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# The Credit Spread Cycle with Matching Friction

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# The Credit Spread Cycle with Matching Friction

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## Abstract

We herein advance a contribution to the theoretical literature on financial frictions and show the significance of the matching mechanism in explaining the countercyclical behavior of interest rate spreads. We demonstrate that when matching friction is associated with a Nash bargaining solution, it provides a satisfactory explanation of the credit spread cycle in response to shocks in production technology or in the cost of banks' resources. During periods of expansion, the credit spread experiences a tightening for two reasons. Firstly, as a result of easier access to loans, entrepreneurs have better opportunities outside a given lending relationship and can negotiate lower interest rates. Secondly, the less selective behavior of entrepreneurs and banks results in the occurrence of fewer productive matches, a fall in the average productivity of matches, and a tightening of the credit spread. Our results also underline the amplification and propagation properties of matching friction, which represent a powerful financial accelerator mechanism.

**Keywords**: Matching Friction, Credit Market Imperfections, Credit Spread, Business Cycle.

**JEL Classification**: C78 ; E32 ; E44 ; G21

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# 1   Introduction

The credit spread cycle may be thought of as the result of the effect of the business cycle on the difference between lending rates and risk-free rates, and is of fundamental importance in finance and macroeconomics. The variations in interest rate spreads that occur during the business cycle are commonly viewed as being the consequence of financial frictions. Through the mechanism of financial acceleration, financial imperfections are well-known for creating[1] specific transmission channels for monetary policy and for amplifying the magnitude and persistence of any fluctuations in the business cycle. Despite a large consensus regarding the empirical robustness of the countercyclical behavior of interest rate spreads[2], some disagreement still exists about the underlying theoretical mechanisms.

This article contributes to the literature on financial frictions and shows the significance of the matching mechanism in the credit spread cycle. We demonstrate how matching friction, when associated with a Nash bargaining solution, explain the countercyclical behavior of the credit spread. This original approach is distinct from previous contributions on the credit spread cycle, which were mainly conducted within the agency paradigm. We herein extend the matching models of the credit market to the prediction of the credit spread cycle.

In previous literature, asymmetry in the flow of information between lenders and borrowers induces an inverse relationship between the borrower's net worth and the relative cost of external finance, usually measured as the spread between the rate of return on the firm's capital and a risk-free rate[3]. Ever since Gomes et al. (2003) showed that models such as those of Carlstrom

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[1] This view has been put forward most notably in the influential series of contributions of Bernanke (1983), Bernanke and Gertler (1989) and Bernanke et al. (1996, 1999).

[2] This fact may be observed by inspecting various data on business cycles and interest rates. Gomes et al. (2003) report lead and lag correlations between two macroeconomic variables (the Total Factor Productivity and the ratio of investment to capital) and two interest rate spreads (the yield spread between Baa and Aaa rated corporate bonds and the spread between prime bank loan and a 3-month commercial paper). Guha and Hiris (2002) and Koopman and Lucas (2005) study the cyclical behavior of the spread between the yields on Baa corporate bonds and on government bonds for long periods. In their analysis, Dueker and Thornton (1997) propose an interpretation of the bank interest rate margin as being a bank markup. This contribution is particularly interesting because it investigates the countercyclical behavior of bank markup due to market imperfections.

[3] This spread is commonly called the external finance premium.

and Fuerst (1997) exhibit a procyclical cost of external finance, several attempts to improve the agency model in this regard have been proposed. Faia and Monacelli (2007) modified the stochastic structure of the shocks in the Carlstrom-Fuerst model in order to shift the sign of the cost of external finance in response to a technological shock. Meeks (2006) preserves the Carlstrom-Fuerst model, but extends the sources of the fluctuations and shows that financial shocks can explain the observed negative correlation between output and the cost of external finance. Walentin (2005) highlighted the significance of the specific assumptions made by Carlstrom and Fuerst (1997) in explaining its undesirable attributes, namely that a procyclical cost of external finance comes from the absence of a link between capital prices and the self-financing ratio of entrepreneurs. He suggests using the Bernanke et al. (1999) model, which encompasses such a link, and thus generates a countercyclical cost of external finance.

There is an important difference between the credit spread, which is the subject of this paper, and the external finance premium, which is usually studied in the agency-based literature[4]. Levin et al. (2004) make the distinction between the two wedges in the well-known model of Bernanke et al. (1999) rather more explicit. The agency-based literature focuses on the external finance premium of physical capital, because the physical capital stock of the borrower forms the collateral in the borrowing relationship. In this literature, the cyclical variation of interest rate spreads results from cyclical variations in the net worth of the borrowers. In our model, unlike those found in the agency-based literature, financial frictions result from the matching process of borrowers and lenders in the credit market. The borrower's net worth does not influence the financial friction mechanism. We herein focus on the cyclical variations of the credit spread[5] showing that it is linked with cyclical variations in the borrower's financing opportunities on the credit market, and

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[4]It is worth mentioning that even if the agency-based models do mainly focus on the external finance premium, the recent contributions of Goodfriend and McCallum (2007), De Fiore and Tristani (2009), and Curdia and Woodford (2009) highlight the importance of the cyclical behavior of the credit spread in monetary policy analysis.

[5]The concept of the credit spread is also important since it is widely used to obtain an empirical measure of the external finance premium, which has no direct empirical counterpart, as explained by de Graeve (2009).

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the reservation productivity of the matches determined by borrowers and lenders.

In departing from the traditional agency-based literature, we reflect a growing interest in the ability of the matching model to explain the consequences of imperfections in the credit market. Dell'Ariccia and Garibaldi (2005) and Craig and Haubrich (2006) constructed databases of credit flows and showed that the credit market in the United States is characterized by large and cyclical flows of credit expansions and contractions that may be explained in terms of matching friction. Den Haan et al. (2003) and Wasmer and Weil (2004) developed theoretical models to describe the powerful amplification and propagation mechanisms associated with matching friction[6]. However, these contributions are not concerned with the implications of matching friction on the cyclical behavior of credit spread. It is worth mentioning that in the model of Den Haan et al. (2003), the loan contract is still based on agency costs and not on a Nash bargaining solution. In this regard, our model is close to that of Wasmer and Weil (2004), who also consider a Nash bargaining solution. The authors emphasize the structural determinants of the credit spread in a double matching process with credit and labor markets, but do not question the business cycle behavior of the credit spread.

In order to address the questions outlined above, we have developed a model that incorporates three separate approaches, as follows. ($i$) An aggregate matching technique identifies the search-and-meet processes taking place in the credit market and then determines the flow of new matches as a function of the mass of unmatched entrepreneurs and the searching intensity of the banks. ($ii$) The financial contract determines the credit interest rate as an outcome of a Nash bargaining solution that takes place between banks and entrepreneurs. ($iii$) The rule of match destruction, which is the consequence of negative idiosyncratic shocks on the entrepreneur's technology production. In our contribution, the Nash bargaining solution plays a critical role in allowing the endogenous dynamics of the credit spread that occur as a result of technological shocks.

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[6]See Besci et al. (2005) and Nicolleti and Pierrard (2006) for more recent research on matching friction in the credit market.

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We show that the response to technological shocks is governed by a combination of the three different effects described below.

1. The first effect of technological shocks on the surplus causes a procyclical credit spread. Banks and entrepreneurs bargain to share the value of the match, which depends on the profits yielded by the production activity and the opportunity cost of the match. Banks obtain a share (precisely equal to their bargaining power) of this value paid by the entrepreneur via the loan interest rate. A positive technological shock increases the profits from the production activity and from the loan interest rate. Since the cost of the bank's resources are independent of the technological shock, an increase in the loan interest rate widens the credit spread. The two other effects act in the opposite sense and lead to a countercyclical credit spread.

2. The second effect is also related to the bank's appropriation of production revenues via the Nash bargaining solution. Following a positive technological shock, the average idiosyncratic productivity of matches decreases[7]. Given the higher efficiency of the aggregate technological productivity, banks and entrepreneurs are less selective and accept matches that have a lower idiosyncratic productivity. The fall in the average idiosyncratic productivity of the matches reduces the profits from the production activity and consequently decreases the loan interest rate. As a result of this downward adjustment of the productivity reservation, the credit spread reacts negatively to positive technological shock.

3. The third effect is a result of modifications to the external opportunities of entrepreneurs. For each agent, the ease of finding another partner determines its threat point and thus its revenues raised as a result of the bargaining process. A positive technological shock increases the average value of its matches, and stimulates the supply of loans to be matched on the credit market. This implies a shorter average delay in entrepreneurs finding loans, and reinforces their threat

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[7]The average productivity of matches is defined as the product of the exogenous technological shock and the average idiosyncratic productivity of the matches.

point in the bargaining process. Since an entrepreneur could find easily an another loan if the bargaining process were to fail, a lower interest rate on the loan may be obtained.

We conduct a numerical analysis to assess the relative importance of these three effects. We conclude that the second and third effects dominate the first, leading to a countercyclical credit spread in the economy. We extend the scope of the model beyond the analysis of the technological shock to that of an additional shock, taking into account an unanticipated exogenous movement in the short-term interest rate. Our results illustrate the countercyclical dynamic of the credit spread induced by the interest rate shock. This results from the adjustments made to the productivity reservation and the external opportunities of the entrepreneurs. Finally, we document how the underlying mechanism amplifies and propagates the effects of the shocks. We demonstrate that both shocks are amplified and propagated in a similar way during five consecutive quarters, which is synonymous with a powerful financial accelerator effect.

The remainder of this article is organized as follows. The model is described in section 2. The equilibrium of the model is defined and studied analytically in Section 3. The results of numerical analysis of the model's predictions for the business cycle are described in section 4. The extension to interest rate shocks, and a discussion of the financial accelerator mechanism are provided in section 5. Some conclusions are given in Section 6.

# 2   The Model

Banks have funds, but no projects. Entrepreneurs have projects, but no funds. Both banks and entrepreneurs therefore search for partners in the credit market. As a result of the phenomenon of search friction, finding a partner on the credit market is rather time-consuming. When matched, banks and entrepreneurs can decide whether to maintain the match or not, depending on the productivity of the funded activity (the project). If they decide to continue with the agreement, they

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then carry out a bargaining process to agree the loan rate.

## 2.1 Credit Market and Matching Frictions

Let us define $E_t$ as the population of entrepreneurs growing at an exogenous and deterministic rate $g_e$. If $N_t$ is the number of entrepreneurs that are matched with banks, it follows that the number of unmatched entrepreneurs is given by $(E_t - N_t)$. The flow of new matches $M_t$ is a function of the numbers of unmatched entrepreneurs, $(E_t - N_t)$, and the banks loan supply, $V_t$. The search friction may be summarized by

$$M_t = m\left(V_t, E_t - N_t\right) \tag{1}$$

The function $m\left(\cdot, \cdot\right)$ increases with both arguments and is strictly concave with constant return to scale. The function satisfies $m\left(V_t, E_t - N_t\right) < \min\left\{V_t, E_t - N_t\right\}$. From the point of view of a bank, the matching probability is given by $q_t = M_t/V_t$. For entrepreneurs, the probability is $p_t = M_t/\left(E_t - N_t\right)$. Given the assumption of constant return to scale, matching probabilities satisfy the following properties

$$q_t = q\left(\theta_t\right) = m\left(1, \frac{1}{\theta_t}\right) = \frac{m\left(\theta_t, 1\right)}{\theta_t} = \frac{p\left(\theta_t\right)}{\theta_t} = \frac{p_t}{\theta_t} \tag{2}$$

where $\theta_t = V_t/\left(E_t - N_t\right)$ is the tightness credit market variable. Using a standard Cobb-Douglas matching function, the matching probability for entrepreneurs may be written as

$$p_t = \overline{m}\theta_t^{\chi} \tag{3}$$

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where $0 < \overline{m} < 1$ is the scale parameter and $0 < \chi < 1$ is the elasticity parameter of the matching function. The rate of matched entrepreneurs, $n_t = N_t/E_t$, evolves according to

$$n_{t+1}(1 + g_e) = (1 - s_{t+1}) \times [n_t + m(v_t, 1 - n_t)] \tag{4}$$

where $s_t$ is the endogenous rate of separation per period. The separation rate concerns both the old matches, $n_t$, and the new matches, $m_t$.

