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Volume Title: Measuring Business Cycles

Volume Author/Editor: Burns, Arthur F. and Wesley C. Mitchell

Volume Publisher: NBER

Volume ISBN: 0-870-14085-X

Volume URL: http://www.nber.org/books/burn46-1

Publication Date: 1946

Chapter Title: Effects of Smoothing on Cyclical Measures

Chapter Author: Arthur F. Burns, Wesley C. Mitchell

Chapter URL: http://www.nber.org/chapters/c2987

Chapter pages in book: (p. 310 - 369)

CHAPTER 8

Effects of Smoothing on Cyclical Measures

As EXPLAINED in Chapter 3, we make cyclical measures directly from the raw data adjusted only for seasonal variations. But since most of our measures cover three or more months, an element of smoothing is inherent in our technique. We use, moreover, simple moving averages to help identify and date the specific cycles of very choppy series, though the cyclical measures proper are invariably computed from the raw data. To some students it might seem that our results would be more trustworthy if the data were first smoothed methodically, and the cyclical measures made from the smoothed figures. It may be argued, for example, that by eliminating 'erratic movements', smoothing exposes unequivocally the underlying form of the cyclical fluctuations; hence that more precise measures of cyclical behavior could be derived from smoothed than from raw data. Or a bolder claim may be staked out: namely, that a cyclical graduation, if carried through properly, will show results of 'systematic' forces alone, all effects of 'random perturbations' being wiped out. To come to grips with the questions thus raised, we have studied the manner in which smoothing—that is, the substitution of smoothed monthly figures for the raw—affects cyclical measures. This chapter presents the main results of our experiments.

I Range of the Tests

Some effects of smoothing time series have already been traced in Chapter 6. Converting monthly data into quarterly form is equivalent to taking every third term of a three-month moving average or total. A quarterly series may therefore be regarded as a smoothed variant of monthly data, but one that contains gaps. Likewise an annual series may be regarded as a smoothed variant of monthly data in which the gaps are wider.

We might begin our analysis by showing what happens to cyclical measures when the gaps are filled. But little can be learned from such calculations. We have seen that the cyclical measures derived from quarterly data are usually very close to those yielded by monthly data, and it is obvious that the use of three-month moving averages would make the margins still narrower. There is more to be learned from experimenting with twelve-month moving averages, but such experiments are bound, in the main, to prove unfavorable to smoothing. Everyone experienced in time series analysis knows that a twelve-month moving average distorts cyclical movements when the phases of expansion and contraction are asymmetrical in the vicinity of cyclical turns, and that even under conditions of symmetry a twelve-month average does not reach far enough up into cyclical peaks or far enough down into cyclical troughs; the result being that cyclical amplitudes are damped and the cyclical patterns distorted.

To evaluate fairly the power of smoothing to improve cyclical measures, we must choose a smoothing 'formula' that meets at least three conditions. First, the formula should cover a period long enough to iron out effectively the seemingly erratic movements which, by changing direction every few months, blur the course of cyclical expansions and contractions. Second, the formula should weight successive months in accordance with a 'smooth' weight diagram; this condition is desirable to insure a smooth result. Third, some of the weights should be negative; this is necessary if the graduation is to reach well up into cyclical peaks and well down into cyclical troughs. Since an indefinite number of formulas will meet these minimum conditions, our experiments of necessity have to be restricted. We have based the experiments largely on one formula, the "43-term summation approximately fifth-degree parabolic graduation" devised by Frederick R. Macaulay. The practical reason for this choice is that Macaulay applied his smoothing formula to several series we have analyzed by our standard technique, and we were able to take over his results. This consideration was reinforced by the fact that Macaulay's method of graduating cyclical data gives results as good for our purpose as any we have yet encountered.

Macaulay's formula is a complicated moving average covering 43 terms of the data. It is alleged to reach well up into cyclical peaks and well down into cyclical troughs. When applied to monthly data, it eliminates seasonal and erratic movements, leaving a curve that is supposed to describe adequately the remaining cyclical and trend components. We selected four American series from the group that Macaulay had smoothed by applying his graduation to logarithms of the data—deflated bank clearings, railroad stock prices, pig iron production, and call money rates. We then compared the measures obtained by applying our technique to each of the four series, first in what we shall call the 'raw' form and second in the 'smoothed' form.[1] The 'raw' data are, of course, adjusted for seasonal variations.

These series form a fair sample for judging the influence of erratic fluctuations on measures of cyclical behavior. One series reports payments aggregating billions of dollars, the second prices, the third production in physical units, and the fourth shows percentages. The erratic movements are pronounced relatively to the cyclical movements in call money rates, but only mild or moderate in the other series. The erratic movements seem to differ in type as well as in size. Bank clearings show a 'saw-tooth' movement with a reversal of direction every month or two. In railroad stock prices such reversals occur at longer intervals. Iron production often rises or falls without interruption for months at a time, but occasionally has sharp 'saw-tooth' movements. In call money rates there are numerous high, narrow peaks, often standing in impressive isolation, but there are few deep, narrow troughs.

Macaulay's formula completely eliminates erratic fluctuations, in the sense that it replaces a 'saw-tooth' movement by a smooth curve, and turns angles, however acute, into gradual curvatures. But it does not necessarily eliminate protracted random movements, such as may last several years. Moreover, smoothing—no matter how skillfully done—may at times distort cyclical movements, besides eliminating erratic fluctuations. We must therefore pay careful attention in this chapter to possible limitations of smoothing, no less than to the presence of erratic components in our cyclical measures. As an aid to the reader, two samples of Macaulay's graduation are shown in Chart 43.

II The Number of Specific Cycles

In Chapter 4 we dwelt at some length, though probably not sufficiently, on the uncertainties connected with the identification of specific cycles. These uncertainties cannot be eliminated by smoothing. For there will always be movements, whether time series are smoothed or not, that seem too small or too brief to qualify as specific cycles. Some uncertainty is therefore bound to surround the results, so long as the attempt is made to distinguish specific cycles, and to fix in time the limits of their expansions and contractions. For example, Table 105 lists every movement of rise and fall in the smoothed figures of pig iron production, irrespective of its length or intensity. If all these movements were treated as specific cycles,

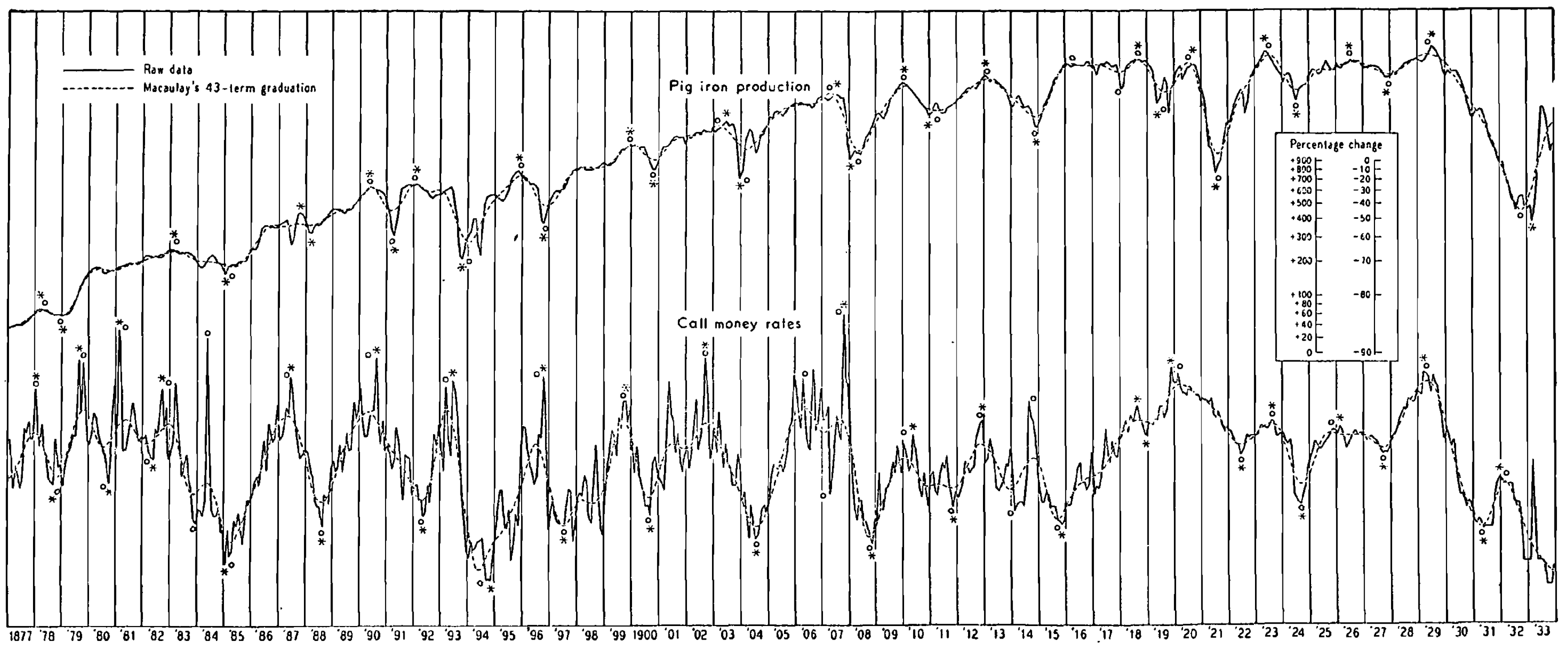
[1] The sources of the raw data are cited in Ch. 6, note 7. The smoothed series come from Frederick R. Macaulay, *Interest Rates, Bond Yields and Stock Prices*, Appendix A, Tables 17, 21, 30, 31, col. 2. The figures are expressed in logarithms, which we converted to natural numbers before analyzing. Macaulay describes his 'cyclical graduation' briefly in the above source, Appendix D, p. A 331, and more fully in *The Smoothing of Time Series* (National Bureau of Economic Research, 1931), pp. 24-6 and 73-5.

There is no difference between data called 'raw' in this chapter and 'unadjusted' in the preceding chapter.

CHART 43
Pig Iron Production and Call Money Rates, United States, 1877–1933
Raw Data and Macaulay's Graduation

CHART 43

Pig Iron Production and Call Money Rates, United States, 1877–1933
Raw Data and Macaulay's Graduation



* Identifies dates of peaks and troughs of specific cycles in raw data.
o Identifies dates of peaks and troughs of specific cycles in Macaulay's graduation.

Adjusted for seasonal variations. For sources of data, see text, note 1. The graduation of pig iron production in the first 21 months is based on estimated monthly figures.

TABLE 105
Full List of Rises and Declines in Monthly Smoothed Data
Pig Iron Production, United States, 1878–1932

Date of		Average daily production (thous. long tons) during month of		Duration in months from	
Peak	Trough	Peak	Trough	Trough on preceding line to peak	Peak to trough
*May 1878	*Dec. 1878	5.7	5.3	..	7
May 1880	Oct. 1880	9.3	9.2	17	5
*Mar. 1883	*Apr. 1885	11.6	9.5	29	25
Oct. 1886	Mar. 1887	15.7	15.5	18	5
Sep. 1887	Feb. 1888	15.9	15.8	6	5
*May 1890	*Mar. 1891	24.6	18.6	27	10
*Jan. 1892	July 1892	25.6	23.3	10	6
Nov. 1892	*Jan. 1894	24.0	12.7	4	14
*Dec. 1895	*Nov. 1896	28.7	19.6	23	11
May 1898	Sep. 1898	31.5	31.3	18	4
*Dec. 1899	*Oct. 1900	41.3	34.5	15	10
*Feb. 1903	*Mar. 1904	52.0	38.9	28	13
*Apr. 1907	*May 1908	77.1	38.4	37	13
*Jan. 1910	*Apr. 1911	84.0	61.3	20	15
*Feb. 1913	*Nov. 1914	90.7	57.0	22	21
*Apr. 1916	Oct. 1916	108.2	105.5	17	6
Mar. 1917	*Dec. 1917	106.8	98.0	5	9
*Sep. 1918	*Aug. 1919	112.8	76.9	9	11
*July 1920	*Aug. 1921	109.8	36.3	11	13
*July 1923	*July 1924	117.7	78.1	23	12
July 1925	Oct. 1925	100.0	99.1	12	3
*July 1926	*Dec. 1927	108.7	94.3	9	17
*May 1929	*Oct. 1932	119.2	18.1	17	41

*Indicates specific-cycle turns recognized in the smoothed data. Concerning the discrepancies between the specific cycles in the raw and smoothed data, see text.

we should be putting into the same class minor ripples whose very existence is unknown to history, with substantial waves for which there is clear historical evidence. To keep to the aims of this investigation, it is necessary to apply the criteria for identifying specific cycles, described in Chapter 4, to the smoothed data just as to the raw data.

The lists of cycles drawn up on this plan for the two sets of data show occasional discrepancies. In railroad stock prices the two lists agree perfectly. In pig iron production the decline in the raw data from October 1887 to March 1888, which we consider a cyclical movement, is virtually ironed out in the smoothed data. The like is true of the decline from August to December 1918 in both bank clearings and call money rates. On the other hand, the sharp drop in pig iron production in the winter of 1917–18, resulting from the extreme cold of that season and best considered as a random movement, is converted by Macaulay's formula into a wave-like fluctuation which we must treat as a specific cycle. In call money rates three 'extra' cycles turn up in the smoothed data, and at least two of them are spurious (Chart 43). The sharp rise in call money rates from July to August 1914, induced by financial panic upon the outbreak of war, and the prompt decline to a level below that of July, are converted by Macaulay's formula into a substantial wave lasting twenty-one months. Again, the sharp isolated peak in 1884, the base of which extends merely from April to July, becomes a seventeen-month wave in the smoothed data. The interval between the peaks in 1906 and 1907 is exactly fifteen months in the smoothed data, and we have marked it as a specific cycle; the corresponding movement in the raw data is too short to qualify as a cycle.[2]

From this summary it is plain that smoothing by no means offers a solution of the difficulties encountered in identifying specific cycles. Sometimes 'marginal' movements appear in smoothed data just as in the raw data, and one is forced to ponder whether to recognize them as cyclical. Sometimes smoothing spreads a high random peak or deep random trough, with a narrow time base, into a wave broad enough to qualify as a specific cycle under our rules. Sometimes, besides eliminating erratic movements, smoothing obliterates a genuine cyclical movement. It should be noted, however, that the three cyclical movements virtually ironed out by smoothing in our sample series are mild even in the raw data, and might not have been recognized as cyclical, were it not our practice to relax rules in treating dubious movements of closely conforming series.

Furthermore, the comparison of cycle lists in the raw and smoothed data is biased in one important respect against smoothing. The specific cycles in the raw data were originally selected with the aid of simple moving averages passed through doubtful portions of the series. The specific cycles in the smoothed data, on the other hand, were selected without reference to the movements of the raw data. As a result, we are able to demonstrate some significant shortcomings of smoothed data when relied on exclusively, but we are not in a position to show the power of smoothing to expose cyclical movements that are hidden from view by erratic fluctuations. To avoid embarrassment arising from this bias, we disregard in the following sections the effects of smoothing on the lists of specific cycles; that is, the effects of smoothing on cyclical measures are analyzed apart from their effects on the determination of specific cycles. This procedure requires that the lists of cycles in the raw and smoothed data be identical.[3] The simplest plan is to treat as 'cyclical' the three faint movements in iron production, bank clearings, and call money rates that correspond in time to cyclical declines in the raw data, and to drop the 'cycles' in the smoothed data, three in call money rates and one in iron production, that have no counterpart in the raw data. Thus put, the materials are in shape for isolating one set of effects of smoothing on

[2] The interval between the peaks in 1901 and 1902 is less than fifteen months in the smoothed data on call money rates; likewise the interval between the troughs in 1897 and 1898.

[3] For a full list of the specific cycles in the raw data and cycle-by-cycle measures, see Appendix Table B1.

cyclical measures. In Section X we shall return briefly to the effects of smoothing *via* changes in the lists of specific cycles.

III Timing and Duration of Specific Cycles

In the preceding chapter we found that the removal of trends shifted less than one-third of the turning points of specific cycles. Table 106 shows that smoothing shifts over four-fifths of the turning points. Except in call money rates, the smoothed data show some tendency to lead at the peaks and to lag at the troughs of the raw data.[4] But most of the shifts are small. Nearly half the turns in the smoothed data come within one month of the turns in the raw data. The discrepancy exceeds four months in only 9, and seven months in only 2, out of 149 instances.

TABLE 106

Shifts in the Timing of Specific Cycles Produced by Smoothing

Four American Series

Lead (−) or lag (+) of smoothed data at turn of raw (mos.)	Number of leads or lags in									
	Deflated clearings (1878–1933)		Pig iron production (1878–1933)		Railroad stock prices (1857–1932)		Call money rates (1857–1932)		Four series	
	Peak	Trough	Peak	Trough	Peak	Trough	Peak	Trough	Peak	Trough
Below −7	..	..	..	..	..	..	1[a]	..	1[a]	..
−7 to −5	1	..	..	1	1	..	..	1	2	2
−4 to −2	3	4	4	..	4	3	8	6	19	13
−1 to +1	8	6	10	9	8	5	10	14	36	34
+2 to +4	2	5	2	6	3	11	6	3	13	25
+5 to +7	..	1	..	..	2	..	..	..	2	1
Over +7	1[b]	..	..	..	..	..	..	..	1[b]	..
Total	15	16	16	16	18	19	25	24	74	75
Leads	8	5	6	5	7	4	14	13	35	27
Coincidences	4	..	5	2	5	1	4	6	18	9
Lags	3	11	5	9	6	14	7	5	21	39
Total	15	16	16	16	18	19	25	24	74	75
Av. lead (−) or lag (+) in months	*−0.2*	*+1.1*	*−0.4*	*+0.9*	*+0.1*	*+1.2*	*−1.2*	*−0.7*	*−0.5*	*+0.5*

The entries in this table run from the first to the last corresponding turn in the raw and smoothed data, whether peak or trough. In a few instances they cover a longer period than the measures in Table 112.

[a] This value is −17 months.

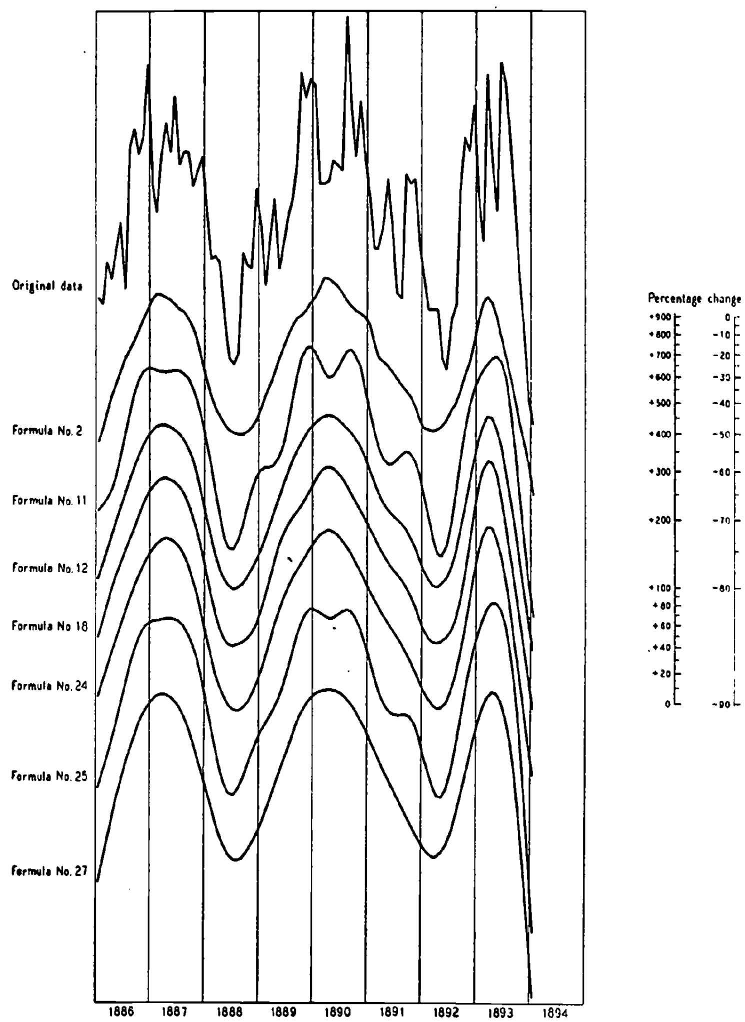
[b] This value is +9 months.

How are the differences in cyclical timing between the raw and smoothed data to be explained? The simplest explanation is that the smoothed curve represents fairly the cyclical movements, and that the discrepancies between the cyclical turns of the raw and smoothed data are therefore due mainly, if not entirely, to erratic movements in the raw data. This explanation, doubtless, is valid in some instances. We know

[4] This feature of the results is explained later: see Tables 110-112, and pp. 324-6.

that the turning points of specific cycles in the raw data are often blurred by erratic movements and that the process of dating cyclical turns sometimes follows a vacillating course. We therefore suspect an appreciable erratic factor in the turning points that we finally select. It is possible, however, that smoothing frequently distorts the underlying cyclical fluctuation, and that turns in the smoothed data miss the 'true' cyclical turns by margins as wide or wider than do the turns in the raw data. For,

CHART 44
Seven Smoothing Formulas
Applied to Call Money Rates on New York Stock Exchange
1886 – 1894



See Table 107, especially the notes; also Table 118.

strictly speaking, smoothing does not eliminate erratic movements; it merely redistributes the values of the original data in a manner predetermined by the particular formula used. If this formula is poorly suited to some or all of the cycles in a series, smoothing may misrepresent the cyclical movements. Just as there is uncertainty in dating cyclical turns from raw data, so there is uncertainty in choosing the method of smoothing. Chart 44 and Table 107 illustrate, for a small range of smoothing formulas, the dependence of turning points on the particular formula selected. Formula No. 24 is Macaulay's 43-term graduation.[5]

The pitfalls of smoothing can be seen most clearly in artificially designed series. The results of an experiment with such data are presented in Chart 45 and Table 108. To gauge the effects of smoothing exactly, we assumed independent knowledge of the true cyclical curve. The second column of the table shows the dates assigned to the peaks and troughs of the pure cycles. The succeeding columns set against this standard the timing of artificial series consisting of an assumed random component besides the assumed cycles. Series S′ is a sine curve with a period of 44 months, to which a series of random numbers (drawn from a distribution with a mean of zero) has been added. Series S″ is the same as S′, except that the random numbers were doubled before being added. Series C′ and C″ are like S′ and S″, respectively, except that they are based on a cusp-shaped cyclical curve instead of a sine curve. The like applies to series T′ and T″, whose underlying cyclical component is triangular. The cusp-shaped curve (series C), the triangular curve (series T), and the sine curve (series S) reach peaks and troughs on exactly the same dates; so that the dates in the second column of Table 108 apply to all three sets of pure cycles.[6]

Several results stand out in the table. As is to be expected, the larger the amplitude of the random component relatively to the pure cycles, the more does the timing of the series tend to deviate from that of the pure cycles. This result is characteristic not only of the raw data, but also of the data smoothed by Macaulay's formula. Smoothing tends to improve the timing of the two series built up from a sine curve, and of one series built up from triangular cycles. But it fails to improve, indeed it worsens, the dating of the cyclical turns in the three remaining series—one built up from a triangular and two from a cusp-shaped cyclical curve.

