GREQAM

**Groupement de Recherche en Economie
Quantitative d'Aix-Marseille - UMR-CNRS 6579
Ecole des Hautes Etudes en Sciences Sociales
Universités d'Aix-Marseille II et III**

Capital Requirements for Operational risk: an Incentive Approach

Mohamed BELHAJ

Juillet 2010

halshs-00504163, version 1 - 20 Jul 2010

DT-GREQAM

CAPITAL REQUIREMENTS FOR OPERATIONAL RISK: AN INCENTIVE APPROACH

MOHAMED BELHAJ

Ecole Centrale de Marseille and GREQAM

ABSTRACT. This paper proposes a simple continuous time model to analyze capital charges for operational risk. We find that undercapitalized banks have less incentives to reduce their operational risk exposure. We view operational risk charge as a tool to reduce the moral hazard problem. Our results show, that only Advanced Measurement Approach may create appropriate incentives to reduce the frequency of operational losses, while Basic Indicator Approach appears counterproductive.

Keywords: Operational Risk, Capital Requirements, Dividends, Basel Accords.

JEL: C61, G28, G32

1. INTRODUCTION

Banking regulation aims to discipline banks and to promote financial stability in the economy on the whole. It relies on three pillars, capital requirements, market discipline, and supervisory review process. Under Basel I, banks are required to hold capital for credit and market risk in order to be insured against these risks. Under basel II, banks should also compute capital charge for operational risk, that is defined as 'the risk of direct or indirect loss resulting from inadequate or failed systems or from external events'. In other words, operational risk can rise as the result of some internal factors like internal fraud, different aspects of business processes (staff, clients, products, workplace safety), physical assets damage, business disruption, as well as external frauds, system failure and execution. Since operational losses have a significant effect on the economy, it can be viewed as one of the crucial subjects of banking regulation.

Actually, in the light of basel II recommendation, operational risk capital charge can be computed using Basic Indicator Approach (BIA), Standardized Approach (STA), or Advanced Measurement Approach (AMA). According to BIA, banks must hold capital for operational risk that is equal to the average over the previous three years of a fixed percentage of positive annual gross income (a proxy of the level of operational risk exposure). Under the STA, that can be considered as a variant of the BIA, banks' activities are divided into eight business lines: corporate finance, trading and sales, retail banking, commercial banking, payment and settlement, agency services, asset management, and retail brokerage. For each business line, the corresponding capital charge is computed by multiplying gross income by a factor assigned to that business line. The total capital charge is obtained by summing capital charges over each business. It is important to note that the first two approaches do not truly reflect bank's exposure to operational risk. Indeed, operational capital charge calculated following these approaches doesn't take into account banks' efforts to manage and to mitigate their operational risk. On the contrary, the capital charge calculated under AMA is more risk-sensitive.[1] For each business line and loss type, the capital charge can be evaluated as percentage of expected loss. Expected losses are equal to the product of an exposure indicator (specified by the supervisor), by the probability of loss event, and by losses given event.

The inclusion of operational risk into the capital requirements framework has been severely criticized. In contrast to credit and market risks, there is no evidence of an excessive exposure to operational risk. Moreover, in order to absorb future losses, banks typically hold cash funds in excess of the required capital. Hence, imposing operational risk capital charge may be counter productive if it does not enhance banks' incentives to manage and mitigate operational risk. Moreover, there is no evidence that capital charge, computed under Basel II, will give banks incentives to reduce their exposure to operational risk.

---

[1]For example: Basel II clause, 677. Under the AMA, a bank will be allowed to recognize the risk mitigating impact of insurance in the measures of operational risk used for regulatory minimum capital requirements. The recognition of insurance mitigation will be limited to 20% of the total operational risk capital charge calculated under the AMA.

In this paper, we develop a continuous time model to examine the issue of operational risk capital requirements. This study can be viewed as a first attempt to quantify capital charge for operational risk based on an incentive approach. We consider a bank that faces two types of risks: credit risk and operational risk. We represent credit risk by a Brownian motion and operational risk by a Poisson process with constant intensity. The bank can reduce the frequency of operational losses by exerting a costly effort.[2] The regulator realizes random audit in order to observe bank's effort and its level of capital. She closes the bank as soon as its capital falls below the required level. Bank's manager chooses a dividend policy and an operational risk exposure to maximize the expected discounted value of future dividend payments.

Firstly, we analyze the case without any operational risk capital charge. We find that the bank holds capital in excess of the required credit risk capital requirements and decides to reduce its operational risk exposure only when it is well-capitalized. The point is that, when the bank's capital is close to the liquidation threshold, the marginal value of internal funds becomes too high. Therefore, in this case, it is not optimal to reduce operational risk exposure, i.e, the bank chooses to keep cash inside to insure itself against credit risk. This provides a quite good rationale for operational risk capital requirements. However, this capital requirements would be counterproductive if it does not encourage banks to monitor and to control operational risk.

Secondly, we analyze bank's behavior under operational risk capital requirements. The regulatory capital charge is chosen in order to reduce the moral hazard problem. In line with BIA, we compute the minimal capital charge for operational risk such that well capitalized banks (banks complying with regulation) always reduce their exposure to this type of risk. We find that BIA does not

---

[2]Basel II clause 663 (a): The bank must have an operational risk management system with clear responsibilities assigned to an operational risk management function. The operational risk management function is responsible for developing strategies to identify, assess, monitor and control/mitigate operational risk; for codifying firm-level policies and procedures concerning operational risk management and controls.

really create incentives to reduce the operational risk exposure. Thereby, BIA can be treated as counterproductive, since banks operate better under no operational risk capital charge.