## 2.2 The Financial Contract

### 2.2.1 Reservation Productivity for Entrepreneurs and Banks

Entrepreneurs produce $y_t$ units of final good (the numeraire) with the quantity $i$ of final good as input according to the following constant return to scale terminology

$$y_t(i; \omega) = z_t \omega i \tag{5}$$

where $z_t$ is the aggregate productivity level, $\omega$ the idiosyncratic productivity level, and $i$ the quantity of input. At each date, all entrepreneurs pick a new value for $\omega$ from the uniform distribution function $G(\omega)$ that satisfies

$$dG(\omega)/d\omega = 1/(\overline{\omega} - \underline{\omega}), \text{ with } \overline{\omega} > \underline{\omega} \tag{6}$$

We denote $J_t(\omega)$, the entrepreneur's value function of being matched with an idiosyncratic productivity level $\omega$. If the entrepreneur accepts the match, he gets the value function $J_t^a(\omega)$, otherwise he turns to the credit market and gets the value function $V_t^e$. Then, the value function $J_t(\omega)$ writes

$$J_t(\omega) = \max\{J_t^a(\omega), V_t^e\} \tag{7}$$

In the remainder, the reservation productivity level $\widetilde{\omega}_t^e$ satisfies the condition $J_t^a\left(\widetilde{\omega}_t^e\right) = V_t^e$, with

$$\max\left\{J_t^a\left(\omega\right), V_t^e\right\} = \begin{cases} J_t^a\left(\omega\right), & \omega \geq \widetilde{\omega}_t^e \\ \\ V_t^e, & \omega < \widetilde{\omega}_t^e \end{cases} \tag{8}$$

For banks the value function, $\Pi_t\left(\omega\right)$, depends also on the idiosyncratic productivity of the entrepreneur's technology, $\omega$, which is perfectly observed by banks unlike in agency-based models. According to the realized value of $\omega$, a bank decides either to accept the match, and obtains the value function $\Pi_t^a\left(\omega\right)$, or to refuse it, and gets $V_t^b$

$$\Pi_t\left(\omega\right) = \max\left\{\Pi_t^a\left(\omega\right), V_t^b\right\} \tag{9}$$

For banks, the reservation productivity level $\widetilde{\omega}_t^b$ satisfies the condition $\Pi_t^a\left(\widetilde{\omega}_t^b\right) = V_t^b$, with

$$\max\left\{\Pi_t^a\left(\omega\right), V_t^b\right\} = \begin{cases} \Pi_t^a\left(\omega_t\right), & \omega \geq \widetilde{\omega}_t^b \\ \\ V_t^b, & \omega < \widetilde{\omega}_t^b \end{cases} \tag{10}$$

Depending on the productivity of the project, matched banks and entrepreneurs will decide either to pursue or to sever the credit relationship. If they choose to maintain their cooperation, they will negotiate a financial contract in which a credit interest rate will be determined.

### 2.2.2 The Nash Bargaining Solution

The financial contract determines the credit interest rate, $R_t^\ell\left(\omega\right)$, as a function of $\omega$, the idiosyncratic productivity of the entrepreneur's technology. The interest rate is the outcome of a Nash bargaining solution, where $\eta$ is the bargaining power of the entrepreneur and $(1 - \eta)$ represents the bank's

bargaining power. Use of the Nash bargaining solution leads to the traditional sharing rule

$$\eta \left( \Pi_t^a \left( \omega \right) - V_t^b \right) = \left( 1 - \eta \right) \left( J_t^a \left( \omega \right) - V_t^e \right) \tag{11}$$

### 2.2.3 The Separation Rule

The outcome of the bargaining process ensures equality between the reservation productivity of the bank and that of the entrepreneur: $\widetilde{\omega}_t = \widetilde{\omega}_t^e = \widetilde{\omega}_t^b$. Indeed, the sharing rule (11) implies that

$$J_t^a \left( \omega \right) - V_t^e = \frac{\eta}{1 - \eta} \left( \Pi_t^a \left( \omega \right) - V_t^b \right) \tag{12}$$

Equation (12) states that, for any $\omega$, if the bank wants to pursue the relationship given by $\Pi_t^a \left( \omega \right) \geq V_t^b$, the implication is that the entrepreneur will also want to stay matched, $J_t^a \left( \omega \right) > V_t^e$. The reservation productivity threshold must satisfy

$$J_t^a \left( \widetilde{\omega}_t \right) - V_t^e = \frac{\eta}{1 - \eta} \left( \Pi_t^a \left( \widetilde{\omega}_t \right) - V_t^b \right) = 0. \tag{13}$$

The endogenous separation rate, which is an increasing function of $\widetilde{\omega}_t$, is computed given the uniform distribution of $\omega$

$$s_t = \int_{\underline{\omega}}^{\widetilde{\omega}_t} dG \left( \omega \right) = \frac{\widetilde{\omega}_t - \underline{\omega}}{\overline{\omega} - \underline{\omega}} \tag{14}$$

## 2.3 Value Functions

### 2.3.1 Entrepreneurs

We assume that entrepreneurs have no personal wealth (internal fund) and borrow a constant amount. In other words, all entrepreneurs borrow the same amount $\ell = i$ to produce. The value functions

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$J_t^a(\omega)$ and $V_t^e$ are defined as follows

$$J_t^a(\omega) = z_t\omega\ell - \ell - R_t^\ell(\omega)\ell - x^e + \beta\mathrm{E}_t\left\{\int_{\underline{\omega}}^{\overline{\omega}} J_{t+1}(\omega)\,dG_{t+1}(\omega)\right\} \tag{15}$$

$$V_t^e = p_t\beta\mathrm{E}_t\left\{\int_{\underline{\omega}}^{\overline{\omega}} J_{t+1}(\omega)\,dG_{t+1}(\omega)\right\} + (1-p_t)\beta\mathrm{E}_t\left\{V_{t+1}^e\right\} \tag{16}$$

where $\beta$ is the discount rate, $z_t\omega$ represent sales, $\ell$ is the cost of input, $R_t^\ell(\omega)\ell$ is the cost of the credit using $R_t^\ell(\omega)$ as the credit interest rate, and $x^e$ represents the fixed costs of production[8]. The probability that an entrepreneur does not find a bank is given by $(1-p_t)$, in which case he must turn to the credit market in the next period. It should be noted that being matched with a bank does not necessarily ensure that the entrepreneur is awarded funds for his project. The decision to finance the project depends on the realized value of $\omega$.

In order to solve the bargaining process, we compute the net surplus of being matched from an entrepreneur's perspective

$$\begin{aligned} J_t^a(\omega) - V_t^e &= z_t\omega\ell - \left(1 + R_t^\ell(\omega)\right)\ell - x^e \\ &\quad + (1-p_t)\beta\mathrm{E}_t\left\{\int_{\underline{\omega}}^{\overline{\omega}}\left[J_{t+1}(\omega) - V_{t+1}^e\right]dG_{t+1}(\omega)\right\} \end{aligned} \tag{17}$$

Equation (17) states that an entrepreneur's net surplus is the sum of the revenue per period (total sales less production and credit costs) plus the expected value of the net surplus in the next period. The entrepreneur gets the net surplus at the new period with a probability of 1 if matched and $p_t$ if unmatched. Here, the term $(1-p_t)$ represents the difference between the matching probabilities of the two states.

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[8]We introduce fixed costs of production to ensure the existence of a positive value for the productivity reservation $\widetilde{\omega}$.

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### 2.3.2 Banks

The value functions $\Pi_t^a(\omega)$ and $V_t^b$ are defined as follows

$$\Pi_t^a(\omega) = R_t^\ell(\omega)\,\ell - R_t^h\ell - x^b + \beta\mathrm{E}_t\left\{\int_{\underline{\omega}}^{\overline{\omega}}\Pi_{t+1}(\omega)\,dG(\omega)\right\} \tag{18}$$

$$V_t^b = -d + q_t\beta\mathrm{E}_t\left\{\int_{\underline{\omega}}^{\overline{\omega}}\Pi_{t+1}(\omega)\,dG(\omega)\right\} + (1-q_t)\,\beta\mathrm{E}_t\left\{V_{t+1}^b\right\} \tag{19}$$

where $R_t^\ell(\omega)\,\ell$ denotes the revenue generated by the credit activity, $R_t^h\ell$ represents the cost of the resources, $x^b$ is the fixed cost of managing the project, $d$ represents the per cost of the search per period, and $q_t$ is the matching probability for a bank. When matched to an entrepreneur, the net surplus of a bank is

$$\begin{aligned}
\Pi_t^a(\omega) - V_t^b &= R_t^\ell(\omega)\,\ell - R_t^h\ell - x^b + d \\
&\quad + (1-q_t)\,\beta\mathrm{E}_t\left\{\int_{\underline{\omega}}^{\overline{\omega}}\left\{\Pi_{t+1}(\omega) - V_{t+1}^b\right\}dG(\omega)\right\}
\end{aligned} \tag{20}$$

Equation (20) states that a bank's net surplus is the sum of the revenue per period (the credit interests, less the costs of ressources and of project management, plus the search cost unpaid when matched) plus the expected value of the net surplus in the next period. The bank gets the net surplus at the new period with a probability of 1 if matched and $q_t$ if unmatched. Here, the term $(1-q_t)$ represents the difference between the matching probabilities of the two states.

## 2.4 Equilibrium Decisions for the Loan Interest Rate, Separation, and Entry

We assume that the costs of searches in the credit market are borne completely by the banks and that all unmatched entrepreneurs search for funds on the credit market. The endogenous entry of

banks determines the tightness of the credit market. The free entry condition on the credit market implies that $V_t^b = 0$. From (19), we deduce that

$$\frac{d}{q_t} = \beta E_t \left\{ \int_{\underline{\omega}}^{\overline{\omega}} \Pi_{t+1}(\omega)\, dG(\omega) \right\} = \beta E_t \left\{ \int_{\widetilde{\omega}_{t+1}^b}^{\overline{\omega}} \Pi_{t+1}^a(\omega)\, dG(\omega) \right\} \tag{21}$$

since $E_t \left\{ V_{t+1}^b \right\} = 0$. The banks' surplus (20) then becomes

$$\Pi_t^a(\omega) - V_t^b = \left( R_t^{\ell}(\omega) - R_t^h \right) \ell - x^b + \frac{d}{q_t}. \tag{22}$$

In equation (22), the new variable is the $d/q_t$, which represents the current average cost of a match. From (21), this equals the expected value of a match for the next period.

