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POST EARNINGS ANNOUNCEMENT DRIFT: MORE RISK THAN INVESTORS CAN BEAR

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Post earnings announcement drift: More risk than investors can bear

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Abstract

This paper shows how post earnings announcement drift may arise in a capital market with rational investors if the firm's earnings in consecutive periods are positively correlated and there is a fixed supply of the firm's shares. This result is driven by the fact that equilibrium share prices depend on the forward looking information contained in current earnings *and* the amount of risk that the fixed supply of shares imposes on the investors. If the latter is sufficiently large, share prices will be relatively rigid with respect to the forward looking information contained in current earnings. Hence, good (bad) news yields an increase (decrease) in the equilibrium price that is too small compared to the information that is released in the earnings announcement, so that positive (negative) abnormal returns are likely to occur again in the next period.

Keywords: rational investors, post earnings announcement drift, capital market efficiency, underreaction, overreaction.
JEL codes: G14, M41.

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1 Introduction

Post earnings announcement drift is generally considered to be a capital market anomaly. It was first documented by Ball and Brown (1968), and refers to the predictability of future abnormal returns based on previous quarterly earnings announcements. In subsequent years, numerous studies aimed at giving an explanation for this phenomenon, e.g. Joy, Litzenberger and McEnally (1977), Watts (1978), Rendleman, Jones and Latane (1982), and Foster, Olsen and Shevlin (1984). Freeman and Tse (1989) and Bernard and Thomas (1989) show that a disproportionately large fraction of the drift is delayed until the next quarterly earnings announcement. Following this observation, Bernard and Thomas (1990) attribute post earnings announcement drift to investors not fully recognizing the implications of current earnings on future earnings. More specifically, investors form their expectations on the basis of quarterly earnings following a seasonal random walk, thereby ignoring the well-documented positive correlation of two subsequent quarterly earnings (e.g. Foster (1977), Brown and Rozeff (1979), Bathke and Lorek (1984), Brown, Griffin, Hagerman and Zmijewski (1987)). In contrast to Bernard and Thomas (1990), Ball and Bartov (1996) claims that investors are aware of the intertemporal correlation of quarterly earnings. Post earnings announcement drift emerges because investors underestimate this correlation. Soffer and Lys (1999) reconciles the two contrasting perspectives of Bernard and Thomas (1990) and Ball and Bartov (1996) by arguing that investors' expectations incorporate more and more of the serial correlation in quarterly earnings as the quarter progresses.

Post earnings announcement drift is part of a more general pattern of asset pricing anomalies. Empirical evidence shows that following a public information event, capital markets underreact in the short run (i.e. positively correlated abnormal returns, see e.g. Grinblatt, Masulis and Titman (1984), Cutler, Poterba and Summers (1991), and Loughran and Ritter (1995)) and overreact in the long run (i.e. negatively correlated abnormal returns, see e.g. DeBondt and Thaler (1985), Fama and French (1988), and Poterba and Summers (1988)). This return predictability is considered to be inconsistent with capital market efficiency. Recently, several behavioral theories have been proposed that feature the aforementioned under- and overreaction in capital markets (see e.g. Daniel, Hirshleifer and Subrahmanyam (1998), Barberis, Shleifer and Vishny (1998), and Hong and Stein (1999)). These theories are all based on bounded rationality of investors.

This paper takes a theoretical approach to explaining capital market under- and overreaction and post earnings announcement drift in particular. It examines the equilibrium behavior of stock prices in a capital market with positive correlation in quarterly earnings. The model considers two periods. At the start of each period (i.e. quarter), investors can trade in a risky asset and a risk free asset. The risky asset represents a share of a particular firm, which quarterly earnings are positively correlated. The number of shares of this firm is exogenously given and the same in both periods. In other words, there is a fixed supply of the risky asset in the capital market. At the end of each period, earnings are publicly announced and paid as dividends to investors . Finally, investors are perfectly rational, constant absolute risk averse expected utility maximizers.

It is shown that in equilibrium, the capital market underreacts to the news in the first period earnings announcement if the autocorrelation in earnings is sufficiently high and if there is a sufficiently large amount of risk in the capital market. The explanation for this result is the following. The second period equilibrium price of the risky asset depends positively on the information that first period earnings provides about second period earnings *and* negatively on the amount of risk in the market. The latter follows from the fact that investors are (increasing relative) risk averse and that the supply of the risky asset is fixed. So, if more risk has to be allocated to the same population of investors, the equilibrium price should decrease to clear the market. In particular, if the risk is sufficiently large, the forward looking information contained in first period earnings has an insignificant effect on price. Hence, if first period earnings are high, the second period price increases, but it increases too little compared to the positive news that high first period earnings provide about second period earnings. Similarly, if first period earnings are low, the second period price decreases too little compared to the negative news of low first period earnings. Since the second period equilibrium price is relatively rigid with respect to new information, second period abnormal returns are likely to have the same sign as in the first period, that is post earnings announcement drift emerges.

It is further shown that capital market overreaction arises if the correlation in earnings is sufficiently low. This may explain the long run reversal in the return pattern. Assuming a sufficiently high amount of risk in the market, short run correlation in earnings can be sufficiently high to induce underreaction. Since the correlation in earnings diminishes as the lag in earnings increases, the short run underreaction will eventually be followed by an overreaction.

Post earnings announcement drift thus arises naturally in a capital market with perfectly rational investors if there is more risk than investors can bear. Too much risk results in a price rigidity that prevents prices from fully reflecting all publicly available information. This return predictability, however, need not imply that capital markets are inefficient. Efficiency requires that no publicly available information is ignored in setting the equilibrium prices. This still holds true for the capital market presented in this paper. In fact, the return predictability of post earnings announcement drift is necessary to adjust the investors' demand so as to meet the exogenous supply. Hence, the abnormal returns do not arise from a delayed price response, they are just to compensate for the risk that investors have to bear.

What is important to observe, is that capital market efficiency does *not* imply that future abnormal returns are not predictable. Most empirical studies on capital market efficiency erroneously rely on the validity of this implication. Extreme caution is therefore required in interpreting the existing empirical evidence on capital market (in)efficiency.

The remainder of this paper is organized as follows. Section 2 presents the theoretical model. Section 2.1 shows under which conditions under-/overreaction occurs in a capital market. Focus will be specifically on short term underreaction in terms of post earnings announcement drift. Section 2.2 then shows how a reversal in the return pattern can arise in equilibrium, while Section 2.3 discusses the robustness of the results. Section 3 deals with the implications on capital market efficiency and Section 4 concludes.

2 Abnormal returns in a capital market

Consider a capital market with one risky and one risk free asset over a horizon of two periods. The risky asset represents a share of a particular firm. At the end of each period $t = 1, 2$, the firm pays its earnings per share $\tilde{y}_t^1$ as dividends to its shareholders. The earnings per share $\tilde{y}_t^1$, $t = 1, 2$, are distributed as follows

$$\tilde{y}_1^1 = \begin{cases} y_h, & \text{with probability } p, \\ y_l, & \text{with probability } 1 - p, \end{cases} \tag{1}$$

and

$$\tilde{y}_2^1|_{\tilde{y}_1^1 = y_l} = \begin{cases} y_h, & \text{with probability } r, \\ y_l, & \text{with probability } 1 - r, \end{cases} \tag{2}$$

$$\tilde{y}_2^1|_{\tilde{y}_1^1 = y_h} = \begin{cases} y_h, & \text{with probability } q, \\ y_l, & \text{with probability } 1 - q, \end{cases} \tag{3}$$

where $0 < p, q, r < 1$ and $y_h > y_l \geq 0$. Observe that earnings can be either high or low and that the distribution of second period earnings depends on first period earnings. More specifically, the covariance in earnings equals $COV(\tilde{y}_1^1, \tilde{y}_2^1) = p(1-p)(q-r)$. Consistent with empirical observations (e.g. Foster (1977), Brown and Rozeff (1979), Bathke and Lorek (1984), and Brown et al. (1987)), I assume that covariance is positive, that is $q > r$. The risk free asset pays $y_t^2 = 1$ at the end of each period $t = 1, 2$. The payoff structures of the risky asset and the risk free asset are common knowledge to all investors. I assume that the firm has issued a fixed number of shares $\bar{z}$, so that supply of the risky asset is the same in each period. The rationale for this assumption arises from the fact that firms issue shares only occasionally, especially compared to the frequency of quarterly earnings announcements. So, for examining the equilibrium price behavior in a two-quarter window, one may consider the number of shares to be fixed. Finally, I assume that investors can borrow the risk free asset at no additional cost.

