Category-based Tail Comovement

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Abstract

Traditional financial theory predicts that comovement in asset returns is due to fundamentals. An alternative view is that of Barberis and Shleifer (2003) and Barberis, Shleifer and Wurgler (2005) who propose a sentiment based theory of comovement, delinking it from fundamentals. In their paper they view comovement under the prism of the standard Pearson's correlation measure, implicitly excluding extreme market events, such as the latest financial crisis. Poon, Rockinger and Tawn (2004) have shown that under such events different types of comovement or dependence may co-exist, and make a clear distinction between the four types of dependence: perfect dependent, independent, asymptotically dependent and asymptotically independent. In this paper we extend the sentiment based theory of comovement so as to cover the whole spectrum of dependence, including extreme comovement such as the one that can be observed in financial crises. One of the key contributions of this paper is that it formally proves that assets belonging to the same category comove too much in the tail and reclassifying an asset into a new category raises its tail dependence with that category.

JEL classification: G12

1 Introduction

During the unfolding financial crisis triggered by the US subprime loan failure, stocks have experienced both extreme movement and comovement in returns. While comovement between asset returns has been generally measured using Pearson's correlation, extreme comovement has been addressed using various statistical measures. For example, using multivariate extreme value theory (EVT), Longin and Solnik (2001) derive extreme correlations for various equity return distributions internationally. Hartmann, Straetmans and de Vries (2004) rely on tail dependence for bivariate linkages between equity and government bond markets in the G-5 industrial countries during market turmoil. While the abovementioned papers and the extant literature implicitly assume asymptotic dependence because of the use of EVT, Poon, Rockinger and Tawn (2004) point out that this is a very simplifying assumption that can lead to estimation errors. They derive a general multivariate framework with two types of extreme value dependence structures that allow for both asymptotic dependence and independence. More specifically they make a clear distinction between the four types of dependence: perfect dependent, independent, asymptotically dependent and asymptotically independent, claiming that "for positively related and asymptotically dependent (independent) variables, large values of each variable will occur simultaneously more often (less often) than if the variables are independent (perfectly dependent)."[1] Most papers in this strand of the literature focus on the statistical measurement of extreme comovement and not on explaining the observed patterns based on a proposition that involves a financial model/theory.

Assuming a frictionless economy and rational investors, traditional financial theory maintains that comovement (extreme comovement) in prices is due to comovement (extreme comovement) in fundamental values. Hence, the reason why the stock prices of a given industry comove is that their earnings and hence their intrinsic values, are related, and when one industry constituent reports good financial results, it is rather likely that other firms in the same industry will too. There is little doubt that under the aforementioned assumptions the traditional fundamentals-based view is able to explain standard patterns of comovement. Nonetheless, in the presence of irrational investors, market imperfections and limits to arbitrage, asset prices and fundamental values become disentangled, rendering traditional theory ineffective and calling for propositions based on behavioral theories of comovement such as investor sentiment. According to Barberis and Shleifer (2003) investors

[1] Bae, Karolyi and Stulz (2003) focus on analyzing the joint occurrences of extreme events using a multinomial logistic model in order to study the propagation of large-return shocks within and across regions.

group information into categories, based on some common characteristic observable across all assets that belong to the same category. Such an approach simplifies the investment process, narrowing down the possibilities and making it much easier to allocate funds across a few categories than among numerous assets. The manifestation of this in financial markets can be found in style investment, where investors group stocks into styles based on some common characteristic that can be related or unrelated to fundamentals and allocate funds at the category level rather than the individual one. Styles are complemented by counter-styles, for example: large cap versus small cap, growth versus value etc. and investors move funds from style to counter style in anticipation of better investment opportunities, that is, styles become in and out-of-favor. If some investors are noise traders with correlated sentiment operating in a market with limited arbitrage, then as they shift funds between categories, they will induce comovement in prices that is not related to fundamentals. For example, if noise traders become irrationally enthusiastic about internet stocks and channel funds into that category driven by sentiment, the common buying pressure will lead to price comovement for such stocks even if their fundamental values are unrelated. Cornell, 2004 used Yahoo and Amazon to illustrate sentiment based comovement related to category fund allocation for two firms with different fundamentals and businesses. A plethora of studies provide evidence that sentiment is correlated across investors and influences comovement in stock prices (see Baker and Wurgler, 2006). Barberis, Shleifer, and Wurgler (2005) provide evidence of style-based comovement associated with the inclusion of stocks in aggregate indices. Finally, Kumar and Lee (2006) have shown that systematic trading by retail investors could lead to stock return comovements beyond the usual risk factors.

