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Marius-Cristian FRUNZA, Dominique GUEGAN

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Risk Assessment for a Structured Product Specific to the CO_2 Emission Permits Market *

Marius-Cristian Frunza† Dominique Guégan‡

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Abstract

The aim of this work is to use a new modelling technique for CO_2 emission prices, in order to estimate the risk associated with a related, structured product. After a short discussion of the specificities of this market, we investigate several modelling methods for CO_2 emission prices. We use these results for risk modeling of the swap between two CO_2 related instruments: the European Union Allowances and the Certified Emission Reductions. We estimate the counterparty risk for this kind of transaction and evaluate the impact of different models on the risk measure and the allocated capital.

Keywords: Carbon, Generalized Hyperbolic Distribution, Value-at-Risk, CER, EUA, Swap.

JEL classification: .

In this paper, we define carbon transactions as contracts whereby one party pays another party in exchange for a given quantity of GHG emission permits that the buyer can use to meet its objectives vis-à-vis climate change mitigation. Carbon transactions can be grouped in two main categories:

- Trades of emission allowances, such as, for example, Assigned Amount Units (AAUs) under the Kyoto Protocol, or allowances under the EU Trading Scheme (EUAs). These allowances are created and allocated by a regulator, usually under a cap-and-trade regime.
- Project-based transactions, i.e. transactions in which the buyer participates in the financing of a project which reduces GHG emissions (Certified Emission Reductions, CER) compared which what would have happened otherwise, and gets emission credits in return. Unlike allowance trading, project-based transactions can occur even in the absence of a regulatory regime: an agreement between a buyer and a seller is sufficient.

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†Sagacarbon, a subsidiary of Caisse des Dépôts, marius.frunza@gmail.com

‡PSE, Centre d'Economie de la Sorbonne, University Paris 1 Panthéon-Sorbonne, MSE, 106 bd de l'Hôpital, 75013 Paris, France. Email: dguegan@univ-paris1.fr.. Tel: +33 1 40 07 82 98.

The directive 2004/101/EC of the European Parliament of 27 October 2004, known as the "linking directive" allows to each EU Trading Scheme industries to use credits from project-based transactions (CERs) for compliance. The maximum allowed usage of project credits is set as a percentage of the allocation per industry, the so called "flexible space". In Exhibit 1 one can find the different flexible space rates per country. The structured product derived from this regulation is the EUA/CER Swap that allows companies to profit from the current price differential between an EUA (Kyoto period) and a CER.

Country	Allowances 2008-2012 (Mt CO_2)	CER/EUA conversion ratio	Potential market for swap (Mt CO_2)
GERMANY	453.1	22 %	99.7
UNITED KINGDOM	246.2	8%	19.696
POLAND	208.5	10%	20.85
ITALY	195.8	15%	29.37
SPAIN	152.3	20%	30.46
FRANCE	132.8	13.50%	17.928
CZECH REPUBLIC	86.8	10%	8.68
NETHERLANDS	85.8	10%	8.58
ROMANIA	75.9	10%	7.59
GREECE	69.1	9%	6.219
BELGIUM	58.5	8.40%	4.914
BULGARIA	42.3	12.60%	5.3298
SLOVAKIA	32.6	7%	2.282
AUSTRIA	30.7	10%	3.07
HUNGARY	26.9	10%	2.69
SLOVENIA	8.3	15.70%	1.3031
Total	1905.6		268.64

Exhibit 1: EUA/CER swap market

The potential volume for the structured swap is estimated to 270 Mt CO_2 which for an average EUA-CER spread of €2 would arose to a 540 million euros market.
In some recent works, a few authors including Paolella and Taschini [2006], Ulrih-Homburg and Wagner [2007], Benz and Truk [2008] and Daskalakis, Psychoyios and Markellos [2008] focused on the econometric modelling of the emission allowances prices, underlining the particularities of this market, like non-Gaussian behavior, auto-regressive phenomena and the presence of the convenience yield. They focus mainly on continuous time modelling and the extreme value approach. Most of their works are based on data concentrated on the period 2005 - 2007.

In the present paper, we consider a new class of models based on Generalized Hyperbolic distributions and we apply the results of price calibration to a financial product, specific to the CO_2 emission permits market, so-called carbon arbitrage, for the period 2008-2012.

Over the past two decades the Generalized Hyperbolic (GH) distributions, have been used to characterize a wide range of asset returns, particularly those with obvious non-Gaussian features (such innovations in electricity or gas prices). We are particularly interested to find the "best" distribution which characterizes the data we consider, and not by the time evolution of the data that we will analyze in an accompanying paper. We compare our new approach with classical type of Brownian motions (discussed for instance in Daskalakis, Psychoyios and Markellos [2008]), and we also introduce jumps in the modeling using mixtures of distributions.

The objective of this paper is to provide a suitable model, for an econometric perspective for historical prices in order to understand the evolution of one of the derivatives built on the CO2 allowances, namely the EUA-CER arbitrage swap. The "'suitability"' is measured as the quantity of information captured by a specific distribution model. This product allows income free of market risk to be generated, by taking the price difference between CER and EUA prices. In this paper, we adapt the carbon arbitrage swap for time horizons between one and five years (2008-2012), until the end of the second Kyoto period. Product's exposure in case of default is the spread between the CER price and EUA prices at the default moment, and it becomes critical if the EUA's price at the moment of the deal is on a bull market. We assessed the exposure to default using the Value At Risk (VaR) as risk indicator and we observe a significant difference if we use classical models (based on Brownian motion) for CO_2 emission allowance prices, or the new model that we propose based on the class of GH distributions.

Next we recall the main features of the emission allowances market. Thus, we introduce the modellings based on classes of Gaussian distributions and Generalized Hyperbolic (GH) distributions. We provide the results of applications of the previous sections on the risk analysis of a structured product typical to carbon market, the swap between two carbon instruments: the EUAs and the CERs. We provide sample results in order to suggest the strengths and the weaknesses of each modelling approach. Finally, we underline the main conclusions of our study.

Overview of emission allowances trading prices

We recall some specific properties of EUA prices which have already been mentioned in several papers, see for instance Daskalakis, Psychoyios and Markellos [2008]. Then, we focus on the way to build future contracts with the specific information set available for carbon prices. We specify the main points which are of interest or the subsequent modelling. The characteristics of the carbon market can be summarized as follows.

- The efficient market hypothesis is a common assumption in traditional financial economics, but as the European emission allowances market (EUA) is a direct consequence of a regulatory system commonly accepted by market actors, some sources of market inefficiency do exist. Indeed, the new information is unequally diffused amongst market players; the EUA is perceived mainly as a financial liability and has no intrinsic value capable of generating economic value added for an investor; finally, the market is heavily influenced by regulators whose actions are not fully determined by price concerns.

- The emission allowance prices can be considered as a commodity representing the right to pollute the environment. This right is underwritten by governments and given to different industries depending on their profile. In this case, the price of an allowance would represent the marginal cost of reducing the GHG emissions. In an efficient market environment this cost could be traded between industries submitted to emission constraints. The price would be established depending on the offer and the demand of the market. Thus, an independent investor would perceive her money as working in a physical mechanism that reduces the emissions. But we can also remark that the allowances are imposed by governments in order to stimulate industries to reduce their emissions. If an industry has more allowances than actual emissions it will cash them out on the market. If it is short of allowances, it will fill the need by buying allowances on the market. This kind of framework would push an independent investor to perceive her money as being held in a regulatory paper, traded between industries without being backed by any physical mechanism. Thereby the price of this security will be determined by the difference between the allowed and real emissions of an industry.

