



Measuring herding intensity: a hard task

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Summary: This paper addresses the traditional Lakonishok, Shleifer and Vishny (LSV) herding measure and points out its lack of internal consistency. Frey, Herbst and Walter (2007) have shown by empirical simulations that LSV is biased. We provide a formal explanation of this bias and propose a more appropriate measure of herding. We then turn to the properties of the new indicator proposed by Frey, Herbst and Walter (2007): we show that it is accurate only under very strong assumptions, and propose a corrected version of their indicator. We also show that the real herding value is within an interval bounded by LSV and FHW. Finally, as both corrected measures require a prior knowledge of some parameters of the distribution, we conclude that measuring accurately herding intensity is a much more difficult task than considered up to now in the empirical literature.

Résumé : Ce travail s'intéresse à la mesure traditionnelle de mimétisme proposée par Lakonishok, Shleifer and Vishny (LSV) et remet en question sa construction. Frey, Herbst and Walter (2007) ont montré par des simulations de Monte Carlo que cet indicateur est biaisé en présence de mimétisme. Nous donnons une explication formelle à ce biais et proposons un indicateur corrigé de celui-ci. Nous examinons ensuite les propriétés de la nouvelle mesure proposée par Frey, Herbst and Walter (2007) pour montrer qu'elle n'est précise que dans certaines configurations très particulières. Nous en proposons une version corrigée qui autorise des configurations plus larges. Mais au total, dans la mesure où les deux indicateurs corrigés dépendent de paramètres inconnus, nous concluons que mesurer le mimétisme est une tâche plus complexe qu'il n'y paraît

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1. Introduction

Over the last 20 years, a vast literature has emerged on the behavior of active investors in financial markets. Their decisions are particularly important, as they can significantly affect financial asset prices. Because of their potential weight on market transactions, institutional investors (pension funds, mutual funds, banks, insurance companies, etc.) have received particular attention from this point of view. Another specificity of these investors lies in their propensity to “follow the crowd”, in other words, to herd¹. First, they can easily observe each other. However, above all, compensation contracts or reputation or career concerns provide fund managers strong incentives to mimic other managers’ strategies. A large theoretical and empirical literature has thus focused on the herding propensity of institutional investors.

In 1992, Lakonishok, Shleifer and Vishny (hereafter LSV) proposed an indicator to empirically assess herding among institutional investors². The LSV indicator uses portfolio data to measure herding as an excessive concentration of transactions of, for example, money managers, on the same side of the market. LSV defines herding as the excess proportion of money managers buying (selling) a given stock in a given quarter. This excess is computed referring to the normal proportion of buyers (sellers) of all market stocks between fund managers. For instance, if the normal proportion of buyers is 50%, a herding of 10% may indicate that 60% of funds are increasing their holdings (buying) of half of the stocks, while for the remaining stocks, only 40% of funds are net buyers.

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¹ An investor is considered to herd when he reverses a planned decision, to buy or sell a stock, for example, after observing others’ actions.

² We focus on the measures concerning stocks. We do not consider other herding measures concerning funds the whole stock market as Hwang and Salmon (2004), Christie and Huang (1995), Chang and al. (2000), Demirer and Kutan (2006), Demirer, Gubo and Kutan (2007), and Gleason and al. (2004).

The LSV measure relies on portfolio data and is easy to implement. It also allows the refinement in the analysis of institutional herding for particular subgroups of investors or stocks. For these reasons, it has been widely used in the empirical literature dedicated to herding by institutional investors, for which portfolio data are easily available (Lakonishok et al., 1992; Grinblatt, Titman and Wermers, 1995; Oehler, 1998, Wermers, 1999; Oehler and Chao, 2000; Borensztein and Gelos, 2003; Voronkova and Bohl, 2005; Wylie, 2005; Walter and Weber, 2006; Lobao and Serra, 2007; Do, Tan and Westerholm, 2008; Puckett and Yan, 2008; Boyd et al., 2009).

This measure has numerous drawbacks, but the main criticisms bear on its ability to provide a relevant measure of institutional herding. It cannot distinguish intentional herding, where investors intentionally imitate the behavior of other investors, from “spurious herding” as defined by Bikhchandani and Sharma (2000), where groups facing similar information sets make similar decisions. Bikhchandani and Sharma (2000) point out two other drawbacks. First, the LSV measure does not take into account the trading intensity because it only uses the number of buyers and sellers in the market regardless of the volume of assets bought and sold (Wermers, 1995; Ohler, 1998). Second, it does not allow to identify inter-temporal trading patterns at a fund level: although it can be used to test whether herding behavior on a particular stock persists over time, it does not enable the detection of herding persistence for a particular fund (Sias, 2004; Pucket and Yan, 2008).

Moreover, Wylie (2005) also indicates another shortcoming of the traditional LSV measure: the probability to buy or sell a stock is not the same if portfolio managers cannot undertake short sales. In the presence of short-sale constraints, only the managers who hold a given stock at the beginning of the period are allowed to sell it. Walter and Weber (2006) also mention that the LSV measure leads to biased results as a consequence of changes in

benchmark index composition. Therefore, some apparent herding can result from the inclusion or exclusion of a certain stock from the benchmark.

As far as we know, there exist only a very few papers that criticize LSV for its lack of internal consistency. Frey, Herbst and Walter (2007) (FHW hereafter) have shown by Monte Carlo simulations that the LSV measure is accurate only if there is no herding and is biased downward otherwise. One of the goals of this paper is to provide an explanation of this bias. Using a theoretical approach, we show that LSV bias is positively linked with the level of herding. This leads us to propose a more appropriate measure of herding.