The equilibrium credit interest rate is deduced from (12), (17) and (22)

$$R_t^{\ell}(\omega)\ell = (1 - \eta)\left[ z_t \omega \ell - (\ell + x^e) \right] + \eta \left( x^b + R_t^h \ell - p_t \frac{d}{q_t} \right) \tag{23}$$

$R_t^{\ell}(\omega)\ell$ represents the bank's revenues, which is equal to the product of the interest rate, $R_t^{\ell}(\omega)$, and the amount of the credit, $\ell$. The equilibrium value of these revenues is an average of two terms weighted according to the bargaining powers represented by $(1 - \eta)$ and $\eta$. The first term is the net production profit, defined as the amount of total output, $z_t \omega \ell$, minus the total cost, $(\ell + x^e)$. The second term represents the fixed cost for the bank, $x^b$, plus the cost of the resources, $R_t^h \ell$, minus the entrepreneur's outside opportunity, $p_t \times d/q_t$. An increase in the credit market tightness (namely $\theta_t = p_t/q_t$) improves the outside opportunity of entrepreneurs, thus diminishing the bank's payoff .

The separation rule is deduced from equations (13) and (22)

$$R_t^{\ell}(\widetilde{\omega}_t)\ell + \frac{d}{q_t} = R_t^h \ell + x^b \tag{24}$$

The term on the LHS is the value of a match for a bank, defined as the sum of the credit activity revenues, $R_t^\ell(\widetilde{\omega}_t)\,\ell$, and the expected value of a match at the next period, $d/q_t$. A bank would maintain a match if, and only if, its value is at least higher than the cost of the match (the term on the RHS), equal to the cost of the loan, $R_t^h \ell$, plus the cost of managing the project, $x^b$.

Given the equilibrium interest rate from (23), the separation rule (24) may then be written as

$$(z_t\widetilde{\omega}_t - 1)\,\ell - x^e + \left(\frac{1-\eta p_t}{1-\eta}\right)\frac{d}{q_t} = R_t^h \ell + x^b \tag{25}$$

and the free entry condition (21) is

$$\frac{d}{q_t} = \beta \mathrm{E}_t\left\{\int_{\widetilde{\omega}_{t+1}}^{\overline{\omega}}\left[\left(R_{t+1}^\ell(\omega) - R_{t+1}^h\right)\ell - x_{t+1}^b + \frac{d}{q_{t+1}}\right]dG(\omega)\right\} \tag{26}$$

The free entry condition implies that banks enter the credit market (bearing a cost $d$ with a probability $q_t$ to find a partner), and remain there until the current expected cost of matching equals the present value of the anticipated bank's surplus for the matches concerned.

## 2.5 Aggregate Variables

We restrict our attention to three aggregate variables, namely $R_t^p$, the average credit spread, $Y_t$, the total output, and, $L_t$, the total credit in the economy. This last variable is obtained by multiplying the number of financed entrepreneurs, $n_t$, by the value of each individual loan

$$L_t = n_t \ell \tag{27}$$

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The total output is the product of the number of financed entrepreneurs, $n_t$, the aggregate productivity level, $z_t$, the value of each loan, $\ell$, and the average productivity of the entrepreneurs[9]

$$Y_t = n_t \ell z_t \left( \frac{\overline{\omega} + \widetilde{\omega}_t}{2} \right). \tag{28}$$

We define the credit spread as the difference between the credit interest rate, $R_t^\ell(\omega)$, and the cost of resources for the bank, $R_t^h$. To account for the heterogeneity of the matches, the credit spread is defined as the average of the individual spreads[10]

$$R_t^p = (1 - \eta) \left[ z_t \frac{(\overline{\omega} + \widetilde{\omega}_t)}{2\ell} - \frac{x^e}{\ell} - \left( 1 + R_t^h \right) \right] + \eta \left( \frac{x^b - d\theta_t}{\ell} \right) \tag{29}$$

Equation (29) is a weighted average of two terms that has coefficients representing the agents' bargaining powers $(1 - \eta)$ and $\eta$. For $\eta = 1$, which corresponds to the extreme case of an absence of bargaining power for a bank, the credit spread depends on two variables, namely the bank's cost, $x^b$, and the credit market tightness, $\theta_t$. In the opposite case where $\eta = 0$, entrepreneurs have no bargaining power and banks earn all the surplus from the production process.

# 3  Theoretical Properties

In this section, we firstly characterize the equilibrium state of the model, and then describe the credit market cycle.

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[9]The expression is derived from $Y_t = n_t z_t \ell \int_{\widetilde{\omega}_t}^{\overline{\omega}_t} \omega dH(\omega)$ where $H(\omega)$ is the distribution function of $\omega$ for the matched entrepreneurs (who have $\omega$ above $\widetilde{\omega}_t$, the productivity of reservation). This function satisfies $dH(\omega)/d(\omega) = 1/(\overline{\omega} - \widetilde{\omega}_t)$.

[10]This equation is deduced by introducing into $R_t^p = \int_{\widetilde{\omega}_t}^{\overline{\omega}} \left[ R_t^\ell(\omega) - R_t^h \right] dH(\omega)$ the expression of $R_t^\ell(\omega)$ given by (23).

## 3.1 Definition, Existence, and Stability of the equilibrium

We firstly define full equilibrium, and then its reduced form. The reduced form yields the conditions of existence, uniqueness, and stability of the equilibrium. In the following section, the fixed amount of the loan is normalized to unity $\ell = 1$, without loss of generality. We also consider a fixed value for the interest rate $R_t^h = R^h$ . Shocks to this variable are introduced in Section 5.1.

**Definition 1** *The equilibrium case is given by the set of endogenous variables $\{Y_t, L_t, R_t^p, n_t, s_t, p_t, q_t, \theta_t, \widetilde{\omega}_t, z_t\}$ that satisfies: (i) the definitions of aggregate variables for output $Y_t$ (28), credit $L_t$ (27), and the credit spread $R_t^p$ (29); (ii) the law of motion of matched entrepreneurs $n_t$ (4) given the rates of matching $p_t$ (2) and $q_t$ (3), and the rate of separation $s_t$ (14); (iii) the equilibrium conditions for credit market tightness $\theta_t$ (26) and for the productivity reservation $\widetilde{\omega}_t$ (25); and (iv) the following process for the exogenous and stochastic variable $z_t$*

$$\log (z_{t+1}) = \rho_z \log (z_t) + (1 - \rho_z) \log (z) + \varepsilon_{z,t+1} \tag{30}$$

*where $z$ is the steady-state value of $z_t$, $\rho_z$ is the persistence parameter, and $\varepsilon_z \sim iid\left(0, \sigma_z^2\right)$ is the innovation with variance $\sigma_z^2$. The set of structural parameters is $\boldsymbol{\xi} = \left\{R^h, x^e, x^b, \overline{m}, \chi, \eta, \beta, d, z, \rho_z, \sigma_z\right\}$.*

In order to establish the existence and uniqueness of the equilibrium, we reduce the equilibrium to a four-dimensional system for the variables $\{\theta_t, \widetilde{\omega}_t, R_t^p, z_t\}$. To this end, we reformulate the free entry condition and separation rule according to the following definition.

**Definition 2** *The reduced model is a set of endogenous variables $\{\theta_t, \widetilde{\omega}_t, R_t^p, z_t\}$ that satisfy four equations, given the set of structural parameters $\boldsymbol{\xi}$ defined in Definition 1. Using the interest rate definition (23), the matching probability (2), and the separation rule (25), the first equation for the*

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*free entry condition (26) becomes*

$$\frac{d}{m}\theta_t^{1-\chi} = \frac{(1-\eta)}{2(\overline{\omega}-\underline{\omega})}\beta E_t \left\{ z_{t+1}(\overline{\omega}-\widetilde{\omega}_{t+1})^2 \right\} \tag{31}$$

*The equilibrium value of the credit market tightness $\theta_t$ depends on the expected values for the technological shock, $E_t\{z_{t+1}\}$, and the productivity reservation, $E_t\{\widetilde{\omega}_{t+1}\}$. The second equation for the separation rule (25) becomes*

$$z_t\widetilde{\omega}_t\ell = x^b + x^e + (1+R^h) - \left(\frac{1-\eta\overline{m}\theta_t^\chi}{1-\eta}\right)\frac{d}{m}\theta_t^{1-\chi} \tag{32}$$

*using the equations for the rates of matching given in (2) and (3). The equilibrium value for the productivity reservation $\widetilde{\omega}_t$ depends on the current values of the technological shock, $z_t$, and the credit market tightness, $\theta_t$. The third equation is (29), which gives the equilibrium value of the credit spread as a function of the credit market tightness, $\theta_t$, the reservation productivity, $\omega_t$, and the technological shock, $z_t$. The fourth, equation is the law of motion of the aggregate technology $z_t$ (30).*

The following proposition establishes the existence and uniqueness conditions of the reduced equilibrium.

**Proposition 1** *The steady-state equilibrium $\{\theta^*, \widetilde{\omega}^*, R^p, z\}$ of the reduced model defined in Definition 2 exists and is unique, if the following condition is satisfied*

$$\frac{x^b + x^e + (1+R^h)}{z} < \overline{\omega} < \frac{x^b + x^e + (1+R^h)}{z} + \left(\frac{(1-\eta)z\overline{m}}{d}\right)^{\chi/(1-\chi)}\eta\overline{m}\left(\frac{\beta\Delta}{2}\right)^{1/(1-\chi)} + \left(\frac{2-\beta}{2}\right)\Delta \tag{33}$$

*in which an additional parameter has been introduced: $\Delta = \overline{\omega} - \underline{\omega}$. This condition is necessary and sufficient.*

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*See Appendix A.1.*

Having established the existence and uniqueness of the equilibrium condition, we now turn to the short-run properties of the model. In order to study the short-run properties, the model is log-linearized around its unique steady-state. In the following section, we define the log-deviation of the variable $x$, denoted $\widehat{x}$, as $\widehat{x} = \log\left(x_t/x\right)$, for $x = \theta, \widetilde{\omega}, R^p$, and $z$.

**Definition 3** *The log-linearized version of the reduced model in Definition 2 is*

$$(1-\chi)\widehat{\theta}_t = \mathrm{E}_t\left\{\widehat{z}_{t+1} - 2\frac{\widetilde{\omega}}{\overline{\omega}-\widetilde{\omega}}\widehat{\widetilde{\omega}}_{t+1}\right\} \tag{34}$$

$$z\widetilde{\omega}\widehat{\widetilde{\omega}}_t = -\left(\frac{1}{\overline{m}}\theta^{-\chi} - \frac{\eta}{1-\chi}\right)(1-\chi)\,d\theta\widehat{\theta}_t - z\widetilde{\omega}\widehat{z}_t \tag{35}$$

$$R^p\widehat{R^p_t} = (1-\eta)\left(\frac{\overline{\omega}+\widetilde{\omega}}{2}\right)z\widehat{z}_t + (1-\eta)\frac{z}{2}\widehat{\widetilde{\omega}}_t - \eta d\widehat{\theta}_t \tag{36}$$

$$\widehat{z}_t = \rho\widehat{z}_{t-1} + \varepsilon_t \tag{37}$$

*We denote $x_z$ as the elasticity of the endogenous variable $x_t$ to the shock $z_t$ that satisfies $\widehat{x}_t = x_z \times \widehat{z}_t$ for $x = \theta, \widetilde{\omega}$, and $R^p$. The elasticities of $\{\theta_t, \widetilde{\omega}_t, R^p_t\}$ are*

$$\theta_z = \left(\frac{\rho}{1-\chi}\right)\left(\frac{\overline{\omega}+\widetilde{\omega}}{\overline{\omega}-\widetilde{\omega}}\right)\left[1 - \frac{2\theta d}{z\left(\overline{\omega}-\widetilde{\omega}\right)}\left(\frac{1}{\overline{m}\theta^\chi} - \frac{\eta}{1-\chi}\right)\rho\right]^{-1} \tag{38}$$

$$\widetilde{\omega}_z = -\left(\frac{1}{\overline{m}\theta^\chi} - \frac{\eta}{1-\chi}\right)\frac{(1-\chi)\,d\theta}{z\widetilde{\omega}}\theta_z - 1 \tag{39}$$

$$R^p R^p_z = (1-\eta)\left(\frac{\overline{\omega}+\widetilde{\omega}}{2}\right)z + (1-\eta)\frac{\widetilde{\omega}}{2}z\widetilde{\omega}_z - \eta d\theta\theta_z \tag{40}$$

*See Appendix A.2 for the detailed calculations.*

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In the following proposition, we state the stability condition of the log-linearized equilibrium, before interpreting the coefficients in Section 3.2.

