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BILATERAL COMPARISONS AND CONSISTENT FAIR DIVISION RULES IN THE CONTEXT OF BANKRUPTCY PROBLEMS

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Bilateral Comparisons and Consistent Fair Division Rules in the Context of Bankruptcy Problems*

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Abstract: We analyze the problem of extending a given bilateral principle of justice to a consistent bankruptcy rule for n-creditor bankruptcy problems. Based on the bilateral principle, we build a family of binary relations on the set of creditors in order to make bilateral comparisons between them. We found that the possibility of extending a specific bilateral principle of justice in a consistent way is closely related to the quasi-transitivity of the binary relations mentioned above.

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1. Introduction

The principle of bilateral comparisons is central in the theory of choice, be it social or individual choice. Clearly, in order to make bilateral comparisons a binary relation is needed. However, if we want these bilateral comparisons to yield a sensible choice rule, we need to impose some restriction on the binary relation on which the pairwise comparisons are based. Indeed, Sen (1969) showed that a necessary and sufficient condition for a complete binary relation to generate a choice function is that this binary relation be quasi-transitive[3]. To interpret this result, note that any complete binary relation induces a choice function for 2-alternative choice problems. Sen's result tells us that this 2-alternative choice function can be extended to any finite-alternative choice function if and only if the binary relation we started with is quasi-transitive.

An alternative use of the principle of bilateral comparisons has been suggested in the context of fair division problems (see for example Harsanyi (1959) and Lensberg (1987)). The idea is that an outcome in an n-person division problem should not be considered fair when it is unfair when considering some pair of individuals. In other words, an outcome of an n-person fair division problem should not be considered fair if there is some pair of individuals in which one individual gets more than his fair share, when compared to the other individual. This alternative use of the principle of bilateral comparisons assumes that when we have a fair division problem involving only two individuals, we know to state what is a fair outcome. That is, that we have a principle that singles out a fair division in two-person problems. The question that arises is whether there are any conditions that allow us to extend this two-person principle to a finite-person fair decision rule, in a consistent way.

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[3]Quasi-transitivity of a complete binary relation is equivalent to the transitivity of its asymmetric part.

Our main result is that this alternative use of the principle of bilateral comparisons is also related to quasi-transitivity when it is applied to allocation rules in the context of bankruptcy problems. We found that in order to be able to extend a 2-person bankruptcy rule to an n-person bankruptcy rule in a consistent manner, the 2-person rule should define a quasi-transitive binary relation on the set of creditors, for any feasible allocation.

In order to prove our result, we define a family of bankruptcy rules, which is interesting in itself. This is the family of bankruptcy rules that are consistent on average. If consistency requires that the outcome should be fair when comparing any two individuals, consistency on average will require that the outcome should be fair only on average, after having carried out all the pairwise comparisons. We show that any given two-person bankruptcy rule has a unique extension which is consistent on average. Further, if the initial 2-person rule happens to have a consistent extension, then the consistent on average rule is precisely this consistent extension. Moreover, given the bilateral rule, the recommendation given by the average consistent extension to any specific bankruptcy problem, can be easily computed based only on the bilateral principle of comparisons that generated it. The importance of this family of rules is that they are a natural generalization of the consistent rules, and in contrast to them, there is always an average consistent rule with respect to any arbitrary bilateral principle of comparisons.

Our paper is related to Young (1987) who shows that a symmetric and continuous bankruptcy rule is consistent if and only if it is representable by a continuous parametric function. From this we can conclude that a symmetric and continuous 2-person rule can be extended in a consistent way if and only if it has a parametric representation. Moreover, a parametric representation of a rule induces an interval presentation a la Fishburn (1970) of our binary relation on the set of creditors. Young (1987) also shows that a symmetric and continuous bankruptcy rule is consistent if and only if its recommendations maximize a symmetric, continuous, additively separable and strictly concave function. All those results give alternative insights on the concept of consistency, which proved to be central in the analysis and characterization of many prominent solution concepts. We believe that the main result of our paper gives another interesting view on the concept of consistency.

The paper is organized as follows. Section 2 gives the formal treatment of bankruptcy problems. Section 3 deals with bilateral principles of justice and shows how they can be used to evaluate the fairness of allocations. The main result on the relation between consistency and the quasi-transitivity of the binary relations generated by the bilateral principles of justice is also stated in this section. Section 4 presents the concept of consistency on average and shows that together with any monotone bilateral principle it characterizes a unique bankruptcy rule, which is further used to prove the main result. Other results concerning strictly monotone rules are also shown. Section 5 concludes.

2. The Axiomatic Bankruptcy Model

A *bankruptcy problem* is a pair (E;**d**) where $\mathbf{d} \in \mathbb{R}_+^I$ is a vector of nonnegative real numbers (the claims), which are indexed by some finite nonempty subset I of natural numbers (the creditors), and $0 \leq E \leq \Sigma_{i \in I} d_i =: D$. E is the estate to be allocated, and D is the sum of the claims.