- Nevertheless, if we apprehend the CO_2 emission allowances as a classic commodity like oil, gas or gold we should find similarities in economic interpretation. On the one hand, the agent has the option of flexibility with regards to consumption (no risk of commodity shortage). On the other hand, the decision to postpone consumption implies storage expenses. The net cost of these services per unit of time is termed the convenience yield δ. Intuitively, the convenience yield corresponds to dividend yield for stocks, thereby the price of a forward contract is given by:

$$F_{t,T} = S_t \cdot \exp((r_{t,T} - \delta_{t,T}) \cdot (T - t)) \tag{1}$$

where $F_{t,T}$ is the value at the moment t of the future contract for the maturity T, S_t is the spot value at time t, $r_{t,T}$ and $\delta_{t,T}$ are respectively the values of the rate and convenience yield for the maturity T. Here, the quota holders will not sell their quotas to realize an arbitrage opportunity (by selling the quota and buying futures contracts). Consequently they "value" their owner-right and the convenience yield is a major element while modeling allowance price. Since its introduction through to the beginning of 2008, the second period market quoted only futures contracts with no price for the spot value of allowances. In order to find the implied convenience yield and spot price we used historical values of both futures contracts and interest rates. The scarcity of relevant data for longer maturities obliged us to consider only the 2008 and 2009 horizons, and to assume that the convenience yield curve is flat for these horizons. Nevertheless the impact of the uncertainty of the convenience yield on the estimation of spot prices are not relevant. The future contracts can be valued using a system of equations with two unknown variables: the implied spot and the convenience yield.

- The main important statistical features of EUA are non-Gaussianity, leptokurtosis, asymmetry and fat tails. All these features have been already described in the previously-cited papers. Here, we illustrate them using the EUA historical prices on the 2005 - 2009 period.

Our dataset contains daily closing prices for the EUA 2009 future contract, between 2005 and 2009. On Exhibit 2, EUA 2008 and 2009 historical prices show high variability regimes and discontinuities in the supply/demand equilibrium. For a significant number of trading days the exchanged volumes of contracts are very small or even zero. In these particular days, the prices are marked by the auction trading systems. Looking at the sample autocorrelation function (ACF) based on the most recent 990 daily log return data of EUA08, we observe a small correlation on the prices (Exhibit 3), while the ACF of squared log return series does show evidence of serial dependence. Nevertheless for both daily and squared daily yields series the Box-Ljung test rejects the null hypothesis of no serial correlation with a 95% confidence level. In Exhibit 4 we give the distribution function of the EUA08: it shows negative skewness and fat tails also reveled by the QQplot diagram. Thus, the preliminary tests reject the hypothesis that EUA08 daily returns are characterized by a Gaussian distribution. More the Jarque-Bera and Kolmogorov-Smirnov tests reject the hypothesis of normality with a 95% confidence level.