Finally, coherent with the spirit of Basel II, we compute capital charges that depend on the bank's operational risk exposure. By adjusting operational risk capital charge to bank's risk exposure, regulator rewards good banks by imposing less capital charge on them. We show that under AMA, regulator can create real incentives to manage operational risk.

The paper is organized as follows. Section 2 briefly reviews related literature. Section 3 sets up the model and provides essential theoretical basis for the further analysis . Section 4 analyzes the behavior of the bank when there are no operational risk capital requirements. Section 5 characterizes operational risk capital charges which ensure that banks, complying with regulation, will always exert effort.

2. Related Literature Review

The literature on bank regulation is huge.[3] Here we briefly review two branches of the literature that are closely related to our work. Built on Merton (1977), a first strand of literature addresses the design of credit capital requirements. Bhattacharya *et al* (2002) assume unobservable bank's capital and random audit. They compute an optimal closure rule that eliminates risk-shifting incentives for well-capitalized banks. Decamps *et al* (2004) analyze the articulation between market capital requirements, supervisory review and market discipline. They compute the minimal capital requirement that pushes banks to monitor their investment and they show that, if the moral hazard problem is not so big, market discipline helps to reduce the required capital. Fries *et al* (1997) examine the interaction between optimal closure rules and bail-out subsidiary policies. They derive a closure rule that minimizes expected discounted social bankruptcy costs and the costs of monitoring banks that continue to operate. Dangl and Lehar (2004) compare Value-at-Risk based risk capital

---

[3]For a recent review of literature see Santos (2001) or Stolz (2002).

requirements and Building Block Approach of the 1988 Basel I accord when the regulator makes random audit. They conclude that Value-at-Risk based risk capital requirements give well capitalized banks stronger incentives to reduce risk. In these studies, banks' assets follow a geometric Brownian motion and bankruptcy is considered as endogenous. In accordance with this literature, we share the view that capital requirements should be set in order to reduce the moral hazard problem. However, the cited works do not consider operational risk and ignore liquidity management problems.

A second branch of literature considers liquidity management. Milne and Robertson (1996) analyze optimal business and dividend policy when firm's cash reserves follows a drift Brownian motion. The firm pays dividend in excess of a certain barrier and switches to a more profitable/risky technology as cash reserves increase. Milne and Whalley (2001) extend this work to make bank's capital endogenous. They assume fixed costs for bank's recapitalization. They find that capital requirements may explain the credit crunch. Belhaj (2010a) models operational risk by jumps with fixed size. He finds that operational risk may push the bank to take more risk and to reduce its excess capital. However, this literature does not address the design of capital requirement, it rather analyze capital in excess of the required level.

Our work is related to the findings of Rochet and Villeneuve (2010) and Belhaj (2010b). They analyze the problem of the industrial firm's insurance against accidents.[4] They show that only cash rich firms choose to insure against shocks. Similarly, a bank with a low level of capital has no incentive to reduce its exposure to operational losses. Operational risk capital requirements will be designed in order to reduce the moral hazard problem.

3. The model

We start by giving the mathematical formulation of the problem. (Ω, F, P) is a probability space, $\{F_t\}_{t>0}$ is a filtration, $\{w_t\}_{t\geq 0}$ is a standard Brownian motion adapted to the filtration and $\{N_t\}_{t\geq 0}$

---

[4]Shocks have fixed size in Rochet and Villeneuve (2010) and are exponentially distributed in Belhaj (2010b).

and $\{A_t\}_{t\geq 0}$ are two independent Poisson processes. The filtration F_t represents the information available at time t and all decisions made are based upon this information.

Consider a bank which holds a portfolio of illiquid assets (loans) B. The bank's cash reserves evolve according to

$$(3.1) \qquad m_t = m_0 + (\mu t + \sigma w_t - \sum_{i=1}^{N_t} Y_i)B,$$

where m_0 is the initial cash reserves level, μ is the expected cash flow per unit of time, and σ is the volatility of the cash flow.[5] We denote the intensity of the Poisson process N_t by $\overline{\lambda}$ and we assume that the stochastic jumps Y_i are independent and have exponential distribution $(e^{-\delta y})$.[6] The Brownian motion corresponds to credit risk and represents small movements of the cash reserves over a small period of time.[7] The one sided Poisson process represents operational risk. We restrict our attention to operational risk that results in immediate losses. In order to keep the model tractable, we assume that cash reserves are not remunerated. In practice banks invest cash in a portfolio market, yielding capital charge for market risk.

The corporate operational function aims to develop policies, procedures and practices to ensure that operational risk is appropriately identified, measured, monitored and controlled. In practice, banks can mitigate their operational risk through appropriate internal processes and controls. However, such activities are costly. Here, we consider that the bank can exert effort in order to reduce the frequency of operational losses to the level $\underline{\lambda}$ $(= \overline{\lambda} - \Delta\lambda)$; it costs c per unit of time. The decision to make an effort is made before knowing the realization of audit. We assume that this effort is

---

[5]The drifted Brownian process can be seen as a limiting process of Poisson processes.

[6]The exponential distribution captures the fact that small losses are more frequent than large losses. We could consider other distributions. This would complicate the model and we should rely on numerical simulations.

[7]In practice, the credit risk may also contains jumps. Here to keep the model simple, we assume that there are no jumps in credit risk.

socially optimal, i.e, the cost of the effort is less than the expected operational losses reduction, $c < \frac{\Delta \lambda}{\delta}$. Let e_t denotes the bank's decision to make effort or not,

$$e_t = \begin{cases} 1 \text{ when the bank exerts effort }, \\ \\ 0 \text{ otherwise.} \end{cases}$$

Further, we assume that the bank's effort and capital are not observable. This will give rise to a moral hazard problem. That is the bank ay find it optimal to shirk for small levels of capital. Therefore, operational risk charge will be designed in order to reduce the moral hazard problem. Finding the optimal contract stays beyond the scope of this work since it may need more than capital requirements. We will only focus on the design the operational risk capital requirements.