An *allocation* in (E;**d**) is a vector $\mathbf{x} \in \mathbb{R}_+^I$ such that $\Sigma_{i \in I} x_i = E$ and $x_i \leq d_i$ for all $i \in I$. The set of all allocations in (E;**d**) will be denoted by A(E;**d**).

Remark: For any list of claims $\mathbf{d} \in \mathbb{R}_+^I$, any vector $\mathbf{x} \in \mathbb{R}_+^I$ with $x_i \leq d_i$ is an allocation of the bankruptcy problem $(\Sigma_{i \in I} x_i;\mathbf{d})$. Therefore, when there is no danger of confusion, we shall call any such vector **x** an allocation without specifying the bankruptcy problem to which it refers.

A *rule* is a function that assigns to each bankruptcy problem a unique allocation.

Examples:

a) The *proportional rule* is defined as follows:

$$\Pr(E;\mathbf{d}) = \lambda\mathbf{d}, \quad \text{where } \lambda D = E.$$

The proportional rule allocates awards proportionally to the claims. The proportionality principle was favored by the philosophers of ancient Greece, and Aristotle even considered it as equivalent to justice.

b) The *constrained equal award (CEA) rule* is defined as follows:

$CEA(E;\mathbf{d}) = \mathbf{x}$ where $x_i = \min(\lambda, d_i)$ and λ solves the equation $\Sigma_{i\in I}\min(\lambda,d_i)=E$.[4]

This rule assigns the same sum to all creditors as long as it does not exceed each creditor's claim. This rule is also very ancient, and was adopted by important rabbinical legislators, including Maimonides.

The CEA rule awards to any creditor who claims more than the whole estate, the same amount it awards to a creditor who claims exactly the whole estate. The excess of a claim over the estate is completely ignored by the CEA rule. The proportional rule, on the contrary, takes into account the entire claim. The following example shows a rule that combines the proportionality principle with the principle that excesses of claims over the estate should be ignored.

c) The modified proportional (MP) rule is defined as follows:

$$MP(E;\mathbf{d})=\Pr(E;(\mathbf{d}\wedge\mathbf{E})),$$

where the ith component of the vector $\mathbf{d}\wedge\mathbf{E}$ is $\min\{d_i,E\}$.

The MP rule allocates awards proportionally to the relevant claims. For a cooperative bargaining motivation of this rule, see Dagan and Volij (1993).

A rule f is *monotone* if for all $(E;\mathbf{d})$ and $0\leq E'\leq E$, $f(E';\mathbf{d}) \leq f(E;\mathbf{d})$. Monotonicity says

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[4]This equation has a unique solution when $D>E$. If $D=E$, any solution λ is greater than or equal to the maximum claim and therefore $x_i = d_i$ for all i.

that a decrease in the estate does not benefit any creditor. A rule f is *strictly monotone* if for all $(E;\mathbf{d})$ and $0 \leq E' < E$, if $d_i > 0$ then $f_i(E';\mathbf{d}) < f_i(E;\mathbf{d})$. Strict monotonicity says that a decrease in the estate makes every non-zero creditor worse off. Clearly, strict monotonicity implies monotonicity.

A rule f is *anonymous* if for all bankruptcy problems $(E;\mathbf{d})$ and for all permutations σ of the set of players I, $f_{\sigma(i)}(E;\mathbf{d}) = f_i(E;\boldsymbol{\sigma}(\mathbf{d}))$ where the vector of claims $\boldsymbol{\sigma}(\mathbf{d})$ is defined by $\sigma_i(\mathbf{d}) = d_{\sigma(i)}$. Anonymity requires that the awards should not depend on the names of the players.

Let $(E;\mathbf{d})$ be a given bankruptcy problem with set of creditors I. For each nonempty subset of creditors J and for each allocation $\mathbf{x}$ in $(E;\mathbf{d})$ *the reduced bankruptcy problem of J with respect to* $\mathbf{x}$ is $(\Sigma_{j\in J}x_j;\mathbf{d}|J)$, where $\mathbf{d}|J$ is the restriction of $\mathbf{d}$ to $\mathbb{R}^J_+$.

A rule f is *consistent* if for any finite nonempty set I of creditors

$$\text{for all } (E;\mathbf{d}),\ \mathbf{d}\in\mathbb{R}^I_+,\ \text{for all } \emptyset \neq J \subset I,$$

$$f(E;\mathbf{d}) = \mathbf{x} \Rightarrow \mathbf{x}|J = f(\Sigma_{i\in J}x_i,\ \mathbf{d}|J). \tag{1}$$

A rule f is consistent if for each bankruptcy problem $(E;\mathbf{d})$ and subset J of creditors, if f chooses $\mathbf{x}$ in $(E;\mathbf{d})$, then f chooses $\mathbf{x}|J$ in the reduced bankruptcy problem of J with respect to $\mathbf{x}$. The interpretation of consistency is as follows. Suppose that a rule f assigns allocation $\mathbf{x}$ to the bankruptcy problem $(E;\mathbf{d})$. Suppose, too, that some subset of creditors wants to reallocate the total amount $\Sigma_{i\in J}x_i$ assigned to them. If we apply the same rule f to allocate this amount among these creditors, each one will get the amount originally assigned to him, provided f is consistent. Consistency in the setup of bankruptcy problems was first discussed by Aumann and Maschler (1985) and further analyzed by Young (1987, 1988). All the rules presented in the above examples are monotone and anonymous. Strict monotonicity is not satisfied by the CEA rule, and the MP rule is not consistent.