This paper complements the previously mentioned literature. More specifically, while the extant literature so far aimed at explaining comovement in normal market states as captured by standard correlation measures, we examine extreme comovement such as the one that can be observed in financial crises. To capture such extreme comovement we employ the two types of tail dependence proposed by Poon, Rockinger and Tawn (2004). More importantly, we take literature further by extending the propositions made by Barberis and Shleifer (2003). Simply speaking, Barberis and Shleifer propose a sentiment based theory of comovement, while we, on the other hand, propose a sentiment based theory of extreme comovement. This is motivated by the fact that during financial crises, sentiment based investment gains prominence, and the tails of the return distribution are gaining importance too. Our contribution can be viewed as complementary to the one of Barberis and Shleifer, as it extends their proposition so as to cover the full spectrum of dependence. More precisely three main contributions have been derived: 1) we formally prove that the category-based comovement model proposed by Barberis and Shleifer (2003) implies that returns are asymptotically independent; 2) we formally prove that the category-based comovement model proposed by Barberis and Shleifer (2003) is also a category-based weak tail-comovement model: in this economy, assets in the same category comove too much in the tail and reclassifying an asset into a new category raises its (weak) tail dependence with that category; 3) we emphasize on the crucial role of the distribution of noise trader sentiment in order to explain both positive correlation and (strong) tail comovement in asset returns. More precisely we show that if the distribution of the change in noise trader sentiment has heavy tails, then assets in the same category comove too much in the tail since tail comonotonicity is found for such assets; and reclassifying an asset into a new category raises its (strong) tail dependence with that category.

The outline of the rest of the paper is as follows. Section 2 introduces . Section 3 details our . In Section ?, we . Section ? concludes and provides directions for future research.

2 Tail Comovement Measures

Quantile dependence is a measure of the dependence in the tails of the distribution. If Z_1 and Z_2 are random variables with distribution functions F_1 and F_2, then there is quantile dependence in the lower tail at threshold α, whenever

$$\mathbb{P}\left(Z_2 \leq F_2^{-1}(\alpha) \mid Z_1 \leq F_1^{-1}(\alpha)\right)$$

is different from zero while there is quantile dependence in the upper tail at threshold α, whenever

$$\mathbb{P}\left(Z_2 \geq F_2^{-1}(\alpha) \mid Z_1 \geq F_1^{-1}(\alpha)\right)$$

is different from zero. Tail dependence obtains as the limit of this probability, as we go arbitrarily far out into the tails. Definitions of tail dependence for multivariate random vectors are mostly related to their bivariate marginal distribution functions. Loosely speaking, tail dependence describes the limiting proportion that one margin exceeds a certain threshold given that the other margin has already exceeded that threshold. The following approach, as provided in the monograph of Joe (1997), represents one of many possible definitions of tail dependence.

The *upper tail dependence* index between Z_1 and Z_2 is defined as

$$\lambda_U = \lim_{\alpha\to 1} \mathbb{P}(Z_1 \geq F_1^{-1}(\alpha) \mid Z_2 \geq F_2^{-1}(\alpha)) = \lim_{\alpha\to 1} \mathbb{P}(Z_2 \geq F_2^{-1}(\alpha) \mid Z_1 \geq F_1^{-1}(\alpha)),$$

and the *lower tail dependence* index is

$$\lambda_L = \lim_{\alpha\to 0} \mathbb{P}(Z_1 \leq F_1^{-1}(\alpha) \mid Z_2 \leq F_2^{-1}(\alpha)) = \lim_{\alpha\to 0} \mathbb{P}(Z_2 \leq F_2^{-1}(\alpha) \mid Z_1 \leq F_1^{-1}(\alpha)).$$

where $0 \leq \lambda \leq 1$, we have that variables are termed *asymptotically dependent* if $\lambda > 0$ and *asymptotically independent* if $\lambda = 0$. However, as pointed out by Poon, Rockinger and Tawn (2004), generally, when $\lambda = 0$ the two random variables are not necessarily exactly independent. Coles, Heffernan and Tawn (1999) have provided a range of extremal dependence models, derived from a different form of multivariate limit theory, that describe dependence but have $\lambda = 0$. Although the random variables are asymptotically independent in this case, different degrees of dependence are attainable at finite levels of α. To this end, they introduce another tail dependence index[2].

The upper tail dependence index between Z_1 and Z_2 is defined as $\eta_U \in (0, 1]$ where

$$\eta_U = \lim_{\alpha\to 1} \frac{\log(1-\alpha)}{\log \mathbb{P}(Z_1 > F_1^{-1}(\alpha), Z_2 > F_2^{-1}(\alpha))},$$

while the lower tail dependence index between Z_1 and Z_2 is defined as $\eta_L \in (0, 1]$ where

$$\eta_L = \lim_{\alpha\to 0} \frac{\log(\alpha)}{\log \mathbb{P}(Z_1 \leq F_1^{-1}(\alpha), Z_2 \leq F_2^{-1}(\alpha))}.$$

Values of $\eta > 0$, $\eta = \frac{1}{2}$ and $\eta < 1$ loosely correspond respectively to when Z_1 and Z_2 are positively associated in the extremes, exactly independent, and negatively associated. $\lambda = 0$ and $\eta \in (0, 1)$ signifies asymptotic independence, in which case the value of η determines the strength of dependence within this class (also known as dependence in independence).

