sum_{s=z-x+1}^{110-x} C(s, z-x, K(z,x,s), L_{z,x,s}), \tag{19}$$

Now, the price of the portfolio of call options follows from (19), with $L_{z,x,s} = \left(\frac{\widetilde{b}_{z,x}}{1-l}\right) {}_sp_x$, and with $K(z,x,s)$ determined by (7) and (18).

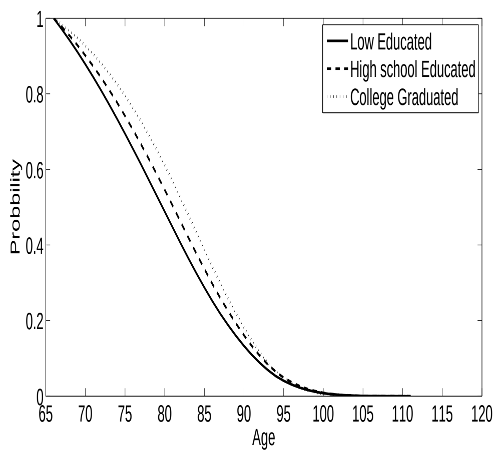
Appendix D Differentiated survival probabilities

In this Appendix we discuss how we determine survival rates differentiated by age, gender, and educational level using the relative mortality factors from Brown (2003). He constructs age-specific relative mortality factors for black, white, and Hispanic men and woman, where the white and black groups are then further differentiated on the basis of education. Three educational levels are distinguished for whites, namely: less than high school, high school plus up to three years of college, and college graduates. To obtain survival probabilities differentiated by educational level, we multiply the relative mortality factors for white men and women with different educational level with the mortality probabilities from the Human Mortality database as described in Appendix A. Let $c_x^{(e)}$ denote the relative mortality factor of an individual with age x with educational level e. The probability that an individual with educational level e is alive over τ years conditional on being alive at age x is given by:

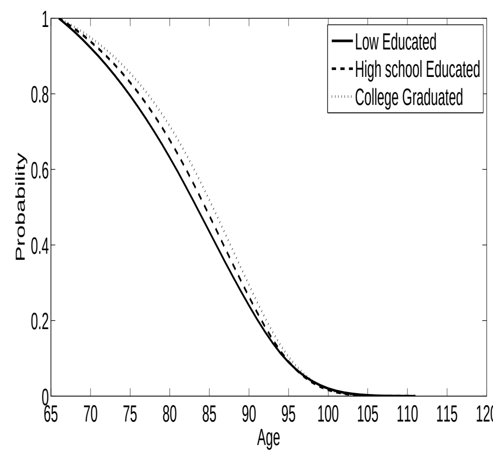
$$_{\tau}p_x^{(e)} = \prod_{\upsilon=1}^{\tau}(1 - q_{x+\upsilon-1} \cdot c_{x+\upsilon-1}^{(e)})$$

The differentiated cumulative survival probabilities for whites are displayed in Figure 10.

Cumulative Survival Probabilities



(a) White Men



(b) White Women

Figure 10: The cumulative survival probabilities differentiated to gender and educational level for men (left) and women (right), conditional on being alive at age 66.

Acknowledgments

We thank James Mahaney, Roel Mehlkopf, Kathrin Nies, Kim Peijenburg, Ralph Stevens, Daniël van Vuuren, and Anthony Webb for helpful suggestions. Useful comments were also received from seminar participants at the 2009 IME meeting, the Netspar Lunch Seminar, Pension Day and Pension Workshop, and the Dutch Central Planning Bureau (CPB).

References

Brigo, D. and Mercurio, F. (2001). *Interest rate models theory and practice*, chapter 3, pages 43–121. Springer Finance. Springer.

Brown, J. R. (2001). Private pensions, mortality risk, and the decision to annuitize. *Journal of Public Economics*, 82(1):29–62.

Brown, J. R. (2003). Redistribution and insurance: mandatory annuitization with mortality heterogenuity. *Journal of Risk and Insurance*, 70(1):17–41.

