Government and the provision of public goods: from equilibrium models to mechanism design[*]

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1 Introduction

After the second world war, most of European governments were currently supposed and required to coordinate and stimulate the reconstruction effort of national economies devastated by the war, to provide the public goods and services in Health, Education, Research and all kinds of public infrastructures which have made possible the substantial economic growth of this period, and, when possible, to regulate this growth by contra-cyclical fiscal and monetary policies. Sixty years after, as a result of the movement of globalization–regionalization of all economies over the world and the construction of large economic areas on the model of the European Union common market, the common representation of governments' tasks has progressively but dramatically changed. Public firms have been privatized, the public goods provision has been largely deregulated, welfare programs are cut and budget deficits are fought. These evolutions have been passively accepted in the general context of the weakening of the concept of 'Etat-Nation', under the increasing influence of liberal and free-trade ideology. Roughly speaking, even if a number of certitudes are nowadays seriously cracked, the paradigm of a central, omniscient, omnipotent (and benevolent) policy maker has been relaxed, replaced by the idea of a multitude of public objectives defined at several, more or less coordinated, levels: national, international, regional, or even branch level. To fulfill these objectives, most of the traditional economic policy instruments, supposed to contradict international agreements, are presented as inadequate, while the others mainly focus on establishing rules that guarantee the fair competition of private firms.

One purpose of this paper is to study how the changes in the *common representation* of governments' tasks are reflected into the evolution of the public goods provision

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theory, especially at its most formalized level. Specifically, recall that the fifties are the years of creation and formalization of the modern general equilibrium theory. During this period, armed with the ambition of providing a coherent framework for understanding market functioning in classical and non-classical environments, mechanism design theory has difficulty in elaborating a general definition for the design of institutions through which individuals interact. In the following decades, simultaneously with the definition of the research program of its initiators, the Arrow–Debreu model[1] has progressively been enriched in order to weaken assumptions and to accommodate, one after the other, most of the different issues tackled by neoclassical economics. The publication by Samuelson in 1954 of an influential paper stating in a general equilibrium framework the optimality problem associated with the provision of public goods and the distribution of tax burdens paved the way for the definition of equilibrium concepts for an economy with public goods.

Such an objective was diversely addressed by the scientific community of general equilibrium. In the seventies some general equilibrium models were studying the equilibrium of "second best" economies where the existence of taxes, lump sum transfers and the governmental provision of public goods were explicitly modeled as exogenous data. More or less at the same period, Russian theorists analyzed general equilibrium of "mixed economies" with some goods publicly provided in fixed quantities and/or at fixed prices and resold at competitive prices on secondary competitive markets. The embedding in general equilibrium models of pricing rules fixed by the government for goods, which are useful from a public policy viewpoint but that market may fail to provide, was also actively worked out. One could evoke in addition the abundant theoretical elaboration in this time on planning of mixed economies. In all these theoretical works, the rationality of public choices is implicitly assumed to be determined outside the functioning of the competitive system. In counterpart, governmental interventions, whose objectives are taken as given by the agents of the competitive system, are assumed to determine the characteristics of consumers' and producers' behavior and, in particular, to influence the production possibilities of firms. The task of general equilibrium theorists is to investigate the possibility of equilibrium, given these public policies, and eventually to look for minimizing the distortions introduced by the government interventions.

This is in contrast with two polar equilibrium models, worked out at the same period, where the provision of public goods enters as an argument in consumer's utility function and so determines the equilibrium amount of their provision and their equilibrium price. In the first one, whose likelihood is attested by the development of charities, consumers "provide" public goods, that is, buy them at their equilibrium market price in order to put them at the disposal of the other consumers. In the second one, consumers pay at personalized prices, called Lindahl prices, their common consumption of public goods pro-

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[1]One generally adds a reference to McKenzie whose responsibility in setting different elements of the general equilibrium framework is undisputed.

duced by competitive producers. Both models, whose consistency is proved under the same standard assumptions, have as a common feature the fact that public goods are provided without intervention of any public institution. Their respective drawbacks are at the origin of the huge development, since the eighties, of the mechanism design literature called for solving the sub-optimality of the private provision equilibrium and the evasion by free-riding of the Lindahl–Foley equilibrium. Replacing equilibrium of competitive economies by equilibrium of mechanisms whose aim is to implement desirable allocations or to allow for a true revealing of preferences has consequences on the role assigned to government institutions by the public goods provision theory. Roughly speaking, in equilibrium models, the government is supposed to stimulate, coordinate, correct, or plan the competitive functioning of private ownership economies as justified by the welfare theorems. With the today ubiquitous mechanism design in the theoretical literature, the completely decentralized functioning of any system has become an objective *per se* that the function of government is to enforce.

In the sequel, we follow this by no way linear evolution beginning with the simultaneous publication of the Arrow–Debreu (1954) paper, birth announcement of the modern general equilibrium theory, and of Samuelson (1954, 1955)'s papers defining public goods. Our objective is to analyze how, after 1954, the theories of general equilibrium and of mechanism design have integrated in their framework the problems of public goods provision and more generally of public policy in response to collective needs of the society. Such an historical analysis is generally missing in books, in other respects important, on the history of general equilibrium (as, for instance, Ingrao and Israel (1990)).[2]

We will see how the theoretical evolution of general equilibrium theory superposes after 1954 two lines of research which overlap, mobilizing sometimes the same researchers, without it be possible to speak of two schools of thought only one of which would accept Samuelson's definition of public goods. As to the elaboration of equilibrium concepts for an economy whose definition incorporates explicitly the presence of Samuelson public goods, two equilibrium definitions persist in the literature, with however a progressive reversal of the relative weight of private provision of public goods relative to their financing with Lindahl prices, at the advantage of public goods private provision equilibrium. Both equilibrium definitions contribute to reinforcing the idea that, ideally, government institutions could (or should) not interfere in the provision of public goods. However, their respective drawbacks confirm the evidence claimed by Samuelson that, in an economy with public goods, no equilibrium concept can be supported as a market mechanism.

Around the same time, building on the extension of planning procedures to the provision of public goods, mechanism design theory finds its first point of application with the resolution of the 'free-rider problem' and the introduction of the notion of 'incentive

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[2]As is still to be done the history of the application of the Invisible Hand paradigm to most of specialized chapters of general equilibrium theory (time, uncertainty, finance, differential information, ...).

compatibility'. Since then, while leaving unsolved the repeated impossibility theorems of general equilibrium with public goods, mechanism design has become the dominant paradigm for the normative analysis of a wide variety of economic and social issues going from social choice theory and voting systems to optimal allocation of indivisible objects and analysis of many other regulation institutions designed at macro or micro levels. In view of the correlative complexity of the system of public policy decision centers, the public policy objectives become for these analyses something on which the economist has little to say, leaving unset and/or unsolved the general issue of a definition and theoretical foundations for public policy in market economies.

In what follows, Section 2 presents the framework and the research agenda of general equilibrium theory. Section 3 introduces Samuelson's definition of public goods and considers different equilibrium models where the tasks of a government for financing or providing public (social, merit or whatsoever) goods are analyzed without reference to this definition. On the contrary, Section 4 formalizes the equilibrium concepts related with Samuelson's definition, while Section 5 describes their respective drawbacks. Section 6 is devoted to the solutions offered by mechanism design theory. In each of these somewhat technical sections, we have tried to ease the formal definitions we give of different equilibrium concepts, in order to allow for reading of this paper by non specialists. At the end of the paper, a conclusion section investigates the meaning of the evolution of the public goods provision theory relative to the issues raised at the beginning of this introduction as well as in the title of the paper.

2   Creation and development of general equilibrium theory

General equilibrium theory is a unified framework for studying, in the Walras tradition, the *general interdependence of economic activities: consumption, production, exchange.* Arrow–Debreu (1954)'s paper is in the same time the seminal definition of a so-called 'private ownership economy' and an equilibrium existence result proving consistency of the model.

The list of data

$$\mathcal{E} = \left(\mathbb{R}^L, (X_i, P_i, e_i)_{i \in I}, (Y_j)_{j \in J}, (\theta_{ij})_{\substack{i \in I \\ j \in J}}\right)$$

is the prototype description of an economy. L is a (finite) set of goods, so that $\mathbb{R}^L$ is the commodity space and the price space of the model. I is a (finite) set of consumers and X_i, P_i and e_i represent respectively the set of possible consumption plans, the preferences and the initial endowment of consumer $i \in I$. J is a (finite) set of producers (firms), and Y_j is the set of possible production plans of firm $j \in J$. For each i and j, θ_{ij} represents the share of consumer i in the profit of firm j. In the above definition, all data of the model may be thought of as historically and socially determined, as the result of

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past evolutions. The distribution parameters (consumers' initial endowments and profit shares) define the *institutional data* of the economy, result of the current social consensus, but also make precise in some sense what is of *the responsibility of State* in the economy under consideration. The competitive (price-taker) behavior of agents defines the model: a *competitive equilibrium* (the solution concept) is the couple of an allocation and of a price vector such that *markets clear* (the allocation is said to be *feasible*), and, at equilibrium prices, *each producer maximizes its profit, each consumer optimizes her preferences in her consumption set under her budget constraint.* In this functioning, the role of firms is purely technical, more suited for 'regulated' than for privately owned firms characterized in the 'real' world by much more complex strategies than the simple profit maximization,[3] and government is absent from the model. Sovereignty of price-taking consumers is the driving force of the standard general equilibrium model.