Investors are assumed to be constant absolute risk averse expected utility maximizers with utility function $U^i(y) = -e^{-\frac{y}{\alpha^i}}$, $y \in \mathbb{R}$, and risk tolerance $\alpha^i > 0$, $i \in N \subset \mathbb{N}$. At the start of each period, each investor $i \in N$ has a capital endowment of $\omega_t^i \geq 0$, $t = 1, 2$. Since investors are constant absolute risk averse, I may assume without loss of generality that $\omega_t^i = 0$ for all $i \in N$ and $t = 1, 2$. Investors can trade at the start of each period $t = 1, 2$. At the start of the second period, the first period earnings $\tilde{y}_1^1$ of the risky asset becomes public information, so that subsequent trade is conditional on this information.

The notion of absolute risk aversion is based on additive changes in risk. Given a risky payoff $\tilde{y}$ of a firm's share and initial wealth ω^i of investor i, let $\omega^i + \tilde{y}$ equal investor i's aggregate payoff. Then the degree of absolute risk aversion measures how an investor's valuation for the share $\tilde{y}$ changes with his initial wealth ω^i. The general opinion is that absolute risk aversion should be nonincreasing in initial wealth, which means that investors become less risk averse the richer they get. Constant absolute risk aversion comprises the special case that the degree of risk aversion is independent of the initial wealth. More important for this study, however, is the notion of relative risk aversion which is based on proportional changes in risk. Let z denote the number of shares that an investor possesses so that his aggregate payoff equals $z\tilde{y}$. Then the degree of relative risk aversion measures how an investor's valuation per share $\tilde{y}$ changes with the number of shares z in his possession. The general opinion is that relative risk aversion should increase with the number of shares. This means that investors become more risk averse if they invest more in the same share. One can show that constant absolute risk aversion implies increasing relative risk aversion.[1]

In a competitive equilibrium, investors are price takers. Given the prices of the two assets, each investor will demand the quantities that maximize his expected utility. Then equilibrium prices are prices for which the aggregate demand for the risky asset equals the exogenous supply.

[1]This implication is not obvious. For a more extensive discussion of absolute and relative risk aversion, see e.g. Eeckhoudt and Gollier (1995).

Let π_t denote the price of the risky asset in period $t = 1, 2$. The price of the risk free asset is normalized to one in both periods. Since second period trade depends on the first period earnings $\tilde{y}_1^1$ of the firm, the same holds true for the second period price π_2 of the risky asset. Let π_{2h} denote the second period equilibrium price for the risky asset if first period earnings are high, that is if $\tilde{y}_1^1 = y_h$. Similarly, let π_{2l} denote the equilibrium price if first period earnings are low, that is if $\tilde{y}_1^1 = y_l$.

Let z_t^{1i} denote the demand of investor $i \in N$ for the risky asset in period $t = 1, 2$, and let z_t^{2i} denote the demand for the risk free asset. Since the second period trade is conditional on the first period earnings $\tilde{y}_1^1$, let $(z_{2h}^{1i}, z_{2h}^{2i})$ and $(z_{2l}^{1i}, z_{2l}^{2i})$ denote the second period demands for the two assets if first period earnings are high and low, respectively.

For a formal statement of the equilibrium conditions, define the indirect utility function $V_l^i(b, \pi_{2l})$ as the maximum expected utility that investor i can obtain in the second period if his budget in the second period equals b, the first period earnings equal $\tilde{y}_1^1 = y_l$, and the price of the risky asset equals π_{2l}, i.e.:

$$\begin{aligned} V_l^i(b, \pi_{2l}) = \max_{(z_{2l}^{1i}, z_{2l}^{2i})} \quad & rU^i\left(z_{2l}^{1i} y_h + z_{2l}^{2i}\right) + (1-r)U^i\left(z_{2l}^{1i} y_l + z_{2l}^{2i}\right) \\ \text{s.t.:} \quad & \pi_{2l} z_{2l}^{1i} + z_{2l}^{2i} = b. \end{aligned} \tag{4}$$

In a similar way, define $V_h^i(b, \pi_{2h})$ if first period earnings equal $\tilde{y}_1^1 = y_h$, i.e.:

$$\begin{aligned} V_h^i(b, \pi_{2h}) = \max_{(z_{2h}^{1i}, z_{2h}^{2i})} \quad & qU^i\left(z_{2h}^{1i} y_h + z_{2h}^{2i}\right) + (1-q)U^i\left(z_{2h}^{1i} y_l + z_{2h}^{2i}\right) \\ \text{s.t.:} \quad & \pi_{2h} z_{2h}^{1i} + z_{2h}^{2i} = b. \end{aligned} \tag{5}$$

Then a price system $(\hat{\pi}_1, \hat{\pi}_{2l}, \hat{\pi}_{2h})$ and demands $(\hat{z}_1^{1i}, \hat{z}_1^{2i}, \hat{z}_{2l}^{1i}, \hat{z}_{2l}^{2i}, \hat{z}_{2h}^{1i}, \hat{z}_{2h}^{2i})$ constitute a competitive equilibrium if the following conditions hold true:

(i) The first period demands maximize expected utility given the second period equilibrium demands and prices, i.e. for each $i \in N$

$$\begin{aligned} (\hat{z}_1^{1i}, \hat{z}_1^{2i}) = \arg\max_{(z_1^{1i}, z_1^{2i})} \quad & pV_h^i\left(z_1^{1i}(y_h + \hat{\pi}_{2h}) + z_1^{2i}, \hat{\pi}_{2h}\right) + \\ & (1-p)V_l^i\left(\hat{z}_1^{1i}(y_l + \hat{\pi}_{2l}) + z_1^{2i}, \hat{\pi}_{2l}\right) \\ \text{s.t.:} \quad & \hat{\pi}_1 z_1^{1i} + z_1^{2i} = 0. \end{aligned} \tag{6}$$

(ii) The first period demands clear the market, i.e. $\sum_{i \in N} \hat{z}_1^{1i} = \bar{z}$.

(iii) Conditional on the first period earnings being low, the second period demands maximize investors' expected utility, i.e. for each $i \in N$

$$\begin{aligned} (\hat{z}_{2l}^{1i}, \hat{z}_{2l}^{2i}) = \arg\max_{(z_{2l}^{1i}, z_{2l}^{2i})} \quad & rU^i\left(z_{2l}^{1i} y_h + z_{2l}^{2i}\right) + (1-r)U^i\left(z_{2l}^{1i} y_l + z_{2l}^{2i}\right) \\ \text{s.t.:} \quad & z_{2l}^{1i}\hat{\pi}_{2l} + z_{2l}^{2i} = \hat{z}_1^{1i}(y_l + \hat{\pi}_{2l}) + \hat{z}_1^{2i}. \end{aligned} \tag{7}$$

Notice that second period budget consists of the initial endowment ω_2^i in period 2, the received dividends $\hat{z}_1^{1i} y_l$, and the value $\hat{z}_1^{1i}\hat{\pi}_{2l} + \hat{z}_1^{2i}$ of the first period portfolio.

(iv) Conditional on the first period earnings being low, the second period demands clear the market, i.e. $\sum_{i \in N} \hat{z}_{2l}^{1i} = \bar{z}$.

(v) Conditional on the first period earnings being high, the second period demands maximize investors' expected utility, i.e. for each $i \in N$

$$\begin{aligned}(\hat{z}_{2h}^{1i}, \hat{z}_{2h}^{2i}) = \; & \arg \max_{(z_{2h}^{1i}, z_{2h}^{2i})} \quad qU^i\left(z_{2h}^{1i} y_h + z_{2h}^{2i}\right) + (1-q)U^i\left(z_{2h}^{1i} y_l + z_{2h}^{2i}\right) \\ & \text{s.t.:} \qquad z_{2h}^{1i}\hat{\pi}_{2h} + z_{2h}^{2i} = \hat{z}_1^{1i}(y_h + \hat{\pi}_{2h}) + \hat{z}_1^{2i}.\end{aligned} \tag{8}$$

(vi) Conditional on the first period earnings being high, the second period demands clear the market, i.e. $\sum_{i \in N} \hat{z}_{2h}^{1i} = \bar{z}$.