3. On Bilateral Comparisons, Justice and Consistency

Since every bankruptcy problem is a legal problem, solutions to it should be guided by the value of justice. It is natural then, to ask what are the necessary properties of bankruptcy rules that make them consistent with the value of justice. Clearly, anonymity is an essential ingredient of justice. An allocation rule can hardly be called just if the names of the creditors are not irrelevant for the division of the estate. Monotonicity is also a property that should be present in any bankruptcy rule consistent with justice. Indeed, our intuition of justice tells us that if, starting from a just position, transferring some amount of money from creditor A to creditor B is unfair, then transferring the same amount from A to B *and* taking some more money from A a fortiori is unfair. It is not clear, however, whether the consistency property of bankruptcy rules has something to do with justice. Now, whatever form the value of justice may take, we should expect it to enable us to determine whether a creditor i received better or worse treatment than creditor j at any given allocation. For example, if we believe that justice is proportionality, then we would say that i is treated better than j at allocation **x**, if i receives a larger proportion of his claim than j does. According to this principle of justice, an allocation will treat i and j equally if they receive the same proportion of their claims. Obviously, we can think of other notions of justice. Yet, in order to make these pairwise comparisons we need only a bilateral principle of justice.

A *bilateral principle* is a function that singles out a unique allocation to every two-person bankruptcy problem. We interpret this unique allocation as the just allocation of the problem according to the bilateral principle. We shall say that any other allocation in the two-person problem treats one creditor better than the other since it awards one creditor more than his "fair" share. Any rule, when applied to two-person problems, is an example of a bilateral principle. Conceptually, however, bilateral principles differ from two-person allocation rules. The former singles out a just allocation that enables us to make pairwise comparisons, while the latter allocates the estate in two-person problems.

Given a bilateral principle f, a list of claims **d** and an allocation $\mathbf{x} \leq \mathbf{d}$, we can define the following binary relation on the set of creditors I:

$$\succ_{\mathbf{x}} = \{(i,j) \in I \times I \mid f_i(x_i + x_j;(d_i,d_j)) < x_i\}.^5$$

$i \succ_{\mathbf{x}} j$ means that **x** treats i better than j according to f, or more shortly, **x** treats i f-better than j. Obviously, if $i \succ_{\mathbf{x}} j$ then $f_j(x_i + x_j;(d_i,d_j)) > x_j$. Note that if $i \succ_{\mathbf{x}} j$, then $i \succ_{\mathbf{y}} j$ for any other allocation **y** in which $y_i = x_i$ and $y_j = x_j$. That is, whether i is treated f-better than j or not is independent of the amounts assigned by **x** to other creditors. Further note that $\succ_{\mathbf{x}}$ is irreflexive, nobody can be treated better than himself.

We define the relations $\succeq_{\mathbf{x}}$ and $\sim_{\mathbf{x}}$ by replacing $<$ in the definition of $\succ_{\mathbf{x}}$ with $\leq$ and $=$ respectively. Clearly, $\succeq_{\mathbf{x}}$ is a complete binary relation. Following Sen (1969), we say that $\succeq_{\mathbf{x}}$ is quasi-transitive if and only if $\succ_{\mathbf{x}}$ is transitive. Transitivity of $\succ_{\mathbf{x}}$ will play a central role in what follows. The above relations have the obvious interpretation. In particular, we shall say that an allocation **x** treats creditors i and j f-*equally* if $i \sim_{\mathbf{x}} j$. An allocation in (E;**d**) is said to be f-*just* if it treats every two creditors f-equally.

There is an interesting relation between the consistency of a bankruptcy rule f and the existence of f-just allocations. It follows directly from the definition of consistency that if f is consistent, then in every bankruptcy problem there exists an f-just allocation. What is not so obvious is that when the bilateral principle is monotone, the converse is also true. Namely, if for every bankruptcy problem there exists an f-just allocation, then the bankruptcy rule f is consistent. This result is a simple extension of Aumann and Maschler (1985) Theorem A and Corollary 3.1.

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[5]For $i=j$, we define $f_i(x_i + x_j;(d_i,d_j))$ to be x_i.

An appealing feature of f-just allocations is that no creditor can complain about being treated worse than any other one. Unfortunately, there are bilateral principles f with no consistent generalization, that is, for some bankruptcy problems f-just allocations cannot be found. Consider for example the bankruptcy problem $(E;\mathbf{d})=((400; 100, 200, 300)$. When the bilateral principle is the MP rule, there is no f-just allocation in it. This will follow from our theorem 3.2 after taking into account that for $\mathbf{x}=(70.42, 134.08, 195.50)$, the relation $\succ_{\mathbf{x}}$ is not transitive. However, when f-just allocations exist, they can be used as a standard of comparison between players, as the following lemma states:

LEMMA 3.1 (Dagan, Serrano and Volij (1993, lemma 3.3)): Let f be a monotone and anonymous principle and let $(E;\mathbf{d})$ be a bankruptcy problem. Assume that there exists an f-just allocation in $(E;\mathbf{d})$ and denote it by $\mathbf{x}^*$. Let $\mathbf{x}$ be an allocation in $(E;\mathbf{d})$ in which there are two creditors i and j with $x_i \leq x_i^*$ and $x_j \geq x_j^*$. Then, $j \succeq_{\mathbf{x}} i$. Moreover, if both inequalities are strict, then $j \succ_{\mathbf{x}} i$.

