Preferences Yielding the "Precautionary Effect"

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Abstract

Consider an agent taking two successive decisions to maximize his expected utility under uncertainty. After his first decision, a signal is revealed that provides information about the state of nature. The observation of the signal allows the decision-maker to revise his prior and the second decision is taken accordingly. Assuming that the first decision is a scalar representing consumption, the *precautionary effect* holds when initial consumption is less in the prospect of future information than without (no signal). Epstein in (Epstein, 1980) has provided the most operative tool to exhibit the precautionary effect. Epstein's Theorem holds true when the difference of two convex functions is either convex or concave, which is not a straightforward property, and which is difficult to connect to the primitives of the economic model. Our main contribution consists in giving a geometric characterization of when the difference of two convex functions is convex, then in relating this to the primitive utility model. With this tool, we are able to study and unite a large body of the literature on the precautionary effect.

Key words: value of information; uncertainty; learning; precautionary effect; support function.

JEL Classification: D83

1 Introduction

Consider an agent taking two successive decisions to maximize his expected utility under uncertainty. As illustrated in Figure 1, after his first decision, a signal is revealed that provides information about the state of nature. The observation of the signal allows the decision-maker to revise his prior and the second decision is taken accordingly. Assuming that the first decision is a scalar representing consumption, the *precautionary effect* holds when initial consumption is less in the prospect of future information than without (no signal).

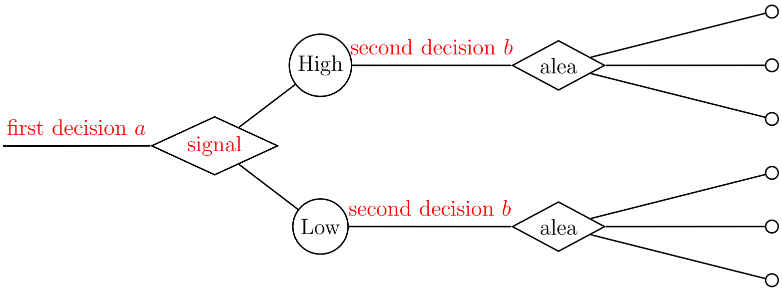

Figure 1: Decision with learning; agent takes decision a; a signal is revealed; agent takes decision b accordingly.

The example above is a stereotype of sequential decisions problems with learning were focus is put on comparison of the optimal initial decisions with different information structures. For instance, should we aim at more reductions of current greenhouse gases emissions today whether or not we assume some future improvement of our knowledge about the climate? Economic analysis has identified effects that go in opposite directions and make the conclusion elusive. This article proposes a characterization of utility functions such that the precautionary effect holds for all signals.

Seminal literature in environmental economics ((Arrow and Fisher, 1974), (Henry, 1974a,b)) focused on the irreversible environmental consequences carried by the initial decision and showed that the possiblity of learning should lead to less irreversible current decisions ("irreversibility effect"). Henry; Arrow and Fisher consider additive separable preferences, and so do (Freixas and Laffont, 1984), (Fisher and Hanemann, 1987), (Hanemann, 1989). Epstein in (Epstein, 1980) studies a more general nonseparable expected utility model, and derives a condition that identifies the direction of the precautionary effect. His contribution remains the most operative tool. Yet, the conditions under which this result holds are difficult to connect to the primitive utility model. Further contributions have insisted on the existence of an opposite economic irreversibility since environmental precaution imply sunk costs that may constrain future consumption ((Kolstad, 1996), (Pindyck, 2000), (Fisher and Narain, 2003)). Risk neutral preferences are studied in (Ulph and Ulph, 1997) for a global warming model. Assuming time separability of preferences, the papers (Gollier, Jullien, and Treich, 2000) and (Eeckhoudt, Gollier, and Treich, 2005) examine risk averse preferences. Gollier, Jullien, and Treich identified conditions on the second-period utility function for the possibility of learning to have a precautionary effect with and alternatively without the irreversibility constraint. By this latter, we mean that the domain of the second decision variable depends on the first decision.

The driving idea for linking the effect of learning and the value of information is the observation that, once an initial decision is made, the value of information can be defined as a *function* of that decision. This is the approach of Jones and Ostroy who define the value of information in this way in their paper (Jones and Ostroy, 1984) where they formalize the notion of flexibility in a sequential decision context, and relate its value to the amount of information an agent expects to receive. Whenever the *second-period value of information*

– namely the value of information measured after an initial commitment is made – is a monotone function of the initial decision, optimal initial decisions can be ranked. All this is recalled in Sect. 2, where we also extend the approach in (Epstein, 1980) and (Jones and Ostroy, 1984) to non necessarily finite sets.