The formalization and the definition adopted by Arrow and Debreu in 1954 for what is often denominated 'Walrasian equilibrium' enabled them to solve the equilibrium existence problem addressed before the second world war by A. Wald (1936)[4] for the Walras formalization and by J. von Neumann (1937) in a somewhat different framework. Once the consistency of the model is guaranteed by an equilibrium existence theorem (an equilibrium exists under reasonable assumptions supposed to represent observable features of the reality), during the twenty following years, the research agenda of general equilibrium theory was progressively made precise, centered on:

- Sufficient conditions for equilibrium existence, a constantly revisited issue with each generalization or extension of the model.

- Optimality properties of equilibrium, in some sense the alpha and omega of general equilibrium theory. According to the first welfare theorem, the equilibrium allocation is optimal from consumers' point of view, a mere tautology for convenient definitions of equilibrium and Pareto optimality. Under continuity, convexity, boundedness assumptions on the economy, local nonsatiation of consumers at any component of a feasible consumption allocation, the second welfare theorem states that, given a total amount of available resources, any optimal feasible allocation can be achieved as an equilibrium, that is decentralized by prices, through a convenient redistribution of consumers' wealth (i.e. endowments and profit shares). Just before 1954, that is before the publication of their joint paper on existence, comparable second welfare theorem results had been separately obtained by Arrow (1951) and Debreu (1954).

- Social and institutional stability properties of equilibrium: the equilibrium allocation belongs to the *core* of the economy, that is to the set of allocations that no coalition

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[3]This remark was recurrently used to stress that the Arrow-Debreu model fits better with a centrally planned economy than with a capitalist economy.

[4]A paper that follows a series of papers published by K. Menger (Ed.) *Ergebnisse eines Mathematischen Kolloquiums*, 1935-36.

of consumers can block using the resources and property rights on the firms of its members. Conversely, under assumptions analogous to the ones used in the second welfare theorem, the Debreu–Scarf theorem shows that an allocation which belongs to the core of all replica economies can be decentralized with prices as an equilibrium allocation. Both results were first proved in Debreu–Scarf (1963) for an exchange economy. They are considered as the translation into the general equilibrium framework of what is called 'Edgeworth conjecture' on the asymptotic functioning of a competitive economy: the set of core allocations shrinks to the set of equilibrium allocations when the number of consumers tends to infinity.

- For the sake of comparative analysis, study of uniqueness and continuity properties with respect to the initial data of the economy.

- Computation of equilibria.

The problems related with dynamic optimization or with the definition of dynamic models of temporary equilibrium are missing in this agenda. The names of Kantorovich and Grandmont are respectively attached with these problems which do not concern the essentially static equilibrium concepts to be reported in Section 4 for an economy with public goods.

After its creation, the general equilibrium model was progressively enriched in order to accommodate, one after the other, most of the different issues successively tackled by neoclassical theory: intertemporal equilibrium, microeconomic foundations of macroeconomics, risk and uncertainty, financial markets, asymmetry of information, to quote only important issues among many others. The extraordinary plasticity of the general equilibrium paradigm explains its longevity. Defining public goods, introducing their production, provision, and consumption in the general equilibrium framework was a natural objective, a way for explaining the rationale of public expenditure and of its financing by individual taxes and subsidies. The publication in 1954-55 of Samuelson's papers was obviously to have an impact on the contribution of general equilibrium theory to public economics.

3 Samuelson's definition of public goods and its limited impact

A 'collective consumption good' is defined by Samuelson (1954,1955) as a good whose each individual consumption (or using in production) leads to no subtraction from any other individual's consumption. The simple definition brought on during more than two decades a host of discussions. Their common characteristic is to call for combining in more flexible or more complex ways (see James 1971, Margolis 1955, Meyer 1971, Sandmo 1973, Weymark 2004) the two characteristics of their consumption (non-excludability, non-rejectibility) assigned by Samuelson to the restrictive definition of (pure) public goods. All

these critics call for considering all kinds of "impure" public goods. Nobody[5] questions the main novelty of the paper which is not to recognize the social character of the benefits associated with the provision of public goods (public utilities) but *to extend the domain of consumers' sovereignty to the choice of the amount of public goods to be provided*. In other words, Samuelson's papers cause Public Economics to shift from a political economy approach to public expenditure to a theory of the demand for and the supply of public goods and services.

The problem dealt with by Samuelson, and by each one of the above quoted critics, is the research of (first order) conditions that guarantee optimality of the public goods provision, from the point of view of consumers (Pareto optimality) or from the point of view of a social planner whose utility is depending on individual consumers' utilities. Conclusions are rather negative. Optimum exists, is multiple depending on the particular form of the social utility function. But the externality in consumer's preferences, inherent to the definition of public goods, prevents any implementation by a market mechanism (consumers have no interest to reveal their preferences, their willingness to pay) or by a planning procedure (which would require from an omniscient planner to know all consumers' marginal rate of substitutions between private and public goods).

The impact of Samuelson's papers on the development of general equilibrium was neither immediate nor complete. As remarked in the introduction, important contributions of general equilibrium theory to the analysis of public interventions have been made independently of Samuelson's public goods provision problem, not necessarily outside the list of other functions that Samuelson (1955) concedes to public expenditure at the end of his article: redistributing income, 'paternalistic policies', provision of goods that market may fail to provide, correction of negative externalities. Let us quote:

- General equilibrium of second best economies Under this sub-title, we refer to a series of papers (Fourgeaud 1969, Mantel 1975, Shafer–Sonnenschein 1976, Shoven 1974, Sontheimer 1971), published in the seventies. Through different hypotheses specific of each paper, they have as a common feature to study mixed or 'second best' economies where the presence of a public sector is explicitly modeled and to consider taxes, lump sum transfers, government consumption of private goods, and the (possible) public provision of certain (non marketed) goods as exogenous data resulting from public policy decisions whose analysis should be kept separated from the analysis of the competitive functioning of the economy.

- Equilibrium of 'mixed economies' with certain goods publicly provided in fixed quantities and/or at fixed prices This literature (see Vasil'ev–Wiesmeth 2008 and its

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[5]With the notable exception of an unpublished paper, written in 1956 and to appear in this issue (Peacock and Wiseman, 2010), which qualifies of futile 'the attempts to explain the economic activities of governments by use of *simplistic* conceptions of welfare economics'.

Russian references) investigates existence and (constrained and unconstrained) efficiency of equilibrium in the context of mixed economies characterized by the possible presence and interplay of dual markets for each commodity. On the first one, market prices are fixed and the allocation of goods is determined by rationing schemes and governmental orders. On the second market, flexible prices resulting from the market mechanism coordinate demand and supply. This model, first elaborated in the eighties by Russian economists, may seem today largely anecdotal, even if for their authors it is supposed to be a still valid approximation to the main features of the government intervention in transition economies as those of some Central and Eastern European countries of the former Soviet Union.

- Embedding in general equilibrium model of pricing rules fixed by the government for goods that market may fail to provide. On the contrary, this theoretical analysis is one of the deepest achievements of general equilibrium theory for the last thirty years, and an important chapter of Public Economics which has little to do with Samuelson's public goods definition. Re-visiting a long and controversial debate (Boiteux 1956, Coase 1946, Hotelling 1938, Lerner 1944, Ramsey 1927) on pricing, regulating and financing the production of public utilities (like public transportation, electric power plants and many other examples) produced by firms with increasing returns (to scale) technologies, this literature[6] extends equilibrium definition, existence and optimality properties to economies which satisfy neither the differentiability assumptions made in all just above quoted papers nor the convexity assumptions of the standard general equilibrium theory. The mathematical tool for this extension is the notion of normal cone whose (not unique) definition always captures in the same time the ideas of profit maximization in the convex case and of 'normal' in the smooth case. Several statements of the second welfare theorem for a production economy have been provided, beginning with Guesnerie (1975). Whatever be the chosen notion of normal cone in the subsequent papers,[7] in order to decentralize Pareto optimal allocations of an economy, *firms must be instructed to behave in conformity with the (necessary) first order conditions of Pareto optimality*, that is to choose prices in the normal cone to their component of the Pareto optimal feasible allocation.