2.1 Capital market underreaction and overreaction

The second period equilibrium price is expected to take into account the information that first period earnings provide about the distribution of second period earnings. The extent to which price reflects this information may lead to under- or overreaction by the capital market. Underreaction implies that price changes are too small in relation to the information that is revealed, yielding a positive correlation in abnormal returns. Similarly, overreaction implies that prices change too much in relation to the information that is revealed, resulting in a negative correlation in abnormal returns.

In equilibrium, the first period return of the risky asset equals

$$\tilde{r}_1 = \begin{cases} \dfrac{y_h + \hat{\pi}_{2h} - \hat{\pi}_1}{\hat{\pi}_1}, & \text{with probability } p, \\ \dfrac{y_l + \hat{\pi}_{2l} - \hat{\pi}_1}{\hat{\pi}_1}, & \text{with probability } 1-p. \end{cases} \tag{9}$$

Recall that first period earnings are paid as dividends to investors. Similarly, the second period equilibrium return equals

$$\tilde{r}_2 = \begin{cases} \dfrac{y_l - \hat{\pi}_{2l}}{\hat{\pi}_{2l}}, & \text{with probability } (1-p)(1-r), \\ \dfrac{y_h - \hat{\pi}_{2l}}{\hat{\pi}_{2l}}, & \text{with probability } (1-p)r, \\ \dfrac{y_l - \hat{\pi}_{2h}}{\hat{\pi}_{2h}}, & \text{with probability } p(1-q), \\ \dfrac{y_h - \hat{\pi}_{2h}}{\hat{\pi}_{2h}}, & \text{with probability } pq. \end{cases} \tag{10}$$

Then abnormal return for each period is defined by $\tilde{R}_t = \tilde{r}_t - E(\tilde{r}_t)$, where $t = 1, 2$. The following proposition concerns the covariance in abnormal returns.

Proposition 1 The covariance between the first and second period abnormal return equals

$$COV(\tilde{R}_1, \tilde{R}_2) = p(1-p)\left(\frac{y_h + \hat{\pi}_{2h} - (y_l + \hat{\pi}_{2l})}{\hat{\pi}_1}\right)\left(E(\tilde{r}_2|\tilde{y}_1^1 = y_h) - E(\tilde{r}_2|\tilde{y}_1^1 = y_l)\right). \quad (11)$$

Capital market underreaction, i.e. positively correlated abnormal returns, arises if the second period conditional expected return is higher if first period earnings are high than if first period earnings are low. Capital market overreaction, i.e. negatively correlated abnormal returns, arises if the opposite holds. Obviously, correlation in abnormal returns is zero if first period earnings do not provide any information about second period earnings. Since in that case the conditional return $\tilde{r}_2|\tilde{y}_1^1$ equals the unconditional return $\tilde{r}_2$, it holds that $E(\tilde{r}_2|\tilde{y}_1^1 = y_h) = E(\tilde{r}_2|\tilde{y}_1^1 = y_l)$.

For determining the conditional expected returns, one requires the second period equilibrium prices. Since investors are constant absolute risk averse, these equilibrium prices are given by[2]

$$\hat{\pi}_{2l} = \frac{re^{-x}}{re^{-x} + 1 - r} y_h + \frac{1-r}{re^{-x} + 1 - r} y_l \quad (12)$$

and

$$\hat{\pi}_{2h} = \frac{qe^{-x}}{qe^{-x} + 1 - q} y_h + \frac{1-q}{qe^{-x} + 1 - q} y_l, \quad (13)$$

where

$$x = \frac{\bar{z}(y_h - y_l)}{\sum_{i \in N} \alpha_i}. \quad (14)$$

The variable x represents the burden that the risky asset puts on the investors. The numerator $\bar{z}(y_h - y_l)$ measures the total risk that the investors have to bear due to the fixed supply $\bar{z}$ of the risky asset. The denominator measures the aggregate risk tolerance of the population of investors. An increase in the number of shares $\bar{z}$ or the dispersion $y_h - y_l$ increases this risk burden, while an increase in the population of investors decreases this risk burden as it increases their risk sharing opportunities.

Observe that the second period equilibrium price depends positively on the probability of high second period earnings and negatively on the risk burden. The latter follows from the fact that investors are increasing relative risk averse. To illustrate, one can show that in equilibrium investor $i \in N$ invests in $\frac{\alpha_i}{\sum_{j \in N} \alpha_j} \bar{z}$ shares, which is the unique Pareto optimal allocation of the

[2]The derivation of $\hat{\pi}_{2l}$ is stated in the appendix. $\hat{\pi}_{2h}$ is derived analogously.

exogenous supply $\bar{z}$ of shares (cf. Wilson (1968)). Since investors' risk aversion increases with the number of shares, the equilibrium price must be lower if more shares have to be allocated to the investors.

Proposition 2 There exists a unique value x^* of the risk burden, namely

$$x^* = \max\left(\log\left(\frac{q}{1-q}\frac{r}{1-r}\frac{y_h}{y_l}\right), 0\right)$$

such that the capital market underreacts if $x > x^*$, and overreacts if $x < x^*$.

Since post earnings announcement drift refers to underreaction, the next result follows straightforwardly.

Corollary 3 Post earnings announcement drift arises in a capital market if investors bear a sufficiently high amount of risk.

For the explanation of this result, recall that post earnings announcement drift arises if the market underreacts to the information provided by first period earnings, that is if $E(\tilde{r}_2|\tilde{y}_1^1 = y_h) > E(\tilde{r}_2|\tilde{y}_1^1 = y_l)$. The two major determinants of the expected return are the probability of high second period earnings and the second period equilibrium price. A higher probability increases the expected return while a higher price decreases the expected return. The equilibrium price also depends on the probability of high earnings but this influence diminishes as the risk burden increases. This observation, which is illustrated in Figure 1, drives Corollary 3 and Proposition 2. For suppose that the risk burden x is relatively high, then the probability of high second period earnings has only little effect on the second period equilibrium price, so that $\hat{\pi}_{2h}$ is approximately equal to $\hat{\pi}_{2l}$. Since the probability of high second period earnings is larger if first period earnings are high than if first period earnings are low, the same holds true for the conditional expected return. Hence, the market underreacts.

This argument no longer holds true if the risk burden is relatively low. In that case, the probability of high second period earnings has a significant effect on the second period equilibrium price. So, although high first period earnings yield a higher success probability in the second period then low first period earnings, it also yields a significantly higher price. Since the success probability and price have opposite effects on the expected return, the total effect is unclear. In fact, if the risk burden is sufficiently low, the negative effect of price dominates and overreaction results.

Observe that Proposition 2 reverses if earnings are negatively correlated. In that case, the market underreacts if the risk burden is sufficiently low, i.e. $x < x^*$, and overreacts if the risk burden is sufficiently high, i.e. $x > x^*$.

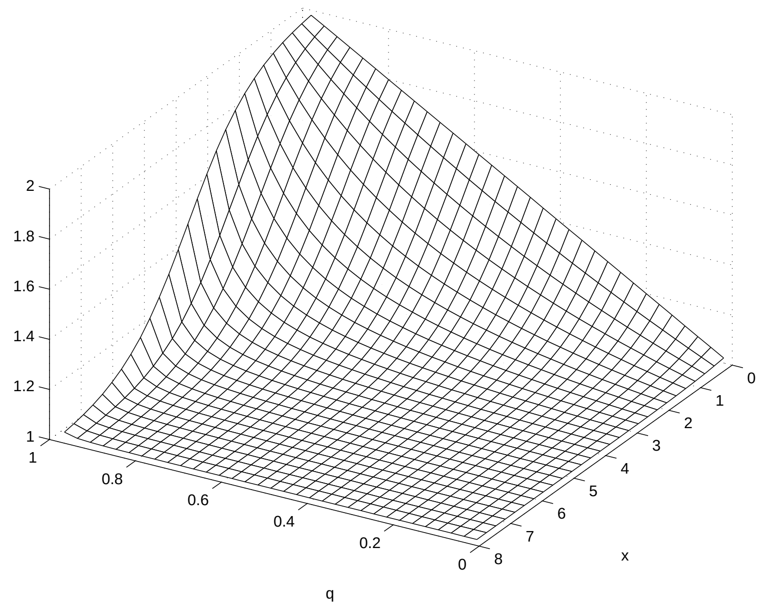

Figure 1: Second period equilibrium price as a function of the probability of high second period earnings and the risk burden on the basis of the parameter values $y_h = 2$, $y_l = 1$, and $\sum_{i \in N} \alpha_i = 1$.

