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STRUCTURAL ANALYSIS OF REAL CAPITAL FORMATION

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Introduction

1. The following observations are intended to supplement in a limited way the investigations, which are the center of discussion at this conference. The observations are limited in three respects. They concentrate on certain issues of real capital formation and completely disregard those of "finance" and "business organization," which are the major theme of most of the other papers. Furthermore, they are concerned with purely analytical problems of "model-building" and have no direct relationship to statistical and descriptive data. Finally, this paper deals with only one aspect of the theory of capital formation, which I call "structural." Since this term has acquired rather diverse meanings in recent writings, I had better explain what it is to signify in the context of this paper.

2. The course, persistence, or change of economic processes can be studied under two different aspects. On the one hand, there exist certain objective-quantitative relations among the components of the system—say, between effective demand and aggregate employment or between the depreciation of existing equipment and the output of capital goods. On the other hand, there are the motivations and behavior patterns of householders, firms, and productive factors, which shape the prevailing objective relations and are shaped by them. No economic analysis is complete that does not take into account the events occurring in both fields and, in particular, their interaction. But if this requirement of "total analysis" is in principle admitted, no harm arises from provisionally studying the phenomena in each field separately.

Much of the distinction between these two fields of inquiry is customarily expressed by the contrast between macroeconomics and microeconomics. But the particular point of difference—namely, between the "impersonal order" and the "personal forces"—which is

I wish to express my gratitude to Julius Wyler for a number of critical suggestions, pertaining in particular to Part 2. Moreover, his own work in the field of structural analysis, still unpublished, has provided a valuable check for some of the propositions established below.

stressed here, has little to do with the degree of aggregation, by which the microeconomic study of firm and industry is traditionally separated from the macroeconomic study of the system as a whole. It is for this reason that I prefer the terms "structural" and "functional." They are neutral to the level of aggregation, and permit the input-output relations among different industries to be considered a "structural" order, just as Leontief considers them, while the motive-behavior complex of the Soviet planning authorities appears as a "functional" problem, even though the ensuing decisions concern the system at large.

3. Among the structural problems thus defined, those which relate to the money flow (income-expenditure-saving, etc.) can be distinguished from those which relate to the physical flow of goods and services (consumer goods–capital goods; natural resources–intermediate goods–finished goods) occurring in and between different "sectors" of the economy. Whereas the former structural relations can be fully described in value terms, the latter have, in addition to the value dimension, a physical-technical dimension.

For both types of structural relations another distinction is important. Interest may be directed to the actual relations between effective demand and the level of aggregate employment, or between the output of capital goods and the level of investment, as these magnitudes appear in an empirical system in historical time. Or attention can be focused on a hypothetical order of either the money flow or the physical flow, which is required to attain a postulated state of the system, such as a certain level of employment or a particular order of distribution, or simply stationary or dynamic equilibrium. According to the viewpoint taken, structural analysis results then either in a number of empirical-statistical relations supplementing the information contained in national income accounts, or in a set of "consistency conditions."

This paper deals almost exclusively with the *physical-technical structure* of industrial systems, insofar as it affects the process of capital formation. And it interprets structural analysis in the "normative" sense, as being concerned with physical-technical *consistency conditions.* These conditions are related, on the one hand, to the sectorial order of stationary and dynamic equilibrium and, on the other hand, to the sectorial adjustment paths required for an industrial system to accomplish, under the impact of economic growth, the formation of real capital in the most "economical" manner. It hardly needs stressing that structural problems of this nature arise under any form of economic organization, individualist or collectivist. It is mainly for reasons of space that the following observations confine themselves to capital formation in a free market system. Moreover, the difference in economic organization affects the results of functional rather than of structural analysis.

4. This line of investigation has been chosen for two reasons. First, the problems encountered in its pursuit touch upon important practical issues pertaining to economic growth in both advanced and backward countries. The specific contribution that "normative" structural analysis can make to the clarification of such practical issues will be discussed at the end of this paper. Second, in contradistinction to its money flows, the physical-technical structure of an industrial economy is still largely unexplored. This is especially true of the manner in which more or less fixed coefficients of production affect the adjustment processes in such a system.

The assumption of fixed technical coefficients is basic for the subsequent exposition. Its practical importance lies in the fact that it reflects the degree of specificity of inputs and outputs. It thus describes the limits set in an industrial system to short-run aggregate expansion as well as to short-run sectorial adjustment. While the former issue is especially relevant in the early stages of industrialization, the latter bears upon the stability of fully industrialized systems. A few cursory remarks must suffice to support this view.

Originally a product of the Industrial Revolution, with its emphasis upon large-scale specialized equipment and differentiated skills, specificity of real capital and labor has varied considerably during the historical stages of industrialization. During the nineteenth century the prevailing tendency was undoubtedly in the direction of increasing specificity. This has somewhat changed during the last generation, and we may well assume that, under purely technical aspects, standardization of equipment parts, further automation, and novel methods of labor training will in the future promote greater flexibility of the industrial structure. But a new trend in economic policy seems to counteract this technological tendency. Paradoxically, during the nineteenth century the instability of a rigid structure provided its own cure, by periodically creating large pools of idle resources that facilitated adjustment and growth. An effective full employment policy is now likely to make for new rigidities. Its very success in stabilizing the structure of money flows may well aggravate the adjustment problems that the physical-technical structure poses. Thus, with all due regard to the dangers arising from insufficient demand, which are now generally admitted, inelasticity of supply owing to technical rigidities stands as another threat to the stability of advanced economic systems. At the same time, in the different "climate" in which economic development is pursued today, the physical-technical bottlenecks that hamper rapid expansion in backward regions will be felt much more strongly than under the earlier conditions of slow initial growth.

The emphasis placed here upon fixed coefficients of production and the ensuing technical rigidity of the system may not, at first sight, seem appropriate to the main topic of this paper. In the Marshallian tradition, changes in real capital are regarded as a problem for long-period or even secular-period analysis, referring to a time span over which the technical coefficients must be treated as perfectly variable. If this is admitted, are we not going to miss our very problem if we argue on the basis of fixed technical coefficients? Brief reflection will show that this apparent concentration on short-run problems, far from conflicting with the study of long-run economic growth, is an indispensable condition for understanding the latter.

Long-run analysis of economic growth describes a sequence of states of the system, which differ with respect to the quantity and/or quality of real capital. But it must be kept in mind that this sequence, except in the limiting and quite unrealistic case of steady exponential growth, is essentially discontinuous. In analogy with comparative statics it depicts successive levels of capital "formed" without regard to the intervening processes by which capital is "being formed." Now it is precisely these intermediary processes that are in the foreground of this investigation. Their systematic place in the larger context of growth analysis can easily be clarified.

First of all, these adjustment processes, through which capital formation occurs, are indeed of a short-period nature in the strict Marshallian meaning of the term. Through them, additional and possibly qualitatively different real capital is created; but this is done with the help of the initially given quantity and quality of real capital. In other words, given full utilization of the available equipment as a typical modern condition in advanced as well as underdeveloped regions in accord with what was stated above, the prevailing technical coefficients can be varied only by a process of production which is conditioned by the existing coefficients.

Second, since the technical structure of the given stock of real capital is unalterable in the short period, the degree of its specificity, and of the prevailing factor specificity generally, has a decisive influence upon the path of the adjustment process as well as upon its duration. Finally, while themselves short-run phenomena, these processes of capital formation are the links between successive stages of growth and thus transform the sequence of discontinuous states into a continuous long-run process.

5. From these considerations one cannot help concluding that the technical structure is a fundamental determinant of the behavior, and especially of the mode of change, of any economic system. Therefore, it is rather surprising that, until quite recently, the whole issue received little attention in academic economics. It is to Leontief's lasting credit that he not only devised a theoretical model for the analysis of these structural relations, but also initiated a comprehensive empirical-statistical test for his matrix, which has greatly deepened our insight into the operation of the productive mechanism. Leontief traces his own work back to Walras's model of general equilibrium. As far as the multiplicity of variables is concerned, Walras's parentage is undeniable. But to the extent to which the input-output matrix concentrates upon the interrelationship of "industries"—that is, aggregates larger than individual firms but smaller than the customary components of macroeconomics—the prototype was established by Marx in his laborious attempts to describe the processes of "simple" and "expanded" reproduction by a quantitative schema. Marx's schema is much more highly aggregated than Leontief's matrix; it distinguishes only between one group producing consumer goods and another producing means of production. But as in modern input-output analysis, Marx's interest was focused upon physical-technical interrelations in and between these two groups rather than upon the value structure of the total process, which was Walras's ultimate concern.

My further observations will be based upon a modified version of the Marxian schema. In view of the much more extensive disaggregation of the input-output model this decision might, at first sight, be likened to the use of a shovel when a bulldozer is available. And this all the more so since the Leontief model is built upon the same technological assumption of constant input coefficients that has been postulated above. If, nevertheless, the simpler model has been employed, this has been done for two reasons. The first is purely pragmatic. For the practical purpose of planning, one can hardly go too far in disaggregating an interindustry model. But this advantage of Leontief's model in all empirical concerns turns into an obstacle when it is applied to the solution of theoretical problems of a general dynamic nature. It proves just too difficult to trace analytically the path of such a large number of variables, especially if they are exposed to several stimuli simultaneously.

