Document de Travail
Working Paper

Lemna

EA 4272

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2009/09

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A Bayesian Real Option Approach to Patents and Optimal Renewal Fees

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This article aims at estimating the optimal profile of renewal fees patent offices should implement. It is at the crossroad of two strands of literature. The first strand is the theoretical literature analysing renewal fees as an optimal revelation mechanism. The second strand is the econometric literature developing real option models of patent renewal decisions to assess the value of patents. Using data from the French patent office, we find that there is little room to lower the social cost of patents without affecting the monetary incentives to apply for a patent and innovate. We show that a menu of optimally defined profiles helps to further discriminate among patents.

JEL Classification : O31, O34, O38
Keywords : Patent Renewal, Renewal Fees, Real Option, Bayesian Learning

1. Introduction

Renewal fees are a common and important feature of the patent system in most countries. Indeed, through the payment of renewal fees, patent offices provide patentees with the entitlement to choose, subject to a maximum period[1], the duration of protection actually granted to their inventions. However, one often notices a lack of interest of policy-makers in this facet of patent policy and a common but wrong wisdom that renewal fees play a limited role in the effectiveness of the patent system. An illustration of this lack of interest can be found in the diversity of patterns and frequencies of renewal fees charged by most patent offices. Looking first at the pattern of renewal fees, it is striking to note the lack of transparency with which renewal fees are set up. In France, for instance, after an experiment between 2001 and 2008 in which the profile of renewal fees was characterised by four stages, the patent office decided to re-implement the previous pattern which was more progressive but without giving any justification for such change. In the same way, in Italy, renewal fees were cancelled in 2006 and then re-introduced in 2007 again without explanation. In turn,

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[1] Art. 33 of the TRIPS Agreement sets a minimum period of protection "*the term of protection available shall not end before the expiration of a period of 20 years counted from the filing date*" but this provision is often considered as the maximum period of protection.

renewal fees may consist of a basic fee or they may consist, like in Japan, of a basic fee plus a fee for each claim. As no extra service is provided at payment of the renewal fee to maintain a patent, one could argue that this may be the ultimate example of a levy or a tax. Lastly, frequencies of payment may also differ. Although in most European countries payment applies annually, in some countries renewal fees are payable less frequently. In the United States, for instance, payment is required at 3.5, 7.5 and 11.5 years after issue but force is to note that such payment is not tied to specific milestones in the patent life cycle. Obviously, the guidelines for setting renewal fees need to be clarified. Indeed, though they are essentially set at defraying the operational costs of patent offices and at subsidizing access to the patent system for SMEs, inconsistencies of the fees could be detrimental to the efficiency of the patent system[2].

Despite this apparent lack of interest, the justification of patent renewal mechanisms is well documented from a theoretical point of view in the economic literature. In their seminal articles, Scotchmer (1999) and Cornelli and Schankerman (1999) argue that renewal mechanisms work as optimal revelation mechanisms when a patent applicant has private information about his invention; information that is observed neither *ex ante* nor *ex post* by the patent office. However, little is said about the exact monetary values at which patent offices should set renewal fees. A reason for this is that theoretical works generally focus on *ex ante* heterogeneity among inventions (i.e. heterogeneity prior a patent is applied for), whereas heterogeneity arising *ex post* from the stochastic nature of the rent associated with a patent is disregarded. Cornelli and Schankerman (1999) or Gans, King and Lampe (2004) acknowledge that the stochastic dynamics of the rent plays a crucial role in practice but it is treated in a simplified way.

From an empirical point of view, the importance of the stochastic dynamics of the rent has been stressed in the econometric literature. Among those econometric works that attempt to infer the value of patents from patents renewal decisions, the real option approach is probably the most convincing one. A key assumption of this approach is that, due to the stochastic dynamics of the rent, patent owners have imperfect but evolving information about the rent associated with their patents. As a result, the annual decision to renew a patent is assimilated to the exercise of a sequence of European type options; the exercise prices of which are the annual renewal fees whereas the stochastic rent is the underlying asset value.

[2] Rassenfosse & van Pottelsberghe (2008) estimate that fee policies adopted over the past three decades explain, to a significant extent, the rising propensity to patent.

Pakes (1986) has been the first to propose this approach and to subsequently develop an estimation method of patent value by using data on observed renewal decisions. More recently, Baudry and Dumont (2006) have used estimation results of a real option model of patent renewal decisions to simulate the impact of a change of renewal fees. However, these authors confine themselves to comparative simulations rather than to fully determine the optimal profile of renewal fees. A reason for this is that real option models of patent renewal decisions lack an explicit link between the value of the rent on the one hand and the social surplus on the other hand.

Clearly, there is a need for a model at the crossing point of the two strands of literature, i.e. combining both an analysis of patent renewals as a revelation mechanism and the assessment of the value of patents on the basis of a real option model. This article aims at developing such a model and at estimating the optimal profile of renewal fees. Two additional points are also addressed by the article. Firstly, should a sole optimal profile of renewal fees be proposed to patent applicants or should patent applicants rather face a menu of profiles and freely choose among the proposed profiles? Secondly, are standard Markovian processes used in real option models relevant to capture uncertainty affecting the dynamics of future markets or, should rather these standard processes be adapted to account for Bayesian learning? These two additional points deserve some comments.

Regarding the first point, i.e. whether similar rules should apply to all patents or not, it is considered as a recurrent debate in intellectual property. The “one size fits all” principle is emblematic of this debate. It refers to the standard statutory life limit of twenty years used by most countries to conform with TRIPS agreements. However, an important but often neglected characteristic of the patent system is that patents are self-screened through the renewal mechanism. In other words, if in theory the “one size fits all” system prevails, in practice a patent holder will decide whether to keep its patent in force or to let it fall into the public domain. This means that if patent applicants have perfect private information about the time path of the rent associated to their patents, then the renewal mechanism may be interpreted as a menu. More precisely, at the date of application, each applicant has to choose the duration of the patent on the one hand and the sum of renewal fees to be paid on the other hand among a set of proposed combinations. The renewal mechanism then discriminates *ex ante* between patents. Cornelli and Schankerman (1999) as well as Scotchmer (1999) are in line with this interpretation. Conversely, if applicants have no private information at the date of their patent application but learn about the value of the rent as time goes, then the renewal mechanism does not discriminate *ex ante* among patents. Discrimination occurs *ex post* and is contingent to the time path of the rent. Gans, King and Lampe (2004) are in line with this view. In practice, patent applicants have private but incomplete information at the date of patent application, a situation this article deals with. For this purpose, we denote by "one profile fits all" (in analogy with the "one size fits all" rule) the case of a sole profile of renewal fees proposed to all patent applicants. By contrast, we denote by "tailor-made profiles" the case of a menu of differentiated profiles of renewal fees among which patent applicants have to choose at the date of application. To our knowledge, "tailor-made profiles" have not yet been proposed nor investigated in the economic literature. They are intended to act as a relevant indirect revelation mechanism when patent applicants have initially private and incomplete information but learn more about the value of the rent as time goes whereas it is impossible or too costly for the patent office to directly obtain the corresponding information. An econometric test of how much could be gained in terms of social welfare by implementing an optimal "tailor-made profiles" mechanism rather than a "one profile fits all" mechanism is proposed in this article.

Regarding the second point, i.e. the question whether standard Markovian processes are relevant to capture the stochastic nature of the dynamics of the rent that accrues from a patent, it may look, at a first glance, rather technical. However, this question does have some economic background once some important assumptions underlying the use of standard Markovian processes in real option models are specified. To be more explicit, recall that the use of Markovian processes in real options is inspired by the practice of financial options. Financial options are derivatives, the underlying asset of which is exchanged on a market since a sufficiently long period. As a result, sufficiently long time series data are available to allow an estimation of the objective probability distribution of identically and independently random shocks affecting the dynamics of the underlying asset. One of the difficulties we are facing here is that the case of the rent associated with a patent does not fit into this context. By definition, a patent involves a new invention so that the dynamics of the rent depends on at least one parameter, the exact value of which is not known with certainty. Uncertainty about the value of the parameters that govern the dynamics of the rent is represented by a subjective probability distribution on the set of possible values. Noisy messages arise from the observation of short term dynamics of the rent and help revising the subjective probability distribution of unknown parameters. Accordingly, this learning process affects expectations about the future values of the rent. The economic literature generally deals with this type of uncertain dynamics in a Bayesian way. Though the importance of learning for the dynamic analysis of patents is outlined by Pakes (1986), Cornelli and Schankerman (1999) or Gans, King and Lampe (2004), it has never been explicitly modelled in a Bayesian way[3]. Therefore, our article goes one step further and proposes an adaptation of the Bayesian treatment of learning to real options models of patent renewal decisions that mixes Bayesian and Markovian dynamics.

The article is structured as follows. Part 2 introduces the model in a general setting. The focus is on obtaining a real option model that provides a realistic representation of patent renewal decisions and allows for social welfare considerations. Formal definitions of the "one profile fits all" and "tailor-made profiles" optimal mechanisms are proposed. The addition of a revenue-generating constraint for the patent office is also examined. Part 3 presents the econometric model. An extensive discussion of the modelling of the Bayesian dynamics of the rent is proposed. The probability distribution of the optimal date of patents withdrawal is then obtained and serves as a basis for estimating the model. Data from the French patent office covering the period 1970-2006 are used. Estimation results are presented in Part 4. They include an estimate of the social cost per patent compared to the monetary incentive to innovate measured by the option value per patent. We also estimate the optimal "one profile fits all" renewal fees and an optimal "tailor-made profiles" menu with two alternative choices. Social gains of the two systems are compared and the incidence of a self-financing constraint for the patent office is examined. In Part 5, concluding remarks summarise the key findings and suggestions for orienting patent offices' practices are made.

2. Definition of an optimal profile of patents renewal fees

An optimal profile of patent renewal fees is a profile that maximises or minimises some objective function of the patent office. The case of a social welfare maximising patent office is more specifically of interest. However, the decision to apply for a patent and then, to keep it in force in order to deter competitors from freely copying the invention is under the control of the inventor. The most recent approach to the modelling of this decision is based on the real option theory. In Section 2.1, we adapt the real option approach to obtain a model of patent renewal decisions more specifically convenient for the analysis of choice of optimal renewal fees by the patent office. Two types of profiles are examined. The first type is the usual "one

[3] An exception is Crampes and Langinier (1998). These authors analyse strategic information disclosure in the renewal of patents with a Bayesian game between a patent holder and challengers but do not consider a learning process about the rent associated with the patent by the patent holder itself.

profile fits all" mechanism introduced in Section 2.2. The second type, presented in Section 2.3, is a more complex menu of "tailor-made profiles" that act as an incentive mechanism.

2.1. Patent renewal decisions as options

A patent typically entitles its owner an exclusive right for a limited period to stop any third party from making, using, or selling the object of the patented invention without his permission. A common way to assess the value of a patent thus consists in defining its value as the additional discounted sum of monetary gains that accrue from this exclusive right. The existence of renewal fees that have to be paid to keep a patent in effect helps measuring the corresponding rent. Indeed, a sufficient condition for a patent to be renewed is that the annual rent exceeds the annual renewal fee. As a consequence, the decision to pay or not renewal fees conveys information about the value of the rent. This sufficient condition may also be a necessary condition when the time path of the rent is purely deterministic. Schankerman and Pakes (1986) have suggested an econometric method that builds on this property to estimate the distribution of the value of patent rights. If the time path of the rent is at least partly stochastic, it is no longer required that the annual rent exceeds the annual renewal fee for a patent to be profitably maintained. Indeed, as originally shown by Pakes (1986), the patent value then includes a speculative component and has to be defined as an option value. More precisely, a patent is assimilated to nested European call options with one year term, the total number of calls being fixed by the statutory life limit of patents. The underlying asset of these European call options is the patent's rent whereas the annual renewal fees correspond to the exercise prices of the different calls.