As FHW proposed a new measure of herding in the line of LSV, we turn to the properties of their indicator: we show that it is accurate only under very strong assumptions about the nature of herding prevailing on the market. In fact, they suppose a symmetrical and integral herding: all the stocks are excessively bought or sold, and the probabilities to be excessively bought or sold are equal. The second result of our paper consists in proposing a corrected indicator that is more robust among various herding configurations. The third result of the paper is to show that the real herding value belongs to an interval defined by LSV (lower bound) and FHW(upper bound).

Finally, the main contribution of this paper is to highlight the difficulties in measuring herding. The corrected measures of both LSV and FHW require a prior knowledge of some parameters of the distribution. However, these parameters are unobservable and thus have to be estimated. This suggests that assessing herding intensity is a more difficult task than considered up to now in the empirical literature.

This paper is organized as follows. Section 2 presents a simple descriptive model of herding, which is used to analyze the properties of herding measures. In section 3, we present the LSV measure, point out its bias and propose a correction. In section 4, we consider the new measure proposed by FHW and again propose a correction. Section 5 concludes.

2. A simple descriptive model of herding structure

To study the properties of herding indicators, we first need to define the herding configuration that can be expected in a standard equity market. As the example given in the introduction - a herding of 10% with a the normal proportion of buyers is 50%, 60% of funds increasing their holdings (buying) of half of the stocks, whereas for the remainder, the proportion of buyers is lower (40%) - to illustrate the LSV measure, most papers (such as FHW in their simulations³) consider what we will call integral and symmetrical herding: all stocks are subject to the same level of herding with equal probabilities of being excessively bought or sold. In the real world, during a given period, some stocks may not be subject to herding, while some sub-groups of stocks are potentially more subject to buy or sell herding than others. For example, herding may be more likely to happen and possibly higher for stocks that are more difficult to evaluate because investors have less information about them. This is in accordance with a large strand of literature findings that small-capitalization stocks have a higher level of herding⁴.

³ In their theoretical model, FHW allow each stock to have an individual level of herding during a given period.

⁴ See Wermers (1999), for example.

To illustrate the bias of herding indicators, we construct a very simplified descriptive model: we consider only three groups of stocks with different herding levels.

First, the stock i has probability $\pi_{o,t}$ to be bought in the same proportion than the market (hence, in this case, there will be no herding). In other words, we allow a situation that we can call “partial herding”, in which some stocks are subject to herding, while others (in a proportion $\pi_{o,t}$) are not.

Second, the probability of being excessively bought relative to the market (denoted $\pi_{b,t}$) is not necessary equal to the probability of being excessively sold (denoted $\pi_{s,t}$). We allow these probabilities to differ from period to period. One can imagine that the proportion of stocks that are excessively sold relative to the whole market can be sensitive to the state of financial markets or to the economic situation. Therefore, the level of herding can be different on both sides of the market: we denote by h_t^b the level of buy-side herding and by h_t^s the level of sell-side herding.

Moreover, denote by $b_{i,t}$ the observed number of buy transactions and by $n_{i,t}$ the total number of transactions in stock i during a period t ; then $b_{i,t}$ follows a binomial distribution with parameters $n_{i,t}$ and $p_{i,t}$: $b_{i,t} \sim B(p_{i,t}, n_{i,t})$ with $p_{i,t}$ define as the probability that the stock i is bought in period t by an active fund manager,

$$\text{where } \begin{cases} p_{i,t} = p_t + h_t^b \text{ (buy-side herding) with probability } \pi_{b,t} \\ p_{i,t} = p_t - h_t^s \text{ (sell-side herding) with probability } \pi_{s,t} \\ p_{i,t} = p_t \text{ (no herding) with probability } \pi_{o,t} \end{cases}, \quad (1)$$

with $\pi_{b,t} + \pi_{s,t} + \pi_{o,t} = 1$,

and as $E(p_{i,t}) = p_t = \pi_{b,t}(p_t + h_t^b) + \pi_{s,t}(p_t - h_t^s) + \pi_{o,t}p_t$, we have

$$\pi_{b,t}h_t^b - \pi_{s,t}h_t^s = 0.$$

The mean level of herding is defined as the weighted sum of the herding levels on each side of the market:

$$h_t = \pi_{b,t}h_t^b + \pi_{s,t}h_t^s.$$

From the two last equations, we can infer that each sub-group herding is linked with the global herding level as an inverse function of its probability:

$$\begin{cases} h_t^b = \frac{h_t}{2\pi_{b,t}} \\ h_t^s = \frac{h_t}{2\pi_{s,t}} \end{cases}. \quad (2)$$

In summary, in this model, three kinds of states (or realizations) can be observed for each stock in period t: the no-herding state (its realizations are called hereafter “no herding stocks”) the buy-herding state (the “buy-side stocks”) and the sell-herding state (the ”sell-side stocks”) each category of stocks having a proportion defined by the probability of each state.

Even in this simple configuration, assessing herding level is a quite difficult task.