In the corresponding equilibrium definition, firms are described by the *pricing rule* they are instructed to follow. In addition, a *wealth structure* defines, in the list of data of the economy, the income of consumers as a function of current prices and of the current production allocation, defined so as to guarantee that, at equilibrium, consumers can finance the possible losses of firms. Under appropriate assumptions,

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[6]One will find a first excellent account in the introduction to a 1988 JME special issue (Cornet 1988) .

[7]Depending on the assumptions made on the production sets, each definition has a different implication for the economic significance of the price decentralization result. Clarke's normal cone is considered as the full generalization of the marginal cost pricing rule.

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equilibrium exists but one should notice that, for nonconvex production economies, the first welfare theorem does not hold: an equilibrium allocation needs not be Pareto optimal. The effect of particular wealth structures, in terms of efficiency of the equilibrium allocation or of alteration of the income distribution, is the translation in the general equilibrium model of the above quoted and still controversial debate on entry fees versus lump sum transfers for financing the deficit of regulated firms.

4 Equilibrium concepts for Samuelson's public goods provision

4.1 A private ownership economy with public goods

Let now L be the set of private goods and, according to Samuelson's definition, K be the set of pure public goods, so that $\mathbb{R}^L \times \mathbb{R}^K$ is the commodity space and the price space of the equilibrium model.

The list of data

$$\mathcal{E} = \left(\mathbb{R}^L \times \mathbb{R}^K, (X_i, P_i, e_i)_{i \in I}, (Y_j)_{j \in J}, (\theta_{ij})_{\substack{i \in I \\ j \in J}} \right)$$

is the prototype description of a *private ownership economy with public goods*, where now each consumer $i \in I$ has her consumption set $X_i \subset \mathbb{R}^L \times \mathbb{R}^K$ and her initial endowment $e_i \in \mathbb{R}^L \times \mathbb{R}^K$, each producer (firm) $j \in J$ jointly produces private and public goods ($Y_j \subset \mathbb{R}^L \times \mathbb{R}^K$). If production technology sets Y_j are assumed to be convex and thus to allow for a profit maximizing behavior of producers, there is no need to reconsider the role in wealth distribution of the profit shares θ_{ij} of consumers on profits of firms.

For defining equilibrium concepts, the difficulty begins with the interpretation of the set X_i for each consumer i and thus of her preferences P_i.

- Either for consumer i, $(x_i, x_i^g) \in X_i$ represents the couple of a consumption of private goods and of a private provision of public goods.

  Then, the utility for consumer i of (x_i, x_i^g) depends on her private goods consumption and on the sum of her own provision of public goods and of the private provisions of the other agents. Until 1976, there will be no general equilibrium existence theorem for such dependent preferences.

- Or $(x_i, G_i) \in X_i$ represents the couple of a consumption of private goods and of a claim for an amount of public goods.

  Then, at equilibrium, all consumers have to agree on a same provision of public goods, and the definition of feasibility for an allocation ("markets clear") has to be reformulated.

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These preliminary considerations explain the multiplicity of equilibrium concepts and the order in which they appear in the literature.

4.2 Lindahl–Foley equilibrium (1970)

The Lindahl–Foley equilibrium corresponds to what is called by Samuelson the Lindahl solution. At equilibrium, consumers consume a same amount $\overline{G}$ of public goods and face personalized prices for public goods, so that an equilibrium is the couple of a consumption-production allocation $\big((\overline{x}_i)_{i\in I}, \overline{G}), (\overline{y}_j, \overline{y}_j^g)_{j\in J}\big)$ and of a non null price vector $\big((\overline{p}, (\overline{p}_i^g)_{i\in I}\big)$ such that

- markets clear for private goods while for public goods, total supply of firms equals consumers' *common demand* (Lindahl–Foley feasibility),

and, at equilibrium prices,

- each producer maximizes its profit $\overline{p} \cdot y_j + (\sum_{i\in I} \overline{p}_i^g) \cdot y_j^g$, using the common private goods price vector and the sum of personalized price vectors for public goods as production price vector for public goods,

- each consumer, using the common private goods price vector and her personalized public goods price vector, optimizes her preferences in her consumption set under her budget constraint.

After some attempts (Johansen 1963, Samuelson 1969) calling for the application of Lindahl's ideas (see Lindahl 1919 and 1928) in the reconstruction of a 'pseudo demand' approach and the definition of a 'pseudo equilibrium' concept,[8] the definition of Lindahl–Foley equilibrium and a proof of its existence emerged in a paper of Foley (1970) and a french working-paper of Fabre-Sender (1969).[9] The now classical equilibrium existence proof (well explained in Milleron (1972)) for a private ownership economy with public goods consists in building an economy with only private goods defined on a commodity space of an increased dimension, by considering each consumer's bundle of public goods

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[8]As emphasized by Johansen, in the Lindahl solution, the extension of expenditures for satisfying public wants and the distribution of the corresponding tax burden must be determined simultaneously. This simultaneity requirement is precisely the one of an equilibrium definition based on the benefit taxation principle. However, for Lindahl (1919), "the concepts of supply and demand as well as the object of exchange are of a special nature ... and equilibrium is reached by agreement between the two protagonists rather than by free competition". For Samuelson (1969), agreement *à la* Lindahl through a parliamentary bargaining among political parties and pseudo equilibrium are subject to the same criticism. In particular, the 'pseudo equilibrium' concept has no relevance to motivated market behavior.

[9]One should also notice that appears, independently but at the same time, the first definition and existence proof for an equilibrium concept with personalized prices of economies with consumption externalities. The model studied by Bergstrom (1970) is that of an exchange economy with only private goods where the utility of each consumer is dependent on the utility gained by all other consumers on their own consumption.

as a separate group of commodities, and applying to the corresponding economy some known equilibrium existence result which will imply equilibrium existence in the original public goods economy.

Besides the usual convexity, continuity, boundedness assumptions for the economy and local no-satiation of consumer's preferences at components of feasible allocations, conditions assumed by Foley for equilibrium existence were quite general with four exceptions which can be partially or totally dispensed with: (1) Consumers have no initial endowment in public goods (for each $i \in I$, $e_i = (\omega_i, 0)$); (2) Public goods are not production inputs; (3) Consumer's utility is increasing in public goods (no public bad); (4) The overall production set is supposed to be a convex cone (constant returns to scale).

More important, the first and second welfare theorems hold. An equilibrium allocation is Pareto optimal and belongs to the core. Conversely, any Pareto optimal Lindahl feasible allocation can be decentralized *using convenient lump sum transfers and consumers' personalized prices for public goods*. This is true even without the second and third assumptions and if constant returns to scale are replaced by the weaker assumption of a convex productive system. Such a result confirms the role generally given in general equilibrium to public redistribution policy for achieving Pareto optimality.

4.3 Wicksell–Foley public competitive equilibrium (1967-1970)

The definition of the Wicksell–Foley public competitive equilibrium is an interesting variation on optimality properties of the Lindahl–Foley equilibrium. A Wicksell–Foley public competitive equilibrium is the triple of a Lindahl–Foley feasible allocation $\big((\overline{x}_i)_{i \in I}, \overline{G}\big)$, a price vector for private and public goods $(\overline{p}, \overline{p}^g)$, and a vector of consumers' taxes $(\overline{t}_i)_{i \in I}$ such that for equilibrium prices $(\overline{p}, \overline{p}^g)$,

- the sum of equilibrium taxes $\sum_{i \in I} \overline{t}_i$ finances the cost of production of the equilibrium public good provision $\overline{G}$,

- each producer maximizes his profit,

- given the equilibrium provision of public goods, each consumer chooses a consumption of private goods so as to optimize her preferences under her after taxes budget constraint $\overline{p} \cdot x_i + \overline{t}_i \leq \overline{p} \cdot \overline{\omega}_i$, and

- There is no other public sector proposal $\big(G, (t_i)_{i \in I}\big)$, in the form of a revised vector of public goods provision together with a vector of taxes on consumers in order to finance together with the sum of equilibrium profits the revised provision of public goods, that makes every consumer better off.

In other words, the grand coalition cannot block with another public sector proposal the equilibrium public sector proposal $\big(\overline{G}, (\overline{t}_i)_{i \in I}\big)$. Even if the realized consensus is rather

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negative, ('a kind of last resort, or worst case', as written by Foley (1967)), this condition can be thought of as capturing and translating into an equilibrium process Wicksell's idea (Wicksell 1896) of an unanimous consent on the couple of a public goods provision together with the set of financing taxes.[10]

Two easy and not surprising properties are the following:

1. A Lindahl–Foley equilibrium allocation is a Wicksell–Foley public competitive equilibrium allocation which is, in turn, Pareto optimal;

2. For an economy $\mathcal{E}$, under the usual conditions for their decentralization with prices, the set of Pareto optimal Lindahl–Foley feasible allocations and the set of Wicksell–Foley public competitive equilibrium allocations coincide.