5 The period covered by this experiment seems to be one in which the different methods are especially likely to harmonize. See Chart 43.

The shape of a graduation depends not only on the formula used, but also on the form of the data to which the formula is applied. If the original figures, instead of the logarithms, were smoothed, the results would be very different in a series like call money rates.

6 Series S, T and C have different cyclical shapes, but their period and amplitude are exactly the same. The period is 44 months. The range is from a maximum of 1200 to a minimum of 800. For the derivation of the series, see the notes on Chart 45 in Appendix C.

TABLE 107

Effects of Fourteen Smoothing Formulas on the Timing of Specific Cycles
Call Money Rates on New York Stock Exchange, 1887–1893

Form of data	Peak	Trough	Peak	Trough	Peak
Raw[a]	June 1887	Aug. 1888	Aug. 1890	May 1892	June 1893
Smoothed by formula[b]					
No. 2	Feb. 1887	Sep. 1888	Mar. 1890	Mar. 1892	Mar. 1893
No. 11	Jan. 1887	July 1888	Dec. 1889	May 1892	May 1893
No. 12	Mar. 1887	July 1888	Apr. 1890	Apr. 1892	Mar. 1893
No. 13	Apr. 1887	Aug. 1888	Apr. 1890	Apr. 1892	Mar. 1893
No. 14	Apr. 1887	Aug. 1888	Apr. 1890	Mar. 1892	Mar. 1893
No. 18	Apr. 1887	July 1888	Apr. 1890	Apr. 1892	Mar. 1893
No. 19	Apr. 1887	Aug. 1888	Apr. 1890	Apr. 1892	Mar. 1893
No. 20	Apr. 1887	Aug. 1888	Apr. 1890	Apr. 1892	Mar. 1893
No. 21	Apr. 1887	Aug. 1888	Apr. 1890	Apr. 1892	Mar. 1893
No. 22	Apr. 1887	Aug. 1888	Apr. 1890	Apr. 1892	Mar. 1893
No. 23	Apr. 1887	July 1888	Apr. 1890	Apr. 1892	Mar. 1893
No. 24	Apr. 1887	Aug. 1888	Apr. 1890	Apr. 1892	Mar. 1893
No. 25	May 1887	July 1888	Dec. 1889	May 1892	May 1893
No. 27	Mar. 1887	Aug. 1888	Apr. 1890	Mar. 1892	Apr. 1893

[a] As usual, adjusted for seasonal variations. Chart 44 shows the data before seasonal adjustment.

[b] The smoothing formulas were applied to the logarithms of the original data, unadjusted for seasonal. The graduations are all taken from Appendix VIII of Macaulay's *Smoothing of Time Series*, where the different formulas are identified. The number of terms covered by the formulas ranges from 13 (No. 2) to the entire body of data (No. 25, 27). But seven of the fourteen (No. 18 through No. 24) are fifth-degree parabolics, covering from 35 to 45 terms.

TABLE 108

Timing of Raw and Smoothed Data of Six Artificial Series
Compared with Timing of the Underlying Pure Cycles

Line	Turning dates of pure cyclical curve	Nature of turn[a]	Lead (−) or lag (+) at turn of pure cyclical curve (mos.) Series S′ Raw	Series S′ Sm.	Series S″ Raw	Series S″ Sm.	Series T′ Raw	Series T′ Sm.	Series T″ Raw	Series T″ Sm.	Series C′ Raw	Series C′ Sm.	Series C″ Raw	Series C″ Sm.
1	Dec. 1892	T	0	..	0	..	0	..	0	..	0	..	0	..
2	Oct. 1894	P	+3	+1	+3	+2	+3	+1	+3	+1	+3	+1	+3	+2
3	Aug. 1896	T	−1	0	−1	−1	−1	0	−1	−1	0	0	0	−1
4	June 1898	P	+1	0	+5	+1	+1	0	+1	+1	0	0	0	+1
5	Apr. 1900	T	0	0	+2	0	0	0	0	0	0	0	0	0
6	Feb. 1902	P	0	0	0	−1	0	0	0	−1	0	0	0	−1
7	Dec. 1903	T	+1	−1	+1	−3	+1	−1	+1	−3	+1	−2	+1	−3
8	Oct. 1905	P	−1	+1	−1	+2	−1	+1	−1	+2	0	+1	−1	+2
9	Aug. 1907	T	+3	−1	+3	−2	−1	−1	+3	−2	−1	−2	+3	−3
10	June 1909	P	−1	−2	−3	−4	−1	−2	−1	−4	−1	−2	−1	−5
11	Apr. 1911	T	+3	+1	+3	+1	+3	+1	+3	+1	0	+1	+3	+1
12	Feb. 1913	P	0	−1	+4	−3	0	0	0	−3	0	0	0	−5
13	Dec. 1914	T	+1	..	+1	..	+1	..	+1	..	+1	..	+1	..
Average, signs ignored														
Lines 1–13			1.2	..	2.1	..	1.0	..	1.2	..	0.5	..	1.0	..
Lines 2–12			1.3	0.7	2.4	1.8	1.1	0.6	1.3	1.7	0.5	0.8	1.1	2.2
Average, signs respected														
Lines 1–13			+0.7	..	+1.3	..	+0.4	..	+0.7	..	+0.2	..	+0.7	..
Lines 2–12			+0.7	−0.2	+1.5	−0.7	+0.4	−0.1	+0.7	−0.8	+0.2	−0.3	+0.7	−1.1

Smoothing by Macaulay's 43-term graduation involves the loss of 21 terms at each end; hence there are no smoothed entries in lines 1 and 13, and their average is confined to lines 2-12. See Chart 45.

[a] T stands for the trough of specific cycles, P for the peak.

It is plain from Chart 45 that besides wiping out the erratic movements, smoothing twists in some measure the cyclical movements. The degree of twist depends partly on the size of the erratic movements, partly on the shape of the cyclical movements. Since Macaulay's graduation is designed to follow the outlines of a sine curve or any other with more or less rounded tops and bottoms, it is most satisfactory in the series derived from a sine curve.[7] But it does not cope adequately with sharp cusps[8] such as characterize series C′ and C″. In both, the graduation is much too low in the immediate vicinity of the true peaks and, usually, much too high in adjacent months. It is likewise too high in the immediate vicinity of the true troughs and too low in adjacent months. Since the tops and bottoms become gently rounded in the graduation, the turning dates can easily deviate from the turns of the underlying cusp cycles. In the raw data, on the other hand, the random component is likely to be especially small relatively to the cusped cyclical component precisely on the dates when the latter reaches a maximum or minimum; hence the timing of the raw data tends to match closely the timing of the pure cycles. Of course, this result will tend to be blurred as the amplitude of the random component is increased relatively to the cusped component. But there can be no escape from the conclusion that a cyclical graduation may seem excellent to the eye, perhaps even satisfy formal tests applied to deviations of the raw data from it, and yet seriously misrepresent the true cyclical movements.

While these experiments demonstrate that the results of smoothing may be surrounded by wide margins of error, they do not argue against smoothing as such. Given the assumptions underlying the artificial series, smoothing is valid in principle, though the right formula may be hard to discover and may need to be varied from series to series. And so it would be also in the case of historical series, if we could assume that they correspond to our experimental model; that is, that they record cyclical effects, or combined secular and cyclical effects, on which random effects are superposed. There is rarely any warrant for such an assumption. In the experimental model cyclical and random forces are distinct; their individual effects can be distinguished precisely, month by month, phase by phase, cycle by cycle. In historical series the effects of cyclical and random forces cannot be separated even over the course of a full cycle. Random factors constantly play on business at large and on each of its many branches, and their effects register in different ways under different circumstances. Some series on prices and central-bank discount rates show step movements of extended duration, free of short-term oscillations. Broad series on employment frequently move in the same direction for many months. Call money rates, on the other hand, show frequent re-

[7] See below, p. 330, and note 24.

[8] This difficulty is by no means peculiar to Macaulay's graduation. See below, especially Sec. IV and VI.

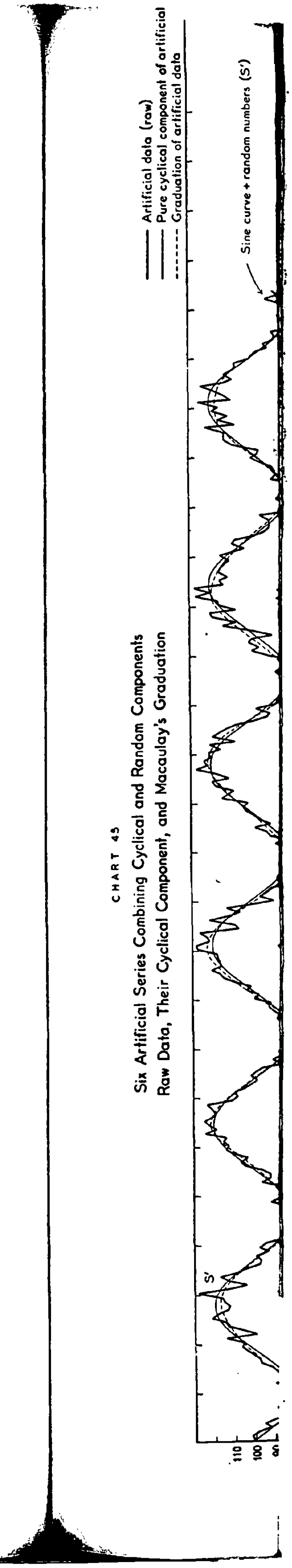
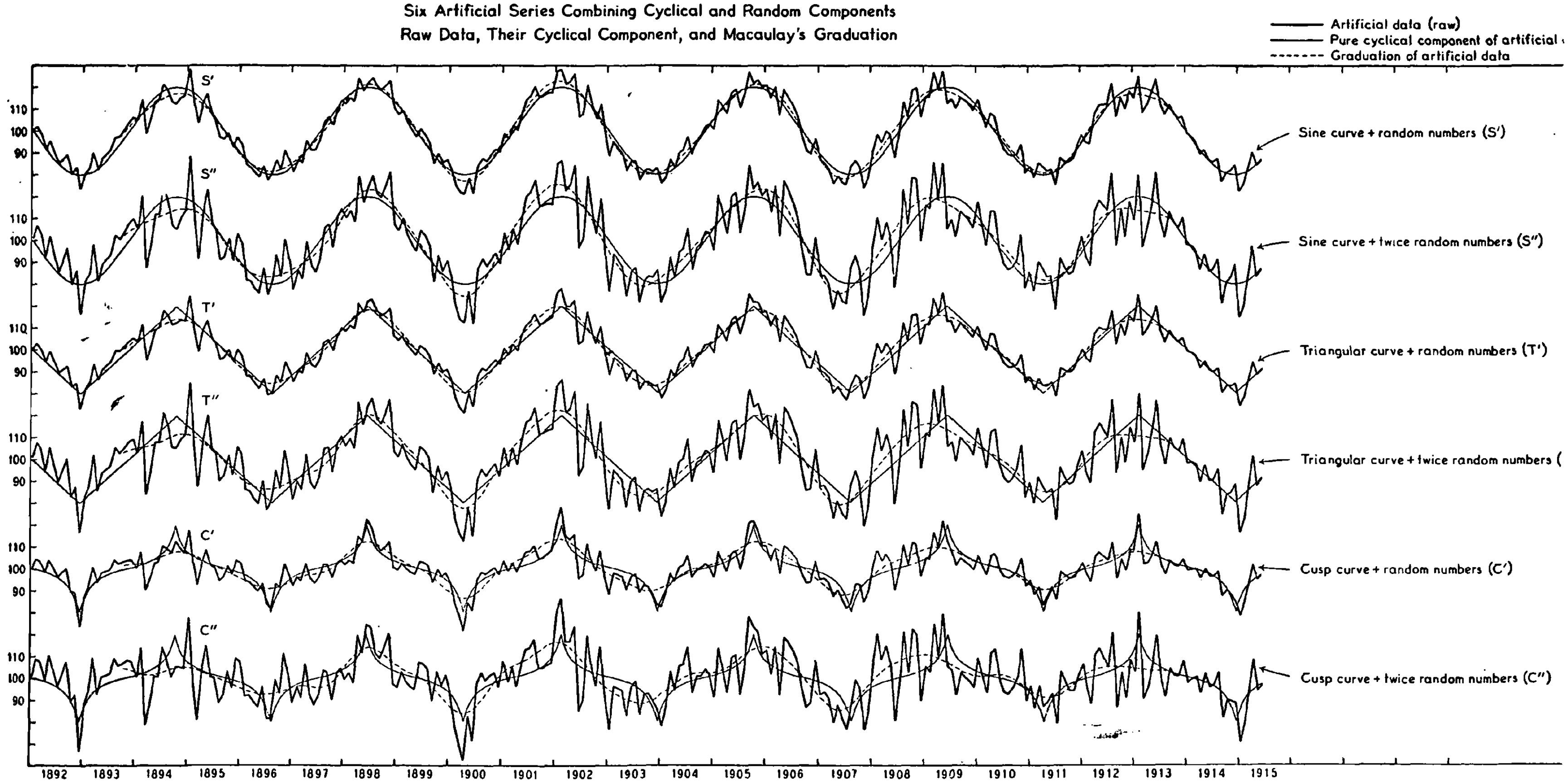


CHART 45
Six Artificial Series Combining Cyclical and Random Components
Raw Data, Their Cyclical Component, and Macaulay's Graduation

CHART 45

Six Artificial Series Combining Cyclical and Random Components
Raw Data, Their Cyclical Component, and Macaulay's Graduation



For derivation of the series and other explanations, see text and Appendix C.

versals of direction which may be mainly the effects of random happenings; though we know, historically, that most of the high narrow peaks in this series mark cyclical, not random, effects. Series like bank clearings and iron production may seem to approximate our experimental model more closely. Yet we may be sure that random factors have played an important part in shaping their specific cycles—now prolonging an expansion, now intensifying as well as prolonging it, now abbreviating a contraction, and so on, in endless variety. 'Erratic movements' therefore cannot be identified with 'effects of random forces' as in our experimental model. The 'erratic movements' that stand out clearly in time series and are obliterated by smoothing are simply the short-term oscillations, other than seasonal variations, that play, so to speak, on the back of specific cycles.[9] Not clearly revealed by the data and not smoothed out by the usual formulas are other effects of random forces. Indeed, the general contours of cyclical movements themselves may have been shaped, in part, by the same forces that produced the short-term oscillations.

These observations instill caution in the use of smoothing devices. If the turning zone of the specific cycles in a series is crisscrossed by erratic movements, it is likely that the historical limits of the expansions and contractions of the specific cycles will be approximated better from smoothed than from raw data. On the other hand, if the cyclical turns in the raw data are naturally 'smooth'—that is, if erratic movements seem absent in the vicinity of cyclical turns[10]—smoothing is an indulgence, the price of which may be distortion of historical cycles. Table 109 presents several illustrations.[11] Even if erratic movements make their influence felt in the vicinity of cyclical turns, smoothing is of doubtful advantage so long as expansion culminates in a clearly defined peak and contraction in a clearly defined trough. For when a cyclical phase happens to be brief but large, smoothing is likely to spread it out, predating the peak and postdating the trough in the case of contractions, predating the trough and postdating the peak in the case of expansions.[12] The strength

9 See pp. 57-8, 87-90, concerning difficulties sometimes encountered in distinguishing these short-term variations, here lumped together as 'erratic movements', from specific or business cycles. The interrelations of the short-term fluctuations, other than seasonal variations or specific cycles, in different economic activities are now being investigated by one of our colleagues, Ruth P. Mack.

10 Here we include curves having cusps, which some readers may not consider as being 'smooth'.

11 To obtain the list of 'smooth turns' in Table 109, we first recorded every trough (peak) that is preceded by at least a five-month decline (rise) and followed by at least a five-month rise (decline). Of course, this criterion of a 'smooth' turn sometimes proves inadequate. For example, if a specific-cycle peak is preceded by a six-month rise and followed by a six-month decline, which in turn is followed by a two-month rise culminating in a value not much lower than the peak, we can hardly say that the cyclical turn is 'smooth'. For reasons of this nature, five instances conforming to the five-month rule are omitted in Table 109. In view of the precautions taken to omit 'double' tops or bottoms, the five-month rule is severe; that is, the table understates the number of 'smooth turns'.

12 If a cyclical phase is brief and mild, smoothing may spread it so much that it disappears—virtually if not entirely. But a movement that is both very brief and very small is not likely to be recognized as cyclical. As a rule, therefore, brief cyclical phases have longer counterparts in smoothed data; although brief phases of small amplitude may have shorter counterparts.

TABLE 109

Effect of Smoothing on Cyclical Turns That Are 'Smooth' in Raw Data

Three American Series

Series and 'smooth' specific-cycle turn in raw data	Nature of turn[a]	No. of months raw data move continuously in same direction		Date of corresponding turn in smoothed data	Lead (−) or lag (+) of smoothed data at turn of raw (mos.)
		Before turn	After turn		
PIG IRON PRODUCTION					
Apr. 1891	T	7	6	Mar. 1891	−1
Nov. 1895	P	7	5	Dec. 1895	+1
Oct. 1896	T	5	5	Nov. 1896	+1
Jan. 1910	P	9	11	Jan. 1910	0
July 1921	T	10	11	Aug. 1921	+1
May 1923	P	9	8	July 1923	+2
RAILROAD STOCK PRICES					
Oct. 1857	T	10	5	Jan. 1858	+3
CALL MONEY RATES					
Sep. 1924	T	5	6	Sep. 1924	0

Based on our four series for the periods shown in Table 112. There are no entries for deflated clearings, since no 'smooth' turns occur in this series in the sense of our rules in note 11.

[a] T stands for the trough of specific cycles, P for the peak.

of these tendencies when Macaulay's formula is used is displayed in Table 110.

An historical illustration may help the reader see more vividly the difficulties that are sometimes caused by smoothing. The contraction of 1907–08 was short but steep, like the contraction of 1937–38 of more recent memory. Signs of recession appeared late in 1906, when stock prices began to sag, trading in securities fell off, and bank clearings in New York turned down. General business continued to expand, however, until May 1907 according to our reference chronology. Whatever decline occurred during the summer was small. But in October a succession of bank closures in New York precipitated a financial panic that spread rapidly to the interior.[13] For several months following, acute depression in industry and trade was general. Table 111 shows the timing of the contraction, as manifested in ten series—the four on which the bulk of the tests of this chapter are based, and six additional series to which Macaulay applied his smoothing formula. The course of the crisis can be followed easily in the raw data, but not in the smoothed data. Macaulay's graduation makes the peak in call money rates come in August 1907,[14] instead of October when panic broke out in the New York financial district. The

[13] See Wesley C. Mitchell, *Business Cycles* (1913), pp. 514-38; or the reprint *Business Cycles and Their Causes*, pp. 74-107.

[14] A still higher peak occurs in May 1906. In Table 111 this peak is ignored, whereas in other comparisons of this chapter the 'minor' peak of Aug. 1907 is ignored. We might have recognized both peaks in our standard analysis; but that course was ruled out by the convention of one-to-one correspondence laid down in Sec. II. The high point in the raw data in Sept. 1906, which corresponds to the high of May 1906 in the smoothed data, could not be recognized as a cyclical peak in the raw series, because it is only thirteen months removed from the higher peak in Oct. 1907. See Chart 43.

TABLE 110
Timing and Duration of 'Brief' Specific-cycle Phases
Four American Series, Raw and Smoothed

Series and specific-cycle phase in raw data[a]	Nature of phase[b]	Number of months lead (−) or lag (+) of smoothed data at		Duration of phase in months	
		Initial turn of raw data	Terminal turn of raw data	Raw data	Smoothed data
DEFLATED CLEARINGS					
June 1881 – Jan. 1882	C	+2	+4	7	9
June 1887 – Mar. 1888	C	0	−1	9	8
Sep. 1890 – Mar. 1891	C	−1	+1	6	8
Feb. 1893 – Aug. 1893	C	−3	+4	6	13
Oct. 1907 – Dec. 1907	C	−6	+4	2	12
Mar. 1910 – Oct. 1910	C	−1	+1	7	9
Aug. 1918 – Dec. 1918	C	−1	−2	4	3
May 1923 – Sep. 1923	C	0	+5	4	9
PIG IRON PRODUCTION					
Oct. 1887 – Mar. 1888	C	−1	−1	5	5
June 1903 – Dec. 1903	C	−4	+3	6	13
July 1907 – Jan. 1908	C	−3	+4	6	13
Sep. 1918 – May 1919	C	0	+3	8	11
RAILROAD STOCK PRICES					
Oct. 1857 – Mar. 1858	E	+3	+5	5	7
Sep. 1860 – May 1861	C	0	+3	8	11
May 1890 – Dec. 1890	C	−2	+3	7	12
Mar. 1895 – Sep. 1895	E	−4	0	6	10
Apr. 1900 – Sep. 1900	C	−7	−4	5	8
CALL MONEY RATES					
Jan. 1876 – Sep. 1876	C	−1	+3	8	12
Jan. 1878 – Sep. 1878	C	0	+2	8	10
Oct. 1880 – Feb. 1881	E	−3	+3	4	10
May 1882 – Sep. 1882	E	−3	+3	4	10
Aug. 1918 – Dec. 1918	C	0	+1	4	5

[a] Includes every expansion or contraction lasting 8 months or less in either the raw or smoothed data during the periods listed in Table 112.
[b] C stands for contraction, E for expansion.

raw data on clearings outside New York show a peak in October and a trough in December 1907; Macaulay's graduation shifts the peak back six months and the trough forward four months. But the raw data tell the tale that is cyclically significant: outside clearings slumped sharply as banks generally suspended or limited cash payments, and rose promptly at the end of the year when normal banking operations were resumed.