In (Jones and Ostroy, 1984), the monotonicity property of the second-period value of information is related to convexity in the prior of a difference of maximal payoffs. This is convexity is far from being granted since the difference of two convex functions is generally not convex. Our main result consist in giving a general condition under which *a difference of maximal payoffs exhibits convexity in the prior*. This is the object of Sect. 3 where, first, we provide a geometric characterization and, second, carry it to the utility model. With this tool, we are able to study in Sect. 4 a large body of the literature on the precautionary effect.

2 The precautionary effect: statement and recalls

We first give a formal statement of the precautionary effect, then recall and extend some results in the literature, upon which we shall elaborate our main contribution in the next section.

We shall assume that all sets are Borel spaces, endowed with the Borel σ-algebra generated by the open subsets, that all mappings have appropriate measurability and integrability properties needed to perform mathematical expectations operations, and that all probabilities are regular to ensure the existence of conditional distributions (Bertsekas and Shreve, 1996), (Kallenberg, 2002).

2.1 Problem statement

Consider an agent taking two successive decisions as in Figure 1. The initial decision a is a scalar belonging to an interval $\mathbb{I}$ of $\mathbb{R}$; the following and final decision b belongs to a set $\mathbb{B}(a)$ which may depend on the initial decision[1] a ($\mathbb{B}(a)$ is a subset of a fixed set $\mathbb{B}$). Uncertainty is represented by states of nature $\omega \in \Omega$ with prior $\mathbb{P}$ on the Borel σ-field $\mathcal{F}$, and by a random variable $X : (\Omega, \mathcal{F}) \to (\mathbb{X}, \mathcal{X})$. Partial information on X is provided by means of a signal (random variable) $Y : (\Omega, \mathcal{F}) \to (\mathbb{Y}, \mathcal{Y})$. A utility function $U(a, b, x)$ is given, defined on $\mathbb{I} \times \mathbb{B} \times \Omega$. The expected utility maximizer solves[2]

$$\max_{a \in \mathbb{I}} \mathbb{E}\big[\max_{b \in \mathbb{B}(a)} \mathbb{E}[U\big(a, b, X\big) \mid Y]\big] . \tag{1}$$

Thus, the second decision b is taken knowing Y.

[1]This may materialize "irreversibility" of the initial decision.

[2]We shall always assume that, for the problems we consider, the *sup* is attained and we shall use the notation *max*.

The evaluation of expected utility right after the first decision a has been taken is conditional on the signal Y and defined as follows:

$$\mathbb{V}^Y(a) := \mathbb{E}\big[\max_{b\in\mathbb{B}(a)} \mathbb{E}[U\big(a,b,X\big) \mid Y]\big] \ . \tag{2}$$

With this notation, the program of the Y-informed agent is $\max_a \mathbb{V}^Y(a)$. Let us assume, for the sake of simplicity, that an optimal solution exists and is unique, denoted by $\bar{a}^Y$.

A signal Y is said to be *more informative* than a signal Y' if the σ-field $\sigma(Y) := Y^{-1}(\mathfrak{Y})$ contains $\sigma(Y')$. It is equivalent to say that Y' is a measurable function of Y, namely $Y' = f(Y)$ where $f : (\mathbb{Y}, \mathfrak{Y}) \to (\mathbb{Y}', \mathfrak{Y}')$.

The *precautionary effect* is said to hold whenever the optimal initial decision is lower with more information that is, if when the signal Y is more informative than the signal Y', then $\bar{a}^Y \leq \bar{a}^{Y'}$.

2.2 Precautionary effect and second-period value of the information monotonicity

We shall recall a sufficient condition under which the presence of learning affects the first optimal decision in a predictable way (see (Jones and Ostroy, 1984), (De Lara and Doyen, 2008, p.229), (De Lara and Gilotte, 2009)).