3. The LSV herding measure: description, bias and correction

3.1. The LSV indicator: definition and assessment under no herding

The herding measure of LSV is defined as

$$HLSV_{i,t} = LSVI_{i,t} - AF_{i,t} = \left| \frac{b_{i,t}}{n_{i,t}} - p_t \right| - AF_{i,t}, \quad (4)$$

where $\frac{b_{i,t}}{n_{i,t}}$ is the observed proportion of buy transactions for stock i in t . Therefore, $LSVI_{i,t}$

measures the absolute gap between this proportion and the expected proportion in the no-herding case. As the estimated herding in a given stock group is the mean of $HLSV_{i,t}$ in this group, the absolute value in the first term avoids a sign compensation between buy- and sell-side herding. $AF_{i,t}$ is an adjustment factor. As explained below, it implies that in the case of no-herding, HLSV is null.

As $b_{i,t}$ follows a binomial law, the realized proportion of buying transactions for stock i in t

is $\frac{b_{i,t}}{n_{i,t}} = p_{i,t} + \varepsilon_{i,t}$ where $\varepsilon_{i,t}$ is an independent error term with a zero mean and a variance

equal to $p_{i,t}(1-p_{i,t})/n_{i,t}$.

In the case of no herding, as $p_{i,t} = p_t$ the indicator can be rewritten as

$$HLSV_{i,t} = |\varepsilon_{i,t}| - AF_{i,t} = |\varepsilon_{i,t}| - E|\varepsilon_{i,t}|.$$

In the no-herding case, even if $\varepsilon_{i,t}$ is centered, its absolute value is not, and even if herding is null, $LSVI$ is always positive. In fact, the adjustment factor $AF_{i,t}$ is the expected value of $LSV1$ in the case of no herding.

Therefore, as in this case $E(HLSV_{i,t})=0$, the LSV measure is unbiased.

Finally, given the law of $b_{i,t}$, the adjustment factor is given by⁵

$$AF_{i,t} = \sum_{k=0}^{n_{i,t}} \text{proba}(b_{i,t} = k) \left| \frac{k}{n_{i,t}} - p_t \right| = \sum_{k=0}^{n_{i,t}} \binom{n_{i,t}}{k} p_t^k (1-p_t)^{n_{i,t}-k} \left| \frac{k}{n_{i,t}} - p_t \right|. \quad (5)$$

⁵ Notice that the adjustment factor has to be computed at the stock level because it depends on the number of transactions of the considered stock during the period.

As illustrated by FHW using Monte Carlo simulations, the HLSV indicator is unbiased then relevant under the null hypothesis of no herding. But FHW also show empirically that in any other configuration, the measure is biased. The aim of the following section is to theoretically proof and explain this point.

3.2. A general expression of adjustment factor

As explained above, the adjustment factor is required because of the absolute value in LSV1, which is designed to avoid sign compensation between buy-side and sell-side herding. Hereafter, we will show that while for each kind of stock (buy and sell herding, no herding),

$\frac{b_{i,t}}{n_{i,t}} - p_t$ is an unbiased estimator of the herding level, its absolute value LSV1 is not: its

expectation is higher than the herding level. Because the spread between the expectation of the absolute value of any random variable and its expectation m , decreases with the absolute value of m , the adjustment factor required to obtain an unbiased estimator should decrease when the herding is increasing. However, as the LSV adjustment factor does not depend on the herding level (see relation (5)) and thus remains constant regardless of the herding intensity, the LSV herding measure is consequently biased.

The aim of this section is to correct this bias, by deriving a more encompassing expression of adjustment factor. We first have to construct three adjustment terms, corresponding to the three possible configurations: buy-side herding, sell-side herding and no herding.

3.2.1. Buy side herding

Let us denote $X = \frac{b_{i,t}}{n_{i,t}} - p_t$.

Here we have $X = p_{i,t} + \varepsilon_{i,t} - p_t = h_t^b + \varepsilon_{i,t}$,

$E_b(X)$ is the conditional expectation of X given the herding is in the buy side.

Because $\varepsilon_{i,t}$ is centered, $E_b(X) = h_t^b$ and then X is an unbiased estimator of h_t^b ⁶. However, the absolute value of X , LSV1 is not.

Let us state the link between $E_b(X)$, the herding level in the buy-side group, and $E_b(|X|)$ to obtain the necessary adjustment factor and to get an unbiased estimator of h_t^b .

From $E_b(X) = E_b(|X|/X \geq 0) \cdot \text{proba}_b(X \geq 0) - E_b(|X|/X < 0) \cdot \text{proba}_b(X < 0)$

and $E_b(|X|) = E_b(|X|/X \geq 0) \cdot \text{proba}_b(X \geq 0) + E_b(|X|/X < 0) \cdot \text{proba}_b(X < 0)$,

it comes that

$$E_b(|X|) = E_b(X) + 2E_b(|X|/X < 0) \cdot \text{proba}_b(X < 0) = h_t^b + 2E_b(|X|/X < 0) \cdot \text{proba}_b(X < 0). \quad (6.1)$$

3.2.2. Sell-side herding

Here we just have to remember that $X = -h_t^s + \varepsilon_{i,t}$ and then $E_s(X) = -h_t^s$. Using the same approach as above, we can show that

$$E_s(|X|) = h_t^s + 2E_s(|X|/X > 0) \cdot \text{proba}_s(X > 0). \quad (6.2)$$

3.2.3. No herding

In this case, we have now $X = \varepsilon_{i,t}$ and $E_0(X) = 0$

$$\text{Again, we have: } E_0(|X|) = h_t^0 + E_0(|X|) \quad (6.3)$$

⁶ The fact that X is unbiased could suggest computing buy-side (resp. sell-side) herding only on a subset of positive (resp. negative) observations of X (Wermers (1997)). This solution is not satisfying because a positive (resp. negative) sign could be alternatively explained by the error term for no-herding stock or sell-side (resp. buy-side) herding stock.