Summarizing, capital market underreaction arises because a high risk burden induces a certain amount of price rigidity for the risky asset with respect to characteristics of the risky asset's distribution. As a result, prices cannot change in such a way so as to fully reflect all the forward looking information that is contained in first period earnings. Following the good news of high first period earnings, the second period equilibrium price will be too low relative to the probability of high second period earnings, so that investors will most likely earn positive abnormal returns in the second period again. A similar argument holds if low first period earnings are announced. Then the second period equilibrium price will be too high relative to the probability of high second period earnings, so that investors will most likely earn negative abnormal returns in the second period again.

2.2 Long run reversals

Proposition 2 shows how under- and overreaction may arise in a capital market. Which of the two actually occurs depends on some firm-specific paramters. Proposition 2 does, however, not explain the reversal of return patterns in the long run. Overreaction follows underreaction if the positive correlation in earnings is followed by a negative correlation. There is, however, no empirical evidence to support such correlation pattern in earnings. Bernard and Thomas

(1990) claim that seasonally differenced quarterly earnings follow such a correlation pattern, but Jacob, Lys and Sabino (2000) show that this is due to overdifferencing of the earnings time-series.

This section shows how for an individual firm the short run underreaction may change into overreaction in the long run. For this purpose, let $\tilde{y}_1^1$ and $y_2^1|_{\tilde{y}_1^1=y_l}$ be distributed as before (cf. (1) and (2)). If, however, first period earnings are high, second period earnings are equal to $\gamma + \tilde{y}_2^2|_{\tilde{y}_1^1=y_h}$, where $\gamma > 0$. So, high first period earnings now entail good news in two ways: it not only increases the probability of high second period earnings, but it also increases the second period earnings with a fixed amount $\gamma > 0$.

Let $\hat{\hat{\pi}}_1$, $\hat{\hat{\pi}}_{2l}$, and $\hat{\hat{\pi}}_{2h}$ denote the new equilibrium prices, so that the equilibrium returns are given by

$$\tilde{r}_1 = \begin{cases} \dfrac{\hat{\hat{\pi}}_{2h} - \hat{\hat{\pi}}_1}{\hat{\hat{\pi}}_1}, & \text{with probability } p, \\ \dfrac{\hat{\hat{\pi}}_{2l} - \hat{\hat{\pi}}_1}{\hat{\hat{\pi}}_1}, & \text{with probability } 1-p, \end{cases} \tag{15}$$

and

$$\tilde{r}_2 = \begin{cases} \dfrac{y_l - \hat{\hat{\pi}}_{2l}}{\hat{\hat{\pi}}_{2l}}, & \text{with probability } (1-p)(1-r), \\ \dfrac{y_h - \hat{\hat{\pi}}_{2l}}{\hat{\hat{\pi}}_{2l}}, & \text{with probability } (1-p)r, \\ \dfrac{\gamma + y_l - \hat{\hat{\pi}}_{2h}}{\hat{\hat{\pi}}_{2h}}, & \text{with probability } p(1-q), \\ \dfrac{\gamma + y_h - \hat{\hat{\pi}}_{2h}}{\hat{\hat{\pi}}_{2h}}, & \text{with probability } pq. \end{cases} \tag{16}$$

Similar to Proposition 1, the market underreacts (positively correlated abnormal returns) if $E(\tilde{r}_2|\tilde{y}_1^1 = y_h) > E(\tilde{r}_2|\tilde{y}_1^1 = y_l)$, while the market overreacts (negatively correlated abnormal returns) if $E(\tilde{r}_2|\tilde{y}_1^1 = y_h) < E(\tilde{r}_2|\tilde{y}_1^1 = y_l)$.

The new payoff structure affects the second period equilibrium price in the following way. If first period earnings are low, second period earnings are distributed as before. Hence, $\hat{\hat{\pi}}_{2l} = \hat{\pi}_{2l}$. If first period earnings are high, the earnings γ will be taken into account by the second period equilibrium price. In fact, due to the assumption of constant absolute risk aversion, it holds that $\hat{\hat{\pi}}_{2h} = \hat{\pi}_{2h} + \gamma$. For the same reason as before, a high risk burden induces underreaction by the capital market. The earnings γ, however, induce overreaction. To see this, observe that if first period earnings are high, then the second period equilibrium price increases with γ to $\hat{\hat{\pi}}_{2h} = \hat{\pi}_{2h} + \gamma$, so that the corresponding return equals

$$\frac{\gamma + \tilde{y}_2^2|_{\tilde{y}_1^1 = y_h} - \hat{\hat{\pi}}_{2h}}{\hat{\hat{\pi}}_{2h}} = \frac{\tilde{y}_2^2|_{\tilde{y}_1^1 = y_h} - \hat{\pi}_{2h}}{\hat{\pi}_{2h} + \gamma}, \tag{17}$$

which is decreasing in γ. Since the negative effect of γ on return only applies to $E(\tilde{r}_2|\tilde{y}_1^1 = y_h)$ and not to $E(\tilde{r}_2|\tilde{y}_1^1 = y_l)$, overreaction arises, i.e. $E(\tilde{r}_2|\tilde{y}_1^1 = y_h) < E(\tilde{r}_2|\tilde{y}_1^1 = y_l)$.

Proposition 4 If $r < \frac{y_l}{\gamma + y_l} q$, then there exists a unique critical value x^* for the risk burden such that the capital market underreacts if $x > x^*$ and overreacts if $x < x^*$. If $r \geq \frac{y_l}{\gamma + y_l} q$, then overreaction occurs for all $x > 0$.

The difference with Proposition 2 is easily explained. Recall that underreaction occurs if the risk burden is sufficiently high. Furthermore, the magnitude of the underreaction increases with the magnitude of the correlation in earnings. The higher the correlation, the more the market will underreact. Then the retained earnings induce overreaction that dominates the underreaction only if the latter is sufficiently low. This is so if the correlation in earnings is sufficiently low, that is if $r \geq \frac{y_l}{\gamma + y_l} q$.

Proposition 4 explains why underreaction may occur in the short run and overreaction in the long run. To see this, extend the model by replicating the second period to T periods. The distribution of earnings $\tilde{y}_t^1$ in period $t = 2, \ldots, T$ depends on the earnings of the previous period in a similar way as in the 2-period model. More specifically, if period $t-1$ earnings are low, then period t earnings $\tilde{y}_t^1|_{\tilde{y}_{t-1}^1 = y_l}$ are distributed as in (2). If period $t-1$ earnings are high, period t earnings equal $\gamma + \tilde{y}_t^1|_{\tilde{y}_{t-1}^1 = y_h}$, where $\gamma > 0$ and $\tilde{y}_t^1|_{\tilde{y}_{t-1}^1 = y_h}$ is distributed as in (3). Then the correlation in earnings of period t and $t+k$ equals $p(1-p)(q-r)^k$. Observe that the correlation decreases as the lag in earnings increases but that the correlation remains positive. Taking the earnings surprise at $t = 1$ as the event date, one can consider the correlation in abnormal returns in period t and the event date $t = 1$. Following Proposition 4, if both the risk burden and the correlation in lag-one earnings is sufficiently high, abnormal returns in period 1 and 2 are positively correlated. Increasing the lag in earnings to k periods, decreases the correlation. Hence, the positive correlation in abnormal returns in period 1 and k also decreases. Increasing the lag in earnings even further will eventually result in such a low level of correlation in earnings that the correlation in abnormal returns becomes negative (cf. Proposition 4). Consequently, short term underreaction is followed by an overreaction.