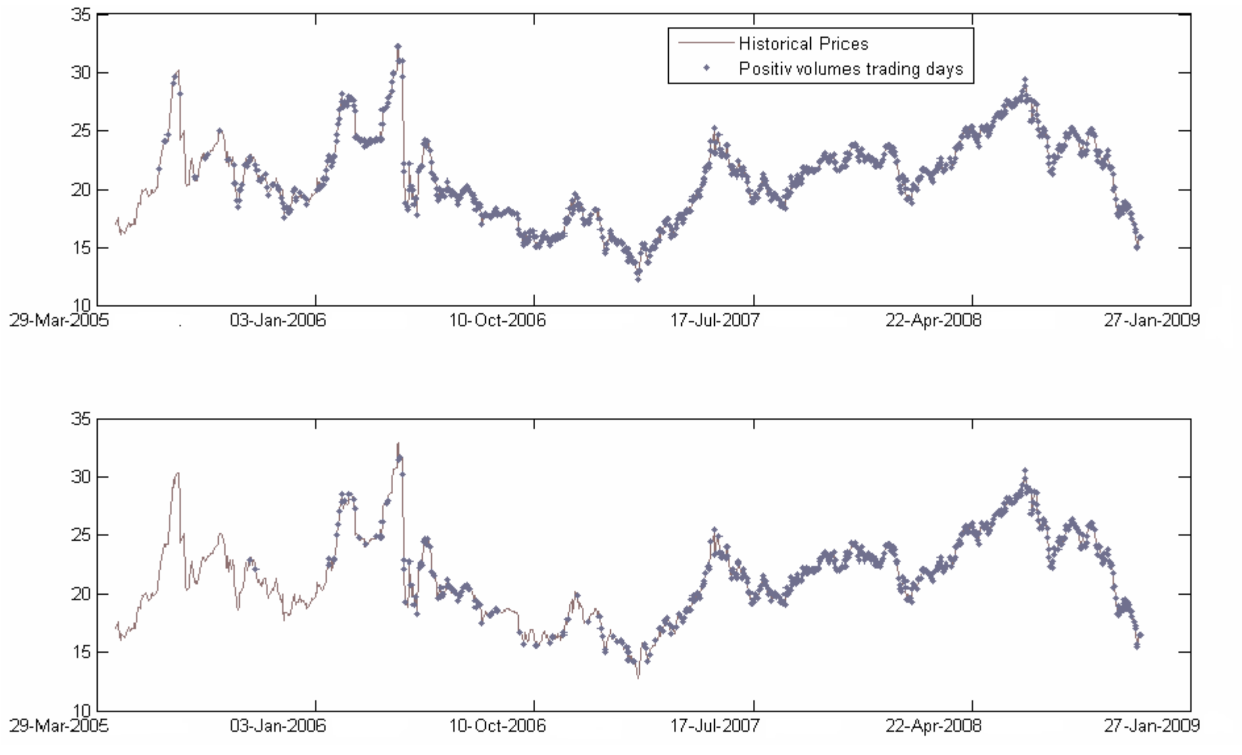

Exhibit 2: EUA08 and EUA09 Price Histories

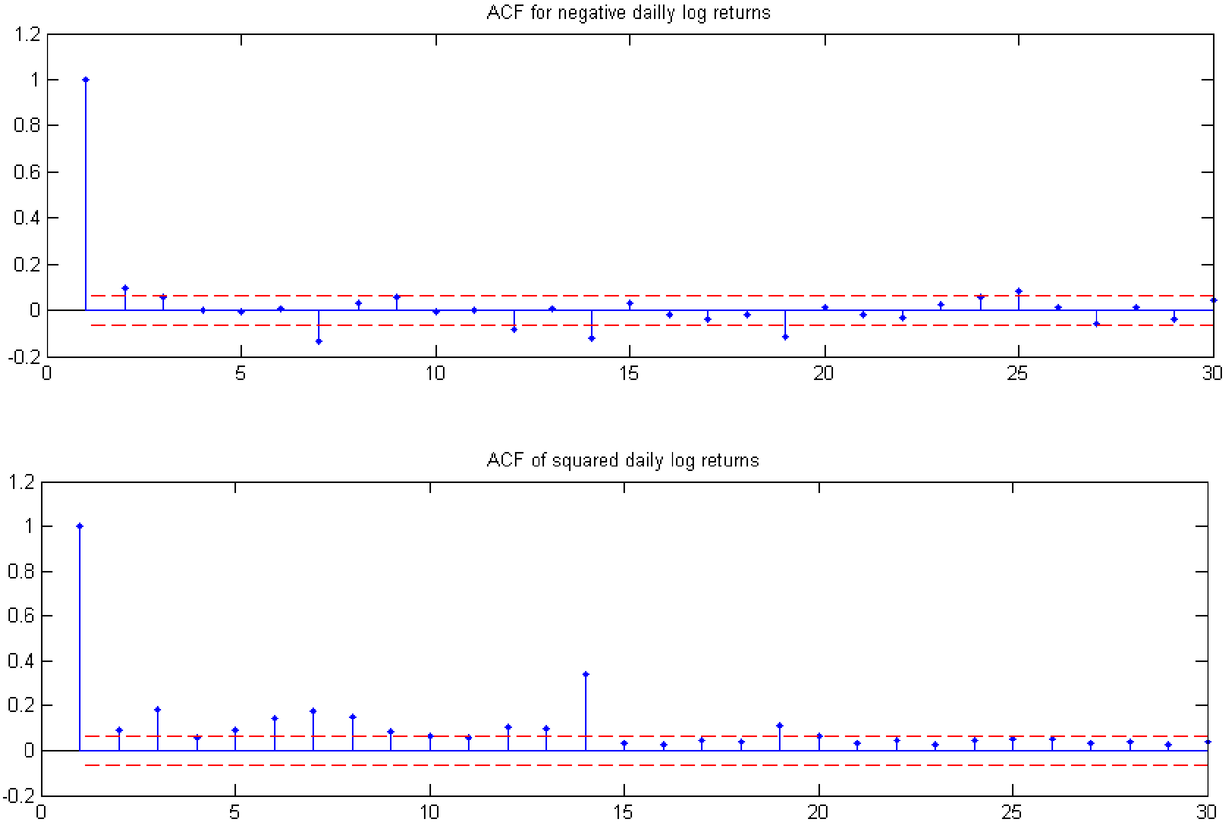

Exhibit 3: Autocorrelation for EUA08: Negative Daily Returns

Model Calibration and Analysis

In this section we calibrate several models for EUA prices for the period 2008 - 2009. Our aim is not to find the "true" model that would explain the behavior of the carbon market but to propose a benchmark of different models commonly used to describe financial assets. Based on the historical time series, we calibrate some models from the classical Brownian diffusion to more sophisticated models based on Generalized Hyperbolic distributions.

Given the fact that the prices show small serial correlations, we use the maximum likelihood as the main criterion to discriminate the fitness of the different models. Indeed, our approach in a first step is mainly one of non-parametric modelling using distribution functions associated with the prices. We do not model here a time evolution through a parametric filter, such filtering will be discussed in an accompanying paper.

We begin by recalling the classical commodity modellings strategies that we use as benchmarks, then we introduce a competitive model based on the Generalized Hyperbolic distribution.

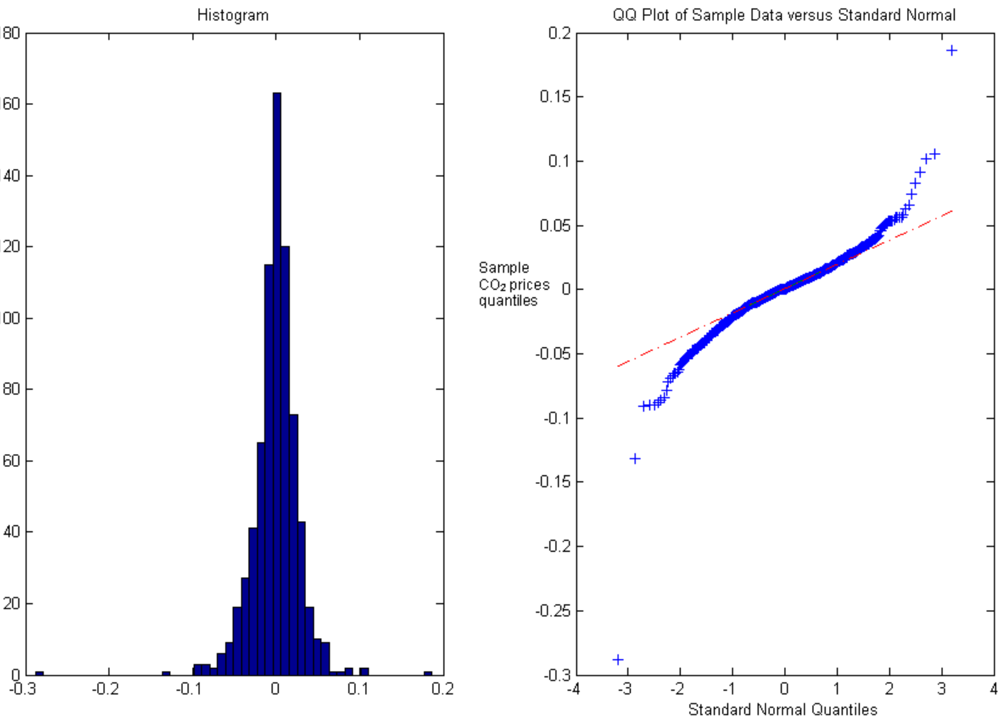

Exhibit 4: Distributions of EUA08: Daily Yields and QQ Plots

A review of classical commodity modelling approaches

In this subsection we search for a model based on a Gaussian return distribution that could fit CO_2 prices behavior. We investigate different hypothesis like mean reversion and jumps, in order to find the factors that could explain the information contained inside the historical time series. We specify now the different approaches.

- A Geometric Brownian Motion (GBM) is a continuous-time stochastic process in which the logarithm of the randomly varying quantity follows a Brownian motion. In this model proportional changes in the asset prices, denoted by S, are assumed to have constant instantaneous drift μ, and volatility σ. The mathematical description of this property is given by the following stochastic differential equation:

$$\frac{dS}{S} = \mu \cdot dt + \sigma \cdot dB. \tag{2}$$

  Here dS represents the increment in the asset price process during a small interval of time dt, and dB is the underlying uncertainly driving the model and represents an increment in a Wiener process during time dt.

- In order to enrich the GBM model we introduce a mean-reversion characteristic. The mean-reverting stochastic behavior (GBMMR) of commodity spot prices can be understood by looking at the one factor model developed by Schwartz [1997] and Campbell, Lo and MacKinlay [1997] and applied to energy prices by Knittel [2005]. It is given by the following equation:

$$\frac{dS}{S} = -\alpha \cdot (ln(S) - m) \cdot dt + \sigma \cdot dB. \tag{3}$$

In this model, the spot price mean reverts to the long-term level $\overline{S}$ with m = ln($\overline{S}$) at a speed given by the mean reversion rate, α and volatility $\sigma > 0$.The consequences of mean reversion can be understood by looking at the first term of the equation (3). If the spot price S is above the long-term level S, then the drift of the spot price will be negative and the price will tend to revert back towards the long-term level. Similarly, if the spot price is below the long-term level, then the drift will be positive and the price will tend to move back towards the long-term level. Defining x = lnS, the conditional distribution of $x_t \mid x_{t-1}$ is given by the following expression:

$$x_t|x_{t-1} \sim N(c + \beta \cdot x_{t-1}, \sigma_\epsilon^2), \tag{4}$$

where $c = m^*(1 - e^{-\alpha})$,$\beta = e^{-\alpha}$, and N denotes the Gaussian law with variance ${\sigma_\varepsilon}^2$ equal to $\sigma^2(1 - e^{-2\alpha})/2\alpha$. We will consider this modelling in the next section.

- To be free of the assumption that the conditional distribution of the logarithm of the commodity spot prices is normal, we extend the mean-reverting model to accommodate large movements (jumps) in the spot prices. Such a popular extension of the standard mean-reverting diffusion process is the mean-reverting jump-diffusion process (GBMMRJ). A relatively simple mean-reverting jump-diffusion model for spot prices is described by the following equation:

$$\frac{dS}{S} = \alpha \cdot (ln(S) - m)dt + \sigma \cdot dB + \kappa \cdot dQ, \tag{5}$$

where the parameters are the same as in the simple mean-reverting model (5), κ represents the jump frequency, and dQ the jump metric. Due to the introduction of jumps we have some extra parameters that come into our model. If we have an arrival of abnormal information, a jump occurs and the log-price is drawn from a conditional normal distribution with mean $c + \beta \cdot x_{t-1} + \mu_\kappa$ and variance $\sigma_\varepsilon^2 + \sigma_\kappa^2$. Hence a mean-reverting jump-diffusion process can be written as a Gaussian mixture:

$$x_t \sim (1 - \lambda) \cdot N(c + \beta \cdot x_{t-1}, \sigma_\epsilon^2) + \lambda \cdot N(\mu_\kappa, \sigma_\kappa^2), \tag{6}$$

where μ_κ is average size of a jump and σ_κ^2 is the variance of the jumps.