Proof:

Case 1: $x_i+x_j \geq x_i^*+x_j^*$. By monotonicity and f-justice,

$$f_i(x_i+x_j;(d_i,d_j)) \geq f_i(x_i^*+x_j^*;(d_i,d_j)) = x_i^* \geq x_i. \text{ Hence, } j \succeq_{\mathbf{x}} i.$$

Case 2: $x_i+x_j < x_i^*+x_j^*$. By monotonicity and f-justice,

$$f_j(x_i+x_j;(d_i,d_j)) \leq f_j(x_i^*+x_j^*;(d_i,d_j)) = x_j^* \leq x_j. \text{ Hence, } j \succeq_{\mathbf{x}} i.$$

This proves the first part of the claim. As for the second part, it is proved analogously and is left to the reader. □

If we are going to use a bilateral principle f for bilateral comparisons, it is desirable that the relations $\succ_{\mathbf{x}}$ it defines should be transitive. For if we find an allocation $\mathbf{x}$ at which creditor i is treated f-better than j, creditor j is treated f-better than k and creditor k is treated f-better than i, it can be argued that the bilateral comparisons that arise from f are meaningless. We feel that, when bilateral comparisons are meaningful, f-just allocations are more appealing than other allocations. This is so since, the injustice of the latter is more evident, in this case. It turns out that to ensure meaningful bilateral comparisons, the bilateral principle f has to have a consistent extension. A consistent extension of a bilateral principle g is a consistent allocation rule that coincides with the bilateral principle for 2-creditor bankruptcy problems. The existence of such a rule is equivalent to the existence of f-just allocations in all bankruptcy problems. The relation between the transitivity of the binary relation and consistency is stated formally as follows:

THEOREM 3.2: Let f be a monotone and anonymous bilateral principle. For each bankruptcy problem $(E;\mathbf{d})$ there exists a unique f-just allocation if and only if for each bankruptcy problem $(E;\mathbf{d})$ and for each allocation $\mathbf{x}$ in it, $\succ_{\mathbf{x}}$ is transitive.

The anonymity of the bilateral principle is actually not needed for the above result, and also for the results in Section 4 below, apart for the "if" part of Theorem 4.8. We assume anonymity , as we find it a natural assumption, and since it simplifies the notation. We can interpret this theorem as saying that a bilateral principle of justice can be extended to a consistent bankruptcy rule if and only if it provides meaningful bilateral comparisons. We leave the proof for the following section.

Young (1987) considered bankruptcy rules that have a parametric representation: Let $g(d,\lambda)$ be a real valued function of two scalar variables d and λ, where $d \geq 0$ and λ ranges over some closed interval $[a,b]$ of the extended reals, and g is continuous and non decreasing in λ, $g(d,a)=0$ and $g(d,b)=d$. A rule f has a parametric representation, if there exists a function g as above, which satisfies:

$$\mathbf{x}=f(E;\mathbf{d}) \text{ iff } \exists\lambda \ \ \forall i\ [x_i=g(d_i,\lambda),\ \Sigma_{i\in I}x_i=E\ ].$$

Young (1987,Theorem 1) showed that the class of rules that have a parametric representation is identical to the class of symmetric, continuous, and consistent rules.This class is identical also to the class of anonymous, monotone, and consistent rules.Given a parametric representation g, of a rule f, a numerical presentation of $\succ_x$ can be constructed. Define for each claim d, and for each amount $x \leq d$ the following values:

$$\lambda_*(d,x) = \inf\{\ \lambda \mid g(d,\lambda)=x\ \}$$

$$\lambda^*(d,x) = \sup\{\ \lambda \mid g(d,\lambda)=x\ \}$$

Now, it is straightforward to verify that:

$$i \succ_x j \text{ iff } \lambda_*(d_i,x_i) > \lambda^*(d_j,x_j).$$

This kind of interval presentation of binary relations was analyzed by Fishburn (1970,pp.18-22).

4. On Average Consistency or Consistency on Average

An appealing feature of f-just allocations is that no creditor can complain about being treated worse than any other creditor. However, as stated above, for some bankruptcy problems f-just allocations cannot be found. In these cases we face two alternatives: either we abandon the bilateral principle or we give up f-justice. There must be a very good reason for a society to abandon a bilateral principle in which it believes and it is not clear to us that the potential non existence of an f-just allocation justifies such a drastic measure. In this section, we present a weaker notion of consistency and f-justice that is compatible with any bilateral principle of justice. If one is willing to accept this weaker notion, then one does not have to give up any bilateral principle.