3.2.4. Expression of the adjustment factor

Because LSV1 is the absolute value of X, we can compute its expectation from relations (6.1), (6.2) and (6.3), using the respective weights $\pi_{b,t}$, $\pi_{s,t}$ and $\pi_{0,t}$:

$$\begin{aligned} E(LSV1) &= E(|X|) = \pi_{b,t} E_b(|X|) + \pi_{s,t} E_s(|X|) + \pi_{0,t} E_0(|X|) \\ &= h_t + 2 \pi_{b,t} E_b(|X|/_{X<0}) \cdot \text{proba}_b(X<0) + 2 \pi_{s,t} E_s(|X|/_{X>0}) \cdot \text{proba}_s(X>0) + \pi_{0,t} (E_0|X|), \end{aligned}$$

$$\text{where } h_t = \pi_{b,t} h_t^b + \pi_{s,t} h_t^s.$$

It easily follows that

$$h_t = E(LSV1) - AFC_{i,t},$$

$$\text{where } AFC_{i,t} = \pi_{0,t} AFO_{i,t} + 2 \pi_{b,t} AFB_{i,t} + 2 \pi_{s,t} AFS_{i,t}. \quad (7)$$

In which $AFO_{i,t}$, $AFB_{i,t}$ and $AFS_{i,t}$ are the adjustment terms associated with no-herding, buy-side herding and sell-side herding, respectively. Replacing X by its value, the expressions of these terms are, respectively:

$$AFB_{i,t} = E_b \left(\left| \frac{b_{i,t}}{n_{i,t}} - p_t \right| / \frac{b_{i,t}}{n_{i,t}} - p_t < 0 \right) \cdot \text{proba}_b \left(\frac{b_{i,t}}{n_{i,t}} - p_t < 0 \right),$$

$$AFS_{i,t} = E_s \left(\left| \frac{b_{i,t}}{n_{i,t}} - p_t \right| / \frac{b_{i,t}}{n_{i,t}} - p_t > 0 \right) \cdot \text{proba}_s \left(\frac{b_{i,t}}{n_{i,t}} - p_t > 0 \right),$$

$$AFO_{i,t} = E_0 \left(\left| \frac{b_{i,t}}{n_{i,t}} - p_t \right| \right).$$

In the case of buy-side herding, the terms to subtract are those where $\frac{b_{i,t}}{n_{i,t}} - p_t$ is negative, that

is, those for which $b_{i,t} < n_{i,t} p_t$. As $b_{i,t}$ follows a binomial law the theoretical expression of the adjustment term in this case:

$$AFB_{i,t} = \sum_{k=1}^{n_{i,t} p_t} \text{proba}_b(b_{i,t} = k) \left| \frac{k}{n_{i,t}} - p_t \right| = \sum_{k=0}^{n_{i,t} p_t} \binom{n_{i,t}}{k} (p_t + h_t^b)^k (1 - (p_t + h_t^b)^{n_{i,t} - k}) \left| \frac{k}{n_{i,t}} - p_t \right|. \quad (8)$$

Concerning the sell-side herding, because the terms to subtract are those for which $b_{i,t} > n_{i,t}p_t$, we have

$$AFS_{i,t} = \sum_{k=n_{i,t}p_t}^{n_{i,t}} \text{proba}_s(b_{i,t} = k) \left| \frac{k}{n_{i,t}} - p_t \right| = \sum_{k=n_{i,t}p_t}^{n_{i,t}} \binom{n_{i,t}}{k} (p_t - h_t^s)^k (1 - (p_t - h_t^s))^{n_{i,t}-k} \left| \frac{k}{n_{i,t}} - p_t \right|. \quad (9)$$

In the no-herding case, we find the « traditional » adjustment term (see relation (5)):

$$AFO_{i,t} = \sum_{k=0}^{n_{i,t}} \binom{n_{i,t}}{k} p_t^k (1 - p_t)^{n_{i,t}-k} \left| \frac{k}{n_{i,t}} - p_t \right|. \quad (10)$$

To conclude, we obtain an unbiased measure of herding intensity with the “corrected” LSV measure:

$$HLSVC_{i,t} = LSVI_{i,t} - AFC_{i,t} = \left| \frac{b_{i,t}}{n_{i,t}} - p_t \right| - AFC_{i,t}, \quad (11)$$

where $AFC_{i,t}$ is defined by relations (7), (8), (9) and (10).

Finally, it is worth noting that, even if the corrected measure is theoretically funded, it requires, even in a very simple model, a prior estimation of not only the probability vector $\{ \pi_{0,t}, \pi_{b,t}, \pi_{s,t} \}$ but also the herding level. Paradoxically, it means that we have to know the herding level to estimate it..... We will go back over this point on the conclusion.

However, the corrected expression of the adjustment factor can shed light on the properties of LSV bias.

3.3. Remarks on the bias of LSV

The LSV bias can now be expressed as the spread between AFC and AF. We illustrate and comment on the magnitude of the LSV bias, and its determinants, on two cases. The first

corresponds to symmetric and integral herding and is designed to make a comparison with the FHW simulations. The second case is more general and encompasses configurations with asymmetric and/or partial herding⁷.