Though the real option approach to patent renewal decisions has proved very useful to assess the value of patents, it is not directly suitable to determine a socially optimal profile of patent renewal fees. As already outlined in the introduction, a reason for this is that no explicit link is made in this approach between the value of the rent on the one hand and the surplus analysis on the other hand. Our article tries to fill this gap by assuming thereafter that each patent entitles its owner a monopolist position on the market for a new good resulting from the patented invention[4]. To keep things computationally tractable, it is more specifically assumed that each buyer of the new good has a linear inverse demand function $p = p_0 - \eta\, q$

[4] As in Cornelli and Schankerman (1986) our presentation focuses on the case of a product innovation rather than a process innovation. This distinction is not a problem as such as a new process can be licensed and thus generate revenues in the same manner as described in our model.

where p_0 and η are invariant parameters whereas p and q respectively denote the price of the new good and the quantity purchased by each buyer. The number of buyers of this new good is supposed to follow a stochastic process N_t. Without loss of generality, the marginal cost of production of each new good is supposed to be zero. As a result, the monopoly rent received at time t by a patent holder is given by

$$R(N_t) = \left(p_0^2 / 4\,\eta\right) N_t \tag{1.a}$$

where N_t may be interpreted as a measure of market size. Meanwhile, the deadweight loss due to the monopoly position amounts to

$$L(N_t) = \left(p_0^2 / 8\,\eta\right) N_t \tag{1.b}$$

In the absence of patent protection, the monopoly position of an inventor could be contested by competitors through reverse engineering for instance. In the case of a competitor succeeding in entering the market, the market structure is supposed to shift to a Bertrand-Nash price equilibrium and the inventor's profit to vanish. The threat of entry is not explicitly formalised but captured by an exogenous probability λ that the monopolist's rent drops to zero. Though profits fall to zero because of entry, competitors are assumed to be interested in supplying the new good either because the good at stake can be used as a loss leader to boost the sales of other products or to lower the inventor's financial resources. In a discrete time context, the value of an invention in the absence of a patent system and prior the entry of a first competitor is then given by

$$V_A(N_t) = R(N_t) + \frac{(1-\lambda)\,\mathrm{E}_t\left[V_A\left(\tilde{N}_{t+1}\right)\right]}{1+\rho_t} \tag{2}$$

where E_t stands for the mathematical expectation conditional on the information available at time t and ρ_t is the interest rate at time t. Thereafter, a tilde above a variable is used to outline it is stochastic. The value function V_A is also the termination payoff of a patent owner when he decides not to renew his patent at time t. The alternative for the patent owner is to renew his patent for one more year by paying the renewal fee c_t in counterpart of the monopolist rent $R(N_t)$ with certainty at time t plus the discounted expected value of the opportunity to renew the patent again at time $t+1$. As long as a patent has not yet been withdrawn and before the statutory term limit T, the value V_B of the patented invention is given by the best alternative between renewing the patent for one more year and withdrawing the patent forever:

$$V_B(N_t, t, c_t, \cdots, c_T) = Max\left\{ R(N_t) - c_t + \frac{E_t\left[V_B(\tilde{N}_{t+1}, t+1, c_{t+1}, \cdots, c_T)\right]}{1+\rho_t} \ , \ V_A(N_t) \right\} \tag{3.a}$$

At the statutory term limit T, the patent owner no longer holds exclusive rights on the invention and it becomes available to commercial exploitation by others. This means that the patent owner only receives the monopolist rent $R(N_T)$ plus the continuation value associated with the end of exclusivity in counterpart of the renewal fee c_T.

$$V_B(N_T, T, c_T) = Max\left\{ R(N_T) - c_T + \frac{E_T\left[V_A(\tilde{N}_{T+1})\right]}{1+\rho_T} \ , \ V_A(N_T) \right\} \tag{3.b}$$

In accordance with the aim of the article to define an optimal profile of renewal fees, the value function V_B is explicitly expressed as a function of the sequence $\{c_t, \cdots, c_T\}$ of current and future renewal fees to be paid if the patent is renewed. Moreover, due to the existence of the statutory life limit T, the value function V_B also depends on time or, more precisely, on the time delay between the current date and the statutory life limit of the patent. The dynamic program (3) is solved backwards. The option value of the patent is the additional revenue generated by the legal protection compared with the absence of a patent system:

$$OV(N_t, t, c_t, \cdots, c_T) = V_B(N_t, t, c_t, \cdots, c_T) - V_A(N_t). \tag{4}$$

Given that the patent has not been withdrawn at time t, the optimal date of withdrawal of the patent is the random stopping time defined as

$$\tilde{\tau}^*(N_t, c_t, \cdots, c_T) = \inf\{\ \tau \in \{t, \cdots, T\}\ ;\ \tilde{N}_\tau \notin \Omega(\tau, c_\tau, \cdots, c_T)\ \} \tag{5}$$

where $\Omega(\tau, c_\tau, \cdots, c_T)$ is the optimal waiting region. At each date τ, this region is defined as the set of values of the rent so that renewing the patent generates a higher value that withdrawing the patent:

$$\Omega(\tau, c_\tau, \cdots, c_T) = \left\{ N \in IR^+ ;\ V_B(N, \tau, c_\tau, \cdots, c_T) > V_A(N) \right\} \tag{6}$$

Both the optimal stopping time and the option value are not only affected by the initial value of the rent but also by the entire sequence of renewal fees to be paid to keep the patent in effect. This is at the core of the definition of an optimal profile for renewal fees.

2.2 The "one profile fits all" optimal renewal fees

Common wisdom holds that while a few patents are very valuable, the majority is worthless. This difference in terms of patent quality is accounted for in econometric works that attempt to assess the value of patents by assuming heterogeneity as regards the initial rent that accrues from a patent at the date it is granted. By contrast, most existing patent renewal systems are based on a "one profile fits all" mechanism of renewal fees[5]. The main feature of such a profile is that it discriminates *ex post* between patents but not *ex ante*. Indeed, all patents face the same profile of renewal fees at the date they are granted but, depending on the bad or good fortune of the patented invention, they are withdrawn more or less early so that the total amount of renewal fees effectively paid substantially varies across patents. In such a context, an optimal profile of renewal fees may be broadly defined as a fixed sequence of renewal fees that balances the social cost of patents on the one hand and the incentives to innovate they generate on the other hand. A comprehensive characterisation of an optimal profile would thus require analysing how the decision to invest in R&D is affected by the option value of a patent which, in turn, depends on the renewal fees sequence. This analysis is beyond the scope of this article. The focus is rather made on the characterisation of a second-best optimal profile in the sense that it minimises the social cost implied by a patent with the constraint to provide a monetary incentive to innovate that amounts at least to an exogenously given value.

The social cost of a patent is defined recursively. When the patent has been withdrawn and a competitor has entered the market, the social cost amounts to zero because the market structure shift to a Bertrand-Nash price equilibrium. When the patent has been withdrawn but no competitor has yet entered the market, the social cost is given by the following value function, the expression of which is close to (2):

$$W_A(N_t) = L(N_t) + \frac{(1-\lambda)\,\mathrm{E}_t\left[W_A(\tilde{N}_{t+1})\right]}{1+\rho_t} \qquad (7.a)$$

Indeed, the current deadweight loss is incurred but there is a probability λ that it vanishes forever at the next date and a probability $1-\lambda$ that it lasts. As long as the patent has not yet been withdrawn, the social cost is finally given by

[5] The Japanese system based on a basic fee plus a per claim fee is a noticeable exception.

$$W_B(N_t, t, c_t, \cdots, c_T) = \begin{cases} L(N_t) + \dfrac{\mathrm{E}_t\left[W_B\left(\tilde{N}_{t+1}, t+1, c_{t+1}, \cdots, c_T\right)\right]}{1+\rho_t} & \text{if} \quad t < \tilde{\tau}^*(N_t, c_t, \cdots, c_T) \\ L(N_t) + \dfrac{\mathrm{E}_t\left[W_A\left(\tilde{N}_{t+1}\right)\right]}{1+\rho_t} & \text{if} \quad t = \tilde{\tau}^*(N_t, c_t, \cdots, c_T) \end{cases} \tag{7.b}$$

Expression (6) makes it explicit that the social cost of a patent depends on the renewal decision of the patent owner. Formally, the optimal profile is thus determined as the outcome of a Stackelberg game with an additional constraint. The patent office is assimilated to the leader and chooses the sequence of renewal fees that minimises the expected discounted sum of deadweight loss induced per a patent where expectation is computed at the date of application. For this purpose, the patent office needs to take into account the reaction function of patent holders as regards their decision to renew or not their patents. The optimal stopping time defined in (5) yields the reaction function for a patent holder facing the initial market size N_0 at the date of application. Though patents are assumed to be heterogeneous in terms of the initial market size N_0, the patent office does not discriminate among patents according to this variable. Instead, its aim is to implement the optimal "one profile fits all" system of renewal fess that minimises the expected discounted sum of deadweight losses where expectation is taken over all possible values of N_0. But, in doing so, the patent office tries to maintain the monetary incentive to innovate generated by the patent system at least equal to an exogenous level $\overline{V}$. The expected option value of a patent, where expectation is taken over the set of possible values of N_0, measures the monetary incentive to innovate provided by the patent system when there is no prior information about the initial market size N_0. The optimal profile then solves the following optimisation program

$$\underset{\{c_0, \cdots, c_T\}}{Min} \; E\left[W_B\left(\tilde{N}_0, 0, c_0, \cdots, c_T\right)\right]. \tag{8.a}$$

Subject to

$$\mathrm{E}\left[V_B\left(\tilde{N}_0, 0, c_0, \cdots, c_T\right)\right] \geq \overline{V} \tag{8.b}$$

If $\overline{V} = 0$, then constraint (8.b) may be interpreted as a participation constraint in the sense that, prior any information about the initial market size, an innovator anticipates he will apply for a patent. If $\overline{V}$ amounts to the actual expected option value of patents, then constraint (8.b) is aimed at making sure the optimal profile is Pareto-improving compared with the actual situation.

At this stage, it is important to take into account the fact that the revenue-generating pressures on patent offices can lead to changes in the social benefits of the patent system. As shown by Gans, King and Lampe (2004), a socially optimal structure of renewal fees would encourage the maximal number of applications while reducing effective patent length. However, when patent offices are required to be self-funding, resources constraints can distort the fee structure. However, these authors consider a simplified two-period model where the fee structure is limited to an application fee at the first period and a single renewal fee at the second period. Imposing a revenue-generating requirement in the optimisation model (8) is straightforward. If $\overline{F}$ denotes the per patent average cost incurred by the patent office, then the self-funding constraint is given by

$$\mathrm{E}\left[F\left(\tilde{N}_0, 0, c_0, \cdots, c_T\right)\right] \geq \overline{F} \tag{9.a}$$

where expectation is taken over all possible values of the initial market size N_0. The value function F is defined iteratively as follows

$$F\left(N_t, t, c_t, \cdots, c_T\right) = \begin{cases} c_t + \dfrac{\mathrm{E}_t\left[F\left(\tilde{N}_{t+1}, t+1, c_{t+1}, \cdots, c_T\right)\right]}{1+\rho_t} & \text{if} \quad t < \tilde{\tau}^*\left(N_t, c_t, \cdots, c_T\right) \\ c_t & \text{if} \quad t = \tilde{\tau}^*\left(N_t, c_t, \cdots, c_T\right) \end{cases} \tag{9.b}$$

Expression (9.b) yields the discounted sum of renewal fees that is expected at time t from a patent associated with a current market size N_t.