When we leave single cycles, where smoothing sometimes improves and sometimes worsens the dating of cyclical turns, and examine averages, it appears that the net effect of smoothing is negligible. Table 106 shows that in four instances the smoothed data lag on the average behind the raw data; in another four instances the smoothed data lead. The maximum average difference at peaks or troughs is only 1.2 months. It thus appears that smoothing influences the average timing of specific cycles less than the removal of secular trends, despite the fact that the former affects single cycles more frequently than the latter. Of course the reason is that the effects of removing trends are cumulative (except when the trend changes direction), while the effects of smoothing are largely self-canceling.

But there is a slight twist to the cancellation. As Table 106 indicates, the smoothed data have some tendency to lead at peaks and lag at troughs. Table 112 shows that in each series smoothing reduces somewhat the average duration of expansions and increases the average duration of con-

TABLE 111

Chronology of Specific-cycle Turns in the Contraction of 1907–1908
Ten American Series, Raw and Smoothed

Series	Specific-cycle turns in raw data		Specific-cycle turns in smoothed data		Lead (−) or lag (+) of smoothed data at turn of raw (mos.)	
	Peak	Trough	Peak	Trough	Peak	Trough
Bank clearings, N.Y.C.	Aug. 1906	Dec. 1907	July 1906	Feb. 1908	−1	+2
Railroad stock prices	Sep. 1906	Nov. 1907	Aug. 1906	Dec. 1907	−1	+1
Pig iron production	July 1907	Jan. 1908	Apr. 1907	May 1908	−3	+4
Call money rates, N.Y. Stock Ex.	Oct. 1907	Nov. 1908	Aug. 1907[a]	Oct. 1908	−2	−1
Bank clearings, outside N.Y.C.	Oct. 1907	Dec. 1907	Apr. 1907	Apr. 1908	−6	+4
Deflated clearings, outside N.Y.C.	Oct. 1907	Dec. 1907	Apr. 1907	Apr. 1908	−6	+4
Wholesale prices	Oct. 1907	Feb. 1908	June 1907	June 1908	−4	+4
90-day money rates, stock ex. loans	Nov. 1907	Sep. 1908	Aug. 1907	Nov. 1908	−3	+2
Railroad bond yields	Nov. 1907	Feb. 1909	Dec. 1907	Mar. 1909	+1	+1
Commercial paper rates, N.Y.C.	Dec. 1907	July 1909	Aug. 1907	Jan. 1909	−4	−6

The smoothed data are all given in Appendix A of Macaulay's *Interest Rates, Bond Yields, and Stock Prices*. For the sources of the original data, see our Appendix C (Table 21). The specific cycles of the 'raw data' are dated from seasonally adjusted figures, except when there seemed to be no seasonal movement (railroad bond yields, railroad stock prices, wholesale prices).

Macaulay's smoothed series on bank clearings outside N.Y.City does not correspond to ours. However, the cyclical turns in 1907 in the 'raw' series underlying Macaulay's smoothed series agree with the turns in the 'raw' series that we have used.

[a] See note 14.

TABLE 112

Average Duration of Specific Cycles in Raw and Smoothed Data
Four American Series

Series and form of data	Period covered[a]	No. of specific cycles[b]	Average duration in months			Average per cent of duration of specific cycles	
			Expansion	Contraction	Full cycle	Expansion	Contraction
DEFLATED CLEARINGS							
Raw	1878–1933	15	32.6	11.4	44.0	75	25
Smoothed	1878–1932	15	31.0	12.5	43.5	71	29
PIG IRON PRODUCTION							
Raw	1879–1933	15	28.8	14.5	43.3	67	33
Smoothed	1878–1932	15	27.0	16.1	43.1	62	38
RAILROAD STOCK PRICES							
Raw	1857–1932	18	28.8	21.0	49.8	56	44
Smoothed	1858–1932	18	27.7	22.0	49.7	54	46
CALL MONEY RATES							
Raw	1858–1931	23	19.9	18.0	37.9	52	48
Smoothed	1858–1931	23	19.2	18.7	37.9	51	49

[a] Unlike Table 106, the periods covered by this table start and end with a trough.
[b] See Sec. II.

tractions. The differences between the average durations are small, as they must be in view of the small effects of smoothing on average timing. Nevertheless, they are of theoretical interest. As we have seen, when a cyclical expansion or contraction is brief, smoothing tends to spread it over a longer period. In three of our series brief contractions are more numerous than brief expansions.[15] On the average, therefore, smoothing tends to lengthen the contractions and to shorten the expansions. In call money rates, where the number of brief expansions and contractions is practically the same, it also happens that smoothing has the smallest influence on the average duration of cyclical phases.

The average duration of full specific cycles is virtually the same in the raw and smoothed data we are using. The result could not be otherwise; for in railroad stock prices the number of specific cycles is the same in the two records, while in the other three series we have forced the two sets of cycles into correspondence.[16]

IV Amplitude of Specific Cycles

If we had a method of dating accurately the crest of a cyclical upswing, we could claim that whatever erratic thrust occurs on that date is as likely to lower the crest as to raise it. But when we select the highest point of data that are composites of both cyclical and erratic movements, that point is more likely to be above than below the cyclical crest. *Mutatis mutandis,* these remarks apply also to the lowest points of cyclical downswings. Since we seek to mark off specific cycles by selecting the highest and lowest months, there is danger that the amplitudes of the cyclical fluctuations would be exaggerated if we measured them by the differences between specific-cycle relatives at the peaks and troughs. To guard against this danger we measure amplitudes, not from the cycle relatives of the highest and lowest months, but from averages covering three months centered on the peaks and troughs.

This protective device is obviously rough. The relations in size among the raw figures in the vicinity of a cyclical turn differ from case to case, and we therefore cannot expect that the use of three-month averages to represent cyclical peaks and troughs will yield a good measure of the cyclical amplitude in every instance. The most that we can expect is that it will do so on the average, and then only if used with circumspection. The method breaks down (1) if a cyclical expansion or contraction is very short, (2) if the top or bottom of a cycle is horizontal, (3) if the month selected as the peak is preceded or followed by an extremely low value, or if the month of trough is preceded or followed by an extremely high value. In these

15 For partial evidence, see Tables 110 and 117.
16 See Sec. X.

TABLE 113
Average Amplitude of Specific Cycles in Raw and Smoothed Data
Four American Series

Series and form of data	No. of specific cycles[a]	No. of months in standing at turns	Average standing in specific-cycle relatives at: Initial trough	Peak	Terminal trough	Average amplitude in specific-cycle relatives: Rise	Fall	Rise & fall
DEFLATED CLEARINGS								
Raw	15	1	83.4	116.0	96.2	32.6	19.8	52.4
Raw	15	3[b]	85.7	112.6	99.2	26.9	13.4	40.2
Smoothed	15	1	86.8	110.8	100.2	24.0	10.6	34.6
PIG IRON PRODUCTION								
Raw	15	1	64.0	131.4	70.7	67.3	60.6	128.0
Raw	15	3	67.3	129.3	74.6	62.1	54.8	116.8
Smoothed	15	1	73.0	126.5	81.6	53.4	44.8	98.3
RAILROAD STOCK PRICES								
Raw	18	1	80.7	120.1	84.8	39.4	35.3	74.7
Raw	18	3	82.8	118.3	86.6	35.6	31.8	67.3
Smoothed	18	1	84.5	116.0	88.0	31.6	28.0	59.6
CALL MONEY RATES								
Raw	23	1	54.6	254.5	54.5	199.8	200.0	399.8
Raw	23	3	62.1	178.0	61.9	115.9	116.1	232.0
Smoothed	23	1	68.2	146.8	68.1	78.6	78.7	157.2
			Relatives of averages based on 3-month standings of raw data					
DEFLATED CLEARINGS								
Raw	15	1	97	103	97	121	148	130
Smoothed	15	1	101	98	101	89	79	86
PIG IRON PRODUCTION								
Raw	15	1	95	102	95	108	111	110
Smoothed	15	1	108	98	109	86	82	84
RAILROAD STOCK PRICES								
Raw	18	1	97	102	98	111	111	111
Smoothed	18	1	102	98	102	89	88	89
CALL MONEY RATES								
Raw	23	1	88	143	88	172	172	172
Smoothed	23	1	110	82	110	68	68	68

See Table 112 for the periods covered.
[a] See Sec. II.
[b] In this series there are several exceptions to the 3-month rule; see Appendix Table B1.

three types of cases we make exceptions from the three-month rule.[17] Yet the exceptions cover only the most glaring defects of the method, and are themselves handled in ways that may raise as many doubts as does the three-month rule.

All these difficulties are swept away by smoothing. Since erratic fluctuations have already been removed in the process of smoothing, amplitudes can be measured directly from the highest and lowest points of the cycles. Measures of cyclical amplitude derived in this way have a simplicity and elegance to which our standard measures can lay no claim.

The use of three-month averages to represent peaks and troughs involves, of course, a form of smoothing. The smoothing effect is substantial, as Table 113 demonstrates. Yet it is smaller on the average than

[17] See p. 132.

the effect produced by Macaulay's formula. The differences between average amplitudes based on three-month standings of the raw data and one-month standings of the smoothed data are sometimes uncomfortably large, especially in call money rates. And since smoothing has a marked effect on average results, it is bound at times to produce still larger effects in single cycles. Table 114 shows the cycle-by-cycle measures for pig iron production. It is plain that smoothing not only tends to reduce the measures of amplitude, but that it does this rather unevenly. Table 115 demonstrates that the uneven effects of smoothing do not disappear completely even when the amplitudes are ranked.

TABLE 114

Amplitude of Specific Cycles in Raw and Smoothed Data
Pig Iron Production, United States, 1878–1933

Form of data	Dates of specific cycles	Standing in specific-cycle relatives at			Amplitude in specific-cycle relatives			Amplitude of smoothed data as relative of raw amplitude		
	Trough – Peak – Trough	Initial trough	Peak	Terminal trough	Rise	Fall	Rise & fall	Rise	Fall	Rise & fall
Raw	Jan. 79 – Feb. 83 – Jan. 85	55.7	119.7	95.1	64.0	24.6	88.6			
Sm.	Dec. 78 – Mar. 83 – Apr. 85	55.3	120.5	99.5	65.2	21.0	86.2	102	85	97
Raw	Jan. 85 – Oct. 87 – Mar. 88	67.0	131.8	106.3	64.8	25.5	90.3			
Sm.	Apr. 85 – Sep. 87 – Feb. 88	68.6	114.1	113.2	45.5	0.9	46.4	70	4	51
Raw	Mar. 88 – May 90 – Apr. 91	74.0	124.2	76.8	50.2	47.4	97.6			
Sm.	Feb. 88 – May 90 – Mar. 91	80.4	125.2	94.7	44.8	30.5	75.3	89	64	77
Raw	Apr. 91 – Feb. 92 – Oct. 93	67.5	112.9	49.2	45.4	63.7	109.1			
Sm.	Mar. 91 – Jan. 92 – Jan. 94	87.9	121.0	60.0	33.1	61.0	94.1	73	96	86
Raw	Oct. 93 – Nov. 95 – Oct. 96	51.5	138.7	77.9	87.2	60.8	148.0			
Sm.	Jan. 94 – Dec. 95 – Nov. 96	57.9	130.8	89.4	72.9	41.4	114.3	84	68	77
Raw	Oct. 96 – Dec. 99 – Oct. 00	52.3	128.3	97.8	76.0	30.5	106.5			
Sm.	Nov. 96 – Dec. 99 – Oct. 00	61.1	128.7	107.5	67.6	21.2	88.8	89	70	83
Raw	Oct. 00 – June 03 – Dec. 03	69.0	118.0	68.0	49.0	50.0	99.0			
Sm.	Oct. 00 – Feb. 03 – Mar. 04	77.2	116.4	87.1	39.2	29.3	68.5	80	59	69
Raw	Dec. 03 – July 07 – Jan. 08	51.1	124.7	62.0	73.6	62.7	136.3			
Sm.	Mar. 04 – Apr. 07 – May 08	65.5	129.8	64.6	64.3	65.2	129.5	87	104	95
Raw	Jan. 08 – Jan. 10 – Dec. 10	59.8	138.4	96.7	78.6	41.7	120.3			
Sm.	May 08 – Jan. 10 – Apr. 11	59.0	129.1	94.2	70.1	34.9	105.0	89	84	87
Raw	Dec. 10 – Jan. 13 – Dec. 14	82.7	127.7	70.9	45.0	56.8	101.8			
Sm.	Apr. 11 – Feb. 13 – Nov. 14	82.6	122.3	76.8	39.7	45.5	85.2	88	80	84
Raw	Dec. 14 – Sep. 18 – May 19	52.5	114.4	74.1	61.9	40.3	102.2			
Sm.	Nov. 14 – Sep. 18 – Aug. 19	59.1	117.0	79.8	57.9	37.2	95.1	94	92	93
Raw	May 19 – Sep. 20 – July 21	90.2	130.9	39.8	40.7	91.1	131.8			
Sm.	Aug. 19 – July 20 – Aug. 21	97.6	139.3	46.1	41.7	93.2	134.9	102	102	102
Raw	July 21 – May 23 – July 24	38.9	147.2	88.6	108.3	58.6	166.9			
Sm.	Aug. 21 – July 23 – July 24	43.1	139.8	92.8	96.7	47.0	143.7	89	80	86
Raw	July 24 – July 26 – Nov. 27	73.5	110.0	91.3	36.5	18.7	55.2			
Sm.	July 24 – July 26 – Dec. 27	79.0	110.0	95.4	31.0	14.6	45.6	85	78	83
Raw	Nov. 27 – July 29 – Mar. 33	123.3	173.3	24.4	50.0	148.9	198.9			
Sm.	Dec. 27 – May 29 – Oct. 32	121.0	152.9	23.2	31.9	129.7	161.6	64	87	81

The amplitudes of the raw data are derived from 3-month averages of specific-cycle relatives centered on the trough and peak dates. The amplitudes of the smoothed data are derived from the specific-cycle relatives at the trough and peak dates.

On a per month basis also the amplitudes are larger on the average in the raw than in the smoothed data (Table 116). But whereas, in proportion to the size of the figures, smoothing affects the fall only slightly more than the rise, it affects the *rate* of fall considerably more than the *rate* of rise. Further, as Table 115 shows, smoothing affects the ranking of the per month amplitudes within a series more than the ranking of the amplitudes proper. These differences are connected with the tendency of smoothing to stretch out and dampen brief cyclical phases; also with the fact that brief contractions are more numerous in our sample than brief expansions. By reducing more often than it prolongs the duration of ex-

TABLE 115

Coefficients of Rank Correlation between Amplitudes of Corresponding Specific Cycles in Raw and Smoothed Data
Four American Series

Series	No. of specific cycles[a]	Coefficient of rank correlation between measures of raw and smoothed data					
		Total amplitude[b]			Per month amplitude[b]		
		Rise	Fall	Rise & fall	Rise	Fall	Rise & fall
Deflated clearings	15	+.95	+.95	+.95	+.94	+.44	+.85
Pig iron production	15	+.91	+.94	+.94	+.92	+.64	+.93
Railroad stock prices	18	+.97	+.97	+.98	+.61	+.79	+.78
Call money rates	23	+.90	+.91	+.88	+.54	+.69	+.82

The periods covered are shown in Table 112.
[a] See Sec. II.
[b] Expressed in specific-cycle relatives. See note to Table 114.

TABLE 116

Average Per Month Amplitude of Specific Cycles in Raw and Smoothed Data
Four American Series

Series and form of data	No. of specific cycles[a]	Per month amplitude in specific-cycle relatives					
		Unweighted average			Weighted average		
		Rise	Fall	Rise & fall	Rise	Fall	Rise & fall
DEFLATED CLEARINGS							
Raw	15	0.8	1.9[b]	0.9	0.8	1.2	0.9
Smoothed	15	0.8	0.7	0.8	0.8	0.8	0.8
PIG IRON PRODUCTION							
Raw	15	2.4	4.7	2.9	2.2	3.8	2.7
Smoothed	15	2.3	2.9	2.5	2.0	2.8	2.3
RAILROAD STOCK PRICES							
Raw	18	1.4	1.7	1.4	1.2	1.5	1.4
Smoothed	18	1.1	1.2	1.2	1.1	1.3	1.2
CALL MONEY RATES							
Raw	23	6.9	7.6	6.3	5.8	6.4	6.1
Smoothed	23	4.3	4.6	4.3	4.1	4.2	4.2

The periods covered are shown in Table 112. See note to Table 114; also Table 113, note 'b'.
[a] See Sec. II.
[b] Becomes 1.2 when the contraction of 1907 is excluded; this contraction was severe but lasted only two months.

pansions, smoothing tends to raise the average rate of rise, offsetting part of the decline it effects in the amount of rise. By prolonging more often than it reduces the duration of contractions, smoothing tends to reduce the average rate of decline, thus reinforcing its effect on the amount of decline. Since the mean duration of full specific cycles is practically the same in the raw and smoothed data, smoothing affects the average per month amplitude of full cycles in approximately the same ratio as the average amplitude proper.[18]

The gap between the raw and smoothed figures in Table 116 is narrower in the weighted than in the unweighted average rates of change for expansions and contractions. Weighting has slight influence on the average per month amplitudes of the smoothed data, but reduces materially the averages of the raw data. The reason for the marked effects of weighting on the raw averages is that short specific-cycle phases tend to have larger per month amplitudes than long phases. The inverse correlation between the per month amplitude and the duration of cyclical phases tends to disappear when the data are smoothed, mainly because smoothing tends to increase the duration and decrease the amplitude of brief cyclical phases.

So much for the differences between the raw and smoothed results. The outstanding fact is that our standard method yields higher measures of amplitude than are derived from smoothed data. We cannot say with certainty which method commonly gives the more valid cyclical measures. It is clear, however, that smoothing by Macaulay's formula often cuts off a part of the 'true' cyclical fluctuations in the process of erasing erratic movements. That result is likely whenever the specific cycles of raw data have high and narrow peaks, or deep and narrow troughs; also when the phases of specific cycles are very brief. Macaulay's formula will preserve almost perfectly the amplitude of a sine curve with a period of 30 months or longer.[19] When the period drops below two years, or when the underlying cyclical curve, whatever its period, is not of sine shape, but has cusped peaks or troughs, Macaulay's formula is sure to understate the cyclical amplitude. Since brief phases are fairly numerous in our test series, and since the cyclical patterns of these series are, as a rule, decidedly more angular than sine curves, we should expect smoothing to yield measures of amplitude that, in the first place, are usually below our standard measures and that, in the second place, are sometimes almost as large

[18] This result is mathematically necessary only in the weighted average per month amplitude. But in our sample, weighting (by duration) makes little difference in the average per month amplitude of the joint rise and fall; hence the relation governing the weighted per month average applies also to the unweighted average.

[19] The percentage of the amplitude preserved is 99.39 for a sine-curve period of 30 months, 100.36 for a period of 36 months, 100.44 for 40 months, 100.33 for 48 months, 100.18 for 60 months; but the percentage is only 94.17 for 24 months, 82.05 for 20 months, 69.76 for 18 months (*The Smoothing of Time Series*, p. 159).

and at other times only a tenth or a fifth as large as our standard measures —precisely the results we have found.

In the preceding section we saw how smoothing tends to draw out brief cyclical phases. When that happens the amplitude of the cyclical phase tends to be reduced. In a cyclical phase that lasts more than a year the amplitude of smoothed data is, typically, only 5 to 15 per cent below our standard measure (Table 117). But when a phase lasts a half year or less, smoothing on the average reduces our standard measure about 50 per cent.[20] The tendency of the margin between the two sets of measures to widen as the duration of cyclical phases becomes shorter is not due to any flaw in our standard method; for, so far as our technique may yield biased measures of amplitude for short phases, that bias is likely[21] to work in the same direction as the bias produced by smoothing.[22]

TABLE 117

Influence of the Duration of Cyclical Phases on the Gap between Amplitudes of Corresponding Phases in Raw and Smoothed Data

Duration of phase in raw data (mos.)	Number of instances			Average ratio of amplitude of smoothed data to corresponding amplitude of raw data		
	Expansion	Contraction	Both phases	Expansion	Contraction	Both phases
Under 7	2	9	11	.43	.57	.55
7 - 12	1	17	18	.73	.77	.76
13 - 18	8	9	17	.84	.85	.85
19 - 24	10	6	16	.89	.84	.87
25 - 36	14	3	17	.85	.95	.87
37 - 48	9	3	12	.92	.93	.92
Over 48	4	1	5	.94	.91	.94

Based on specific cycles in deflated clearings, pig iron production, and railroad stock prices during the periods shown in Table 112. See Sec. II and note to Table 114.

The relations exhibited by Table 117 appear in call money rates, as well as in the three series covered by the table.[23] But in call money rates the chief reason for the wide gap between the amplitude measures derived from raw and smoothed data is the large number of high and narrow

20 Of course, the gap between the two measures for a phase depends (among other factors) on the duration of adjacent phases as well as on its own duration. This consideration is important in long phases. The average ratio of the smoothed to the raw amplitude is .90 for the 34 phases that lasted more than two years. Of these 34 phases, 12 were preceded or/and followed by a phase lasting 6 months or less; the average ratio is .86 for these cases and .92 for the rest.

21 Except in phases lasting less than 4 months (see p. 132); but such extremely short phases are very rare.

22 Compare the effects of the shift from one-month to three-month standings on the average amplitude of expansions and contractions in the raw data on clearings and iron production (Table 113) with the average duration of these phases (Table 112).

23 The averages for call money rates corresponding to the first six lines in the last column in the table are as follows: .36(3), .71(7), .71(15), .71(9), .76(11), .57(1). The figures in parentheses show the number of observations included in the averages. No phase lasting over 48 months occurs in call money rates.

cyclical peaks. Given the organization of the New York money market, particularly as it stood before the adoption of the Federal Reserve system, and the character of the demand for call loans, cyclical forces themselves have tended under certain circumstances to produce high and narrow peaks, and at other times broad and low troughs. Macaulay's formula turns the high, narrow peaks into broad, low hills, and thus misrepresents the cyclical behavior that is characteristic of the call loan market.[24] Nor is Macaulay's formula by any means peculiar in this respect, as Table 118 demonstrates.[25] It seems not unlikely that even our three-month averages understate the 'true' cyclical amplitudes of this series.