The regulator has to audit the bank in order to observe it's capital and effort. Following Merton (1978), we model the random audit by a Poisson process A_t with intensity ψ. The probability that an audit takes place in the period of time dt is equal to ψdt. We also assume that the audit probability is independent of the previous audit result;

$$dA_t = \begin{cases} 1 \text{ with probability } \psi dt, \\ \\ 0 \text{ with probability } (1 - \psi)dt. \end{cases}$$

We assume that equity issue is prohibitively costly.[8] Therefore, the bank is closed when its capital becomes less than the sum of operational risk capital charge (ORC) and credit risk capital charges (CRC): $m + B - D \leq CRC + ORC$. To focus on operational risk capital charge, we consider for the further analysis that the book value of bank's illiquid asset B net of its debt D is equal to the credit risk capital requirements.[9] Hence, absent operational risk capital requirements, the bank would be closed by the manager the first time when its level of cash reserves falls below zero (liquidity problem). In the presence of operational risk capital charge, the bank will be closed either by the

---

[8]It is more realistic to assume assume fixed costs for equity issue as Milne and Whalley (2001). We would obtain qualitatively the same results, however we should rely on numerical simulations

[9] Because of credit risk capital requirements, banks should choose a level B-D larger than CRC. But, banks want to minimize B-D because the cost of capital is larger than the return on internal funds. Therefore, the assumption that B-D=CRC seems reasonable.

manager if it's capital falls below zero or by the regulator if an audit reveals its incompliance with operational risk capital requirements. We also assume limited liability of the bank and consider bank's asset B to be normalized to one. Henceforwards, cash reserves parameter m should be interpreted as the ratio of the book value of excess bank's capital (with respect to required credit risk capital) to the total illiquid asset.

The bank's manager has control over the dividend payments. A dividend policy L_t represents the cumulated dividends payed up to time t. L is an *adapted* right-continuous non decreasing process with $0 \leq \Delta L_t \leq m_t$ for all $t \geq 0$ $P - a.s.$ This condition states that the manager cannot pay an amount of dividends larger than cash reserves. We denote Π the set of all admissible dividend and effort strategies.

Under the manager's control, the cash reserves evolve according to

$$(3.2) \qquad dm_t^{L,\, e} = (\mu - e_t.c)dt + \sigma dw_t - \lambda(e_t)dN_t Y_{N_t} - dL_t,$$

We consider that the bank's manager acts in the interest of shareholders. She chooses a dividend policy and the effort level to maximize the expected value of future dividend payments. We denote by v the optimal value function,

$$(3.3) \qquad v(m) = \sup_{(L,\, e) \in \Pi} \mathbb{E} \int_0^{\tau(L,e)} e^{-\rho t} dL_t / m_0 = m,$$

where ρ is the discount factor and $\tau_L = \inf\{t/m_t \leq I_{\{A_t=1\}} ORC\}$ is the bankruptcy time.

Let us introduce the following operators

(3.4)

$$A(e)v(m) = \frac{1}{2}\sigma^2 v''(m) + (\mu - ec)v'(m) - (\rho + \psi I_{\{m \leq ORC\}})v(m) + \lambda(e)\delta \int_0^{+\infty} (v(m-y) - v(m)) \exp(-\delta y) dy.$$

and

$$(3.5) \quad DV(m) = A(1)V(m) - A(0)V(m) = \Delta\lambda\delta \int_0^{+\infty} (V(m) - V(m-y)) \exp(-\delta y) dy - cV'(m).$$

Function DV measures the benefits from the efforts. They are equal to the gain from the risk-exposure reduction net of the efforts cost multiplied by the marginal value of cash.

Using stochastic optimal control techniques, we can show that if the optimal value function v is C^2, then it satisfies the following HJB equation:

$$(3.6) \qquad Max(A(0)v, A(1)v, 1 - v') = 0,$$

with $v(0) = 0$.

However, finding a solution of (3.6) does not guarantee that it is the value function. The following lemma gives necessary conditions for optimality. The proof is omitted, since it relies on standard verification techniques.

Lemma 3.1. *Let V a twice continuously differentiable concave solution of (3.4) with bounded first derivative then V is the optimal value function.*

The following result will be useful.[10] Consider the following integro-differential equation

$$(3.7) \qquad \frac{1}{2}\sigma^2 V''(m) + \mu V'(m) - (\rho + \lambda)V(m) + \lambda\delta \int_0^{+\infty} (V(m-y))\exp(-\delta y)dy = 0.$$

Differentiating this equation yields

$$(3.8) \qquad \frac{1}{2}\sigma^2 V'''(m) + \mu V'' - (\rho + \lambda)V'(m) - \lambda\delta^2 \int_0^{+\infty} (V(m-y))\exp(-\delta y)dy + V(m)) = 0.$$

Combining these equalities, we obtain

$$(3.9) \qquad \frac{1}{2}\sigma^2 V'''(m) + (\mu + \frac{1}{2}\delta\sigma^2)V''(m) - (\delta\mu - (\rho + \lambda)V'(m) - \delta\rho V(m) = 0.$$

We define the polynomial

$$(3.10) \qquad P_{(\mu,\rho,\lambda)}(d) = \frac{1}{2}\sigma^2 d^3 + (\mu + \frac{1}{2}\delta\sigma^2)d^2 + (\mu\delta - (\rho + \lambda))d - \delta\rho.$$

---

[10]This result is derived in Belhaj (2010b).