2.3. "Tailor-made optimal profiles" as an incentive mechanism

A key assumption underlying the "one profile fits all" definition of an optimal profile of renewal fees is that an innovator applies for patent without having any knowledge about the state of nature. In reality, it is doubtful that an inventor does not have prior information about the initial market size associated with his patented invention, notably because divergence in industry perspectives on the values and uses of patents has been documented by large-scale R&D surveys conducted over several decades. Moreover, patent applicants also have private information about how much money has been spend to develop an innovation and whether it may be considered as a drastic innovation or an incremental innovation, a feature that clearly influences the magnitude of demand which again is different across sectors. This means that the present "unitary" patent system is limited in its ability to account for the ways that patents are used and viewed in different sectors. But most importantly, this also means that it is not efficient for a social surplus maximising patent office to adopt a "one profile fits all" system when patent owners have private information. Instead, as shown by Cornelli and Schankerman (1999) and Scotchmer (1999), the patent office should rather try to design a renewal mechanism that aims at optimally screening patents. However, most of theoretical works that deal with such screening mechanisms disregard the stochastic nature of the dynamics of the rent associated to a patent. Stated another way, they consider that a patent owner is fully informed about the characteristics of his patent. The real option approach to patent renewal decisions allows a more realistic setting where a patent owner is only partially informed about the characteristics of the patent. Information is partial in the sense that the initial market size N_0 is known by the patent holder but not its exact future time path. The option to renew or not the patent is precisely aimed at introducing some flexibility in counterpart of this uncertainty. Because the role of uncertainty as regards the future values of the rent diminishes as the patent get closer to the statutory term limit, intuition suggests that renewal fees should vary with the age of the patent. Hence, what we try to determinate is a menu of renewal fees profiles in which an innovator will select a profile given the partial private information at his disposal at the date of his patent application.

For the sake of simplicity, the optimal design of a menu of renewal fees profiles is developed in the context of a discrete distribution of the initial market size N_0 over a finite set of I possible values. The probability of each possible value N_0^i $(i \in \{1, \cdots, I\})$ of the market size is denoted by $pr[N_0^i]$. The patent office is still assumed to minimise the expected social cost of a patent where expectation is taken over all possible values N_0^i $(i \in \{1, \cdots, I\})$ of the initial market size because the state of nature is a private information and is not known by the patent office. By doing so, the patent office now faces I participation constraints or Pareto improvement constraints. Each of these constraints is similar to (8.b) except that the expected values on the left hand side and right hand side are replaced by exact realisations for each possible value of N_0. Last but not least, incentive compatibility constraints have to be added. The optimal "tailor-made profiles" are thus obtained as the solution of the following optimisation program:

$$\underset{\{c_0^1, \cdots, c_T^1, \cdots, c_0^I, \cdots, c_T^I\}}{Min} \sum_{i=1}^{I} pr[N_0^i]\; W_B(N_0^i, 0, c_0^i, \cdots, c_T^i). \qquad (10.a)$$

Subject to

$$V_B\left(N_0^i, 0, c_0^i, \cdots, c_T^i\right) \geq \overline{V}^i \quad \forall i = 1, \cdots, I \tag{10.b}$$

$$V_B\left(N_0^i, 0, c_0^i, \cdots, c_T^i\right) \geq V_B\left(N_0^i, 0, c_0^j, \cdots, c_T^j\right) \quad \forall i = 1, \cdots, I \text{ and } \forall j = 1, \cdots, I \tag{10.c}$$

Constraints of type (10.b) are participation constraints (if $\overline{V}^i = 0$) or Pareto improvement constraints (if $\overline{V}^i$ is the current option value for patents of type i). Constraints of type (10.c) are incentive compatibility constraints. When patent offices are required to be self-funding, then the additional constraint reads

$$\sum_{i=1}^{I} pr\left[N_0^i\right] \; F\left(N_0^i, 0, c_0, \cdots, c_T\right) \geq \overline{F} \tag{11}$$

This additional constraint does not depart from (9) because, as assumed by Gans, King and Lampe (2004), the administrative cost of a patent is considered as fixed, regardless the type of the patent.

3. The econometric model

So far, the concepts and definitions of the optimal "one profile fits all" and "tailor-made profiles" of renewal fees have been introduced in a rather general setting. More specifically, no assumption has been made about the exact form of the stochastic process used to describe the dynamics of the market size. However, such assumptions are necessary to proceed with an econometric application. As a result, Section 3.1 first introduces the stochastic process of the market size. Short term shocks affecting the dynamics of the market size are treated as informative messages (though noisy) that convey information about key parameters of the long term dynamics. Therefore, an emphasis is made on Bayesian learning about long term tendencies of the market size. Probability distributions of interest from an econometric point of view are then derived and presented in Section 3.2. Firstly, the probability distribution of future values of the market size conditional on the observed current market size is derived from the stochastic process defined in Section 3.1. Secondly, the probability distribution of optimal stopping times, conditional on the sequence of renewal fees, is obtained. This probability distribution is at the core of the econometric method presented in section 3.3 for the estimation of the parameters involved in the model.

3.1. Bayesian learning and market size dynamics

A key idea in the analysis of patent renewals due to Pakes (1986) is that holding a patent is a bet on future but uncertain revenues. Accordingly, the dynamics of the associated rent is supposed to follow a Markovian stochastic process. Such a representation of the dynamics of the rent mimics the representation usually used for most financial or real assets. Markovian processes rely on the assumption that there exist identically and independently distributed random shocks that affect the evolution of the rent associated to a patent. Therefore, Markovian processes are adapted to the representation of risk affecting the dynamics of the rent but disregard a fundamental feature of the real option problem at stake here, namely uncertainty. Uncertainty does not refer to a situation where the value of the underlying asset changes as time goes but to a situation where the knowledge that economic agents have of this value changes with information over time. As information is often noisy, time is needed for a patent owner to learn about the correct value of the rent and to obtain a more accurate knowledge of it. The evolution of knowledge is represented by changes in subjective probabilities associated to each possible value of the rent. These changes obey Bayes' theorem so that the dynamics underlying the option problem departs from more standard Markovian processes and the resulting real option problem may be referred to as a Bayesian real option problem by contrast with the usual Markovian real option problems extensively treated in the literature (see e.g. Dixit and Pindyck, 1994).

The model of patent renewal choice developed in this article actually mixes Bayesian and Markovian dynamics[6]. Uncertainty affects the long term value N_{LT} of market size and, as a consequence, the long term value of the rent. Though the correct value of long term market size is invariant, it is not directly observed. The simple model considered here involves two possible long term market sizes denoted by $N_{LT}^{\max}$ and $N_{LT}^{\min}$ with $N_{LT}^{\min} < N_{LT}^{\max}$. The two corresponding scenarios are respectively referred to as the optimistic scenario ($N_{LT} = N_{LT}^{\max}$) and the pessimistic scenario ($N_{LT} = N_{LT}^{\min}$). The short term or observed market size N_t at time t is a discrete time stochastic process, the evolution of which is partly governed by the long term market size. We use the broad class of discrete time stochastic processes proposed by Baudry and Dumont (2006) to represent the link between short term and long term market sizes. Following Baudry and Dumont (2006), we assume that the time unit is one year and

[6] Mixes of Markovian and Bayesian processes have already been used to analyse the optimal allocation of time between activities with uncertain returns in a continuous time approach (Bolton and Harris, 1999; Moscarini and Smith, 2001). Kelly and Kolstad (1999) used a discrete time equivalent of these mix processes in their study of optimal investment strategies to curve global warming.

divide each annual period $[t, t+1]$ in M subintervals of equal length $\Delta t = 1/M$. The dynamics of the short term market size is then defined by

$$N_{t+\Delta t} = N_t \, e^{(2\,\Delta Z_t - 1)\,\Delta h} \tag{12.a}$$

with

$$\Delta Z_t = \begin{cases} 1 & \text{with probability} \quad p_t^{\max} \\ 0 & \text{with probability} \quad 1 - p_t^{\max} \end{cases} \quad \text{if} \quad N_{LT} = N_{LT}^{\max} \tag{12.b}$$

$$\Delta Z_t = \begin{cases} 1 & \text{with probability} \quad p_t^{\min} \\ 0 & \text{with probability} \quad 1 - p_t^{\min} \end{cases} \quad \text{if} \quad N_{LT} = N_{LT}^{\min} \tag{12.c}$$

and

$$p_t^{\max} = 1/\left(1 + \exp\left(\alpha + \beta\left(N_t - \overline{N}^{\max}\right)\right)\right) \tag{12.d}$$

$$p_t^{\min} = 1/\left(1 + \exp\left(\alpha + \beta\left(N_t - \overline{N}^{\min}\right)\right)\right) \tag{12.e}$$

Δh, $\overline{N}^{\max}$ and $\overline{N}^{\min}$ are positive parameters whereas α is a real parameter without predefined sign. The expected sign of β will be discussed latter on. The functional form used in (12.d) and (12.e) satisfies three important properties. Firstly, it is consistent with the description of a probability. Indeed, the values of $p_t^{\max}$ and $p_t^{\min}$ range between zero and one whatever the value of parameters and the value of the short term market size. Secondly, the likelihood of a positive shock (i.e. the probability that $\Delta Z_t = 1$) on the short term market size is higher (respectively lower) than the likelihood of a negative shock (i.e. the probability that $\Delta Z_t = 0$) if and only if $\alpha + \beta\left(N_t - \overline{N}\right) > 0$ (respectively $\alpha + \beta\left(N_t - \overline{N}\right) < 0$) with $\overline{N} = \overline{N}^{\max}$ or $\overline{N} = \overline{N}^{\min}$. The additional constraint $\beta > 0$ implies that the stochastic process described in (12) may be thought of as a mean reverting process with reversion to $\overline{N}^{\max} - (\alpha/\beta)$ or $\overline{N}^{\min} - (\alpha/\beta)$, depending on the correct scenario. In this sense, the long term market sizes associated to each scenario are respectively given by $N_{LT}^{\max} = \overline{N}^{\max} - (\alpha/\beta)$ and $N_{LT}^{\min} = \overline{N}^{\min} - (\alpha/\beta)$. The gap between the short term and the long term market sizes then clearly influences the dynamics of the short term market size. If $\beta = 0$, then the dynamic process described in (12) resumes to a basic random walk and the distinction between the two scenarios no longer matters[7]. Thirdly, as long as $\beta > 0$, the likelihood of a positive shock on

[7] Conversely, the case where $\beta < 0$ does not have much sense. It corresponds to what could be called a "mean repulsing" process (by analogy with the well known "mean reverting" process) with $N_{LT}^{\max}$ and $N_{LT}^{\min}$ as repulsing values of the short term market size.

the short term market size is always higher when the correct scenario is the optimistic one (i.e. $N_{LT}=N_{LT}^{\max}$) rather than the pessimistic one (i.e. $N_{LT}=N_{LT}^{\min}$). An important consequence of this third property is that shocks affecting the dynamics of the observed short term market size are noisy messages that convey information about the unobserved long term market size. This is a key element of the learning process.