TABLE 118

Effects of Fourteen Smoothing Formulas on Amplitude of Specific Cycles
Call Money Rates on New York Stock Exchange, 1887–1893

Form of data	Per cent change from			
	Peak in 1887 to trough in 1888	Trough in 1888 to peak in 1890	Peak in 1890 to trough in 1892	Trough in 1892 to peak in 1893
Raw	−73	+304	−76	+256
Smoothed by formula				
No. 2	−57	+154	−60	+121
No. 11	−66	+234	−71	+229
No. 12	−62	+183	−64	+177
No. 13	−62	+182	−63	+160
No. 14	−63	+192	−65	+180
No. 18	−63	+193	−65	+199
No. 19	−64	+191	−65	+186
No. 20	−64	+190	−65	+182
No. 21	−64	+190	−65	+180
No. 22	−64	+188	−64	+170
No. 23	−64	+196	−66	+204
No. 24	−64	+196	−66	+198
No. 25	−65	+203	−67	+219
No. 27	−63	+177	−63	+168

In contrast to our standard method, the measures in this table express the change between dates of cyclical turn as a percentage of the standing at the earlier turn. One-month standings are used in the smoothed data, 3-month standings in the raw data. The months of cyclical turn in the raw and smoothed data are given in Table 107. For supplementary information, see Chart 44 and the notes to Table 107.

If the differences between the peaks of the raw data and of Macaulay's graduation were due mainly to erratic movements, we should not expect these differences to differ significantly from one period of call money rates to another. In fact, as Table 119 shows, the average difference between one-month standings at peaks of raw and smoothed data drops from 133

[24] One of the criteria for satisfactory smoothing set up by Macaulay is that "if applied to successive points on a sine curve . . . , the graduation should fall as close as possible to the points on the sine curve". His 43-term graduation meets this test satisfactorily for sine curves with a period of 30 months or longer (see note 19). Macaulay recognizes that "if the underlying curve have cusps or be discontinuous", any continuous smoothing, whether by his 43-term or some other formula, will "somewhat obscure such characteristics"; and he cites the behavior of call money rates "during and after a financial panic" as an example. See *The Smoothing of Time Series,* pp. 21-2, 104-5.

[25] Macaulay's formula is No. 24. In view of the wide use of a twelve-month moving average to represent the combined trend and cyclical fluctuations of a series, it is worth noting that formula No. 2 is a two-month moving average of a twelve-month moving average.

points in 1858–1915 to 18 points in 1915–31; the average difference between one-month and three-month standings of raw data drops from 94 to 13; the average difference between three-month standings of raw data and one-month standings of smoothed data drops from 38 to 5. The reason for the drop is, of course, that after the introduction of the Federal Reserve system, the high and narrow peaks in call money rates practically disappeared. There can be little doubt that this represents a change in the cyclical behavior of the New York money market, and that we would not be justified in treating the smoothed figures of call money rates before 1915 as even approximate measures of cyclical fluctuations.[26]

TABLE 119
Average Amplitude of Specific Cycles
Call Money Rates on New York Stock Exchange
Before and after Inauguration of Federal Reserve System

Period and form of data	No. of specific cycles[a]	No. of months in standing at turns	Average standing in specific-cycle relatives at			Difference from average on preceding line		
			Initial trough	Peak	Terminal trough	Initial trough	Peak	Terminal trough
1858–1915								
Raw.......	18	1	52.1	281.8	50.6	....	....	....
Raw.......	18	3	60.8	187.6	59.1	+8.7	−94.2	+8.5
Smoothed...	18	1	67.0	149.1	65.0	+6.2	−38.5	+5.9
1915–1931								
Raw.......	5	1	63.7	156.0	68.6	....	....	....
Raw.......	5	3	66.8	143.3	72.2	+3.1	−12.7	+3.6
Smoothed...	5	1	72.6	138.2	79.1	+5.8	−5.1	+6.9

[a] See Sec. II.

In iron production there are numerous instances of deep and narrow troughs, though they are not nearly so deep as the peaks of call money rates are high. In 1891, 1896, 1900, 1914 and 1921, when the troughs are fairly narrow but seemingly free from erratic movements, smoothing raises the troughs by larger amounts than in 1910 and 1927, when the troughs are relatively broad but marked by conspicuous erratic movements (Chart 43). The troughs in 1900 and 1914 are not included in the list of 'smooth' turns in Table 109, but that list is unduly conservative.

From this evidence we judge that the differences between our standard measures of amplitude and those derived from Macaulay's graduations result in considerable part from a tendency of smoothing to eliminate a part of the cyclical movements. But can we be sure that our standard method is free from bias? As in the problem of timing, one seemingly objective way of judging the relative merits of the two methods is to see what happens when each is applied to series whose underlying cyclical shapes are known *a priori*. The results for several artificial series are recorded in Chart 45 and Table 120. It is evident, in the first place, that smoothing gives only rough approximations, sometimes very poor

[26] See below, Chart 50 and pp. 352, 356.

approximations. The relative merits of the two methods are more difficult to judge, partly because the sample covered by the experiments is small. Taking the figures in Table 120 as they come, it seems that when the underlying cyclical movement is a sine curve, smoothing yields somewhat better approximations of the true amplitude than does our standard method. Results of this character may be generally expected, inasmuch as Macaulay's 43-term graduation is designed to fit sine curves closely. On the other hand, when the underlying cycles are cusp-shaped, smoothing yields decidedly poorer approximations than our method. This result, too may be generally expected, except perhaps when the amplitude of the random component is especially violent. When the underlying cycles are triangular, the merits of the two methods depend more closely on the size of the random movements; our method is likely to give better results when the random movements are 'mild', but poorer results when the random movements are 'pronounced'. On the whole, our method surely fares no worse than smoothing in the circumstances envisaged by the experiment.[27]

We must recall, moreover, that the theoretical model underlying the artificial series is not strictly applicable to actual time series. So far as the specific cycles of actual series are affected by cumulations of random movements, we do not want to ignore the random movements. Our aim in measuring the amplitude of a specific cycle is to allow for the random movements that blur its contours, not for the part played by random forces in the observed fluctuation—as is the case with the artificial series.[28] The practice of representing peaks and troughs by three-month standings promotes this objective unevenly, but so too would smoothing. In brief phases, three-month standings may well tend to understate cyclical amplitudes; the use of one-month or two-month standings might be better.[29] In very choppy series, five or seven months may not suffice to get rid of the erratic component at the turns.[30] On the other hand, in a series like call money rates, which is choppy but has sharply cusped cyclical peaks,

27 It may be of interest to note that the use of one-month standings at cyclical turns of the raw data, instead of three-month standings, would exaggerate the 'true' amplitude of full cycles in every series. The averages based on one-month standings are as follows:

Series S'	104.7	Series T"	124.8
" S"	132.6	" C'	86.9
" T'	98.2	" C"	109.6

The total rise and fall of the pure cyclical component is 80, in units of specific-cycle relatives.

28 See above, pp. 320-2.

29 Note the discontinuity in our rules (p. 132) for handling phases less than four months, and those four months or longer.

30 Imagine a strong case: a random series in which specific cycles are somehow chosen by a cycle-conscious investigator. Then the three-month standing at the peak is sure to be above the three-month standings at the adjacent troughs; if it were otherwise, these turns would not have been recognized in the first place. The case is not very different if we imagine, next, a series having a cyclical component of very small amplitude but large erratic movements; here too the use of three-month standings is likely to overstate the cyclical amplitude and produce cusped patterns.

TABLE 120

Amplitude of Specific Cycles in Raw and Smoothed Data of Six Artificial Series Compared with Amplitude of the Underlying Pure Cycles

Phase	Nature of phase[a]	Excess over amplitude of pure cyclical curve (in specific-cycle relatives)											
		Series S′		Series S″		Series T′		Series T″		Series C′		Series C″	
		Raw	Sm.	Raw	Sm.	Raw	Sm.	Raw	Sm.	Raw	Sm.	Raw	Sm.
1	E	+0.3	...	+3.0	...	−4.3	...	−1.6	...	−14.1	...	−11.6	...
2	C	−1.5	−4.1	−0.4	−8.2	−6.5	−10.5	−5.4	−14.7	−16.9	−23.1	−17.1	−27.3
3	E	+1.8	0.0	+0.7	+0.3	−1.2	−6.3	+1.0	−6.1	−8.1	−18.8	−7.8	−18.8
4	C	+8.8	+4.5	+6.9	+9.1	+6.2	−1.8	+15.2	+2.8	−0.2	−14.3	+8.0	−9.8
5	E	+11.9	+5.3	+15.3	+10.3	+9.7	−0.9	+21.6	+4.2	+3.6	−13.4	+15.3	−7.9
6	C	+6.4	+2.3	+12.2	+5.6	+3.8	−4.0	+10.4	−0.6	−3.0	−16.3	+3.6	−12.5
7	E	+2.9	+1.0	+6.2	+4.1	−0.6	−5.2	+3.2	−2.6	−5.0	−17.5	−4.3	−14.7
8	C	+3.8	+2.9	+9.5	+7.4	+0.2	−3.3	+4.5	+0.9	−4.1	−15.7	−6.3	−11.3
9	E	+0.9	+0.6	+7.1	+3.4	−1.5	−5.9	−0.8	−3.0	−9.3	−18.2	−11.3	−14.8
10	C	−2.2	−2.0	+0.9	−2.5	−6.7	−8.5	−7.0	−8.9	−13.1	−20.8	−17.5	−20.8
11	E	−6.0	−4.3	−2.6	−7.7	−10.6	−10.7	−14.2	−14.2	−15.7	−23.1	−24.2	−26.0
12	C	−1.8	...	+4.4	...	−4.2	...	−5.3	...	−10.7	...	−12.0	...
Av., signs ignored													
Phases 1-12.....		4.0	...	5.8	...	4.6	...	7.5	...	8.6	...	11.6	...
Phases 2-11.....		4.6	2.7	6.2	5.9	4.7	5.7	8.3	5.8	7.9	18.1	11.5	16.4
Av., signs respected													
Phases 1-12.....		+2.1	...	+5.3	...	−1.3	...	+1.8	...	−8.0	...	−7.1	...
Phases 2-11.....		+2.7	+0.6	+5.6	+2.2	−0.7	−5.7	+2.8	−4.2	−7.2	−18.1	−6.2	−16.4

For explanations of the artificial series, see p. 318 and Appendix C (Chart 45). The specific cycles are treated positively. The amplitudes of the raw data are measured from 3-month standings at cyclical turns; the amplitudes of the smoothed data and of the pure cycles are measured from 1-month standings. The amplitude of the expansion or contraction of the pure cycles is invariably 40. The amplitude of phase 1 in the raw data of Series S′ is 40.3; hence the entry +0.3 in the table, and so on. Since smoothing involves the loss of a phase at each end of the series, there are no entries for phases 1 and 12 in the smoothed data, nor averages based on phases 1-12.

[a] E stands for specific-cycles expansion, C for contraction.

even three-month standings at peaks may understate the cyclical amplitudes. Ideally, the method should be varied from case to case depending on the character of the cyclical and erratic movements; but this counsel of perfection applies with equal force to smoothing. We believe that our rough method yields amplitude measures that are simpler to interpret and more trustworthy, on the whole, than those yielded by Macaulay's graduations or similar devices.

V The Secular Component of Specific Cycles

Since Macaulay's graduation cannot influence much the level about which cyclical fluctuations play, smoothing makes practically no difference in average measures of the percentage change between the levels of successive cycles (Table 121). It affects the percentage changes between successive cycles chiefly by shifting the dates of cyclical troughs or peaks: the former if a series is analyzed on a positive basis, as are our present samples, the latter in inverted series. Though the effects of these shifts are sometimes large in cycle-by-cycle measures, they cancel out in the averages.

TABLE 121

Average Measures of Secular Movements of Raw and Smoothed Data
Four American Series

Series and form of data	No. of specific cycles[a]	Average per cent change from: Preceding phase: Contraction to expansion	Preceding phase: Expansion to contraction	Preceding cycle on base of: Preceding cycle: Total	Preceding cycle: Per month	Average of given and preceding cycle: Total	Average of given and preceding cycle: Per month: Unweighted	Per month: Weighted
DEFLATED CLEARINGS								
Raw	15	+8.9	+7.3	+16.9	+0.40	+15.2	+0.36	+0.37
Smoothed	15	+8.8	+6.9	+16.8	+0.40	+15.1	+0.37	+0.37
PIG IRON PRODUCTION								
Raw	15	+9.5	+7.1	+18.2	+0.43	+14.5	+0.34	+0.35
Smoothed	15	+13.8	+3.3	+18.2	+0.44	+15.0	+0.36	+0.36
RAILROAD STOCK PRICES								
Raw	18	+7.4	+4.7	+13.4	+0.26	+10.4	+0.20	+0.21
Smoothed	18	+9.1	+2.8	+13.3	+0.27	+10.2	+0.20	+0.21
CALL MONEY RATES								
Raw	23	+9.8	−4.9	+4.1	+0.09	−0.1	−0.02	0.00
Smoothed	23	+1.5	+2.7	+4.2	+0.10	−0.1	−0.03	0.00

See Table 112 for the periods covered. Except for the last three columns, the average percentages are subject to an upward bias, explained in Ch. 5, Sec. V. This bias is immaterial in a comparison of raw and smoothed data.

[a] See Sec. II.

The effects of smoothing are more prominent in the phase-to-phase changes than in the cycle-to-cycle changes. The former are affected by shifts of both troughs and peaks. Since the number of months shared by corresponding phases in raw and smoothed data is sometimes a small fraction of the full phase in either body of data, large differences can easily arise. Furthermore, smoothing influences the phase-to-phase changes by altering the form of the cyclical pattern as well as the timing of cyclical turns.

VI Specific-cycle Patterns

Our practice of replacing the full set of monthly values in each specific cycle by nine average standings facilitates comparisons among cycles of different durations, and at the same time is a powerful device for reducing erratic fluctuations. Whereas the monthly figures of time series usually have extremely rough contours, the nine-stage patterns tend to rise continuously to the peak and decline continuously to the trough. A further reduction of erratic movements is effected when the standings in each stage are averaged for all the cycles covered by a series.

Our method, however, is far from perfect. In deflated clearings, for example, only 3 out of 15 cycles rise continuously from stage I to V, and decline continuously from stage V to IX. Corresponding numbers in call

TABLE 122
Average Specific-cycle Patterns of Raw and Smoothed Data
Four American Series

Series and form of data	No. of specific cycles[a]	I Initial trough	II Expansion: First third	III Expansion: Middle third	IV Expansion: Last third	V Peak	VI Contraction: First third	VII Contraction: Middle third	VIII Contraction: Last third	IX Terminal trough
		Average in specific-cycle relatives at stage								
DEFLATED CLEARINGS										
Raw	15	85.7	90.5	99.2	106.7	112.6	108.7	106.0	101.9	99.2
Smoothed	15	86.8	90.6	99.0	106.6	110.8	109.4	105.5	101.4	100.2
PIG IRON PRODUCTION										
Raw	15	67.3	82.5	103.7	116.5	129.3	122.6	108.2	88.4	74.6
Smoothed	15	73.0	82.9	103.5	118.1	126.5	120.7	102.8	86.2	81.6
RAILROAD STOCK PRICES										
Raw	18	82.8	88.0	98.8	110.8	118.3	112.4	103.2	94.0	86.6
Smoothed	18	84.5	88.4	100.1	111.2	116.0	112.2	102.2	91.7	88.0
CALL MONEY RATES										
Raw	23	62.1	80.3	104.7	123.2	178.0	120.4	89.9	71.8	61.9
Smoothed	23	68.2	76.0	98.5	130.4	146.8	130.4	99.0	76.4	68.1
		Excess of the raw average over the smoothed								
Deflated clearings	15	−1.1	−0.1	+0.2	+0.1	+1.8	−0.7	+0.5	+0.5	−1.0
Pig iron production	15	−5.7	−0.4	+0.2	−1.6	+2.8	+1.9	+5.4	+2.2	−7.0
Railroad stock prices	18	−1.7	−0.4	−1.3	−0.4	+2.3	+0.2	+1.0	+2.3	−1.4
Call money rates	23	−6.1	+4.3	+6.2	−7.2	+31.2	−10.0	−9.1	−4.6	−6.2

The standings in stages I, V and IX cover 3 months in the raw data, 1 month in the smoothed data; but see Table 113, note 'b'. For the periods covered, see Table 112.

[a] See Sec. II.

money rates are 10 out of 23, railroad stock prices 13 out of 18, pig iron production 11 out of 15.[31] In these four series the *average* pattern rises without interruption from stage I to V and declines without interruption from stage V to IX (Table 122 and Chart 46). Our efforts to measure cyclical patterns are not always attended with such conspicuous success, and even in these instances success is partly an illusion. The fact that the pattern of a single cycle, or the average pattern, is free from reversals of direction within the expansion or contraction may mean that erratic movements have disappeared; but it is much more likely to mean that whatever erratic component remains is merely not large enough to produce reversals of direction.[32] Substantial erratic components may be expected to remain in the patterns of individual cycles during brief phases, and even in the average patterns of short series characterized by sharp erratic fluctuations. The difficulty is most acute at the turning stages (I, V and IX), since the three-month standings may yield a biased framework of the patterns.

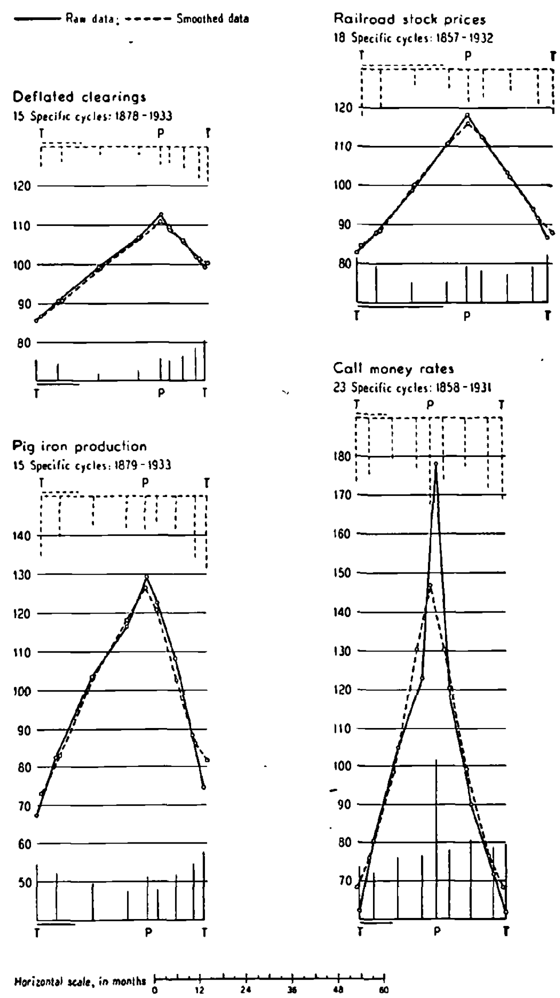
[31] See the cycle-by-cycle measures in Appendix Table B1.

[32] Some reversals of direction may even be cyclical phenomena, as when the rise of a series is regularly retarded, sometimes interrupted by an actual dip, near the center of expansion. However, few reversals of direction in cyclical patterns can be confidently put in this class.

CHART 46

Average Specific-cycle Patterns of Raw and Smoothed Data

Four American Series



See Table 122. The patterns are spaced so as to show average leads or lags between the raw and smoothed data. For other explanations of chart, see Ch. 5, Sec. VI.

So far as an erratic component remains in the standings of the raw data at individual stages of the cycles, it is bound to count more heavily in the rates of change between successive standings than in the standings themselves. Moreover, we should expect the erratic factor to vary inversely with the length of the interval between the cycle stages. Of the eight intervals, those during contraction tend to run shorter, and therefore are more susceptible to erratic movements, than those during expansion. The shortest intervals are usually from the peak to the first third of contraction (V-VI) and from the last third of contraction to the trough (VIII-IX). Also the intervals from the trough to the first third of expansion (I-II) and from the last third of expansion to the peak (IV-V) are short relatively to the other intervals into which we divide expansions.[33] The measures in our standard Table S5 therefore have uneven degrees of reliability as representatives of cyclical behavior, and the question how far measures of cyclical patterns could be improved by preliminary smoothing of the data by Macaulay's formula, or some similar device, assumes considerable importance.

The differences between the raw and smoothed figures showing rates of change from stage to stage of the specific cycles (Table 123) are much larger in proportion to the items compared than the corresponding differences between the standings in successive stages (Table 122). The differences between the rates of change (signs disregarded) in the raw and smoothed data are on the average larger during contractions than during expansions, except in call money rates. They average larger also during the turning intervals of the cycles (that is, during stages I-II, IV-V, V-VI and VIII-IX) than during other intervals. Again, they tend to be larger during the turning intervals within contractions (V-VI and VIII-IX) than during the turning intervals within expansions (I-II and IV-V). In all these respects the figures behave as we should expect them to behave if the smoothed data yielded faithful representations of cyclical patterns, while an appreciable erratic component remained in our standard averages.

But there are serious obstacles to this interpretation. If erratic movements and nothing else stood between the raw and smoothed figures, we should indeed find larger differences between their rates of change in the turning intervals of the cycles than in other intervals, but it would be unlikely that the signs of the differences in the turning intervals would agree from series to series. Table 123 shows a plus difference in every series for the first and last intervals of expansion, and a minus difference for the first and last intervals of contraction. This uniformity of signs reflects, of course, the tendency of smoothing to make cyclical amplitudes smaller than the amplitudes yielded by our standard method. If our judgment that smoothing tends to understate the 'true' cyclical amplitudes in our sample is sound, erratic movements cannot account wholly, perhaps not even largely, for the differences between the rates of change in the raw and smoothed data in the turning intervals of the cycles, or for the larger difference in these intervals than in others. There is a similar difficulty in interpreting the larger differences between the raw and smoothed rates during the contraction than during the expansion stages. As shown in Section IV, smoothing influences the average rates of change more during contractions than during expansions of specific cycles—a difference con-

[33] The intervals for our four series are shown at the bottom of Table 122.

nected with the tendency of smoothing to reduce cyclical amplitudes and to change the relative duration of expansions and contractions. These distortions of the cyclical movements are inevitably reflected in larger differences, generally, between the raw and smoothed rates in the contraction than in the expansion intervals of the cycles.