Let us compare the programs of the Y-informed and Y'-informed agent by writing

$$\max_a \mathbb{V}^Y(a) = \max_a \{\mathbb{V}^{Y'}(a) + \big(\mathbb{V}^Y(a) - \mathbb{V}^{Y'}(a)\big)\} \ . \tag{3}$$

It appears that the decision maker who expects more information optimizes the same objective as the less informed decision maker *plus* what we shall coin the *second-period value of information*

$$\Delta\mathbb{V}^{YY'}(a) := \mathbb{V}^Y(a) - \mathbb{V}^{Y'}(a) \tag{4}$$

which depends on his initial decision. The more-informed agent initial optimal decision achieves a trade-off: it can be suboptimal from the point of view of the less-informed decision maker but compensates for this by an increase of the second-period value of information.

Proposition 1 *Assume that the programs* $\max_a \mathbb{V}^Y(a)$ *and* $\max_a \mathbb{V}^{Y'}(a)$ *have unique*[3] *optimal solutions* $\bar{a}^Y$ *and* $\bar{a}^{Y'}$. *Whenever the second-period value of the information is a decreasing function of the initial decision, namely*

$$\Delta\mathbb{V}^{YY'} : a \mapsto \mathbb{V}^Y(a) - \mathbb{V}^{Y'}(a) \ \textit{ is decreasing,} \tag{5}$$

[3]Unicity is for the sake of simplicity. If the programs $\max_a \mathbb{V}^Y(a)$ and $\max_a \mathbb{V}^{Y'}(a)$ do not have unique optimal solutions, denote by $\arg\max_a \mathbb{V}^Y(a)$ and $\arg\max_a \mathbb{V}^{Y'}(a)$ the sets of maximizers. If $\Delta\mathbb{V}^Y(a)$ is decreasing, the upper bounds of these sets can be ranked: $\sup\arg\max_a \mathbb{V}^Y(a) \leq \sup\arg\max_a \mathbb{V}^{Y'}(a)$. If $\Delta\mathbb{V}^Y(a)$ is strictly decreasing, we obtain that $\sup\arg\max_a \mathbb{V}^Y(a) \leq \inf\arg\max_a \mathbb{V}^{Y'}(a)$ that is, $\bar{a}^Y \leq \bar{a}^{Y'}$ for any $\bar{a}^Y \in \arg\max_a \mathbb{V}^Y(a)$ and any $\bar{a}^{Y'} \in \arg\max_a \mathbb{V}^{Y'}(a)$.

then

$$\bar{a}^Y \leq \bar{a}^{Y'} . \tag{6}$$

In the case where Y' is constant (no information since $\sigma(Y')$ is the trivial σ-field $\{\emptyset, \Omega\}$), the effect of learning is *precautionary* in the sense that the optimal initial decision is lower with information than without.

2.3 Second-period value of information and Epstein functional

We extend the approach of Epstein in (Epstein, 1980) and (Jones and Ostroy, 1984) to non necessarily finite sets. We denote by $\mathcal{P}(\mathbb{X})$ the Borel space of probability measures on $\mathbb{X}$, with its Borel σ-field; the same holds for $\mathcal{P}(\mathbb{Y})$.

Following Epstein, let us define what we shall coin the *Epstein functional* by[4]:

$$J(a, \rho) := \sup_{b \in \mathbb{B}(a)} \mathbb{E}_\rho\big[U\big(a, b, \cdot\big)\big] = \sup_{b \in \mathbb{B}(a)} \int_{\mathbb{X}} U\big(a, b, x\big) d\rho(x) , \quad \forall \rho \in \mathcal{P}(\mathbb{X}) . \tag{7}$$

Denote by ν and ν' the unconditional distributions of the signals Y and Y' on their image set $\mathbb{Y}$. The conditional distribution of X knowing Y is a mapping $\mathbb{P}_X^Y : \mathbb{Y} \to \mathcal{P}(\mathbb{X})$.

The following result may be found in (Jones and Ostroy, 1984). We give its proof in the general case of non necessarily finite sets.

Proposition 2 *Assume that*

1. *for any pair of initial decisions* $a_1 \geq a_0$, $\rho \in \mathcal{P}(\mathbb{X}) \mapsto J(a_1, \rho) - J(a_0, \rho)$ *is convex (resp. concave),*
2. Y *is more informative than* Y'.