3.3.1 LSV bias with symmetrical and integral herding.

We made some computations in order to depict the LSV bias for the case of symmetric and integral herding ($\pi_{b,t} = \pi_{s,t} = 0.5$) for different values of real herding and of the number of transactions.

The theoretical expectation of the first term of the measure: LSV1 (using the binomial law applied to model (1)) is

$$\begin{aligned}
 E(\text{LSV1}) = & \pi_{b,t} \sum_{k=0}^{n_{i,t}} \binom{n_{i,t}}{k} (p_t + h_t^b)^k (1 - (p_t + h_t^b))^{n_{i,t}-k} \left| \frac{k}{n_{i,t}} - p_t \right| \\
 & + \pi_{s,t} \sum_{k=0}^{n_{i,t}} \binom{n_{i,t}}{k} (p_t - h_t^s)^k (1 - (p_t - h_t^s))^{n_{i,t}-k} \left| \frac{k}{n_{i,t}} - p_t \right| \\
 & + \pi_{0,t} \sum_{k=0}^{n_{i,t}} \binom{n_{i,t}}{k} (p_t)^k (1 - (p_t))^{n_{i,t}-k} \left| \frac{k}{n_{i,t}} - p_t \right| .
 \end{aligned} \tag{12}$$

In Table 1, we compute this expectation for different value of h_t and $n_{i,t}$, with $\pi_{b,t} = \pi_{s,t} = 0.5$ and $\pi_{0,t} = 0$ and $h_t = h_t^b = h_t^s$. The adjustment factors without and with correction (4th and 5th column of Table 1) are computed from, respectively, relation (5) and relations (7), (8), (9) and (10).

⁷ The main properties of LSV measure reported here were confirmed by Monte Carlo simulations that are not reported here.

These computations have to be linked with the Monte Carlo simulations⁸ made by FHW. As their empirical work is very complete, we do not reproduce it. However, it can be useful to mention their three main results.

- First, the LSV measure is very well suited to test the null hypothesis of no herding.
- Second, when herding is prevalent, LSV systematically underestimates it, and the bias is increasing with the herding level.
- Third, the LSV bias decreases when the transaction number increases.

One important contribution of our approach is to highlight the driving factors behind these three properties. This can be done by comparing the traditional LSV adjustment factor with the corrected one.

First, it is obvious that the “no-herding” configuration is a special case of the corrected measure with $\pi_{0,t}=1$, $\pi_{b,t}=\pi_{s,t}=0$. In this case, as $AF=AFC$, the LSV measure is then correctly built, and the estimator is unbiased.

Second, the LSV bias is increasing with herding. Formally, one can observe

that $\frac{\partial AFB_{i,t}}{\partial h_t^b} < 0$ and $\frac{\partial AFS_{i,t}}{\partial h_t^s} < 0$. This property is simply explained by the fact that the

probability that a buy-side (respectively a sell-side) herding stock has a negative (respectively

a positive) value for $\frac{b_{i,t}}{n_{i,t}} - p_t$ decreases when herding is increasing. Then, given $n_{i,t}$, $p_{i,t}$, $\pi_{b,t}$

and $\pi_{s,t}$, $AFC_{i,t}$ are decreasing with h_t , and as the adjustment factor of LSV remains constant,

the bias rises.

⁸ They report simulations for varying values of parameters n_{it} , h_t and for the number of stocks, but they always set probabilities (to be buy or sell for a stock) to be $(\frac{1}{2}, \frac{1}{2})$. Because the bias of LSV does not depend on this hypothesis, this choice does not matter in the evaluation of LSV, and we keep it for our computations (Table 1). However, as section 3 will show, the choice of this set of probability is crucial when they assess the statistical properties of their new measure.

Table 1: Illustration of the bias of LSV

in case of total and symmetric herding

with $p_t=0.5$

The two first columns describe the different configurations (real herding level and transaction number $n_{i,t}$). E(LSV1) is computed from the binomial law using relation (12). AF is the adjustment factor proposed by LSV (relation (5)), and AFC is the corrected adjustment factor that takes into account the level of herding (relation (7) (8) (9) (10). E(LSV) is computed as the difference between E(LSV1) and AF, and the bias is the spread between real herding en E(LSV).

$n_{i,t}$	<i>Real herding</i>	<i>E(LSV1)</i>	<i>AF</i>	<i>AFC</i>	<i>E(HLSV)</i>	<i>HLSV bias</i>
10	0.00%	12.30%	12.30%	12.30%	0.00%	0.00%
	10.00%	14.70%	12.30%	4.70%	2.40%	7.60%
	20.00%	21.19%	12.30%	1.19%	8.89%	11.11%
	30.00%	30.15%	12.30%	0.15%	17.84%	12.16%
	40.00%	40.00%	12.30%	0.00%	27.70%	12.30%
50	0.00%	5.61%	5.61%	5.61%	0.00%	0.00%
	10.00%	10.47%	5.61%	0.47%	4.86%	5.14%
	20.00%	20.01%	5.61%	0.01%	14.39%	5.61%
	30.00%	30.00%	5.61%	0.00%	24.39%	5.61%
	40.00%	40.00%	5.61%	0.00%	34.39%	5.61%

For instance, with $n_{i,t} = 10$ and a 10% herding intensity, the true adjustment factor is almost divided by three, turning from 12.3% with no herding to 4.7% with a 10% herding. Thus HLSV underestimate about the three-fourths of herding.