The second reason is substantive and more basic. All subdivisions of the productive structure are not equally important for the study of particular dynamic processes. One can, in principle, conceive of different patterns of disaggregation, each one appropriate to a specific problem. Now, with certain modifications to be explained presently, Marx's schema seems to be suited especially well to the study of real capital formation. There is an a priori presumption that the theoretical problems associated with the building up and wearing down of the capital stock, with the relation between capital stock and output flow, with the processes of "widening" and "deepening," and with the effects of innovations upon capital formation are basically the same in every industry. But their solution is bound to differ according to whether we study "capital-producing" or "capital-using" processes, a distinction which is central for the Marxian schema. I speak of an "attempt," because in its original form the schema is defective in at least three respects.

The first defect refers to the relation between capital stock and output flow. In spite of his continuous preoccupation with "capital," the equations that Marx presents in his structural analysis are meaningful only if understood as describing flows. Appropriate stock variables must be added to make the schema an analytical tool for the study of capital formation.

Another defect is that Marx's distinction between two industrial groups focuses upon fixed capital goods only. If the schema is to apply also to working capital goods as goods in process, each of the groups must be disaggregated into "vertical" stages, depicting the process by which natural resources are technically transformed into finished consumer or equipment goods.

Finally, certain essential "circular" processes can be clearly described only if the equipment goods group is further disaggregated into one subgroup which produces the equipment for the consumer goods group, and another subgroup which produces the equipment for both subgroups of the equipment goods group.

This is, in its most general features, the model I shall use as a tool for the study of real capital formation under the impact of growth.[1] I shall begin with a brief analysis of the structural conditions that determine stationary equilibrium. Then follows an exposition of certain dynamic relations: first, as they arise under the impact of once-over changes; second, as they take shape under continuous change. In the latter category the structural conditions of a constant rate of change, viz. dynamic equilibrium, are distinguished from those of varying rates of change with more complicated adjustment paths, as they emanate, for example, from non-neutral technical changes. During the exposition itself little will be said about the practical relevance for advanced and backward countries of the problems discussed. A brief conclusion will suggest possible applications of this analytical technique.

1. *Structural Conditions of Stationary Equilibrium*

GROUP MODEL AND STAGE MODEL

6. We start out from an elementary set of relations describing the flow of production over the period t:

$$(1)\qquad \begin{aligned} (F_a \cdot d_a \equiv f_{at}) \times n_{at} \times r_{at} &\longrightarrow a_t \\ (F_b \cdot d_b \equiv f_{bt}) \times n_{bt} \times r_{bt} &\longrightarrow b_t \\ (F_z \cdot d_z \equiv f_{zt}) \times n_{zt} \times r_{zt} &\longrightarrow z_t \end{aligned}$$

At this stage of the argument we deal only with physical magnitudes. Therefore, the relationship between inputs and outputs is expressed as no more than a causal nexus, symbolized by arrows. Similarly, the multiplication signs stand for the technical combination of the input factors in fixed proportions.

To the right of the arrows, a signifies the output of a units of equipment goods which are intended to make equipment goods, henceforth called "primary equipment," whereas b signifies the output of b units of equipment goods intended to make consumer

[1] The model has been described in greater detail in my paper "A Structural Model of Production," *Social Research*, June 1952, pp. 135-176. There reference is also made to the earlier literature on the subject, including a critical comparison between the Marxian concept and the so-called Austrian concept of a "linear" structure of production that underlies much of modern theoretical reasoning in economic dynamics. In view of the subsequent application of the model to dynamic problems, Hans Neisser's *Some International Aspects of the Business Cycle* (University of Pennsylvania Press, 1936), Appendix to Chapter I, deserves special mention.

goods, henceforth called "secondary equipment"; z denotes the aggregate output of z units of consumer goods.[2]

To the left of the arrows are the corresponding inputs for each of the three groups, the relevant group being specified by a subscript. These inputs are subdivided into the basic factors of production: fixed capital goods, f; labor, n; and natural resources, r. For the input of fixed capital goods two expressions are given: one, denoted by f, is a direct expression of the input flow entering the corresponding output flow; the other expresses the same magnitude in terms of the existing stock of fixed capital goods, F, multiplied by the prevailing rate of depreciation, d. In principle, similar stock magnitudes could be added to the flow expressions for the other two factors, and they are quite useful for the study of certain dynamic problems. In a study of the dynamics of capital formation they can be disregarded. (Whenever stocks appear in our models and equations, they are symbolized by capital letters, whereas flows are described in lower-case letters.)

7. In the above form the group relations are the result of a far-reaching aggregation. Each group describes the output of a given period in terms of *finished* goods, ready for use either as means of consumption in the households or as means of production in the firms. These outputs of finished goods are the technical result of the productive process, which transforms natural resources with the help of labor and equipment goods. This process of transformation can, and for the solution of certain problems must, be disaggregated into a number of "vertical stages." A second set of relations describes such a stage model for the group of consumer goods:

$$
(2) \qquad
\begin{array}{l}
(F_{z_1} \cdot d_{z_1} \equiv f_{z_1 t}) \times n_{z_1 t} \times r_{z_1 t} \longrightarrow w_{z_1 t} \\
(F_{z_2} \cdot d_{z_2} \equiv f_{z_2 t}) \times n_{z_2 t} \times r_{z_2 t} \times w_{z_1 t} \longrightarrow w_{z_2 t} \\
(F_{z_3} \cdot d_{z_3} \equiv f_{z_3 t}) \times n_{z_3 t} \times r_{z_3 t} \times w_{z_2 t} \longrightarrow w_{z_3 t} \\
(F_{z_4} \cdot d_{z_4} \equiv f_{z_4 t}) \times n_{z_4 t} \times r_{z_4 t} \times w_{z_3 t} \longrightarrow z_t \\
\hline
(F_z \cdot d_z \equiv f_{zt}) \times n_{zt} \times r_{zt} \longrightarrow z_t
\end{array}
$$

[2] In order to simplify the subsequent exposition the physical distinction among the outputs of the three groups is treated as absolute. This is, of course, not so in reality. Certain products, and the industries producing them, belong in more than one group (e.g. coal, steel, even certain machine tools), though in every concrete instance one can always determine where a specific commodity should be placed. For a more refined exposition see my paper, *op. cit.*, pp. 144-146.

This model is based on the assumption, to be examined more closely, that the technical process of production by which the natural resource r_{z_1} is transformed into the finished consumer good z can be subdivided into four stages. The outputs w (working capital goods) of each stage appear as inputs in the subsequent stage, down to the "stage of completion," whose output is the finished good. The total for the inputs of each factor in all stages must be equal to the consolidated expression that appears in model 1 and is restated at the bottom of model 2.

8. What precise meaning can be attached to the notion of a *technical* sequence of stages of production? There is no doubt that, from the point of view of business organization, a number of successive interfirm exchanges can be distinguished where the buying firm uses its purchases for further manufacturing. Though useful as an indicator of business differentiation and also important for the solution of the "transaction problems" in the theory of money, from the technical point of view separate stages of production are arbitrary. Only for the first stage, where the "gifts of nature" are "seized," and for the last stage, when the finished product is handed over to the prospective user (or speculator), can a definite meaning be attached to such a distinction.

The fact that from a technical point of view the production flow is indivisible has always been recognized in attempts to find a quantitative expression for the stock and the flow of working capital goods. The total of all "stage outputs," which can be derived from any given order of business differentiation, cannot be used for this purpose, since it contains unavoidable double counting. Therefore, proper measures for working capital can be established only by referring to factor inputs.[3] But if we are interested in the manner

[3] Any such measure presupposes a genuine summation of all inputs, which itself is conditional upon the comparability of physically different inputs in a homogeneous value dimension. A simple measure can be devised if we assume that the inputs of fixed capital goods, labor, and natural resources are evenly distributed over all stages as given by the state of business differentiation. Denoting aggregate input of all factors over a stated period by i, the *flow* of working capital goods which move during that period in the vertical direction toward completion equals $i/2$, since under the conditions assumed half the factor input must serve the replacement of the working capital used in each stage. If the inputs are unevenly distributed over the stages, the fraction $i/2$ changes to i/q, where $0 < q < 1$. The *stock* of working capital goods, on the other hand, which must be maintained in the interest of continuous production is then im/qp, where p records the period of observation over which we measure i, and m expresses the "period of maturation," that is, the time it takes with given technology to transform a unit of natural resources into a finished good.

in which a *given* order of business differentiation is affected by vertical shifts of factors—because of, for example, alterations in physical returns or nonneutral technical changes—a stage model of the kind described in model 2 is a useful analytical tool. The particular problems of capital formation discussed below refer to fixed capital only, so that the stage model can be disregarded in favor of the consolidated group as formulated in model 1 (see, however, section 19 below).

9. This conclusion is likely to meet with strong objections from those who conceive of the whole productive process in "linear" fashion. A schema like that described in model 2 underlies the so-called Austrian concept of the structure of production. Originally devised by Eugen v. Boehm-Bawerk, it was introduced into the Anglo-American tool chest by Wicksell and his followers, especially Hayek, eclipsing the much sounder notions of J. B. Clark.[4] The discussion ended in stalemate with the well-known controversy between F. H. Knight and Nicholas Kaldor,[5] to the complete disregard of the work of F. A. Burchardt,[6] in which a happy synthesis between a "circular" and a "linear" model had been achieved.