Then, the value $\Delta\mathbb{V}^{YY'}(a) = \mathbb{V}^Y(a) - \mathbb{V}^{Y'}(a)$ *of substituting* Y *for* Y', *is increasing (resp. decreasing) with initial decision* $a \in \mathbb{I}$. *Hence, Proposition 1 applies.*

Proof. We have:

$$\begin{aligned}
\mathbb{V}^Y(a) &= \mathbb{E}_{\mathbb{P}}\big[\max_{b \in \mathbb{B}(a)} \mathbb{E}_{\mathbb{P}}[U\big(a, b, X\big) \mid Y]\big] \text{ by definition (2)} \\
&= \mathbb{E}_\nu\big[\max_{b \in \mathbb{B}(a)} \mathbb{E}_{\mathbb{P}_X^Y}[U\big(a, b, \cdot\big)]\big] \text{ by using the conditional distribution} \\
&= \mathbb{E}_\nu\big[J(a, \mathbb{P}_X^Y)\big] \text{ by (7).}
\end{aligned}$$

Let $a_1 \geq a_0$, and suppose that $\varphi(\rho) = J(a_1, \rho) - J(a_0, \rho)$ is convex in $\rho \in \mathcal{P}(\mathbb{X})$. We have

$$\begin{aligned}
\Delta\mathbb{V}^{YY'}(a_1) - \Delta\mathbb{V}^{YY'}(a_0) &= \mathbb{E}_\nu\big[J(a_1, \mathbb{P}_X^Y) - J(a_0, \mathbb{P}_X^Y)\big] - \mathbb{E}_{\nu'}\big[J(a_1, \mathbb{P}_X^{Y'}) - J(a_0, \mathbb{P}_X^{Y'})\big] \\
&= \mathbb{E}_\nu\big[\varphi(\mathbb{P}_X^Y)\big] - \mathbb{E}_{\nu'}\big[\varphi(\mathbb{P}_X^{Y'})\big] \geq 0
\end{aligned}$$

since Y is more informative than Y'. Indeed, it is known (see (Dellacherie and Meyer, 1975), (Artstein and Wets, 1993), (Artstein, 1999)) that if Y is more informative than Y', then $\mathbb{E}_\nu[\varphi(\mathbb{P}_X^Y)] \geq \mathbb{E}_{\nu'}[\varphi(\mathbb{P}_X^{Y'})]$ for all convex function φ on $\mathcal{P}(\mathbb{X})$.[5] The converse holds for concave. □

[4] We denote by $U(a, b, \cdot)$ the mapping $\mathbb{X} \to \mathbb{R}$ given by $x \mapsto U(a, b, x)$.

[5] Notice that $\mathbb{E}_\nu[]$ denotes a mathematical expectation taken on the probability space $(\mathbb{Y}, \mathfrak{Y}, \nu)$.

3 Conditions on the primitive utility for the precautionary effect

Epstein's condition for the precautionary effect in (Epstein, 1980) relies upon convexity (or concavity) of $\rho \mapsto \frac{\partial J}{\partial a}(\bar{a}^Y, \rho)$. In the same vein, Jones and Ostroy rely upon convexity of the mapping $\rho \in \mathcal{P}(\mathbb{X}) \mapsto J(a_1, \rho) - J(a_0, \rho)$ in (Jones and Ostroy, 1984). With the expression (7) of $J(a, \rho)$, it is not easy to see how this relates to the primitive of the model, namely the utility U.

We shall proceed in two steps to characterize utility functions such that $\rho \in \mathcal{P}(\mathbb{X}) \mapsto J(a_1, \rho) - J(a_0, \rho)$ is convex. The difficulty comes from the fact that this latter function is the difference of two convex functions, hence has no reason to be convex. We shall, first, provide a geometric characterization and, second, carry it functionally to the utility model by means of so-called *support functions*.

3.1 A geometric characterization

The following characterization of when $\rho \in \mathcal{P}(\mathbb{X}) \mapsto J(a_1, \rho) - J(a_0, \rho)$ is convex relies upon the notion of sum of subsets of a vector space. Recall that, for any subsets Λ_1 and Λ_2 of a vector space, $\Lambda_1 + \Lambda_2 = \{x_1 + x_2 \ , \ x_1 \in \Lambda_1 \text{ and } x_2 \in \Lambda_2\}$ is their so called direct sum, or *Minkowsky* sum.

Let us define, for any initial decision $a \in \mathbb{I}$,

$$\Lambda^-(a) := \{f : \mathbb{X} \to \mathbb{R} \mid \text{ there exists } b \in \mathbb{B}(a) \text{ such that } f(x) \leq U(a, b, x) \ , \quad \forall x \in \mathbb{X}\} \tag{8}$$

the set of maximal possible random rewards when the initial decision is a. Our first main result is the following.