A patent owner is supposed to have beliefs about what is the correct value of the long term market size only. Thereafter, we denote by X_t (respectively $1-X_t$) the subjective probability attributed by the patent owner at time t that the correct scenario is the optimistic (respectively pessimistic) one. In order to reflect the absence of prior information at the initial time $t=0$ we set $X_0=1/2$. Subjective probabilities are revised by implementing Bayes' theorem to the noisy information provided by the observation of the random shocks affecting the short term market size on each subinterval of length Δt. Using (12.b) (12.c) and Bayes' theorem more specifically yields

$$X_{t+\Delta t}=\begin{cases} X_t\, p_t^{\max}\big/\mathrm{Pr}_{X_t}^{\Delta Z=1} & \text{if} \quad \Delta Z_t=1 \\ X_t\left(1-p_t^{\max}\right)\big/\mathrm{Pr}_{X_t}^{\Delta Z=0} & \text{if} \quad \Delta Z_t=0 \end{cases} \tag{13.a}$$

where

$$\mathrm{Pr}_{X_t}^{\Delta Z=1}=X_t\, p_t^{\max}+\left(1-X_t\right)p_t^{\min} \tag{13.b}$$

$$\mathrm{Pr}_{X_t}^{\Delta Z=0}=X_t\left(1-p_t^{\max}\right)+\left(1-X_t\right)\left(1-p_t^{\min}\right) \tag{13.c}$$

are the unconditional probabilities of observing respectively a positive and a negative shock given the sole beliefs at time t and without full information as regards the correct scenario.

3.2. Probability distribution of shocks and optimal stopping times

All value functions in (2), (3), (7) and (9) involve mathematical expectations of future market size for which computation is not straightforward. Indeed, a distinctive feature of the stochastic process described in (13) compared to usual discrete time stochastic processes is that the probabilities of positive and negative shocks are themselves functions of the stochastic process. This difference apart, the usual tree form representation (Cox, Ross and Rubinstein, 1979) displayed in Figure 1 and used for the analysis of discrete time options applies to the dynamics of the number of positive shocks observed from time t until time $t+K\,\Delta t$.

Insert Figure 1

Let define $\Pr_t\left[\sum_{\tau=t}^{t+K\,\Delta t}\Delta Z_\tau = k\right]$ as the probability of observing k ($k \in \{0,\cdots,K\}$) additional positive shocks forwards on K subintervals of length Δt given the number Z_t of positive shocks already observed at time t. This probability is computed backwards with the following iterative formula

$$\Pr_t\left[\sum_{\tau=t}^{t+K\,\Delta t}\Delta Z_\tau = k\right] = \begin{cases} \Pi_K & \text{if} \quad k = K \\ \Pi_k & \text{if} \quad k \in \{1,\cdots,K-1\} \\ \Pi_0 & \text{if} \quad k = 0 \end{cases} \tag{14.a}$$

with

$$\Pi_K = \Pr_t\left[\sum_{\tau=t}^{t+(K-1)\,\Delta t}\Delta Z_\tau = K-1\right]\Pr_{X_{t+(K-1)\Delta t}}^{\Delta Z=1} \tag{14.b}$$

$$\Pi_t = \Pr_t\left[\sum_{\tau=t}^{t+(K-1)\,\Delta t}\Delta Z_\tau = k-1\right]\Pr_{X_{t+(K-1)\Delta t}}^{\Delta Z=1} + \Pr_t\left[\sum_{\tau=t}^{t+(K-1)\,\Delta t}\Delta Z_\tau = k\right]\Pr_{X_{t+(K-1)\Delta t}}^{\Delta Z=0} \tag{14.c}$$

$$\Pi_0 = \Pr_t\left[\sum_{\tau=t}^{t+(K-1)\,\Delta t}\Delta Z_\tau = 0\right]\Pr_{X_{t+(K-1)\Delta t}}^{\Delta Z=0} \tag{14.d}$$

Figure 1 illustrates formula (14.c). Observing two positive shocks on three subintervals of length Δt (i.e. observing $\sum_{\tau=t}^{t+3\,\Delta t}\Delta Z_\tau = 2$) arises as the outcome of two mutually exclusive events. The first event is a positive shock on the subinterval $[t+2\,\Delta t\,,t+3\,\Delta t]$ from a situation with $\sum_{\tau=t}^{t+2\,\Delta t}\Delta Z_\tau = 1$. The probability of this event is thus $\Pr_{X_{t+2\Delta t}}^{\Delta Z=1}$ times the probability of $\sum_{\tau=t}^{t+2\,\Delta t}\Delta Z_\tau = 1$. The second event is a negative shock on the subinterval $[t+2\,\Delta t\,,t+3\,\Delta t]$ from a situation with $\sum_{\tau=t}^{t+2\,\Delta t}\Delta Z_\tau = 2$. The probability of this event is $\Pr_{X_{t+2\Delta t}}^{\Delta Z=0}$ times the probability of $\sum_{\tau=t}^{t+2\,\Delta t}\Delta Z_\tau = 2$. The probability of observing $\sum_{\tau=t}^{t+3\,\Delta t}\Delta Z_\tau = 2$ is then obtained as the sum of these two events. Combined with expressions (12) and (13) defining the dynamics of market size and the dynamics of beliefs, the iterative formula (14) is more specifically of interest to compute the mathematical expectations characterising the value functions defined in (2), (3), (7) and (9) and solve the real option problem. In turn, as outlined by (5) and (6), the value functions defined in (2) and (3) directly affect the probability distribution of optimal stopping time.

We follow the method proposed by Baudry and Dumont (2006) for computing the exact probability distribution of optimal stopping times in a discrete time real option model of patent renewal decisions. Firstly, note that according to (12.a), the variation of the market size N_t from time t until time $t + K\,\Delta t$ only depends on the sum of positive shocks observed between the two dates. As a consequence, the market size N_t at time t may be expressed as a function $N(Z_t)$ of the total number $Z_t = \sum_{\tau=0}^{t} \Delta Z_\tau$ of positive shocks observed between the initial date $t = 0$ and the current date. This property also induces that the optimal waiting region in (6) may be defined indifferently in terms of market size N_t or in terms of total number Z_t of positive shocks. Secondly, the following indicator variable is defined:

$$II_t = \begin{cases} 1 & \text{if} \quad N(Z_t) \in \Omega(t\,, c_t\,, \cdots, c_T) \\ 0 & \text{if} \quad N(Z_t) \notin \Omega(t\,, c_t\,, \cdots, c_T) \end{cases} \tag{15}$$

where $\Omega(t\,, c_t\,, \cdots, c_T)$ is the optimal waiting region defined in (6). Subscript t is used to stress that II_t depends on all the variables that appear in the left hand side of (15), and more specifically on Z_t. Thirdly, we denote by Φ_t^{t-s} the probability of the event "the patent is renewed up to date t" conditional on the information available at date $t - s$ (i.e. conditional on the sum of observed positive shocks Z_{t-s}, on the current date $t - s$ and on the sequence of renewal fees $c_{t-s}\,, \cdots, c_T$). This probability is defined recursively as follows (See Baudry and Dumont, 2006):

$$\Phi_t^{t-s} = II_{t-s} \begin{cases} \sum_{k=1}^{M} \Pr_{t-s}\left[\sum_{\tau=0}^{M\,\Delta t} \Delta Z_{(t-s)+\tau} = k\right] \; II_{(t-s)+1} & \text{if} \quad s = 1 \\ \sum_{k=1}^{M} \Pr_{t-s}\left[\sum_{\tau=0}^{M\,\Delta t} \Delta Z_{(t-s)+\tau} = k\right] \; \Phi_t^{(t-s)+1} & \text{if} \quad s > 1 \end{cases} \tag{16}$$

M is the number of shocks, either positive or negative, between two renewal dates. The probability distribution of the optimal stopping time (i.e. the optimal date of withdrawal of a patent) is then given by:

$$\Pr\left[\tilde{\tau}^*(N_0\,, c_0\,, \cdots, c_T) = t\right] = \begin{cases} 1 - II_0 & \text{for} \quad t = 0 \\ \Phi_{t-1}^0 - \Phi_t^0 & \text{for} \quad 0 < t \le T \\ \Phi_T^0 & \text{for} \quad t = T + 1 \end{cases} \tag{17}$$

3.3. Data and estimation method

We use in this article data on French patent renewals for the period 1970-2006, broken down by patent application date (cohort)[8]. Our panel is balanced for about 16 cohorts (1970-1986). For each cohort, the data include the number of patent applications, whatever the nationality of the patentee and the number of patent renewals at each available age[9]. Unfortunately, no information was available either on individual patents or on the breakdown of cohorts by technology field or type of patent.

Renewal fee schedules in France are published in the *Official Journal* and were changed frequently during the sample period but most recent schedules apply to all patents regardless of the year in which the patent has been applied for. Renewal fees start at very low levels and rise monotonically as the patent ages (*Cf.* Figure 2). Since 2008, the French patent office (INPI) is using a renewal fee structure that is more progressive after seven years of experiment in which the profile of the renewal fees was characterised by four stages. For the needs of the article, these nominal renewal fees obtained in nominal domestic currency were converted to real costs by using the country's own implicit GDP deflator and then converted into euros.

Insert Figure 2

Looking now at patent renewals, one notices (Figure 3) that, contrary to all expectations, many patents are dropped out even for a small amount of renewal fees. Thus, more than 50% of the patents granted by the French patent office were dropped before the age of eight years and only 25% were maintained over the age of thirteen. Given the relatively low renewal fees, this clearly indicates a concentration of low-value patents. Two phenomena can lead to such a proportion of drop-out (mortality rate):

- The first is technical. Every patent application will not automatically give rise to a patent grant. The patent office may refuse to grant a patent following the patent review

[8] Data for the interest rate have been collected on the website of the French Finance Ministry. They correspond to the legal interest rate calculated as a moving average of French treasury bills rates. The corresponding values have been deflated by the national consumer price index.

[9] We thank Dominique Deberdt from INPI who provided us with data.

process by considering that the innovation at stake does not fulfil the criteria of patentability[10].

- The second is more of an economic nature. Failure to pay renewal fees results in an automatic lapse of the patent.

Another important aspect underlined by Figure 3 is a tendency of decline in the frequencies of drop out by age (from around 8% in the first periods to 2% at the end of the patent life). The last value represents the percentage of patents renewed to the statutory maximum, i.e. approximately 8% in average with a drop to 6-7% for some cohorts. At this stage, it is important to take two aspects into consideration: first of all, a patent grant enters into force most of the time 24 to 48 months after a patent application. This means that the frequencies for the first four years encompass a component which is independent from the willingness of the patentee and which, in turn, may also explain the artificially high proportions of drop outs over this period. Secondly, it is worth noting that the proportion of renewals sensibly varies according to the cohort taken into consideration.