Third, the LSV bias decreases (but remains positive) for higher numbers of transactions. As

the variance of $\frac{b_{i,t}}{n_{i,t}}$ is decreasing with $n_{i,t}$ the required adjustment should decline when the

number of transactions on a stock grows, which is true for both adjustment factors. But again, when herding grows, the adjustment should decrease. However, this is not true for HLSV.

3.3.2 LSV Bias with asymmetrical and/or partial herding.

Table 2 is designed to illustrate the LSV bias in more general configurations, that is, with potentially asymmetric and/or partial herding. To simplify, we just present the value of HLSV in this table. The first three columns indicate the different configurations in terms of probability vector. The first column defines four different blocks, each containing three rows. The first block corresponds to no herding, and the following blocks consider partial herding with an increasing probability of no herding⁹. The first row of each block presents the symmetric case. In other rows, an increasing asymmetry is introduced with values of $\pi_{b,t} / \pi_{s,t}$ equal to 1 (no asymmetry), 1.5 (slight asymmetry) or 4 (strong asymmetry). The first row of the first block is then the case illustrated in table 1 with symmetrical and integral herding.

Theoretical values of LSV are again computed using relation (12). Because LSV is sensitive to the number of transactions, Table 2 reports computations for 10 and 50 transactions.

First, one can observe that the LSV bias decreases with the weight of no herding stocks. This result is easy to explain, as LSV is designed for the no herding case. When the probability of no herding grows, the adjustment factor required (AFC) comes closer to the LSV adjustment factor AF and both converge as $\pi_{o,t}$ approaches one.

⁹ The “diagonal” structure of the table means that the fact that for a given set of probabilities of buy and sell herding a higher proportion of no herding stocks or/and higher asymmetry leads to a decrease in the mean herding level. For example, in the symmetrical case, with $p_{it} = 0.9$ in the buy side and $p_{it} = 0.1$ in the sell side, the mean herding level is 40% if herding is integral but decreases to 16% when the proportion of no-herding stocks became 80%.

Table 2: Theoretical values of LSV

The table reports theoretical values of LSV computed using relation (12). For convenience, the value p is fixed to 0.5. The first three columns (in bold) indicate the different configurations in terms of the probability set. The following corresponds to different herding level for 10 transactions. The first row is the configuration with symmetrical and integral herding; the following rows correspond to an increasing asymmetry and following blocks correspond to partial herding. For each value of $\pi_{o,t}$ varying degrees of asymmetry are considered: no asymmetry, slight asymmetry with $\pi_{b,t} / \pi_{s,t} = 1.5$ and strong asymmetry, where $\pi_{b,t} / \pi_{s,t} = 4$. The “diagonal” structure of the table is due to the fact that for one set of probabilities of buy and sell herding, a higher proportion of no herding stocks or/and higher asymmetry decreases the mean herding level.

			n=10					n=50				
Set of probabilities			Real herding values					Real herding values				
$\pi_{o,t}$	$\pi_{b,t}$	$\pi_{s,t}$	40%	30%	20%	10%	5%	40%	30%	20%	10%	5%
0%	50%	50%	27.70%	17.84%	8.89%	2.40%	0.61%	34.39%	24.39%	14.39%	4.85%	1.35%
	60%	40%		17.98%	9.08%	2.48%	0.63%		24.39%	14.41%	4.94%	1.40%
	80%	20%				3.39%	0.93%				5.53%	1.85%
20%	40%	40%		20.17%	10.53%	2.95%	0.76%		25.51%	15.51%	5.66%	1.65%
	48%	32%		20.21%	10.69%	3.05%	0.79%		25.51%	15.51%	5.72%	1.70%
	64%	16%				3.99%	1.14%				6.10%	2.15%
50%	25%	25%			13.85%	4.44%	1.20%			17.19%	7.20%	2.43%
	30%	20%				4.54%	1.24%				7.20%	2.47%
	40%	10%					1.70%					2.76%
80%	10%	10%					2.63%					3.87%
	12%	8%					2.67%					3.88%
	16%	4%					2.91%					3.89%

Second, the LSV bias reduces also with asymmetry. Technically, the LSV1 expectation for a given herding level increases with asymmetry, and as the AF of LSV does not change, the

downward bias is reduced. With the same level of global herding, higher asymmetry increases the weight of one type of herding stocks, say, overbought stocks, while the herding level in this group is reduced (as $h_t^b = \frac{h_t}{2\pi_{b,t}}$, see relation (2)). It decreases the weight of sell-side herding stocks, while, for the same reason, the herding level in this group is higher. Because the required adjustment factors for herding groups (AFB and AFS) are decreasing with the herding level, AFB becomes higher and AFS becomes lower. The fact that AFB has a higher weight in AFC (see relation (7)) implies that the former effect dominates. Then, for the same herding level, the required adjustment factor AFC is increasing with asymmetry. The spread between AFC and AF, in other words, the LSV bias, is therefore reduced by asymmetry. As with a higher number of transactions, the required adjustment is reduced and becomes very weak for high herding values, the effect of introducing asymmetry remains but is less tangible than it is for 10 transactions.

4. The FHW herding measure: description, bias and correction

4.1 Description

In their paper, FHW (2007) conceive a new indicator that enables the correction of the LSV bias. They use square values instead of absolute values to avoid the compensation effect between both types of herding. Their indicator can be seen as the spread between the empirical variance and the expected one under no herding.