Proposition 3 *Let $a_1 > a_0$. If there exists a subset K of functions defined on $\mathbb{X}$ such that*

$$\Lambda^-(a_1) = \Lambda^-(a_0) + K \ , \tag{9}$$

then $\rho \in \mathcal{P}(\mathbb{X}) \mapsto J(a_1, \rho) - J(a_0, \rho)$ is convex. Hence, the first hypothesis of Proposition 2 is satisfied.

Proof. The proof comes from the observation that the Epstein functional $J(a, \rho)$ is a so-called *support function*[6] as a function of its argument ρ. Recall that, to any set Λ of bounded measurable functions $f : \mathbb{X} \to \mathbb{R}$ is attached the support function σ_Λ, defined on the Banach space of finite signed measures on $\mathbb{X}$, by:

$$\sigma_\Lambda(\rho) := \sup_{\lambda \in \Lambda} \int_{\mathbb{X}} \lambda(x) d\rho(x) \ . \tag{10}$$

Indeed, (7) may be written as

$$J(a, \rho) = \sup_{b \in \mathbb{B}(a)} \mathbb{E}_\rho\big[U\big(a, b, \cdot\big)\big] = \sup_{\lambda \in \Lambda(a)} \mathbb{E}_\rho\big[\lambda\big] = \sigma_{\Lambda(a)}(\rho) \ , \tag{11}$$

[6] It is well known that an expected utility is convex in the prior. It seems less noticed that the expected utility, seen as a function of the prior, is the support function of the set of payoffs indexed by actions.

where

$$\Lambda(a) := \{f : \mathbb{X} \to \mathbb{R} \mid \text{ there exists } b \in \mathbb{B}(a) \text{ such that } f(x) = U(a,b,x) , \quad \forall x \in \mathbb{X}\} . \tag{12}$$

Now, since ρ belongs to cone of positive measures ($\rho \in \mathcal{P}(\mathbb{X})$), we also have that $J(a,\rho) = \sigma_{\Lambda(a)}(\rho) = \sigma_{\Lambda^-(a)}(\rho)$, because the polar cone of nonnegative functions on $\mathbb{X}$ is the set of measures (Aubin, 1982, p.31,p.107).

Support functions have the nice property to transform a Minkowsky sum into a sum of functions (Aubin, 1982):

$$\sigma_{\Lambda_1+\Lambda_2} = \sigma_{\Lambda_1} + \sigma_{\Lambda_1} . \tag{13}$$

Thus, whenever $\Lambda^-(a_1) = \Lambda^-(a_0) + K$, we obtain that

$$\begin{aligned} J(a_1,\rho) - J(a_0,\rho) &= \sigma_{\Lambda^-(a_1)}(\rho) - \sigma_{\Lambda^-(a_0)}(\rho) \\ &= \sigma_{\Lambda^-(a_0)+K}(\rho) - \sigma_{\Lambda^-(a_0)}(\rho) \\ &= \sigma_{\Lambda^-(a_0)}(\rho) + \sigma_K(\rho) - \sigma_{\Lambda^-(a_0)}(\rho) = \sigma_K(\rho) . \end{aligned}$$

Hence, $\rho \mapsto J(a_1,\rho) - J(a_0,\rho) = \sigma_K(\rho)$ is convex since it is a support function.[7] □

3.2 Characterization of utility functions ensuring the precautionary effect

We shall now show how the above geometric characterization (9) translates into a condition on the utility function U.

Consider two initial decisions $a_1 > a_0$. To any mapping $\phi : \mathbb{B}(a_0) \to \mathbb{B}(a_1)$ between second decision sets, associate the following set of minimizers

$$\mathbb{B}_\phi(a_1,a_0,x) := \underset{b\in\mathbb{B}(a_0)}{\arg\min} \Big(U(a_1,\phi(b),x) - U(a_0,b,x) \Big) , \quad \forall x \in \mathbb{X} . \tag{14}$$

and

$$\mathbb{B}_\phi(a_1,a_0) := \bigcap_{x\in\mathbb{X}} \mathbb{B}_\phi(a_1,a_0,x) . \tag{15}$$

When this latter set is not empty, there exists at least one minimizer $b \in \mathbb{B}(a_0)$ of $U(a_1,\phi(b),x) - U(a_0,b,x)$ independent of the alea x. Our second main result is the following.