The point at issue is the place of fixed capital goods in the structure of production. Because they are the result of the productive process, they cannot be treated as data side by side with labor and natural resources. On the other hand, the attempt to "dissolve" their contribution into inputs of labor and natural resources fails since, to make fixed capital goods, other fixed capital goods are needed in addition to labor and natural resources. Therefore it is not possible to treat fixed capital goods as the output of some intermediary stage in the vertical model, as Boehm-Bawerk and his followers have suggested. In other words, all attempts to describe the process of production in purely linear fashion, tracing all finished

In particular, the last concept will prove useful when the process of capital formation is studied more closely. For a generalization of the above results see Julius Wyler, "Working Capital and Output," *Social Research*, Spring 1953, pp. 91-99.

[4] See his *Distribution of Wealth*, Macmillan, 1926, Chaps. XVIII-XX.

[5] Nicolas Kaldor, "Annual Survey of Economic Theory: The Recent Controversy on the Theory of Capital," *Econometrica*, July 1937, pp. 201-233; F. H. Knight, "On the Theory of Capital: In Reply to Mr. Kaldor," *Econometrica*, January 1938, pp. 63-82; and Nicholas Kaldor, "On the Theory of Capital: A Rejoinder to Professor Knight," *Econometrica*, April 1938, pp. 163-176.

[6] "Die Schemata des stationaeren Kreislaufs bei Boehm-Bawerk und Marx," *Weltwirtschaftliches Archiv*, Kiel, Vol. 34, 1931, pp. 525-564, and Vol. 35, 1932, pp. 116-176.

goods technically back to nothing but man and nature, not only are unrealistic but involve an infinite regress.

A solution of this apparent paradox that is in accordance with both facts and logic is possible only if it is recognized, as Marx and J. B. Clark did many decades ago, that fixed capital goods are replaced and multiplied by a process of physical self-reproduction analogous to the maintenance and increase of the stock of the organic factors of production: men, animals, and plants. Not all fixed capital goods have this technical capacity. It is the characteristic of a particular group, called machine tools. In conjunction with one another and with labor and natural resources (in the form of special working capital goods), these tools are capable of making other equipment as well as their own kind. This circular process raises group *a* to a strategic position in the technical structure of every industrial economy, whatever its social organization. In other words, group *a* is the bottleneck which any process of rapid expansion must overcome, a problem which a linear concept of the structure of production cannot even locate, let alone study, in substantive terms.

10. Returning now to the original group schema described in model 1, we can transform it into a set of equations by interpreting its variables (with the exception of *d*) as price-sum magnitudes pertaining to the output of three physically distinct aggregates. Such a "two-dimensional" determination enables us to establish certain equilibrium conditions of a continuing stationary process, and to express them in marketing terms as well as in physical-technical terms.

For the system to continue in stationary equilibrium, the outputs of period t_1 must become the inputs of period t_2. Thus we have

$$(3) \qquad \begin{aligned} a_{t_1} &= f_{at_2} + f_{bt_2} \\ b_{t_1} &= f_{zt_2} \\ z_{t_1} &= n_{at_2} + n_{bt_2} + n_{zt_2} + r_{at_2} + r_{bt_2} + r_{zt_2} \end{aligned}$$

In words, the primary machinery produced during period t_1 reappears physically in period t_2 as the fixed capital goods operating in the two equipment goods groups, whereas the secondary machinery b_{t_1} becomes the fixed capital goods operating during period t_2 in the consumer goods group. By this physical application, the outputs of both primary and secondary machinery replace the wear and tear of equipment which occurred in the act of their own production as well as of the simultaneous production of consumer goods. In the same manner, the consumer goods output during period t_1 can be said to "replace" the "wear and tear" of the prime factors N and R—in our context a helpful interpretation of the stationary income claims of their owners—and to serve their maintenance over the period t_2.

To bring about the proper physical "relocation" the three groups must behave in the manner of countries involved in a triangular exchange relationship. The whole output of secondary machinery of group b moves to group z in exchange for an equivalent amount of consumer goods. Part of these "imports" into group b are used to feed the prime factors employed there, whereas the rest are "re-exported" to group a for primary equipment required to replace f_{bt_1}. The "imports" of consumer goods into group a go to the prime factors of this group, whereas the wear and tear of its equipment f_{at_1} is replaced from its own output at_1. In the same manner the prime factors employed in group z are fed out of their own output of consumer goods.

The fact that, as in international trade, these exchanges of non-substitutable goods occur through the marketing actions of individual firms and households in no way reduces the importance of the structural relations among the groups at large. Their significance for macroeconomic analysis is analogous to that of balances of payments in international exchange. The equilibrium conditions of a stationary flow can then be expressed in the following, more comprehensive form:

$$(4) \qquad \begin{aligned} f_{zt_2} &= b_{t_1} \\ &= z_{t_1} - n_{zt_2} - r_{zt_2} \\ &= n_{at_2} + n_{bt_2} + r_{at_2} + r_{bt_2} \end{aligned}$$

and

$$(5) \qquad \begin{aligned} f_{bt_2} &= a_{t_1} - f_{at_2} \\ &= z_{t_1} - n_{bt_2} - n_{zt_2} - r_{bt_2} - r_{zt_2} \\ &= n_{at_2} + r_{at_2} \end{aligned}$$

In words, the secondary machinery required per period in group z must equal, physically and in value terms, the output of such equipment produced previously in group b. But it must also equal in value terms a definite amount of consumer goods, namely, the total previous output in group z minus that group's present requirements, a difference which must equal the amount presently demanded by all income-receivers in groups a and b. Furthermore, the primary machinery required per period in group b must equal, physically and in value terms, the surplus of such equipment produced previously in group a over and above that group's own present requirements, and must also equal the value of consumer goods at present demanded by the income-receivers in group a. This latter magnitude, in turn, must equal the total previous output in group z minus the present requirements of consumer goods in both group z and group b.

From these relations a simple inequality can be derived which defines in structural terms the nature of all dynamic processes in the sense of growth or decline of the system. In such a situation there is for any period during which aggregate change occurs:

$$f_z \lesseqgtr n_a + n_b + r_a + r_b$$

$$f_b \lesseqgtr n_a + r_a$$

Model 1 and equations 3 to 5 tell us all that is relevant for the group structure in a continuing stationary process. Not only do they describe the physical transformation of inputs into outputs and conversely, but they also express the changing meaning of the corresponding money flows in subsequent periods. In model 1 interpreted as equality between input values and output values, the n, r, and f variables must be understood not only as costs to the firms, but also as the money receipts that accrue, in the form of income and amortization, to the holders of the respective stocks. In equations 3 to 5 the same symbols represent the expenditures on consumer and equipment goods respectively. On the other hand, the a, b, and z variables in model 1 denote receipts from sales, while they measure aggregates of expenditure in the later equations.

This "ambiguity" has a parallel in the Keynesian model, which also has a "supply" as well as a "demand" meaning. In the first interpretation Keynes deals with aggregate output, consumer goods output, and investment goods output. His second interpretation refers to aggregate income, divided into one part spent on consumer goods and another part called savings, which is spent on investment goods. Far from prejudicing analytical clarity, this change of meaning of the basic variables in successive transactions only emphasizes the circular nature of the exchange process.

STATIONARY GROUP RELATIONS IN TERMS OF CAPITAL COEFFICIENTS

11. Our next step is to derive some analytical tools, with the help of which the "group ratios"—that is, the relative size of the three groups and their components in any given state of the system—can be expressed. For this purpose certain concepts relating to a system's capital structure must be defined.

a. "Total value productivity" is defined as the unit cost of a given output:

$$\epsilon = \frac{o_v}{i_v}$$

that is, output value over input value. "Labor productivity" is then consistently defined as

$$\epsilon_n = \frac{o_v}{n_v}$$

that is, output value over payrolls; and "capital productivity" as

$$\epsilon_o = \frac{o_v}{f_v}$$

that is, output value over replacement value.

For problems other than those discussed in this paper, the relation of input value to output *quantity* is a preferable measure. This is especially true of all investigations having to do with the effect of technical changes on physical productivity, a problem with which we are not concerned here. With technology constant, the two measures are, of course, identical.

In recent studies it has become customary to express capital productivity by the ratio o/F—that is, output (quantity or value) over the value of the capital *stock*—or the reciprocal measure. However convenient these measures are for practical purposes, three objections must be raised against the use of a stock magnitude in the definition of productivity. First, while there is a corresponding measure for labor productivity (o/L, that is, output over the number of workers in man-hours), there is obviously no symmetrical measure for total productivity. Second, the same capital stock may yield different outputs according to the prevailing rate of depreciation. This is the reason why the above measure o_v/f_v explicitly introduces the rate of depreciation, f_v being equal to $F \cdot d$. Third, by combining a flow magnitude with a stock magnitude, we introduce a time dimension into the measure of productivity, a dimension which is happily absent when two flow magnitudes with the same time coefficient are combined.

b. Our second concept is "capital depth" in the sense in which the term was first used by Hawtrey. This concept we define by the ratio

$$k = \frac{F_v}{o_v}$$

that is, the value of the capital stock over the value of output.[7] The measure has a time dimension and can therefore be applied only to clearly defined periods of observation. As in the definition of productivity, it is possible to relate the quantity, rather than the value, of output to the value of capital stock. Accordingly, "value capital depth" can be distinguished from "quantity capital depth." Throughout this paper k refers to value capital depth.