This polynomial satisfies the following conditions:

$$\lim_{\theta \to -\infty} P(\theta) = -\infty, \; P(-\delta) = \delta\lambda > 0, \; P(0) = -\delta\rho < 0, \text{ and } \lim_{\theta \to +\infty} P(\theta) = +\infty.$$

Thus, it has three real zeros d_1, d_2, and d_3 such that $d_1 < -\delta < d_2 < 0 < d_3$. In such a way, solutions of eq (3.9) can be written as linear combinations of $e^{d_1 m}$, $e^{d_2 m}$ and $e^{d_3 m}$.

Solutions of $A(0)V(m) = 0$ with $V(0) = 0$ are proportional to the following function

$$(3.11) \qquad\qquad f(m) = a_1 e^{\theta_1 m} + a_2 e^{\theta_2 m} + a_3 e^{\theta_3 m},$$

where θ_1, θ_2, and θ_3 are the three reals zeros of $P_{(\mu, \rho + \psi, \overline{\lambda})}$, $a_1 = \frac{1}{2}\sigma^2(\theta_2^2 - \theta_3^2) + \mu(\theta_2 - \theta_3)$, $a_2 = \frac{1}{2}\sigma^2(\theta_1^2 - \theta_3^2) + \mu(\theta_1 - \theta_3)$, and $a_3 = \frac{1}{2}\sigma^2(\theta_2^2 - \theta_1^2) + \mu(\theta_2 - \theta_1)$. We will see later on that for small cash reserves, the value function is proportional to f.

Solutions of $A(1)V(m) = 0$ are linear combinations of $e^{\alpha_1 m}$, $e^{\alpha_2 m}$, and $e^{\alpha_3 m}$, where α_1, α_2, and α_3 are the three real zeros of $P_{(\mu - c, \rho, \underline{\lambda})}$.

The following two lemmas will be useful for the purpose of our analysis.

Lemma 3.2. *The equation $DU(m) = 0$, has at most one solution if U is a C^2 increasing and concave function.*

Let $\bar{m} = \inf\{m \geq 0, f''(m) = 0\}$. This level corresponds to the optimal dividend barrier of an unregulated bank that has no control over its operational risk exposure.

Lemma 3.3. *The equation $Df(m) = 0$ with $m \in [0, \bar{m}]$, has exactly one solution.*

The previous two lemma give us an idea about the optimal exercise of effort. That is, if the value function is C^2 and concave, it is optimal to exert effort only for cash reserves larger than a critical level.

4. No Operational Risk Capital Charge

In this section, we will analyze the bank's optimal policies when there is no operational risk capital requirements ($ORC = 0$). Bank is liquidated only when the level of cash reserves falls below

zero. This problem has been studied by Belhaj (2010b) for $c = \frac{\bar{\lambda}}{\delta}$ and $\underline{\lambda} = 0$, i.e, the effort fully

eliminates operational risk. Here we extend his results considering $\underline{\lambda} \geq 0$ and for $c \leq \frac{\Delta \lambda}{\delta}$.

First of all, note that it is not optimal to exert effort near liquidation threshold, since $Dv(0) = -\frac{\lambda}{\delta} v'(0) < 0$. Now, let us define f_0 as in eq (3.11) with $\psi = 0$ and let m_e be the solution of

$Df_0(m) = 0$. At the threshold m_e, the costs of effort is equal to its benefits. Now, consider the

following strategy: the bank pays dividends in excess of a level m^* and exerts effort only when cash

reserves become larger than m_e.

The corresponding value function V is the solution of

$$(4.1) \qquad \begin{cases} A(0)V(m) = 0 \ for \ m \leq m_e \\[2mm] A(1)V(m) = 0 \ for \ m \in [m_e, m^*] \\[2mm] V'(m) = 1 \ for \ m \geq m^*. \end{cases}$$

The solution of (4.1) can be written as

$$(4.2) \qquad V(m) = \begin{cases} \frac{f_0(m)}{g_0'(m^*)} \ for \ m \in [0, \ m_e] \\[2mm] \frac{g_0(m)}{g_0'(m^*)} \ for \ m \in]m_e, \ m^*] \\[2mm] \frac{g_0(m^*)}{g_0'(m^*)} + m - m^* \ for \ m \geq m^*, \end{cases}$$

with

$$(4.3) \qquad g_0(m) = b_1 e^{\alpha_1 m} + b_2 e^{\alpha_2 m} + b_3 e^{\alpha_3 m}.$$

The parameters b_1, b_2, and b_3 are chosen in order to ensure that V is C^2, that is $g_0(m_e) = f_0(m_e)$,

$g_0'(m_e) = f_0'(m_e)$, and $g_0''(m_e) = f_0''(m_e)$. Let B and M be two 3×3 matrices with $b_{ij} = \alpha_j^{i-1} e^{\alpha_j m_e}$

and $m_{ij} = \theta_j^{i-1} e^{\theta_j m_e}$. Therefore, we obtain $b = B^{-1} M \times a$.

Optimizing V with respect to the dividend barrier m^* yields $g_0''(m^*) = 0$. The following lemma

ensures that the dividend barrier exists.

Lemma 4.1. *The equation $g_0''(m) = 0$ has at least one solution.*

Now, let $m_0^* = \inf\{m, g_0''(m) = 0\}$ and let V be the corresponding value function. We obtain the following result.