Proposition 4 *Assume that*

1. *the set* $\Phi(a_1,a_0) := \{\phi : \mathbb{B}(a_0) \to \mathbb{B}(a_1) \mid \mathbb{B}_\phi(a_1,a_0) \neq \emptyset\}$ *is not empty,*

[7] The condition $\Lambda^-(a_1) = \Lambda^-(a_0) + K$ is almost necessary for $J(a_1,\rho) - J(a_0,\rho)$ to be convex in $\rho \in \mathcal{P}(\mathbb{X})$. Indeed, (Hiriart-Urrurty and Lemaréchal, 1993, p. 92-93) states that the closed convex hull $\overline{co}(\sigma_{\Lambda_2} - \sigma_{\Lambda_1})$ is equal to $\sigma_{\Lambda_2 \overset{\star}{-} \Lambda_1}$, where the *star-difference* $\Lambda_2 \overset{\star}{-} \Lambda_1 := \{x \mid x + \Lambda_1 \subset \Lambda_2\}$. Recall that the *closed convex hull* $\overline{co}f$ of a function f minorized by an affine function is the largest closed convex function minorizing f.

2. to any $b_1 \in \mathbb{B}(a_1)$ can be associated at least one $\phi \in \Phi(a_1, a_0)$ and one $b_0 \in \mathbb{B}_\phi(a_1, a_0)$ such that $b_1 = \phi(b_0)$.

Then there exists a subset K of functions defined on $\mathbb{X}$ such that $\Lambda^-(a_1) = \Lambda^-(a_0) + K$. Hence, the assumption of Proposition 3 is satisfied.

Proof. Let us define the set of functions $K = \cup_{\phi\in\Phi(a_1,a_0)} K_\phi$, where

$$K_\phi = \{x \in \mathbb{X} \mapsto U(a_1, \phi(b), x) - U(a_0, b, x) \text{ for } b \in \mathbb{B}_\phi(a_1, a_0)\} . \tag{16}$$

1. We first show the inclusion $\Lambda^-(a_1) \supset \Lambda^-(a_0) + K$. Indeed, for any $\phi \in \Phi(a_1, a_0)$ and $b \in \mathbb{B}(a_0)$, we have by definition of K_ϕ and $\mathbb{B}_\phi(a_1, a_0)$:

$$k(x) \leq U(a_1, \phi(b), x) - U(a_0, b, x) , \quad \forall x \in \mathbb{X} , \quad \forall k \in K_\phi .$$

Hence, $k(x)+U(a_0, b, x) \leq U(a_1, \phi(b), x)$. Thus, $\Lambda(a_0)+K \subset \Lambda^-(a_1)$, and therefore $\Lambda^-(a_0)+K \subset \Lambda^-(a_1)$.

2. We now show the reverse inclusion $\Lambda^-(a_1) \subset \Lambda^-(a_0) + K$. Let $b_1 \in \mathbb{B}(a_1)$. By assumption, there exist $\phi \in \Phi(a_1, a_0)$ and $b_0 \in \mathbb{B}_\phi(a_1, a_0)$ such that $b_1 = \phi(b_0)$. We have that

$$U(a_1, b_1, x) = U(a_1, \phi(b_0), x) = U(a_0, b_0, x) + \underbrace{U(a_1, \phi(b_0), x) - U(a_0, b_0, x)}_{k(x)} \tag{17}$$

where the function k belongs to K_ϕ since $b_0 \in \mathbb{B}_\phi(a_1, a_0)$.

□

The following Corollary provides practical conditions on the utility function U which ensure that the assumptions of Propositions 4 and 3 are satisfied, hence that the first hypothesis of Proposition 2 is satisfied.

What is more, the first-order condition (19) has proximities with the second-order one in (Salanié and Treich, 2007). This opens the way for comparison between our approach and the invariance approach of Salanié and Treich.