Combining the measure for capital depth with the rate of depreciation, we have the reciprocal of the measure proposed above for capital productivity:

$$kd = \frac{F}{o}d = \frac{f}{do}d = \frac{f}{o}$$

This coefficient kd will be a principal tool in the subsequent structural analyses.

c. Our third concept is "capital intensity," which is the ratio

$$c = \frac{F_v}{n_v}$$

[7] If we were to add to the value of the capital stock the value of the *working* capital stock, the above measure would change to

$$k_{f+w} = \frac{F_v}{o_v} + \frac{W_v}{o_v}$$

Since, as shown in footnote 3 above,

$$W_v = \frac{im}{qp}$$

we obtain by equating the period of observation with the period of maturation

$$k_{f+w} = k_f + \frac{1}{q}$$

or the value of the capital stock over the value of payrolls of the workers operating it over a stated period. Having also a time dimension, the measure is formally analogous with capital depth. It will prove important in the analysis of labor-displacing and labor-attracting technical changes.

12. The coefficient *kd*, the reciprocal of capital productivity, will now be used for a reformulation of the structural relations which prevail among the three strategic groups in a stationary process. To simplify the exposition, I shall from now on drop the variable *r* from the set of input factors, assuming either that natural resources are "free gifts" or that the factor *n* stands for all income-receiving factors. The latter assumption disposes also of any discussion of how to treat interest in a stationary system.

As a first approximation, capital depth coefficients and depreciation rates are taken to be the same in all groups. The sum of outputs $a+b+z$ over the given period is denoted by *o*. The total capital stock of the system can then be expressed by *ok*, and the flow of depreciation over the stated period by *okd*, to which the condition attaches that $k < 1/d$; *okd* is equal to $a+b$, that is, the output of equipment goods in the given period. This yields

$$z = o - (a+b) = o(1-kd)$$

$$b = f_z = F_z d = o(1-kd)kd$$

$$a = o - (z+b) = ok^2d^2$$

Therefore,

$$z : b : a = 1 : kd : \frac{k^2d^2}{1-kd} \tag{6}$$

which also measures the ratio of the three capital stocks

$$F_z : F_b : F_a$$

With unequal *k*'s and *d*'s for the three groups, the expression changes to

$$z : b : a = 1 : k_zd_z : \frac{k_zd_zk_bd_b}{1-k_ad_a} \tag{7}$$

and

$$F_z : F_b : F_a = 1 : k_bd_z : \frac{k_ad_zk_bd_b}{1-k_ad_a} \tag{7a}$$

Thus once the capital depth coefficients and depreciation rates for the respective groups are known, the relative share of the output of the groups in the gross national product and the relative size of net national income, which under stationary conditions coincides with the output of consumer goods, are determined. So is the ratio of the capital stocks, and the distribution of prime and supplementary factors over the groups.

If a stationary process is studied, all capital problems are reduced to replacement problems, that is, to the maintenance of the existing ratios among the three groups. When we turn to the analysis of certain dynamic processes, the notions of both "group ratios" and "intergroup equilibrium conditions" will prove useful in dealing with the concomitant problems of capital formation.

2. *The Dynamics of Once-Over Changes*

13. This is not the place to undertake a critical review of all the definitions of economic "dynamics" that have been put forth in recent years (Frisch, Hicks, Samuelson, Harrod). In what follows the term includes any economic process which is exposed to change, whether bipolar as in the case of a shift in tastes, or aggregate as under conditions of growth or decline of the system as a whole. For the purpose of this paper bipolar changes are disregarded. Among aggregate changes it is convenient to distinguish between once-over and continuous changes.

Harrod defines a once-over change as a single act of change in one or more of the data of the system, such as an increase in labor supply owing to the influx of a certain number of immigrants at a given point of time, or the introduction of a particular technical improvement.[8] After the absorption of such a once-over change the system is supposed to operate again under the previously prevailing (zero or constant) rate of data changes. Unlike Harrod I regard such once-over changes as legitimate problems for dynamic analysis. In the first place, far from dealing with "trivial matter" "satisfactorily to be handled by the apparatus of static theory" (*ibid*, p. 7), all the practically relevant cases of once-over changes occur in the framework of otherwise continuous change, and the interplay between the two types of change can hardly be studied in terms of comparative statics. Second, and more important, a description of the adjustment path that the system pursues under the impact of once-over changes

[8] R. F. Harrod, *Toward a Dynamic Economics*, Macmillan, 1948, Lecture One.

points up particularly well the structural problems of capital formation, which beset all dynamic processes operating under any rate of change other than constant.

I shall concentrate upon one particular type of once-over change, since the basic problems can best be thrown into relief by an isolating analysis. For this purpose a change in labor supply will be selected and the results of the analysis supplemented by a few remarks on "neutral" changes in productivity.

To highlight the issues relating to capital formation, severely restrictive assumptions have been made. The system is assumed to move initially in stationary equilibrium under conditions of pure competition, with firms of equal size operating in continuous production under minimum average costs. Constant returns are to prevail on all natural resources, which are treated as free goods, and the capital depth coefficients and rates of depreciation are to be equal in all groups.

All these conditions can easily be relaxed. There is also no difficulty in analyzing a simultaneous change in labor supply and productivity, as will be indicated below. It is even more important to realize from the outset that the results, though gained within a stationary framework, are fully applicable to a dynamic framework. In other words, changes in a positive rate of change can be analyzed by "superimposing" our results upon a process in dynamic equilibrium with a steady rate of growth.

A ONCE-OVER CHANGE IN LABOR SUPPLY

14. Our stationary equilibrium is supposed to be disturbed by an increment a of labor supply, expressing the potential increment Δn_v in the value of labor effort as a fraction of the value of the stationary labor effort n_v per period of observation. Our problem, then, concerns those aggregate changes of, and structural shifts among, the three groups which describe the "optimal" path for the absorption of the increment or, what amounts to the same thing, restore in the most economical manner stationary equilibrium on a higher level of input and output.

It may be advisable to emphasize once more that the concomitant "functional" problems—namely, the particular behavior patterns and motivations of the actors in the market that are the condition for the realization of the structural adjustment—are not discussed in this paper. One fundamental functional assumption referring to savings and investment will be stated below. For the rest little can be said about the functional aspects of dynamic processes before the structural conditions have been clarified.[9]

There are, in principle, two ways in which a labor increment can be absorbed: either by utilizing the existing equipment at more than its optimum intensity—in other words, by expanding output all through the system beyond the point of minimum average costs through a change in factor proportions in favor of labor—or by building new equipment. In accordance with my basic assumption of fixed technical coefficients, I shall concentrate upon the second alternative.[10]

15. In order to provide the labor increment with working places and real income, the stationary factors must perform a temporary act of net saving and net investment. Its monetary aspect, in which we are not interested here, consists in the transfer of appropriate purchasing power from the stationary income-receivers to the entrepreneurs, by an act of voluntary saving, by credit inflation, or even by a general fall in wages resulting from the increased competition in the labor market.[11] Its real aspect, in which alone we are interested, consists in all three cases in the displacement of factors in the consumer goods group, factors which—given the required mobility—can be transferred to the equipment goods groups for the purpose of expanding the output of equipment. As a first approximation, it is assumed that such mobility exists for labor, so that a weaver can in the very short run be used as, say, a steel worker. However, the whole problem to be studied would be eliminated if the same notion of mobility were applied to fixed capital goods. I therefore assume that the displaced looms cannot be used in steel-

[9] The ultimate reason for the dependence of functional analysis upon structural analysis lies in the fact that all dynamic behavior patterns are shaped by expectations, which in their turn are related to the prevailing structure. For a more detailed exposition see my paper "On the Mechanistic Approach in Economics," *Social Research*, December 1951, pp. 403-434.

[10] Even if some short-period variability of factor proportions is admitted, the concomitant rise in user costs provides "a motive for increasing the stock of equipment." See J. R. Hicks, *A Contribution to the Theory of the Trade Cycle*, Oxford, Clarendon Press, 1950, pp. 39-40. Therefore, for all but very small increments in labor supply the alternative chosen above seems the only realistic one.

[11] Whether, and under what conditions, such a decrease in wages frees business funds and thus provides the equivalent of savings, or rather leads to a proportional price fall that leaves real incomes unchanged, is a functional problem whose answer clearly depends on prevailing expectations. The same is true of the effect that such a fall in wages has upon the capital depth of the subsequent investment.

making, the expansion of which forms part of the process of real capital formation.

16. This process of equipment-building requires closer examination in accord with what was said above about the technical relationship between primary and secondary machinery. Obviously, once the total labor increment has been finally absorbed, all three groups will have expanded by a rate equal to α under our assumption of unchanged factor proportions. But since under the same assumptions there is no reserve capacity in the field of primary machinery, the addition to secondary machinery presupposes the prior expansion of the output of primary machinery. We saw above that it is the circular nature of production in group *a* which makes such expansion possible. The fall in output of consumer goods has freed capacity not only in the respective group *z*, but also in group *b*, which supplies the replacement of secondary machinery, and in group *a* itself, which provides the replacement of capacity in group *b*.