For the stock i in period t , they compute

$$H_{ii}^2 = \left(\left(\frac{b_{it}}{n_{i,t}} - p_t \right)^2 - E_{h_0} \left(\frac{b_{it}}{n_{i,t}} - p_t \right)^2 \right) \frac{n_{i,t}}{n_{i,t} - 1} = \left(\left(\frac{b_{it}}{n_{i,t}} - p_t \right)^2 - p_t(1 - p_t) / n_{i,t} \right) \frac{n_{i,t}}{n_{i,t} - 1}. \quad (13)$$

The terms $H_{i,t}^2$ are added up in any subgroup g of the stock period:

$$H_g^2 = 1/n_g \sum_{(i,t) \in g} H_{i,t}^2. \quad (14)$$

The herding level in this group is estimated by the square root of H_g^2 :

$$HFHW_g = \sqrt{H_g^2}.$$

4.2 Intuition of the measure and of its bias

When the herding level increases, the gap increases between the two extreme components of the binomial law. This implies a greater level of the variance for the whole distribution.

In their Monte Carlo simulations, FHW always consider that the probabilities to be excessively sold and to be excessively bought are equal to $1/2$. For varying values of other parameters (herding level, number of transactions, and number of stocks/period), they show that their measure is unbiased and has good properties (as regards the power and the size of the tests).

However, it is easy to show that their measure is correct only under the hypothesis that $\pi_{b,t} = \pi_{s,t} = 1/2$ and is biased otherwise. The problem results from the aggregation step required to obtain the mean level of herding (relation (14)). More precisely, the bias stems from the fact that the squared root of a sum is not -except in very special cases- the sum of square root values.

FHW show that $H_{i,t}^2$ is unbiased for a stock-period for a very general model, where the herding is defined at a stock level $h_{i,t}$ ¹⁰.

That implies that

$$H_{i,t}^2 = h_{i,t}^2 + u_{i,t},$$

where $E(u_{i,t})=0$

For example, if we try to evaluate mean herding for all stock in period t, we then have

$$H_t^2 = \frac{1}{I} \sum_{i=1}^I H_{i,t}^2 = \frac{1}{I} \sum_{i=1}^I h_{i,t}^2 + \frac{1}{I} \sum_{i=1}^I u_{i,t},$$

where I is the total number of stocks in period t.

For convenience, we use the probability limit of this mean: with a sufficiently large sample, we can neglect the last term, which is supposed to converge to zero. We assume that the empirical weight of these groups converges to the probability (respectively $\pi_{b,t}$, $\pi_{0,t}$ and $\pi_{s,t}$).

We can rewrite

$$\text{plim } H_t^2 \simeq \pi_{b,t} (h_t^b)^2 + \pi_{s,t} (h_t^s)^2.$$

Therefore, if we replace the levels of sell-side and buy-side herding estimations by their value as a function of global herding value (relation (2)), we have

$$H_t^2 = \pi_{b,t} \left(\frac{h_t}{2\pi_{b,t}} \right)^2 + \pi_{s,t} \left(\frac{h_t}{2\pi_{s,t}} \right)^2 = h_t^2 \left(\frac{1}{4\pi_{b,t}} + \frac{1}{4\pi_{s,t}} \right). \quad (15)$$

The squared root of H_t^2 is then

$$H_t = \sqrt{\frac{1}{4\pi_{b,t}} + \frac{1}{4\pi_{s,t}}} h_t. \quad (16)$$

¹⁰ In their model, each stock can have its own herding level.

It is obvious that the only “good” case for FHW is the configuration they retain in their paper, that is, the case of integral and symmetrical herding. In other cases, their measure leads to bias in the herding estimation.

The bias is easy to correct theoretically, knowing the set of probabilities¹¹. From relation (16) it easily follows that the corrected expression of this indicator HFHWC should be

$$HFHWC_g = \frac{1}{\sqrt{\frac{1}{4\pi_{b,t}} + \frac{1}{4\pi_{s,t}}}} HFHW_g. \quad (17)$$

4.3. Illustration of the bias

Table 3 illustrates the FHW bias. The first three columns indicate the different configurations in terms of probability vector. The first column defines four different blocks, the first (containing five rows) corresponding to no herding and the following considering partial herding with an increasing probability of no herding¹². The first row of each block corresponds to the symmetric case. In other rows, an increasing asymmetry is introduced. The first row of the first block is then the FHW case with symmetrical and integral herding.

¹¹ One can imagine that it may be appropriate to use the sum of square root individual herding estimation instead of the square root of the sum. This solution seems to be more correct, but the resulting estimation might be disturbed when the variance of the terms u is high.

¹² The “diagonal” structure of the table means that the fact that for a given set of probabilities of buy and sell herding a higher proportion of no herding stocks or/and higher asymmetry leads to a decrease of the mean herding level. For example, in the symmetrical case, with $p_{it} = 0.9$ in the buy side and $p_{it} = 0.1$ in the sell side, the mean herding level is 40% if herding is integral but decreases to 16% when the proportion of no herding stocks became 80%.