**Proposition 2** *The log-linear equilibrium of $\left\{\widehat{\theta}_t, \widetilde{\widehat{\omega}}_t, \widehat{R^p}_t, \widehat{z}_t\right\}$ defined by equations (34)-(35)-(36)-(37) is stable if the following condition holds*

$$\left| \frac{2\theta d}{z\left(\overline{\omega} - \widetilde{\omega}\right)} \left( \frac{1}{\overline{m}\theta^{\chi}} - \frac{\eta}{1-\chi} \right) \right| < 1 \tag{41}$$

*This condition is sufficient.*

*See Appendix A.2 .*

## 3.2 The Credit Market Cycle

In this section, we discuss the theoretical properties of the credit market cycle. This discussion is restricted to the endogenous variables of the reduced model defined in Definition 2. The dynamic properties of other variables, such as the output and the total credit, are studied using model simulations in Section 4.2.1.

**Proposition 3** *The elasticity of the credit market tightness to technological shock is positive: $\theta_z > 0$. The sign of the elasticity of the productivity reservation to the technological shock is ambiguous. A sufficient (and not necessary) condition for $\widetilde{\omega}_z < 0$ is*

$$\eta \leq (1-\chi) \tag{42}$$

*The sign of the elasticity of the credit spread to technological shock is ambiguous. The credit spread*

*reacts negatively to the shock ($R_z^p < 0$) if*

$$(1 - \eta) \left( \frac{\overline{\omega} + \widetilde{\omega}}{2} \right) z < \eta d\theta\theta_z + (1 - \eta) \frac{\widetilde{\omega}}{2} z \left( -\widetilde{\omega}_z \right) \tag{43}$$

*This condition is necessary and sufficient.*

*See Appendix A.2.*

**Comments on the condition (42).** We hereafter assume condition (42) holds. We will check the robustness of our results against this condition using numerical analysis. It is worth mentioning that this condition can be related to the famous condition of Hosios (1990) used in models with matching friction. This condition states that the trading externalities induced by matching friction are efficiently internalized by the Nash bargaining solution, provided that the bargaining powers of the agents are equal to their marginal contribution to the matching process – see also Pissarides (2000). In our setup, the Hosios (1990) condition implies $\eta = 1 - \chi$. This condition is then more restrictive than the condition (42) imposed in Proposition 3.

### 3.2.1 Credit Market Tightness

We may describe the full effect of technological shock on credit market tightness using three terms. In order to characterize explicitly the effects of technological shock $z_t$ on credit market tightness $\theta_t$, we introduce Equation (35) into (34) and obtain

$$(1 - \chi) \widehat{\theta}_t = E_t \left\{ \widehat{z}_{t+1} + \frac{2\widetilde{\omega}}{\overline{\omega} - \widetilde{\omega}} \widehat{z}_{t+1} + \left( \frac{1}{m} \theta^{-\chi} - \frac{\eta}{1 - \chi} \right) \frac{2 (1 - \chi) d\theta}{(\overline{\omega} - \widetilde{\omega}) z} \widehat{\theta}_{t+1} \right\} \tag{44}$$

Given $\widehat{\theta}_t = \theta_z \widehat{z}_t$ and $E_t \{\widehat{z}_{t+1}\} = \rho \widehat{z}_t$, the three identifiable terms are

$$\theta_z = \underbrace{\frac{\rho}{1-\chi}}_{\text{First term}} + \underbrace{\frac{2\widetilde{\omega}}{\overline{\omega}-\widetilde{\omega}}\frac{\rho}{1-\chi}}_{\text{Second term}} + \underbrace{\left(\frac{1}{m}\theta^{-\chi} - \frac{\eta}{1-\chi}\right)\frac{2d\theta}{(\overline{\omega}-\widetilde{\omega})z}\rho\theta_z}_{\text{Third term}} \tag{45}$$

The value $\theta_z$ given in Equation (38) is the solution of Equation (45).

The first term is positive and equal to $(\rho/1-\chi)$ in Equation (38). Its magnitude depends on the persistence parameter, $\rho$, and the elasticity parameter of the matching function, $(1-\chi)$, introduced in Equations (30) and (3), respectively. The more persistent the shock, the higher the impact of $z_t$ on $\theta_t$. This property results from the forward-looking characteristic of the tightness of the credit market – see Equation (31). The current entry of banks into the credit market is driven by the expectation of future revenues that may be earned if matching occurs. In the extreme case of an absence of persistence, $\rho = 0$, since future revenues are independent of the current technology, the coefficient, $\theta_z$, is null and the tightness of the credit market does not respond to technological shocks[11].

With regard to the second (elasticity) parameter, the higher the elasticity of the matching function with respect to the mass of unmatched entrepreneurs, denoted $(1-\chi)$, the lower the impact of the shock on the tightness of the credit market. In order to gain further insight into this relationship, by inspection of Equation (31) it may be seen that $(1-\chi)$ may also be interpreted as the elasticity of the equilibrium duration of a vacant position for a bank with respect to the tightness of the credit market (i.e. $1/q = \theta^{1-\chi}/\overline{m}$). In response to the expectation of future revenues that may be associated with a positive shock, banks accept a longer average duration of vacant positions ($1/q$ rises). In order to achieve an increase in $1/q$, $\theta$ must rise with an amplitude that depends on $(1-\chi)$. For values of $\chi$ close to unity, the average duration of vacant positions shows a slight sensitivity to

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[11]If the shocks are not persistent, any improvement of technology does not last and cannot therefore stimulate the entry of banks into the credit market.

$\theta$. Consequently, large variations of $\theta$ are required in response to technological shocks ($\theta_z$ is high, indeed $\lim_{\chi \to 1} \theta_z = \infty$). At the other extreme, for $\chi$ close to zero, small variations of $\theta$ are sufficient in response to technological shocks and the tightness of the credit market is less sensitive to these shocks ($\theta_z$ is low, for $\chi = 0$).

The second and third terms are the consequence of the negative response of the productivity reservation $\widetilde{\omega}_t$ to technological shocks. The expected productivity reservation for the next period has a negative influence in the equation of the free entry condition that determines the current tightness of the credit market – see Equation (34). A high value of $\widetilde{\omega}_t$ implies a highly selective process of matches, with a low probability of being matched on the credit market with a sufficiently productive entrepreneur[12]. Consequently, banks are less willing to enter the credit market, since total matching costs are higher, and the tightness of the credit market $\theta_t$ falls. As explained below, productivity reservation responds negatively to technological shocks. The second term in Equation (45) corresponds to the current impact of the expected variation of the productivity reservation on the tightness of the credit market. The coefficient $(\rho/1 - \chi) \times [(\overline{\omega} + \widetilde{\omega})/(\overline{\omega} - \widetilde{\omega})] > 1$ in Equation (38) corresponds to a combination of the first and second terms.

The third and final term identified in Equation (45) corresponds to the final term in brackets of Equation (38). In response to a positive technological shock, given a future increase in $\theta_{t+1}$ induced by a decrease in $\omega_{t+1}$, there is a smaller current increase in $\theta_t$. Given the stability condition (41), the sign of the third term in Equation (45) is strictly positive, but it can be either above or below unity. When greater than one, the third term reinforces the first and second terms (thus increasing the total effect of shocks). Under condition (42), this term is less than unity. Hence, this effect weakens the full effect of technological shocks. In order to understand this point, it must be remembered that condition (42) implies that productivity reservation reacts negatively to a positive shock and

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[12]A high value of $\widetilde{\omega}_t$ also implies a higher average idiosyncratic productivity of matches. This productivity effect could act in the opposite sense, by stimulating the supply of loans on the credit market. However, in our model this productivity effect is strictly dominated by the effect of $\widetilde{\omega}_t$ on the probability of match acceptance described in the text.

depends inversely on the tightness of the credit market. In the intertemporal equation (34), this means that a current positive shock increases the expected value of the tightness of the credit market for the following period, which lowers the current response of the tightness of the credit market to the shock. The response is weakened, but still positive.

### 3.2.2 Reservation Productivity

Equilibrium reservation productivity depends on technological shock via two mechanisms. The first follows from the perfect interchangeability of the aggregate productivity and the idiosyncratic productivity. This effect corresponds to the coefficient $-1$ in Equation (39). From the perspective of banks and entrepreneurs, the aggregate productivity, $z$, and the idiosyncratic productivity, $\omega$, are perfect substitutes for one another. Indeed, in the term on the LHS of Equation (32), which defines the equilibrium rule of separation , the amount of the loan is multiplied by the product of the two productivity variables $(z_t \times \widetilde{\omega}_t)$. This term defines the lower production level of a match that banks and entrepreneurs can accept. If the term on the RHS of this equation is constant (which is the case for $\theta_z = 0$), there is a one-to-one relation between $z_t$ and $\widetilde{\omega}_t$. An increase of 1% in the aggregate productivity induces a reduction of 1% in the productivity reservation in the economy.

The second mechanism by which technological shock affects reservation productivity arises from the response of the tightness of the credit market. This effect is represented by the coefficient of $\theta_z$ in Equation (39). The sign of this coefficient is determined by condition (42). Condition (42) implies that the reservation productivity depends inversely on the tightness of the credit market and the sign of the coefficient of $\theta_z$ in Equation (39) is negative. In fact, the tightness of the credit market $\theta_t$ influences the reservation productivity $\widetilde{\omega}_t$ in two different ways - see Equation (32). The first arises from the free entry condition. Recalling that with free entry the expected value of a match for banks is equal to the average cost of a match[13]: $(d/\overline{m})\,\theta_t^{1-\chi}$, it follows that a high value of $\theta_t$ means

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[13]The average cost of a match is the per-period search cost, $d$, times the average duration of a vacant position for

that the value of a match is high. In this case, banks and entrepreneurs are willing to accept lower idiosyncratic productivity to preserve the match ($\widetilde{\omega}_t$ decreases with $\theta_t$). The second way results from the bargaining process. For a fixed value of match (equal to $d/q_t$), the equation of the separation rule (24) implies that the equilibrium revenues for the bank, given by (23), are constant. According to Equation (23), as the tightness of the credit market increases, better external opportunities emerge for entrepreneurs, which provokes a decline in the loan interest rate. Finally, high values of $\theta_t$ make banks more selective and the productivity reservation is also higher[14].