It is this freed capacity in groups *a* and *b*[12] which forms the nucleus for the "self-expansion" of primary machinery and thus for the "widening" of productive capacity in all three groups. And it is in the gradual self-expansion of group *a* that the prime factors initially displaced in groups *b* and *z* will have to find employment during this first phase of adjustment. The second phase—namely, the process of net absorption—begins when output in group *a* has risen to a point that permits aggregate employment to rise above the stationary level. This then induces the gradual expansion of groups *b* and *z* beyond the level of employment and output to which saving on the part of the stationary factors had reduced them originally, to the new equilibrium level.[13]

[12] Since the physical goods used as primary machinery operate in some instances also as secondary machinery (e.g. extracting machinery and steel mills providing the material for certain household articles or transportation services, and even machine tools "shaping" the final consumer good), the fall in real consumption also "frees" some capacity in group *z* which can be utilized for the expansion of group *a*. We shall soon see that the size of the freed capacity in groups *a* and *b* can be easily determined. To do so for group *z* presupposes a detailed knowledge of the income elasticities of demand. For this reason the issue is disregarded in what follows, an omission that gives the subsequent conclusions as to "waste" and as to the length of the construction period a slightly "pessimistic" bias.

[13] To clinch the argument I would have to demonstrate the complementary functional processes in a step-by-step analysis. These processes refer, above all, to the mechanism that transfers aggregate savings to the entrepreneurs in group *a*; to the consequent attraction of idle prime factors to that group; to

This sketchy description of the process of real capital formation under the conditions assumed forms the background for the structural analysis to follow. More specifically, it will now be our task to formulate, in terms of a minimum number of independent variables, certain strategic relationships which characterize the process of capital formation. They refer to the dynamic group ratios, to the danger of "overbuilding" the capital stock, and, above all, to the length of the "period of construction."

The independent variables, which are to serve as data for our analysis, are four. Two of them—namely, k (capital depth) and d (depreciation)—are directly related to the size of the capital stock. The other two are α, the rate of growth of the labor increment, and s, the ratio of aggregate planned savings to aggregate income in the various phases of the adjustment process. Since in our "normative" model, planned savings always equal planned investment, s also measures realized savings and investment.

Group Ratios

17. The structural shifts occurring during the process of capital formation can best be read in the variation of the group ratios. Not all the possible variations will be discussed at this point. There is, however, one intermediate phase in the adjustment process that requires closer examination. It concerns that stage of expansion when an amount of prime factors, equal to the amount originally displaced in groups b and z, has found re-employment in group a. Or, as we can also say, it describes the maximum expansion

the "multiplier" process by which groups b and z gradually expand, once expansion of group a begins to raise aggregate employment and income above the stationary level; and, last but not least, to the expectational conditions, upon which the postulated behavior patterns depend.

Moreover, a definite pattern of saving and investment must be postulated. The pattern assumed here requires constant (ex ante) ratios equal for savings and investment up to the point where the stationary level of employment is again reached; from there on, as absorption proceeds, the savings ratio must fall in proportion to the fall in investment demand, which, with the completion of absorption, reaches zero. It has been stated above that groups b and z will not expand before employment in group a has been increased to the point where aggregate employment equals again the stationary level. This conclusion follows from the premise that the (ex ante) savings and investment ratios are equal and constant during the first phase of the adjustment process.

If some of these postulates appear rather unrealistic, the reader should remember that we are not concerned here with the description of empirical processes, but with the analysis of the conditions for "optimum" adjustment within the framework of a free market system.

of group a which, with a given savings ratio, can be achieved with the available amount of stationary prime factors. Output at this stage, denoted by the subscript $_{00}$, compares with the stationary magnitudes denoted by $_0$ as follows:

$$(8)\qquad \begin{aligned} z_{00} &= z_0(1-s) = o_0(1-s)(1-kd) \\ b_{00} &= f_{z_{00}} = z_0(1-s)kd = o_0(1-s)(1-kd)kd \\ a_{00} &= o_0 - (z_{00} + b_{00}) = o_0[1-(1-s)(1-k^2d^2)] \end{aligned}$$

Therefore,

$$(9)\qquad z_{00} : b_{00} : a_{00} = 1 : kd : \frac{k^2d^2(1-s)+s}{(1-s)(1-kd)}$$

It agrees with common sense that group a expands with the size of the savings ratio and the capital depth coefficient and rate of depreciation of the original capital stock. We saw above that expansion of group a presupposes that savings "free" part of the available capacity of primary equipment for the purpose of self-reproduction. The variables k and d determine the potential range within which capacity can be "freed," and s determines the size of capacity actually freed within that range. In this manner it is the savings ratio which fixes, within the limit of the technical variables k and d, the point of maximum expansion of group a in the sense defined, that is, before net absorption starts. In an empirical free market, in which the (ex ante) savings ratio is the result of many independent decisions, this may lead to the overbuilding of the capital stock and subsequent waste of part of the addition to primary equipment which is produced during the phase of expansion. We encounter here a possible conflict between the goal of economizing resources and the institutional order of a free market system.

The Overbuilding of Primary Equipment

18. The problem referred to is customarily dealt with under the heading of the acceleration principle. In particular, the older "accelerationists" (Aftalion, J. M. Clark) felt troubled about the "waste" of equipment stock, which seemed to them bound up with any decline in the rate of increase in consumption. Our example of an isolated, once-over change provides a good test for this proposition, since after a single act of expansion the rate of growth of consumption falls back to zero.

The foregoing analysis supplies all the necessary tools for such a test. First of all, it tells exactly where the alleged overbuilding will occur. Since, as the above group ratios show, the demand for secondary equipment always moves in proportion to the demand for consumer goods, the critical sector can only be group a, the current output of which, after the expansion, may indeed exceed the final *replacement* demand in both group a and group b. Second, we can calculate in terms of our independent variables the size of the additional primary equipment required in the new state of equilibrium, as well as the size of the actual supply of such equipment at the point of maximum expansion described by equation 8.

The first magnitude, primary equipment stock required (StR_{a+b}), can be ascertained in the following manner. *Total* additional investment—that is, the increment of both primary and secondary equipment—must amount to

$$StR_{a+b+z} = ako \tag{10}$$

Of this addition to total capital stock an amount of

$$StR_{a+b} = ak^2do \tag{10a}$$

must consist of primary equipment, equivalent to the replacement demand for both primary and secondary equipment.

The second magnitude, net addition to primary equipment stock actually supplied in advance of absorption (StS_a), can be calculated on the basis of equation 8. It amounts to the total stock of primary equipment in operation at the maximum point of expansion as defined above, minus the stationary stock of primary equipment. Or in symbolic language:

$$\begin{aligned} StS_a &= (a_{00} + b_{00} - a_0 - b_0)k \\ &= sk(1 - kd)o \end{aligned} \tag{11}$$

Potential waste W of primary equipment built during the process of expansion then amounts to

$$\begin{aligned} W &= StS_a - StR_{a+b} \\ &= [s(1 - kd) - akd]o \end{aligned} \tag{12}$$

We can generalize this result by dropping the assumption that the coefficients of capital depth and depreciation valid for the new capital stock are equal to those of the stationary stock. By denoting the new coefficients as k' and d', we obtain

(12a) $$W = [sk(1 - kd) - \alpha k'^2 d']o$$

It is obvious that the expression within the brackets can, but need not, be positive. The result depends on the relative size of the variables s, k, and d, on the one hand, and α, k', and d', on the other. A high ex ante savings ratio, which under our assumptions is identical with a high "investment ratio," and a high capital depth coefficient of the original equipment make for waste, whereas a high rate of growth coupled with a high capital depth coefficient of the equipment increment counteract it. Depreciation rates for both original and additional capital stock affect waste inversely.

Assuming a savings-investment ratio of 10 per cent, and on both sides capital depth coefficients of 3 and depreciation rates of 10 per cent, a rate of once-over growth in the neighborhood of 25 per cent would avoid any waste. This example assumes, in accord with our model, that all planned savings are actually invested in the service of a *change* in the rate of growth. In any empirical market system observed over the last century, a large part of s has always been needed to sustain a positive *constant* rate of growth, and it is hardly surprising that Kuznets and Tinbergen have found little empirical evidence for any investment waste.

This is not the place to dwell upon the flaws which mar the conventional exposition of the process of "overbuilding."[14] Apart from faulty notions about the structure of production, which obscure the process of expansion, the pessimistic conclusions arise from the assumption that the entire addition to equipment must be built in one arbitrarily chosen period of construction, an assumption that implies a fantastically high savings ratio. In reality the independent variable is not the construction period, but the savings-investment ratio in conjunction with the coefficients of capital depth and depreciation and the period of maturation of the capital increment (see section 19 below). In other words, in principle the choice is between speedy adjustment by means of a high savings-investment ratio involving the danger of waste, and slow adjustment by a low savings-investment ratio. The latter alternative keeps the building of capital stock in line with subsequent replacement demands, though at the price of making the consumer "wait" longer for the fruits of investment. Of course, in practice the choice between these two alternatives

[14] See, e.g., Gottfried Haberler's *Prosperity and Depression*, Columbia University Press, 1946, Chap. 3; pars. 17-24, esp. par. 19.

is open only to a planned system, which can manipulate s in such a manner that, with given k, k', d, and d', it just satisfies the requirements of a. In a free market system, in which none of these variables is subject to over-all control, overbuilding, though by no means inevitable, is a possible danger.