A more interesting property, stated and proved by Foley in his (1967) PhD dissertation (Foley 1967) under restrictive assumptions, but not reported in his Econometrica paper (Foley 1970) and today forgotten, shows that Pareto optimality of the allocation is compatible with an equitable repartition of the tax burden: there exists a Wicksell–Foley public competitive equilibrium where the tax paid by each consumer is proportional to his equilibrium income. Foley even conjectures the existence of a public competitive equilibrium for an arbitrary progressive income tax.

4.4   Public goods private provision equilibrium (1976-1986)

The idea of private provisions of public goods corresponds to the idea of private donations to charity, campaign funds of political parties and so on. And papers abound during the period on theoretical (and experimental) analysis of voluntary contributions to schools, churches, etc., in relation with the free-rider problem.

The equilibrium definition in the public goods private provision model differs from the standard general equilibrium definition only in the fact that the utility for a consumer $i \in I$ of a couple (x_i, x_i^g) of a private goods consumption and a public goods private provision depends on her private goods consumption and on the sum of all individual provisions of public goods. For the same private ownership economy with public goods $\mathcal{E}$ as above, an equilibrium is the couple of an allocation $\left((\overline{x}_i, \overline{x}_i^g)_{i\in I}, (\overline{y}_j, \overline{y}_j^g)_{j\in J}\right)$ and of a non null price vector $\left(\overline{p}, \overline{p}^g\right)$ for private and public goods such that

- markets clear for private goods and private provisions of public goods (total supply of firms equals consumers' demand),

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[10]For Wicksell (and later Lindahl), agreement should be achieved through parliamentary discussions, the meaning of which was the object of controversial comments (Silvestre 2003). In the equilibrium process, one may imagine a kind of Walrasian auctioneer drawing up, together with private and public goods prices, successive public sector proposals until an agreement is reached.

and, at equilibrium prices,

- each producer j maximizes its profit $\overline{p} \cdot y_j + \overline{p}^g \cdot y_j^g$,

- each consumer i, *taking as given the equilibrium public goods provisions of other consumers*, $\sum_{h \neq i} \overline{x}_h^g$, optimizes (from the point of view of her preferences and under her budget constraint) her consumption of private goods and her own provision of public goods.

As, at equilibrium, consumers take as given the equilibrium behavior of other consumers with respect to their public goods private provision, this equilibrium may be considered as an 'equilibrium *à la* Nash'.

Equilibrium exists under the same standard conditions as in the standard general equilibrium model. But it has no reason to be optimal. It is only 'constrained' optimal, that is, optimal for each agent given the public goods provisions of the other agents. The intellectual tool for the equilibrium existence result is an equilibrium existence result for an *abstract economy*[11] where preferences of players formally depend on the current strategies of the other players, to be found in a Shafer–Sonnenschein paper (Shafer–Sonnenschein 1976) for which public goods private provision was not the concern. Actually, this paper was belonging to and following a series of equilibrium existence results for second best economies. Public goods, evoked at the very end of the paper, were, in the ideas of Shafer–Sonnenschein, to be publicly provided. For this reason, the interest of the paper for public goods private provision equilibrium was ignored, as was and is still ignored its potential interest for "warm glow" and any other form of benevolent, malevolent, or 'concerned for face' public goods provision.

As far as equilibrium existence is only concerned, the celebrated paper "On the private provision of public goods" (Bergstrom–Blume–Varian 1986), represents a regression with respect to the Shafer–Sonnenschein equilibrium model. The B–B–V model allows for one public good (extended to several, at the end of the paper) and one private good. Equilibrium existence in this simple model may be seen as a consequence of equilibrium existence in Shafer–Sonnenschein's paper. But Bergstrom, Blume and Varian study also uniqueness and sensitivity of equilibrium to wealth distribution. This makes for their

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[11]An abstract economy is a 'generalized game' $\Gamma = \left((X_i, \alpha_i, P_i)_{i \in N} \right)$ where in addition to his strategy set and preferences defined on his strategy set, a constraint correspondence defines for each agent the set of strategies he can choose given the strategies of the other agents. An equilibrium of the generalized game is a t-uple of individual strategies where each agent optimizes his preferences in his constraint set. The idea of deducing equilibrium existence in an economy from an equilibrium existence result in an associated abstract economy where an additional agent, the 'Walrasian auctioneer', chooses prices so as to maximize the total excess demand traces back to Arrow–Debreu (1954). The decisive improvement of S–S's paper was to allow preferences in the abstract economy to depend on the strategies chosen by other agents. In the original economy, preferences may thus depend on the current allocation and prices.

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epigones,[12] the interest of their paper. To know if and to what extent private provision of public goods can be (Pareto) improved by government interventions, is still today an issue and an object of research.

5 Drawbacks of the different equilibrium concepts

Existence and optimality properties of the previous equilibrium concepts strongly depend on convexity assumptions on consumers' preferences and production. On the consumption side, nonconvexity may come from indivisibility in the quantities to be consumed, or is related with lack of risk aversion in preferences. We will neglect these causes of nonconvexity. On the production side, most of so-called collective goods (public utilities) are classical examples of decreasing costs and are goods that market may fail to provide. This is one of the main drawbacks of the results reported in Section 4.

As we have seen in Section 3, even if such goods are considered as private goods from consumers' point of view, non convexity on the production side requires public intervention for enforcing pricing rules and designing income distribution rules allowing consumers to survive and to finance a possible deficit in their production. Regulation of firms and the necessity of this alteration of distribution rules with respect to the institutional definition of private ownership economies set the issue of new foundations for public policy of market economies without providing means for calculating and implementing an appropriate public policy.

When goods produced under nonconvex technologies are considered as Samuelson public goods, one can rely on Gourdel (1995) for conditions of existence of public goods private provision equilibrium in nonconvex production economies,[13] on Bonnisseau (1991) for conditions of existence of Lindahl equilibria in the same model, on Kahn–Vohra (1987) for the extension of the second welfare theorem to economies with nonconvexities and public goods, on Murty (2006) for the extension to the nonconvex case of the Wicksell–Foley public competitive equilibrium concept. As in the convex case, public goods private provision equilibrium exists but is not optimal. Lindahl equilibrium exists but, unlike in the convex case, is not necessarily Pareto optimal. As in the convex case, consumers' Lindahl prices and lump sum transfers exist for decentralization of optimal Lindahl–Foley feasible allocations.[14]

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[12]Some of them as Villanacci and Zenginobuz (2005) have completely generalized the B-B-V model and its conclusions. See also subsequent papers of Villanacci–Zenginobuz and their references.

[13]Like Shafer–Sonnenschein (1976), when extending equilibrium existence in nonconvex production economies to dependent consumers' preferences, Gourdel does not refer to the externality inherent to the public goods private provision but to more general dependences of preferred sets on the current allocation and prices.

[14]It is in this framework that one should re-visit the old controversy, referred to at the end of Section 3, on entry fees versus lump sum transfers for financing the deficit of regulated firms.

Under convexity on the consumption side, whether public goods be produced by a convex or a nonconvex technology, the other drawbacks of the equilibrium concepts for their provision are today exactly the ones pointed out in 1954 by Samuelson.

On the one hand, Pareto improved or not by government interventions, public goods private provision equilibrium fails, by definition, to be Pareto optimal. And this holds true whatever be the incentives one may invoke for more Pareto satisfactory individual consumers' donations.

On the other hand, general equilibrium theorists are unanimously ready to stress, like Mas-Colell (1980), Silvestre (2003), the implausibility of competitive markets with Lindahl prices for public goods. Revealing his 'willingness to pay" at Lindahl–Foley equilibrium is not a plausible equilibrium consumers' behavior, since it is not a best strategy for every consumer. In other words, according to the terminology of mechanism design theory, the main drawback of Lindahl–Foley equilibrium is not to be 'incentive compatible'.