- We know that existence of jumps create switches, thus we can also consider the simplest model to be as the previous one, considering a special case of the regime switching model introduced by Hamilton [1989] with a mixture of Gaussian distributions (GBMJ). Such modeling means that, at each time period, if we do not have an arrival of "abnormal" information (an event with a probability (1-λ)) the next logprice is drawn by a conditional normal distribution with mean μ and variance σ^2, and if we do have an arrival of "abnormal" information, a jump occurs and the log-price is drawn from a conditional normal distribution with mean $\mu + \mu_\kappa$ and variance $\sigma^2 + \sigma_\kappa^2$, then the model is:

$$x_t \sim \lambda \cdot N(\mu, \sigma^2) + (1 - \lambda) \cdot N(\mu_\kappa, \sigma_\kappa^2), \quad (7)$$

where μ_κ is average size of a jump and σ_κ^2 is the variance of the jumps.

We now apply these four models to our data set.

Results for classical commodities models

In order to compare the adequacy of the four previous modellings to the EUA historical prices on period 2005-2009, we use here the Bayesian Information Criteria (BIC) (in our writing, the lower the value of the BIC, the better fit we obtained). Range restrictions are imposed to model parameters in order to obtain relevant estimators and to avoid biases due to isolated maxims . All the results are given with a confidence level of 95% using the asymptotic covariance matrix for maximum likelihood estimators. We can make the following comments on data summarized in Exhibit 5.

- The results of the GBM calibration show that the historical volatility of the CO_2 market is around 47 % and that the null hypothesis of the drift cannot be rejected. Thus this modelling appears very poor.

Model	**GBM**	**GBMMR**	**GBMJ**	**GBMMRJ**
μ	0.29	-	-0.121	-
	[-0.82, 0.25]	-	[-0.55, 0.31]	-
σ	0.46	0.46	0.28	0.0267
	[0.43, 0.48]	[0.44, 0.49]	[0.24, 0.32]	[0.24, 0.32]
β	-	0.99	-	0.998
	-	[0.98, 1.005]	-	[0.989, 1.006]
c	-	0.009	-	0.007
	-	[-0.01,0.03]	-	[-0.011,0.031]
μ_{jump}	-	-	2.28	2.23
	-	-	[-0.35, 4.92]	[-0.65, -4.87]
σ_{jump}	-	-	0.82	0.82
	-	-	[0.66,0.99]	[0.410, 0.995]
λ	-	-	0.20	0.20
	-	-	[0.09, 0.29]	[0.097, 0.302]
BIC	-3332	-3328	**-3522**	-3516

Exhibit 5: Calibration results for Gaussian distribution based models

- We observe that the mean reverse Brownian diffusion model does not capture more information about the allowance prices than the previous Brownian motion. The mean reversion hypothesis is common in the commodity analysis, due to the fact that there are production and consumption cycles. As the carbon market is driven by the annual environmental compliance obligation, the existence of a cycle could be an underlying hypothesis. The other factors that determine the market inefficiency have a strong influence on the EUA price and erode the mean reversion behavior, thereby showing an informational level lower than expected.

- The results obtained using the Gaussian mixtures modelling seem to describe in a better way the evolution of allowance prices. This model permits, the asymmetric distribution of the information in the market between the big and small players to be captured. It should be noted that such an informational broken symmetry was observed in the first three months of 2009 when the market was long due to an overallocation of some major industries. This event along with general turbulence on the energy market drove the allowances to the lowest historical price around €7-8, well below the economic limit of €10 which represents the marginal cost of depoluting one tonne of CO_2. We notice that the drift parameters for both Gaussian and jump model are not statistically significant. In fact the presence of drift in carbon prices returns is not economically proven, as it is for other underlings, like US equities as shown by Schwert [1990] and Siegel [2002]. Nevertheless our focus is on searching proofs for volatility regime switching in emission prices taking into account in a different way that Daskalakis [2009], the modelling of jumps in carbon prices.

- The calibration results for the mean-reversion jump diffusion modelling capture less information than the previous jump models. Nevertheless given the large number of parameters the model does not provide a robust choice. Once again the mean reversion model does not seem to be a suitable modelling.

Generalized Hyperbolic models

We introduce now a new modelling technique which permits both skewness and kurtosis features to be taken in account. Indeed, these features cannot be taken by the previous modellings. Following the works of Eberlein and Prause [2002] and Barndorff-Nielsen [1977] done on financial assets, we are going to calibrate the class of Generalized Hyperbolic distributions on our data set. This very flexible class of distributions is able to capture heavy tails and asymmetry. It is characterized by 5 parameters with a shape parameter which permits very specific shapes to be obtained. The four other parameters are linked in an easy way with the first four moments of the distribution.

Presentation of the Generalized Hyperbolic Distribution

We make a brief review of the Generalized Hyperbolic distribution functions focusing on the Normal Inverse Gaussian one. The generic form of a Generalized Hyperbolic model is:

$$f(x;\lambda;\chi;\psi;\mu;\sigma;\gamma) = \frac{(\sqrt{\psi\chi})^{-\lambda}\psi^{\lambda}(\psi+\frac{\gamma^2}{\sigma^2})^{0.5-\lambda}}{\sqrt{2\pi}\sigma K_{\lambda}(\sqrt{\psi\chi})} \times \frac{K_{\lambda-0.5}(\sqrt{(\chi+\frac{(x-\mu)^2}{\sigma^2})(\psi+\frac{\gamma^2}{\sigma^2})})e^{\frac{\gamma(x-\mu)}{\sigma^2}}}{(\sqrt{(\chi+\frac{(x-\mu)^2}{\sigma^2})(\psi+\frac{\gamma^2}{\sigma^2})})^{\lambda-0.5}},$$

where $K_{\lambda}(x)$ is the modified Bessel function of the third kind:

$$K_{\lambda}(x) = \frac{1}{2}\int_0^{\infty} y^{\lambda-1}e^{-\frac{x}{2}(y+y^{-1})}dy. \tag{8}$$

Among the Generalized Hyperbolic family, we focus on the Normal Inverse Gaussian distribution obtained by setting $\lambda = -\frac{1}{2}$ in the previous equation. Thus:

$$f(x;-\frac{1}{2};\chi;\psi;\mu;\sigma;\gamma) = \frac{\chi^{\frac{1}{2}}(\psi+\frac{\gamma^2}{\sigma^2})}{\pi\sigma e^{\sqrt{-\psi\chi}}} \times \frac{K_1(\sqrt{(\chi+\frac{(x-\mu)^2}{\sigma^2})(\psi+\frac{\gamma^2}{\sigma^2})})e^{\frac{\gamma(x-\mu)}{\sigma^2}}}{(\sqrt{(\chi+\frac{(x-\mu)^2}{\sigma^2})(\psi+\frac{\gamma^2}{\sigma^2})})}.$$

By changing the variables of the previous equation $c = \frac{1}{\sigma^2}$; $\beta = \frac{\gamma}{\sigma^2}$; $\delta = \sqrt{\frac{\chi}{c}}$; $\alpha = \sqrt{\frac{\psi}{\sigma^2}+\beta^2}$ we obtain a more popular representation, and the density of the *NIG*(α,β,μ, δ) distribution is equal to:

$$f_{NIG}(x;\alpha;\beta;\mu;\delta) = \frac{\delta\alpha \cdot exp(\delta\gamma+\beta(x-\mu))}{\pi \cdot \sqrt{\delta^2+(x-\mu)^2}}K_1(\alpha\sqrt{\delta^2+(x-\mu)^2}).$$

The moments (mean, variance, skewness and kurtosis) are respectively equal to:

$$\mathtt{E}(X) = \mu + \delta\frac{\beta}{\gamma} \tag{9}$$

$$\mathtt{V}(X) = \delta\frac{\alpha^2}{\gamma^3} \tag{10}$$

$$\mathtt{S}(X) = 3\frac{\beta}{\alpha \cdot \sqrt{\delta\gamma}} \tag{11}$$

$$\mathtt{E}(K) = 3 + 3(1+4(\frac{\beta}{\alpha})^2)\frac{1}{\delta\gamma}. \tag{12}$$

Thus, the NIG distribution allows for behavior characterized by heavy tails and strong asymmetries, depending on the parameters α, β and δ.