The Period of Construction

19. The assertion that the appearance or nonappearance of waste is related to the size of the savings ratio, deserves further investigation, which will also supply us with an exact measure of the minimum period of construction for the capital increment.

Given a system in which labor and equipment are fully utilized, how long will it take to produce an additional unit of a consumer good? Under the assumption of full utilization of the available capital stock, a prior increase in the output of primary and secondary equipment is the condition for an increase in the output of consumer goods. Thus in all three groups some units of natural resources must undergo the process of transformation into a finished good described in our stage schema (model 2). The total construction period that must elapse before the additional consumer good is available consists then of the sum of the "maturation periods," m_a, m_b, and m_z, required in the three groups to move the respective natural resources down to the stage of completion.

The notion of a "period of maturation" was introduced in discussion of the concept of a stock of working capital goods (section 8). With a given technology, m is an empirical constant which of course differs not only from group to group but from industry to industry. Therefore, if we want to apply this concept to a general process of expansion rather than to the increase in output of a single consumer good, it seems that we must introduce the notion of an "average" period of maturation, a concept that would be difficult to establish empirically. In fact, however, what is needed in order to measure the period of construction is the *longest* rather than the *average* period of maturation in each group, because the new equilibrium cannot be attained before the good with the longest period of maturation reaches the state of completion. It does not appear impossible to ascertain empirically such "maximum" m's.

These considerations seem to yield a measure for the period of construction (PC) required to expand the output of consumer goods in a system in which initially all resources are fully utilized. We obtain[15]

$$PC = m_a \text{ max.} + m_b \text{ max.} + m_z \text{ max.}$$

This measure is based entirely upon empirical technical constants without any reference to economic variables such as s. However, if examined more closely, the expression is defective in two respects.

First of all, we have so far implicitly assumed that the initial stock of primary equipment necessary to expand such output is fully available from the outset. Our exposition of the expansion process (section 16) indicated that this is not so. The initial stock of primary equipment must be "freed," and, as was shown above, the amount "freed" depends upon the fall in replacement demand, which in its turn is determined by the prevailing savings ratio. Thus under the conditions assumed—with real capital formation throughout the system—the actual period of maturation in group a exceeds m_a max. as defined above. It is a multiple of m_a max., the multiplicand g measuring the number of maturation periods required to expand the equipment stock "freed" (StF) to the size of the equipment "required" for the subsequent over-all increase in output (StR).

Secondly, a multiplicand has also to be added to m_b max. Once the required addition to the stock of primary equipment is available, it has to be used for maintaining itself and, above all, for raising the stock of secondary equipment to the required level. The latter aim could be achieved in *one* maturation period only if the size of the additional primary equipment were geared to the continuous *increase* rather than to the current maintenance of the additional secondary equipment. For any size of the additional primary equipment smaller than indicated, a period of $j \cdot m_b$ max. is required to construct the addition to secondary equipment.

The true value for the period of construction is then

$$PC = g \cdot m_a \text{ max.} + j \cdot m_b \text{ max.} + m_z \text{ max.} \tag{13}$$

and we have now to determine the magnitudes of g and j.

[15] Since small variations of the initially available capacity are in practice possible, the measure yields the upper limit for the construction period necessary to accommodate a *small* rate of growth. For *large* rates of growth the result gives the lower limit.

It may be objected that to increase consumer goods output, the system need not "wait" until the last piece of primary and secondary equipment comes from the assembly line. This is certainly true of a partial expansion of group z. But such "staggering" obviously cannot affect the length of the adjustment period for aggregate expansion. We can get *some* consumer goods earlier, but only at the price of having to wait longer for the remainder.

Whereas the three *m*'s are technical constants, *g* is a variable. To determine it, we need a measure for *StF*, for *StR*, and for the rate of growth of *StF*. It will now be shown that all these variables are related to the independent variables of our system.

Primary equipment freed, *StF*, consists of capacity freed in both group *b* and group *a*. The size of capacity freed in group *b* is determined by the fall of replacement demand on the part of group *z*. The latter being equal to $skdz_0$, the stock of primary equipment freed in group *b* amounts to sk^2dz_0.

On the same principle, an additional stock amounting to $\frac{sk^3d^2}{1-kd}z_0$ is freed in group *a*.[16] And the total stock of primary equipment freed equals

$$StF_{a+b} = \frac{sk^2d}{1-kd}z_0 = sk^2do_0 \tag{14}$$

StR, the aggregate size of the primary equipment stock *required* (stock freed plus increment) can be determined in accord with equation 10a above:

$$StR = (s + \alpha)k^2do$$

Finally, the rate of growth of *StF* is a function of *k*. Each unit of stock freed produces an output equal to *StF*/*k*, which itself is added to the equipment stock and, allowing for its own depreciation, is ready to produce additional equipment in the next maturation period. Thus the rate of growth of *StF* equals[17]

$$\frac{k(1-d)+1}{k}$$

Thus we obtain

$$StF\left[\frac{k(1-d)+1}{k}\right]^g = StR$$

[16] The stock freed in group *a* equals *aks*. Since, according to equation 6 above,

$$a = \frac{k^2d^2}{1-kd}z$$

then

$$aks = \frac{sk^3d^2}{1-kd}z_0$$

[17] Since *g* refers to m_α max., all variables such as *k*, *k'*, *d*, and *d'*, which have a time dimension, must be related to this period.

and

$$(15) \qquad g = \frac{\log \frac{s+a}{s}}{\log \frac{k(1-d)+1}{k}} \quad [18]$$

We have finally to determine the magnitude of j, which measures the number of maturation periods required for the addition to secondary equipment. From equation 10a we can derive the size of the secondary equipment stock required as

$$StR_b = ak(1-kd)o_0$$

The amount of primary equipment additionally available for the production of secondary equipment (over and above replacement of primary equipment) follows from equation 12 as

$$StS_{a \text{ for } b} = sk(1-kd)(1-kd)o_0$$

which produces per period of maturation m_b max. an output of secondary equipment

$$\Delta o_b = \frac{sk(1-kd)(1-kd)}{k} o_0 = s(1-kd)^2 o_0$$

Therefore,

$$(15a) \qquad j = \frac{StR_b}{\Delta o_0} = \frac{ak(1-kd)}{s(1-kd)^2} = \frac{ak}{s(1-kd)}$$

The upshot of all this is that, given the three periods of maturation as technical constants, the minimum period of construction of the total additional equipment stock required can be calculated on the basis of our independent variables. As one would expect on common sense grounds, both g and j, and thus the construction period in its entirety, are directly related to the rate of growth and the depth

[18] The term g measures the number of maturation periods necessary to build the additional stock of primary equipment. Assuming $k = 2$, $d = 1/10$, $s = 1/10$, and $a = 1/5$, equation 15 yields $g =$ circa 3.3. How are we to interpret the decimal .3? It can mean either that, after the lapse of three maturation periods, the then-available capacity need be employed only over another three-tenths of one period to reach the desired aggregate output, or that only three-tenths of the capacity available after three periods need be utilized for another full period. Since with a given technology the maturation period is fixed, only the second interpretation makes economic sense. From this it follows that g must always be rounded upward to the next highest integer before the minimum period of construction can be established.

and depreciation rate of the additional capital stock, and inversely related to the savings ratio and the depth and depreciation rate of the original capital stock.

We saw above that the savings ratio may well prove excessive in view of the goal of optimum utilization of resources. We now have it confirmed by quantitative analysis that the higher the savings ratio the shorter is the minimum period of construction. Thus, as was indicated above, a conflict of goals may arise in a free market system between "maximum speed of adjustment" and "minimum waste of resources." Considering their low savings ratio, the economies of underdeveloped countries are unlikely to be caught in this dilemma.

SOME REMARKS ON "NEUTRAL" CHANGES IN PRODUCTIVITY

20. Besides a change in labor supply it is, above all, a change in technology that may induce a once-over change in capital formation. Since the most important type of technical change—namely, factor-displacing and factor-attracting innovations—will be discussed in the context of continuous change, only "neutral" changes in productivity will be discussed here. Harrod defines a neutral advance as one that does not alter the coefficients of capital depth.[19] In other words, output rises in the same proportion as capital stock, so that the group ratios in the new equilibrium do not differ from those in the original equilibrium.

As an analogy to the growth rate α of the labor force, a measure for the once-over rise in productivity is needed. This measure, π, is best defined as the difference between the coefficient of total productivity prevailing in the original equilibrium and that prevailing in the final equilibrium:

$$\pi = \frac{\epsilon_1}{\epsilon_0} - 1$$

where both ϵ's are related to i_0, that is, the unit input of the stationary process (see section 11). Equations 10, 10a, 12, 12a, 15, and 15a can then be rewritten by substituting π for α. By inserting $\alpha + \pi$ in the respective terms, we obtain expressions for the required capital stocks, for waste, and for the period of construction under conditions of a simultaneous once-over change in both labor supply and productivity.

[19] *Op. cit.*, p. 23.

3. *The Dynamics of Continuous Change*

21. Turning now to the capital problems related to continuous change, we have first to make a convenient breakdown of this large topic. To explore all structurally relevant cases, from the conditions for dynamic equilibrium to the multitude of dynamic disequilibria and the conditions for re-equilibration, would require a book. Therefore, it seems advisable to subdivide this part according to a formal principle which is applicable to all cases and can serve as a sort of "cadre," into which the reader may fit those other problems which cannot be discussed in this paper.