Let (E;**d**) be an n-creditor bankruptcy problem and let f be a monotone and anonymous bilateral principle. An allocation $\mathbf{x}^* \in A(E;\mathbf{d})$ is said to be f-*just on average* (or *average* f-*just*) if for all creditors i

$$\Sigma_{j \neq i}[f_i(x_i^* + x_j^*;(d_i,d_j)) - x_i^*] = 0.$$

In order to understand this definition, note that x_i^* is the amount awarded to creditor i at allocation $\mathbf{x}^*$ and similarly for x_j^*. $f_i(x_i^* + x_j^*;(d_i,d_j))$ is the amount i should receive if i and j were to divide the amount $x_i^* + x_j^*$ they received f-equally between them. Hence $f_i(x_i^* + x_j^*;(d_i,d_j))$-$x_i^*$ is the amount that creditor j should give creditor i in order to divide justly the sum assigned to them. It can be interpreted as the amount j owes i. f-justice on average requires from an allocation that the total debt of each creditor is zero. Clearly, this is a weaker condition than f-justice that requires that no one owes anything to anyone. The concept of average f-justice is inspired by Maschler and Owen's (1989) 2-consistency in hyperplane games.

The following result will allow us to provide a well-defined bankruptcy rule:

PROPOSITION 4.1: Let f be a monotone and anonymous bilateral principle. For every bankruptcy problem there exists a unique f-just on average allocation.

This proposition assures that when we speak about an allocation that is f-just on average we are not speaking about an empty concept. That is, finding an f-just on average allocation is always possible. Moreover, the proposition states that choosing among such allocations is not a problem since there is never more than one. This enables us to define the *average f-just rule* as the rule that assigns each bankruptcy problem its unique average f-just allocation. The next proposition states that the average f-just rule inherits the basic properties of the bilateral principle f.

PROPOSITION 4.2: Let f be a monotone and anonymous bilateral principle. Then, the average f-just rule is monotone and anonymous.

***Proof of proposition* 4.1**:

Lemma 4.3: If a rule is monotone then it is continuous in the estate.

Proof: Let $\{(E_n;\mathbf{d})\}_{n=1}^{\infty}$ be a sequence of bankruptcy problems that converges to $(E;\mathbf{d})$ and let f be a monotone rule. By monotonicity, for all $n\geq 1$ and for all $i\in I$, $0\leq|f_i(E_n;\mathbf{d})-f_i(E;\mathbf{d})|\leq|E_n-E|$. Since $\{E_n\}_{n=1}^{\infty}$ converges to E, it follows that $\{f_i(E_n;\mathbf{d})\}_{n=1}^{\infty}$ converges to $f_i(E;\mathbf{d})$ for all $i\in I$ and therefore $\{f(E;\mathbf{d})\}_{n=1}^{\infty}$ converges to $f(E_n;\mathbf{d})$. □

Let $(E;\mathbf{d})$ be an n-creditor bankruptcy problem and let f be a monotone and anonymous bilateral principle. Define the following function:

$$T:A(E;\mathbf{d})\rightarrow A(E;\mathbf{d})\,|\,T(\mathbf{x})=\mathbf{t}$$

where $$t_i=(n-1)^{-1}\Sigma_{j\neq i}f_i(x_i+x_j;(d_i,d_j)),$$

and n is the number of creditors in $(E;\mathbf{d})$.

To see that T maps allocations of $(E;\mathbf{d})$ into allocations, first note that for all i and j in I and for all $\mathbf{x}\in A(E;\mathbf{d})$, $0\leq f_i(x_i+x_j;(d_i,d_j))\leq d_i$. Hence, $0\leq(n-1)^{-1}\Sigma_{j\neq i}f_i(x_i+x_j;(d_i,d_j))\leq d_i$. Moreover, $\Sigma_{i\in I}(n-1)^{-1}\Sigma_{j\neq i}f_i(x_i+x_j;(d_i,d_j)) = (n-1)^{-1}\Sigma_{i<j}[f_i(x_i+x_j;(d_i,d_j))+f_j(x_i+x_j;(d_i,d_j))] = (n-1)^{-1}\Sigma_{i<j}(x_i+x_j) = \Sigma_{i\in I}x_i = E$. Clearly, the set of allocations $A(E;\mathbf{d})$ is compact and convex. Moreover, since f is monotone lemma 4.3 implies that T is continuous. Hence by Brouwer's fixed point theorem, T has a fixed point. Clearly, any fixed point of T is an average f-just allocation of $(E;\mathbf{d})$ and conversely, any allocation in $(E;\mathbf{d})$ that is f-just on average, is a fixed point of T.

For any $\mathbf{x}$ in $\mathbb{R}^I$ let $\|\mathbf{x}\|=\Sigma_{i\in I}|x_i|$. The uniqueness part will follow from the following lemma:

Lemma 4.4: Let **x** and **y** be two allocations in (E;**d**). If $\mathbf{x} \neq \mathbf{y}$ then $\|T(\mathbf{x})-T(\mathbf{y})\| < \|\mathbf{x}-\mathbf{y}\|$.