Proposition 4.2. *The function V is the optimal value function. The threshold m_0^* is the optimal dividend policy barrier. And, it is optimal for the bank to exert effort only when the level of cash reserves exceeds the threshold m_e.*

The optimal dividend policy is a barrier strategy. The bank keeps cash inside for small levels of capital and distributes everything in excess of m_0^* as dividends. The bank mitigates its operational risk by holding capital to absorb losses and also by reducing its operational risk exposure. However, only well-capitalized banks $(m > m_e)$ reduce their operational risk exposure. For under-capitalized banks, the marginal value of cash is too high and the bank is more likely to go bankrupt because of credit risk rather than operational risk. Therefore, the bank does not exert effort and prefers to keep cash (instead of finance effort costs) in order to insure itself against downside credit risk.

5. Operational Risk Capital Charges

In the previous section, we have found that, absent operational risk capital charge, banks with small levels of capital have no incentive to reduce their exposure to operational risk . This gives a rationale for operational risk capital requirements. We also have shown that banks optimally hold capital in excess of the required credit capital to absorb future operational and credit losses. Therefore, imposing a capital charge would be counter productive if it does not encourage banks to reduce their operational risk exposure. Here, we adopt the view that operational risk capital charges should be computed in a way that ensures that banks, complying with the regulatory capital requirements (well capitalized banks), will choose to reduce their exposure to operational risk.[11]

---

[11]The same approach has been used in Bhattacharya *et al* (2002) to determine a closure rule that eliminates risk shifting for banks with asset values larger than the required level.

5.1. **Basic Indicator Approach.** First, we start by analyzing the case when operational risk capital is fixed. This means that reducing operational risk exposure does not affect the required capital. This idea is coherent with the BIA. Since bank's capital is unobservable, capital requirements cannot prevent shirking for all banks. In order to prevent shirking for well capitalized banks, the level of operational risk capital charge k should be chosen in such way that the benefits from efforts is larger than its costs for those banks. That is for all $m > k$, $Dv(m) > 0$. Let $k = inf\{m > 0, \ Df(m) = 0\}$ with f defined in equation (3.11). As in Proposition (4.2), we can show that the bank exerts effort only when cash reserves are larger than k and that the optimal value function is given by

$$(5.1) \qquad V(m) = \begin{cases} \dfrac{f(m)}{g'(m_1^*)} \ for \ m \in [0, \ k] \\[2mm] \dfrac{g(m)}{g'(m_1^*)} \ for \ m \in [k, m_1^*] \\[2mm] \dfrac{g(m_1^*)}{g'(m_e^*)} + m - m_1^* \ for \ m \geq m_1^*, \end{cases}$$

with

$$(5.2) \qquad g(m) = c_1 e^{\alpha_1 m} + c_2 e^{\alpha_2 m} + c_3 e^{\alpha_3 m}.$$

Parameters c_1, c_2, and c_3 are chosen in order to ensure that V is C^2, that is $g(k) = f(k)$, $g'(k) = f'(k)$, and $g''(k) = f''(k)$. The level m_1^* is the optimal dividend barrier, it is defined as the $\inf\{m > 0, g''(m) = 0\}$. Technically, compared to the previous section, the discount rate will be increased by the amount ψ in the region $[0, \ k]$. We obtain the following:

Proposition 5.1. *The level k is the minimal operational risk capital charge ensuring that well capitalized banks always exert effort.*

Note that operational risk capital requirements cannot induce under capitalized banks to reduce their operational risk exposure since, when bank capital falls below the level k, the marginal value of internal funds becomes larger than the benefits of operational risk reduction.[12]. The point is

---

[12]Dangl and Lehar (2004) find that regulatory capital requirements do not prevent distressed banks from increasing their asset risk.

that operational risk capital charge artificially increases the discount rate in the region $[0, \ k]$ by the amount ψ. The latter means that the bank becomes more impatient, discounting future cash at a higher rate. This decreases both the marginal value of internal cash and the benefits from effort.

Proposition (5.1) allows us to analyze the effect of an increase of the operational risk capital requirements on bank's behavior. If the initial required capital is less than k, an increase of its level to k would make the banking industry more stable, i.e, surviving banks will be less exposed to large losses. However, when required capital is larger than k, the bank always starts exerting efforts at the level k (since we still have $Df(k_1) = 0$). Thereby, the rise of operational risk capital requirements will increase the probability of failure and will have no effect on bank's operational risk exposure. Hence, the level k seems to be the best candidate for operational risk capital charge from an incentive perspective.

However, an important question is raised. Does operational risk capital charge really creates incentive to exert effort under BIA? In other words, is the level k less than m_e (the level at which a bank would start exerting effort in the absence of operational risk capital requirements)? Numerical simulations shows that k is larger than m_e. This means that an unregulated bank operates better than a BIA-regulated one. Therefore, this approach can be seen as counterproductive, since it does not really create incentives to exert effort. Under BIA, banks will consider operational risk capital charge as an extra cost.

5.2. **Advanced Measurement Approach.** So far, we have considered that operational risk capital requirements is independent from the bank's operational risk exposure. Under AMA, the regulator can adjust operational risk capital charge to the bank's operational risk exposure. He rewards goods banks, that make risk-preventing effort, by imposing on them less capital charge. Let ORC_1 be the operational risk charge for good banks (e=1) and ORC_0 be the operational risk capital charge for bad banks. It is important to note that ORC_1 should be less than than k, otherwise there is no need to adjust the required capital to bank's operational risk exposure.

The gain from effort is now given by

$$(5.3) \qquad \Delta V(m) = DV(m) + I_{\{m \in [ORC_1, \ ORC_0] \text{ and } e=1\}} \psi V(m)$$

By exerting effort in the region $[ORC_1, \ ORC_0]$, good banks obtain an extra gain measured by ψV.