The intuition of this coefficient is the following. If Z_1 and Z_2 are independent (in tails), for α large enough

$$\mathbb{P}(Z_1 > F_1^{-1}(\alpha), Z_2 > F_2^{-1}(\alpha)) = \mathbb{P}(Z_1 > F_1^{-1}(\alpha)) \cdot \mathbb{P}(Z_2 > F_2^{-1}(\alpha)) = (1-\alpha)^2,$$

[2]In Poon, Rockinger & Tawn (2002), λ is denoted χ, while η is $(1+\overline{\chi})/2$. This weak dependence function is closely related to bivariate regular variation (as defined in Resnick (?)).

or equivalently,

$$\log \mathbb{P}(Z_1 > F_1^{-1}(\alpha), Z_2 > F_2^{-1}(\alpha)) = 2\log(1-\alpha),$$

while if Z_1 are comonotonic (in tails), for α large enough

$$\mathbb{P}(Z_1 > F_1^{-1}(\alpha), Z_2 > F_2^{-1}(\alpha)) = \mathbb{P}(Z_1 > F_1^{-1}(\alpha)) = 1-\alpha,$$

or equivalently,

$$\log \mathbb{P}(Z_1 > F_1^{-1}(\alpha), Z_2 > F_2^{-1}(\alpha)) = \log(1-\alpha).$$

Thus, the limit of the ratio $\log P(Z_1 > F_1^{-1}(\alpha), Z_2 > F_2^{-1}(\alpha))/\log(1-\alpha)$ can be seen as a tail dependence measure, with tail independence when the limit is 2 and tail independence when the limit is 1. In order to have an index increasing with the strength of the dependence, it becomes more natural to consider the inverse of that ratio.

3 Tail Comovement in a Category-based Comovement Model

There are a number of interesting stock market patterns that don't fit neatly into the traditional fundamental view of comovement. One example concerns so-called "twin stocks," which are stocks that are claims to the same cash-flow stream, but are primarily traded in different locations. The best-known example is Royal Dutch and Shell. They used to be completely independent companies, but in 1907 they agreed to merge their interests while remaining separate entities. Today, shares of Royal Dutch are traded primarily in the United States and in the Netherlands, and are a claim to 60 percent of the combined firm's cash flow, while Shell shares, traded primarily in the United Kingdom, are a claim to the remaining 40 percent. Since the two shares are claims to exactly the same cash-flow stream, the fundamentals-based view of comovement argues that the prices of the two shares should move in lock-step with one another. In reality, the two stocks seem to have minds of their own-Royal Dutch moves closely with the S&P index, while Shell's movements are closely tied to those of the FTSE index of U.K. stocks. A related example concerns closed-end country funds, whose assets are traded in a different location from the funds themselves. For example, there are closed-end funds invested entirely in German equities, but whose shares trade primarily in New York. Since a closed-end fund and the assets it holds are claims to very similar cash-flow streams, the price of the closed-end fund and the value of its holdings should move together very closely. However, this is not often the case.

Closed-end country funds tend to move more closely with the national market where they are traded than with the national market where their holdings are traded. In this example, a Germany fund would tend to move more closely with the U.S. market even though all its holdings are German equities. Finally, there is strong evidence that small-cap stocks tend to move together, as do value stocks.

The framework developed by Barberis and Shleifer (2003) for analyzing trading based comovement is straightforward. Consider an economy that contains a riskless asset in perfectly elastic supply and with zero rate of return, and also $2n$ risky assets in fixed supply. Risky asset i is a claim to a single liquidating dividend $D_{i,T}$ to be paid at some later time T. This eventual dividend equals

$$D_{i,T} = D_{i,0} + e_{i,1} + ... + e_{i,T}$$

where $D_{i,0}$ and $e_{i,t}$ are announced at time 0 and time t, respectively, and where

$$e_t = (e_{1,t}, ..., e_{2n,t})' \,\tilde{}\, N(0, \Sigma_D)\,,\ i.i.d. \text{ over time}$$

Barberis and Shleifer (2003) assume that the cash-flow covariance matrix Σ_D takes a specific form, although the predictions also hold for more general structures. In particular, we suppose that the cash-flow shock to an asset has three components: a marketwide cash-flow shock, a group-specific cash-flow shock that affects assets in one group but not the other, and a completely idiosyncratic cash-flow shock specific to the asset. Formally, for $i \in X$,

$$e_{i,t} = \psi_M f_{M,t} + \psi_S f_{X,t} + \sqrt{1 - \psi_S^2 - \psi_M^2}\varepsilon_{i,t}$$

and for $j \in Y$,

$$e_{j,t} = \psi_M f_{M,t} + \psi_S f_{Y,t} + \sqrt{1 - \psi_S^2 - \psi_M^2}\varepsilon_{j,t}$$

where $f_{M,t}$ is the market-wide shock, $f_{X,t}$ and $f_{Y,t}$ are the group-specific shocks, and $\varepsilon_{i,t}$ and $\varepsilon_{j,t}$ are idiosyncratic shocks; ψ_M and ψ_S are constants that control the relative importance of the three components. Each shock has unit variance and is orthogonal to the other shocks. The price of a share of risky asset i at time t is $P_{i,t}$ and the return on the asset between time $t-1$ and time t is