Table 3: Theoretical values of FHW

Theoretical values of FHW are computed using relation (17), where HFHWC is equal to real herding values. The first columns (in bold) indicate the different configurations in terms of the probability set. The first row is the configuration considered by FHW with symmetrical and integral herding; the following rows correspond to an increasing asymmetry, and the following blocks correspond to partial herding. For each value of $\pi_{o,t}$, varying degrees of asymmetry are considered: no asymmetry, slight asymmetry with $\pi_{b,t}/\pi_{s,t}=1.5$ and strong asymmetry where $\pi_{b,t}/\pi_{s,t}=4$. For each value of $\pi_{o,t}$ varying degrees of asymmetry are considered: no asymmetry, slight asymmetry with $\pi_{b,t}/\pi_{s,t}=1.5$ and strong asymmetry, where $\pi_{b,t}/\pi_{s,t}=4$. The “diagonal” structure of the table is due to the fact that for one set of probabilities of buy and sell herding, a higher proportion of no-herding stocks and/or higher asymmetry decrease the mean herding level.

Set of probabilities			Real herding values				
$\pi_{o,t}$	$\pi_{b,t}$	$\pi_{s,t}$	h=40%	h=30%	h=20%	h=10%	h=5%
0%	50%	50%	40.00%	30.00%	20.00%	10.00%	5.00%
	60%	40%		30.62%	20.41%	10.21%	5.10%
	80%	20%				12.5%	6.25%
20%	40%	40%		33.54%	22.36%	11.18%	5.59%
	48%	32%		34.23%	22.82%	11.41%	5.71%
	64%	16%				13.97%	6.99%
50%	25%	25%			28.28%	14.14%	7.07%
	30%	20%				14.43%	7.20%
	40%	10%					8.83%
80%	10%	10%					11.18%
	12%	8%					11.41%
	16%	4%					13.98%

Theoretical values of FHW are computed using relation (17), where HFHWC is equal to real herding values¹³. Here, we do not need to specify the number of transactions: unlike LSV, the bias of FHW only depends on the probability vector.

Table 3 reveals interesting facts. First, we can easily observe that the FHW estimator overestimates herding intensity when there is asymmetric or/and partial herding. More precisely the effect of asymmetry and of the no herding probability are reversed.

Second, a higher level of asymmetry leads to greater bias. In the case of integral herding ($\pi_{o,t}=0$), the bias is very weak when the asymmetry is low. It increases with the degree of asymmetry. As expected, it is proportional to the real level of herding. When we consider partial herding, the bias rises sharply with the weight of no-herding stocks. For example, if herding affects only 20% of the stocks, as in the two last rows of Table 3, the theoretical value of FHW reaches more than twice the real herding level.

4.4. Comparison with LSV

A comparison between results reported in Tables 1, 2 and 3 shows that the herding characteristics influence in opposite ways the bias of LSV and FHW measures. First, even if this effect is slight, while a higher degree of asymmetry reduces the HLSV bias, it increases the overestimation of HFHW. Conversely, and this effect is more substantial, the LSV measure becomes more reliable when the weight of no-herding stocks is growing, whereas the HFHW bias is increasing.

¹³ The main properties of FHW measure were confirmed by Monte Carlo simulations, which are not reported here.

Finally, our bias analysis of both indicators shows that real herding values can be bounded by both indicators, HLSV as the minimum value and HFHW as the maximum value. However, as it can be interesting to know which range of values is likely to include the herding level, this property may not allow a precise estimation of herding: the spread between both measures is potentially very wide: for 10 transactions it can reach, in many cases, values higher than 10%.

5. Conclusion

Our main conclusion is that, if checking for the presence of herding in the financial markets is possible using LSV, measuring it accurately seems to be a harder task than it is usually considered in the literature to be.

In this work, we have shown that previous herding measures in the line of LSV are not as encompassing as their authors seem to consider. Notably, as illustrated in FHW, the well-known LSV measure is relevant only in the case of no herding.

One of the main contributions of this paper is to provide a theoretical rationale for this property. We use a simple descriptive model of herding that allows us to derive the exact value of adjustment term required to properly estimate herding. However, the correction we propose is not very tractable. It requires a prior knowledge of the herding level and the probabilities to belong to each category of stocks (sell-side, buy-side, no herding)¹⁴.

¹⁴ A current work shows that estimating herding level could be done using recursive methods: using the fact that the global adjustment term is decreasing with the herding level, one can compute the AFC term with the LSV estimation for the first step of a recursive estimation of LSVC. However, it is more difficult to estimate the probabilities or weights of each group of stocks.

In this paper, we also show that the measure proposed by FHW loses relevance when herding is not integral or not symmetric. Here again, a theoretical correction is proposed for more general cases. Our corrected indicator does not need to know the level of herding, and for this reason, it is probably easier to implement than LSV. But, as the LSV indicator, it requires to know the weights of each type of stocks.

Our work calls for further investigations. First, can our descriptive model be considered a good approximation of the reality? If it does, is it possible to provide a reliable estimation of these parameters? It is well known that the extraction of individual component of a mixture of laws is not easy and that this step can significantly increase the complexity in the evaluation process of herding.

In the plausible case of more than three groups, it is necessary to check by Monte Carlo simulations whether a reliable estimation can be obtained using our corrected measure that supposes only three groups. If simulations prove that the real number of groups of herding is important from an empirical point of view, the corrected measures have to be amended to include more groups. Therefore, the process of herding evaluation will be made much more complex with the prior estimation of the number of groups and of their weights.

All these questions suggest that there is much work left to be done concerning the empirical evaluation of herding behavior, and they undoubtedly offer a stimulating research agenda.

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