2.3 Robustness of the results

The model assumes that the supply of the risky asset is exogenous and independent of any new information in the second period. One can endogenize the supply level by extending the model and obtain as such an equilibrium value for the supply level. Then post earnings announcement drift may cease to exist if the equilibrium value of supply yields a risk burden x that coincides with the critical value x^*, for in that case correlation between abnormal returns will be zero. Although one cannot exclude this possibility in the present model, one can construct a model where the supply is endogenously determined but constant over all periods, in which the equilibrium value of x differs from x^*. For this, one needs to extend the state space by introducing some uncertainty about the probabilities q and r. For instance, before first period trade starts, let nature determine the probabilities q and r from the possible states $(q_1, r_1), (q_2, r_2), \ldots, (q_k, r_k)$. Furthermore, suppose that the supply of the risky asset is determined endogenously before the state of nature is determined, but that investors learn the state of nature before first period trade starts. In this regard one can interpret the choice of nature as the 'history' of the risky asset between the date of issue of the risky asset and the two periods that are explicitly considered in the model. Then zero correlation in abnormal return arises if the equilibrium value for the risk burden x equals $x^*(q_j, r_j)$ for all $j = 1, 2, \ldots, k$. Since $x^*(q, r)$ varies with the probabilities q and r, this condition is violated. Hence, the exogenous supply level is not crucial for the results.

An implicit assumption of the model is that the risk burden does not change over the two periods and, particularly, that it is independent of the first period earnings. By allowing the risk burden to vary, zero correlation in abnormal returns may arise in equilibrium. However, the factors that determine the risk burden do not change frequently over time. Firms issue shares to acquire capital from investors. The frequency of such an event is relatively low compared to the production of information like quarterly earnings figures. A similar argument holds for the number of investors. Risk sharing arguments yield that, in equilibrium, all potential investors will actually invest in the risky asset. Hence, new information will have no effect on the population of investors.

The assumption of constant absolute risk aversion is not vital for supporting the results of this paper. Constant absolute risk aversion enabled me to explicitly determine the equilibrium prices. What drives the results is that the influence of the probability distribution of the risky asset on the equilibrium price becomes insignificant as the risk burden increases (cf. Figure 1). This is due to the increasing relative risk aversion of investors. For constant relative risk averse investors, for instance, this no longer holds true as investors' valuation for the risky shares is independent of the quantity they receive. Hence, an increase in the risk burden through an increase in the supply $\bar{z}$ would have no effect on the equilibrium price.

3 Capital market efficiency

The results of this paper also shed a new light on the implications of capital market efficiency. Following Fama (1976), capital market efficiency requires that all publicly available information is used in setting the equilibrium prices. More formally, let I_a denote the information that is available to the capital market and let I_m denote the information that the capital market actually uses in setting the equilibrium prices. Then a capital market is efficient if prices are set as if $I_m = I_a$, that is no available information is ignored. Capital market efficiency is generally phrased as prices fully reflect all available information in the market. There is, however, no formal definition of what 'fully reflect' means in this respect. The general interpretation is that prices fully reflect all available information I_a if one cannot predict future abnormal returns on the basis of this available information I_a. Empirical evidence claiming (in)efficiency of capital markets is commonly based on this predictability argument.

As this paper shows, capital market efficiency does not automatically imply that future abnormal returns are not predictable. In a capital market, prices are used to reflect information about future payoffs *and* to clear the market. Since the latter must occur in equilibrium, there may be too few degrees of freedom left to accomplish the former. That this may give rise to predictable future abnormal returns is not an inefficiency of the market. On the contrary, the market creates this predictability to increase demand for the risky asset so as to meet the fixed supply. In this regard it is important to observe that the gains resulting from predictable abnormal returns are limited. If first period earnings are high, investors are willing to invest more in the risky asset because of the existing positive correlation. Although the rate of return of the risky asset does not depend on the size of the investment z_{2h}^{1i} - the rate of return is constant at $\frac{\tilde{y}_2^1 - \hat{\pi}_{2h}}{\hat{\pi}_{2h}}$ - the payoff $z_{2h}^{1i}(\tilde{y}_2^1 - \hat{\pi}_{2h})$ of the risky asset does depend on the demand z_{2h}^{1i}. Increasing the demand z_{2h}^{1i} will increase the risk of the investor. Since investors are increasing relative risk averse, the increase in risk will ultimately outweigh the abnormal return. In equilibrium, the abnormal returns that investors earn just compensate for the additional risk that investors have to take. Hence, post earnings announcement drift may not be driven by a delayed price response, the explanation that currently prevails in the literature (e.g. Bernard and Thomas (1990), Ball and Bartov (1996), and Soffer and Lys (1999)).

Since capital market efficiency is not equivalent to stating that future abnormal returns are not predictable, one should exercise caution in interpreting the results of empirical studies on capital market efficiency, as these studies may be based on a false assumption. This paper shows that even if it is possible to predict future abnormal returns, this need not be an indication of capital market inefficiency. Similarly, if it is not possible to predict future abnormal returns, this need not support capital market efficiency. The latter claim is easily explained as follows. Suppose the capital market is not efficient, that is it sets equilibrium prices on the basis of the information set $I_m \subset I_a$. Further, suppose that it is not possible to predict future abnormal returns on the basis of the information I_m that the market uses. Then it is also not possible to predict future abnormal returns on the basis of the information set I_a; By assumption, I_m does not predict future abnormal returns and since the unused information $I_a - I_m$ is uncorrelated with equilibrium prices, it also cannot predict future abnormal returns. Hence, although future abnormal returns cannot be predicted on the basis of all available information, the capital market is inefficient as it ignores some of the available information in setting the equilibrium prices. To illustrate with an (extreme) example, suppose the market sets prices at random. Obviously, such a market would not be efficient. However, future abnormal returns are also not predictable in this market.

Although efficient capital markets use all available information in setting the equilibrium prices, not all of this information need to be 'reflected' in prices. Consequently, returns or market prices may not be the appropriate instruments to empirically test capital market efficiency. For this purpose, other measures are needed that capture the use of information better than prices do.

4 Conclusions

This paper shows how post earnings announcement drift may arise in a capital market with rational investors if the firm's earnings in consecutive periods are positively correlated and the supply of the firm's shares is fixed. The fixed supply of shares imposes a risk burden on the population of investors as the risky payoffs of the shares have to be allocated among the investors. If this risk burden is large, asset prices become rigid with respect to forward looking information. As a result, forward looking information contained in a current earnings announcement is not fully reflected in the equilibrium price and a 'drift' emerges. The paper further shows how short run underreaction is followed by long run overreaction. Reason for this is that underreaction requires a sufficiently high amount of correlation in earnings. Since correlation decreases as the lag in earnings increases, overreaction will eventually arise.

Capital market under- and overreaction do not indicate capital market inefficiencies as no available information is ignored in setting the equilibrium prices. What is important to observe, is that capital market efficiency does *not* imply that future abnormal returns are not predictable. Most empirical studies on capital market efficiency erroneously rely on the validity of this implication. Extreme caution is therefore required in interpreting the existing empirical evidence on capital market (in)efficiency.

5 Appendix

PROOF OF PROPOSITION 1: There are four states of the world $(\tilde{y}_1^1, \tilde{y}_2^1)$, namely (y_l, y_l), (y_l, y_h), (y_h, y_l), and (y_h, y_h), that occur with probability $(1-p)(1-r), (1-p)r, p(1-q)$, and pq, respectively.