Corollary 5 *Assume that the second decision variable b belongs to $\mathbb{B} = \mathbb{R}^n$ and that the minimizers in (14) are characterized by the first-order optimality condition*

$$\phi'(b)\frac{\partial U}{\partial b}(a_1, \phi(b), x) - \frac{\partial U}{\partial b}(a_0, b, x) = 0 , \quad \forall x \in \mathbb{X} . \tag{18}$$

Suppose that, to any vector $b_1 \in \mathbb{B}(a_1)$ can be associated at least one vector $b_0 \in \mathbb{B}(a_0)$ and one square matrix $M \in \mathbb{R}^{n\times n}$ such that

$$M\frac{\partial U}{\partial b}(a_1, b_1, x) - \frac{\partial U}{\partial b}(a_0, b_0, x) = 0 , \quad \forall x \in \mathbb{X} . \tag{19}$$

If, in addition, we have $b_1 + M(b - b_0) \in \mathbb{B}(a_1)$ for all b in a neighbourhood of b_0 in $\mathbb{B}(a_0)$,[8] *then the assumptions of Proposition 4 are satisfied.*

[8]This condition is meaningless if b_1 belongs to the interior of $\mathbb{B}(a_0)$. Hence this condition has to be verified only when an irreversibility constraint bites.

Proof. By assumption, to any $b_1 \in \mathbb{B}(a_1)$ can be associated at least one mapping defined by $\phi(b) = b_1 + M(b - b_0)$ in a neighbourhood of b_0 (and smoothly prolongated outside). The first-order optimality condition (18) attached to (14) admits the solution $b = b_0$ by (19).

□

4 Analysis of examples in the literature

We shall examine a large body of the literature, and see that Corollary 5 applies in all cases and explains the precautionary effect.

At first, we shall assume that the second decision set $\mathbb{B}(a) = \mathbb{B}$ does not depend on the initial decision a, to concentrate on the precautionary effect and to try to disentangle it from the irreversibility effect. Second, we shall relax this assumption and attempt to point out the impact that the second decision set $\mathbb{B}(a)$ indeed depends upon the initial decision a.

In the sequel, we consider two initial decisions $a_1 > a_0$.

4.1 Additive separable preferences

The case of additive separable preferences may be found in (Arrow and Fisher, 1974), (Henry, 1974a), (Epstein, 1980), (Freixas and Laffont, 1984), (Fisher and Hanemann, 1987), (Hanemann, 1989) and is formalized by

$$U(a, b, x) = u(a, x) + v(b, x) \ .$$

It can be seen that $\Lambda(a)$ defined in (12) may be written as

$$\Lambda(a_1) = \{u(a_1, x) + v(b, x) \ , \ b \in \mathbb{B}\} = u(a_1, x) - u(a_0, x) + \{u(a_0, x) + v(b, x) \ , \ b \in \mathbb{B}\} = K + \Lambda(a_0)$$

where K is the singleton $\{u(a_1, x) - u(a_0, x)\}$. Hence $J(a_1, \rho) - J(a_0, \rho) = \sigma_{u(a_1,x)-u(a_0,x)}(\rho) = \int_{\mathbb{X}} \big(u(a_1, x) - u(a_0, x)\big) d\rho(x)$ is linear in the prior ρ, hence both concave and convex. Thus, the precautionary effect holds true in a strong sense since the initial optimal decision does not depend on the amount of information the agent expects to receive.

The above analysis is confirmed by the first-order optimality condition (19) which is

$$M \frac{\partial v}{\partial b}(b_1, x) = \frac{\partial v}{\partial b}(b_0, x) \ , \quad \forall x \in \mathbb{X} \ .$$

Since $M = \mathrm{Id}_{n \times n}$ and $b_0 = b_1$ are solutions, the precautionary effect holds true.

However, when $\mathbb{B}(a)$ indeed depends upon a, we no longer have that $\Lambda(a_1) = K + \Lambda(a_0)$. We also observe that the additional conditions of Corollary 5 related to the irreversibility constraints, namely $\mathbb{B}(a_1) \subset \mathbb{B}(a_0)$ and $b_1 \in \mathbb{B}(a_1)$ for all b in a neighborhood of $b_1 \in \mathbb{B}(a_0)$, are not generaly satisfied and this may prevent the precautionary effect to hold true.