$$\Delta P_{i,t} = P_{i,t} - P_{i,t-1}$$

The innovation introduced by Barberis and Shleifer (2003) is to assume that noise traders are attracted to certain groups of assets and that they allocate their funds across those groups rather than at the level of individual assets. For instance, value stocks, small cap stocks and technology stocks could be examples of such groups. If arbitrage is limited, change in noise trader sentiment regarding any one group will lead to price movements that push prices for that group of assets away from their fundamental value. However, this movement, and the subsequent return to fundamental value, are common across all assets in the group. Suppose that, to simplify their decision-making, some investors group the $2n$ risky assets into two categories, X and Y, and then allocate funds at the level of these categories rather than at the individual asset level. In particular, they place assets 1 through n in category X and assets $n+1$ through $2n$ in category Y. A simple representation for asset returns is then

$$\begin{aligned}\Delta P_{i,t} &= P_{i,t} - P_{i,t-1} = e_{i,t} + \Delta u_{X,t}, \text{ for } i = 1, ..., n \\ \Delta P_{i,t} &= P_{i,t} - P_{i,t-1} = e_{i,t} + \Delta u_{Y,t}, \text{ for } i = n+1, ..., 2n\end{aligned}$$

Here, $u_{X,t}$ can be thought of as time t noise trader sentiment about the securities in category X. Since the noise traders allocate funds by category, this sentiment level is the same for all securities in category X. The return on a security in category X is affected not only by news about cash flows, $e_{i,t}$, but also by the change in sentiment about X, $\Delta u_{X,t}$: when noise traders become more bullish about old economy stocks, these stocks go up in price. This model can also be thought of as a reduced-form model for the habitat view of comovement. In this case, X and Y simply have to be reinterpreted as habitats, not categories: instead of representing groups of assets that some investors do not distinguish between when allocating funds, they represent groups of assets that are the sole holdings of some investors. Specifically, we can think of assets 1 through n as U.S. stocks and assets $n+1$ through $2n$ as U.K. stocks; there are many investors in both countries who trade only domestic securities. Under the habitat interpretation, $u_{X,t}$ tracks the risk aversion, sentiment, or liquidity needs of investors who invest only in the securities in X. The return of an asset in habitat X is affected not only by news about cash flows but also by the change in risk aversion, say, of these specific investors.

In the propositions 1, 2 and 3, we will assume that

$$\begin{pmatrix} u_{X,t} \\ u_{Y,t} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \sigma_u^2 \begin{pmatrix} 1 & \rho_u \\ \rho_u & 1 \end{pmatrix}\right) \; i.i.d. \text{ over time}$$

Proposistion 1. Under the category-based comovement model, for two assets i and j we have $\lambda\left(\Delta P_i, \Delta P_j\right) = \lambda\left(e_i, e_j\right) = 0$

Proposistion 2. Under the category-based comovement model, if two assets i and j, $i \neq j$, belong to the same category, $i, j \in X$ or Y, then

$$\eta\left(\Delta P_{i,t}, \Delta P_{j,t}\right) > \eta\left(e_{i,t}, e_{j,t}\right)$$

Proposistion 3. Under the category-based comovement model, suppose that asset j, previously a member of category Y, is reclassified as belonging to X. Then $\eta\left(\Delta P_{j,t}, \Delta P_{X,t}\right)$ increases after j is added to category X where $\Delta P_{X,t} = \frac{1}{n}\sum\limits_{l \in X} \Delta P_{l,t}$.

The intuition is straightforward, whether X and Y are categories, habitats, or groups of stocks that incorporate information at similar rates. When asset j enters category X, it is buffeted by noise traders' flows of funds in and out of that category.

In the propositions 4 and 5, we will assume that u_X has heavy tails.

Proposition 4. Under the category-based comovement model, if two assets i and j, $i \neq j$, belong to the same category, $i, j \in X$ or Y, then

1. if u_X has right-heavy tails

$$1 = \lambda_U\left(\Delta P_{i,t}, \Delta P_{j,t}\right) > \lambda_U\left(e_{i,t}, e_{j,t}\right) = 0$$

2. if u_X has left-heavy tails

$$1 = \lambda_L\left(\Delta P_{i,t}, \Delta P_{j,t}\right) > \lambda_L\left(e_{i,t}, e_{j,t}\right) = 0$$

We thus observe that even if cash-flow shocks are tail independent, ΔP_i and ΔP_j are comonotonic in tails. Indeed if $\lambda = 1$, there is *tail comonotonicity.*[3]

[3] Z_1 and Z_2 are said to be *comonotonic* if there exists φ strictly increasing such that $Z_2 = \varphi(Z_1)$. Then one variable increases iff the other increases with probability one. If Z_1 and Z_2 are finite variances, then the correlation between between Z_1 and Z_2 exists, and it is maximal when Z_1 and Z_2 are *comonotonic* (so called Hoeffding upper bound for correlation).