The modelling of emission prices with Generalized Hyperbolic distributions

We now apply, the GH Distribution to the EUA. In Exhibit 6, we obtain the fit of NIG distribution for the EUA09, with the Q-Q plot. The fit is quite good and much better than the fit obtained using a Gaussian law (Exhibit 4). The calibration of a GH distribution using the full historical data set with various values for λ provides the results given in the Exhibit 7. The parameters are estimated using both maximum likelihood algorithm and Gibbs sampling techniques. The results are coherent, but the likelihood maximization converges faster. The 95% confidence range for the estimators is calculated with a bootstrap method. This modelling shows better likelihood figures than those obtained with the previous class of diffusion models.

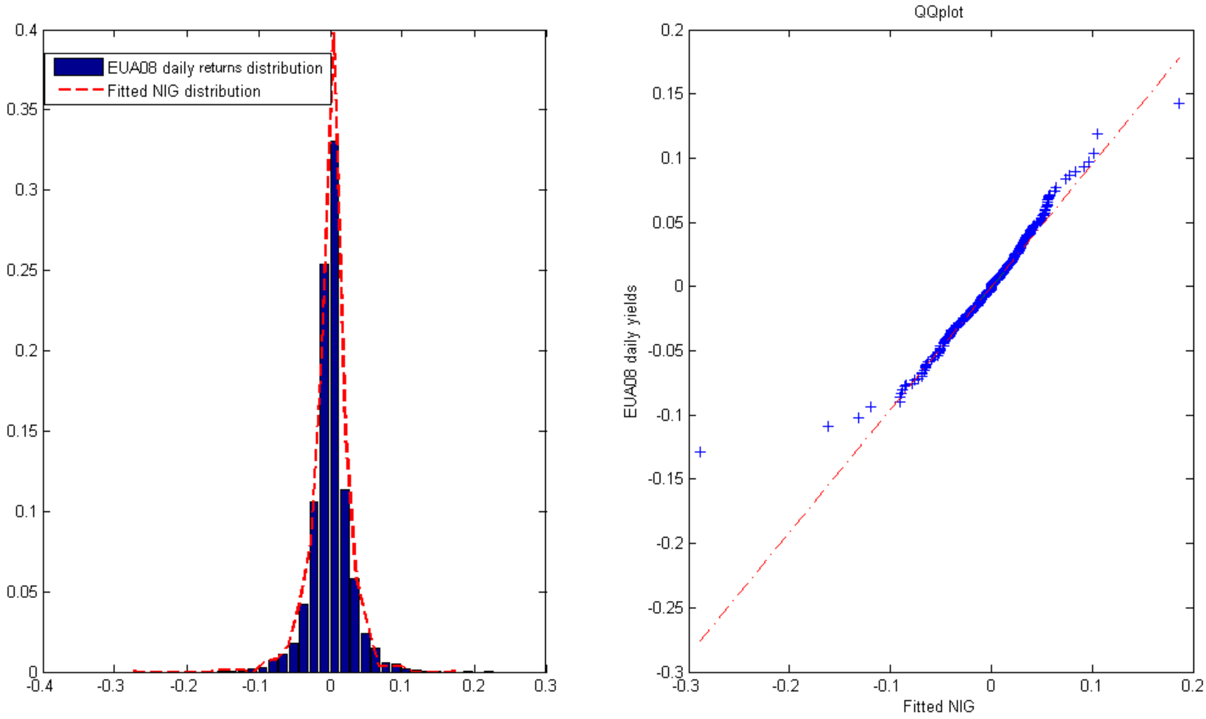

Exhibit 6: NIG Distribution adapted on EUA08: on the left returns histogram and the fitted NIG; on the right the QQ plot.

Amongst the previous GH models the case of the NIG modelling provides with the best fitting in terms of informations. Compared to the jump modelling the GH distribution captures more features of emission prices and improves the figures of the BIC estimator.

In the previous subsection we have observed that a mixture of distributions could improve the fit of the data as soon as some jumps or explosions exist in the data set. Here, we also tried to extend this idea mixing a Gaussian distribution with an NIG distribution, and we introduce the following model:

$$\frac{dS}{S} = \lambda \cdot \boldsymbol{dNIG}(\alpha, \beta, \delta, \mu) \cdot dt + (1-\lambda) \cdot \sigma \boldsymbol{dB}. \tag{13}$$

Nevertheless the incremental information given by this last modelling compared with the NIG calibration is still insufficient and the confidence interval of the jump frequency shows limited relevance. Its also known that the NIG distribution can be assimilated to a Levy process and a mixture-like modelling NIG/GBM would not improve the statistical framework.

The conclusion of this part is that carbon allowances need both non-zero higher moment distributions and broken symmetry regimes if we want to be able to capture most of their features. Future developments to improve these modellings would include switching regimes with bilinear terms, autoregressive volatility behavior, and Levy processes.

Model	α	β	μ	δ	**BIC**
GH (λ =0.5)	41.65	3.33	-0.001	0.0076	-3546
	[39.47, 43.84]	[1.78, 4.88]	[-0.002, -0.0005]	[0.006, 0.009]	
GH (λ =0)	31.79	-3.08	-0.001	0.013	-3548
	[26.09, 37.30]	[-4.86, -1.31]	[-0.002, -0.0004]	[0.011, 0.014]	
GH (λ = -0.5)	23.32	-2.96	-0.001	0.0187	**-3549**
NIG	[20.51, 26.13]	[-4.19, -1.73]	[-0.0019, -0.0005]	[0.017, 0.019]	
GH (λ = -1)	14.48	-2.91	-0.001	0.025	-3549
	[11.27, 17.69]	[-4.31, -1.55]	[-0.002, -0.0004]	[0.023, 0.026	
GH (λ = -1.5)	5.30	61.2307	-0.001	0.0305	-3546
	[1.70, 8.90]	[-4.30, -1.55]	[-0.002, -0.0004]	[0.0298, 0.0311]	
GH (λ = -2)	3.60	-3.60	-0.0015	0.0377	-3539
	[1.90, 5.26]	[-5.26, -1.93]	[-0.0022, -0.0006]	[0.0367, 0.0386]	

Exhibit 7: Calibration results for NIG based models

Model stationarity focus

We investigated also the stationarity of time series through Phillips-Perron and Augmented Dickey-Fuller tests. Both tests rejected with a 95% confidence level the stationarity null hypothesis of the carbon prices returns. These results confirm the previous findings about switching regimes and underlines the neccesity of searching for stationarity model proofs. In fact the fat tails and negative skewness features identified over the considered period (2005-2009) are heavily driven by a large market jump that occured in May 2006 (Exhibit 2). In order to validate the conclusions of the previous section we have to asses if the NIG and jump processes ouperform the Gaussian model on different time subsamples.

We estimated the statistical features of three models (GBM, GBMJ and NIG) through an in-the-sample/out-of-the-sample basis on 100, 250 and 500 days moving windows. We compared the BIC for each estimation along the testing period and we benchmarked the candidate models for each subsample. The results are shown in Exhibits 10, 11 and 12. In order to compare the discriminative power amongst the candidate models we used a BIC ratio test. The BIC ratio test is a generalization of the likelihood ratio test that allows us to assess if the alternative model outperforms the null model within a certain confidence level. It could be exposed as follows

$$\Lambda = sup(-BIC_{null}(\theta|x); \theta \in \Theta_{null}) - sup(-BIC_{alternative}(\theta|x); \theta \in \Theta_{alternative}) \quad (14)$$

The BIC ratio test rejects the null hypothesis if the value of this statistic is smaller than the significance level of the test (0.05). If the null hypothesis is true, then Λ will be asymptotically χ^2 distributed with degrees of freedom equal to the difference in dimensionality of hypothesis $\Theta_{alternative}$ and Θ_{null}.