4.2 Risk neutral preferences

Examples in (Epstein, 1980; Ulph and Ulph, 1997) present the general structure

$$U(a,b,x) = u(a,b) + v(a,b)x = u(a,b) + \sum_{i=1}^{p} v_i(a,b)x_i \ .$$

The first-order optimality condition (19) is

$$\begin{cases} M\dfrac{\partial u}{\partial b}(a_1,b_1) & = & \dfrac{\partial u}{\partial b}(a_0,b_0) \\ M\dfrac{\partial v_i}{\partial b}(a_1,b_1) & = & \dfrac{\partial v_i}{\partial b}(a_0,b_0) \ , \quad i = 1,\ldots,p \ . \end{cases}$$

This is a system of $n+np$ equations with $n+n^2$ unknown (M, b_0). Thus, when the dimension p of the noise is less than the dimension n of the second decision variable, the precautionary effect generally holds true.

4.3 Risk averse preferences

We shall examine three models where preferences exhibit risk aversion.

A consumption-savings problem (Epstein, 1980)

A two-periods consumption-savings problem is modelled by

$$U(a,b,x) = u_1(w-a) + \beta u_2(ra-b) + \beta^2 u_3(bx) \ ,$$

with savings a, b, and irreversibility constraint $\mathbb{B}(a) = [0, ra]$.

The first-order optimality condition (19) is

$$M\beta x u_3'(b_1 x) - \beta x u_3'(b_0 x) = M u_2'(ra_1 - b_1) - u_2'(ra_0 - b_0) \ , \quad \forall x \in \mathbb{X} \ .$$

If there exists a solution (M, b_0), this implies that there must exist constants α, γ and δ such that u_3' satisfies an equation of the form

$$x u_3'(\alpha x) = \gamma x u_3'(x) + \delta \ , \quad \forall x \in \mathbb{X} \ .$$

A candidate is $u_3'(x) = x^{-\gamma}$, yielding a solution $M = \gamma$ and $b_0 = \alpha b_1$, with the compatibility condition $\gamma u_2'(ra_1 - b_1) - u_2'(ra_0 - \alpha b_1) + \delta = 0$. Hence, the precautionary effect holds true.

Global warming and emissions (Gollier, Jullien, and Treich, 2000)

With pollution emissions a, b, a two-periods model with benefits and costs of emitting pollutions is modelled by

$$U(a,b,x) = u(a) + v\big(b - x(a+b)\big) \ .$$

The first-order optimality condition (19) is

$$Mv'\big(b_1 - x(a_1 + b_1)\big) = v'\big(b_0 - x(a_0 + b_0)\big) \ , \quad \forall x \in \mathbb{X} \ .$$

If there exists a solution (M, b_0), this implies that there must exist constants α, β and M such that v' satisfies an equation of the form

$$v'(\alpha x + \beta) = Mv'(x) \ , \quad \forall x \in \mathbb{X} \ .$$

In this case, $b_0 = b_1 a_0 / a_1$. The utility $v(x) = \frac{\gamma}{1-\gamma}\left[\eta + \frac{x}{\gamma}\right]^{1-\gamma}$ in (Gollier, Jullien, and Treich, 2000) precisely satisfies $v'(\alpha x + \gamma\eta(\alpha - 1)) = \alpha^{-\gamma} v'(x)$, which explains why the precautionary effect holds true.

Eating a cake with unknown size (Eeckhoudt, Gollier, and Treich, 2005)

The following model from (Eeckhoudt, Gollier, and Treich, 2005) is qualified of the problem of "eating a cake with unknown size" in (Salanié and Treich, 2007):

$$U(a, b, x) = u(a) + v(b) + w(x - a - b) \ .$$

The first-order optimality condition (19) is

$$Mv'(b_1) - v'(b_0) = Mw'(x - (a_1 + b_1)) - w'(x - (a_0 + b_0)) \ .$$

If there exists a solution (M, b_0), this implies that there must exist constants β, κ and M such that w' satisfies an equation of the form $w'(x + \beta) = Mv'(x) + \kappa$. Then, we find that $\beta + a_1 + b_1 = a_0 + b_0$ and that $Mv'(b_1) - v'(b_0) + \kappa = 0$. Thus, β, κ and M must satisfy some compatibility constraint, so that the precautionary effect holds true.

5 Conclusion

We have provided general conditions for the precautionary effect to hold true, including a condition that bears directly on the primitive utility of the economic model. We have examined a large body of the literature, and seen how operative is this condition to explain the precautionary effect. Preferences yielding the precautionary effect for all signals appear to belong to a restricted class. This is related to the strong conditions, that we provide, needed to have a difference of maximal payoffs exhibit convexity in the prior. The connexion with the invariance approach of (Salanié and Treich, 2007) deserves to be studied and clarified.

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