Condition (42) implies that the first way in which the tightness of the credit market influences the reservation productivity, strictly dominates the second. The elasticity $\widetilde{\omega}_z$ defined by (39) is then negative and less than $-1$ (which is the value associated with the first effect only). The positive response of the tightness of the credit market to a positive technological shock reinforces the first effect of the shock on $\widetilde{\omega}_t$. A positive technological shock increases the tightness of the credit market, hence a higher expected value of a match leads banks and entrepreneurs to accept lower idiosyncratic productivity values.

### 3.2.3 Credit Interest Spread

We now identify three distinct effects of technological shocks on the credit spread, which correspond to the coefficients of $z$, $\widetilde{\omega}_z$, and $\theta_z$ in Equation (40).

The first two depend on the entrepreneurs' profits in the production sector. Banks earn a share $(1 - \eta)$ of the profits equal to the banks' bargaining power in the Nash bargaining solution. If they have no bargaining power ($\eta = 1$), the loan interest rate paid by the entrepreneurs covers the costs of the banks[15] and are independent of profits. The first effect of a positive technological

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a bank, which is equal to the inverse of the bank matching probability $1/q_t = (d/\overline{m})\,\theta_t^{1-\chi}$.

[14]This last effect vanishes for $\eta = 0$, i.e. if entrepreneurs have no bargaining power.

[15]More precisely, the loan interests cover the fixed costs of banking activity, denoted $x^b$, minus the external opportunities of entrepreneurs, equal to $d\theta_t$. See Equation (29) with $\eta = 1$ and note that $d\theta_t = p_t \times (d/q_t)$, where $p_t$ is the matching probabiliy of entrepreneurs and $d/q_t$ is the expected value of a match on the credit market.

shock increases profits for the entrepreneur. For the same costs of production, profits grow with technological improvement. This corresponds to the coefficient of $z$ in Equation (40). The size of this first effect depends on the banks' bargaining power, and the average productivity of matches, equal to $(\overline{\omega} + \widetilde{\omega}) z/2$. The greater the bargaining power of the bank, the higher the impact of the technological shock on the credit spread. In order to understand why the average productivity of matches forms part of the expression for $R_z^p$, it must be remembered that credit spread is an aggregate variable. The impact of the shock on the production of a given match depends on the idiosyncratic productivity of this match. Since the credit spread is an average of the individual credit interest rate spreads in the economy, the impact of the shock depends on the average idiosyncratic productivity of all the matches. This first effect leads to a procyclical credit spread.

The second effect is based on the average productivity of matches, and acts in the opposite sense to the first effect, generating a countercyclical credit spread. It corresponds to the coefficient $(1 - \eta) z\widetilde{\omega}$ of $\widetilde{\omega}_z$ in (40). A fall in the productivity reservation $\widetilde{\omega}_t$, in response to a positive shock, diminishes the average productivity of matches as measured by credit spread– see Equation (29) again.

The third effect is based on the threat point of entrepreneurs and, as for the second effect, acts to generate a countercyclical credit spread. This effect corresponds to the coefficient $\eta d\theta$ of $\theta_z$ in the expression of $R_z^p$ given by (40). In order to better understand this, it should be noted that $\eta d\theta = p \times \eta \times (d/q)$. It may be observed that, with probability $p$, the entrepreneur can find another match and get a share $\eta$ of the value of this match, equal to $d/q$ as a result of the free entry condition. Consequently, as the tightness of the credit market increases in response to a positive shock, better external opportunities occur for entrepreneurs, which increase their threat point and provoke a decline in the credit spread – see Equation (29).

In concluding this section on the credit spread, it is noted that in the extreme case of $\eta = 1$, banks have no bargaining power. In this case, the first two effects of a shock disappear. In

consequence, the credit spread is necessarily countercyclical, given the third effect (condition (43) reduces to $\theta_z > 0$). In the general case of $0 < \eta < 1$, if the second and third effects are sufficiently large (i.e. having high values for $\theta_z$ and $-\widetilde{\omega}_z$) to verify condition (43), the credit spread is countercyclical, since it reacts negatively to a positive productivity shock. We now turn to numerical simulations in order to assess the plausibility of a countercyclical credit spread in this model.

# 4  Numerical Analysis

We use numerical analysis to clarify the cyclical properties of the credit spread, which are theoretically ambiguous, in order to assess the robustness of our theoretical results to condition (42), and to describe the dynamic behavior of other aggregate variables, such as the output and the total credit.

## 4.1  Calibration

The model is calibrated by choosing the available empirical counterparts for the interest rates and the average rates of credit flows creation and destruction. Because the previous variables do not allow us to calibrate all the structural parameters, we must make additional assumptions on the values of these parameters. We restrict their range using the conditions of existence, uniqueness, and stability of the equilibrium. A unit of time corresponds to a quarter.

The calibration constraints on interest rates are as follows[16]: the quarterly interest rate on bank resources is $R^h = 1.020^{1/4}$ and the quarterly interest rate on loans is $R^\ell = 1.039^{1/4}$. The rate of

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[16] The interest rates are obtained using data generated by the Federal Reserve in their "Survey of terms of business lending". Since our model is designed for business loans, and not for loans for household or real estate, we use the series entitled "Commercial and Industrial Loan Rates Spreads over intended federal funds rate" (available on the website http://www.federalreserve.gov/releases/e2/e2chart.htm). Data are available only after 1986(3). For the period before this, we generate the spread directly, as the difference between the bank prime loan rate and the effective federal funds rate. Both these series are available from 1955 at monthly frequencies on the *FRED* website (http://research.stlouisfed.org/fred2 , Table H.15 Selected Interest Rates, series ID are MPRIME and FEDFUNDS respectively). Data are converted to quarterly frequency by taking the average for each quarter. Finally, the sample is 1955(1)-2008(4).

creation and destruction of credit flows are taken from the database of commercial loan constructed by Dell'Ariccia and Garibaldi (2005) (see Table 3, p. 675). This implies a steady state for the rate of credit destruction $s = NEG = 0.0111$, which corresponds to the variable NEG of Dell'Ariccia and Garibaldi (2005). The theoretical counterpart of the variable $POS$ of Dell'Ariccia and Garibaldi (2005) is given by $POS = [(1 - s) \times m(v, 1 - n) \times \ell] / L$ i.e. the flow of new credit matches divided by the total loan in the economy. We impose $POS = 0.0179$ in the calibration procedure, and assume that the matching probability of entrepreneurs is $p = 0.4$[17]. The condition of Hosios (1990) is imposed in the case of a symmetric Nash bargaining i.e. $\chi = \eta = 0.5$. The scale parameters of the production and matching technologies are set as follows: $\underline{\omega} = 0.95$, $\overline{\omega} = 1$, $z = 4$, and $\overline{m} = .01$. Finally, the discount rate is set to a conventional value $\beta = 0.999$.

We then deduce from steady-state restrictions the values[18] of $g, \widetilde{\omega}, \theta, q, x^e, x^b, n, Y$, and $L$. In the following section, we provide a discussion of the sensitivity of our results to these assumptions. Before that, we describe the business cycle behavior of the model for this calibration.

## 4.2 The Cyclical Behavior of the Credit Spread

The previous theoretical analysis emphasizes the interactions between several mechanisms that determines the behavior of the business cycle of the credit spread, which have a number of distinct implications. Some of the effects of technological shock induce a procyclical credit spread dynamic, while in contrast others lead to a countercyclical credit spread. In what follows, we perform numerical exercises to assess and to quantify the relative importance of these mechanisms according to the values of the structural parameters used.

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[17]On average, it takes 2.5 quarters for an entrepreneur to be financed.

[18]The values are $\widetilde{\omega} = 0.9506$, $\theta = 0.65$, $x^e = 1.871$, $x^b = 0.004$, $n = 0.493$, $Y = 1.923$, $L = 0.493$, $g = 0.006$, and $d = 0.031$.

### 4.2.1 IRFs

Figure 1 depicts the impulse response functions (IRFs) to a positive technological shock of the output, the credit spread, the credit market tightness, the reservation productivity, the total credit, and the average productivity of matches.

A positive technological shock leads to an expansion of the credit in the economy by two means. Firstly, the improvement of the aggregate technology leads banks and entrepreneurs to accept a lower idiosyncratic productivity level. Since our calibration respects condition (42), the elasticity coefficient $\omega_z$ is negative and the IRF of $\widetilde{\omega}_t$ is negative. This fall in reservation productivity decreases the rate of match destruction in the economy and leads to a credit expansion. Secondly, the improvement in aggregate technology stimulates the entry of banks into the credit market. In Proposition 3, we showed that the elasticity $\theta_z > 0$, and thus the IRF of $\theta_t$ is positive. This rise in credit market tightness facilitates the financing of entrepreneurs by increasing their matching probability, and thus contributes to a credit expansion.

The IRFs of the credit market tightness and the productivity reservation return monotonically to zero as the shock disappears. However, they induce a radically different pattern for the IRFs of the total credit. In our economy, the short-run behavior of the total credit replicates the behavior of the matching rate of entrepreneurs[19], as defined by equation (4). The total credit variable adjusts rather gradually, because it takes time for banks to find new entrepreneurs on the credit market. The IRF of the total credit depicted in Figure 1 is indeed hump-shaped with growing values during the first two years after the shock. The hump-shaped response of the total credit is very similar to the output behavior, even though the definitions of the two variables differ substantially – see Equations (27) and (28). The difference between output and total credit is the average productivity of matches, denoted $\omega_t^{*}$[20]. In Figure 1, we plot the IRFs of the average productivity of matches. The response

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[19]It would be different with a variable amount of loan per match or for an endogenous arrival of entrepreneurs.

[20]More precisely, $Y_t = L_t \times \omega_t^{*}$ where $Y_t$ is the output, $L_t = n_t \ell$ is the total loan variable, and $\omega_t^{*} = z_t \left( \overline{\omega} + \widetilde{\omega}_t \right) / 2$

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is negative and very close to zero. The weak response of this variable explains the high similarity of the IRFs of output and total loan. The sign of the response signifies that the negative values for the average idiosyncratic productivity of matches outweighs the positive values for the aggregate technological shock[21]. To conclude this section on the IRFs, we now turn to the dynamics of the credit spread .

The credit spread reacts negatively to positive technological shocks, implying that $R_z^p < 0$. For all horizons, the sign of the IRF of the credit spread is opposite to that of the IRF of the output. From this we conclude that this model generates a clear countercyclical credit spread. Since the first effect of shocks induces a procyclical credit spread, it may be seen that the second and third effects described above dominate the first. To conclude, for this calibration, matching friction on the credit market supports countercyclical behavior in the credit spread.

### 4.2.2  Sensitive analysis

In order to assess the robustness of the previous results, we conduct a sensitivity analysis of the values of the structural parameters. For each parameter, we use the conditions of existence, uniqueness, and stability (33) and (41) to define the range of admissible values. Given the lack of empirical information for the matching rate of entrepreneurs, we also consider different steady-state values for $p$. For each value of $p$, we again apply the entire calibration procedure described in section 4.1. We simulate the model for values within this range and compute the correlation between output and credit spread and between output and the average productivity of loans. Results are reported in Figure 2 for the two key parameters $\chi$ and $\eta$ and for the variable $p$[22].

The mechanism based on the market tightness is however sufficiently strong to generate a coun-

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is the average productivity of matches.