Table 123 thus confirms what we have said about the distortions produced by smoothing; it also focuses attention upon a feature of smoothing which, although correlated with the tendency to reduce peaks

TABLE 123

Average Rate of Change from Stage to Stage of Specific Cycles
Four American Series, Raw and Smoothed

Series and form of data	No. of specific cycles	Average change per month in specific-cycle relatives between stages							
		I-II	II-III	III-IV	IV-V	V-VI	VI-VII	VII-VIII	VIII-IX
		Expansion				Contraction			
		Trough to first third	First to middle third	Middle to last third	Last third to peak	Peak to first third	First to middle third	Middle to last third	Last third to trough
DEFLATED CLEARINGS									
Raw	15	+0.8	+0.9	+0.7	+1.0	−2.6	−2.1	−1.7	−1.5
Smoothed	15	+0.7	+0.9	+0.8	+0.7	−0.4	−0.9	−0.9	−0.4
PIG IRON PRODUCTION									
Raw	15	+3.2	+2.7	+1.5	+2.7	−2.3	−3.0	−7.2	−6.2
Smoothed	15	+1.9	+2.7	+2.3	+1.9	−1.9	−3.8	−3.4	−1.4
RAILROAD STOCK PRICES									
Raw	18	+1.3	+1.4	+1.6	+1.4	−1.7	−1.6	−1.5	−2.2
Smoothed	18	+0.8	+1.4	+1.3	+0.8	−0.9	−1.5	−1.4	−0.7
CALL MONEY RATES									
Raw	23	+6.6	+5.8	+1.0	+16.9	−20.2	−6.2	−4.5	−2.3
Smoothed	23	+2.1	+4.1	+5.7	+4.4	−4.7	−6.3	−4.4	−2.2
		Excess of the raw average over the smoothed							
Deflated clearings	15	+0.1	0.0	−0.1	+0.3	−2.2	−1.2	−0.8	−1.1
Pig iron production	15	+1.3	0.0	−0.8	+0.8	−0.4	+0.8	−3.8	−4.8
Railroad stock prices	18	+0.5	0.0	+0.3	+0.6	−0.8	−0.1	−0.1	−1.5
Call money rates	23	+4.5	+1.7	−4.7	+12.5	−15.5	+0.1	−0.1	−0.1
		Average interval in months between midpoints of stages							
DEFLATED CLEARINGS									
Raw	15	5.8	10.5	10.5	5.8	2.3	3.4	3.4	2.3
Smoothed	15	5.5	10.0	10.0	5.5	2.4	3.8	3.8	2.4
PIG IRON PRODUCTION									
Raw	15	5.1	9.3	9.3	5.1	2.7	4.6	4.6	2.7
Smoothed	15	4.8	8.7	8.7	4.8	3.0	5.0	5.0	3.0
RAILROAD STOCK PRICES									
Raw	18	5.1	9.2	9.2	5.1	3.8	6.7	6.7	3.8
Smoothed	18	5.0	8.9	8.9	5.0	3.9	7.1	7.1	3.9
CALL MONEY RATES									
Raw	23	3.7	6.3	6.3	3.7	3.3	5.7	5.7	3.3
Smoothed	23	3.5	6.1	6.1	3.5	3.5	5.8	5.8	3.5

The average rates of change are unweighted; weighted averages are given in Table 125. See notes to Table 122.

and raise troughs, becomes evident only in the full patterns. In every series covered by our sample the smoothed data rise fastest on the average from stage II to III or III to IV and fall fastest on the average from VI to VII or VII to VIII; that is, the rise is fastest around mid-expansion and the fall is fastest around mid-contraction. In no series does the average rate of change reach a numerical maximum within the turning intervals of the cycles. Numerical minima, on the contrary, come invariably within a turning interval. That is exactly what we should expect of a curve with gently rounded tops and bottoms. The raw patterns are free from this technical compulsion, and have notably different characteristics. The maximum rates of change are no longer concentrated in the middle range of expansions and contractions; they frequently come in one of the turning intervals. Nor are the phases of expansion and contraction as symmetrical as in the smoothed data. These differences be-

TABLE 124

Position of Fastest and Slowest Rates of Change in Specific-cycle Patterns
Four American Series, Raw and Smoothed

Series and form of data	No. of specific cycles	Distribution of rates of change between stages							
		I-II	II-III	III-IV	IV-V	V-VI	VI-VII	VII-VIII	VIII-IX
		No. of expansions in which rise is fastest				No. of contractions in which fall is fastest			
DEFLATED CLEARINGS									
Raw	15	5	1	2	7	6.5	4.5	3	1
Smoothed	15	1.5	5	5	3.5	0.25	7.25	7.25	0.25
PIG IRON PRODUCTION									
Raw	15	7	1	2	5	1	2	4	8
Smoothed	15	4	4	5	2	1	10.5	3.5	...
RAILROAD STOCK PRICES									
Raw	18	2	4	4	8	5	2	4	7
Smoothed	18	...	9.5	5.5	3	...	11	6	1
CALL MONEY RATES									
Raw	23	2	1	3	17	16	2	3	2
Smoothed	23	1	3	14	5	2	13	5	3
		No. of expansions in which rise is slowest				No. of contractions in which fall is slowest			
DEFLATED CLEARINGS									
Raw	15	6	4	3	2	...	7	5.5	2.5
Smoothed	15	7	1	2	5	5.25	0.25	0.25	9.25
PIG IRON PRODUCTION									
Raw	15	3	1	11	...	5.5	4.5	2	3
Smoothed	15	6	...	5	4	4	1	1	9
RAILROAD STOCK PRICES									
Raw	18	6	2	6	4	5	4	8	1
Smoothed	18	8.5	1	...	8.5	4	2	1	11
CALL MONEY RATES									
Raw	23	3	7	12	1	1	8	4	10
Smoothed	23	15	3	2	3	2	4	4	13

To minimize the number of ties the rates of change were computed to extra decimals. But some ties remained; hence the fractions. Cf. Table 76, and see notes to Table 122.

TABLE 125

Weighted Average Rate of Change from Stage to Stage of Specific Cycles
Four American Series, Raw and Smoothed

Series and form of data	No. of specific cycles	Weighted average change per month in specific-cycle relatives between stages							
		I-II	II-III	III-IV	IV-V	V-VI	VI-VII	VII-VIII	VIII-IX
DEFLATED CLEARINGS									
Raw	15	+0.8	+0.8	+0.7	+1.0	−1.7	−0.8	−1.2	−1.2
Smoothed	15	+0.7	+0.8	+0.8	+0.8	−0.6	−1.0	−1.1	−0.5
PIG IRON PRODUCTION									
Raw	15	+3.0	+2.3	+1.4	+2.5	−2.5	−3.1	−4.3	−5.2
Smoothed	15	+2.1	+2.4	+1.7	+1.7	−1.9	−3.6	−3.3	−1.5
RAILROAD STOCK PRICES									
Raw	18	+1.0	+1.2	+1.3	+1.5	−1.6	−1.4	−1.4	−2.0
Smoothed	18	+0.8	+1.3	+1.3	+1.0	−1.0	−1.4	−1.5	−0.9
CALL MONEY RATES									
Raw	23	+5.0	+3.9	+2.9	+15.0	−17.4	−5.4	−3.2	−3.0
Smoothed	23	+2.3	+3.7	+5.2	+4.7	−4.7	−5.4	−3.9	−2.4
		Excess of the raw average over the smoothed							
Deflated clearings	15	+0.1	0.0	−0.1	+0.2	−1.1	+0.2	−0.1	−0.7
Pig iron production	15	+0.9	−0.1	−0.3	+0.8	−0.6	+0.5	−1.0	−3.7
Railroad stock prices	18	+0.2	−0.1	0.0	+0.5	−0.6	0.0	+0.1	−1.1
Call money rates	23	+2.7	+0.2	−2.3	+10.3	−12.7	0.0	+0.7	−0.6

See notes to Table 122.

tween the raw and smoothed averages in Table 123 are broadly confirmed by the summary of individual cycles in Table 124.[34] They are again confirmed by Table 125, in which weighted average rates of change replace the unweighted averages of Table 123.[35]

An independent check on the preceding analysis is provided by Table 126 and Chart 47, which present cyclical patterns of several artificial

[34] Although the slowest rates of change in the smoothed data are concentrated in the turning zones, the concentration is heavier in stages I-II and VIII-IX than in stages IV-V and V-VI. This result is partly due to the fact that Macaulay converted the data to logarithms before smoothing. We converted his smoothed logarithms to natural numbers before making cyclical measures.

[35] The averages in Table 125 correspond precisely to the slopes of the patterns in Chart 46 (see p. 151). They are significant for that reason, and also because they reduce erratic movements more effectively than do the unweighted averages.

The differences (signs ignored) between the raw and smoothed averages in Table 125 are smaller in most instances than the corresponding differences in Table 123. The reduction of the gaps is due chiefly to the effects of weighting on the raw averages. Extreme figures come, as a rule, in very short phases, precisely the instances in which our method handles erratic movements least effectively. For example, in the raw data on call money rates the unweighted average change from stage III to IV is +1.0 points per month; the weighted average is +2.9. The discrepancy is mainly attributable to the expansion from Oct. 1880 to Feb. 1881. Since this expansion lasted only four months, the standing in stage III covers Dec. 1880 and the standing in stage IV Jan. 1881. Between the two months there was a sharp erratic decline in interest rates, which is indicated by a rate of change of −81.5 points from stage III to IV of the cycle. In the unweighted average of Table 123, this figure gets the same weight as any other. But in Table 125 its influence on the average is reduced; for the rates of change in individual cycles are weighted by the interstage intervals. In this instance and several others, the weighted average exceeds numerically the unweighted average. But the opposite relation is the rule, since the rate of change and the length of the interstage interval tend to be correlated inversely.

series, before and after smoothing. These exhibits should be studied in conjunction with Chart 45. The reader should note, especially, how smoothing imposes a 'sine shape' on cyclical fluctuations, regardless of their true characteristics. Many theorists are prone to reason on the assumption that economic cycles are of 'sine shape'; meaning by this phrase not that economic activities fluctuate precisely like sine curves, but that their rates of rise increase in the early stages of a cyclical expansion and taper off towards its close, and that the rates of fall behave similarly in contraction. This belief has deep roots in intellectual history: the extensive use of sine curves in the physical sciences, the ease of fitting a sine curve to observational data, the vogue of the 'principle of continuity', have all contributed to it. But perhaps the most important reason is the image created in the minds of men over several generations by the practice of smoothing. The statistician faced with jerky figures commonly resorts to smoothing. As a rule he relies on simple, unweighted moving averages. Sometimes he uses more sophisticated devices; for example, moving averages having a more or less smooth weight diagram, with some weights negative, such as are described in Macaulay's *Smoothing of Time Series*. In either case he obtains a curve with rounded tops and bottoms.[36] The 'sine shapes' are sometimes faithful representatives of the specific cycles of economic time series. Perhaps more often, they are merely technical creatures of the smoothing process.[37]

The jerky contours of time series present a dilemma. If the data are left as they come, the shapes of cyclical patterns cannot be measured. On the other hand, if smoothing of the ordinary type is employed, it is practically inevitable that the patterns will be 'sine shaped'; for 'sine shapes' emerge the moment the terms included in a moving average cross cyclical phases. One way of escaping from the dilemma is to avoid moving averages altogether, and that is the route we have followed in measuring cyclical patterns. Another way is to use hypothetical values, based on projections of the actual data, in getting moving averages for the months in the vicinity of a cyclical turn. Still another possibility, theoretically, is to vary the smoothing formula from one stretch of the data to the next, the formula being always properly adjusted to the underlying cyclical movement.[38] But this method breaks down when the underlying cyclical curve is triangular or cusp shaped: there is no set of weights which, if applied

36 See the illustrations of this effect in Charts 44, 45 and 47.

37 In this connection it is important to bear in mind the distinction between specific and business cycles. For example, the patterns of the specific cycles in most activities might be triangular, yet the pattern of the cycles in aggregate business activity might be 'sine shaped'; though, in fact, we have found little evidence of the latter.

38 Appropriate devices for parabolic curves of different orders are described by Macaulay in *The Smoothing of Time Series*, and by Max Sasuly, *Trend Analysis of Statistics* (Brookings Institution, 1934). Appendix VII of Macaulay's book shows the closeness with which 19 graduation formulas fit sine series of different periods.

TABLE 126

Average Specific-cycle Patterns of Raw and Smoothed Data of Six Artificial Series Compared with Patterns of the Underlying Pure Cycles

Series and form of data	No. of specific cycles	No. of mos. in stage I, V & IX	Average in specific-cycle relatives at stage I	II	III	IV	V	VI	VII	VIII	IX
(S) Sine curve	..	1	80.0	83.9	100.0	116.1	120.0	116.1	100.0	83.9	80.0
(S′) Sine curve + random											
Raw	6	3	78.7	87.8	102.3	116.0	120.7	114.4	97.5	82.6	78.4
Raw	4	3	78.1	87.6	101.2	116.6	122.5	115.9	97.6	82.2	78.3
Smoothed	4	1	79.1	83.9	100.0	116.4	120.8	116.8	100.1	82.7	78.9
(S″) Sine curve + 2 random											
Raw	6	3	77.9	91.2	104.1	115.7	122.9	111.1	95.3	81.3	77.3
Raw	4	3	77.1	91.4	102.4	117.6	124.4	112.6	95.9	80.2	77.0
Smoothed	4	1	77.6	82.9	99.2	116.8	122.2	117.5	100.9	82.3	77.2
(T) Triangular curve	..	1	80.0	87.3	100.0	112.7	120.0	112.7	100.0	87.3	80.0
(T′) Triang. curve + random											
Raw	6	3	80.4	89.7	101.1	112.7	119.0	111.0	98.9	86.2	80.2
Raw	4	3	79.4	88.7	100.1	113.2	121.0	112.6	99.4	86.0	80.2
Smoothed	4	1	82.2	87.0	100.2	113.3	117.6	113.7	99.9	85.8	82.0
(T″) Triang. curve + 2 random											
Raw	6	3	78.6	93.4	102.1	112.0	120.2	110.0	97.6	85.4	78.1
Raw	4	3	77.0	93.2	101.0	113.3	123.3	112.2	97.3	84.6	77.5
Smoothed	4	1	80.8	86.0	99.8	113.9	119.0	114.4	100.4	85.2	80.4
(C) Cusp curve	..	1	80.0	93.4	100.0	106.6	120.0	106.6	100.0	93.4	80.0
(C′) Cusp curve + random											
Raw	6	3	83.7	94.9	100.8	106.8	115.6	105.4	99.6	92.1	83.6
Raw	4	3	82.6	94.6	100.2	106.8	117.9	106.6	99.3	92.3	83.0
Smoothed	4	1	88.4	92.0	100.4	107.9	111.4	108.2	100.0	91.6	88.2
(C″) Cusp curve + 2 random											
Raw	6	3	83.4	99.4	101.3	105.6	116.1	104.8	98.8	90.4	83.0
Raw	4	3	81.5	99.2	100.5	106.4	119.5	106.9	98.1	90.0	82.5
Smoothed	4	1	86.9	91.2	100.4	108.6	112.8	109.2	99.8	90.4	86.4

Smoothing involves the loss of 2 positive cycles, as explained in Appendix C (Chart 45). The averages for 4 cycles include corresponding cycles in the raw and smoothed data.

to successive terms in a moving average, will reproduce triangular or cusped cycles.[39]

Granted that our method of measuring cyclical patterns avoids the rounding bias characteristic of smoothing, may it not be subject to other types of bias? Surely, the patterns of the raw artificial series deviate materially from the patterns of the underlying pure cycles. The former tend to have higher standings in expansion and lower standings in contraction; the excess in successive thirds of expansion and the deficiency in successive thirds of contraction tends to shrink; hence the rate of rise during stages II-III and III-IV is moderated, and likewise the rate of fall during VI-VII and VII-VIII. Furthermore, the pattern of the artificial series is asymmetrical in the turning zones: the rate of change from stage V to VI exceeds numerically the rate from IV to V, and the rate from I to

[39] We disregard 'fitting' by drawing on many terms of a parabolic expansion, or by means of harmonic analysis. There is nothing to be said in favor of expending much labor on reproducing, approximately, the original data, unless the process yields a truly helpful classification of different types of fluctuation.

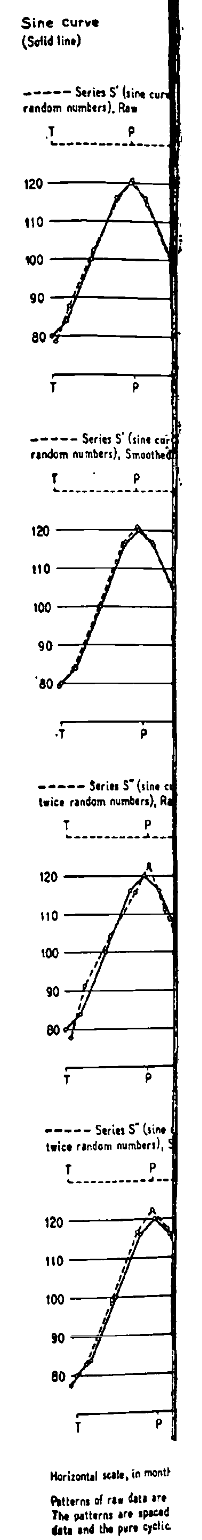
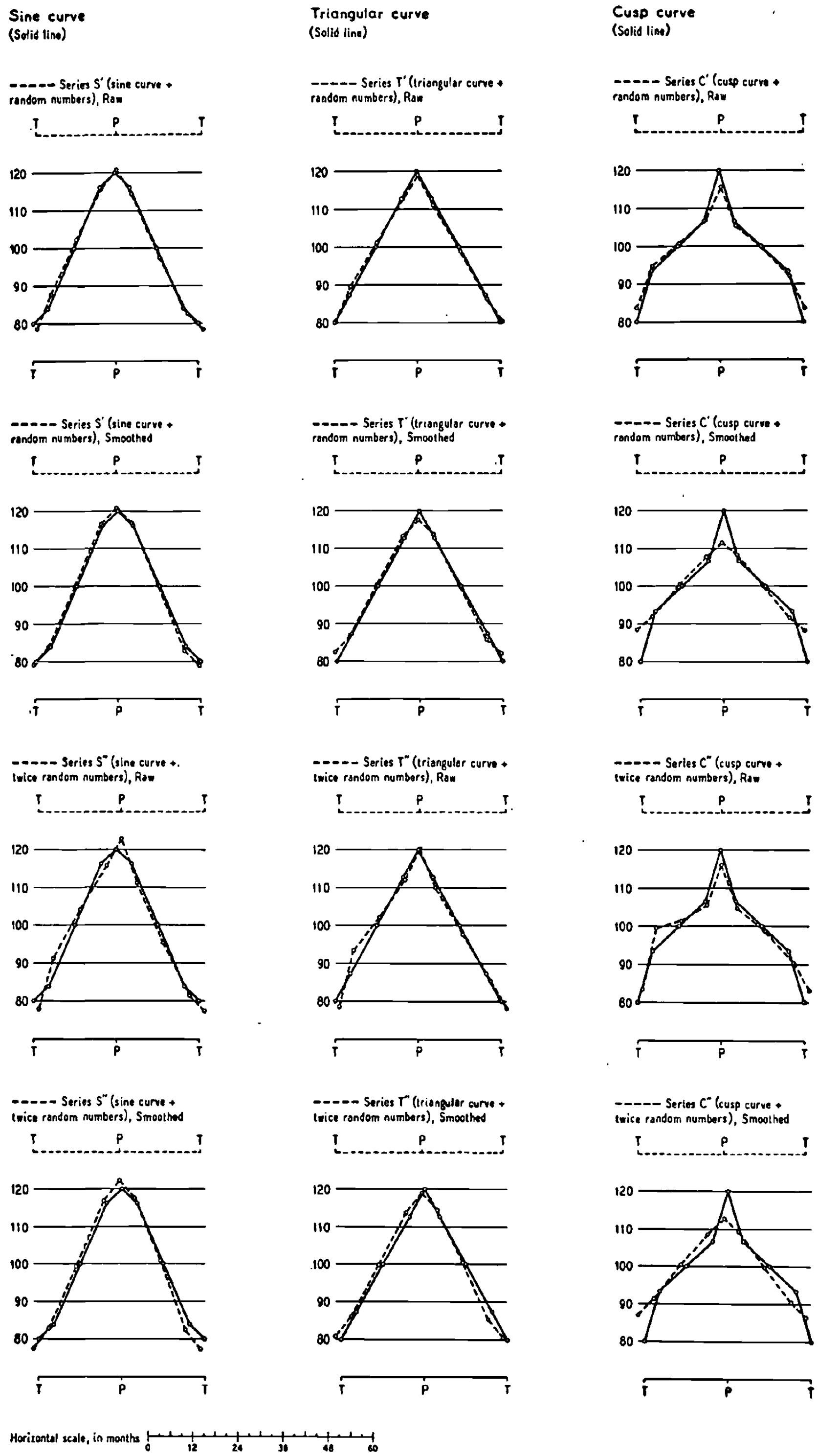


CHART 47

Average Specific-cycle Patterns of Raw and Smoothed Data of Six Artificial Series Compared with Patterns of the Underlying Pure Cycles



Patterns of raw data are based on six cycles, patterns of smoothed data on four cycles. The patterns are spaced so as to show average leads or lags between the raw or smoothed data and the pure cyclical component. See Chart 45 and Table 126.

II exceeds numerically the rate from VIII to IX. All these departures from the true cyclical pattern *could* arise from our practice of dating cyclical turns towards the close of the period of transition from one phase to the next, as when a trough is 'flat' or 'double-bottomed', or a peak is 'flat' or 'double-topped'.

But they could arise also from the peculiarities of the particular set of random numbers that enters into all six artificial series.[40] The latter, in fact, is largely responsible for the asymmetry.[41] Our rule regarding transitions is used sparingly in dating cyclical turns. It serves only as a last resort, when other rules fail to yield a decision. In the great majority of series the rule is of slight importance. If the rule were reversed, one or two turns, at most, would be shifted in our present sample of four series. The rule has serious consequences, however, in series with perfectly flat tops or bottoms, and it may have a telling effect whenever erratic movements are exceptionally pronounced in the vicinity of cyclical turns.[42]

Not a few of the cyclical patterns yielded by our method of analysis seem 'cusp shaped', in the sense that they show maximum rates of change in the turning zones. This feature may lead some readers to inquire whether our method imparts a bias to cyclical patterns opposite to the bias imparted by smoothing: that is, if smoothing tends to create 'sine shapes', may not our method create 'cusp shapes'? As far as we can judge, there is nothing in our method to produce such a bias in series free from pronounced erratic movements. True, three-month standings at cyclical turns may be poorly suited to the underlying cyclical movements. But if this factor introduces a bias towards cusp shapes in some series, it introduces a bias towards sine shapes in others.[43] In very choppy series, on the other hand, our method does promote cusps. In such series there is danger of recognizing spurious cycles. When that happens, a cusp-shaped 'cycle' is likely to emerge.[44] A similar result may arise if a genuine cyclical rise or decline is overlooked, for the skipped movement will then depress the rate of change in one or more intervals outside the turning zones of the cycles that are recognized. Even if specific cycles are marked off properly, the use of three-month standings at peaks and troughs is likely to exaggerate the amplitude of very choppy series, and thereby tend to produce cusp-

[40] We have six series, not six independent tests. See p. 318 and Appendix C.