Moreover, as also pointed out by Samuelson (1954), implementing with Lindahl prices and lump-sum transfers or taxes a desired Pareto optimal Lindahl–Foley feasible allocation would require, for their calculation by a hypothetical coordinating center, government's perfect information on consumers' preferences. An analogous difficulty holds true with planning procedures à la Drèze–La Vallée Poussin (Drèze–La Vallée Poussin 1971) or Malinvaud (Malinvaud 1972). Such planning procedures rule the exchange of information between a 'central board' (central agency, government, whatever) and the economic agents, consumers and (convex) producers,[15] in a process operating in continuous time whose convergence should lead to an equilibrium relative to a system of private goods prices and Lindahl prices for public goods, thus to a Pareto optimal Lindahl–Foley feasible allocation. In a kind of 'Walras tâtonnement', at each stage of such procedures, the indicators issued by the board are an amount of public goods provision together with production prices of all goods and an amount of numéraire left to consumers for their expenditure on private goods. In response, consumers declare their private goods demand and their marginal willingness to pay for the proposed public goods consumption, while producers declare a profit maximizing net supply of private and public goods. Rules of revision of next stage board's indicators as a function of consumers and producers' proposals complete the specification of the dynamic procedure. Besides desirable properties of the different processes, in particular their convergence towards a satisfactory allocation, the possibility of implementing the procedure requires for the board knowledge, at each stage of the process, of consumers' marginal willingness to pay for the proposed public goods provision, an information that consumers may have incentives for under-reporting. The possibility and the design of procedures implying incentives for correct reporting of these marginal rates is thus an important concern for the study of planning procedures in the management

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[15]Mas-Colell (1980) specifies an analogous procedure in the more general case of public projects produced with nonconvex technologies.

of public goods provision, as it is more generally an important concern for mechanism design theory and its contribution to understanding and modeling the provision of public goods.

6   Are mechanisms providing better foundations for public policy of public goods provision in market economies?

6.1   What is mechanism design theory?

Mechanism design theory traces back to discussion papers of Hurwicz at the Cowles Foundation in the first years of development of general equilibrium theory,[16] successively summarized in some seminal papers (Hurwicz 1960, 1972, 1979). A constant reference is done in the beginning to the Barone, Mises, von Hayek, Lange and Lerner debates concerning the economics of socialism, as well as to the different formalizations of Walrasian tâtonnement process. When applied to the resource allocation problem in a society, a constant objective of this new formalization of social and economic interaction is to provide a coherent framework encompassing competitive equilibrium as a particular example of mechanism.

What is first meant under different names given for mechanisms in Hurwicz's papers is the design of a *process* leading, through the collection of decentralized information, to a desired resource allocation. The conceptual framework $\left(N, (\mathcal{M}_i)_{i \in N}, Z, \Phi, (f_i)_{i \in N}\right)$ consists of a *set of agents N* with or without a coordination center, a *space of messages* $\mathcal{M} = \prod_{i \in N} \mathcal{M}_i$, a *space of outcomes Z*, for example the set of all (not necessarily feasible) resource allocations in some economy, on which agents have individual preferences, a pre-specified function $\Phi \colon \mathcal{M} \to Z$ assigning outcomes for every collection of received messages. With $\mathcal{M}$ and Φ, *rules for revision of individual messages*, $f_i \colon \mathcal{M} \to \mathcal{M}_i$, complete the definition of the process and define a *dynamic adjustment process* which should converge towards an element $\overline{m} \in \mathcal{M}$, the equilibrium of the process, with an image $\Phi(\overline{m})$ belonging to a certain class of desirable outcomes. But there is also a more static definition of the equilibrium of a mechanism given by Hurwicz (1979). The mechanism is the game form $\left(N, (\mathcal{M})_{i \in N}, Z, \Phi\right)$. Given individual agents' preferences on Z and, according to a given game theoretic equilibrium notion, the equilibrium should yield a desirable outcome. In both cases, processes and mechanisms should be, as much as possible, *informationally decentralized*, which implies an initial dispersion of information among the agents and limited communication. The resulting outcome should have certain *optimality properties* relative to individual agents' preferences on the different outcomes. Finally, the rules prescribed by the mechanism should be *compatible with either individual or group incentives* based on their own interest, and individual participation to the mechanism should be

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[16]Available at cowles.econ.yale.edu/P/ccdp/ccdp1.htm

voluntary. In view of these requirements, one understands that, as written by Hurwicz , "unlike in the more traditional approach, the mechanism becomes (for its designer) the unknown of the problem rather than a datum" (Hurwicz 1960).

According to the previous definitions, it is obvious that competitive equilibrium of a private ownership economy is, under classical assumptions, a kind of Nash equilibrium for a static mechanism involving the addition of an artificial player called Walrasian 'auctioneer'. The same is true for Lindahl–Foley equilibrium of an economy with public goods. The interest of the static mechanisms proposed in Hurwicz (1979) is not to require any auctioneer. The various dynamic models of market processes and also the different decentralized planning models, including the planning procedures studied above for an optimal provision of public goods, are examples of dynamic mechanisms. All these processes and mechanisms are informationally decentralized (the message sent by each agent requires only information on its own characteristics and does not allow any agent to infer complete information on other agents' characteristics), satisfy static or dynamic optimality (the first and second welfare theorems are satisfied), participation is voluntary (the resulting outcome is individually rational, that is, preferable to the initial allocation), at least under the Foley assumptions, for mechanisms leading to a Lindahl–Foley equilibrium. But they need not satisfy individual *incentive compatibility*, a requirement present in the first papers of Hurwicz but whose formal statement was of difficult elaboration. Roughly speaking, the response function prescribed to each player by each of these mechanisms is not a best strategy for agents who may have interest to give false signals on their own characteristics.[17]

6.2   Dominant strategy mechanisms for public goods provision (1970–80)

The search for incentive compatible mechanisms for public goods provision has been one of the first points of application of mechanism design theory, in relation with the free-rider problem.[18]

The condition for incentive compatibility corresponds to the requirement that equilibrium in dominant strategies be the solution concept adopted for the mechanism. For an agent, a strategy is *dominant* if it is a best strategy no matter what are the strategies

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[17]As explained by Samuelson, this is obvious for Lindahl equilibrium. But, as noticed several times by Hurwicz, this is also true in some sense for competitive equilibrium of an economy with only private goods and a finite number of agents, since these agents would be better off manipulating prices rather than being price takers. Consumers' myopia is necessary to the general equilibrium welfare analysis.

[18]Later, Danziger and Schnytzer (1991) will define a public good economy whose Lindahl–Foley equilibrium emerges, under rather restrictive assumptions, as the perfect equilibrium of a two-stage non cooperative game where consumers *freely* announce subsidy prices for the public goods private provision of the other players and the quantity of public goods they wish to purchase on a private basis. As individual preferences are common knowledge, this kind of mechanism - a good candidate for solving the free rider problem while satisfying the voluntary participation criterion - is not informationally decentralized.

chosen by the other agents. And a mechanism is *dominant strategy incentive compatible* or *strongly individually incentive compatible* if truthfully reporting is for each agent a dominant strategy, that is, an optimal strategy *irrespective of the strategies chosen by the other agents.*

For public goods provision, dominant strategy incentive compatibility is generally stated in the partial equilibrium setting of a decision on a public project with a (already known) cost in numéraire to be financed by collecting contributions (taxes and transfers) from different agents having quasi-linear utility functions on the public goods provision versus their allocation of numéraire.

Namely, X is a set of possible public projects in an economy with a unique private good (the numéraire) and a set N of individuals. Simultaneously with the public goods provision corresponding to the public project, one considers positive or negative monetary transfers $(t_i)_{i \in N}$. Each individual $i \in N$ has preferences on X represented by a *valuation function* $v_i \colon X \to \mathbb{R}$, net of the imputed costs, and a utility function $u_i \colon X \times \mathbb{R} \to \mathbb{R}$:

$$u_i(x, t_i) = v_i(x) + t_i$$

sum of his valuation and of the monetary transfer he receives. A *revelation mechanism* is a mechanism in which each agent $i \in N$ communicates a valuation function, not necessarily his true one, belonging to some set V_i of admissible valuation functions. The message space is thus $\mathcal{M} = \prod_{i \in N} V_i$. The outcome rule, $\Phi \colon \mathcal{M} \to X \times \mathbb{R}^N$, associates with each list of messages $w = (w_i) \in \mathcal{M}$, $\Phi(w) = \big(\Phi_x(w), \Phi_t(w)\big)$, where $\Phi_x(w)$ specifies the accepted project and $\Phi_t(w)$ specifies a list of monetary transfers $(t_i(w))_{i \in N}$ between a central agency and the agents $i \in N$. The revelation mechanism is a *direct revelation mechanism* if, for each list of messages $w = (w_i)_{i \in N}$, the corresponding level of public goods $\Phi_x(w)$ maximizes on X the total social value $\sum_{i \in N} w_i(x)$. The mechanism is said to be *strongly incentive compatible* if truth telling is a dominant strategy for each individual, that is, if for each $i \in N$

$$u_i\big(\Phi(w_{-i}, v_i)\big) \geq u_i\big(\Phi(w_{-i}, w_i)\big) \ \forall w_{-i} \in \prod_{j \neq i} V_j, \ \forall w_i \in V_i.$$

Clarke in 1971 (Clarke 1971), Groves in 1973 (Groves 1973), Groves and Loeb in 1975 (Groves–Loeb 1975) demonstrated that a class of direct revelation mechanisms[19] exists in which truth telling is a dominant strategy for each agent.