[21] The average idiosyncratic productivity of matches is $(\overline{\omega} + \widetilde{\omega}_t)/2$, which is compared with the aggregate technological shock $z_t$.

[22] For parameters $\beta$, $\overline{\omega}$, $\underline{\omega}$, $z$, and $\overline{m}$, the coefficients of correlation do not show significative variations (the additional figures are available upon request).

tercyclical credit spread for all the values considered. The variations of the coefficient of correlation between the credit spread and the output are very small for $\chi$ and $\eta$, but sizeable for $p$. However, even in the extreme case of $p$ close to zero, the coefficient of correlation is still clearly negative (about $-0.60$). Finally, it is worth mentioning that even if condition (42) does not hold for all simulations, we do not observe shifts in the sign of the response $\widetilde{\omega}_t$ to technological shock. We then conclude that our main results are robust to changes in the values of parameters.

# 5    Extension and Discussion

This final section extends the model to the case of shocks in the short term interest rate, and discusses the financial accelerator properties of our model.

## 5.1    The Interest Rate Shock

We have hitherto only considered a unique source of fluctuations, namely one of technological shock. However, the literature on credit spread also focuses on interest rate shocks, specifically those involving unanticipated exogenous movement in the short term interest rate.

We first define the response of the reduced model to interest rate shocks and also describe its log-linearized version.

**Definition 4** *The reduced model with interest rate shocks may be described using the set of endogenous variables $\left\{\theta_t, \widetilde{\omega}_t, R_t^p, R_t^h\right\}$ that satisfy four equations, given the set of structural parameters $\boldsymbol{\xi}$ defined in Definition 1. The first equation is the free entry condition (26), which becomes*

$$\frac{d}{m}\theta_t^{1-\chi} = \frac{(1-\eta)\,z}{2\,(\overline{\omega} - \underline{\omega})}\beta\mathrm{E}_t\left\{(\overline{\omega} - \widetilde{\omega}_{t+1})^2\right\} \tag{46}$$

*using equations for the interest rate (23), the matching probability (2), the separation rule (25), and*

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*given the assumption $z_t = z$. The equilibrium value of the credit market tightness $\theta_t$ depends on the expected values for the productivity reservation, $E_t\{\widetilde{\omega}_{t+1}\}$. The second equation is the separation rule (25), which becomes*

$$z\widetilde{\omega}_t = x^b + x^e + \left(1 + R_t^h\right) - \left(\frac{1 - \eta\overline{m}\theta_t^\chi}{1 - \eta}\right)\frac{d}{\overline{m}}\theta_t^{1-\chi} \tag{47}$$

*using equations for the rates of matching (2) and (3) and given the assumption $z_t = z$. The equilibrium value for the productivity reservation $\widetilde{\omega}_t$ depends on the current values for the interest rate, $R_t^h$, and for the credit market tightness, $\theta_t$. The third equation is (29), which becomes*

$$R_t^p = (1 - \eta)\left[z\frac{(\overline{\omega} + \widetilde{\omega}_t)}{2} - x^e - \left(1 + R_t^h\right)\right] + \eta\left(x^b - d\theta_t\right) \tag{48}$$

*given the assumption $z_t = z$. The equilibrium value of the credit spread is a function of the credit market tightness, $\theta_t$, the reservation productivity, $\omega_t$, and the interest rate shock, $R_t^h$. The fourth, and last, equation is the law of motion of the interest rate $R_t^h$*

$$\log\left(R_{t+1}^h\right) = \rho_{R^h}\log\left(R_t^h\right) + (1 - \rho_{R^h})\log(z) + \varepsilon_{R^h, t+1} \tag{49}$$

*where $R^h$ is the steady-state value of $R_t^h$, $\rho_{R^h}$ the persistence parameter and $\varepsilon_{R^h} \sim iid\left(0, \sigma_{R^h}^2\right)$ the innovation with variance $\sigma_{R^h}^2$ .*

Condition (33) of existence and uniqueness of the equilibrium still applies. The model is log-linearized around its unique steady-state[23].

**Definition 5** *The loglinearized reduced model defined in Definition 4 may be written as*

$$(1 - \chi)\widehat{\theta}_t = E_t\left\{-2\frac{\widetilde{\omega}}{\overline{\omega} - \widetilde{\omega}}\widehat{\widetilde{\omega}}_{t+1}\right\} \tag{50}$$

---

[23] The log-deviation of the variable $x$, denoted $\widehat{x}$, is $\widehat{x} = \log(x_t/x)$, for $x = \theta, \widetilde{\omega}, R^p$, and $R^h$.

$$z\widetilde{\omega}\widehat{\widetilde{\omega}}_t = R^h\widehat{R}_t^h - \left(\frac{1}{m}\theta^{-\chi} - \frac{\eta}{1-\chi}\right)(1-\chi)\,d\theta\widehat{\theta}_t \tag{51}$$

$$R^p\widehat{R}_t^p = -(1-\eta)R^h\widehat{R}_t^h + (1-\eta)\frac{z}{2}\widehat{\widetilde{\omega}}_t - \eta d\theta\widehat{\theta}_t \tag{52}$$

$$\widehat{R}_t^h = \rho_{R^h}\widehat{R}_{t-1}^h + \varepsilon_{R^h,t} \tag{53}$$

*It may be assumed that $x_{R^h}$ is the elasticity of the endogenous variable $x_t$ to the shock $z_t$ that satisfies $\widehat{x}_t = x_{R^h} \times \widehat{R}_t^h$ for $x = \theta, \widetilde{\omega}$, and $R^p$. The elasticities of $\{\theta_t, \widetilde{\omega}_t, R_t^p\}$ are*

$$\theta_{R^h} = -\left(\frac{\rho_{R^h}}{1-\chi}\right)\left(\frac{2R^h}{z\left(\overline{\omega}-\widetilde{\omega}\right)}\right)\left[1 - \frac{2d/z}{(\overline{\omega}-\widetilde{\omega})}\left(\frac{1}{m\theta^\chi} - \frac{\eta}{1-\chi}\right)\right]^{-1} \tag{54}$$

$$z\widetilde{\omega}\widetilde{\omega}_{R^h} = R^h - \left(\frac{1}{m}\theta^{-\chi} - \frac{\eta}{1-\chi}\right)(1-\chi)\,d\theta\theta_{R^h} \tag{55}$$

$$R^p R_{R^h}^p = -(1-\eta)R^h + (1-\eta)\frac{z}{2}\widetilde{\omega}_{R^h} - \eta d\theta\theta_{R^h} \tag{56}$$

The condition of equilibrium stability (41) defined in the model with technological shock still applies with interest rate shocks.

### 5.1.1 The Interest Rate-driven Credit Market Cycle

The following proposition characterizes the credit market cycle induced by interest rate shocks.

**Proposition 4** *The elasticity of the tightness of the credit market to interest shock is negative, $\theta_{R^h} < 0$. The sign of the elasticity of the productivity reservation to interest shock is ambiguous. Condition (42) is sufficient (but not necessary) to ensure $\widetilde{\omega}_{R^h} > 0$. The credit spread reacts positively to a positive shock ($R_{R^h}^p > 0$) if*

$$(1-\eta)R^h < (1-\eta)\frac{z}{2}\widetilde{\omega}_{R^h} - \eta d\theta\theta_{R^h} \tag{57}$$

*This is a necessary and sufficient condition.*

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*See Appendix A.3.*

In order to explain the effects of interest rate shocks on the credit market, it is useful to distinguish between several different effects. The analysis of these effects takes less times than for technological shock, because the economic mechanisms operating here are similar to those presented in Section 3.2.

**Reservation Productivity.** Interest rate shocks have two effects on reservation productivity. The first effect of the interest rate shock is positive and equal to $R^h$ in the expression of $\widetilde{\omega}_{R^h}$ given by (55). A positive interest shock increases the cost of resources for banks, and as a consequence it decreases the profits arising from the match. In response to this higher cost, banks and entrepreneurs become more selective, leading to an increase in reservation productivity $\widetilde{\omega}$.

The second effect of interest rate shocks results from the adjustment of the tightness of the credit market. Under condition (42), the coefficient of $\theta_{R^h}$, in the expression of $\widetilde{\omega}_{R^h}$ given by (55), is negative. Given that $\theta_{R^h} < 0$ (as explained below), this implies that the second effect of interest rate shocks reinforces the first by again increasing the reservation productivity. The decrease in the tightness of the credit market lowers the value of a match[24]. In this context, banks and entrepreneurs are less willing to accept matches with low idiosyncratic productivity to maintain the value of the match. Consequently, the reservation productivity increases in response to positive interest rate shocks.

**Credit Market Tightness.** Interest rate shocks do not directly affect credit market tightness, instead the relationship arises as a result of variations in reservation productivity. As shown in (50), the current credit market tightness depends only (and negatively) on the expected value of tomorrow's reservation productivity. A high reservation productivity implies a high destruction rate

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[24]If condition (42) does not hold, the indirect effect could act in the opposite sense. See the comments on condition (42) in section 3.2.

of matches, which deters the entry of banks into the credit market. The interpretation of the first term of the expression of $\theta_{R^h}$ in (54) is the same as in Section 3.2. The persistence parameter $\rho_{R^h}$ comes from the forward-looking characteristic of the credit market tightness, whose sensitivity to changes in economic environment is determined by the parameter $(1 - \chi)$. The second term in (54), $2R^h / (z(\overline{\omega} - \widetilde{\omega}))$, is a consequence of the effect of the shock on reservation productivity on credit market tightness. This effect is unambiguously positive because it induces a positive response in reservation productivity following a positive interest shock. The third and last term in brackets in Equation (54) corresponds to the response of the credit market tightness to tomorrow's expected interest rate shock. The condition of stability (41) implies that this term is positive.

**Credit Interest Rate Spread.** The sign of the elasticity $R_{R^h}^p$ defined by (56) is ambiguous, because the effect associated with $R^h$ acts in the opposite sense to the two effects associated with $\widetilde{\omega}_{R^h}$ and $\theta_{R^h}$. These two last effects result from the endogenous responses of the credit market tightness and the productivity reservation. If these endogenous responses are sufficiently large (entailing high values for $\widetilde{\omega}_{R^h}$ and $-\theta_{R^h}$) to verify condition (57), the credit spread is countercyclical, since it reacts positively to a positive interest rate shock.

The first effect of a positive interest shock on the credit spread is negative, and corresponds to $(1 - \eta) \times R^h$ in the elasticity $R_{R^h}^p$ defined in Equation (56). The credit spread falls, because the cost of funds for the banks are taken into account in the bargaining process. The increase in $R^h$ diminishes the entrepreneurs' profits and thus the loan interest paid to the bank. This effect disappears for the case of null bargaining power for the banks $\eta = 1$.

The second and third effects of the interest rate shocks on the credit spread results from the same mechanisms as for technological shocks. Indeed the coefficients of the elasticities $\widetilde{\omega}_{R^h}$ and $\theta_{R^h}$ in the expression of $R_{R^h}^p$ given by (56) are identical to the coefficients of the elasticities $\widetilde{\omega}_z$ and $\theta_z$ in the expression of $R_z^p$ given by (40). The difference originates from the signs of these coefficients and elasticities. Because Section 3.2 provides a detailed interpretation of the coefficients, we merely

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comment here on the mechanisms associated with these effects.