We illustrate the results of the test for different lengths of the moving window and three different benchmarks: NIG versus GBM, GBMJ versus GBM and NIG versus GBMJ. The Exhibit 8 gives for each case the percentage of subsamples where the alternative model outperforms the null model and the Exhibit 9 shows the periods when the alternative model outperforms the null model.

We observed that for the small subsamples of 100 days the discriminative power of alternative models is limited compared to the classic Gaussian model. The exception is for the subsamples that include the big jump that occurred in May 2006. For those cases the NIG and the jump models capture better the extreme value effect. As the subsample window increases the alternative models show better performances. Large subsamples include more than one behavioral regime and the switch is better captured by the jumps and NIG modellings.

The conclusion of this part is that NIG and jumps modelings provide with better fitting over large estimation subsamples and the discriminative power is stationary. The results of the previous section which emphasized the advantage of Generalized Hyperbolic distributions and switching regime modellings are not only the consequence of some isolated market ruptures as witnessed in 2006 but are confirmed on a robust basis. Hence one could use these results for pricing and hedging of carbon linked financial instruments.

Alternative vs null model / Window length	**GBMJ vs GBM**	**NIG vs GBM**	**NIG vs GBMJ**
100 days	11.6%	17.4%	16.8%
250 days	32.1%	62.4%	61.6%
500 days	99.9%	99.9%	99.8%

Exhibit 8: BIC ratio test results: Percentage of subsamples where the alternative model outperforms the null model

Alternative vs null model / Window length	GBMJ vs GBM	NIG vs GBM	NIG vs GBMJ
100 days	24/05/06-30/10/06	04/05/06-30/10/06 04/06/07-24/06/07 29/02/08-31/03/08 09/03/09-24/03/09	04/05/06-30/10/06 05/06/07-24/06/07 03/03/08-31/03/08 12/03/09-19/03/09
250 days	08/09/06-30/08/07 12/03/09-31/03/09 13/11/09-25/11/09	08/09/06-21/09/07 01/11/07-18/03/08 26/12/08-21/01/09 10/02/09-21/05/09 06/08/09-14/12/09	08/09/06-21/09/07 05/11/07-18/03/08 29/12/08-21/01/09 05/03/09-21/05/09 13/08/09-14/12/09
500 days	29/08/07-14/12/09	29/08/07-14/12/09	29/08/07-14/12/09

Exhibit 9: BIC ratio test results: Time windows of the subsample when the alternative model outperforms the null model

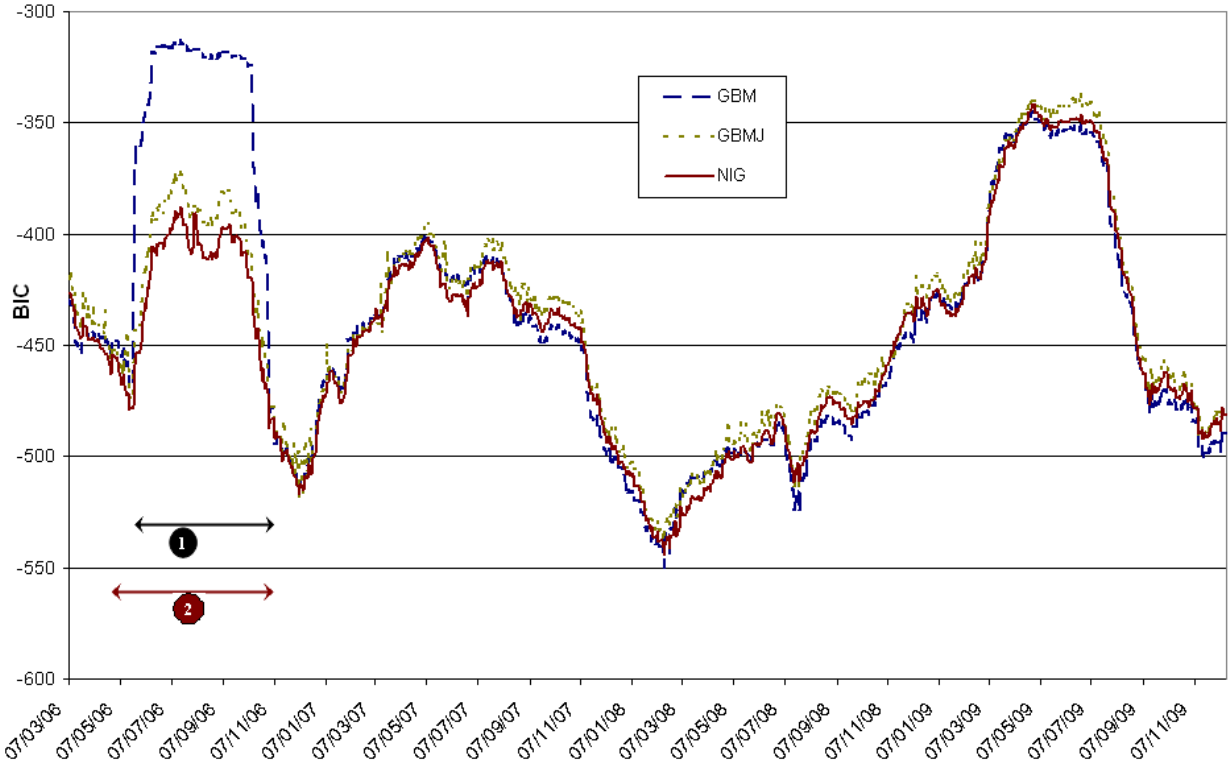

Exhibit 10: Evolution of the BIC for the GBM, GBMJ and NIG models estimated on a 100 days moving window; **Period 1:** GBMJ outperforms the GBM model; **Period 2:** NIG outperforms both the GBMJ and the GBM models.