Assume that ORC_0 is sufficiently large. Let $k_1 = \inf\{m > 0, \ \Delta f(m) = 0\} = \inf\{m > 0, Df(m) = -\psi f(m)\}$. Note that $k_1 \in]0, k[$ since $\Delta f(0) < 0$ and $\Delta f(k) > 0$. We can show that, if $ORC_1 = k_1$, the bank exerts effort only for m larger than k_1, and chooses its dividend barrier policy at the level m_2^*. Consequently, the value function is given by

$$(5.4) \qquad V(m) = \begin{cases} \dfrac{f(m)}{h'(m_2^*)} \ for \ m \in [0, \ k_1] \\[2mm] \dfrac{h(m)}{h'(m_2^*)} \ for \ m \in]k_1, \ m_2^*] \\[2mm] \dfrac{h(m_2^*)}{h'(m_2^*)} + m - m_2^* \ for \ m \geq m_2^*, \end{cases}$$

with

$$(5.5) \qquad h(m) = h_1 e^{\alpha_1 m} + h_2 e^{\alpha_2 m} + h_3 e^{\alpha_3 m}.$$

The parameters h_1, h_2, and h_3 are chosen in order to insure that $h(k_1) = f(k_1)$, $h'(k_1) = f'(k_1)$, and $h''(k_1) = f''(k_1)$. The level m_2^* is defined as the $\inf\{m > 0, h''(m) = 0\}$.

We obtain the following:

Proposition 5.2. *The capital charge k_1 is the minimal operational risk capital charge for good banks ensuring that well capitalized banks always exert efforts.*

The level k_1 is less than k since $Df(k_1) < 0$. This result is due to the adjustment of the required capital to the bank's operational risk exposure. In that way, the regulator lowers the required capital for good banks with respect to required capital computed under BIA. Thus, setting operational risk charges for good banks equal to k_1 ensures that all banks with capital levels larger than k_1 will exert effort. However, since bank's effort is not observable, capital requirements cannot give full

incentives for banks to always reduce their operational risk exposure , i.e, under-capitalized banks have no incentives to monitor their operational risk.

Now we discuss the choice of the level ORC_0. Some shirking incentives may appear for levels of capital larger than ORC_0 since the extra gain ψV vanishes. The level ORC_0 has to be chosen in order to prevent these shirking incentives. In other words, the difference between ORC_0 and ORC_1 should be sufficiently large in order to ensure that the gain from the effort at ORC_0^+ is positive, $DV(ORC_0) \geq 0$. We have that $Dh(k_1) = Df(k_1) < 0$, $Dh(m_2^*) > 0$, and h is concave in the interval $]k_1, m_2^*[$. Hence, by lemma 3.2, the equation $Dh(m) = 0$ has exactly one solution k_0 in $]k_1, m_2^*[$. We obtain the following straightforward result.

Proposition 5.3. *The capital charge k_0 is the minimal operational risk capital charge for bad banks ensuring that well capitalized banks always exert effort.*

Differentiating operational risk capital requirement in accordance with risk exposure, the regulator provides banks a real incentive to monitor their operational risk. The regulator rewards good banks by imposing less capital charge on theme. Note that no banks would be closed for a cash reserves larger than k_1, since the adequate choice of operational risk capital charges will induce good behavior when the level of cash exceeds k_1. It is worthwhile to note that the choice of k_0 is not crucial here. It is sufficient to close the bank if an audit reveals that it does not exert effort. However, in that case, the regulator needs to know k_0 in order to avoid unnecessary audit of bank's effort if an audit reveals that the bank's capital level is larger than k_0. To summarize, the best candidates for operational risk capital requirements satisfy $\Delta f(k_1) = 0$ and $Dh(k_0) = 0$.

The following example illustrates that capital requirements under AMA may create incentives to reduce risk exposure, whereas BIA is counterproductive. The level of capital requirements k is computed under BIA and k_1 is computed under AMA. Both of these capital requirements ensure that well-capitalized banks always exert operational risk preventive effort.

Example: $\mu = 0.01$, $\delta = 1000$, $c = 0.001$, $\overline{\lambda} = 2$, $\underline{\lambda} = 0$, $\sigma = 0.0035$, $\rho = 0.05$.

ψ	0 (no ORC)	0.5	1	2	5	6
k	4.45%	4.47%	4.50%	4.57%	4.78%	4.86%
k_1	4.45%	3.51%	2.93%	2.23%	1.32%	1.16%

In this example, expected operational losses represent 20% of expected cash flow. In the absence of operational risk capital requirements, the bank starts exerting effort at the 4.45% level of excess capital to total illiquid asset. We remark that k is slowly increasing with the frequency of audit, whereas k_1 is decreasing with the level of audit .

6. Conclusion

This paper develops a framework to analyze operational risk capital requirements. Under-capitalized banks have no incentives to monitor their operational risk. Operational risk charge can be computed in order to enhance banks' incentives to manage operational risk. Under BIA, operational risk capital is counterproductive. Aligning the required capital with bank's risk is more efficient in reducing the moral hazard problem. This work can be straightforwardly extended to cover a richer setting. We can allow for the remuneration of cash reserves, other distribution of jumps and the introduction of two types of operational risk; one that can be controlled and the other not. We can also introduce fixed recapitalization costs. However, it is not not possible to obtain closed form solutions for the value function. Therefore we should rely on numerical solutions.