<u>Insert Figure 3</u>

Because patent withdrawals may result either from a rejection of the application or from a voluntary decision by the applicant, the real option model developed in part 2 has to be slightly modified. More precisely, the value function of an invention for which a patent is pending is given by

$$\overline{V}_B(N_t, t, c_t, \cdots, c_T) = Max\{\theta_t V_A(N_t) + (1-\theta_t)H(N_t, t, c_t, \cdots, c_T)\ ,\ V_A(N_t)\} \qquad (18.a)$$

with

$$H(N_t, t, c_t, \cdots, c_T) = R(N_t) - c_t + \frac{E_t[\overline{V}_B(\widetilde{N}_{t+1}, t+1, c_{t+1}, \cdots, c_T)]}{1+\rho_t} \qquad (18.b)$$

This value function is obtained by introducing the probability θ_t of rejection of the application at age t in the value function (3.a) characterising a granted patent. We do not have

[10] i.e. the criteria of novelty, inventiveness and industrial application in Europe.

detailed data on patent rejections by age but only aggregate data on patent rejections per cohort. For each cohort, the average rate of patent applications that are rejected by the patent office amounts to about 30%. This rate approximately corresponds to the sum of observed average frequencies of withdrawals for the first fourth ages (see Figure 2). This is consistent with the maximum delay of examination of about four years observed for a patent application to be accepted or rejected. Therefore, we assume that the average frequencies of withdrawal during the first fourth years correctly approximate the probability of rejection and set $\theta_1 = 0.105$, $\theta_2 = 0.083$, $\theta_3 = 0.063$, $\theta_4 = 0.064$ and $\theta_t = 0$ for all $t > 4$. Expression (18) is used in place of (3.a) to compute the probability distribution (17) of the optimal stopping time. Finally, the unconditional probability for a patent to be withdrawn at date t on a voluntary basis or because of rejection is

$$\Pr_t^w(N_0, c_0, \cdots, c_T) = \begin{cases} \left(\prod_{s=1}^{t-1}(1-\theta_s)\right)\left(\theta_t + \Pr\left[\tilde{\tau}^*(N_0, c_0, \cdots, c_T) = t\right]\right) & \text{for} \quad t \le 4 \\ \left(\prod_{s=1}^{4}(1-\theta_s)\right)\Pr\left[\tilde{\tau}^*(N_0, c_0, \cdots, c_T) = t\right] & \text{for} \quad t > 4 \end{cases} \tag{19}$$

The probability of observing for cohort i some values n_{ti} $(t = 1, \cdots, T+1)$ of the theoretical number N_{ti} of withdrawals at age t can be written as

$$\Pr\left[N_{1i} = n_{1i}, \cdots, N_{1i} = n_{1i} \;/\; N_0\right] = \prod_{t=1}^{T+1} \Pr_t^w\left(N_0, c_0, \cdots, c_T\right)^{n_{ti}} \tag{20}$$

This probability depends on observed renewal fees and is conditional on the unobserved initial market size N_0. Therefore, a probability distribution has to be defined for N_0. A discrete approximation of the log normal distribution has been chosen. A first reason for this choice is that, compared to continuous probability distributions, it has the advantage of not requiring time consuming simulation methods to compute the likelihood of observed withdrawal frequencies. An exact computation of this likelihood is possible instead. A second reason is that a discrete probability distribution for N_0 implies a finite number of alternative classes of patents and thus simplifies the computation of an optimal revelation mechanism. A discrete approximation of the log normal probability distribution has been obtained by dividing the range of values between 0 and the 995$^{\text{th}}$ quantile in intervals of equal length and affecting the corresponding density f_m to the middle N_0^m of each interval. The likelihood of observed numbers n_{ti} $(t = 1, \cdots, T+1)$ of withdrawals at the different age for cohort i if then given by

$$L_i = \sum_{m=1}^{10} f_m \Pr\left[N_{1i} = n_{1i}, \cdots, N_{1i} = n_{1i} \,/\, N_0^m\right] \tag{21}$$

The treatment of renewal fees also deserves some comments. Indeed, as already outlined, changes of values for renewal fees are frequent and apply to all patents regardless of the date of application. Nevertheless, it does not seem possible to correctly forecast the date and magnitude of adjustments of patent renewal fees. Therefore, these changes have been dealt with by assuming that patent holders adjust instantaneously their decisions on the basis of current renewal fees rather than on expected future renewal fees.

4. Results

A major interest of the model developed in this paper is the contrast between its relatively low data requirement and the importance of numerical results that may be inferred from. Data on patent renewal frequencies by age and cohort are available in most developed countries. They have been extensively used to assess the value of patents, including on the basis of real option models of patent renewal decisions but, to our knowledge, they have never been used to assess the social cost imposed to consumers by the patent system. An immediate contribution of this part is to provide with estimates of not only parameters involved in the model but also estimates of the option value of patents, the social cost of patents and the revenues they generate for the patent office (section 4.1). Besides this first contribution, estimation results are also used to determine what the optimal "one profile fits all" renewal fees look like and examine implications in terms of social cost for consumers and revenues for the patent office (section 4.2)[11]. We finally give some insights about the interest and limits of a menu of optimal "tailor-made profiles" for renewal fees (section 4.3).

4.1. Estimation results

Given values of all parameters in the model, the exact value of the total likelihood of withdrawal obtained for cohorts 1970 to 2006 may be computed[12]. However, we are not able to find the analytical expression of the log-likelihood function. Following Pakes [1986] and Baudry and Dumont [2006], a numerical method has therefore been used. The steepest

[11] Social effects are mainly characterised at two levels: at the level of the efficiency of the technical progress in the industry and at the level of the social surplus. Only the second aspect is studied here.

[12] The number of shocks per year has been set to $M = 4$.

gradient method has been retained to maximise the likelihood[13]. Estimation results are displayed in Table 1. Δh, α and β are parameters involved in expression (12) defining the dynamics of the rent. λ is the probability of entry of competitors in the absence of a patent introduced in (2). According to expressions (1.a) and (1.b), both the rent that accrues to the patent holder and the corresponding deadweight loss are linear functions of the expression $p_0^2 N_t/\eta$ which captures the state of demand for the new product. As a result, we do not need to distinguish between the different components of this expression and directly denote it S_t. The dynamics of the rent and the deadweight loss are directly expressed in terms of S_t and all value functions as well. A discrete approximation of the log-normal probability distribution is assumed for S_0 to capture heterogeneity between patents as regards initial conditions. μ_{S_0} and σ_{S_0} stand for the expected value and standard deviation of this distribution. Ten threshold values, associated with eleven intervals of equal length, between 0 and the 995th quantile of S_0 have been used to obtain the discrete approximation. Due to the heterogeneity between patents as regards their initial conditions, it is easier to define $\overline{N}^{\max}$ and $\overline{N}^{\min}$, or equivalently $\overline{S}^{\max}$ and $\overline{S}^{\min}$, in (12.d) and (12.e) as $N_0(1+\delta^{\max})$ and $N_0(1-\delta^{\min})$ respectively where $\delta^{\max}$ and $\delta^{\min}$ are two parameters to be estimated. The expected option value and social cost of a patent are also reported in Table 1. The significance of estimated parameters is tested on the basis of log-likelihood ratios rather than usual t-statistics. The reason for this is that, as already outlined by Pakes [1986] and Baudry and Dumont [2006], implementing numerical methods to maximise the likelihood of real option models of patent renewal decisions yields excessively high standard deviations of estimated coefficients. Moreover, the null hypothesis retained to implement the log-likelihood ratio test is a restriction of the estimated coefficient to half its value rather than to zero. Indeed, setting coefficients to zero does not have much sense in the model for most of the parameters and may lead to a null likelihood due to some frequencies of withdrawal that subsequently amount to zero.

Insert Table 1

All estimated coefficients are highly significant. Moreover, the assumption that coefficients β, $\delta^{\min}$ and $\delta^{\max}$ are simultaneously significantly different from half their

[13] The steepest gradient method has actually first been implemented to minimise the mean square error of withdrawal frequencies. Results for this first estimation have then been used as initial values to maximise the likelihood. Indeed, a shortcoming of the maximum likelihood estimation method for real option models of patent renewal decisions is that the likelihood amounts to zero as long as at least one estimated frequency is equal to zero, a case which often appears with arbitrary values of parameters.

estimated value, and thus significantly different from zero, is strongly rejected. The log-likelihood ratio statistic associated to the test amounts to 25325.09, which is higher than all conventional threshold values for the khi-square statistic with three degrees of liberty. We conclude that the stochastic process for the rent cannot be reduced to a simple random walk and that Bayesian learning about the long term market size actually takes place. From a policy point of view, it is particularly of interest to note that the estimated expected social cost of a patent reported in Table 1 is more that ten times the estimated value of the expected option value of a patent. The burden of the charge borne by society to give monetary incentives to private inventors to patent their inventions and diffuse knowledge is thus quite consequent. It is not compensated by revenues collected by the patent office through renewal fees paid by patent holders. Indeed, these revenues per patent are estimated to amount to one tenth of the option value. These results make it urgent to reflect upon the question of whether the gains to society in terms of promotion of innovation through intellectual property are sufficiently high to justify such a social cost.

A convenient way to assess the global quality of the regression consists in plotting observed and predicted frequencies of withdrawals. This is done in Figure 4 for all cohorts and in Figure 5 for the cohort 1986 which is the last cohort in our database for which withdrawals are observed for all ages. In both cases, the general shape of withdrawal frequencies is correctly predicted. Average frequencies over all cohorts are slightly overestimated at middle ages of patents whereas they are slightly underestimated at the last ages. This tendency does not appear when examining predicted frequencies for the cohort 1986. For this cohort, the ratio of patents that are renewed until the legal limit is almost perfectly predicted. Interestingly, Figure 5 suggests that error predictions are not systematically higher when examined for a specific cohort rather than across all cohorts.

<u>Insert Figure 4</u>

<u>Insert Figure 5</u>

4.2. Estimation of the optimal "one profile fits all" renewal fees

Recent adjustments of renewal fees reveal that the French patent office is looking for a way to improve the system without apparently having a clear guideline to define its policy. The theoretical framework proposed in Part 2 offers such a guideline. The Pareto improving version of the optimal "one profile fits all" system defined in (8) more specifically allows the determination of alternative renewal fees that lower the social cost of patents without deterring innovation compared with the current profile. For this purpose, we need to solve program (8) where all coefficients in value functions are set at their estimated value and $\overline{V}$ is given by the estimated current option value of a patent. However, because we do not have analytical expressions for the value functions involved in program (8), a numerical approximation of these functions is required. For this purpose, we limit the degrees of liberty for the general shape of profiles by focussing on those profiles with a constant rate of growth or decay denoted by γ. It is more specifically convenient for graphical purposes to express this rate of growth or decay as a function of the initial and final renewal fees (respectively c_0 and c_T). We thus consider profiles of the form

$$c_t = c_0 (1+\gamma)^t \qquad \text{with} \qquad \gamma = \left(\frac{c_T}{c_0}\right)^{\frac{1}{T}} - 1 \tag{22}$$

As a result, value functions involved in program (8) only depend on c_0 and c_T and the minimisation problem is solved only with respect to these two renewal fees. We do not impose that renewal fees increase with the age of a patent as currently observed (i.e. that $c_0 < c_T$) but rather try to determine whether such a shape is optimal or not. Therefore we numerically approximate the value functions for cases with either $c_0 < c_T$ (i.e. increasing profiles) or $c_0 > c_T$ (i.e. decreasing profiles) but also cases with constant renewal fees. Cases with an annual subsidy to maintain the patent instead of an annual renewal fee are ruled out so that we only consider positive values for c_0 and c_T. More precisely, we use a lower bond of 1€ for c_0 and c_T because setting one of these two parameters to zero in (22) induces that all renewal fees also amount to zero so that patents are granted for free.