The second effect is related to the threat point of entrepreneurs. A positive interest rate shock lowers the tightness of the credit market ($\theta_{R^h} < 0$). As a consequence, entrepreneurs have access to poor levels of external opportunity, which weakens their threat point, and the credit spread increases since banks can negotiate a higher interest rate on the loan. The third effect is related to the average productivity of matches. A positive interest rate shock increases the reservation productivity ($\widetilde{\omega}_{R^h} > 0$), so that the profits from final production activity are higher and the credit spread increases since banks get a share $(1 - \eta)$ of these profits.

To conclude this section on the credit spread, it should be noted that in the extreme case of the banks having no bargaining power ($\eta = 1$), the first and third effects disappear. The credit spread is in consequence necessarily countercyclical, given the second effect via the response of the credit market tightness (the condition (57) reduces to $\theta_{R^h} < 0$). This case is naturally too restrictive to enable sensible conclusions to be drawn. To this end, we turn to numerical simulation to assess the plausibility of a countercyclical credit spread.

### 5.1.2 Numerical Analysis

The model simulation is carried out using the calibration described in Section 4.1 with the additional constraint $\rho_{R^h} = 0.95$ for the persistence of interest rate shocks. Figure 3 shows the IRFs produced by a negative interest rate shock for the following variables: the output, the credit spread, the credit market tightness, the reservation productivity, the total credit, and the average productivity of matches. These IRFs are very similar to the IRFs associated with technological shock, which confirms the results of the previous theoretical analysis.

Negative interest shocks lead to an expansion in the credit market (i.e. the credit variable increases) that results from two mechanisms, namely a fall in the rate of match destruction (i.e. the reservation productivity variable decreases) and a rise in the matching probability of entrepreneurs

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(i.e. the credit market tightness variable increases). This credit market expansion leads to an expansion in output that gradually diffuses. The IRFs of the total credit and output variables are hump-shaped. Since the aggregate technology $z$ is constant in this case, the average productivity of matches (previously denoted $\omega_t^*$) depends only on the reservation productivity and reacts negatively to negative interest rate shocks.

The credit spread reacts negatively to interest shock, which implies that $R_{R^h}^p < 0$. In a similar way to technological shocks, the sign of the IRF of the credit spread is the opposite of the sign of the output's IRF for all horizons. Because the first effect of interest shocks induces a procyclical credit interest spread, this countercyclical property results from the fact that the two other effects dominate the first. We perform a sensitivity analysis, for a range of parameter values identical to these of Section 4.2.2. We do not observe any cases of procyclical credit interest spread[25]. We therefore conclude that this model generates a robust countercyclical credit spread in response to both technological and interest rate shocks.

## 5.2    The Financial Accelerator

As defined by Bernanke et al. (1996), the financial accelerator refers to the amplification of initial shocks brought about by changes in conditions in the credit market. Ever since Bernanke and Gertler (1989), the agency-based view of the credit market has been by far the most popular approach used to describe the role of the financial accelerator. Under such reasoning, the financial accelerator arises from variations in agents' net worth, which is directly related to cyclical movements in cash flow. Our model uses an alternative approach based on matching friction, originally developed by Den Haan et al. (2003) and Wasmer and Weil (2004). Given the particular nature of our model, it is useful to document how matching frictions on the credit market amplify and propagate the effects of shocks in the economy.

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[25]Corresponding figures are available upon request.

In order to explain the financial accelerator mechanism, we introduce a wedge between output $Y_t$ and the exogenous aggregate productivity $z_t$, to represent a measure of the financial accelerator. Using equation (28). We define this wedge $\lambda_t$ as follows

$$\log\left(\lambda_t\right) = \log\left(\frac{Y_t}{z_t}\right) = \log\left(n_t\right) + \log\left(\frac{\overline{\omega} + \widetilde{\omega}_t}{2}\right) \tag{58}$$

the RHS of which is simply the sum of the log-deviations of the mass of financed entrepreneurs and the reservation productivity, i.e. $\widehat{\lambda}_t = \widehat{n}_t + \widehat{\widetilde{\omega}}_t$. As a result of technological shock in $z_t$, $\lambda_t$ measures the output's response that is not directly attributable to the shock, but rather to friction in the credit market. As a result of interest shocks in $R_t^h$, with $z_t = z$, this wedge corresponds to the output's response, which is entirely attributable to credit market friction.

In response to an expansionary shock (that is $\widehat{z}_t > 0$ or $\widehat{R}_t^h < 0$), the mass of financed entrepreneurs increases, while the reservation productivity falls[26]. As a consequence, the two variables have different implications for the financial accelerator, which may be associated with two different mechanisms. The first is based on the rate of financed entrepreneurs in the economy and the second is linked, without being equivalent, to the reservation productivity. It is noteworthy that the fluctuations in $n_t$ result partially from the fluctuations in $\widetilde{\omega}_t$. The effect of $\widetilde{\omega}_t$ on $\lambda_t$ in equation (58), i.e. the second mechanism, does not correspond to the full effect of this variable, but only to its effect through the average idiosyncratic productivity of matches (excluding its effects on the mass of financed entrepreneurs).

The mechanism related to the average idiosyncratic productivity of matches weakens the financial accelerator. In response to an expansionary shock (i.e. $\widehat{z}_t > 0$ or $\widehat{R}_t^h < 0$), banks and entrepreneurs are willing to accept matches with lower idiosyncratic productivity. This fall in the average idiosyncratic productivity of matches decreases the amplitude of the output's response to

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[26]The cyclical behavior of $n_t$ and $\widetilde{\omega}_t$ are described in the Sections 4.2 and 5.1.

shocks, hence weakening the financial accelerator.

The mechanism related to the rate of financed entrepreneurs reinforces the financial accelerator, by amplifying and propagating the effects of shocks. In response to an expansionary shock (i.e. $\widehat{z}_t > 0$ or $\widehat{R}_t^h < 0$), because banks agree to enter the credit market with a lower probability of matching, the relative supply of loans grows in the economy (i.e. the credit market tightness increases). In addition, as explained above, since banks and entrepreneurs are more willing to accept matches with a lower idiosyncratic productivity (i.e. the reservation productivity decreases), the match destruction rate decreases. As a result of these two effects, matching friction amplifies the effects of shocks on the economy as a result of changes in the rate of financing of entrepreneurs.

This relationship has interesting dynamic properties. Figures 1 and 3 show that the IRFs of the reservation productivity decreases monotonically, whereas the IRFs of the rate of financing for entrepreneurs is hump-shaped. These Figures also show that the responses of $n_t$ are larger than those of $\widetilde{\omega}_t$. Hence, the amplification and propagation mechanisms associated with the rate of financing of entrepreneurs strongly dominate the stabilization mechanism associated with the effect due to the average idiosyncratic productivity of matches. Figure 4 makes this point explicitly by plotting the IRFs of $\lambda_t$ for identical (symmetric) shocks on the aggregate technology $z_t$ and on the interest rate $R_t^h$.

Both shocks are propagated in a similar way, with a growing value of $\lambda_t$ during the five quarters following the shock. It may be seen that the responses of $\lambda_t$ remain above their initial values for some time (more than twenty quarters). The amplification is stronger for the technological shock than for the interest rate shock. In response to a 1% improvement in the aggregate technology $z_t$, the instantaneous response of $\lambda_t$ is about 43% and reaches a maximum of 89% after five quarters. The amplification effect of the interest rate shock is not as strong, but still significant. In response to a 1% cut in the interest rate $R_t^h$, the instantaneous response of $\lambda_t$ is about 11% and reaches a maximum of 23% after five quarters. Clearly, with matching friction on the credit market, small

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shocks induce large and persistent fluctuations in the economy.

# 6    Conclusion

This study was motivated by evidence of the countercyclical behavior of the credit spread. We have explored the consequences of matching friction in the credit market in order to explain this evidence. To this end, we departed from the traditional view on agency costs and instead followed den Haan et al. (2003) and Wasmer and Weil (2004), who developed models of the credit market based on matching friction. We have proposed an original model with Nash bargaining on the credit interest rate, entry decisions of banks on the credit market, and separation decisions between banks and entrepreneurs.

Although some of the effects of technological or interest rate shocks induce a procyclical credit spread, additional effects associated with the responses of endogenous variables (namely credit market tightness and reservation productivity) lead to countercyclical behavior. The model simulations suggest that the latter effects dominate the former, implying a robust countercyclical credit spread.

We also discussed the properties of the financial accelerator in our model, and confirmed the conclusion of den Haan et al. (2003) and Wasmer and Weil (2004) that matching friction in the credit market induces a powerful persistence mechanism in the economy. Persistence is a long-standing puzzle in macroeconomics, which is of concern both in the literature on business cycles and on monetary policy. The key issue in this literature is to understand the delay in the reaction to shocks of endogenous variables such as output in Cogley and Nason (1995), or loan, in Bernanke and Blinder (1992). An interesting feature of our model is its ability to generate a strong persistence in these variables merely with credit market friction in the economy, without considering any additional mechanisms.

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# A Appendix

## A.1 Proof of Proposition

In order to prove Proposition 1, we first define the function $\theta^* \left( \widetilde{\omega}; \boldsymbol{\xi}_\theta \right)$ that gives $\theta^*$ as a function of $\widetilde{\omega}$ and a set of structural parameters $\boldsymbol{\xi}_\theta = \{\beta, \eta, z, \overline{m}, \overline{\omega}, \underline{\omega}\}$

$$\theta^* \left( \widetilde{\omega}; \boldsymbol{\xi}_\theta \right) = \left[ \frac{\beta \left( 1 - \eta \right) z \overline{m}}{2 \left( \overline{\omega} - \underline{\omega} \right) d} \left( \overline{\omega} - \widetilde{\omega} \right)^2 \right]^{1/(1-\chi)} \tag{A.1}$$

This function is obtained from the steady–state expression of (31). The limit values for $\theta^*$ are

$$
\begin{aligned}
\lim_{\widetilde{\omega} \to \underline{\omega}} \theta^* \left( \widetilde{\omega}; \boldsymbol{\xi}_\theta \right) &= \left. \theta^* \left( \widetilde{\omega}; \boldsymbol{\xi}_\theta \right) \right|_{\widetilde{\omega} \to \underline{\omega}} = \left[ \frac{\beta \left( 1 - \eta \right) z \overline{m}}{2 \left( \overline{\omega} - \underline{\omega} \right) d} \Delta^2 \right]^{1/(1-\chi)} > 0 \\
\lim_{\widetilde{\omega} \to \overline{\omega}} \theta^* \left( \widetilde{\omega}; \boldsymbol{\xi}_\theta \right) &= 0
\end{aligned}
$$