Proof: Let **x** and **y** be two distinct allocations in (E;**d**). By definition of T and by the triangle inequality we have:

$$|T_i(\mathbf{x})-T_i(\mathbf{y})| = |(n-1)^{-1}\Sigma_{j\neq i}f_i(x_i+x_j;(d_i,d_j))-(n-1)^{-1}\Sigma_{j\neq i}f_i(y_i+y_j;(d_i,d_j))|$$

$$\leq (n-1)^{-1}\Sigma_{j\neq i}|f_i(x_i+x_j;(d_i,d_j))-f_i(y_i+y_j;(d_i,d_j))|.$$

Summing over all creditors i we have:

$$\|T(\mathbf{x})-T(\mathbf{y})\| \leq (n-1)^{-1}\Sigma_{i\in I}\Sigma_{j\neq i}|f_i(x_i+x_j;(d_i,d_j))-f_i(y_i+y_j;(d_i,d_j))|$$

$$\leq (n-1)^{-1}\Sigma_{i\in I}\Sigma_{j<i}[|f_i(x_i+x_j;(d_i,d_j))-f_i(y_i+y_j;(d_i,d_j))| + |f_i(x_i+x_j;(d_i,d_j))-f_i(y_i+y_j;(d_i,d_j))|].$$

By monotonicity, the terms inside the absolute values have the same sign, hence

$$\leq (n-1)^{-1}\Sigma_{i\in I}\Sigma_{j<i}[|f_i(x_i+x_j;(d_i,d_j))-f_i(y_i+y_j;(d_i,d_j))+f_i(x_i+x_j;(d_i,d_j))-f_i(y_i+y_j;(d_i,d_j))|],$$

and since f assigns allocations,

$$\leq (n-1)^{-1}\Sigma_{i\in I}\Sigma_{j<i}|(x_i+x_j)-(y_i+y_j)|.$$

By the triangle inequality, $|(x_i+x_j)-(y_i+y_j)| \leq |x_i-y_i| + |x_j-y_j|$ and since $\mathbf{x}\neq\mathbf{y}$, for some pair of creditors i and j this last inequality is strict. Therefore,

$$\|T(\mathbf{x})-T(\mathbf{y})\| < (n-1)^{-1}\Sigma_{i\in I}\Sigma_{j<i}[|x_i-y_i| + |x_j-y_j|]$$

$$= 1/2(n-1)^{-1}\Sigma_{i\in I}\Sigma_{j\neq i}[|x_i-y_i| + |x_j-y_j|]$$

$$= 1/2(n-1)^{-1}[\Sigma_{i\in I}(n-1)|x_i-y_i| + \Sigma_{i\in I}\Sigma_{j\neq i}|x_j-y_j|]$$

$$= 1/2(n-1)^{-1}[\Sigma_{i\in I}(n-2)|x_i-y_i| + \Sigma_{i\in I}\Sigma_{j\in I}|x_j-y_j|]$$

$$= 1/2(n-1)^{-1}[(n-2)\|\mathbf{x}-\mathbf{y}\| + n\|\mathbf{x}-\mathbf{y}\|]$$

$$= \|\mathbf{x}-\mathbf{y}\|. \ \square$$

It follows immediately from lemma 4.4 that if **x** and **y** are fixed points of T, they must be equal. This completes the proof of proposition 4.1. □

Proof of Proposition 4.2: Since the anonymity part is trivial, we shall only prove the monotonicity of the average f-just rule. Let (E;**d**) be a bankruptcy problem and let $0 \leq E' < E$. Denote by **x** and by **y** the average f-just allocations of (E;**d**) and (E';**d**) respectively. Assume by contradiction that $y_k > x_k$ for some creditor k. By an argument analogous to the one in lemma 4.4, we have

$$\|T(\mathbf{x})-T(\mathbf{y})\| \leq (n-1)^{-1}\Sigma_{i\in I}\Sigma_{j<i}|(x_i+x_j)-(y_i+y_j)|.$$

By the triangle inequality, $|(x_i+x_j)-(y_i+y_j)| \leq |x_i-y_i| + |x_j-y_j|$ for each pair of creditors i and j. But since for some creditor k $y_k > x_k$, and since $\Sigma_{i\in I}x_i > \Sigma_{i\in I}y_i$, for some pair of creditors i and j the triangle inequality is strict. Therefore,

$$\|T(\mathbf{x})-T(\mathbf{y})\| < (n-1)^{-1}\Sigma_{i\in I}\Sigma_{j<i}[|x_i-y_i| + |x_j-y_j|].$$

Again, by the same argument as in lemma 4.4, we get

$\|T(\mathbf{x})-T(\mathbf{y})\| < \|\mathbf{x}-\mathbf{y}\|$ contradicting the fact that **x** and **y** are both fixed points. □

Since f-just allocations are also f-just on average, it follows from proposition 4.1 the well-known result that if f-just allocations exist, they are unique. The next proposition shows that we can use the operator T defined in the proof of proposition 4.1 to define a dynamic process that always converges to the average f-just allocation.