Unfortunately, maximization of the social surplus does not mean Pareto optimality of the outcome $\Big(\Phi_x\big((v_i)_{i \in N}\big), \Phi_t\big((v_i)_{i \in N}\big)\Big)$. Indeed, recall that individual utility functions have two arguments and express the personal trade off of each agent between the utility

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[19] A class that Green and Laffont have characterized in 1977 (Green–Laffont 1977) when the set X of public projects is compact and every continuous function $v \colon X \to \mathbb{R}$ is an admissible valuation function.

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(net of cost) of the public project and the individual amount of money t_i he receives ($t_i \geq 0$) from the central agency or pays ($t_i < 0$) to the central agency. Since quasi-linear utility functions are strictly monotone with respect to numéraire, Pareto optimality requires the equilibrium transfers to be *balanced*, that is, $\sum_{i \in N} \Phi_t\big((v_i)_{i \in N}\big) = 0$. The Clarke-Groves mechanisms need not satisfy this condition and thus do not necessarily lead to a Pareto optimal outcome.

6.3 Nash incentive compatibility as a solution to the free-rider problem (1977–80)

With a less restrictive condition for incentive compatibility, Groves and Ledyard (1977) address this problem in the classical model of a private ownership economy with public goods defined in Section 4. As explained in the introduction to their paper, the mechanism designed by Groves–Ledyard consists of appending to this model an explicit procedure for determining consumers' demand for public goods and their tax burdens. It requires the definition of a space of messages $M = \prod_{i \in I} M_i$ for consumers and of an hypothetical agent, the 'government',[20] which sets the provision $G(m, p, p^g)$ of public goods and the individual contributions of consumers $t_i(m, p, p^g)$ as a function of the collection of messages $m = (m_i)_{i \in I}$ sent by consumers and the current market prices (p, p^g) for private and public goods. An *equilibrium of the mechanism* is a list $\big(\overline{m}, (\overline{p}, \overline{p}^g), ((\overline{x}_i)_{i \in I}, \overline{G}), (\overline{y}_j, \overline{y}_j^g)_{j \in J}\big)$ of a collection of messages, a non null price vector and an allocation of the public goods economy such that

- markets clear for private goods and the provision $\overline{G} = G(\overline{m}, \overline{p}, \overline{p}^g)$ of public goods

- each producer j maximizes at $(\overline{y}_j, \overline{y}_j^g)$ his profit $(\overline{p} \cdot y_j + \overline{p}^g \cdot y_j^g)$

- each consumer i optimizes his preferences relative to private and public goods in the budget set[21]

$$\left\{ (x_i, m_i) \in X_i \times M_i \colon \overline{p} \cdot x_i + t_i\big((m_i, \overline{m}_{-i}), \overline{p}, \overline{p}^g\big) \leq \overline{p} \cdot \omega_i + \sum_{j \in J} \theta_{ij}(\overline{p} \cdot \overline{y}_j + \overline{p}^g \cdot \overline{y}_j^g) \right\}.$$

Groves and Ledyard give first two examples of mechanisms, one with a so-called 'naive' government leading to a public goods private provision equilibrium, a second one with a so-called 'Lindahl' government leading to a Lindahl–Foley equilibrium whose achievement requires from the consumers to truthfully report their marginal rate of substitution between each public good and some numéraire private good. Each one of these examples

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[20]In probable addition to the Walrasian auctioneer which should guarantee in the existence proof the feasibility of the equilibrium allocation.

[21]It is worth noticing that in consumer i's budget set, $t_i\big((m_i, \overline{m}_{-i}), \overline{p}, \overline{p}^g\big)$ represents for i the equilibrium value of her own message given the equilibrium messages $\overline{m}_{-i}$ of the other consumers.

suffers the drawbacks of the two equilibrium concepts: sub-optimality of the resulting allocation in the first case, lack of incentive compatibility in the second one. This motivates the main result of the paper consisting in the definition of two 'optimal' mechanisms[22] proved to lead to Pareto optimal allocations, and such that, *at equilibrium*, it is each consumer's individual self-interest to reveal his true valuation for or demand of the public goods. In addition, the second mechanism allows for a decentralization of Pareto optimal allocations through the redistribution of initial endowments and shares of profits. These results are by no means immediate and are completed by a mechanism equilibrium existence result which can be found in (Groves–Ledyard 1980).[23]

The decade 1970-80 will finish with various impossibility theorems for public goods provision mechanisms (Green–Laffont 1979, Hurwicz–Walker 1990, Walker 1980), deeply connected with the general impossibility theorems in voting and social choice theory. In the partial equilibrium setting of a decision on a public project and the schemes of transfers for the distribution of its cost, a truth-dominant mechanism either fails to be social value maximizing or fails to be budget balancing; under suitable assumptions, this negative result is 'generic', that is, holds for an open and dense set of valuation profiles in $\prod_{i \in N} V_i$. In the general equilibrium setting of public goods provision, the Groves–Ledyard mechanism might require some agents to be coerced into participating.

Such negative results created interest in less demanding notions of incentive compatibility based on the intermediary solution concept of Nash equilibrium in a model where participants have beliefs concerning each other and are expected-utility maximizers.

6.4 Bayesian mechanisms for public goods provision (1979–90)

Let us first come back to the partial equilibrium setting of a decision on a public project studied in the paragraph 6.2 and denote by $v_i(x, \theta_i)$ the valuation function of agent $i \in N$ for the public project x, depending on the value of a parameter $\theta_i \in \Theta_i$. The space Θ_i represents the set of possible types of agents i; $\widehat{\theta_i} \in \Theta_i$ identifies agent i's tastes for of the agent i. The message space can be rewritten $\mathcal{M} = \prod_{i \in N} \Theta_i$ and the outcome rule, $\Phi \colon \mathcal{M} \to X \times \mathbb{R}^N$, associates with each $\theta = (\theta_i) \in \mathcal{M}$, $\Phi(\theta) = \big(\Phi_x(\theta), \Phi_t(\theta)\big)$ where $\Phi_x(\theta)$ specifies the accepted project and $\Phi_t(\theta)$ a list of monetary transfers $(t_i(\theta)$ to agents $i \in N$. The incentive compatibility constraint of the previous paragraph can be rewritten:

$$v_i\big(\Phi_x(\theta_{-i}, \widehat{\theta_i}), \widehat{\theta_i}\big) + \Phi_t(\theta_{-i}, \widehat{\theta_i}) \geq v_i\big(\Phi_x(\theta_{-i}, \theta_i), \widehat{\theta_i}\big) + \Phi_t(\theta_{-i}, \theta_i) : \forall i \in N; \ \forall \theta \in \prod_{i \in N} \Theta_I.$$

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[22]In the first one, each message space is a space of differentiable and strictly concave functions $m_i \colon \mathbb{R}_+^K \to \mathbb{R}$ representing individual valuation functions for public goods; in the second one, $M_i = \mathbb{R}^K$, each m_i representing an individual claim for public goods provision.

[23]Equilibrium is proved to exist under standard assumptions on the public goods economy, with in addition a condition on the parameters of the mechanism and a technical condition whose role is to prevent consumers to go to bankruptcy under the government taxation when other consumers are in equilibrium.

In the Bayesian model associated with this formalization, the agents have beliefs on the other agent's characteristics conditional to their own type, that is, subjective probabilities $p_i(\cdot|\theta_i)$ on $\prod_{j\neq i}\Theta_j$ known only when their own type θ_i is also known. A strategy is a decision rule associating a unique strategy choice $m_i(\theta_i)$ to each of his possible types. A *Bayesian Nash equilibrium* is a n-uple of strategies such that, whatever be his type, each agent maximizes the mathematical expectation of the utility of the corresponding outcome, assuming that the other agents will not change their strategy. Finally, a mechanism is *Bayesian incentive compatible* if the n-uple of strategies $\overline{m} = (\overline{m}_i)_{i\in N}$ defined by for all i, $\overline{m}_i(\theta_i) = \theta_i$ is a *Bayesian Nash equilibrium*, that is, if

$$E[v_i\big(\Phi_x(\widetilde{\theta}_{-i},\theta_i),\theta_i\big) + \Phi_t(\widetilde{\theta}_{-i},\theta_i)|\theta_i] \geq E[v_i\big(\Phi_x(\widetilde{\theta}_{-i},\theta_i'),\theta_i\big) + \Phi_t(\widetilde{\theta}_{-i},\theta_i')|\theta_i]$$

for all i, $\widetilde{\theta}_{-i} \in \prod_{j\neq i}\Theta_j$ and $\theta_i' \in \Theta_i$. In other words, for every agent $i \in N$ and every possible type $\theta_i \in \Theta_i$, sending as a message this information to the center dominates every other possible message *whenever the other agents have presumably the same behavior.*[24]

The existence of a Bayesian incentive compatible mechanism enabling to solve efficiently a collective decision problem and to ensure budget balancing was proved by D'Aspremont and Gerard-Varet (1979) when the sets Θ_i are finite, under a compatibility condition on individual beliefs and other mild conditions. However, as remarked later (Maylath–Postlewaite 1990), their mechanism typically violates the voluntary participation constraint of individual rationality. If participation is voluntary, the problem of free-riding may become severe, which extends to incentive compatible and individually rational direct revelation mechanisms the negative Samuelson conjecture on the impossibility of a decentralized optimal provision of public goods.