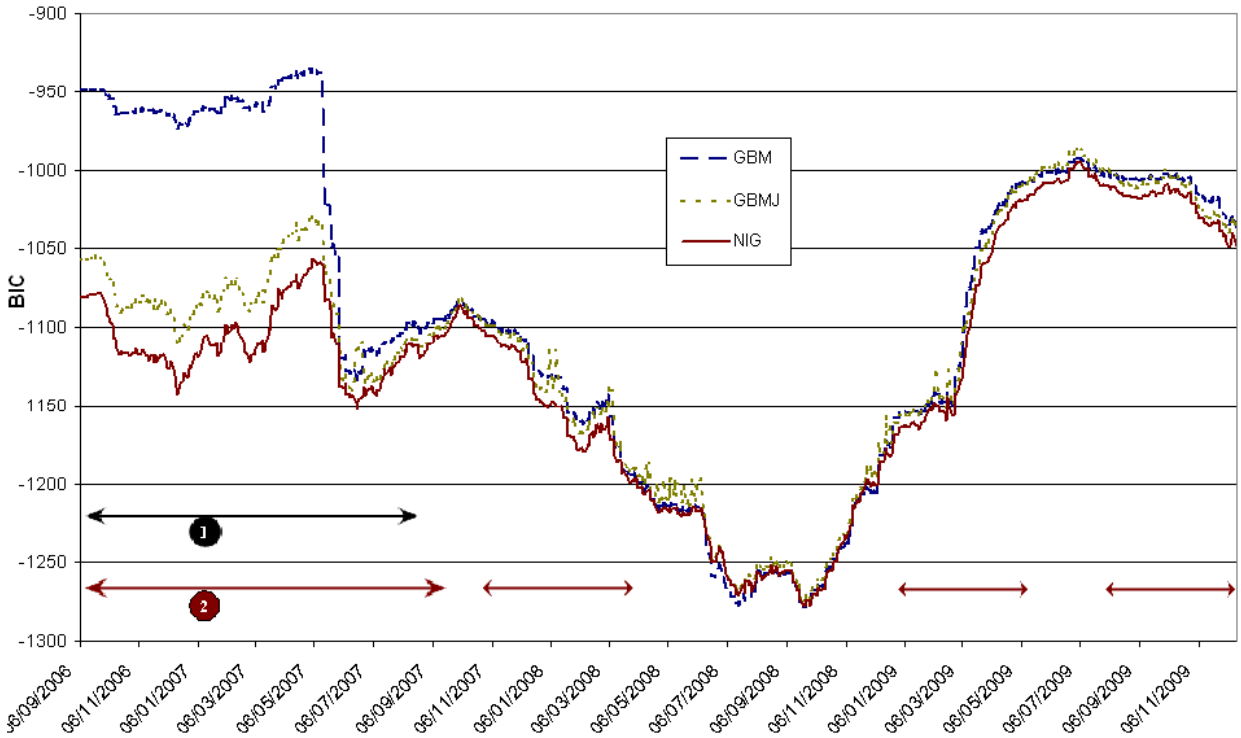

Exhibit 11: Evolution of the BIC for the GBM, GBMJ and NIG models estimated on a 250 days moving window; **Period 1:** GBMJ outperforms the GBM model; **Period 2:** NIG outperforms both the GBMJ and the GBM models.

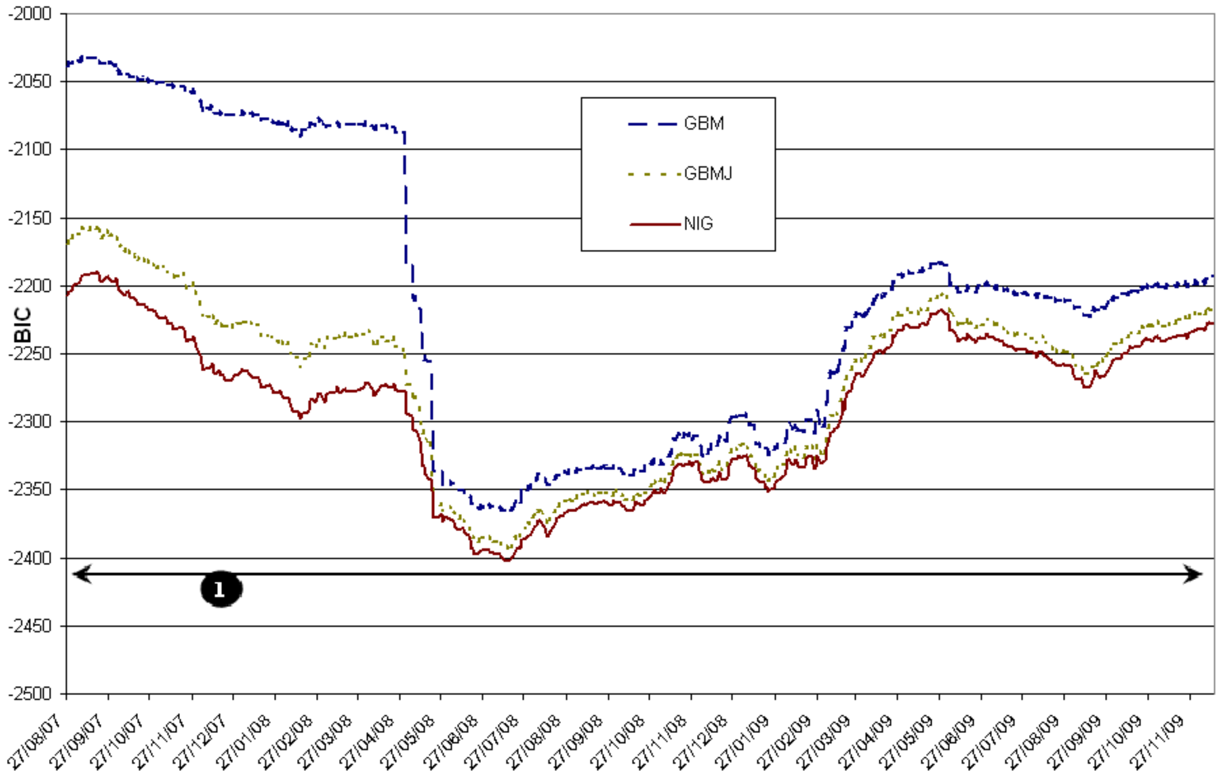

Exhibit 12: Evolution of the BIC for the GBM, GBMJ and NIG models estimated on a 500 days moving window; **Period 1:** NIG outperforms both the GBMJ and the GBM models over the hole testing period

Application to Risk Modelling of a CO_2 Derivative

At the drawn of the carbon market, a wide range of specific financial products were put forward in order to answer to the different needs and to profit from arbitrage opportunities. From forward contracts to exotic options and structured products, the financial institutions cover all the spectrum of derivatives mainly by leveraging on their commodity markets experience. Amongst these products, one of the most popular is the EUA-CER arbitrage swap, a CO_2 structured strategy developed by most of the financial institutions in carbon finance. This product allows riskless income to be generated, by taking the price difference between CER and EUA prices. It is important to mention that under the European Environmental Compliance directives, a company may be compliant if owns the necessary amount of allowances. Nevertheless it is also allowed to own Kyoto credits (CER) instead of EUA, in order to be compliant at a level between 10% and 20%, depending on local regulations. The total physical swap market of 270 Mt is breakdowned amongst various players, and industries and utilities. The volumes engaged in a swap could vary between few thousand tonnes per year for a small industry to tens of millions per year for an electricity producer. Given the important nominal exposure of this kind of product a risk assessment and quantifcations is neccessary.

The arbitrage swap

The difference between prices of the EUA and CER may vary over time. The carbon arbitrage swap creates profitability from the immobilized allowances and income from the prices difference between EUA and CER, without any consequences associated with the price fluctuation.

The carbon arbitrage swap can be adapted for time horizons between one and five years 2008-2012, until the end of the second Kyoto period. It is tailored in such manner that the client is compliant at each regulatory deadline. Hence the client receives the credits each year before its compliance date.

At the beginning of the transaction the industrial company delivers to the broker the quotas that are cashed out on the market via a financial institution (credit company). At the same time, the broker locks the prices for future deliveries of the credits by an agreement with the credit company. In the following years, the credit company will deliver periodically before the compliance date the equivalent credits for the received quotas.

Let us given an example. Consider an industrial company with a CER limitation of 10% and suppose that the company has 1 million allocated 2008 quotas. The industry can thus surrender 100,000 CERs per year. Over 5 years therefore it has recourse to 500,000 CERs. In March 2008 the company transfers 500,000 of the 1 million EUAs which it receives from the Commission into CERs. Since the CER trades at a lower price than the EUA, the difference in price in the exchange releases a premium. This difference in price fluctuates today between €4 and €5,5. In our example, the premium would be fixed for example at €4 per tonne of

CO_2 given up for conversion. The customer will receive 500,000 * €4 = €2 million at the date of the signature of the contract. The delivery of the EUA between the broker and the industry can be done immediately.

Economic capital allocated to an Arbitrage Swap product

It is obvious that the financial counterparty could be under default any time between the date of the contract and the effective deliveries dates of the CERs. In this case, the broker must replace the CER, by buying them at the market price. Under a scenario of a rising trend of CERs, the broker should fill the difference between the negotiated price at the beginning of the contract and the market price at the moment of default.