Proposition 5. Under the category-based comovement model, suppose that asset j, previously a member of category Y, is reclassified as belonging to X. Then

1. if u_X has right-heavy tails, $\lambda_U\left(\Delta P_{j,t}, \Delta P_{X,t}\right)$ increases after j is added to category X where $\Delta P_{X,t} = \frac{1}{n}\sum\limits_{l\in X}\Delta P_{l,t}$.

2. if u_X has left-heavy tails, $\lambda_L\left(\Delta P_{j,t}, \Delta P_{X,t}\right)$ increases after j is added to category X where $\Delta P_{X,t} = \frac{1}{n}\sum\limits_{l\in X}\Delta P_{l,t}$.

4 Conclusion

Traditional financial theory predicts that comovement in asset returns is due to fundamentals. An alternative view is that of Barberis and Shelifer (2003) and Barberis, Shelifer and Wurgler (2005) who propose a sentiment based theory of comovement, delinking it from fundamentals. In their paper they view comovement under the prism of the standard Pearson's correlation measure, implicitly excluding extreme market events, such as the latest financial crisis. Poon, Rockinger and Tawn (2004) have shown that under such events different types of comovement or dependence may co-exist, and make a clear distinction between the four types of dependence: perfect dependent, independent, asymptotically dependent and asymptotically independent. In this paper we extend the sentiment based theory of comovement so as to cover the whole spectrum of dependence, including extreme comovement such as the one that can be observed in financial crises. One of the key contributions of this paper is that it formally proves that assets belonging to the same category comove too much in the tail and reclassifying an asset into a new category raises its tail dependence with that category.

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5 Annex A

Proof of Proposition 1: Since $\boldsymbol{e} = (e_{i,t}, e_{j,t})$ is a Gaussian random vector, independent of $\Delta u_{X,t}$ which is also Gaussian, $\boldsymbol{\Delta P} = \boldsymbol{e} + \Delta u_X$ is also a Gaussian random vector: since $\varepsilon_{i.}$ and u_X are independent. Strong tail dependence is null for non comonotonic Gaussian random vectors, i.e.

$$\lambda(\boldsymbol{\Delta P}) = \begin{cases} 0 \text{ if } corr(\boldsymbol{\Delta P}) \in [-1, +1), \\ 1 \text{ if } corr(\boldsymbol{\Delta P}) = +1. \end{cases}$$

thus, $\lambda(\Delta P_{i,t}, \Delta P_{j,t}) = 0 = \lambda(e_{i,t}, e_{j,t})$. This finishes the proof of Proposition 1.

We now prove the following lemma, which will be useful in the proofs 2 and 3.

Lemma. If $X = (X_1, X_2)$ has a Gaussian distribution with correlation ρ, then $\eta_U = \eta_L = (1 + \rho)/2$.

Proof of Lemma. η_U is the upper tail dependence index of (X_1, X_2) if

$$\mathbb{P}(Z_1 > z_1, Z_2 > Z_2) \sim [z_1 z_2]^{-1/2\eta} \mathcal{L}(z_1, z_2)$$

if Z_1 and Z_2 have unit Fréchet distributions, and where $\mathcal{L}$ is bivariate slowly varying function, i.e.

$$\lim_{t\to\infty} \frac{\mathcal{L}(tz_1, tz_2)}{\mathcal{L}(t,t)} = h\left(\frac{z_1}{z_1 + z_2}\right),$$

for some function h defined on $[0, 1]$ such that $h(1/2) = 1$.

The expression of the weak tail dependence index for Gaussian random vectors can be obtain heuristically as follows. For the bivariate normal distribution, an asymptotic development of Mills ratio gives

$$\frac{\mathbb{P}(X > x, Y > y)}{\phi(x, y; \rho)} \sim \frac{(1 - \rho^2)^2}{(x - \rho y)(y - \rho x)}.$$

Set $U = -1/\log\Phi(X)$, $V = -1/\log\Phi(Y)$, and $u = -1/\log\Phi(x)$, $v = -1/\log\Phi(y)$ (so that U and V have unit Fréchet distributions),

$$\mathbb{P}(U > u, V > v) \sim \phi(\Phi^{-1}(e^{-u^{-1}}), \Phi^{-1}(e^{-v^{-1}}); \rho)\frac{(1 - \rho^2)^2}{(x - \rho y)(y - \rho x)}$$

since $x = \Phi^{-1}(e^{-u^{-1}})$ and $y = \Phi^{-1}(e^{-v^{-1}})$. A quick limit development yields

$$x \sim \Phi^{-1}(1-u^{-1}) \sim (2\log u)^{1/2},$$

thus

$$\begin{aligned}
\phi(\Phi^{-1}(e^{-u^{-1}}),\Phi^{-1}(e^{-v^{-1}});\rho) &\sim \phi((2\log u)^{1/2},(2\log v)^{1/2};\rho) \\
&\propto \exp\left(-\frac{(2\log u)+2\rho\sqrt{(2\log u)(2\log v)}+(2\log v)}{2(1+\rho^2)}\right) \\
&= [uv]^{1/(1+\rho^2)}\exp\left(\frac{2\rho\sqrt{(2\log u)(2\log v)}}{2(1+\rho^2)}\right)
\end{aligned}$$

where function on the right is slowly varying.