First, let me derive the first period abnormal return $\tilde{R}_1 = \tilde{r}_1 - E(\tilde{r}_1)$. If $\tilde{y}_1^1 = y_l$ then

$$\begin{aligned}\tilde{r}_1 - E(\tilde{r}_1) &= \frac{y_l + \hat{\pi}_{2l} - \hat{\pi}_1}{\hat{\pi}_1} - p\frac{y_h + \hat{\pi}_{2h} - \hat{\pi}_1}{\hat{\pi}_1} - (1-p)\frac{y_l + \hat{\pi}_{2l} - \hat{\pi}_1}{\hat{\pi}_1} \\ &= -p\frac{y_h + \hat{\pi}_{2h} - y_l - \hat{\pi}_{2l}}{\hat{\pi}_1}. \end{aligned} \tag{18}$$

Similarly, if $\tilde{y}_1^1 = y_h$, then

$$\begin{aligned}\tilde{r}_1 - E(\tilde{r}_1) &= \frac{y_h + \hat{\pi}_{2h} - \hat{\pi}_1}{\hat{\pi}_1} - p\frac{y_h + \hat{\pi}_{2h} - \hat{\pi}_1}{\hat{\pi}_1} - (1-p)\frac{y_l + \hat{\pi}_{2l} - \hat{\pi}_1}{\hat{\pi}_1} \\ &= (1-p)\frac{y_h + \hat{\pi}_{2h} - y_l - \hat{\pi}_{2l}}{\hat{\pi}_1}. \end{aligned} \tag{19}$$

Next, let me derive the second period abnormal return $\tilde{R}_2 = \tilde{r}_2 - E(\tilde{r}_2)$. Observe that

$$\begin{aligned}E(\tilde{r}_2) &= (1-p)(1-r)\left(\frac{y_l - \hat{\pi}_{2l}}{\hat{\pi}_{2l}}\right) + (1-p)r\left(\frac{y_h - \hat{\pi}_{2l}}{\hat{\pi}_{2l}}\right) + \\ &\quad p(1-q)\left(\frac{y_l - \hat{\pi}_{2h}}{\hat{\pi}_{2h}}\right) + pq\left(\frac{y_h - \hat{\pi}_{2h}}{\hat{\pi}_{2h}}\right) \\ &= (1-p)r\left(\frac{y_h - y_l}{\hat{\pi}_{2l}}\right) + pq\left(\frac{y_h - y_l}{\hat{\pi}_{2h}}\right) + \\ &\quad (1-p)\left(\frac{y_l - \hat{\pi}_{2l}}{\hat{\pi}_{2l}}\right) + p\left(\frac{y_l - \hat{\pi}_{2h}}{\hat{\pi}_{2h}}\right).\end{aligned}$$

If $(\tilde{y}_1^1, \tilde{y}_2^1) = (y_l, y_l)$ then $\tilde{r}_2 = \frac{y_l - \hat{\pi}_{2l}}{\hat{\pi}_{2l}}$, so that

$$\begin{aligned}\tilde{r}_2 - E(\tilde{r}_2) &= -(1-p)r\left(\frac{y_h - y_l}{\hat{\pi}_{2l}}\right) - pq\left(\frac{y_h - y_l}{\hat{\pi}_{2h}}\right) + \\ &\quad p\left(\frac{y_l - \hat{\pi}_{2l}}{\hat{\pi}_{2l}}\right) - p\left(\frac{y_l - \hat{\pi}_{2h}}{\hat{\pi}_{2h}}\right). \end{aligned} \tag{20}$$

If $(\tilde{y}_1^1, \tilde{y}_2^1) = (y_l, y_h)$ then $\tilde{r}_2 = \frac{y_h - \hat{\pi}_{2l}}{\hat{\pi}_{2l}}$, so that

$$\begin{aligned}\tilde{r}_2 - E(\tilde{r}_2) &= \frac{y_l - \hat{\pi}_{2l}}{\hat{\pi}_{2l}} + \frac{y_h - y_l}{\hat{\pi}_{2l}} - E(\tilde{r}_2) \\ &= (1-(1-p)r)\left(\frac{y_h - y_l}{\hat{\pi}_{2l}}\right) - pq\left(\frac{y_h - y_l}{\hat{\pi}_{2h}}\right) + \\ &\quad p\left(\frac{y_l - \hat{\pi}_{2l}}{\hat{\pi}_{2l}}\right) - p\left(\frac{y_l - \hat{\pi}_{2h}}{\hat{\pi}_{2h}}\right). \end{aligned} \tag{21}$$

If $(\tilde{y}_1^1, \tilde{y}_2^1) = (y_h, y_l)$ then $\tilde{r}_2 = \frac{y_l - \hat{\pi}_{2h}}{\hat{\pi}_{2h}}$, so that

$$\begin{aligned}\tilde{r}_2 - E(\tilde{r}_2) &= -(1-p)r\left(\frac{y_h - y_l}{\hat{\pi}_{2l}}\right) - pq\left(\frac{y_h - y_l}{\hat{\pi}_{2h}}\right) - \\ &\quad (1-p)\left(\frac{y_l - \hat{\pi}_{2l}}{\hat{\pi}_{2l}}\right) + (1-p)\left(\frac{y_l - \hat{\pi}_{2h}}{\hat{\pi}_{2h}}\right). \end{aligned} \tag{22}$$

If $(\tilde{y}_1^1, \tilde{y}_2^1) = (y_h, y_h)$ then $\tilde{r}_2 = \frac{y_h - \hat{\pi}_{2h}}{\hat{\pi}_{2h}}$, so that

$$\begin{aligned}\tilde{r}_2 - E(\tilde{r}_2) &= \frac{y_l - \hat{\pi}_{2h}}{\hat{\pi}_{2h}} + \frac{y_h - y_l}{\hat{\pi}_{2h}} - E(\tilde{r}_2) \\ &= -(1-p)r\left(\frac{y_h - y_l}{\hat{\pi}_{2l}}\right) + (1-pq)\left(\frac{y_h - y_l}{\hat{\pi}_{2h}}\right) - \\ &\quad (1-p)\left(\frac{y_l - \hat{\pi}_{2l}}{\hat{\pi}_{2l}}\right) + (1-p)\left(\frac{y_l - \hat{\pi}_{2h}}{\hat{\pi}_{2h}}\right). \end{aligned} \tag{23}$$

Since $E(\tilde{R}_t) = 0$ for $t = 1, 2$ it follows that $COV(\tilde{R}_1, \tilde{R}_2) = E(\tilde{R}_1 \tilde{R}_2)$. Since $\tilde{R}_1$ is proportional to $\frac{y_h + \hat{\pi}_{2h} - (y_l + \hat{\pi}_{2l})}{\hat{\pi}_1}$, it holds that

$$\begin{aligned}\frac{\hat{\pi}_1}{y_h + \hat{\pi}_{2h} - (y_l + \hat{\pi}_{2l})} E(\tilde{R}_1 \tilde{R}_2) &= -p(1-p)r\left(\frac{y_h - y_l}{\hat{\pi}_{2l}}\right) + p(1-p)q\left(\frac{y_h - y_l}{\hat{\pi}_{2h}}\right) - \\ &\quad p(1-p)\left(\frac{y_l - \hat{\pi}_{2l}}{\hat{\pi}_{2l}}\right) + p(1-p)\left(\frac{y_l - \hat{\pi}_{2h}}{\hat{\pi}_{2h}}\right) \\ &= p(1-p)\left(q\frac{y_h - y_l}{\hat{\pi}_{2h}} + \frac{y_l - \hat{\pi}_{2h}}{\hat{\pi}_{2h}}\right) - \\ &\quad p(1-p)\left(r\frac{y_h - y_l}{\hat{\pi}_{2l}} + \frac{y_l - \hat{\pi}_{2l}}{\hat{\pi}_{2l}}\right) \\ &= p(1-p)\left(E(\tilde{r}_2 | \tilde{y}_1^1 = y_h) - E(\tilde{r}_2 | \tilde{y}_1^1 = y_l)\right). \quad \Box \end{aligned}$$

PROOF OF EXPRESSION (12): Take the first period equilibrium demands $(\hat{z}_1^{1i}, \hat{z}_1^{2i})$ and equilibrium prices $\hat{\pi}_1$ and $\hat{\pi}_{2l}$ as given. Using the substitution $z_{2l}^{2i} = \hat{z}_1^{1i}(y_l + \hat{\pi}_{2l}) + \hat{z}_1^{2i} + \omega_2^i - \hat{\pi}_{2l} z_{2l}^{1i}$, the second period maximization problem equals

$$\begin{aligned}\max_{z_{2l}^{1i}} \; & rU^i\left(z_{2l}^{1i}(y_h - \hat{\pi}_{2l}) + \hat{z}_1^{1i}(y_l + \hat{\pi}_{2l}) + \hat{z}_1^{2i} + \omega_2^i\right) + \\ & (1-r)U^i\left(z_{2l}^{1i}(y_l - \hat{\pi}_{2l}) + \hat{z}_1^{1i}(y_l + \hat{\pi}_{2l}) + \hat{z}_1^{2i} + \omega_2^i\right). \end{aligned}$$