Insert Figure 6

Insert Figure 7

Insert Figure 8

Figures 6 and 7 display the expected social cost and the expected option value of a patent as functions of the initial and final renewal fees. A striking property of the two functions is that they co-vary. More precisely, both the expected social cost and the option value of a patent decrease with respect to the initial and final renewal fees. Though rather intuitive, this property implies that standard second order conditions for a solution of the constrained minimisation problem to be obtained as a solution of the first order conditions are not fulfilled. A corner solution is rather expected[14]. This is confirmed by Figure 8. The optimal solution is obtained at $c_0 = 1$ Euro and $c_T = 3695.07$ Euros. As shown by Figure 9.a, compared to the current profile, the corresponding optimal profile is characterised by lower renewal fees until age 14 and then a sharp increase to reach a final renewal fee that amounts to six times the current one. Exact values of renewal fees at each date are reported in Table 2. The optimal "one profile fits all" renewal fees let the expected option value of patents unchanged but yield an expected social cost of 59881.80 Euros per patent which is 127.1 Euros less than the current expected social cost. Conversely, the expected discounted sum of renewal fees paid to the patent office falls to 238.972 Euros per patent, which is less than half the value estimated for the current profile. Table 3 displays detailed simulation results for the option value, social cost for consumers and revenues raised by the patent office by class of patent. A detailed examination of these results reveals an important drawback of the optimal "one profile fits all" system of renewal fees: the expected option value of patents is let unchanged compared to the current system at the cost of an implicit monetary transfer from patents with initially high option values to patents with an initially low option value. Say another way, the suggested optimal "one profile fits all" system of renewal fees favours patents of low value compared to patents with high value. This seems rather inconsistent with the willingness to favour high value patents and to stop the current patent backlog to balloon further.

If a revenue-generating constraint is imposed to make sure that revenues from renewal fees that accrue to the patent office do not fall behind their current level, the optimal profile becomes closer to the current one. The expected social cost of a patent then rises to 60022.40

[14] We used *Mathematica®* software and the instruction *Minimize* to solve all constrained optimisation programs in this paper.

Euros which is slightly higher than the current one. This result follows on from the fact that optimal profiles are constrained to be exponentially increasing or decreasing functional forms of the age of a patent whereas current profiles have a more flexible form. Figure 10 shows how the revenue-generating constraint affects the optimal solution. A third iso-curve is added to Figure 8 to represent all couples of initial and final renewal fees that are compatible with the constraint. The new constraint is steeper than the iso-curve associated with the expected option value so that the solution depicted in Figure 8 is located behind the constraint. The minimal increase of c_0 and decrease of c_T required to fulfil the constraint is given by the crossing point between the constraint and the iso-curve for the expected option value in Figure 10. Again, Table 3 highlights some distributional effects of the revenue-generating constraint. Interestingly, the effects as regards option values are opposite to those outlined in the unconstrained optimal profile. The optimal profile with revenue-generating constraint strengthens the asymmetry between patents in favour of high value patents compared with the current system and *a fortiori* compared with the unconstrained optimal profile. There are also distributional effects in terms of revenues raised by the patent office in spite of the constraint to let expected revenues unchanged compared with the current system. Indeed, the optimal profile with a revenue-generating constraint is characterised by slightly higher renewal fees at the first ages and lower renewal fees at the last ages compared with current renewal fees. It thus transfers the burden of financing the patent system from high option value patents to low option value patents. Stated another way, there is a transfer of the burden from high to low-quality patents where quality is assessed by the option value of patents. Indeed, patents with a high option value are renewed for a long period whereas patents that are withdrawn early have a lower option value.

Broadly speaking, the gain in terms of social cost from implementing the optimal “one profile fits all” system without imposing a revenue-generating constraint is low. Moreover, this system induces negative distributional ancillary effects that make it of little interest compared with the current system. By contrast, the optimal “one profile fits all” system with a revenue-generating requirement does not generate any gain in terms of expected social cost per patent but has positive distributional ancillary effects both in terms of option value of patents and on the burden of financing the patent office. As a result, one may wonder whether it is possible to conciliate the respective advantages of the unconstrained and constrained profiles. A way to do so could consist in discriminating between patents *ex ante* by proposing a menu of alternative profiles. This suggestion is examined in the next section.

Insert Figure 9

Insert Table 2

Insert Figure 10

Insert Table 3

4.3. Estimation of the optimal "tailor-made profiles" renewal fees

The optimal system of "tailor-made profiles" for renewal fees defined in (10) assumes that one profile is determined for each class of patent. However, several problems have been encountered when trying to determine such an optimal system. First of all, it has not been possible to numerically find a solution to (10). More specifically, it seems particularly uneasy to find a solution that fulfils all constraints simultaneously. Secondly, when relaxing some of the constraints, it appears that the optimal profiles for most of the different classes of patents are very close. Therefore, we have finally opted for a system that involves only two different profiles. The first profile (profile A) is intended to be chosen by applicants if their patents belong to the first two classes of patents, those with the lowest initial market size. The second profile (profile B) is intended to be chosen by all other applicants. Program (10) has to be modified in accordance. There are now one Pareto improvement constraint (10.b) and one incentive compatibility constraint (10.c) for each class of patent. The Pareto improvement constraint states that the option value of a patent with the profile of renewal fees specially designed for its class is at least as high as its option value with the current system of renewal fees. At worst, the monetary incentives to innovate are thus let unchanged. The incentive compatibility constraint states that the value of a patented invention is higher with profile A than with profile B if the patent belongs to class 1 or class 2 and conversely if the patent belongs to other classes. As in the previous section, the analysis is restrained to the case of exponentially increasing or decreasing profiles for renewal fees. The expected social cost in (10.a) has thus to be minimised with respect to the initial and final renewal fees c_0^A and c_T^A characterising profile A and the initial and final renewal fees c_0^B and c_T^B characterising profile B.

Insert Figure 11

Insert Table 4

The solution for profile A and profile B are illustrated by Figure 11. The detailed renewal fees are reported in Table 4. Again, we consider both the case with and without a revenue-generating constraint. In each case, the general shape of profile A is close to that of the optimal "one profile fits all" system obtained in the previous section, except that renewal fees at the last ages are systematically higher. Unsurprisingly, profile A is thus clearly designed to encourage patent applications for inventions with initially low market size but, in counterparts, it imposes high renewal fees in case of success. Profile B without a revenue-generating requirement is almost similar to the current profile of renewal fees. This profile imposes higher renewal fees than profile A at the first ages but compensates by lower renewal fees at the last ages. For this reason, it is expected to be chosen by applicants whose patents belong to classes 3 to 11, i.e. patents with a sufficiently high initial market size to justify the payment of renewal fees for a long period. The decrease of renewal fees characterising profile B with a revenue-generating constraint is more surprising. Nonetheless, the impact of the revenue-generating constraint on profile B is in line with the impact already observed for profile A and for the optimal "one profile fits all" system examined in section 4.2. Indeed, the revenue-generating constraint systematically flattens the profiles of renewal fees and increases initial renewal fees in counterpart. The impact of the constraint on profile B corresponds to an extreme version of this tendency. The resulting profile may be interpreted as a profile which is essentially intended to act as an application fee. Indeed, due to the exponential functional form used for the profile, it is not possible to obtain a system of application fees only but it is possible to approximate such a system by imposing decreasing renewal fees. Simulation results thus suggest that setting application fees to zero or at a low level and imposing increasing renewal fees is relevant for patents characterised by unfavourable initial conditions (i.e. low initial market size) whereas a system of application fees without renewal fees may be preferred for patents characterised by favourable initial conditions.

There is little to gain to expect from the optimal "tailor-made profiles" system compared to the optimal "one profile fits all" system in terms of expected option value, expected social cost and expected discounted sum of fees paid per patent when examined across all classes of patents. Indeed, there is a slight increase of both the expected option value and the expected revenue perceived per patent whereas the expected social cost per patent is very close to that obtained with the current system. Nevertheless, there are interesting distributional effects between classes of patents. Table 3 gives detailed results on the option value, social cost for consumers and revenues perceived per patent and by class of patent with profiles A and B. A comparison of option values with those obtained with current renewal fees helps identifying for which classes of patents the Pareto improvement constraint (10.b) is binding. The expected option value per patent obtained with the "tailor-made profiles" mechanism is equal to the expected option value per patent obtained with the current profile for class 2 (with profile A) and class 4 (with profile B) in the absence of a revenue-generating constraint and class 2 (with profile A) and class 3 (with profile B) when a revenue-generating constraint is imposed. All other classes have a net gain in terms of patent's option value whether a revenue-generating requirement is imposed or not. More interestingly, the corresponding increase of the monetary incentive to innovate compared to the current system is higher for classes 5 to 11 when a revenue-generating constraint is imposed. Hence, the constraint does not affect the incentive to innovate for patented inventions with a high initial market size. Conversely, it lowers the monetary incentive to innovate for patented inventions with initially low market size, i.e. for patents belonging to class 1. Broadly speaking, similar tendencies are observed in terms of expected discounted sum of fees paid per patent: the revenue-generating constraint lowers the amount paid for patented inventions with an initially high market size and increases the amount paid for those with an initially low market size compared to both the optimal "tailor-made profiles" system with no revenue-generating requirement and the current system of renewal fees[15]. There are no important changes in terms of the social cost per patent observed by class of patents between the optimal "one profile fits all" system, the optimal "tailor-made profiles" system and the current system.

[15] The evolution class by class is less smooth than the one observed for the option value or the social cost. A reason for this may be that, contrary to the option value or the social cost per patent, revenues generated by renewal fees do not depend on the stochastic rent and may therefore be subject to some threshold effects.

6. Concluding Remarks

Since the early 1980s, patent policy in most industrialised countries has been strengthened, broadened and extended to areas where earlier patenting was relatively low. A consequence of these reforms is that there is an increasing debate on how to improve patent quality and on the ways to reduce the backlog of most patent offices. The main difficulty in fixing the problems with the patent system is that by doing so, we need to preserve the essential innovation incentives which patent property rights were originally designed to provide and to formulate reforms that recognize the informational limitations under which patent offices will inevitably operate. Among the suggested directions for reform, a change in the profile of patent renewal fees is of particular importance. However, the main barrier to this reform is that patent offices in a variety of countries are self-financed and are therefore facing a revenue constraint which requires them to raise funds. If a patent office seeks to raise revenue, then it will not in general set the socially optimal schedule of patent renewal fees and may have an incentive to encourage too many patent renewals from a social point of view.

Considering this constraint, the main objective of this article was to use renewal fees to improve the efficiency of the innovation incentives generated by the patent system and therefore to study alternative renewal fee structures to determinate the optimal renewal fees profile. The econometric analysis yields two important results. First, it is not possible to reduce the deadweight loss without reducing the incentive to innovate. This is a problem as such as the estimated expected social cost of a patent is more that ten times the estimated value of the expected option value of a patent. The burden of the charge borne by society to give monetary incentives to private inventors to patent their inventions and diffuse knowledge is thus quite consequent. We should hence consider the social costs of the patenting system as well as its advantages in order to guide decisions. Second, there is room for an improvement of the renewal fee structure if one adds some criteria of quality to the social cost. Indeed, for a given incentive to innovate and for a same social cost, a menu of profile rather than a “one profile fits all” system could increase the asymmetry between patents in terms of option value and, by doing so, it would clearly encompass a quality premium.