[41] The evidence for this statement is partly indirect, partly direct. Table 108 shows that the artificial series, before smoothing, tend to lag at both peaks and troughs of the pure cycles. That is what we should expect, if the rule concerning transitions made an appreciable difference. In that event, however, a reversal of the rule should produce leads. Actually, an experiment with reversing the rule left the turns practically unchanged.

This indirect evidence was checked by constructing 'cyclical' patterns of the random numbers entering into the artificial series, the dates of the pure cycles serving as a framework for the patterns. The patterns clearly indicated that the particular random numbers that we have used produce the asymmetry.

[42] See p. 148 for a method of handling such cases.

[43] See above, pp. 333-5.

[44] See above, note 30.

shaped patterns.[45] But the existence of a bias towards cusp shapes cannot be safely inferred from the mere fact that erratic movements are pronounced, for the combination of cyclical and erratic movements varies from series to series. Call money rates, for example, are subject to pronounced erratic movements. Yet the cusped cyclical peaks of this series are genuine; so far as our method errs, it probably does so in the direction of moderating the cusps.

TABLE 127
'Special' Cyclical Patterns
Pig Iron Production, United States, 1879–1933

Stage	Average specific-cycle pattern		Average reference-cycle pattern	
	Cumulative no. of months to stage	Standing in cycle relatives at stage	Cumulative no. of months to stage	Standing in cycle relatives at stage
1 (Trough)	0.0	65.3	0.0	74.0
2	3.2	77.3	2.6	83.5
3	6.4	85.7	5.2	93.9
4	9.6	94.2	7.8	99.6
5	12.8	101.2	10.4	102.0
6	16.0	106.1	13.0	103.6
7	19.2	109.0	15.5	106.9
8	22.4	113.5	18.1	111.3
9	25.6	120.4	20.7	116.4
10 (Peak)	28.8	130.9	23.3	122.1
11	30.4	125.2	25.5	119.2
12	32.0	121.2	27.7	116.7
13	33.7	114.9	29.9	112.0
14	35.3	108.2	32.1	103.8
15	36.9	103.1	34.4	95.3
16	38.5	98.2	36.6	88.7
17	40.1	90.7	38.8	86.3
18	41.8	81.0	41.0	80.7
19 (Trough)	43.4	72.2	43.2	81.7

Based on 15 specific and reference cycles. See the explanations in the text.

A difficulty of another sort is that our technique yields only nine observations on cyclical patterns. Additional detail would be desirable, especially in the neighborhood of cyclical turns. But to provide much detail is not feasible, in view of the brevity of many cyclical phases and the prevalence of erratic movements. There are no such limitations on cyclical patterns made from data smoothed by Macaulay's formula, but in their case fine detail would often be spurious. One way of steering between these difficulties is to use a smoothing formula of much shorter time span than Macaulay's, and to protect the patterns against the 'rounding effect' of smoothing by special treatment of the data in the vicinity of peaks and troughs. The results of an experiment along these lines on pig iron production are presented in Table 127 and Chart 48. The 'special'

45 A partial tendency towards cusped patterns is also imparted by the rule on cyclical dating, discussed in the preceding paragraph, which tends to exaggerate the rates of change during stages I-II and V-VI: but this rule imparts an opposite bias to the rates of change during stages VIII-IX and IV-V.

specific-cycle pattern consists of nineteen standings, in contrast to the 'standard' pattern of nine standings.[46]

The 'special' specific-cycle pattern was derived in the following steps. (1) A five-month centered moving average was taken of the raw data. (2) Since a 'rounding effect' emerges when the span of a moving average laps over two phases, hypothetical values were used to get the moving average for each month of cyclical turn and for the month just preceding and following the turn. The hypothetical values were obtained by projecting the actual movements of the data [47] in the vicinity of the turns. (3) The interval from the middle of the trough to the middle of the peak month in each specific cycle was divided into nine equal segments, and the standing at the end of each segment was computed by linear interpolation of the moving averages. (4) The contraction was divided likewise into nine segments, and the standing at the end of each segment computed. (5) The nineteen standings for the cycle were then expressed as relatives of the cycle base, computed from the raw monthly values on our standard plan. (6) Next, the cycle relatives at the end of each segment were averaged for all cycles. (7) Likewise the intervals between successive standings were averaged for all cycles. (8) Finally, the successive average standings of the cycle relatives were plotted on Chart 48 against the cumulatives of the average intervals.

The detailed 'special' pattern derived by this method is surprisingly similar to our standard pattern. The principal difference is the slightly

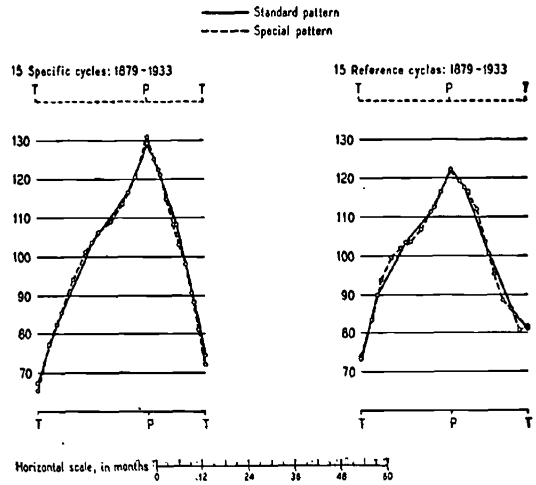
46 The 'special' reference-cycle pattern is considered at the close of Sec. VII.

47 Let a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, etc. be the actual values for a run of months, where the subscript identifies the month. Let a_{15} be the value in the peak (or trough) month. Then the hypothetical value (a'_{16}) substituted for a_{16} is $(a_{15} + a_{14} - a_{13})$; and the hypothetical value (a'_{17}) substituted for a_{17} is $(a'_{16} + a_{15} - a_{14})$, or $(2a_{15} - a_{13})$. The five-month moving average is therefore $\frac{a_{12} + 2a_{14} + 2a_{15}}{5}$ for month 14, and $\frac{-a_{13} + 2a_{14} + 4a_{15}}{5}$ for month 15. The movement following the peak (or trough) is projected backward in a similar manner, which results in a five-month moving average of $\frac{2a_{15} + 2a_{16} + a_{18}}{5}$ for month 16, and $\frac{4a_{15} + 2a_{16} - a_{17}}{5}$ for month 15. Since two values for the peak (or trough) month are obtained by this process, we take their average: $\frac{-a_{13} + 2a_{14} + 8a_{15} + 2a_{16} - a_{17}}{10}$. This value is not likely to differ much from the actual value at the peak or trough. It may even exceed the former and fall short of the latter. For example, it will exceed the actual value at the peak if $a_{14} - a_{13} > a_{15} - a_{14}$ and $a_{16} - a_{17} > a_{15} - a_{16}$. In such cases we have used the formula $\frac{2a_{14} + 8a_{15} + 2a_{16}}{12}$. Obviously a makeshift, it emphasizes the difficulty of making sensible projections.

An alternative method is to project forward a straight line connecting $\frac{a_{10} + a_{11} + a_{12}}{3}$ and $\frac{a_{13} + a_{14} + a_{15}}{3}$ to obtain hypothetical values for months 16 and 17, and to project backward a straight line connecting $\frac{a_{15} + a_{16} + a_{17}}{3}$ and $\frac{a_{18} + a_{19} + a_{20}}{3}$ to obtain hypothetical values for months 13 and 14. This method raises slightly the standing at the peak of the pattern of iron production, and reduces slightly the standing at the terminal trough.

CHART 48

Special and Standard Average Cyclical Patterns
Pig Iron Production, United States



For special patterns, see Table 127 and accompanying text. The standard pattern is our usual nine-point pattern based on the raw data; see Tables 122 and 128.

larger amplitude of the special pattern.[48] Since both patterns are based on turning points dated from the raw data, they share the defect of a chronological framework that is somewhat warped by erratic fluctuations. To reduce this difficulty, the method could be modified so that cyclical turns marred by erratic fluctuations would be dated from, say, a five-month moving average.

The 'special' pattern represents one of many experiments that might well be undertaken to improve the measurements of cyclical patterns we are now making. Further work in this direction is desirable in view of the importance of economic thinking in terms of the rate of change of different variables from stage to stage of business cycles.

VII Reference-cycle Patterns

In the course of the preceding analysis we have seen that our method of measuring specific-cycle patterns eliminates erratic movements imperfectly, and occasionally imparts a systematic twist to the patterns. If the original data were subjected to smoothing by Macaulay's formula before making cyclical patterns, erratic movements would be wiped out, but the cyclical fluctuations would be distorted with intolerable frequency. The defects of both methods count for less in reference-cycle than in specific-

48 The reason for this difference is apparent from note 47. In the special pattern the standing at the peak (trough) stage is dominated by the actual value at the peak (trough). In the standard pattern the actual value at the peak (trough) gets a weight no larger than that of the two adjacent months.

TABLE 128
Average Reference-cycle Patterns of Raw and Smoothed Data
Four American Series

Series and form of data	No. of reference cycles	I	II	III	IV	V	VI	VII	VIII	IX
		Average in reference-cycle relatives at stage								
		Initial trough	Expansion: First third	Expansion: Middle third	Expansion: Last third	Peak	Contraction: First third	Contraction: Middle third	Contraction: Last third	Terminal trough
DEFLATED CLEARINGS										
Raw	15	88.1	94.0	98.4	105.2	107.5	106.7	102.3	99.5	100.6
Smoothed	15	88.6	93.1	98.9	105.2	107.5	106.2	102.8	100.1	101.5
PIG IRON PRODUCTION										
Raw	15	73.3	90.0	103.5	112.5	122.2	117.6	100.4	84.8	81.1
Smoothed	15	77.8	88.6	103.8	113.8	119.8	117.1	99.3	85.0	86.7
RAILROAD STOCK PRICES										
Raw	19	91.0	96.9	104.0	109.4	106.9	104.3	97.7	92.5	94.7
Smoothed	19	91.7	96.4	104.7	109.3	107.5	104.6	97.8	93.0	95.7
CALL MONEY RATES										
Raw	19	77.5	82.4	98.4	128.2	159.5	128.5	103.9	81.1	76.2
Smoothed	19	78.4	82.8	101.0	127.2	138.6	128.7	99.7	82.5	76.6
		Excess of the raw average over the smoothed								
Deflated clearings	15	−0.5	+0.9	−0.5	0.0	0.0	+0.5	−0.5	−0.6	−0.9
Pig iron production	15	−4.5	+1.4	−0.3	−1.3	+2.4	+0.5	+1.1	−0.2	−5.6
Railroad stock prices	19	−0.7	+0.5	−0.7	+0.1	−0.6	−0.3	−0.1	−0.5	−1.0
Call money rates	19	−0.9	−0.4	−2.6	+1.0	+20.9	−0.2	+4.2	−1.4	−0.4

See Chart 49 for the periods covered. Three-month standings are used in stages I, V and IX of both the raw and smoothed patterns. The use of 1-month standings in the smoothed specific-cycle patterns was necessary to obtain a valid measure of amplitude, a reason that has no force in the case of reference cycles.

cycle patterns. In the first place, since the months entering into each reference-cycle stage are the same in the raw and smoothed analyses, there can be no differences in duration, whether due to erratic movements or the perversity of smoothing, such as occur in the specific cycles.[49] In the second place, each distortion of the tempo of cyclical movements by smoothing, instead of being concentrated in the same stage of different specific cycles, is dispersed over several stages of the reference cycles—the dispersion being large or small according as the differences in time between the specific and reference cycles are considerable or slight. The like is true of any systematic distortions of cyclical movements that may be produced by our standard method.

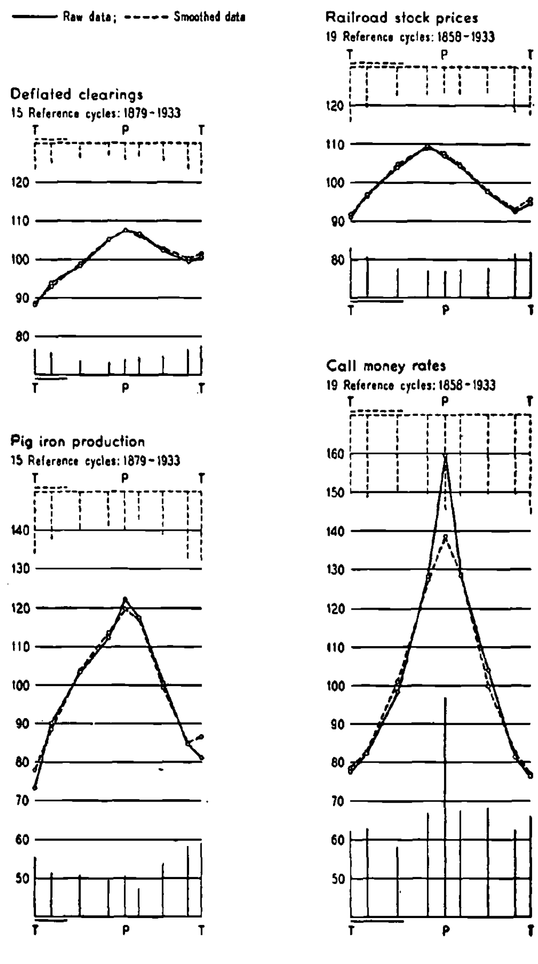
The average reference-cycle patterns of the raw and smoothed data of our four series are shown in Tables 128-129 and Chart 49. As expected, the differences between the standings of the two sets of patterns are, on the whole, considerably smaller than the corresponding differences be-

[49] Of course, reference dates determined from smoothed data would not match exactly our present reference dates. But it is neither feasible nor (if the analysis of Sec. III is sound) desirable to investigate the influence of smoothing on cyclical measures *via* its influence on reference dates. The practical question is how smoothed data behave relatively to raw data within the periods of reference cycles, as now marked off.

CHART 49

Average Reference-cycle Patterns of Raw and Smoothed Data

Four American Series



See Table 128. For explanation of chart, see Ch. 5, Sec. VII.

tween the specific-cycle patterns. Of course, the rates of change from stage to stage of the reference cycles differ more in proportion to the size of the figures than do the corresponding standings of the patterns, but they differ less than the rates of change from stage to stage of specific cycles. We no longer find a tendency for the differences between the rates of change in the raw and smoothed data to be larger during contractions than during expansions. There is still a tendency for the rates of change to differ more in the turning intervals than in the others, but it is not as

TABLE 129

Average Rate of Change from Stage to Stage of Reference Cycles
Four American Series, Raw and Smoothed

Series and form of data	No. of reference cycles	Change per month in reference-cycle relatives between stages							
		I-II	II-III	III-IV	IV-V	V-VI	VI-VII	VII-VIII	VIII-IX
		Expansion				Contraction			
		Trough to first third	First to middle third	Middle to last third	Last third to peak	Peak to first third	First to middle third	Middle to last third	Last third to trough
		Unweighted averages							
DEFLATED CLEARINGS									
Raw	15	+1.4	+0.5	+0.9	+0.6	−0.2	−0.9	−0.3	+0.5
Smoothed	15	+1.0	+0.8	+0.8	+0.5	−0.3	−0.6	−0.3	+0.5
PIG IRON PRODUCTION									
Raw	15	+3.7	+2.2	+1.1	+2.9	−1.2	−2.7	−3.0	−1.5
Smoothed	15	+2.2	+2.1	+1.6	+1.6	−0.7	−3.0	−2.4	+0.3
RAILROAD STOCK PRICES									
Raw	19	+1.3	+0.7	+0.7	−0.4	−1.0	−0.7	−0.5	+0.6
Smoothed	19	+1.0	+0.9	+0.6	−0.3	−0.8	−0.8	−0.3	+0.6
CALL MONEY RATES									
Raw	19	+1.4	+2.0	+4.8	+6.8	−3.2	−4.2	−5.1	−0.8
Smoothed	19	+1.2	+2.7	+3.8	+3.1	−1.0	−4.5	−3.7	−1.7
		Weighted averages							
DEFLATED CLEARINGS									
Raw	15	+1.4	+0.6	+0.9	+0.5	−0.2	−0.7	−0.4	+0.3
Smoothed	15	+1.0	+0.8	+0.8	+0.5	−0.4	−0.5	−0.4	+0.4
PIG IRON PRODUCTION									
Raw	15	+3.9	+1.8	+1.2	+2.3	−1.3	−2.7	−2.5	−1.0
Smoothed	15	+2.5	+2.1	+1.4	+1.4	−0.7	−2.8	−2.3	+0.5
RAILROAD STOCK PRICES									
Raw	19	+1.3	+0.9	+0.7	−0.6	−0.6	−0.9	−0.7	+0.5
Smoothed	19	+1.0	+1.1	+0.6	−0.4	−0.7	−1.0	−0.7	+0.7
CALL MONEY RATES									
Raw	19	+1.1	+2.0	+3.8	+6.9	−7.8	−3.5	−3.2	−1.2
Smoothed	19	+1.0	+2.3	+3.3	+2.5	−2.5	−4.1	−2.4	−1.5

See note to Table 128.

pronounced as in the specific cycles. Moreover, as Table 130 demonstrates, the ranking of the rates of change from stage to stage of the reference cycles in the smoothed data is very similar to the ranking of the rates of change in the raw data—a condition not found in the specific cycles.

It would not be unreasonable to infer from the similarity between the patterns of the raw and smoothed data that erratic movements play a relatively minor role in reference-cycle patterns made by our standard method. But there is stronger evidence to support this conclusion. If the month-by-month differences between the raw data and Macaulay's graduation were akin to a series of random numbers, we would expect the differences to be uncorrelated with business cycles. This expectation may

TABLE 130

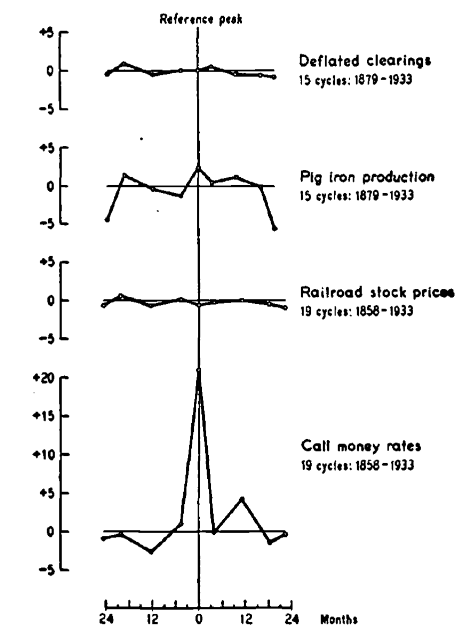
Ranks of Average Rates of Change from Stage to Stage of Reference Cycles
Four American Series, Raw and Smoothed

Series and form of data	No. of reference cycles	Rank of average change per month between reference-cycle stages							
		I-II	II-III	III-IV	IV-V	V-VI	VI-VII	VII-VIII	VIII-IX
DEFLATED CLEARINGS									
Raw	15	1	3	2	4	6	8	7	5
Smoothed	15	1	3	2	4	6	8	7	5
PIG IRON PRODUCTION									
Raw	15	1	3	4	2	6	8	7	5
Smoothed	15	1	2	4	3	6	8	7	5
RAILROAD STOCK PRICES									
Raw	19	1	2	3	5	6	8	7	4
Smoothed	19	2	1	4	5	7	8	6	3
CALL MONEY RATES									
Raw	19	4	3	2	1	8	7	6	5
Smoothed	19	4	3	1	2	7	8	6	5

Derived from weighted average rates, given in the lower section of Table 129. A rank of 1 is assigned to the highest figure, a rank of 8 to the lowest (taken algebraically). In case of a tie the computations were carried to an additional place before ranking.

CHART 50

Differences between Average Reference-cycle Patterns
of Raw and Smoothed Data
Four American Series



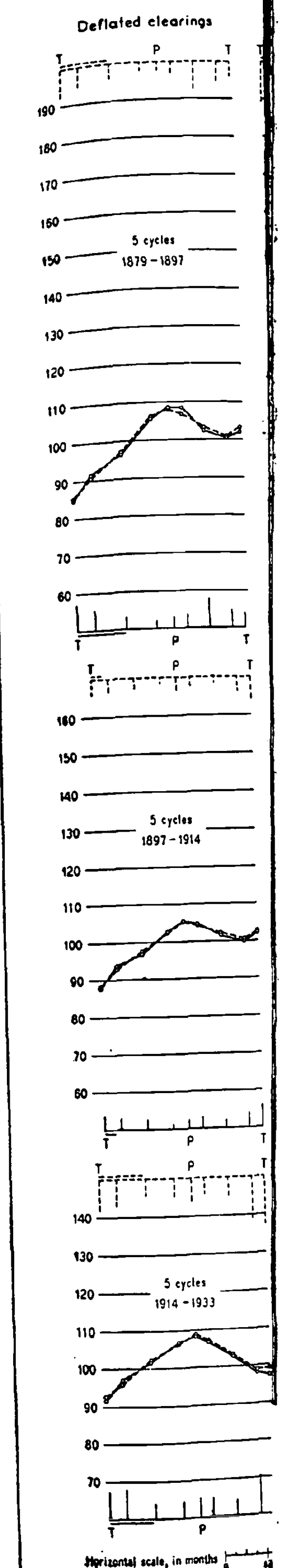
The vertical scale shown here is twice the standard scale used in other charts.
See Table 128 and Chart 49.