Since investors are constant absolute risk averse, the fixed income $\hat{z}_1^{1i}(y_l + \hat{\pi}_{2l}) + \hat{z}_1^{2i} + \omega_2^i$ does not influence the second period demand z_{2l}^{1i}, so that the first order conditions are given by

$$-\frac{1}{\alpha_i} r(y_h - \hat{\pi}_{2l}) e^{-\frac{1}{\alpha_i} z_{2l}^{1i}(y_h - \hat{\pi}_{2l})} - \frac{1}{\alpha_i}(1-r)(y_l - \hat{\pi}_{2l}) e^{-\frac{1}{\alpha_i} z_{2l}^{1i}(y_l - \hat{\pi}_{2l})} = 0.$$

Rearranging terms yields that

$$z_{2l}^{1i} = -\frac{\alpha_i}{y_h - y_l} \log\left(\frac{(1-r)(\hat{\pi}_{2l} - y_l)}{r(y_h - \hat{\pi}_{2l})}\right)$$

for each $i \in N$. Using that $\sum_{i \in N} z_{2l}^{1i} = \bar{z}$, gives

$$\frac{(1-r)(\hat{\pi}_{2l} - y_l)}{r(y_h - \hat{\pi}_{2l})} = e^{-x},$$

where $x = -\frac{\bar{z}(y_h - y_l)}{\sum_{i \in N} \alpha_i}$. Hence,

$$\begin{aligned} \hat{\pi}_{2l} &= y_h - \frac{y_h - y_l}{1 + \frac{r}{1-r} e^{-x}} \\ &= y_h - \frac{1-r}{re^{-x} + 1 - r}(y_h - y_l) \\ &= \frac{re^{-x}}{re^{-x} + 1 - r} y_h + \frac{1-r}{re^{-x} + 1 - r} y_l. \end{aligned}$$

□

PROOF OF PROPOSITION 2: Recall that the correlation in abnormal returns is postive if and only if $E(\tilde{r}_2 | \tilde{y}_1^1 = y_h) > E(\tilde{r}_2 | \tilde{y}_1^1 = y_l)$. Rearranging terms yields that this inequality is equivalent to

$$\frac{ry_h + (1-r)y_l}{qy_h + (1-q)y_l} < \frac{\hat{\pi}_{2l}}{\hat{\pi}_{2h}} = \frac{\frac{re^{-x}}{re^{-x}+1-r} y_h + \frac{1-r}{re^{-x}+1-r} y_l}{\frac{qe^{-x}}{qe^{-x}+1-q} y_h + \frac{1-q}{qe^{-x}+1-q} y_l}. \tag{24}$$

Next, define the parametric function

$$f(x) = (f_q(x), f_r(x)) = \left(\frac{qe^{-x}}{qe^{-x} + 1 - q}, \frac{re^{-x}}{re^{-x} + 1 - r}\right), \tag{25}$$

for $x \geq 0$ and define

$$h(q, r) = \frac{ry_h + (1-r)y_l}{qy_h + (1-q)y_l}, \tag{26}$$

for $q, r \in (0, 1)$ and $q > r$. Observe that expression (24) is equivalent to the inequality $h(q, r) < h(f_q(x), f_r(x))$. So, correlation is positive if the point $(f_q(x), f_r(x))$ belongs to the hyperplane $\{(q', r') | h(q', r') > h(q, r)\}$ and correlation is negative if the point $(f_q(x), f_r(x))$ belongs to the hyperplane $\{(q', r') | h(q', r') < h(q, r)\}$ (see Figure 2(a)).

The parametric function $f(x) = (f_q(x), f_r(x))$ yields a curve in $\mathbb{R}^2$ that starts in the point $(f_q(0), f_r(0)) = (q, r)$ and ends in the point $\lim_{x \to \infty}(f_q(x), f_r(x)) = (0, 0)$. Observe that both $f_q(x)$ and $f_r(x)$ are continuous and decreasing in x and that the graph of $(f_q(x), f_r(x))$ is convex (see Lemma 5 in the Appendix). Convexity implies that the parametric function

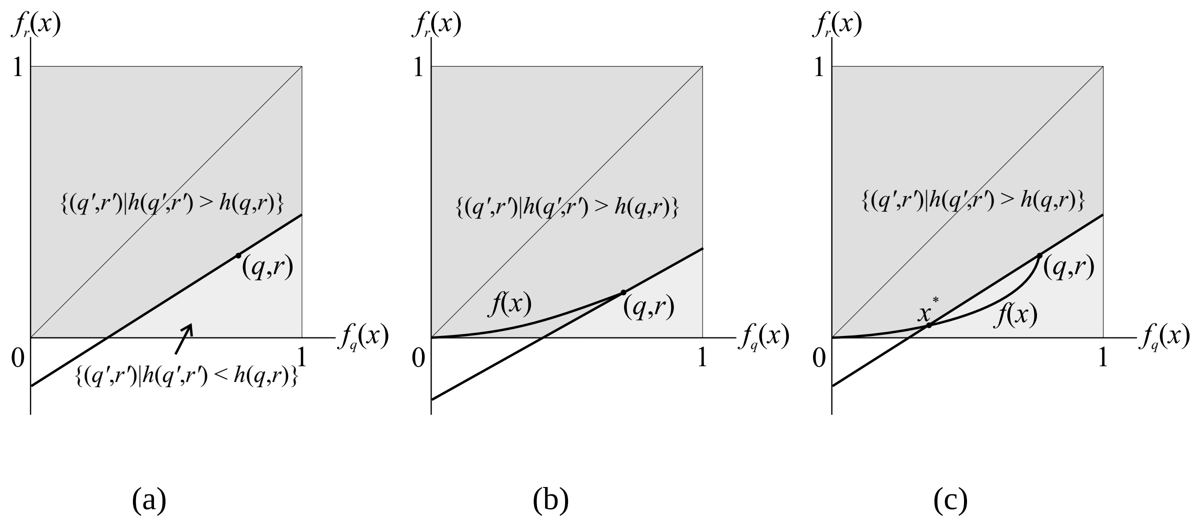

Figure 2: The influence of the risk burden x on the correlation between abnormal returns.

$(f_q(x), f_r(x))$ intersects the isoquant $\{(q', r')|h(q', r') > h(q, r)\}$ at most once. Figure 2(b) illustrates the situation that no such intersection exists. Since $h(0, 0) = 1 > h(q, r)$, it follows that $h(f_q(x), f_r(x)) > h(q, r)$ for all $x > x^* = 0$. Figure 2(c) illustrates the situation that precisely one intersection exists. In that case, let x^* be such that $h(f_q(x^*), f_r(x^*)) = h(q, r)$. Since $h(0, 0) = 1 > h(q, r)$, positive correlation in abnormal return arises if $h(f_q(x), f_r(x)) > h(q, r)$, i.e. if $x > x^*$, and negative correlation arises if $h(f_q(x), f_r(x)) < h(q, r)$, i.e. if $x < x^*$.

To determine the value of x^*, recall that x^* satisfies $h(f_q(x^*), f_r(x^*)) = h(q, r)$, i.e.

$$\frac{ry_h + (1-r)y_l}{qy_h + (1-q)y_l} = \frac{f_r(x^*)y_h + (1-f_r(x^*))y_l}{f_q(x^*)y_h + (1-f_q(x^*))y_l}.$$

Observe that $x^* = 0$ satisfies this equality as $(f_q(0), f_r(0)) = (q, r)$. Rearranging terms yields that

$$\frac{q-r}{f_q(x^*) - f_r(x^*)} = \frac{y_l + q(y_h - y_l)}{y_l + f_q(x^*)(y_h - y_l)}.$$

Substituting

$$f_q(x^*) - f_r(x^*) = \frac{e^{-x^*}(q-r)}{\left(qe^{-x^*} + 1 - q\right)\left(re^{-x^*} + 1 - r\right)}$$

and rearranging terms gives

$$r\left(e^{-x^*} - 1\right)\left(qe^{-x^*}y_h + (1-q)y_l\right) = (1-q)y_l\left(e^{-x^*} - 1\right).$$

This equation is satisfied if $e^{-x^*}-1=0$ or if $r\left(qe^{-x^*}y_h+(1-q)y_l\right)=(1-q)y_l$. The former condition yields $x^*=0$. The latter one yields $rqe^{-x^*}y_h=(1-q)(1-r)y_l$ so that

$$x^* = \log\left(\frac{q}{1-q}\frac{r}{1-r}\frac{y_h}{y_l}\right).$$

Observe that x^* may be negative. Since the risk burden is nonnegative by definition, a lower bound of zero is imposed on x^*. □

Lemma 5 Let $g:[0,q]\to[0,r]$ be the function described by the parametric function $(f_q(x), f_r(x))$, $x\geq 0$. Then g is a convex function.