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<u>*Figure 1*</u>
Tree form representation of the stochastic process for the number of positive shocks affecting market size

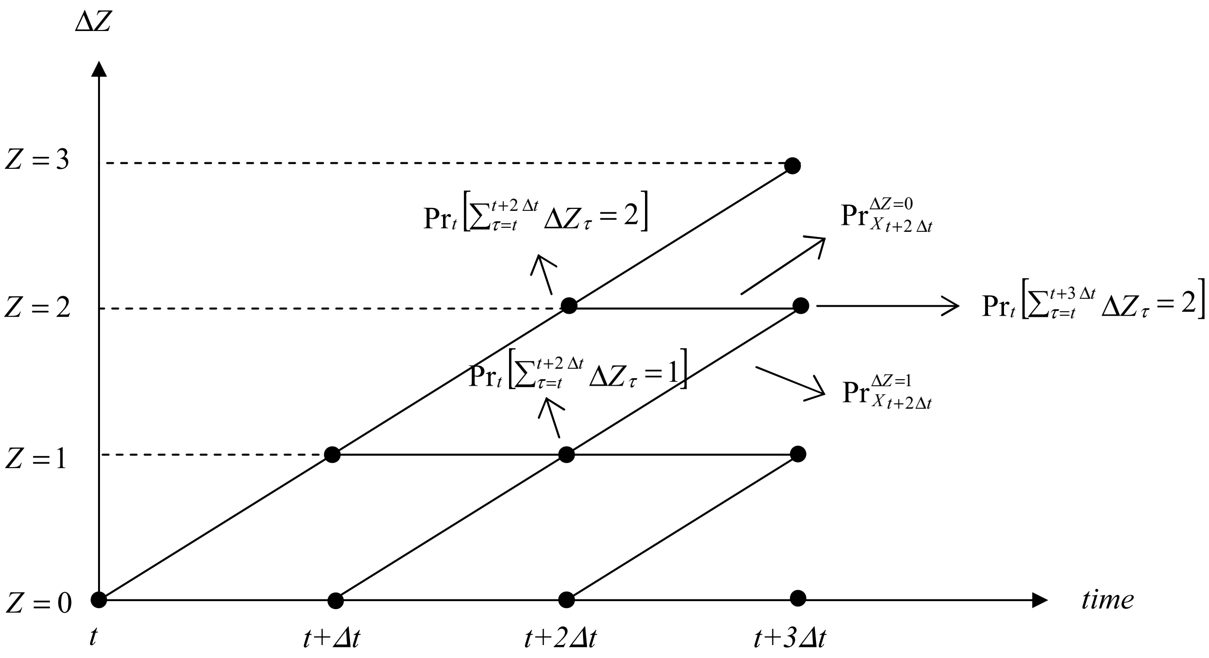


Figure 2
Observed profiles of renewal fees at different dates (in constant 2000 euros)

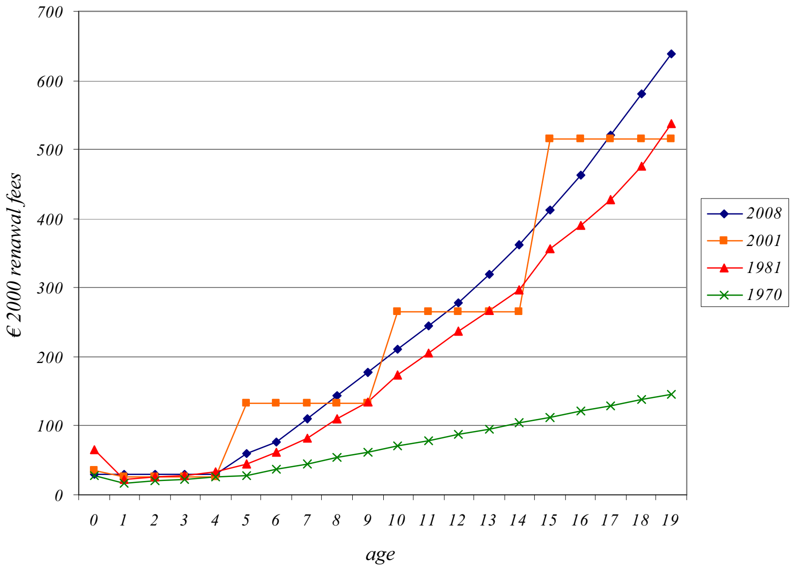
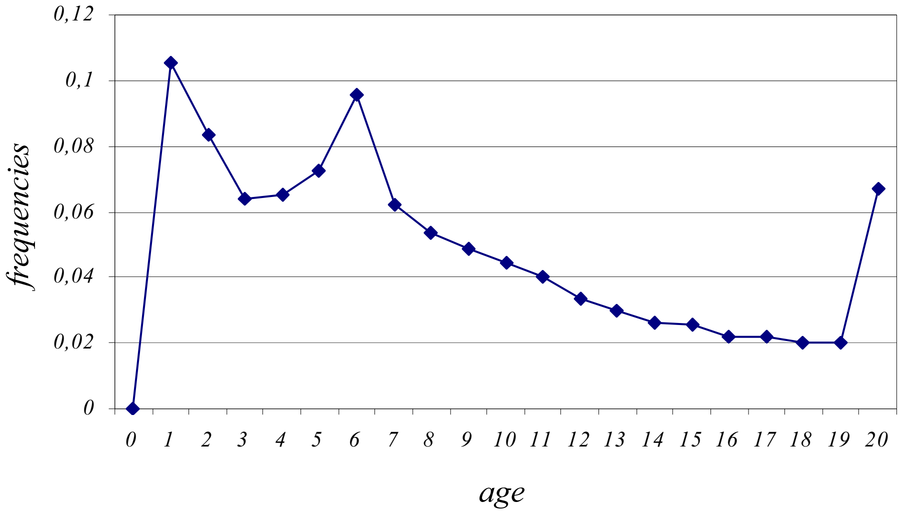


Figure 3
Average Frequencies of withdrawals by age (cohorts 1970-1986)



<u>*Figure 4*</u>

Observed (continuous line) and predicted (dashed line) average frequencies of withdrawals by age (cohorts 1970-2006)

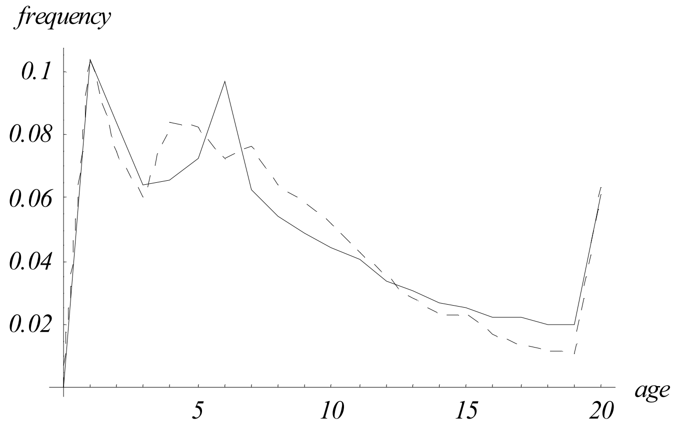


Figure 5
Observed (continuous line) and predicted (dashed line) frequencies of withdrawals by age (cohorts 1986)

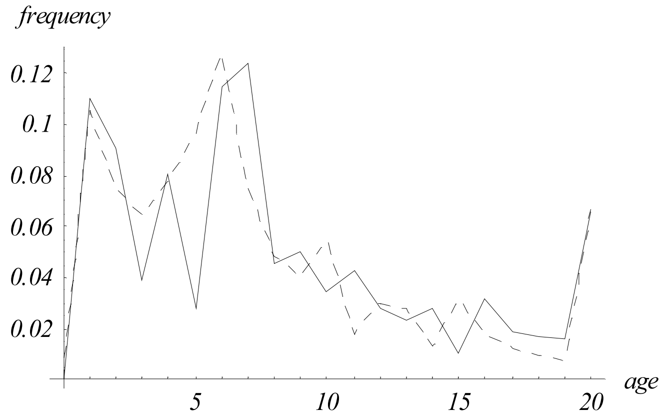
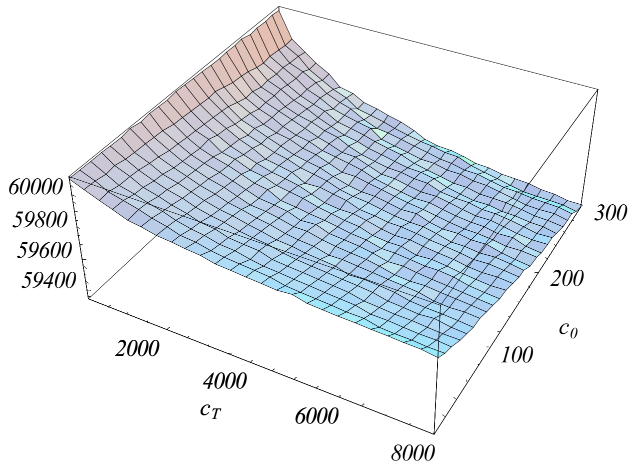


Figure 6

Expected social cost (in Euros) of a patent as a function of the initial and final renewal fees



<u>Figure 7</u>
Expected option value (in Euros) of a patent as a function of the initial and final renewal fees

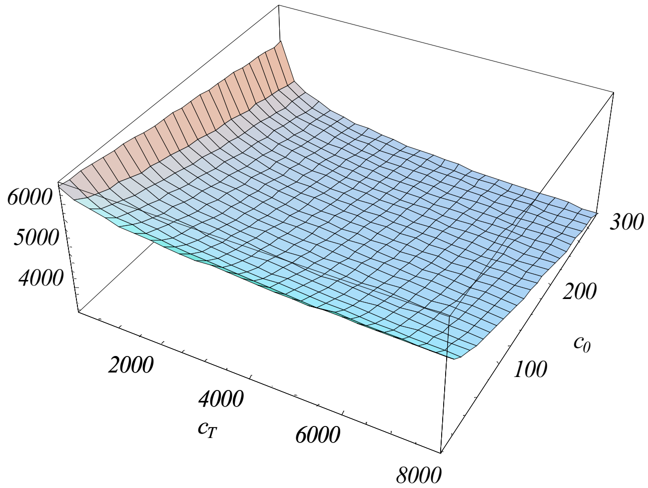
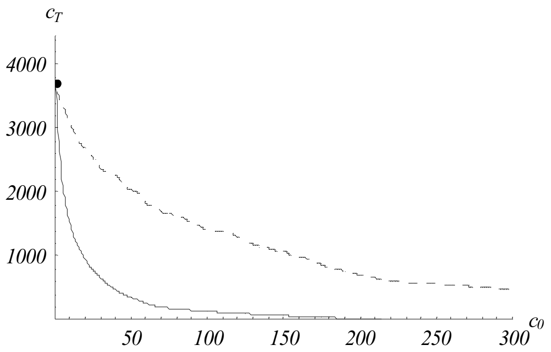