Thus

$$\mathbb{P}(U>u,V>v) = [uv]^{-1/(1+\rho^2)}\mathcal{L}(u,v),$$

where it can be proved that $\mathcal{L}(u,v)$ is a slowly varying function. $>$From the definition of η the power value is necessarily $1/2\eta$, and thus, for a Gaussian random vector

$$\eta = \frac{1+\rho}{2}.$$

Proof of Propositions 2. Since $\boldsymbol{e} = (e_{i,t}, e_{j,t})$ is a Gaussian random vector, independent of $\Delta u_{X,t}$ which is also Gaussian, $\boldsymbol{\Delta P} = \boldsymbol{e} + \Delta u_X$ is also a Gaussian random vector. From the above lemma,

$$\eta_U(\Delta P_{i,t},\Delta P_{j,t}) = \eta_L(\Delta P_{i,t},\Delta P_{j,t}) = \frac{1+corr(\Delta P_{i,t},\Delta P_{j,t})}{2} \text{ for } i,j\in X$$

Here,

$$cov(\Delta P_i,\Delta P_j) = cov(e_i,e_j)+\sigma_u^2$$

and

$$var(\Delta P_i) = var(e_i)+\sigma_u^2.$$

The proposition therefore follows if.

$$cov(e_i,e_j) < var(e_i)$$

Using

$$e_{i,t} = \psi_M f_{M,t} + \psi_S f_{X,t} + \sqrt{1 - \psi_S^2 - \psi_M^2} \varepsilon_{i,t}$$

it is easily checked that

$$\begin{aligned} cov(e_i, e_j) &= \psi_S^2 + \psi_M^2 < 1 \\ var\,(e_i) &= 1 \end{aligned}$$

which means that inequality $corr(\boldsymbol{\Delta P}) > corr(\boldsymbol{\varepsilon})$ does indeed hold and thus following the above lemme that the proposition also hold.

Function h is said to be regularly varying at ∞ with index $\alpha \in \mathbb{R}$, denoted $h \in RV_\alpha^\infty$ if

$$\lim_{t \to \infty} \frac{h(tx)}{h(t)} = x^{-\alpha}, \text{ for all } x > 0.$$

If $\alpha = 0$, then h is said to be *slowly varying*. If $\alpha = +\infty$, then h is said to be *rapidly varying* at ∞, i.e. $\lim_{t\to\infty} \frac{h(tx)}{h(t)} = 0$ for all $x \in (0,1)$ and $\lim_{t\to\infty} \frac{h(tx)}{h(t)} = \infty$ for all $x > 1$. A random variable Z is said to be *right regularly varying* with index $\alpha \in \mathbb{R}$ if $\overline{F}_Z \in RV_\alpha^{+\infty}$, and *left regularly varying* with index $\alpha \in \mathbb{R}$ if $F_Z \in RV_\alpha^{-\infty}$. From classical results on extreme values (see e.g. Embrechts *et al.* (1997)) we will say that Z has *right heavy tails* if its distribution is in the max-domain of attraction of the Fréchet distribution, i.e. $\overline{F}_Z \in RV_\alpha^{+\infty}$ with $\alpha \in (0, \infty)$. But unfortunately, rapid variation is not sufficient to characterize light tails. We will say that Z has *right light tails* if its distribution is in the max-domain of attraction of the Gumbel distribution. A necessary condition is that $\overline{F}_Z \in RV_\infty^{+\infty}$. A sufficient condition is that $(1/h(x))' \to 0$ as $x \to +\infty$ where h denotes the hazard rate of Z, i.e. $h(x) = f_Z(x)/\overline{F}_Z(x)$. For instance, the Student t distribution has heavy tails (with degrees of freedom equal to the tail index α), while the Gaussian distribution has light tails.

FINIR DE REFERENCER LES LOIS CLASSIQUES

- Power laws For regularly varying distributions, far out in the tail $t \to \infty$ the distribution behaves *like* a Pareto distribution. For power laws, distributions have exact Pareto tails.

- α-stable laws α-stable (or Lévy) distributions with infinite variance $\alpha \in (0, 2)$ have heavy tails. They appear naturally when studying sum of random variables.

- Elliptical distributions Tails of standard elliptical distributions can be simply characterized. For instance, the Gaussian distribution has light tails, while the Student t

distribution has heavy tails, its tail regular variation index α is the number of degrees of freedom (????).

- Markov switching processes Consider a random coefficient autoregressive model, e.g. $X_t = \alpha_n X_{t-1} + \varepsilon_t$, where (α_n) is a series of random variables. Then X_t has heavy tails (see e.g. section 8.4.3 in Embrechts, Kluppelberg & Mikosh (1997)).