PROOF: The function g satisfies $g(f_q(x)) = f_r(x)$. Differentiating both sides to x yields

$$g' = \frac{\frac{df_r(x)}{dx}}{\frac{df_q(x)}{dx}} = \frac{r(1-r)}{q(1-q)}\left(\frac{qe^{-x}+1-q}{re^{-x}+1-r}\right)^2,$$

so that

$$g'' = \frac{r(1-r)}{q(1-q)}\frac{d}{dx}\left(\frac{qe^{-x}+1-q}{re^{-x}+1-r}\right)^2\left(\frac{df_q(x)}{dx}\right)^{-1}.$$

Since

$$\frac{df_q(x)}{dx} = -\frac{q(1-q)}{(qe^{-x}+1-q)^2} < 0$$

and

$$\begin{aligned}
&\frac{d}{dx}\left(\frac{qe^{-x}+1-q}{re^{-x}+1-r}\right)^2 \\
&= 2\left(\frac{qe^{-x}+1-q}{re^{-x}+1-r}\right)\left(\frac{-qe^{-x}\left(re^{-x}+1-r\right)+re^{-x}\left(qe^{-x}+1-q\right)}{(re^{-x}+1-r)^2}\right) \\
&= -2e^{-x}(q-r)\frac{qe^{-x}+1-q}{(re^{-x}+1-r)^3} < 0
\end{aligned}$$

it follows that $g''>0$. □

PROOF OF PROPOSITION 4: Similar as in the proof of Proposition 2, abnormal returns are positively correlated if and only if $h(f_q(x), f_r(x)) > h(q,r)$, except that h is now defined as

$$h(q,r) = \frac{ry_h+(1-r)y_l}{\gamma+qy_h+(1-q)y_l}. \tag{27}$$

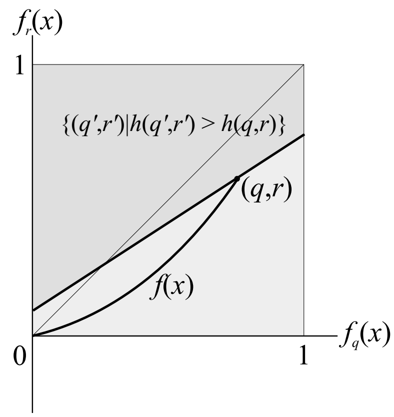

Figure 3: Negative correlation for all $x > 0$.

The parametric function $(f_q(x), f_r(x))$ has the same characteristics: it starts in (q, r), ends in $(0, 0)$, is decreasing in both dimensions, and its graph is convex. Hence, it intersects at most once with the isoquant $\{(q', r')|h(q', r') = h(q, r)\}$. However, $h(0, 0) = \frac{y_l}{\gamma + y_l} < 1$. This means that besides the cases (b) and (c) illustrated in Figure 2, which occur if $h(q, r) < h(0, 0)$, a third case arises if $h(q, r) \geq h(0, 0)$ (see Figure 3). In that case, since $h(q, r) \geq h(0, 0)$ and the convexity of the graph of $(f_q(x), f_r(x))$, it follows that $h(f_q(x), f_r(x)) < h(q, r)$ for all $x > 0$. So, there is negative correlation in abnormal returns whatever the value of the risk burden x. Finally, observe that $h(q, r) > h(0, 0)$ is equivalent to $r > \frac{y_l}{\gamma + y_l} q$. $\square$

References

Ball, R. and Bartov, E.: 1996, How naive is the stock market's use of earnings information?, *Journal of Accounting and Economics* **21**, 319–337.

Ball, R. and Brown, P.: 1968, An empirical evaluation of accounting income numbers, *Journal of Accounting Research* **6**, 159–178.

Barberis, N., Shleifer, A. and Vishny, R.: 1998, A model of investor sentiment, *Journal of Financial Economics* **49**, 307–343.

Bathke, A. and Lorek, K.: 1984, The relationship between time-series models and the security market's expectation of quarterly earnings, *The Accounting Review* **59**, 163–176.

Bernard, V. and Thomas, J.: 1989, Post earnings announcement drift: delayed price response or risk premium?, *Journal of Accounting Research* **27**, 1–36.

Bernard, V. and Thomas, J.: 1990, Evidence that stock prives do not fully reflect the implications of current earnings for future earnings, *Journal of Accounting and Economics* **13**, 305–340.

Brown, L. and Rozeff, P.: 1979, Univariate time series models of quarterly accounting earnings per share: A proposed model, *Journal of Accounting Research* **17**, 179–189.

Brown, L., Griffin, R., Hagerman, R. and Zmijewski, M.: 1987, An evaluation of alternative proxies for the market's assessment of unexpected earnings, *Journal of Accounting and Economics* **9**, 159–194.

Cutler, D., Poterba, J. and Summers, L.: 1991, Speculative dynamics, *Review of Economic Studies* **58**, 529–546.

Daniel, K., Hirshleifer, D. and Subrahmanyam, A.: 1998, Investor psychology and security market under- and overreactions, *Journal of Finance* **53**, 1839–1885.

DeBondt, W. and Thaler, R.: 1985, Doest the stock market overreact?, *Journal of Finance* **40**, 793–808.

Eeckhoudt, L. and Gollier, C.: 1995, *Risk: Evaluation, Management, and Sharing*, Harvester Wheatsheaf, New York.

Fama, E.: 1976, *Foundations of Finance*, Basic Books, New York.

Fama, E. and French, K.: 1988, Permanent and temporary components of stock prices, *Journal of Political Economy* **96**, 246–273.

Foster, G.: 1977, Quarterly accounting data: Time series properties and predictive-ability results, *The Accounting Review* **52**, 1–21.

Foster, G., Olsen, C. and Shevlin, T.: 1984, Earnings releases, anomalies, and the behavior of security returns, *The Accounting Review* **59**, 574–603.

Freeman, R. and Tse, S.: 1989, The multi-period information content of earnings announcements: Rational delayed reactions to earnings news, *Journal of Accounting Research* **27**, 49–79.

Grinblatt, M., Masulis, R. and Titman, S.: 1984, The valuation effects of stock splits and stock dividends, *Journal of Financial Economics* **13**, 97–112.

Hong, H. and Stein, J.: 1999, A unified theory of underreaction, momentum trading, and overreaction in asset markets, *Journal of Finance* **54**, 2143–2184.

Jacob, J., Lys, T. and Sabino, J.: 2000, Autocorrelation structure of forecast errors from time series models: Alternative assessments of the causes of post earnings announcement drift, *Journal of Accounting and Economics* **28**, 329–358.

Joy, O., Litzenberger, R. and McEnally, R.: 1977, The adjustment of stock prices to announcements of unanticipated changes in quarterly earnings, *Journal of Accounting Research* **15**, 207–225.

Loughran, T. and Ritter, J.: 1995, The new issues puzzle, *The Journal of Finance* **50**, 365–394.

Poterba, J. and Summers, L.: 1988, Mean reversion in stock prices: Evidence and implications, *Journal of Financial Economics* **22**, 27–59.

Rendleman, R., Jones, C. and Latane, H.: 1982, Empirical anomalies based on unexpected earnings and the importance of risk adjustments, *Journal of Financial Economics* **10**, 269–287.

Soffer, L. and Lys, T.: 1999, Post earnings announcement drift and the dissemination of predictable information, *Contemporary Accounting Research* **16**, 305–331.

Watts, R.: 1978, Systematic abnormal returns after quarterly earnings announcements, *Journal of Financial Economics* **6**, 127–150.

Wilson, R.: 1968, The theory of syndicates, *Econometrica* **36**, 119–132.