PROPOSITION 4.5: Let $\mathbf{x}_0$ be an allocation in (E;**d**) and define inductively $\mathbf{x}_t = T(\mathbf{x}_{t-1})$ for $t > 0$. $\{\mathbf{x}_t\}$ converges to the average f-just allocation.

Proof[6]: Since $\mathbf{x}_t \in A(E;\mathbf{d})$ and $A(E;\mathbf{d})$ is bounded, $\{\mathbf{x}_t\}$ has a convergent subsequence $\{\mathbf{x}_{t(k)}\}$. Let **y** be the limit of this subsequence. Since A(E;**d**) is closed, $\mathbf{y} \in A(E;\mathbf{d})$. It is sufficient to show that **y** is the unique f-just on average allocation in (E;**d**), which will be denoted by **x**. It follows from lemma 4.4 that $\{\|\mathbf{x}_t-\mathbf{x}\|\}$ is a non increasing sequence of non negative real numbers, hence

[6]This proof is due to Sjaak Hurkens.

it must have a limit, i.e, $\|\mathbf{x}_t-\mathbf{x}\|\to a$. Since $\{\mathbf{x}_{t(k)}\}$ and $\{T(\mathbf{x}_{t(k)})\}$ are subsequences of $\{\mathbf{x}_t\}$ we must have $\|\mathbf{x}_{t(k)}-\mathbf{x}\|\to\|\mathbf{y}-\mathbf{x}\|=a$ and by continuity of T, $\|T(\mathbf{x}_{t(k)})-\mathbf{x}\|\to\|T(\mathbf{y})-\mathbf{x}\|=a$. But then $\|\mathbf{y}-\mathbf{x}\|=\|T(\mathbf{y})-\mathbf{x}\|=\|T(\mathbf{y})-T(\mathbf{x})\|$ and lemma 4.4 implies that $\mathbf{y}=\mathbf{x}=T(\mathbf{x})$. □

We can now proceed to the proof of theorem 3.2.

Proof of theorem 3.2: The proof will follow from the following lemmas:

Lemma 4.5: Let f be a monotone and anonymous bilateral principle. Let (E;**d**) be a bankruptcy problem with at least three creditors and **x** an allocation in it. If there exists an f-just allocation in each 3-creditor reduced problem with respect to **x** then $\succ_x$ is transitive.

Proof: Let (E;**d**) be a bankruptcy problem and let **x** be an allocation in it. Let i, j and k three creditors such that $i\succ_x j$ and $j\succ_x k$. Let (E';**d'**) be the reduced bankruptcy problem of {i,j,k} with respect to **x**, i.e., $(E';\mathbf{d}'):=(x_i+x_j+x_k;(d_i,d_j,d_k))$. Let $\mathbf{x}^*$ be the f-just allocation of this problem. It must be the case that $x_k<x_k^*$. For otherwise, since $j\succ_x k$, lemma 3.1 implies $x_j>x_j^*$ and since $i\succ_x j$, the same lemma implies $x_i>x_i^*$ contradicting the fact that $x_i+x_j+x_k=x_i^*+x_j^*+x_k^*$. Hence, $x_k<x_k^*$. Analogously, it must be that $x_i>x_i^*$. Hence by lemma 3.1 $i\succ_x k$. This completes the proof of lemma 4.5.

Lemma 4.6: Let f be a monotone and anonymous bilateral principle. Let (E;**d**) be an n-creditor bankruptcy problem and let **x** be the average f-just allocation in it. If $\succ_x$ is transitive, then **x** is an f-just allocation.

Proof: Assume by contradiction that **x** is not f-just and $\succ_x$ is transitive. Since **x** is not f-just, there are two creditors i and j such that $i\succ_x j$ and since **x** is f-just on average, there exists a creditor k such that $j\succ_x k$. By transitivity $i\succ_x k$ and by irreflexivity, $i\neq k$. Again, since **x** is f-just on average there exists a creditor m such that $k \succ_x m$. The above arguments show that m is different from the previous creditors. Applying these arguments n times will contradict the fact that there are n creditors. This completes the proof of lemma 4.6.

To see that theorem 3.2 follows from lemmas 4.5 and 4.6, let $(E;\mathbf{d})$ be a bankruptcy problem. If for each allocation $\mathbf{x}$ in it $\succ_x$ is transitive, then in particular it is transitive when $\mathbf{x}$ is an f-just on average allocation, which by proposition 4.1 exists and is unique. But in this case lemma 4.6 ensures that $\mathbf{x}$ is the f-just allocation of $(E;\mathbf{d})$. Conversely, if $\mathbf{x}$ is an f-just allocation in $(E;\mathbf{d})$, then $\mathbf{x}|J$ is an f-just allocation in the reduced bankruptcy problem of J with respect to $\mathbf{x}$ for all J containing exactly 3 creditors. Therefore, by lemma 4.5 $\succ_x$ is transitive for all allocations $\mathbf{x}$.
□

COROLLARY 4.7: Let f be a monotone and anonymous bilateral principle. For each 3-creditor bankruptcy problem there exists an f-just allocation if and only if for each bankruptcy problem there exists an f-just allocation.