References

1. Barth J.R., 2004. Bank regulation and supervision: what works best?. *Journal of Financial Intermediation, vol 13(2), 205-248.*

2. Belhaj M., 2010. Excess Capital, Operational Risk, and Capital Requirements for Banks, *Forthcoming in quantitative Finance.*

3. Belhaj M., 2010. Optimal dividend policy when cash reserves follow a jump-diffusion process, *Mathematical Finance, Vol. 20, Issue 2, pp. 313-325, April 2010*

4. Bhattacharya, S. , Plank, M., Zechner, J., and G. Strobl, 2002. Bank capital regulation with random audits. *Journal of Economic Dynamics and Control (26) 1301-1321.*

5. Blum J., 1999. Do capital adequacy requirements reduce risks in banking?. *Journal of Banking and Finance, 755-771.*

6. Choulli, T., Taksar, M., and X.Y. Zhou (2000). A Diffusion Model for Optimal Dividend Distribution for a Company with Constraints on Risk Control. *SIAM Journal on Control and Optimization, 41 (6), 1946-1979.*

7. Dangl T. and A. Lehar, 2004. Basel accord vs building block regulation in banking. *Journal of Financial Intermediation, 13(2), 96-131.*

8. Decamps, J.P. , Rochet, J.C. and B. Roger , 2004. The three pillars of basel II: optimizing the mix. *Journal of Financial Intermediation, vol. 13(2), 132-155.*

9. De Fontnouvelle, P, De Jesus-Rueff, V., Jordan, J. S. and E. S. Rosengren, 2003. Capital and Risk: New Evidence on Implications of Large Operational Losses. *Journal of Money, Credit and Banking, Blackwell Publishing, vol. 38(7), 1819-1846.*

10. Diamond, D. and Dybwig, P. (1983), Bank runs, deposit insurance, and liquidity. *Journal of Political Economy, 91, 401-419.*

11. Fries, S., Mella-Barral, P., and, W. Perraudin, 1997. Optimal bank reorganization and the fair pricing of deposit guarantees. *Journal of Banking and Finance, 441-468.*

12. Froot K. A. and J. C. Stein, 1998. Risk management, capital budgeting and capital structure policy for financial institutions: an integrated approach. *Journal of Financial Economics, 47, 55-82.*

13. Jaffe, D.M. and T. Russell (1997). Catastrophe Insurance, Capital Markets and Uninsurable Risks. *Journal of Risk and Insurance, 64, 205 - 230.*

14. Hoojgaard, B. and M. Taksar (2004). Optimal Dynamic Portfolio Selection for a Corporation with Controllable Risk and Dividend Distribution Policy. *Quantitative Finance, 3, 315-327.*

15. Hellmann, T. F. and K.C. Murdock, 2000. Liberalization, moral hazard in banking, and prudential regulation: are capital requirements enough?. *The Americain Economic Review, 147-164.*

16. Hovakimian A. and E J. Kane, 2000. Effectiveness of capital regulation at US commercial banks. *Journal of Finance, vol 55(1), 451-468.*

17. Jeanblanc, P. and A.N Shiryaev, 1995. Optimization of the flow of dividends. *Russian Mathematical Surveys, 50, 257-277.*

18. Merton, R., 1977. An analytic derivation of the cost of deposit insurance and loan guarantees: an application of modern option pricing theory. *Journal of Banking and Finance, 1, 3-11.*

19. Merton, R., 1978. On the cost of deposit insurance when there are surveillance costs. *Journal of Business, 51, 439-452.*

20. Milne, A. and D. Robertson, 1996. Firm behavior under the threat of liquidation. *Journal of Economic Dynamics and Control, 20(8), 1427-1449.*

21. Milne A. and Whalley A E , 2001. Bank capital regulation and incentives for risk-taking. *Bank of England working paper no 90.*

22. Opler T., Pinkowitz, L., Stulz, R., and R. Williamson (1999). The Determinants and Implications of Corporate Cash Holdings. *Journal of Financial Economics, 52, 3-46.*

23. Peura, S. and J. Keppo (2006). Optimal bank capital with costly recapitalization. *The Journal of Business,79, 2163-2201 .*

24. Prescot E. S., 2001. Regulating bank capital structure to control risk. *Economic Quarterly, 35-52.*

25. Radner, R. and L. Shepp (1996). Risk vs. profit potential: A model for corporate strategy. *Journal of Economic Dynamics and Control, 20, 1373-1393.*

26. Rochet J.C. and S. Villeneuve, 2010. Liquidity risk and corporate demand for hedging and insurance. *IDEI Working Paper, n254, mai 2004.*

27. Santos, J , 2001. Bank capital regulation in contemporary theory: a review of the literature. *Financial Markets, Institutions and Instruments, 41-84.*

28. Stolz, S, 2002, The relationship between bank capital, risk-taking, and capital regulation: a review of the literature, Kiel Working Paper No. 1105

7. Appendix

Proof of lemma 3.2 Let U a C^2 increasing and concave function. We have $(DU)'(m) = -\delta DU(m) + (\Delta\lambda - \delta c)U'(m) - cU''(m)$. As we have $\Delta\lambda - \delta c \geq 0$ then we obtain that if $DU(m) = 0$ then $(DU)'(m) > 0$. This means that the equation $DU(m) = 0$ has at most one solution.