6.5 The shift to second best analysis and the correlative dissolution of the figure of social planner (1979–)

Obviously, the papers reported in the three previous paragraphs are in some sense seminal. Subsequently, the public goods provision problem has been studied based on other frameworks (voting games, perfect equilibrium of two-stage games, ...) and/or applied to several extended settings allowing for large economies, more complex systems of information parameters, the possibility of exclusion or costly access to public goods, and diverse possibilities of renegotiation of the allocation corresponding to the government message. The recurrent theme that comes out of this huge literature is still the difficulty of finding informationally decentralized, incentive compatible mechanisms that simultaneously result in decisions maximizing the total welfare, the voluntary participation of the individuals, and balanced transfers.

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[24]A stronger incentive compatibility notion can be defined in terms of dominant strategy equilibrium.

Around the same time, and still in the context of decentralized information, a 'second best' formulation, using the principal–agent framework, allows for more successful applications of mechanism design theory. In contrast to the first generation of incentive models which were concerned with the possibility or impossibility of implementing ('first best') Pareto optimal outcomes, the principal–agent analysis begins with the identification of a set of incentive compatible (for the agents) mechanisms, before choosing among them according to the objectives of the principal.[25] As the principal may be as well a social planner caring about efficiency and distributive criteria as a private self-interested agent maximizing some payoff, the mechanism design theory has henceforth largely deserted the field of optimal public goods provision analysis, and more generally the early questions raised by Hurwicz concerning the optimal allocation of resources in classical and non-classical environments of an economy, for focussing on the solution of a great variety of useful applications in different branches of economics.

The main applications concern the design of auctions, bargaining, exchange and regulation mechanisms. A fruitful application of the study of regulation mechanisms is its contribution to foundations of a theory of the firm. But mechanism design may also be used for the regulation by a social planner of a monopoly utility provider[26], the control of a polluting producer, and more generally in several problems where externalities are present. Mechanism design is thus not absent from normative analyses in public economics.

7   Concluding

At their true beginning, mechanism design, under the influence of Hurwicz, and, to a slightly lesser degree, general equilibrium had both a lucid vision of the fact that modern capitalist economies are mixed economies. It is for the public policy of such economies that Musgrave (1959) defined the three levels of planning (allocation, distribution, stabilization) which correspond to the government tasks that we have summarized in the introduction under the words of 'common representation' in this time of the role of the State. Fifty years after, despite the sum of theoretical efforts to prove the contrary, the first obvious conclusion of this historical survey is the impossibility of a total absence of state intervention in the optimal production and/or provision of public goods. From equilibrium to mechanism design, the difference is in the role and the objectives assigned to this intervention.

Whatever be the normative or positive character of their analysis, the general equilibrium models studied in Section 4 without reference to Samuelson's definition of public

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[25]In his survey of mechanism design theory and his explanation of the role played by Leonid Hurwicz, Eric Maskin and Roger Meyerson in the development of this field, Mookerjee (2008) attributes to the large number of impossibility results this shift in the way the key questions have to be posed and the reversal of the sequence of analysis.

[26]Privately informed of its (variable) cost of production.

goods assume or call for state intervention. For instance, the implementation of pricing rules associated in the nonconvex production sector with optimal allocations calls for public production of public utilities or for appending procedures of mechanism design to the general equilibrium model in order to guide the behavior of agencies in charge of their production.

When starting from Samuelson's definition of public goods, neither general equilibrium nor mechanism design have taken up the challenges raised by Samuelson's paper. In the equilibrium process that determines simultaneously the public goods optimal provision and financing, the notion of the consumer so distributing his income as to maximize satisfactions that originate from private and public goods consumption is maybe the 'original error'.[27] Focussing more and more on information revelation problems and making of incentive compatibility of essentially selfish individual behaviors the main requirement for an appropriate public policy of market economies, equilibrium theory and mechanism design have led, for one half of century, to the same negative conclusion on the impossibility of a fully decentralized (first best) optimal public goods provision through market or market-like institutions.

Called for defining signals and procedures, appended to the general equilibrium model, which could guide the decentralized behavior of the agents, the mechanism design theory locates the role of the government in the design and the enforcement of such procedures. The difficulties of their implementation, in particular concerning the public goods provision, are the subject of a last paper of Hurwicz (Hurwicz 2008). As noted by Lee (2006), far from replacing general equilibrium theory, "the mechanism design theory provides the dominant Walrasian general equilibrium tradition with a new transpersonal, algorithm-centered vision of markets/organizations/institutions", leaving unsolved the lingering problems associated with this tradition (here, the impossibility of a fully decentralized public goods provision).

What is insufficiently quoted by Lee, and explains for us the shift in the vision of public policy offered today by the general equilibrium and mechanism design theories, is the increasing influence of the principal–agent setting and the agency theory. To the idea of a central planner is substituted the proliferation of decentralized principals which reflects the multiplicity today of (eventually) public objectives quoted in the introduction. The elaboration of these objectives could contribute to redefining the role of the State. Such an effort is neither in the ambition nor in the abilities of this paper.

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[27]Recall that for Galbraith (Galbraith 1967, chapter XIX, The Revised Sequence), the assumption that sovereignty of consumers is the driving force of economic equilibrium was not of original error but of obsolescence.

References

Arrow, K.J. (1951) An extension of the basic theorems of the classical welfare economics. In *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability*, University of California Press: 507–532.

Arrow, K.J. and Debreu, G. (1954) Existence of an equilibrium for a competitive economy. *Econometrica* 22: 265–290.

Bergstrom, T.C. (1970) A "Scandinavian consensus" solution for efficient income distribution among non-malevolent consumers. *Journal of Economic Theory* 2: 383–398.

Bergstrom, T., Blume L. , Varian, H. (1986) On the private provision of public goods. *Journal of Public Economics* 29: 25–49.

Boiteux, M. (1956) Sur la gestion des monopoles publics astreints à l'équilibre budgétaire. *Econometrica* 24: 22–44. English translation: On the management of public monopolies subject to budgetary constraints, *Journal of Economic Theory* 3 (1971): 219–240.

Bonnisseau, J-M. (1991) Existence of Lindahl equilibria in economies with nonconvex production sets. *Journal of Economic Theory* 54: 409–416.

Coase, R.H. (1946) The marginal cost controversy. *Economica* 13: 169–189.

Clarke, E.H. (1971) Multipart pricing of public goods. *Public Choice* 8: 19–33.

Cornet, B. (ed.) (1988) General equilibrium theory and increasing returns. *Journal of Mathematical Economics* 17, Special Issue: 103–315.

d'Aspremont, C. and Gérard-Varet, L-A. (1979) Incentives and incomplete information. *Journal of Public Economics* 11: 22–45.

Danziger, L. and Schnytzer, A. (1991) Implementing the Lindahl voluntary-exchange mechanism. *European Journal of Political Economy* 7: 55–64.

Debreu, G. (1954). Valuation equilibrium and Pareto optimum. *Proc. Natl. Acad. Sci. USA* 40: 588–592.

Debreu, G. and Scarf, H. (1963) A limit theorem on the core of an economy. *Int. Econ. Rev.* 4, 235–246.

Drèze, J.H. and de la Vallée Poussin, D. (1971) A tâtonnement process for public goods. *The Review of Economic Studies* 38: 133–150.

Fabre-Sender, D. (1969) Biens collectifs et biens à qualité variable. *CEPREMAP Working paper*, Paris (France).

Florenzano, M. (2009) Walras–Lindahl–Wicksell: What equilibrium concept for public goods provision? I- The convex case. *CES Working Papers*, Paris (France).

Foley, D.K. (1967) Resource allocation and the public sector. *Yale Economic Essays* 7: 45–98.

Foley, D.K.(1970) Lindahl's solution and the core of an economy with public goods. *Econometrica* 38: 66–72.

24

Fourgeaud, C. (1969) Contribution à l'étude du rôle des administrations dans la théorie mathématique de l'équilibre et de l'optimum. *Econometrica* 37: 307–323.

Galbraith, J.K. (1967) *The New Industrial State*. London: Hamish Hamilton.

Gourdel, P. (1995) Existence of intransitive equilibria in nonconvex economies. *Set-valued Analysis* 3: 307-337.

Green, J. and Laffont, J.J. (1977) Characterization of satisfactory mechanisms for the revelation of preferences for public goods. *Econometrica* 45: 427–438.