Figure 8

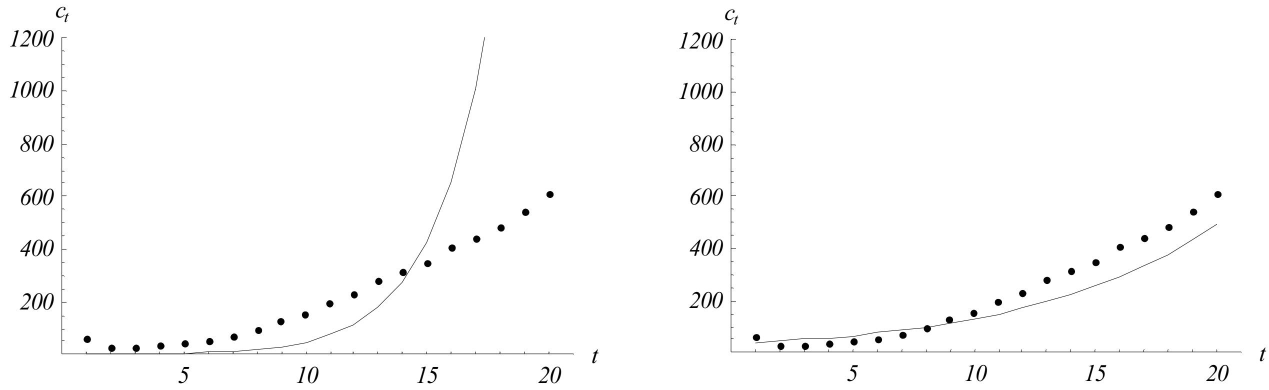
Iso-curves for the expected option value (continuous line) and expected social cost (dashed line) of a patent at the corner solution for the optimal "one profile fits all" system.



<u>*Figure 9*</u>

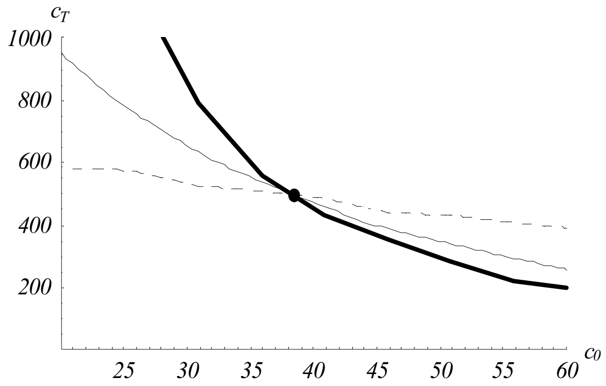
Current renewal fees (points) and optimal "one profile fits all" renewal fees (continuous line)

a) without revenue-generating requirement *b) with a revenue-generating requirement*



<u>*Figure 10*</u>

Iso-curve for the expected option value (slim continuous line), iso-curve for the expected social cost (dashed line) of a patent and revenue-generating constraint (thick continuous line) at the solution for the optimal "one profile fits all" system with a revenue-generating constraint.



<u>*Figure 11*</u>

Current renewal fees (points), optimal profile A for the first two classes of patents (continuous line) and optimal profile B for other classes of patents (dashed line)

a) without revenue-generating requirement *b) with a revenue-generating requirement*

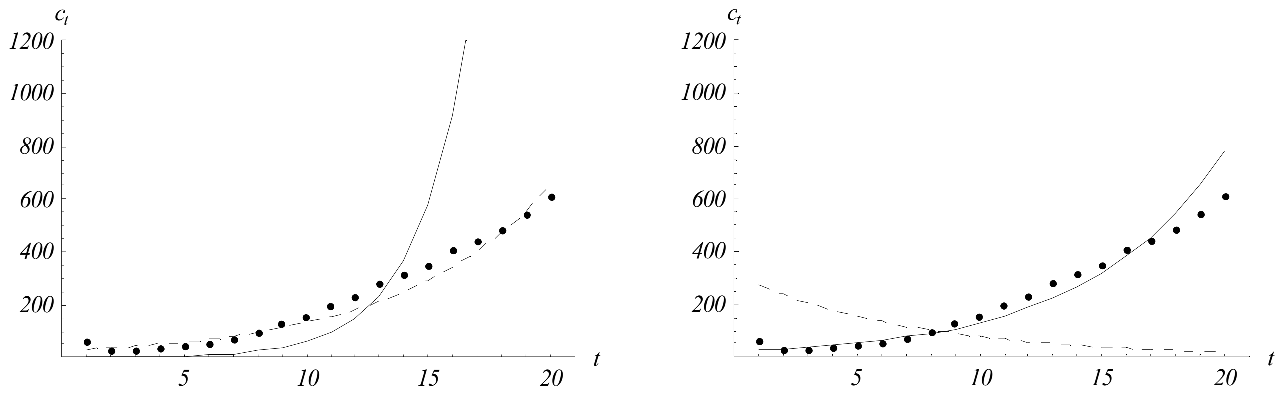


Table 1: estimated coefficients (cohorts 1970 to 2004)

	coefficient	Log likelihood ratio statistic
Δh	0.37477	477578.42
α	0.542073	1.1006657×10^{6}
β	2.15401×10^{-6}	37105.65
$\delta^{\min}$	0.63837	44347.18
$\delta^{\max}$	0.22226	8054.90
λ	0.0163875	39278.29
μ_{S0}	101153	97402.87
σ_{S0}	121275	74099.63
Likelihood	$9.88442499\times10^{-924031}$	
Mean Square Error	0.345986	
Expected option value of a patent (in Euros)	5767.76	
Expected social cost of a patent (in Euros)	60008.9	
Expected revenues of the patent office from a patent (in Euros)	520.262	

<u>Table 2</u>: Comparison of renewal fees (in Euros) with the current system and the optimal "one profile fits all" system

	Current renewal fee	Optimal "one profile fits all" renewal fee	Optimal "one profile fits all" renewal fee with a revenue-generating requirement
Age 0	53.4894	1.	38.5463
Age 1	28.8843	1.54088	44.0894
Age 2	31.0239	2.37432	50.4296
Age 3	33.1635	3.65855	57.6817
Age 4	43.8613	5.63739	65.9765
Age 5	57.7686	8.68656	75.4643
Age 6	77.0248	13.385	86.3164
Age 7	103.77	20.6247	98.7291
Age 8	136.933	31.7802	112.927
Age 9	166.887	48.9696	129.166
Age 10	212.888	75.4564	147.741
Age 11	249.261	116.269	168.987
Age 12	295.262	179.158	193.288
Age 13	332.704	276.061	221.083
Age 14	369.077	425.377	252.876
Age 15	430.055	655.457	289.241
Age 16	469.637	1009.98	330.835
Age 17	514.568	1556.26	378.411
Age 18	571.267	2398.02	432.828
Age 19	647.222	3695.07	495.07

<u>Table 3</u>: Comparative simulation data

Class of patent			All classes	Class 1	Class 2	Class 3	Class 4	Class 5	Class 6	Class 7	Class 8	Class 9	Class 10	Class 11
Probability of occurrence			1.	0.4491	0.3047	0.1238	0.0558	0.0278	0.0149	0.0085	0.0051	0.0032	0.0021	0.005
Value of S_0			101153	31740.5	95221.5	158702.	222183.	285664.	349145.	412626.	476107.	539588.	603069.	666550.
Current system		OV	5767.76	1688.2	5374.89	10128.3	15168.5	18081.6	18600.7	18845.6	19459.7	20414.4	21637.2	23085.4
		SC	60008.9	20467.4	57887.7	96955.2	135981.	168631.	193484.	215427.	236672.	257769.	278940.	300292.
		SF	520.262	268.612	570.62	876.955	963.9	947.189	890.728	890.34	925.085	1020.66	1077.16	1145.1
Optimal "one profile fits all" system	No revenue-generating requirement	OV	5767.76	1863.23	5452.17	9853.97	14487.2	17420.9	18263.1	18662.4	19273.3	20155.6	21266.	22576.9
		SC	59881.8	20463.2	57764.4	96653.	135524.	168201.	193148.	215155.	236404.	257445.	278544.	299832.
		SF	238.972	95.7574	212.78	449.172	666.605	681.33	479.494	420.716	477.62	492.927	532.728	622.704
	With revenue-generating requirement	OV	5767.76	1651.2	5375.64	10175.2	15236.6	18145.6	18653.4	18897.6	19519.5	20484.6	21719.6	23180.
		SC	60022.4	20472.3	57907.7	96982.4	136001.	168648.	193508.	215456.	236693.	257796.	278966.	300305.
		SF	520.262	326.407	575.762	786.682	871.371	850.593	808.33	788.56	809.852	867.383	924.206	965.77
Optimal "tailor-made profiles" system	No revenue-generating requirement	OV	5835.39	1837.21	5374.89	10130.6	15168.5	18084.5	18609.1	18857.6	19473.1	20427.3	21649.1	23094.8
		SC	59957.5	20445.8	57750.3	96959.	135980.	168629.	193481.	215429.	236670.	257766.	278932.	300275.
		SF	322.786	86.8609	261.75	784.49	897.286	872.963	799.136	774.068	810.037	863.156	926.029	989.507
	With revenue-generating requirement	OV	5779.23	1693.17	5374.89	10128.3	15252.5	18145.3	18622.9	18874.5	19527.	20531.5	21806.7	23307.
		SC	60028.1	20470.6	57879.9	97041.1	136052.	168701.	193568.	215530.	236776.	257871.	279034.	300375.
		SF	520.262	280.673	521.011	950.904	957.496	957.759	958.751	958.056	968.44	970.352	973.55	978.122

OV: per patent option value (Euros) SC: per patent social cost (Euros) SF: per patent discounted sum of fees (Euros)

Table 4: Comparison of renewal fees (in Euros) with the current system and the optimal "tailor-made profiles" system

	Current renewal fee	Optimal "tailor-made profile" renewal fee		Optimal "tailor-made profile" renewal fee with a revenue-generating requirement	
		Profile A	Profile B	Profile A	Profile B
Age 0	53.4894	1.00077	30.4743	25.7315	271.047
Age 1	28.8843	1.5768	35.8058	30.7909	235.455
Age 2	31.0239	2.48439	42.0702	36.8451	204.536
Age 3	33.1635	3.91437	49.4305	44.0897	177.678
Age 4	43.8613	6.16744	58.0785	52.7587	154.346
Age 5	57.7686	9.71735	68.2395	63.1323	134.078
Age 6	77.0248	15.3105	80.1782	75.5456	116.472
Age 7	103.77	24.1231	94.2057	90.3996	101.178
Age 8	136.933	38.0081	110.687	108.174	87.8915
Age 9	166.887	59.8851	130.052	129.444	76.3501
Age 10	212.888	94.3542	152.805	154.895	66.3243
Age 11	249.261	148.663	179.539	185.351	57.615
Age 12	295.262	234.232	210.95	221.796	50.0493
Age 13	332.704	369.054	247.857	265.406	43.4771
Age 14	369.077	581.477	291.22	317.591	37.768
Age 15	430.055	916.168	342.17	380.037	32.8085
Age 16	469.637	1443.5	402.033	454.761	28.5003
Age 17	514.568	2274.37	472.37	544.177	24.7578
Age 18	571.267	3583.46	555.013	651.175	21.5068
Age 19	647.222	5646.07	652.114	779.212	18.6826