Brown, J. R., Mitchell, O. S., Poterba, J. M., and Warshawsky, M. J. (2001). *The role of annuity markets in financing retirement*, chapter Taxing retirement income: nonqualified annuities and distributions from qualified accounts, pages 185 – 226. Massachusetts Institute of Technology.

Brugiavini, A. (1993). Uncertainty resolution and the timing of annuity purchases. *Journal of Public Economics*, 50(1):31–62.

Coile, C., Diamond, P., Gruber, J., and Jousten, A. (2002). Delays in claiming social security benefits. *Journal of Public Economics*, 84(3):357–385.

Davidoff, T., Brown, J. R., and Diamond, P. A. (2005). Annuities and individual welfare. *American Economic Review*, 95(5):1573–1590.

Desmet, R. and Jousten, A. (2003). The decision to retire: iindividual heterogeneity and actuarial neutrality. *CEPR Working Paper 593*.

Diamond, P. A. (2005). Reforming public pensions in the us and the uk. *MIT Working paper 05-19*.

Duggan, J. E. and Soares, C. J. (2002). Actuarial nonequivalence in early and delayed social security benefit claims. *Public Finance Review*, 30(3):188–207.

Gerrard, R., Højgaard, B., and Vigna, E. (2010). Choosing the optimal annuitization time post retirement. *Quantitative Finance*, Forthcoming.

Gupta, A. and Li, Z. (2007). Integrating optimal annuity planning with consumption-investment selections in retirement planning. *Insurance: Mathematics and Economics*, 41(1):96–110.

Horneff, W., Maurer, R., Mitchell, O., and Dus, I. (2006). Optimizing the retirement portfolio: Asset allocation, annuitization, and risk aversion. *NBER working paper 12392*.

Horneff, W. J., Maurer, R., and Stamos, M. Z. (2008). Optimal gradual annuitization: Quantifying the costs of switching to annuities. *Journal of Risk & Insurance*, 75(4):1019–1038.

Hull, J. C. (2003). *Options, Futures and Other Derivativess*, chapter 23, pages 537–570. Prentice Hall, 5 edition.

Hurd, M. D., Smith, J. P., and Zissimopoulos, J. M. (2004). The effects of subjective survival on retirement and social security claiming. *Journal of Applied Econometrics*, 19(6):761–775.

Jamshidian, F. (1989). An exact bond option formula. *Jounal of Finance*, 44(1):205–209.

Milevsky, M. A. (2001). Optimal annuitization policies: Analysis of the options. *North American Actuarial Journal*, 5(1):57–69.

Milevsky, M. A. and Young, V. R. (2007a). Annuitization and asset allocation. *Journal of Economic Dynanimc & Control*, 31(9):3138–3177.

Milevsky, M. A. and Young, V. R. (2007b). The timing of annuitization: Investment dominance and mortality risk. *Insurance: Mathematics and Economics*, 40(1):135–144.

Mitchell, O. S., Poterba, J. M., Warshawsky, M. J., and Brown, J. R. (1999). New evidence on the money's worth of individual annuities. *American Economic Review*, 89(5):1299–1318.

Queisser, M. and Whitehouse, E. (2006). Neutral or fair? actuarial concepts and pension-system design. *OECD Social Employment and Migration Working Papers*, 40.

Song, J. G. and Manchester, J. (2007). New evidence on earnings and benefit claims following changes in the retirement earnings test in 2000. *Journal of Public Economics*, 91(3-4):669–700.

Sun, W. and Webb, A. (2009). How much do households really lose by claiming social security at age 62. *Center for Retirement Research at Boston College Unpublished Working Paper*.

Vasicek, O. (1977). An equilibrium characterization of the term structure. *Journal of Financial Economics*, 5(2):177–188.

Yaari, M. E. (1965). Uncertain lifetime, life insurance, and the theory of the consumer. *Review of Economic Studies*, 32(2):137–150.