Proof: Lemma 4.5 says that if for each 3-creditor bankruptcy problems there exists an f-just allocation then $\succ_x$ is transitive for all allocations $\mathbf{x}$ and theorem 3.2 ensures that in this case there exists an f-just allocation in each bankruptcy problem. The other direction is immediate. □

This corollary says that the answer to the question of the existence of a consistent extension to a specific bilateral principle, lies in the family of 3-creditor problems. If there is no consistent extension of a bilateral principle f, then we must be able to find a 3-creditor problem with no f-just allocation.

Theorem 3.2 states that there is a connection between the existence of f-just allocations and the transitivity of the relations $\succ_x$. On the other hand, a rule may well be consistent while the weak relations $\succeq_x$ are not transitive. To see this, consider the following bankruptcy problem: (E;**d**):=(400;(300, 200, 100)) and the following allocation: **x**=(160, 140, 100). When f is the constrained equal award rule it is easy to see that $2 \succeq_x 3$, $3 \succeq_x 1$ but $1 \succ_x 2$. However, the following theorem states that there is some relation between the transitivity of the weak relations $\succeq_x$ and the strict monotonicity of the bilateral principle f.

THEOREM 4.8: Let f be a monotone and anonymous bilateral principle. f is strictly monotone and for each bankruptcy problem there exists an f-just allocation if and only if for each bankruptcy problem with no zero creditors and for all allocations **x** in it $\succeq_x$ is transitive.

Proof: Only if: Let (E;**d**) be a bankruptcy problem, let **x** be an allocation in it and let f be a strictly monotone rule. We shall show that $\succeq_x$ is transitive whenever there is an f-just allocation in (E;**d**). Assume that there are three players i, j and k such that $i \succeq_x j$ and $j \succeq_x k$. Consider the reduced bankruptcy problem of i,j and k with respect to **x** and denote by **x*** its f-just allocation (by assumption, this allocation exists). Since $i \succeq_x j$, lemma 3.1 implies that either $x_i \geq x_i^*$ or $x_j \leq x_j^*$. Since $j \succeq_x k$, lemma 3.1 implies that either $x_j \geq x_j^*$ or $x_k \leq x_k^*$. Assume by contradiction that $k \succ_x i$. Then again by lemma 3.1 either $x_i < x_i^*$ or $x_k > x_k^*$. But it follows from the previous inequalities that in either case $x_j = x_j^*$. So if $x_i < x_i^*$, then $x_i + x_j < x_i^* + x_j^*$ and by strict monotonicity of f we have $f_j(x_i + x_j) < f_j(x_i^* + x_j^*) = x_j^* = x_j$, contradicting the fact that $i \succeq_x j$. But if $x_k > x_k^*$ then $x_j + x_k > x_j^* + x_k^*$. Hence by strict monotonicity of f we have $f_j(x_j + x_k) > f_j(x_j^* + x_k^*) = x_j^* = x_j$, contradicting the assumption that $j \succeq_x k$.

If: Assume $\succeq_x$ is transitive for every bankruptcy problem (E;**d**) with no zero creditors and allocation x in it, and that f is not strictly monotone. Then there are two 2-creditor bankruptcy problems $(E;\mathbf{d})$ and $(E';\mathbf{d})$ with $0 \leq E' < E$, $I=\{i,j\}$, $\mathbf{d}>\mathbf{0}$ with $f_i(E;\mathbf{d})=f_i(E';\mathbf{d})=x_i$ and $x_j=f_j(E;\mathbf{d})>f_j(E';\mathbf{d})=y_j$. Let $(E^*;\mathbf{d}^*)=(x_i+x_j+y_j;(d_i,d_j,d_j))$ and let $\mathbf{x} \in A(E^*;\mathbf{d}^*)$ be the allocation (x_i,x_j,y_j). By construction $2 \succeq_{\mathbf{x}} 1$, $1 \succeq_{\mathbf{x}} 3$ and $3 \succ_{\mathbf{x}} 2$ which contradicts the transitivity of $\succeq_{\mathbf{x}}$. Finally note that the transitivity of $\succeq_{\mathbf{x}}$ implies the transitivity of $\succ_{\mathbf{x}}$. Hence by theorem 3.2, if $\succeq_{\mathbf{x}}$ is transitive, there is an f-just allocation in each bankruptcy problem $(E;\mathbf{d})$. □

5. CONCLUSION

The motivation for this paper is to find conditions on bilateral principles, that assure the possibility of extending them to a consistent n-creditor rule. However, an alternative motivation can be found. Since a characterization of the bilateral principles mentioned above is equivalent to a characterization of the consistent rules induced by them, our main result can be viewed as an equivalence theorem for the class of monotone and consistent rules. This equivalence theorem, in contrast to Young's, characterizes the monotone and consistent rules by properties that relate to allocations that the rules do not recommend. Although different in nature, our results provides a new interpretation of parametric representations of bankruptcy rules.

Many bilateral principles do not have a consistent extension (for example the 2-creditor MP rule). The introduction of consistency on average seems a natural alternative. We believe that further study of these rules may be of interest.

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