If $\widetilde{\omega}^* \in ]\underline{\omega}, \overline{\omega}[$ exists and is unique, the Equation (A.1) implies that $\theta^*$ exists, is unique, and satisfies $\theta^* \in \left] 0, \left. \theta^* \left( \widetilde{\omega}; \boldsymbol{\xi}_\theta \right) \right|_{\widetilde{\omega} \to \underline{\omega}} \right[$. In order to establish the existence and uniqueness of $\widetilde{\omega}^*$, we introduce the steady–state expression of Equation (32) into Equation (A.1) and deduce that $\widetilde{\omega}^* \in ]\underline{\omega}, \overline{\omega}[$ is the solution of

$$T \left( \widetilde{\omega}^*; \boldsymbol{\xi}_\omega \right) = 0 \tag{A.2}$$

where $\boldsymbol{\xi}_\omega = \left( x^b, x^e, a, R^h, \ell, z, \eta, \overline{m}, d, \beta, \overline{\omega}, \underline{\omega} \right)$ is a set of structural parameters and the function $T \left( \widetilde{\omega}^*; \boldsymbol{\xi}_\omega \right)$ is

$$
\begin{aligned}
T \left( \widetilde{\omega}; \boldsymbol{\xi}_\omega \right) &= \frac{x^b + x^e + \left( 1 + R^h \right)}{z} - \frac{\beta \left( \overline{\omega} - \widetilde{\omega} \right)^2}{2 \left( \overline{\omega} - \underline{\omega} \right)} - \widetilde{\omega} \\
&\quad + \eta \overline{m} \left( \frac{\left( 1 - \eta \right) z \overline{m}}{d} \right)^{\chi/(1-\chi)} \left( \frac{\beta}{2\Delta} \right)^{1/(1-\chi)} \left( \overline{\omega} - \widetilde{\omega} \right)^{2/(1-\chi)}
\end{aligned}
\tag{A.3}
$$

In order to find the solution for Equation (A.2), we first note that $T(\widetilde{\omega}; \boldsymbol{\xi}_\omega)$ is strictly decreasing with respect to $\widetilde{\omega}$. The first order derivative of $T(\widetilde{\omega}^*; \boldsymbol{\zeta})$ with respect to $\widetilde{\omega}$ is

$$
\begin{aligned}
T_1(\widetilde{\omega}; \boldsymbol{\zeta}) = & -\left[1 - \beta\left(\frac{\overline{\omega} - \widetilde{\omega}}{\overline{\omega} - \underline{\omega}}\right)\right] \\
& -\left(\frac{(1-\eta)\,z\overline{m}}{d}\right)^{\chi/(1-\chi)} \eta\overline{m}\left(\frac{\beta}{2(\overline{\omega} - \underline{\omega})}\right)^{1/(1-\chi)} \frac{2}{1-\chi}(\overline{\omega} - \widetilde{\omega})^{\frac{1+\chi}{1-\chi}}
\end{aligned}
$$

since $\beta\left(\frac{\overline{\omega}-\widetilde{\omega}}{\overline{\omega}-\underline{\omega}}\right) < 1$, the two terms of the derivative are negative and $T_1(\widetilde{\omega}; \boldsymbol{\zeta}) < 0$. Hence, the existence and uniqueness of the equilibrium value $\widetilde{\omega}^*$ requires that $\lim_{\widetilde{\omega}\to\underline{\omega}} T(\widetilde{\omega}; \boldsymbol{\zeta}) > 0$ and $\lim_{\widetilde{\omega}\to\overline{\omega}} T(\widetilde{\omega}; \boldsymbol{\zeta}) < 0$. The deduction of (33) is then straightforward given the two following expressions of the limits of the function $T(\widetilde{\omega}; \boldsymbol{\zeta})$

$$
\begin{aligned}
\lim_{\widetilde{\omega}\to\underline{\omega}} T(\widetilde{\omega}; \boldsymbol{\zeta}) = & \frac{x^b + x^e + a + (1 + R^h)}{z} - \overline{\omega} \\
& + \left(\frac{(1-\eta)\,z\overline{m}}{d}\right)^{\chi/(1-\chi)} \eta\overline{m}\left(\frac{\beta(\overline{\omega} - \underline{\omega})}{2}\right)^{1/(1-\chi)} + \left(\frac{2-\beta}{2}\right)\Delta
\end{aligned}
$$

$$
\lim_{\widetilde{\omega}\to\overline{\omega}} T(\widetilde{\omega}; \boldsymbol{\zeta}) = \frac{x^b + x^e + a + (1 + R^h)}{z} - \overline{\omega}
$$

## A.2  Proof of the Proposition

In order to prove Proposition 2, we solve the recursive equilibrium of the credit market tightness. Introducing Equation (35) into Equation (34) gives

$$
(1-\chi)\widehat{\theta}_t = \mathrm{E}_t\left\{\frac{\overline{\omega} + \widetilde{\omega}}{\overline{\omega} - \widetilde{\omega}}\widehat{z}_{t+1} + \frac{2d/z}{\overline{\omega} - \widetilde{\omega}}\left(\frac{1-\chi}{\overline{m}\theta^\chi} - \eta\right)\theta\widehat{\theta}_{t+1}\right\}
$$

The current log-deviation of the credit market tightness depends on the expected values at the next period for the technological shock and the credit market tightness. Given the autoregressive process

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for $z_t$ defined in (30) and assuming $\mathrm{E}_t\{\varepsilon_{t+1}\} = 0$, we obtain

$$\widehat{\theta}_t = \frac{2\theta d}{z\,(\overline{\omega} - \widetilde{\omega})} \left(\frac{1}{\overline{m}\theta^\chi} - \frac{\eta}{1 - \chi}\right) \mathrm{E}_t\left\{\widehat{\theta}_{t+1}\right\} + \left(\frac{\overline{\omega} + \widetilde{\omega}}{\overline{\omega} - \widetilde{\omega}}\right)\left(\frac{\rho}{1 - \chi}\right)\widehat{z}_t$$

This is a standard intertemporal equation for $\widehat{\theta}_t$ that can be solved by iterating over the future period. Let us simplify the equation as follows

$$\widehat{\theta}_t = \mathbf{a}\mathrm{E}_t\left\{\widehat{\theta}_{t+1}\right\} + \mathbf{b}\widehat{z}_t$$

The assumption $|\mathbf{a}| < 1$ is sufficient to guarantee the stability of the equilibrium. We then deduce the value of the coefficient $\theta_z$ that satisfies

$$\widehat{\theta}_t = \theta_z \times \widehat{z}_t = \frac{\mathbf{b}}{1 - \mathbf{a}\rho} \times \widehat{z}_t$$

For $|\mathbf{a}\rho| < 1$, the condition $(1 - \mathbf{a}\rho) > 0$ is always satisfied and since $\mathbf{b} > 0$, we conclude that $\theta_z > 0$. The explicit expression of $\theta_z$ is

$$\theta_z = \left(\frac{\rho}{1 - \chi}\right)\left(\frac{\overline{\omega} + \widetilde{\omega}}{\overline{\omega} - \widetilde{\omega}}\right)\frac{1}{1 - \frac{2\theta d}{z(\overline{\omega} - \widetilde{\omega})}\left(\frac{1}{\overline{m}\theta^\chi} - \frac{\eta}{1 - \chi}\right)\rho}.$$

For the reservation productivity, we assume $\widehat{\widetilde{\omega}}_t = \widetilde{\omega}_z z_t$ and deduce from (35) the following expression for $\widetilde{\omega}_z$

$$\widetilde{\omega}_z = -\left(\frac{1}{\overline{m}\theta^\chi} - \frac{\eta}{1 - \chi}\right)\frac{(1 - \chi)\,d\theta}{z\widetilde{\omega}}\theta_z - 1$$

The case of $\widetilde{\omega}_z < 0$ requires

$$-\left(\frac{1}{\overline{m}\theta^\chi} - \frac{\eta}{1 - \chi}\right)\frac{(1 - \chi)\,d\theta}{z\widetilde{\omega}}\theta_z < 1$$

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since $\theta_z > 0$, a sufficient but not necessary condition of $\widetilde{\omega}_z < 0$ is

$$\frac{1}{\overline{m}\theta^\chi} > \frac{\eta}{1 - \chi}.$$

## A.3   Proof of the Proposition

This section describes the model analysis under interest rate shocks. We first define the reduced model, and then its log-linear version.

Condition (41) still ensures the stability of the model. In order to obtain the values of the elasticities, we introduce Equation (51) into Equation (50)

$$(1 - \chi)\widehat{\theta}_t = \mathrm{E}_t \left\{ -\frac{2}{z\left(\overline{\omega} - \widetilde{\omega}\right)} R^h \widehat{R}^h_{t+1} + \frac{2d/z}{\left(\overline{\omega} - \widetilde{\omega}\right)} \left( \frac{1 - \chi}{\overline{m}\theta^\chi} - \eta \right) \theta\widehat{\theta}_{t+1} \right\}$$

The current log-deviation of the credit market tightness depends on the expected values of interest rate shock and credit market tightness at the next period. Given the autoregressive process for $R^h_t$ defined in (49) and assuming $\mathrm{E}_t \left\{ \varepsilon_{R^h,t+1} \right\} = 0$, we obtain

$$\widehat{\theta}_t = \frac{2\theta d}{z\left(\overline{\omega} - \widetilde{\omega}\right)} \left( \frac{1}{\overline{m}\theta^\chi} - \frac{\eta}{1 - \chi} \right) \mathrm{E}_t \left\{ \widehat{\theta}_{t+1} \right\} - \frac{2R^h}{z\left(\overline{\omega} - \widetilde{\omega}\right)} \left( \frac{\rho_{R^h}}{1 - \chi} \right) \widehat{R}^h_t$$

This is a standard intertemporal equation for $\widehat{\theta}_t$ that can be solved by iterating over the next period, as for the technological shock. We simplify the equation as follows:

$$\widehat{\theta}_t = \mathbf{a}\mathrm{E}_t \left\{ \widehat{\theta}_{t+1} \right\} + \mathbf{b}\widehat{z}_t$$

The assumption $|\mathbf{a}| < 1$ is sufficient to guarantee the stability of the equilibrium. The solution for

$\theta_{R^h}$ such that

$$\widehat{\theta}_t = \theta_{R^h} \times \widehat{R}_t^h = \frac{\mathbf{b}}{1 - \mathbf{a}\rho_{R^h}} \times \widehat{R}_t^h$$

For $|\mathbf{a}\rho| < 1$, the condition $1 - a\rho > 0$ is always satisfied and since $\mathbf{b} < 0$, $\theta_{R^h} < 0$.

$$\theta_{R^h} = -\frac{2R^h}{z\left(\overline{\omega} - \widetilde{\omega}\right)} \left(\frac{\rho_{R^h}}{1 - \chi}\right) \left[1 - \frac{2d/z}{(\overline{\omega} - \widetilde{\omega})} \left(\frac{1}{\overline{m}\theta^\chi} - \frac{\eta}{1 - \chi}\right)\right]^{-1} < 0$$

For the reservation productivity, we impose $\widehat{\widetilde{\omega}}_t = \widetilde{\omega}_{R^h}\widehat{R}_t^h$ and deduce from (51) the following expression for $\widetilde{\omega}_{R^h}$

$$z\widetilde{\omega}\widetilde{\omega}_{R^h} = R^h - \left(\frac{1}{\overline{m}}\theta^{-\chi} - \frac{\eta}{1 - \chi}\right)(1 - \chi)\,d\theta\theta_{R^h}$$

since $\theta_{R^h} < 0$, the condition (42) here implies that $\widetilde{\omega}_{R^h} > 0$.

Finally, for the credit spread, we obtain

$$R^p R_{R^h}^p = -(1 - \eta)\,R^h + (1 - \eta)\,\frac{z}{2}\widetilde{\omega}_{R^h} - \eta\,d\theta\theta_{R^h}$$

Given $\theta_{R^h} > 0$ and $R^h > 0$, condition (42) is sufficient to ensure $R_{R^h}^p > 0$, because this condition implies $\widetilde{\omega}_{R^h} < 0$.

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## 4a. IRFs of the financial accelerator wedge λ to shocks



## 4b. The shocks