The distinction between "constant" and "varying" rates of change supplies such a principle. It restates, in a manner, the criterion for distinguishing between a stationary process and a once-over change, but now introducing this criterion into a framework of continuous change. The "equilibrium norm" of a constant rate of change, which in a stationary process implies a *zero* rate of change, now implies a *positive* rate of change. And, as has already been stated (section 13), processes involving varying rates of change can then be analyzed by the technique applied to once-over changes.

THE DYNAMICS OF A CONSTANT RATE OF CHANGE (DYNAMIC EQUILIBRIUM)

22. Certain structural conditions for dynamic equilibrium have received wide attention during the last decade. Considering the Keynesian origin of most of this work, it is not surprising that all the representative formulations (Harrod, Domar, Hicks) are in aggregate terms. Therefore, they describe the conditions which relate to the structure of the income-expenditure flow (section 3 above) rather than those which concern the physical structure of production. One of our tasks will be to show that, within the frame of reference assumed by the originators, the Harrod-Domar conditions, though necessary, are not sufficient to assure dynamic equilibrium.

For this purpose these conditions themselves must be formulated in terms of our independent variables. Dynamic equilibrium can prevail only if, for any given period, the supply of real capital equals the demand for real capital. Assuming a savings (investment) ratio of s, supply of real capital in terms of aggregate output equals $s(1-kd)o$. Demand for real capital can be determined with the help of equation 10 by interpreting a as a constant rate of growth. Adding to the constant growth in labor supply a constant increase in neutral technical advances, $(\alpha + \pi)ko$ measures the aggregate demand for real capital.

Therefore, in dynamic equilibrium

$$s(1 - kd) = (\alpha + \pi)k$$

or

$$\frac{(\alpha + \pi)k}{1 - kd} = s \tag{16}$$

This condition for dynamic equilibrium differs from the "no waste" condition formulated in equation 12 only by the absence of the variable d in the expression for the stock required. The modification indicates what, with respect to capital formation, is the difference between the two types of growth. In "wasteless" once-over growth, stock-building is precisely geared to subsequent replacement demand, whereas in continuous growth the demands of both stock replacement and stock expansion must be satisfied.

It is immediately clear that equation 16 is equivalent to both

$$G_n C_r = s \qquad \text{(Harrod's condition)}$$

and

$$\frac{\Delta I}{I\sigma} = s \qquad \text{(Domar's condition)}$$

The terms $\alpha + \pi$ in equation 16 have the same meaning as Harrod's G_n ("natural rate of growth") and Domar's addition to investment, $\Delta I/I$, whereas our k is identical with Harrod's C_r ("required capital coefficient") and is the inverse of Domar's σ ("investment productivity"). The only difference is that Harrod and Domar relate s to net output or income, whereas equation 16 relates s to gross output.

Thus there is general agreement that dynamic equilibrium cannot persist unless income and investment change at a rate equal to the product of the prevailing savings ratio and Domar's coefficient of investment productivity. But for dynamic equilibrium to be assured in an industrial system with factor specificity, the rate of growth of net output and investment, that is, $\alpha + \pi$, not only must equal the critical product, but must remain constant. Furthermore, given such a constant rate of growth, not only the critical product itself, but also the factors composing it, will have to remain constant. If the rate of growth changes, persistence of dynamic equilibrium will be conditional upon complicated shifts among the groups of production, *even if the product of the savings ratio and the investment productivity ratio spontaneously and simultaneously adjusts itself to the new rate of growth.* Similar shifts will be required if s or $1/k$ change, *even if their product remains constant.* The fact that, and the manner in which, such shifts are likely to destabilize the system will be demonstrated below.[20] In preparation for this discussion the more restrictive conditions, which are both necessary and sufficient for the maintenance of dynamic equilibrium, will now be established.

23. For this purpose we must, first of all, formulate structural equations of production, equivalent to the set of stationary equations formulated in model 1, but now appropriate to dynamic equilibrium:

(17)

$$[(1+a)^{t_n} + (1+\pi)^{t_n}][(F_{at_0}d \equiv f_{at_0}) + n_{at_0} + \sigma_{at_0}] = a_{t_n}$$

$$[(1+a)^{t_n} + (1+\pi)^{t_n}][(F_{bt_0}d \equiv f_{bt_0}) + n_{bt_0} + \sigma_{bt_0}] = b_{t_n}$$

$$[(1+a)^{t_n} + (1+\pi)^{t_n}][(F_{zt_0}d \equiv f_{zt_0}) + n_{zt_0} + \sigma_{zt_0}] = z_{t_n}$$

The set of equations in 17 differs from the stationary equations in two respects. First, the number of input items, contained in the right-hand brackets, is increased by the factor σ, representing savings over the chosen period. Second, the left-hand brackets contain a multiplicand, namely, the rate of growth by which the system expands from period to period as a result of increase in labor supply and of neutral advances. Therefore, the equations describe the level of *real* output attained in period t_n, expressed as the compounded level of real output in period t_0.

From set 17 we can derive the structural conditions of dynamic equilibrium, equivalent to those described for stationary equilibrium in equations 4 and 5:

(18)
$$\begin{aligned} f_{zt_2} + \sigma_{zt_2} &= b_{t_1} \\ &= z_{t_1} - n_{zt_2} \\ &= n_{at_2} + n_{bt_2} \end{aligned}$$

[20] It is true that both Harrod and Domar regard dynamic equilibrium as described by their equations as extremely unstable. But the initiating shock is always seen in a discrepancy between planned savings and the rate of growth times capital depth, whereas the above propositions include the case of parallel movement and even some cases of constancy of these two rates.

$$(19) \qquad f_{bt_2} + \sigma_{bt_2} = a_{t_1} - f_{at_2} - \sigma_{at_2}$$
$$= z_{t_1} - n_{bt_2} - n_{zt_2}$$
$$= n_{at_2}$$

These equilibrium conditions contain implicitly also the condition formulated in equation 16, that is, the Harrod-Domar condition for aggregate dynamic equilibrium. But they point up, above all, the physical-technical relations that must persist among certain strategic components of the three groups if today's outputs are to serve as tomorrow's inputs in a steadily expanding process. The underlying principle is that the Harrod-Domar conditions must apply not only to aggregate output and investment, but also to output and investment in each of the three groups.[21]

In this respect a comment is in order with regard to the variable σ. As is the case with all the other variables, σ has a twofold meaning, one monetary, one real (see section 10 above), that is, it measures both savings and investment. As measures of investment σ_a, σ_b, and σ_z express that distribution of total investment over the three groups on which persistence of dynamic equilibrium depends. As measures of savings they express the relative amounts of funds *available* for investment in each group rather than the relative amounts *accumulated* there. In other words, for the maintenance of dynamic equilibrium it does not matter how much of the income of each group is saved by the respective income-receivers, so long as the aggregate savings ratio remains constant and the "oversavings" of one or more groups are transferred to the "undersaving" group(s) for investment there.

[21] It may well be asked whether the above revision of the Harrod-Domar equilibrium conditions goes far enough. Factor specificity is likely to obstruct short-run shifts not only *among* the equipment goods and consumer goods groups, but also *within* each one of .these groups. From this consideration the ultimate conclusion can be drawn: to assure stability, the structure of demand as well as the rate of growth must remain constant.

Indeed, this conclusion eliminates large and sudden changes in demand. This, incidentally, is a type of change which as a rule involves also some change in the system's total capital structure, that is, a shift *among* the groups. For small and slow changes in demand the less severe conditions as formulated in equations 18 and 19 remain valid, since such changes would seem to fall within the range of tolerance for frictions that exists even in an industrial economy.

For a related problem see Robert M. Solow and Paul A. Samuelson, "Balanced Growth under Constant Returns to Scale," *Econometrica*, July 1953, pp. 412-424. Hans Neisser's criticism of their paper in *Econometrica*, October 1954, pp. 501-503, does not affect the above conclusions, which are based on the assumption of fixed input proportions.

24. In order to explore fully the structure of dynamic equilibrium, we must now determine the prevailing group ratios. Starting from the group ratios that characterize stationary equilibrium (equation 6), it is easy to see that output of consumer goods in dynamic equilibrium (z_{dy}) is reduced below the stationary level by the amount of net savings. Therefore,

$$z_{dy} = z_0(1-s)$$
$$= (1-s)(1-kd)o_0$$

It will be apparent that this expression is identical with the one given in section 17 for the initial change in output of consumer goods under the impact of a once-over change. The difference is that in dynamic equilibrium with constant money supply, only the *money* expression for z_{dy} lies permanently below the money expression for z_0, but real output rises continuously.