- GARCH processes ARCH and GARCH processes (with a Gaussian noise) are heavy tailed (see e.g. section 8.4.3 in Embrechts, Kluppelberg & Mikosh (1997) for ARCH(1) processes).

Some distributions have heavier tails than the Gaussian distribution, but will not necessarily be called *heavy tailed*.

nonexistence of exponential moments A first class of distribution with heavier tails than the Gaussian distribution is the class of distributions such that $\mathbb{E}(e^X) = \infty$. Then tail probability $\mathbb{P}(X > x)$ declines faster than exponentially.

Subexponential distribution A famous class of heavy tailed distribution is obtained when the sum of n random variables is likely to be large if and only if their maximum is likely to be large, i.e.

$$\lim_{x\to\infty} \frac{\mathbb{P}(X_1 + \cdots + X_n > x)}{\mathbb{P}(\max\{X_1, \cdots, X_n\} > x)} = 1.$$

Here the tails decrease more slowly than any exponential distribution.

Proof of Propositions 3. Let $\Delta P_{X,t} = \frac{1}{n}\sum_{l\in X} \Delta P_{l,t}$, and assume that $(\Delta u_{X,t}, \Delta u_{Y,t})$ is a Gaussian vector, then $(\Delta P_{j,t}, \Delta P_{X,t})$ is a Gaussian vector, for any j ($j \in X$ or $j \in Y$). Suppose that asset $n+1$ is reclassified from style Y into style X, and that at the same time, asset 1 is reclassified from style X into style Y. Berfore reclassification, we have

$$cov(\Delta P_X, \Delta P_{n+1}) = \psi_M^2 + \rho_u^2 \sigma_u^2$$

and after

$$cov(\Delta P_X, \Delta P_{n+1}) = \psi_M^2 + \psi_S^2 + \sigma_u^2$$

Therefore, $cov(\Delta P_X, \Delta P_{n+1})$ does indeed increase after addition.Thus $corr(\Delta P_X, \Delta P_{n+1})$ does also increase after addition and thus following the above lemme that the proposition also hold..

Proof of Proposition 4: A proof will be given here only for right tail, and upper tail dependence, since the proof remains mainly unchanged for left tail and lower tail dependence.

$$\mathbb{P}(\Delta P_i > F^{-1}_{\Delta P_i}(t), \Delta P_j > F^{-1}_{\Delta P_j}(t)) = \int_{F^{-1}_{u_X}(t)}^{\infty} f_{u_X}(u) \cdot \overline{F}_{e_i,e_j}\left(F^{-1}_{\Delta P_i}(t) - u, F^{-1}_{\Delta P_j}(t) - u\right) du. \tag{1}$$

With a change of variable $u = F^{-1}_{u_X}(t)x$, we get

$$\begin{aligned} & \mathbb{P}(\Delta P_i > F^{-1}_{\Delta P_i}(t) | \Delta P_j > F^{-1}_{\Delta P_j}(t)) \\ = & \int_1^{\infty} \frac{F^{-1}_{u_X}(t)}{1-t} f_{u_X}(F^{-1}_{u_X}(t)x) \cdot \overline{F}_{e_i,e_j}\left(F^{-1}_{\Delta P_i}(t) - F^{-1}_{u_X}(t)x, F^{-1}_{\Delta P_j}(t) - F^{-1}_{u_X}(t)x\right) dx. \end{aligned}$$

Let g_t denote the integrated function, so that

$$\mathbb{P}(\Delta P_i > F^{-1}_{\Delta P_i}(t) | \Delta P_j > F^{-1}_{\Delta P_j}(t)) = \int_1^{\infty} g_t(x) dx,$$

then

$$\lambda_U(\Delta P_i, P_j) = \lim_{t \to 1} \int_1^{\infty} g_t(x) dx = \int_1^{\infty} \lim_{t \to 1} g_t(x) dx.$$

Recall that

$$g_t(x) = \underbrace{\frac{F^{-1}_{u_X}(t)}{1-t} f_{u_X}(F^{-1}_{u_X}(t)x)}_{\text{first term}} \cdot \underbrace{\overline{F}_{e_i,e_j}\left(F^{-1}_{\Delta P_i}(t) - F^{-1}_{u_X}(t)x, F^{-1}_{\Delta P_j}(t) - F^{-1}_{u_X}(t)x\right)}_{\text{second term}}.$$

For the first term, note that

$$\frac{F^{-1}_{u_X}(t)}{1-t} f_{u_X}(F^{-1}_{u_X}(t)x) = \frac{s}{1 - F_{u_X}(s)} f_{u_X}(sx) \text{ where } s = F^{-1}_{u_X}(t),$$

and therefore

$$\lim_{t \to 1} \left[\frac{F^{-1}_{u_X}(t)}{1-t} f_{u_X}(F^{-1}_{u_X}(t)x)\right] = \lim_{s \to \infty} \left[\frac{s}{1 - F_{u_X}(s)} f_{u_X}(sx)\right]$$