TABLE 131

Average Patterns of Small Groups of Reference Cycles
Four American Series, Raw and Smoothed

Series, period and form of data	No. of reference cycles	Average in reference-cycle relatives at stage								
		I	II	III	IV	V	VI	VII	VIII	IX
Deflated Clearings										
1879–1897										
Raw	5	85.0	91.4	96.6	105.8	108.6	108.6	102.4	100.3	102.0
Smoothed	5	85.4	90.6	97.1	106.1	108.3	106.9	103.2	100.8	103.1
1897–1914										
Raw	5	87.8	93.7	96.9	102.8	105.3	104.6	101.7	100.0	102.2
Smoothed	5	87.9	92.7	97.6	102.7	105.3	104.4	102.0	100.5	102.3
1914–1933										
Raw	5	91.6	97.0	101.8	106.9	108.5	106.9	102.9	98.3	97.7
Smoothed	5	92.6	95.9	102.2	106.7	108.8	107.2	103.1	99.0	99.2
Pig Iron Production										
1879–1897										
Raw	5	67.9	89.1	99.8	110.8	117.3	113.1	97.8	91.9	92.0
Smoothed	5	73.7	86.1	101.9	112.1	115.9	110.9	96.6	93.4	98.8
1897–1914										
Raw	5	72.5	88.3	99.4	109.7	117.5	115.8	100.0	86.7	85.4
Smoothed	5	75.4	86.6	100.7	110.2	117.0	114.9	98.8	87.1	90.1
1914–1933										
Raw	5	79.5	92.7	111.2	117.0	131.9	124.0	103.4	75.9	66.0
Smoothed	5	84.3	93.2	108.6	119.0	126.3	125.5	102.5	74.5	71.2
Railroad Stock Prices										
1858–1888										
Raw	6	82.8	89.0	101.5	112.0	106.2	105.6	102.8	95.3	96.5
Smoothed	6	83.8	88.7	101.9	111.5	107.8	106.7	102.3	95.8	98.0
1888–1908										
Raw	6	89.5	96.8	101.5	107.7	105.8	102.3	92.5	93.7	98.7
Smoothed	6	90.4	96.0	102.3	107.4	105.8	101.3	93.5	94.2	99.3
1908–1933										
Raw	7	99.3	103.7	108.4	108.5	108.4	105.0	97.8	89.1	89.6
Smoothed	7	99.5	103.3	109.1	109.0	108.8	105.6	97.6	89.6	90.8
Call Money Rates										
1858–1888										
Raw	6	80.4	90.6	106.4	139.3	193.7	116.5	80.0	83.8	76.3
Smoothed	6	81.7	92.2	106.1	135.4	133.7	113.3	88.7	81.6	75.8
1888–1908										
Raw	6	67.3	71.1	88.4	107.8	145.1	151.4	137.4	58.8	63.2
Smoothed	6	63.6	70.8	89.4	114.0	142.5	152.8	113.0	69.1	61.2
1908–1933										
Raw	7	83.8	85.1	100.2	136.3	142.5	119.3	95.7	98.0	87.3
Smoothed	7	88.2	85.1	106.6	131.5	139.5	121.2	97.8	94.7	90.4

The periods mark off successive thirds of the reference cycles covered by a series. Where the full number is not exactly divisible by 3, the odd item is placed in the last period. Three-month standings are used in stages I, V and IX of both the raw and smoothed patterns.

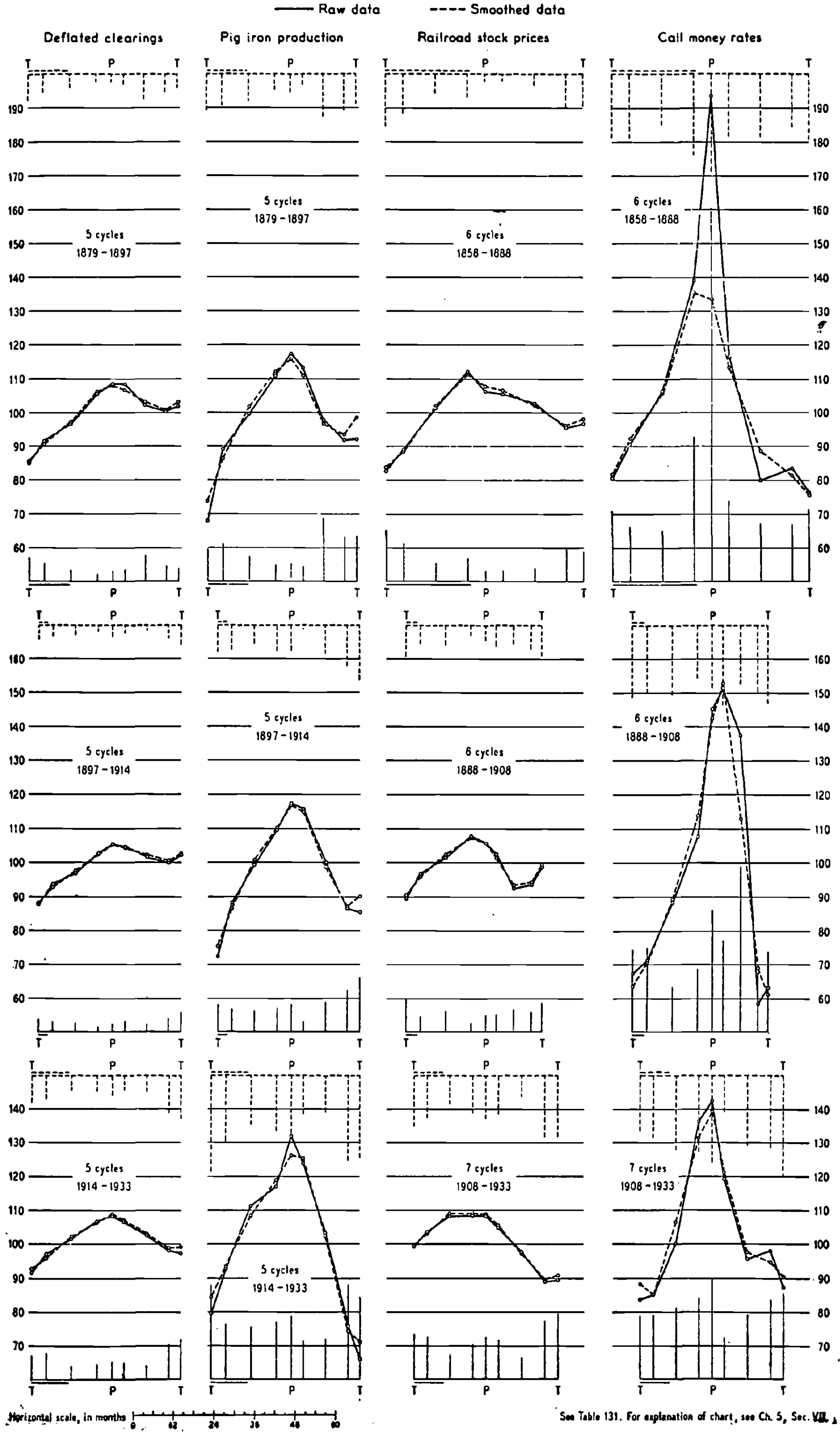


I	IX
.3	102.0
.8	103.1
.0	102.2
.5	102.3
.3	97.7
.0	99.2
.9	92.0
.4	98.8
.7	85.4
.1	90.1
.9	66.0
.5	71.2
.3	96.5
.8	98.0
.7	98.7
.2	99.3
.1	89.6
.6	90.8
.8	76.3
.6	75.8
.8	63.2
.1	61.2
.0	87.3
.7	90.4

umber is not
stages I, V

CHART 51

Average Patterns of Small Groups of Reference Cycles
Four American Series, Raw and Smoothed



See Table 131. For explanation of chart, see Ch. 5, Sec. VIII.

be tested by constructing reference-cycle patterns of the differences between the raw and smoothed data, as we do in Chart 50.[50] The patterns are irregular in the expansion and contraction stages, as they should be on the hypothesis of randomness; but in each series the standing of the differences is higher at the reference peak than at the reference troughs. The margin is slight in railroad stock prices, considerable in the other series. This striking feature appears in the reference cycles taken singly, not only in the averages, as indexes of full-cycle conformity for the differences demonstrate:[51] deflated clearings +38, iron production +93, railroad stock prices +32, call money rates +53. The marked conformity of the raw-smoothed differences to business cycles confirms a judgment voiced repeatedly in preceding pages: namely, that smoothing by Macaulay's formula or by similar devices, besides eliminating erratic movements, tends to distort cyclical fluctuations. It seems reasonable to infer that whatever erratic components remain in the raw reference-cycle patterns are smaller than the differences between the raw and smoothed patterns.

Of course, the conclusion that erratic movements play a relatively minor role in the reference-cycle patterns must be interpreted with care. It is more likely to be true of our standard Table R1 than of Table R2. The conclusion does not apply to patterns of single reference cycles, and may not apply to average patterns that are based on few cycles, especially if the erratic movements of a series are pronounced. Our comparisons between raw and smoothed patterns have been made from series that cover from 15 to 19 reference cycles. But as Chapter 1 shows, relatively few of our time series are so long; a large fraction cover no more than five cycles and many cover a smaller number. The degree of mutual canceling of erratic movements is likely to be smaller in reference-cycle patterns based on a half-dozen observations than in patterns based on three times that number. To throw some light on this matter, Table 131 and Chart 51 present raw and smoothed patterns for successive thirds of the reference cycles in our four long test series. In a few instances the differences between the raw and smoothed patterns run much larger than any encountered in the patterns for all cycles. There are, moreover, some irregularities in the subgroup patterns of raw data, whereas none occur in the patterns based on all cycles.[52] On the whole, however, the reduction in the

[50] Strictly speaking, the chart shows the differences between the reference-cycle patterns of the raw and smoothed data (see the bottom section of Table 128). This is not a mathematical equivalent of reference-cycle patterns of the differences between the raw and smoothed data. However, the shapes of the patterns yielded by the two methods are very similar. The method used in the chart avoids the need to express the results for different series in the original units.

[51] The indexes of conformity were computed, on the plan of Table R3, from the differences between the raw and smoothed data, with the former as minuend.

[52] A reference-cycle pattern may be considered 'irregular' if it contains more than one peak and trough. A stricter definition would be that the pattern is irregular if any movement runs counter

number of cycles included in the patterns makes less difference than might have been expected.

As in the case of specific cycles, our standard method yields only nine-point patterns for reference cycles. The lack of greater detail would be a source of some embarrassment in tracing the sequence of revivals and recessions in different activities, were it not for the monthly timing measures in Table S1. Even so, greater refinement in the reference-cycle patterns would be an advantage if the additional detail were trustworthy. Table 127 and Chart 48 present the results of an experiment with a nineteen-point pattern. The 'special' pattern for reference cycles was made on exactly the same principle as the special pattern for specific cycles, described towards the end of Section VI. The special reference-cycle pattern is somewhat less regular than the standard pattern. It suggests, however, a slight lead (2.2 months) of pig iron production at the reference trough, which is confirmed by the measure of average timing in our Table S1 (3.4 months).

VIII Measures of Conformity to Business Cycles

In view of the comparatively small differences between the reference-cycle patterns of raw and smoothed data, we should expect their measures of conformity to be similar. Table 132 shows that in each series smoothing by Macaulay's formula tends to reduce (numerically) the average rates of change during the stages matched with reference expansions and with reference contractions. But the differences made by smoothing run smaller in these measures than in the average rates of change during expansions and contractions of specific cycles.

Of the 12 indexes of conformity, smoothing leaves 7 unchanged, raises 2 and lowers 3.[53] None of the changes are of much consequence. Some improve the measure of conformity slightly, others worsen it. In clearings, for example, the smaller contraction index of the smoothed data is due to the virtual disappearance of the decline in 1918, which we consider a genuine cyclical contraction. In call money rates, on the other hand, the expansion index is higher in the smoothed data, in part because erratic movements in the raw data are very prominent in the vicinity of the boundaries of the reference expansion of 1891–93.

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to the division of stages in our standard Table R4. Note that while irregular movements in specific-cycle patterns are always indicative of erratic movements, subject only to the qualifications in note 35 of Ch. 5 and note 32 of this chapter, a reference-cycle pattern may be irregular for two reasons: (1) erratic movements in the raw data, (2) erratic behavior of specific cycles with respect to business cycles.

[53] The count is 6, 2 and 4, if indexes based on stages VIII-V are used for the smoothed data on iron production.

TABLE 132
Conformity to Business Cycles of Raw and Smoothed Data
Four American Series

Series and form of data	No. of reference cycles[a]	Stages matched with reference expansion	Average change per month in reference-cycle relatives during stages matched with reference		Index of conformity to reference		
			Expansions	Contractions	Expansions	Contractions	Cycles
DEFLATED CLEARINGS							
Raw	15	VIII-V	+0.78	−0.50	+100	+73	+86
Smoothed	15	VIII-V	+0.75	−0.39	+100	+60	+93
PIG IRON PRODUCTION[b]							
Raw	15	I-V	+2.26	−2.27	+100	+100	+100
Smoothed	15	I-V	+1.87	−1.79	+100	+87	+100
Smoothed	15	VIII-V	+1.61	−2.26	+87	+87	+100
RAILROAD STOCK PRICES							
Raw	19*	VIII-IV	+0.77	−0.61	+79	+60	+74
Smoothed	19*	VIII-IV	+0.73	−0.57	+79	+60	+74
CALL MONEY RATES							
Raw	19*	I-V	+3.62	−3.57	+68	+100	+100
Smoothed	19*	I-V	+2.83	−3.07	+89	+80	+100

[a] See Chart 49 for the periods covered; but note that where an asterisk appears, the contraction and full-cycle indexes cover an additional reference contraction (1857–58) at the beginning of the series.

[b] The conformity measures for the smoothed data are shown for two sets of intervals because there is little to choose between them. See pp. 195-6.

In Table 133 we show separate indexes of conformity for each of the eight intervals into which Table R2 divides reference cycles. These indexes are computed in the same way as our standard measures. In the group of stages matched with reference expansions, the conformity index for a given interval (for example, stage I to II in call money rates, or stage VIII to IX in clearings) is the excess of the number of rises over the number of falls, expressed as a percentage of the number of cycles. In the group of stages matched with reference contractions, a fall signifies positive conformity and a rise inverted conformity; hence the excess of the number of falls over the number of rises for a given interval is expressed as a percentage of the number of cycles. Of course this method of computation reduces the indexes; for a movement opposed to the cyclical tide in any one of the three to five stages that are grouped together as 'expansions' or 'contractions' will affect the entries in the present table, whereas in the broader averages of Table 132 that movement is often more than offset by conforming changes in the preceding or following stage. In these stage-by-stage indexes, the smoothed figures stand higher than the raw figures in 17 cases, the same in 7 cases and lower in 8. This drift in the results is to be expected; for smoothing tends to eliminate erratic movements counter to the cyclical tide, which reduce the raw indexes.

TABLE 133

Stage-by-stage Indexes of Conformity to Business Cycles
Four American Series, Raw and Smoothed

Series and form of data	No. of reference cycles	Stages matched with reference expansion	Index of conformity of changes between reference-cycle stages								Average index of conformity		
			I-II	II-III	III-IV	IV-V	V-VI	VI-VII	VII-VIII	VIII-IX	Turning zones[b]	Other intervals	All intervals
DEFLATED CLEARINGS													
Raw	15	VIII-V	+100	+80	+87	+47*	+33*	+33	+13*	+73*	+42	+75	+58
Smoothed	15	VIII-V	+100	+100	+100	+73*	+33*	+47	+20*	+87*	+53	+87	+70
PIG IRON PRODUCTION[a]													
Raw	15	I-V	+87*	+100	+60	+100*	+20*	+73	+73	+47*	+63	+77	+70
Smoothed	15	I-V	+87*	+87	+73	+73*	+7*	+73	+73	−7*	+40	+77	+58
Smoothed	15	VIII-V	+87	+87	+73	+73*	+7*	+73	+73*	+7*	+40	+80	+60
RAILROAD STOCK PRICES													
Raw	19	VIII-IV	+89	+68	+37*	+16*	+26	+16	+16*	+16*	+21	+50	+36
Smoothed	19	VIII-IV	+58	+79	+47*	+26*	+53	+63	+5*	+5*	+21	+63	+42
CALL MONEY RATES													
Raw	19	I-V	+16*	+47	+58	+58*	+26*	+79	+16	+26*	+32	+50	+41
Smoothed	19	I-V	+16*	+68	+79	+47*	+26*	+89	+47	+47*	+34	+71	+53

See Chart 49 for the periods covered.

[a] See Table 132, note 'b'.

[b] Here treated as two-interval overlaps on the selected expansion and contraction segments of the series. Indexes of conformity for these intervals are marked by an asterisk (*) in the preceding columns.

The addition that Table 133 makes to our knowledge is that the conformity of a series to business cycles is higher outside than within its turning zones. Of course the turning zones vary from series to series. Two of the present four usually turn up in the last third of reference contraction, but call money rates and iron production defer the upturn until the reference trough. The downturn usually occurs in the last third of reference expansion in railroad stock prices, but not until the reference peak in the other series. In Table 133 the indexes for the turning zones are starred, and averages are shown of these four indexes and also of the four indexes outside the turning zones. Without exception, in both the raw and the smoothed data, the average for the turning zones is lower than the average for the other intervals.

We attribute the low conformity in the turning zones to irregularities in the timing of specific-cycle revivals and recessions. If a series is a little prompter or a little slower than usual in responding to a cyclical turn in general business, its turn may not come in the usual reference-cycle stage. That is true whether a series is analyzed in raw or smoothed form. Since our plan of computing indexes of conformity assigns the same stages of reference cycles to expansion in all the cycles covered by a series, and other fixed stages to contraction, any departure from the timing of cyclical turns characteristic of the series as a whole may affect our standard indexes in Table 132, and is much more likely to affect the stage-by-stage indexes of Table 133 that cover briefer periods. Smoothing tends to increase the stage-by-stage indexes outside the turning zones, but has no systematic influence on the indexes within the turning zones.

TABLE 134

Average Deviations from Average Cyclical Measures

Four American Series, Raw and Smoothed

Measure	Deflated clearings		Pig iron production		Railroad stock prices		Call money rates	
	Raw	Sm.	Raw	Sm.	Raw	Sm.	Raw	Sm.
(I) DURATION OF SPECIFIC CYCLES								
Expansion	9.9	9.3	9.0	8.8	16.0	15.1	7.6	6.4
Contraction	6.5	4.6	7.1	6.4	12.1	10.4	6.4	6.7
Full cycle	11.6	10.6	9.9	9.7	22.6	21.3	8.6	8.0
Expansion as % of cycle	11	9	13	14	13	11	14	11
(II) AMPLITUDE OF SPECIFIC CYCLES								
Rise	10.6	9.9	15.7	16.1	20.5	20.5	52.0	33.2
Fall	8.7	8.0	21.2	23.0	17.0	15.9	56.7	40.7
Rise & fall	13.8	13.1	26.8	26.6	35.9	34.6	105.9	67.2
Rise per month	0.2	0.2	0.9	0.9	0.6	0.4	3.3	1.5
Fall per month	1.7	0.3	2.1	1.3	0.8	0.5	4.0	2.3
Rise & fall per month	0.2	0.2	0.9	0.9	0.4	0.4	2.4	1.7
(III) SECULAR MOVEMENTS								
Change from								
Contraction to expansion	6.4	5.6	12.6	11.2	14.0	14.6	15.8	16.8
Expansion to contraction	8.2	7.3	15.0	13.4	10.4	9.4	17.3	16.8
Cycle to cycle, total	6.2	5.8	17.4	15.1	16.7	16.7	23.8	23.6
Cycle to cycle, per month	0.15	0.13	0.40	0.33	0.36	0.36	0.68	0.68
(IV) SPECIFIC-CYCLE PATTERN								
Standing								
I	5.5	5.3	14.6	15.3	11.6	12.3	13.7	16.6
II	4.2	4.2	12.1	10.7	9.6	9.9	12.3	14.5
III	2.0	2.1	9.5	7.4	5.3	4.4	16.3	10.7
IV	2.8	2.3	7.5	8.3	5.4	5.1	16.8	13.1
V	5.9	4.9	11.2	8.7	9.5	8.6	41.8	22.7
VI	5.2	4.4	7.9	6.7	8.4	7.3	18.1	16.1
VII	6.6	5.6	11.9	8.5	7.3	5.8	20.7	12.8
VIII	8.8	8.3	14.6	15.9	9.4	9.2	18.9	18.6
IX	10.6	9.1	17.8	18.6	12.3	11.5	19.5	21.2
Rate of change								
I-II	0.5	0.2	1.6	0.7	0.8	0.3	4.1	0.8
II-III	0.4	0.3	1.2	1.1	0.8	0.6	6.2	1.7
III-IV	0.3	0.3	1.1	1.5	1.1	0.5	7.9	2.6
IV-V	0.4	0.4	1.4	0.7	0.8	0.4	11.7	2.6
V-VI	2.6	0.3	1.3	0.9	1.1	0.3	14.6	2.7
VI-VII	3.3	0.4	2.4	2.1	0.9	0.7	5.4	4.2
VII-VIII	2.0	0.4	5.2	1.5	1.2	0.8	3.7	2.7
VIII-IX	1.3	0.2	3.9	0.6	1.4	0.4	2.7	0.8
(V) REFERENCE-CYCLE PATTERN								
Standing								
I	6.6	6.8	15.5	16.1	13.8	14.1	22.3	20.3
II	5.7	5.2	11.4	12.2	10.7	10.8	22.8	21.3
III	3.7	3.8	10.9	9.7	7.7	7.5	18.2	19.4
IV	3.2	3.2	9.4	9.0	7.0	7.4	26.9	20.3
V	4.0	4.2	10.7	10.1	7.0	6.7	56.9	24.6
VI	4.5	3.5	7.3	7.2	7.4	7.2	27.6	21.1
VII	4.8	4.8	13.9	11.3	7.9	6.9	28.3	20.6
VIII	6.6	6.9	18.2	17.2	11.5	11.8	22.4	20.5
IX	7.6	7.8	19.0	17.9	12.2	12.3	26.0	25.8

TABLE 134—*Continued*
Average Deviations from Average Cyclical Measures
Four American Series, Raw and Smoothed

Measure	Deflated clearings		Pig iron production		Railroad stock prices		Call money rates	
	Raw	Sm.	Raw	Sm.	Raw	Sm.	Raw	Sm.
(V) REFERENCE-CYCLE PATTERN—*Cont.*								
Rate of change								
I-II	0.5	0.3	2.2	1.3	0.8	0.9	4.8	3.2
II-III	0.3	0.2	1.3	1.1	0.8	0.7	2.7	2.6
III-IV	0.4	0.3	1.9	1.3	0.9	0.8	4.3	2.2
IV-V	0.6	0.4	2.1	1.0	1.4	1.1	9.2	3.8
V-VI	0.7	0.6	2.1	2.4	1.5	0.9	10.5	4.4
VI-VII	1.0	0.6	2.8	1.7	1.4	1.0	6.7	3.1
VII-VIII	0.5	0.5	2.7	1.9	1.1	0.9	7.4	4.2
VIII-IX	0.6	0.5	2.6	1.7	1.2	0.9	4.1	2.7
(VI) CONFORMITY								
Rate of change[a] during reference								
Expansions	0.18	0.16	0.87	0.79[b]	0.57	0.60	2.37	1.38
Contractions	0.44	0.45	1.35	1.22[b]	0.58	0.61	2.08	2.09

The entries in group (I) are expressed in months, except on the last line. The entries in groups (II) and (IV) are expressed in specific-cycle relatives, and in groups (V)-(VI) in reference-cycle relatives. The entries in group (III) correspond to the plan of computation in col. 5, 6, 9 and 10 of our standard Table S3 (see Table 33). The periods and numbers of cycles covered are given in Table 112 for the specific-cycle measures, in Chart 49 for the reference-cycle measures.