Proof of lemma 3.3 Define $V(m) = \frac{f(m)}{f'(\bar{m})}$ for $m \leq \bar{m}$. The function V is concave[13] and satisfies the following properties $V''(\bar{m}) = 0$ and $V'(\bar{m}) = 1$. We have $\mu - (\rho + \psi)V(\bar{m}) + \lambda\delta \int_0^{+\infty}(V(m - y) - V(\bar{m}))\exp(-\delta y)dy = 0$, then $DV(\bar{m}) = \Delta\lambda\frac{-(\rho+\psi)V(\bar{m})+\mu}{\overline{\lambda}} - c$. Since $c \leq \frac{\Delta\lambda}{\delta}$, we obtain $DV(\bar{m}) \geq \Delta\lambda\frac{-(\rho+\psi)V(\bar{m})+\mu-\frac{\overline{\lambda}}{\delta}}{\overline{\lambda}} > 0$. As V is concave and $V'(m) < 1$ for $m < \bar{m}$, we can easily show that $(\rho + \psi)V(\bar{m}) < \mu - \frac{\overline{\lambda}}{\delta}$. Therefore, we obtain $DV(\bar{m}) > 0$. We also have $DV(0) = -\frac{\lambda}{\delta}V_0'(0) < 0$. As DV is continuous, there is $m_e \in [0, \bar{m}]$ such that $DV(m_e) = 0$ (or $Df(m_e) = 0$. Note that f is C^2, increasing, and concave in $[0, \bar{m}]$. Using lemma 3.2, we obtain that the equation $Df(m) = 0$ with $m \in [0, \bar{m}]$, has exactly one solution.

Proof of lemma 4.1 Consider the barrier policy at $\tilde{m} = \frac{\mu-c}{\rho+\psi}$, and assume that for all $m < \tilde{m}$, $g_0''(m) < 0$. Then the corresponding value function $V(m)$ (defined similarly as in eq (4.2)) is concave. Since $A(1)V(\tilde{m}) = 0$, then we have $\mu - c - (\rho + \psi + \lambda)V(m) + \lambda\delta \int_0^{+\infty}(V(m - y))\exp(-\delta y)dy = 0$. We obtain that $V(\tilde{m}) < \tilde{m}$. This contradicts $V'(m) > 1$ for $m < \tilde{m}$.

Proof of Proposition 4.2 First, we show that it is optimal to exert effort only for cash reserves larger than m_e. Note that V is twice continuously differentiable and concave by construction. By Lemma (3.2) and (3.3), we have that $DV(m) = 0$ has only one solution. Since $DV(0) < 0$, it follows that $A(0)V(m) > A(1)V(m)$ for $m < m_e$, and $A(0)V(m) < A(1)V(m)$ for $m > m_e$.

---

[13]V is the optimal value function when the bank has no control over its operational risk.

Second, we show that the barrier policy at m^* is optimal.

For $m > m^*$, we have $V'(m) = 1$. Then, we need to show that the function V satisfies $A(1)V(m) \leq 0$ for $m > m^*$.

Let $m > m^*$, we have $V(m) = V(m^*) + m - m^*$ and $A(1)V(m) = \mu - c - (\rho + \underline{\lambda})V(m) + \underline{\lambda}\delta \int_0^{+\infty}(V(m-y)\exp(-\delta y)dy$.

We differentiate the equation above, we obtain

$A(1)V'(m) = -(\rho + \underline{\lambda}) + \underline{\lambda}\delta \int_0^m (V'(m-y)\exp(-\delta y)dy$.

We use integration by part, we find

$A(1)V'(m) = -(\rho + \underline{\lambda}) + \underline{\lambda}\delta(V(m) - \delta \int_0^m V(m-y)\exp(-\delta y)dy)$.

Combining these two equations we obtain

$A(1)V'(m) + \delta A(1)V(m) = \delta(\mu - c) - (\rho + \underline{\lambda}) - \delta\rho V(m)$

Since $A(1)V(m^*) = 0$, we obtain $A(1)V'(m) + \delta A(1)V(m) = A(1)V'(m^*) - \delta\rho(m - m^*)$.

Using equation (3.9) (we replace μ by $\mu - c$ and ρ by $\rho + \psi$ in eq (3.9)), we obtain $A(1)V'(m^*) = \frac{1}{2}\sigma^2 V'''((m^*)^-) < 0$. Therefore we have $A(1)V'(m) + \delta A(1)V(m) < 0$ and $A(1)V'(m^*) < 0$. It can be easily shown (by contradiction) that $A(1)V(m) < 0$ for all $m > m^*$.

Proof proposition 5.1 Let z the smallest level at which the bank exerts effort. We have that the optimal value function v is proportional to the function f in the interval $[0, min(z, ORC)]$.

First case: $\bar{m} > ORC > k$, as f is concave and $Df(k) = 0$, then we obtain $Df(m) < 0$ for all $m < k$. Since the value function is concave, we obtain by lemma 3.2 that $Dv(m) > 0$ iff $m > k$. That is the bank exerts effort only for cash reserves larger than k.

Second case: $ORC < k$, we have $Df(ORC) < 0$ or equivalently $Dv(ORC) < 0$. Therefore, the bank does not exert effort in the neighborhoods of ORC.

Proof proposition 5.2

Let z the smallest level at which the bank exerts effort. We distinguish between two cases:

First case: $k > ORC_1 \geq k_1$; We will show that $z = ORC_1$. We have that, for all $m \leq ORC_1$, the gain form effort is given by $Dv(m) < 0$ ($Df(m) < 0$ since $m \leq ORC_1 < k$). Therefore we obtain

$z \geq ORC_1$. As we have $ORC_1 > k_1$ and $\Delta f(k_1) = 0$, applying lemma **??** (f is concave in the interval [0, k]) we obtain that $\Delta f(ORC_1) > 0$ or equivalently $\Delta v(ORC_1) > 0$. Therefore, the bank starts exerting effort at the level ORC_1.

Second case: $ORC_1 < k_1$; For all $m \leq ORC_1$ the gain form effort is given by $Dv(m) < 0$ since $ORC_1 < k_1$. Therefore, we obtain $z > ORC_1$.