Green, J. and Laffont, J.J. (1979) *Incentives in Public Decision Making*. Amsterdam: North-Holland.

Groves, T. (1973) Incentives in teams. *Econometrica* 41: 617–631.

Groves, T. and Ledyard, J. (1977) Optimal allocation of public goods: A solution to the "free rider problem". *Econometrica* 45,: 783–809.

Groves, T. and Ledyard, J. (1980) The existence of efficient and incentive compatible equilibria with public goods. *Econometrica* 48: 1487–1506.

Groves, T. and Loeb, M. (1975) Incentives and public inputs. *Journal of Public Economics* 4: 211–226.

Guesnerie, R. (1975) Pareto optimality in non-convex economies. *Econometrica* 43: 1–29.

Hotelling, H. (1938) The general welfare in relation to problems of taxation and of railway and utility rates. *Econometrica* 6: 242–269.

Hurwicz, L. (1960) Optimality and informational efficiency in resources allocation processes. In Arrow, K.J., Karlin, S. and Suppes, P. (eds.) *Mathematical Methods in the Social Sciences*, Stanford University Press: 27–46.

Hurwicz, L. (1972) On informationally decentralized systems. In Radner, R. and McGuire, B. (eds.) *Decision and Organization*, Amsterdam: North Holland Press.

Hurwicz, L. (1979) Outcome functions yielding Walrasian and Lindahl allocations at Nash equilibrium points. *The Review of Economic Studies* 46: 217–225.

Hurwicz, L. (2008) But who will guard the guardians? *American Economic Review* 98 (3): 577–585.

Hurwicz, L. and Walker, M. (1990) On the generic nonoptimality of dominant-strategy allocation mechanisms: A general theorem that includes pure exchange economies. *Econometrica* 58: 683–704.

Ingrao, B., Israel, G. (1990) *The Invisible Hand: Economic Equilibrium in the History of Science*. Cambridge: The MIT Press.

Jackson, M.O. (2003) Mechanism theory. *Optimization and Operations*, Derigs, U. (ed.) in *Encyclopedia of Life Support Systems*, EOLSS Publishers:Oxford, UK. A more complete draft is available on the web page of Mattew O. Jackson.

James, E. (1971) Joint products, Collective goods, and external effects: Comment. *Journal of political economy* 79: 1129–1135.

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Johansen, L. (1963) Some notes on the Lindahl theory of determination of public expenditures. *International Economic Review* 4: 346–358.

Khan, M.A. and Vohra, R. (1987) An extension of the second welfare theorem to economies with nonconvexities and public goods. *The Quarterly Journal of Economics* 102: 223–241.

Lee, K.S. (2006) Mechanism Design Theory Embodying an Algorithm-Centered Vision of Markets/Organizations/Institutions. *History of Political Economy* 38 (annual suppl.): 283–304.

Lerner, A.P. (1944) *The Economics of Control*, New york: Macmillan.

Lindahl, E. (1919) Just taxation–A positive solution. Translated from German: Positive Lösung, in *Die Gerechtigkeit der Besteuerung* (Lund), and published in R.A. Musgrave and A.T. Peacock, eds., *Classics in the Theory of Public Finance*, New York: Macmillan and Co. (1958): 168–176.

Lindahl, E.(1928) Some controversial questions in the theory of taxation. Translated from German: Einige strittige Fragen der Steuertheorie, in *Die WirtschaftstheorievderbGegenwart* (Vienna), and published in R.A. Musgrave and A.T. Peacock (eds.) (1958) *Classics in the Theory of Public Finance*, New York: Macmillan and Co.: 214–232.

Mailath, G. and Postlewaite, A. (1990) Asymmetric bargaining problems with many agents. *Review of Economic Studies* 57: 351–367.

Malinvaud, E. (1972) Prices for individual consumption, quantity indicator for collective consumption. *The Review of Economic Studies* 39: 385–405.

Mantel, R.R. (1975) General equilibrium and optimal taxes. *Journal of Mathematical Economics* 2: 187–200.

Margolis, J. (1955) A comment on the pure theory of public expenditure. *The Review of Economics and Statistics* 37: 347–349

Mas-Colell, A. (1980) Efficiency and decentralization in the pure theory of public goods. *The Quarterly Journal of Economics* 94: 625–641.

Meyer, R.A. (1971) Private costs of using public goods. *Southern Economic Journal* 37: 479–488.

Milleron, J-C. (1972) Theory of value with public goods. *Journal of Economic Theory* 5: 419–477.

Mookherjee, D. (2008) The 2007 Nobel Memorial Prize in Mechanism Design Theory. *Scand. J. of Economics* 110(2): 237–260.

Musgrave, R.A. (1959) *The Theory of Public Finance A Study in Public Economy*. New York: Mcgraw-Hill, Book Company, Inc.

Murty, S. (2006) Externalities and fundamental nonconvexities: A reconciliation of approaches to general equilibrium externality modelling and implications for decentralization. *Warwick Economic Research Papers* 756, The University of Warwick, Coventry (UK)

von Neumann, J. (1937) A Model of General Economic Equilibrium, in K. Menger (ed.) *Ergebnisse eines mathematischen Kolloquiums, 1935-36*. (Translated and reprinted in 1945 in *The Review of Economic Studies* 12 (2): 1–9.

Peacock, A. T. and J. Wiseman (2010) On the theory of the public economy. *Euro. J. History of Economic Thought*, this issue.

Ramsey, F.P. (1927) A contribution to the theory of taxation. *The Economic Journal* 37: 47–61.

The Royal Swedish Academy of Sciences (2007), Mechanism design theory. Compiled by the Prize Committee of the Royal Swedish Academy of Sciences, Stockolm (Sweden).

Samuelson, P.A. (1954) The pure theory of public expenditure. *The Review of Economics and Statistics* 36: 387–389.

Samuelson, P.A. (1955) Diagrammatic exposition of a theory of public expenditure. *The Review of Economics and Statistics* 37: 350–356.

Samuelson, P.A. (1969) Pure theory of public expenditure and taxation. In Margolis, J. and Guitton, H.(eds.), *Public Economics. Proceedings of a Conference held by the International Economic Association*, London: Macmillan: 98–123.

Sandmo (1973) A., Public goods and the technology of consumption. *The Review of Economic Studies* 40: 517–528.

Shafer, W. and H. Sonnenschein (1976) Equilibrium with externalities, commodity taxation, and lump-sum transfers. *International Economic Review* 17: 601–611.

Shoven, J.B. (1974) General equilibrium with taxes. *Journal of Economic Theory* 8: 1–25.

Silvestre, J. (1984) Voluntariness and efficiency in the provision of public goods. *Journal of Public Economics* 25: 249–256.

Silvestre, J. (2003) Wicksell, Lindahl and the theory of public goods. *Scand. J. of Economics* 105 (4): 527–553.

Sontheimer, K.C. (1971) An existence theorem for the second best. *Journal of Economic Theory* 3: 1–22.

Vasil'ev, V. and Wiesmeth, H. (2008) Equilibrium in a mixed economy of Arrow–Debreu type. *Journal of Mathematical Economics* 44: 132–147.

Villanacci, A. and Zenginobuz, E. Ü. (2005) Existence and regularity of equilibria in a general equilibrium model with private provision of a public good. *Journal of Mathematical Economics* 41: 617–636.

Wald, A. (1936) Über einige Gleichungssysteme der mathematischen Ökonomie. *Zeitschrift für Nationalökonomie* 7: 637–679. (Translated as 'On some systems of equations of Mathematical Economics', *Econometrica* 19 (1951): 368–403).

Walker, M. (1980) On the non-existence of a dominant-strategy mechanism for making optimal public decisions. *Econometrica* 48: 1521–1540.

Weymark, J.A.,(2004) Shared consumption: A technological analysis. *Annales d'Economie et de Statistiques* 75/76: 175–195 (Substantially revised version of the first chapter of author's PhD dissertation defended in 1976, Department of Economics, University of Pennsylvania).

Wicksell, K. (1896) A new principle of just taxation. Extracts translated from: Ein nues prinzip der gerechten besteuerung, in *Finanztheoristizche Unterschungen*, iv-vi, 76–87 and 101–159, and published in R.A. Musgrave and A.T. Peacock (eds.) *Classics in the Theory of Public Finance*, New York: Macmillan and Co. (1958): 72–118.

Abstract

This paper investigates to what extent the seminal contribution of Samuelson (1954) has been or not incorporated by the theories of general equilibrium and mechanism design in their analysis of optimal public good provision and more generally of optimal public policy. Our conclusion is that, far from taking up the challenges raised by Samuelson's contribution, both paradigms lead to the negative conclusion of the impossibility of a fully decentralized optimal public goods provision through market or market-like institutions, without giving a key for (re)defining the role of state in market economies.

JEL Classification Code: B2, H4

Keywords

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