$>$From Karamata's theory (see e.g. Proposition 1.5.8 in Bingham *et al.* (1987), also called Von Mises' conditions), since u_X is right regularly varying with index $\alpha \in (0, \infty)$, this term is simply

$$\lim_{s\to\infty}\left[\frac{s}{\overline{F}_{u_X}(s)}\overline{F}'_{u_X}(sx)\right]=\underbrace{\lim_{s\to\infty}\left[\frac{s\overline{F}'_{u_X}(s)}{\overline{F}_{u_X}(s)}\right]}_{=\alpha}\cdot\underbrace{\lim_{s\to\infty}\left[\frac{\overline{F}'_{u_X}(sx)}{\overline{F}'s_{u_X}(s)}\right]}_{=x^{-(\alpha+1)}}$$

For the second term,

$$F^{-1}_{\Delta P_i}(t)-F^{-1}_{u_X}(t)x=F^{-1}_{u_X}(t)\left(\frac{F^{-1}_{\Delta P_i}(t)}{F^{-1}_{u_X}(t)}-x\right)$$

Let $q_i=\lim_{t\to 1}\frac{F^{-1}_{\Delta P_i}(t)}{F^{-1}_{u_X}(t)}$. Since ε and u_X are independent, the quantile function of $\varepsilon+u_X$ is lower than the quantile function of $\varepsilon+u_X$ in the case of comonotonicity, for u large enough. Hence, there is $t_0<1$ such that for all $t\in(t_0,1)$,

$$F^{-1}_{\Delta P_i}(t)=F^{-1}_{\varepsilon_i^{\perp}+u_X^{\perp}}(t)\leq F^{-1}_{\varepsilon_i^{+}+u_X^{+}}(t)=F^{-1}_{\varepsilon_i}(t)+F^{-1}_{u_X}(t),$$

where the $^+$ exponent is for comonotonic pairs, while $^\perp$ denotes independent pairs. The last equality is obtained from the property of additivity of the quantile function for comonotonic variables. Thus,

$$\frac{F^{-1}_{\Delta P_i}(t)}{F^{-1}_{u_X}(t)}\leq\frac{F^{-1}_{\varepsilon_i}(t)+F^{-1}_{u_X}(u)}{F^{-1}_{u_X}(t)}.$$

Since u_X has heavier tails than ε_i, then (see Proposition 1.5.7 in Bingham *et al.* and Proposition VIII.8.1 in Feller (1971)),

$$\lim_{t\to\infty}\frac{F^{-1}_{\varepsilon_i}(t)+F^{-1}_{u_X}(t)}{F^{-1}_{u_X}(t)}=1.$$

Hence,

$$q_i=\lim_{t\to 1}\frac{F^{-1}_{\Delta P_i}(t)}{F^{-1}_{u_X}(t)}\geq 1.$$

Thus,

$$\begin{cases}\text{for all } x>1, \lim_{t\to\infty}\left[F^{-1}_{\Delta P_i}(t)-F^{-1}_{u_X}(t)x\right]=-\infty,\\ \text{for all } x<1, \lim_{t\to\infty}\left[F^{-1}_{\Delta P_i}(t)-F^{-1}_{u_X}(t)x\right]=+\infty,\end{cases},$$

and therefore

$$\overline{F}_{\varepsilon_i,\varepsilon j}\left(F^{-1}_{u_X}(t)-F^{-1}_{\Delta P_i}(t)-F^{-1}_{u_X}(t)x,F^{-1}_{u_X}(t)-F^{-1}_{\Delta P_j}(t)-F^{-1}_{u_X}(t)x\right)=\mathbf{1}(x>1).$$

So finally

$$\lambda_U(\Delta P_i, \Delta P_j) = \int_1^{\infty} \left[\lim_{t\to 1} g_t(x)\right] dx = \int_1^{\infty} \left[\frac{\alpha}{x^{1+\alpha}}\right] dx = 1.$$

Moreover since e_i are gaussian $\lambda_U(e_i, e_j) = 0$, we have $\lambda_U(\Delta P_i, \Delta P_j) > \lambda_U(e_i, e_j)$.

Proof of Proposition 5. It is possible to rewrite $\Delta P_{X,t}$ as follows

$$\Delta P_{X,t} = \frac{1}{n}\sum_{i\in X} \Delta P_{l,t} = \underbrace{\frac{1}{n}\sum_{i\in X} \Delta e_{l,t}}_{e_{X,t}} + \Delta u_{X,t}$$

where $e_{X,t}$ is normal (as a weighted sum of components of a Gaussian vector) and independent of $\Delta u_{X,t}$ (since $\boldsymbol{e}_{.,t}$ is independent of $\Delta u_{X,t}$).

So the proof of the previous Proposition can be used to prove that $\lambda(\Delta P_{X,t}, \Delta P_{n+1,t})$ is equal to one after reclassification while $\lambda(\Delta P_{X,t}, \Delta P_{n+1,t})$ was zero before reclassification. Thus the proposition hold.