In the new, heavily regulated environment each financial establishment should have enough capital to cover the extreme risk undertaken by its operations. In our case, the broker should have put aside enough economic capital to cover the consequences of a probable default of the financial counterparty. A classical metric of the economic capital used by many financial institutions for a structured product is the value at risk (VaR). VaR is a very intuitive measure (with some limitations, Artzner[1998]), and is defined as follows:
For a given probability level α, $0 < \alpha < 1$, VaR_α is simply the maximum loss that is exceeded over a specific period with a level of confidence 1-α:

$$P[X \leq VaR_\alpha] = \alpha \tag{15}$$

The risk undertaken by the broker is the difference between the negotiated credit price and the observed price in case of a default of the credit company. Hence in order to evaluate the Value at Risk of the product three problems arise:

1. to estimate the default risk over the product time horizon ;
2. to estimate the spread behavior between the EUA and CER ;
3. to cumulate the credit and the market risks.

The default risk is estimated via the default probabilities and transition matrices given by the rating agencies. Here, we model the spread with a fixed discount rate to the EUA prices. The linkage of both credit and market risk is made via a CreditMetrics-like approach. Otherwise we simulate the exposure scenarios in case of counterparty default, thereby obtaining the aggregated risk distribution.

We used a Monte Carlo VaR based on Gaussian and non-Gaussian distributions, calibrated on the historical times series, through the models estimated in the previous section.

Value at Risk: classical versus GH models

In the following, we compare the economical capital figures for the carbon arbitrage swap measured via the different models calibrated in the previous sections. We use the example given in the previous paragraph with 1 million quotas per year, for each delivery horizon on the product, thereby providing a Value at Risk figure for each year.

The product risk exposure depends on two factors: the nominal exposure of the swap and the volatility of the market. It is obvious that the nominal exposure is diminishing with the passage of time because periodic deliveries are made. On the other hand, the passage of time amplifies the volatility of the market and indirectly the product risk exposure. The two factors have different sensitivities depending on the time horizon and as a consequence the maximum marginal exposure is at the point half way to the time horizon. In Exhibit 13, we provide the average euros loss for different horizons and in Exhibits 14, 15, and 16 the results for the Monte Carlo VaR (euros) with α values equal to 99.5%, 99.9% and 99.99%.

It appears that for a given α the VaR results have heterogeneous values depending on the models. The GH and jumps models give the most conservative results due to the fact that they contain more information about the tail behavior than the classical models, hence emphasizing the potential extreme events. For an investor underwriting an EUA/CER swap the reserve capital and implicitly her return over capital for the operation would be significantly different depending on her risk adversity and on her view in terms of market behavior.

For different values of α, the variations of the risk measures also depend on the chosen model. Hence models with strong kurtosis tend to show bigger variations for different percentiles that classic models. This depends on the capacity of the model to enclose significant information that could characterize the extreme percentiles. In these cases, figures of GH and jump models consume more capital than Brownian diffusion models.

The horizon plays an essential role for this type of product. As we have already shown, the risk horizon for the arbitrage swap is measured in years and goes far beyond the classic "10 days", used by the option desks. For longer horizons the VaR becomes bigger but the incremental VaR from one year to another has a maximum value between the second and the third year. It appears that the averages of the lost distributions stay in the same range of values for all models for a given horizon. Nevertheless the models with fat tails show a bigger average, due to the extreme losses that could appear. The VaR results are more heterogeneous and show big differences between Gaussian and Generalized hyperbolic models, and jump models are more conservative in terms of capital allocation.

Model	2008	2009	2010	2011	2012
GBM	0.0148	0.0620	0.0820	0.0783	0.051
GBMMR	0.0174	0.0276	0.0315	0.0267	0.0156
GBMMRJ	0.0217	0.0362	0.0394	0.0324	0.0194
GBMJ	0.0242	0.0473	0.0554	0.0484	0.0272
NIG	0.0411	0.0991	0.1482	0.1511	0.1092
NIG-GBM	0.0461	0.1245	0.1921	0.1952	0.1402

Exhibit 13: Average Euro loss based on different models and for different horizons

Model	2008	2009	2010	2011	2012
GBM	0.2358	2.4458	4.3021	4.3385	2.9806
GBMMR	0.2156	1.3789	1.9544	1.7293	1.0188
GBMMRJ	0.2345	1.4850	2.2347	2.0264	1.2299
GBM	0.1553	1.3879	2.2468	2.2581	1.3059
NIG	0.5770	5.2909	8.9410	9.1141	6.3059
NIG-GBM	0.5937	6.2808	11.3493	11.4375	8.0929

Exhibit 14: Computation of the VaR using Monte Carlo simulations based on different models and for different horizons with $\alpha = 99.5$

Model	2008	2009	2010	2011	2012
GBM	9.2113	16.7318	19.3572	17.4601	10.4790
GBMMR	5.5986	7.1933	7.0370	5.1112	2.7391
GBMMRJ	7.1489	10.1157	9.3211	7.0537	3.8999
GBMJ	8.0263	13.0906	14.0142	11.5880	6.4137
NIG	13.3518	24.7028	31.6558	28.0115	19.1064
NIG-GBM	15.3028	30.4945	39.6635	37.6635	24.6062

Exhibit 15: Computation of the VaR using Monte Carlo simulations based on different models and for different horizons with $\alpha = 99.9$

Model	2008	2009	2010	2011	2012
GBM	25.6763	45.5230	51.7478	49.0629	28.6319
GBMMR	16.5090	16.0876	14.7093	9.9976	5.3056
GBMMRJ	19.4343	24.9377	21.6928	15.3175	8.4437
GBM	26.0841	41.3185	43.0824	35.0128	18.9109
NIG	33.0043	61.2307	75.3453	75.7261	52.8432
NIG-GBM	37.9612	84.4808	99.6631	97.9891	6.8784

Exhibit 16: Computation of the VaR using Monte Carlo simulations based on different models and for different horizons with $\alpha = 99.99$

Conclusions

Understanding the emission allowances market goes beyond the classic stochastic apprehension of financial assets like commodities, and enters in a more subjective area of behavioral finance.

The main topic of this paper is to propose a modelling approach that better fits the historical time series, using the likelihood function as a discriminating factor to rank models' relevance. The CO_2 allowance prices show pronounced non-Gaussian behavior with fat tails and negative skewness. The NIG distribution outperforms the classic Brownian models in terms of quantity of information. It appears clearly that jumps are a necessary hypothesis for an accurate modeling of CO_2 prices. We applied the results of the model calibration on the risk estimation of a financial product specific to the CO_2 market, the EUA -CER swap. The economic capital allocated for carbon transactions is more conservative when we use NIG models than with classic Brownian models.

In terms of calibration (Exhibit 6) the NIG distribution captures far more information than the classic Brownian models. The main reason is the ability of the GH models to be customized, to different skews and tails forms simultaneously. In our case, the carbon market is far from being Gaussian and the Gaussian mixture evoked previously makes up partially for this handicap, but still keeps the behavior in a "normal universe". The NIG distribution brings another dimension with more parameters that are less intuitive than the Brownian process, but that are more suitable to asymmetric distributions with fat tails. This GH distribution will be favored in further modellings.

Indeed, in the perspective of this work, further natural developments will include Markov switching models with Bilinear terms and memory effects in the model calibration, (Diongue, Guégan and Wolff [2010]), the econometric study of the EUA - CER spread and the macroeconomic model of the CO_2 market, taking in account fundamental factors.

This work appears also as a first step given the evolution of a commodity like the CO_2, under historical measures: this work could be used to develop a robust theory for derivatives based on this commodity, under the risk neutral measure. Indeed, in recent papers, a new option pricing theory has been developed under incomplete market assumptions, based on discrete time series models which are closer to the evolution of the market than all the models developed in continuous time, using classes of Brownian diffusions, (Christoffersen and Jacobs [2002], Badescu, Kulperger and Lazar [2008], and Chorro, Guégan and Ielpo [2010]). These works could be extended usefully to the issues discussed here.

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