[a] That is, during the stages matched with reference expansion and contraction, shown in Table 132.
[b] Based on the assumption that stages I-V are characteristic of expansion. See Table 132, note 'b'.

IX Variability of Cyclical Measures

Table 134 shows in detail how smoothing by Macaulay's formula affects the average deviations about the averages given in the preceding tables, and Table 135 summarizes these effects. Smoothing as a rule reduces the average deviations. The reduction is, typically, some ten or twenty per cent, except in the measures of stage-to-stage changes, where the reduction is usually greater because these measures are especially susceptible to erratic disturbances.

The large average deviations that remain in the smoothed figures are among the more significant results established by this series of tests. One reason for the large average deviations is that while smoothing eliminates erratic movements that change direction every few months, it does not eliminate random movements that spread over years. Strictly speaking, smoothing merely redistributes the original values. In this redistribution the degree to which a movement in the raw data is changed depends upon the extent of its departure from the general level, its duration, and the formula used. As said above, high peaks or low troughs with a time base of two or three months may disappear in Macaulay's graduation or become converted into rather gentle undulations; movements of equal amplitude lasting two or three years are changed much less. Now there are random factors that continue in force for periods reckoned in years; for example, a great war. There are also random events that sometimes

TABLE 135

Effect of Smoothing on Average Deviations from Average Cyclical Measures

Measure	Number of observations	Number of instances in which smoothing			Average ratio of average deviation of smoothed data to average deviation of raw[a]
		Reduces average deviation	Increases average deviation	Leaves average deviation unchanged	
I Duration of specific cycles	16	14	2	..	.91
II Amplitude of specific cycles	24	16	2	6	.82
III Secular movements	16	11	2	3	.94
IV Specific-cycle pattern					
Standing	36	25	10	1	.91
Rate of change	32	29	1	2	.50
V Reference-cycle pattern					
Standing	36	21	13	2	.94
Rate of change	32	29	2	1	.72
VI Conformity	8	4	4	..	.92
Total	200	149	36	15	.81

Derived from Table 134.

[a] The ratio of the average deviation of smoothed data to the average deviation of raw data was computed separately for each measure in each series. The ratios were then averaged for all measures assigned to a group.

recur in two or more successive years; for example, harvest failures. Smoothing by Macaulay's method will not remove the great bulges in American price and value series in 1862–67 and in 1915–21. It will moderate the effects of two bad harvests upon agricultural prices less than it will moderate the effects of speculative maneuvers associated with monthly crop reports, or even the effects of two bad seasons separated by a good season. Random effects of considerable size thus remain in the smoothed forms of the series we have used in our tests, and contribute toward making the average deviations nearly as large in the results[54] obtained from the smoothed as in those obtained from the raw data.

All that Macaulay claims for smoothing is that it eliminates erratic movements. But some students of this problem have made much bolder claims. Anderson, for example, believes that his method of smoothing, which is based upon analysis of the variances of successive differences, makes possible the separation of the 'casual' from the 'essential' components of a time series.[55] But since Anderson's method proceeds on the assumption that 'casual' forces operate independently on each value of a time series, it does not cope with random perturbations that shape the general features of a time series over many months.[56]

[54] Except in the measures of stage-to-stage changes in cyclical patterns.

[55] See Oskar Anderson, Die Korrelationsrechnung in der Konjunkturforschung (*Veröffentlichungen der Frankfurter Gesellschaft für Konjunkturforschung,* Heft 4, 1929), particularly pp. 72-80.

[56] In *Prices in the Trade Cycle* (J. Springer, Vienna, 1935), Gerhard Tintner describes the Anderson method in nontechnical language and applies it to numerous price series. His own work, as he recognizes, is based upon approximations that do not make the fullest use of the Anderson criteria of graduation. Tintner merely uses an unweighted moving average containing from three to eleven monthly items, the period of the moving average being fixed for any one series. For a further development of Anderson's method, see Tintner's recent book, *The Variate Difference Method* (Principia Press, 1940).

X Uncertainties in Identifying Specific Cycles

Our analysis of specific cycles has been restricted by the assumption that Macaulay's graduation leaves the lists of specific cycles unchanged. This assumption is in a sense invalid. As Section II shows, if the specific cycles in smoothed data are marked off independently of the specific cycles in the raw data, the lists of cycles disagree in three of our four series. But it is neither necessary nor wise to follow so artificial a plan in practice. Whether an investigator prefers to make cyclical measures from raw or from smoothed data, he must try to pick 'real' specific cycles. The specific cycles identified in the raw data of the present sample were originally selected with the aid of simple moving averages passed through doubtful portions of the series, as is our usual practice. Study of the raw data can be no less helpful in this regard to other investigators who may prefer to make cyclical measures from smoothed data. Thus viewed, the correspondence forced in Section II between the cycles in raw and smoothed data is more than an analytic convenience; it is a device for making the analysis run, as we best can, in terms of 'real' cycles.

But 'real' cycles can be very elusive. Since some uncertainty surrounds the selection of specific cycles in many series, the question of practical interest is how erratic movements affect cyclical averages, by leading us either to recognize a spurious cycle or to ignore a genuine cycle. The simplest way to illustrate these effects is to compare two sets of averages, one set being based on the cycles we actually recognize, the other on some modification of that list. In line with this plan, we may treat the mild contraction of 1918 in deflated clearings and in call money rates and that of 1887–88 in iron production, which are the least certain of the cyclical movements we recognize in the raw data, as samples of the uncertainties to which erratic movements frequently give rise in the process of identifying specific cycles. Table 136 compares our standard averages of cyclical duration and amplitude both for all cycles and small groups of cycles with averages in which one cycle instead of two is taken during 1885–91 in iron production, during 1914–21 in clearings, and during 1915–22 in call money rates. The standard averages are entered on the lines marked 'Raw I', those omitting a cycle on lines 'Raw II'.

To rationalize the influence of a dubious 'extra' cycle, let us assume, for simplicity, that the 'Raw II' averages are 'true', that the 'Raw I' averages mistakenly include an extra cycle, that the extra movement is mild and brief, and that it comes during a true expansion. It is obvious that the extra cycle must reduce the average duration of expansions; for the sum of the lengths of the expansions is reduced while their number is increased. It must likewise reduce the average duration of full cycles, since their number is larger while the sum of their lengths is unchanged.

TABLE 136

Effect of a 'Dubious' Cycle on Average Duration and Amplitude of Specific Cycles
Three American Series

Series, period, and form of analysis	No. of specific cycles	Average duration in months			Average in specific-cycle relatives: Amplitude of			Per month amplitude of		
		Expansion	Contraction	Full cycle	Rise	Fall	Rise & fall	Rise	Fall	Rise & fall
DEFLATED CLEARINGS										
1878–1933										
Raw I	15	32.6	11.4	44.0	26.9	13.4	40.2	0.8	1.9	0.9
Raw II	14	35.2	11.9	47.1	28.4	14.1	42.4	0.8	2.0	0.9
1910–1933										
Raw I	5	36.8	17.0	53.8	23.7	17.6	41.3	0.7	1.1	0.8
Raw II	4	47.0	20.2	67.2	28.2	21.1	49.3	0.6	1.0	0.7
PIG IRON PRODUCTION										
1879–1933										
Raw I	15	28.8	14.5	43.3	62.1	54.8	116.8	2.4	4.7	2.9
Raw II	14	31.2	15.2	46.4	64.8	57.5	122.3	2.4	4.7	2.9
1879–1896										
Raw I	5	28.6	14.0	42.6	62.3	44.4	106.7	2.6	3.8	2.8
Raw II	4	37.0	16.2	53.2	72.1	51.3	123.4	2.7	3.7	2.7
CALL MONEY RATES										
1858–1931										
Raw I	23	19.9	18.0	37.9	115.9	116.1	232.0	6.9	7.6	6.3
Raw II	22	21.0	18.6	39.6	119.3	120.8	240.2	6.9	7.6	6.4
1904–1931										
Raw I	8	20.4	19.9	40.2	95.6	91.2	186.8	4.7	5.7	4.5
Raw II	7	23.9	22.1	46.0	103.4	102.5	205.9	4.5	5.5	4.5

'Raw I' stands for the raw data as treated in preceding tables; 'Raw II' for the raw data treated in the manner explained in the text.

The average duration of contractions, on the other hand, may or may not be modified; but it is likely to be lower, since the addition made by the extra cycle to the sum of contractions is likely to be below the 'true' average of this measure. For a similar reason, the average amplitudes of rise, fall, and combined rise and fall are likely to be lowered.[57] The per month amplitudes are affected differently. Consider, first, the fall per month. This average is as likely to be raised as lowered, since there is no more reason for supposing that the fall per month of the 'extra' cycle is below than for supposing that it is above the 'true' average. It may seem that the average rise per month, on the other hand, is more likely to be raised than lowered, for the average rate of rise during the two expansions carved out of one 'true' expansion is practically sure to exceed the rate of rise during the true expansion. But in order that the average rise per month for all cycles actually increase, the sum of the two rates during the true expansion must exceed the rate during this expansion by more than

[57] This and the following remarks concerning amplitudes ignore the complicating influence of specific-cycle bases.

TABLE 137

Effect of a 'Dubious' Cycle on Average Specific-cycle Patterns
Three American Series

Series, period, and form of analysis	No. of specific cycles	Average in specific-cycle relatives at stage								
		I	II	III	IV	V	VI	VII	VIII	IX
DEFLATED CLEARINGS										
1878–1933										
Raw I	15	85.7	90.5	99.2	106.7	112.6	108.7	106.0	101.9	99.2
Raw II	14	84.8	90.0	98.7	107.0	113.2	109.2	106.1	101.9	99.1
1910–1933										
Raw I	5	88.3	92.1	100.8	107.4	112.0	107.2	104.1	97.6	94.4
Raw II	4	85.7	90.8	99.6	108.5	113.8	108.7	103.9	96.8	92.7
PIG IRON PRODUCTION										
1879–1933										
Raw I	15	67.3	82.5	103.7	116.5	129.3	122.6	108.2	88.4	74.6
Raw II	14	65.9	82.3	103.6	117.3	130.8	123.7	108.8	88.3	73.3
1879–1896										
Raw I	5	63.1	78.7	101.5	112.7	125.5	121.4	112.9	96.6	81.1
Raw II	4	57.4	77.1	100.7	114.5	129.5	124.8	116.2	98.4	78.2
CALL MONEY RATES										
1858–1931										
Raw I	23	62.1	80.3	104.7	123.2	178.0	120.4	89.9	71.8	61.9
Raw II	22	60.9	79.4	105.2	123.5	180.2	120.4	88.6	69.9	59.4
1904–1931										
Raw I	8	60.9	76.2	107.1	123.3	156.5	123.5	97.4	76.2	65.3
Raw II	7	57.0	73.1	108.8	124.1	160.4	123.8	94.2	70.8	57.8

See note to Table 136.

the true average rate. The average joint rise and fall per month is affected in much the same way as the average rise per month.[58]

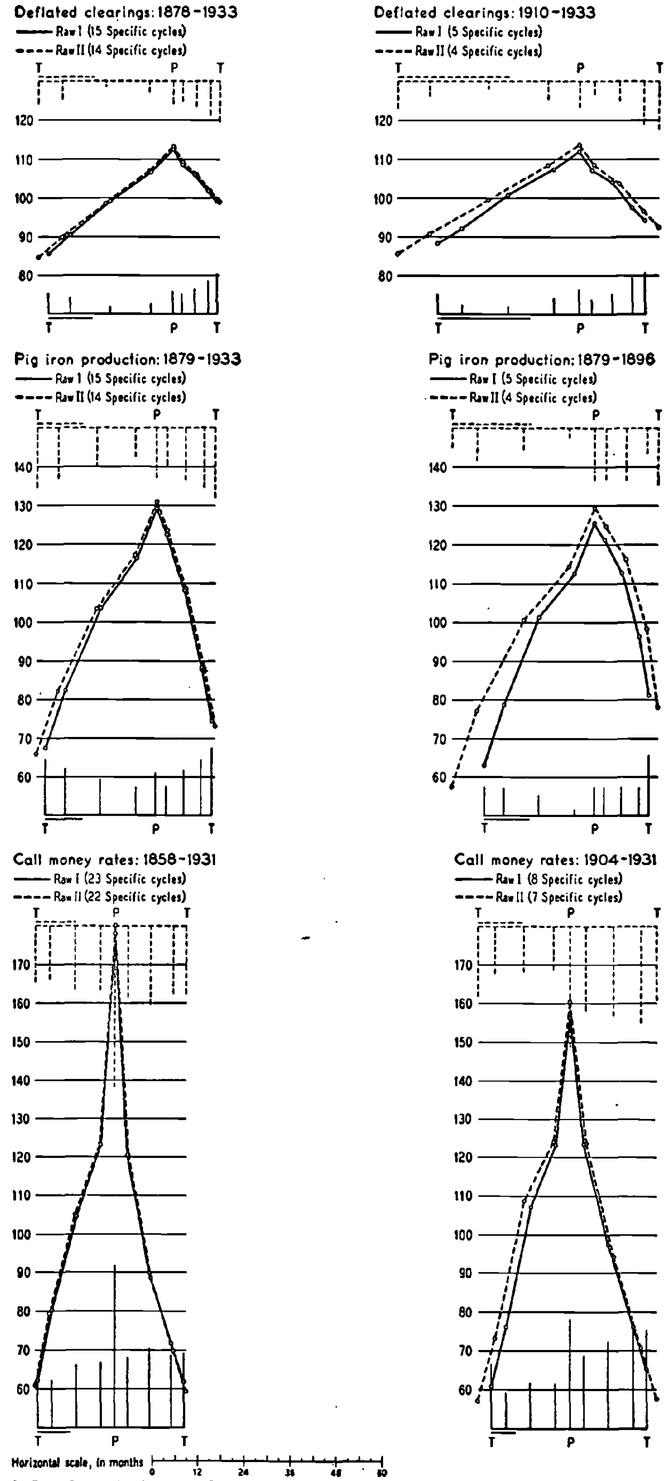
These expectations could be worked out in greater detail and on altered assumptions, but that is not necessary. The significant results are indicated in Table 136. A single doubtful cycle affects average durations more than average amplitudes, and average total amplitudes much more than average per month amplitudes. When the number of cycles is rather large, the effect on all averages in the table is small. When the number of cycles is only about a half dozen, a single doubtful cycle naturally has larger effects. Even so, really prominent effects are confined to several measures of duration and total amplitude; the per month amplitudes are remarkably stable. Results of this character are fairly typical measures of the influence that doubtful decisions concerning specific cycles exercise on our averages. Of course, when erratic fluctuations are so violent that many cycles are obscured, the effects may be much greater than any in the table. Although such instances are relatively rare, they require cautious handling, and in later monographs we strive to call attention to them as they come up.

58 Given the assumptions in the text, if the per month amplitudes are weighted by durations, the extra cycle will necessarily increase the average rise per month, and the average joint rise and fall per month, but not the average fall per month.

CHART 52

Effect of a 'Dubious' Cycle on Average Specific-cycle Patterns

Three American Series



See Table 137 and explanations in text. For construction of chart, see Ch. 5, Sec. VI.

TABLE 138

Effect of a 'Dubious' Cycle on
Average Rate of Change from Stage to Stage of Specific Cycles
Three American Series

Series, period, and form of analysis	No. of specific cycles	Change per month in specific-cycle relatives between stages							
		I-II	II-III	III-IV	IV-V	V-VI	VI-VII	VII-VIII	VIII-IX
		Unweighted averages							
DEFLATED CLEARINGS 1910–1933									
Raw I	5	+0.5	+0.9	+0.6	+0.7	−1.4	−0.6	−1.3	−1.0
Raw II	4	+0.6	+0.6	+0.6	+0.7	−0.9	−1.6	−0.6	−1.2
PIG IRON PRODUCTION 1879–1896									
Raw I	5	+4.3	+3.2	+1.1	+2.5	−1.8	−2.6	−5.6	−5.4
Raw II	4	+4.9	+3.2	+1.1	+2.2	−1.9	−1.8	−4.8	−7.2
CALL MONEY RATES 1904–1931									
Raw I	8	+4.1	+4.8	+3.1	+8.1	−11.1	−5.5	−4.5	−2.8
Raw II	7	+4.1	+5.1	+2.9	+6.7	−11.2	−5.4	−3.4	−3.3
		Weighted averages							
DEFLATED CLEARINGS 1910–1933									
Raw I	5	+0.6	+0.7	+0.6	+0.7	−1.5	−0.6	−1.2	−1.0
Raw II	4	+0.6	+0.6	+0.6	+0.7	−1.4	−0.8	−1.1	−1.1
PIG IRON PRODUCTION 1879–1896									
Raw I	5	+3.1	+2.5	+1.2	+2.5	−1.6	−1.9	−3.6	−6.2
Raw II	4	+3.0	+2.0	+1.2	+2.3	−1.7	−1.6	−3.4	−7.0
CALL MONEY RATES 1904–1931									
Raw I	8	+4.1	+4.8	+2.5	+8.9	−9.1	−4.1	−3.4	−3.0
Raw II	7	+3.7	+4.7	+2.0	+8.3	−9.2	−4.2	−3.3	−3.2

See note to Table 136.

Tables 137-138 and Chart 52 supply comparisons of specific-cycle patterns on the plan of Table 136. The 'Raw I' pattern lies inside the 'Raw II' pattern, which merely means that its average duration and amplitude are both smaller. An 'extra' cycle sometimes makes a very considerable difference in the unweighted average rates of change from stage to stage of the specific cycles, but the weighted average rates are tolerably stable even when the number of cycles in the sample is only about half a dozen.

XI Conclusions

The reader who has gotten so far in this book must be sufficiently aware of the difficulties that surround our efforts to provide an authentic description of cyclical behavior. The 'facts' of cyclical experience do not tell their own story. They must be wrung from data by tools of observation. If the tools are coarse and the investigator is blind to their defects, his vision of cyclical fluctuations may be distorted. A serious investigator does not save time in the long run by neglecting to understand his tools or to improve upon them.

In this chapter we have examined the influence of erratic movements on our measures of cyclical behavior, and investigated the possibility of refining the measures through preliminary smoothing of the raw data by Macaulay's formula or some similar device. We have found that smoothing does indeed refine cyclical measures, in the sense of freeing them from the influence of erratic, saw-tooth movements of brief duration. But smoothing does not eliminate random movements that extend over years, and it frequently misrepresents cyclical fluctuations.

We do not claim that smoothing by so sensitive a graduation formula as Macaulay's would be valueless in our studies. On the contrary, we believe that it could be a material aid in identifying and, to a lesser extent, in dating specific cycles. Nor do we claim that cyclical measures made from smoothed data are necessarily less trustworthy than those yielded by our rough method. Theoretically, the results may be better. But if that end is to be attained, the investigator will usually need to vary the smoothing formula not only from series to series, but also from part to part of a given series. Furthermore, he will have to devise methods that avoid spurious rounding of cyclical tops and bottoms. Experimentation along these lines is highly desirable. But if we attempted it on any scale, we would be forced to confine our statistical analysis to a small fraction of the time series we can cover by our present methods. Experience has convinced us that knowledge of cyclical fluctuations based upon a small sample of time series is untrustworthy, and that as the range of activities analyzed becomes broader, the need for highly refined measurements is much reduced.[59]

Although our measures for individual cycles are marred by erratic fluctuations, it is important to observe that our technique involves a certain element of smoothing. In most measures made from monthly data, we use, not the entries for single months, but averages covering three or more months. For example, in making Tables S4 and R1, we determine the standing of the series at the initial trough, peak, and terminal trough of each cycle by averaging the cycle relatives for the three months centered on each of these dates. Also the standings during successive thirds of expansion and of contraction are averages for whatever months are included in these stages. Averaging of a similar sort is used in Table S2 which gives the amplitudes of specific cycles, in Table S3 which measures secular movements, in Tables S5 and R2 which show rates of change from stage to stage of specific and reference cycles, in Table R3 which shows rates of change during reference expansions and contractions, and in Table R4 which shows rates of change during the reference-cycle stages characteris-

[59] Cf. Ch. 9, Sec. III, and Ch. 12, Sec. V.

tic of expansions and contractions in each series. Indeed the only measures in which such averaging is not used are the measures of cyclical duration and of leads and lags in Table S1.

When averages are struck for all cycles covered by a series, the erratic factors in the measures for single cycles have an additional opportunity to cancel out. In the course of the tests in this chapter we have found that in some measures the averages made from raw data are practically indistinguishable from the averages made from Macaulay's graduations. In other measures the differences are very large; but these seem to be precisely the cases where Macaulay's formula represents cyclical movements least successfully. We believe, therefore, that so far as erratic movements do not obscure specific cycles, our averages, as a rule, are not gravely affected by erratic movements in the data from which they are made. In our judgment this statement holds for averages based on only half a dozen cycles as well as for averages based on many cycles, except perhaps the averages in Tables S5 and R2, so long as the erratic movements are not especially pronounced.

The tendency of erratic movements to obscure specific cycles is a more serious matter than their influence on the timing and outlines of the cycles that stand out clearly in the data. This factor is very troublesome in some series and its importance should not be minimized. But we also must not exaggerate this difficulty relatively to others—the defects of the original data in most economic time series, the roughness of many seasonal adjustments, the relatively short time span covered by most series, the marked variability of cyclical behavior. The effect upon our averages of uncertainties in identifying specific cycles is certainly no greater—as a rule it is probably smaller—than the effect of 'extreme' cycles that stand out clearly in the records, as the reader may judge by comparing the discrepancies between the averages in Table 136 with the discrepancies between averages based upon different periods or made by different methods, which appear in Tables 180, 186, 187 and 190. The influence of the period covered by a time series on average measures of its cyclical behavior is by far the most important question raised by our technique. The remaining four chapters are devoted to this intricate problem.