In establishing the size of b_{dy}, a certain complication must be considered. During the first phase of a once-over change, b_{00} simply serves as replacement of f_{z00}. In dynamic equilibrium, b_{dy} must fulfill the same function but must also provide for an additional stock of secondary equipment in accord with the prevailing rate of growth. Thus

$$b_{dy} = f_{zdy} + F_{zdy}(a+\pi)$$
$$= \left[(1-s)(1-kd)kd + (1-s)(1-kd)\frac{kd}{d}(a+\pi) \right] o_0$$

From equation 16 we know that $s(1-kd)/k$ can be substituted for $a+\pi$. This yields

$$b_{dy} = (1-s)(1-kd)[kd(1-s)+s]o_0$$

Finally,

$$a_{dy} = o_0 - (b_{dy} + z_{dy}) = [kd(1-s)+s]^2 o_0$$

so that

$$(20) \quad z_{dy} : b_{dy} : a_{dy} = 1 : kd(1-s)+s : \frac{[kd(1-s)+s]^2}{(1-s)(1-kd)}$$
$$= F_{zdy} : F_{bdy} : F_{ady}$$

Introducing different capital depth coefficients, different rates of depreciation, and different savings ratios, the three group ratios become

$$(20a)\quad z_{dy} : b_{dy} : a_{dy} = 1 : (1 - s_z)k_z d_z + s_z : \frac{[(1 - s_z)k_z d_z + s_z][(1 - s_b)k_b d_b + s_b]}{(1 - s_a)(1 - k_a d_a)}$$

The ratio $F_{zdy} : F_{bdy} : F_{ady}$ can be obtained by multiplying the members on the right side of equation 20a with k_z, k_b, and k_a, respectively.

25. It may be appropriate at this point to demonstrate how, with the help of equation 20 (or 20a), the actual structure of the system can be derived if the values of o, s, k, and d are given. The group ratios and capital stock ratios appropriate to the given values of s, k, and d follow from equation 20. The absolute values of the group outputs can then be derived by subdividing the value of o accordingly. The absolute values for the capital stocks equal the absolute values of the respective group outputs multiplied by the respective k's. The absolute values of the inputs f_z, f_b, and f_a follow from the respective group outputs multiplied by kd. The income magnitudes $(n + \sigma)$ equal the residual of $o - f$ in each group, and with the help of s we can divide the respective sums of $n + \sigma$ into their components. The equilibrium conditions, equations 18 and 19, provide a final check.

THE DYNAMICS OF A VARYING RATE OF CHANGE

26. Once we admit variations in the rate of change, any one of our independent variables, a, π, k, d, and s, and any combination of these variables can undergo such a change. A complete survey might have to include also shifts in taste, at least to the extent to which the bi-polar changes in capital stock bound up with such shifts do not fully balance, and also changes in the supply of natural resources, especially diminishing returns on land.

This discussion will be confined to two examples, which illustrate some of the structural problems of capital formation that arise in the context of varying rates of change. One deals with shifts in the demand and supply functions for investment; the other is concerned with nonneutral—that is, factor-displacing—technical changes.

Shifts in the Investment Functions

27. What is at stake here can best be understood in terms of the Harrod-Domar conditions as formulated in equation 16:

$$\frac{(a+\pi)k}{1-kd}=s$$

A shift relating to the variables of this equation can mean two things. On the one hand, and this is the usual interpretation, it can refer to a shift of the demand for investment relative to the supply of investment—in popular language, to either undersaving or oversaving. But it can also mean a proportionate change on both sides of the equation by a simultaneous parallel change in the savings ratio and the output of capital goods. This alternative has greater realistic significance than may appear at first sight. It covers, for example, the case of a mature economy in which the community responds to a falling rate of population increase with a rise in the average propensity to consume. Even more important is the case of an overinvestment boom in Hayek's sense, in which rising real wages cut into savings and enforce a reduction of aggregate investment.

It is interesting to note that Harrod, Domar, and all their critics are exclusively concerned with the first alternative, that is, a relative change between the two functions. Parallel shifts do not seem to pose any problems to them, though these are the real test for any equilibrium condition which is formulated in aggregate terms only.[22] The analysis of such shifts will now serve to complete the argument about the conditions of dynamic equilibrium, which was started in section 22 above.

28. The case of a parallel shift in the demand and supply functions for investment can be expressed in the following modification of equation 16:

$$h\frac{(a+\pi)k}{1-kd}=hs \tag{16a}$$

where h measures the change in the rate of growth and in the savings ratio as the ratio between the new and the old level of these two magnitudes. If interpreted as an "ex post" relationship, the above equation is of course a truism. The problem to which I want to draw attention is posed by the structural shifts within the aggregate, which are enforced by the transition from the state described in equation 16 to that described in equation 16a.

[22] "All that is required for the argument immediately to follow is that any changes in s, i.e. savings expressed as a fraction of income, should be small by comparison with experimental changes in G" (Harrod, *op. cit.*, p. 79). The same position is taken by E. D. Domar, "The Problem of Capital Accumulation," *American Economic Review*, December 1948, p. 779, though the inherent assumption of factor mobility is realized.

To follow up these shifts I assume that, with a constant coefficient of capital depth, the rate of population growth falls and that the savings ratio adjusts itself immediately. Even though these shifts are simultaneous, the structure of production is now in disequilibrium. To regain equilibrium, the output of consumer goods and of secondary equipment must rise (in accord with the rise in the average propensity to consume) relative to the output of primary equipment. This change in the output ratio between groups z and b, on the one hand, and group a, on the other hand, depends upon a corresponding reshuffling of factors. Some primary equipment and labor, which under the previous higher rate of absorption produced additional primary machinery, will now have to produce more secondary machinery. This, however, will not keep the total previous stock of primary equipment fully employed. It is true that, once the new dynamic equilibrium has been reached, the *aggregate* stock of primary and secondary equipment equals the aggregate stock that would exist had the rate of growth not fallen. But the share of the secondary equipment stock in the aggregate is now higher, and the share of the primary equipment stock is reduced correspondingly. Therefore, some primary equipment must be scrapped or at least kept idle until the system grows into the existing capacity, always assuming that, for reasons of specificity, it cannot be used as secondary equipment.

A similar though less drastic friction arises in the labor market. More labor is now needed to mind machines in group z, whereas some labor formerly employed in making machines in group a will be displaced. Labor is never so specialized that gradual transfer cannot take place. But whether and when such adjustment, and with it the approach to a new equilibrium, will occur depends upon what effect capital devaluation and temporary unemployment in group a will have on entrepreneurs' expectations and behavior.

Here the limits of structural analysis have been reached and functional analysis, the study of behavior patterns, must take over if the actual stability conditions are to be established. In a general way one may venture the guess that adjustment hangs in the balance, to say the least, if the magnitude of h is considerable and the shift occurs suddenly. The coincidence of both circumstances in a "strong" boom makes this phase of the cycle highly unstable.[23]

[23] In this context see my critique of Hicks in "A Structural Model of Production," *Social Research*, June 1952, pp. 168-173. Though I still maintain my objections to Hicks' explanation of the downturn, in the light of the above

Thus dynamic equilibrium through time is assured only if major shifts among the groups can be excluded. This is equivalent to saying that the Harrod-Domar conditions must be fulfilled for each one of the three groups—the postulate contained in the above formulation of the equilibrium conditions in equations 18 and 19.

A measure for the waste in primary equipment, which a parallel downward shift of the savings and the investment functions creates, can easily be ascertained by comparing the size of the stock necessary to sustain the higher rate of growth with that required to sustain the lower one. The former equals

$$St\,A(\text{vailable})_{a+b} = (a_{dy} + b_{dy})k$$

Using equation 20, we can transform this into

$$St\,A_{a+b} = [(1-s)kd + s]ko$$

If the change in the rate of growth, expressed as the ratio between the new and the old rate of growth, equals h, the new stock of primary equipment required equals

$$St\,R_{a+b} = [(1-sh)kd + sh]ko$$

Waste, which is equal to the difference between the two expressions, then is

$$(21) \qquad W' = sk(1-h)(1-kd)o$$

W' varies directly with the size of the capital coefficient and the original savings ratio, and inversely with the change in the rate of growth. If h is positive—that is, if the rate of growth increases—waste is "negative." Then equation 21 measures the additional amount of primary equipment required for the system to adjust to the new rate.

A comment is in order on the relationship between the two kinds of waste, which are determined by equations 12 and 21 respectively. They refer to quite different phenomena. The term W, waste due to "overbuilding," indicates that actual investment overshoots the fixed target of required investment. The term W', waste due to a parallel shift in the demand and supply functions for investment, indicates that the investment target itself changes in the downward direction. If it changes in the upward direction, W' is

considerations my own conclusions as stated in the article seem to me now in need of some more "pessimistic" modification.

negative. But there is then no positive W either, since no "overbuilding" can occur so long as the new, higher rate of growth is maintained.

It may well be asked whether the functional obstacles which obstruct adjustment to a falling rate of growth interfere also with adjustment to a rising rate of growth. At the beginning of such a change some secondary equipment and the labor operating it will certainly be displaced. Absolute specificity completely prevents the former from being shifted to group a, and the latter can at best be shifted with some delay. One might argue that the effect of structural frictions upon expectations is less destabilizing in an expanding than in a contracting system. But in the former case an investment decision must be made, while in the latter case output can follow the price signals of a rising consumer demand. Whatever the ultimate conclusion may be, a parallel shift of the savings and investment functions establishes an interesting mechanism. It brings about a change in the income-expenditure structure which as such does not distort the structure itself. This lack of distortion is probably the reason why Harrod and Domar pay no attention to such parallel shifts. However, the change in the income-expenditure structure indirectly causes a distortion of the technical structure. This secondary distortion may in its turn destabilize the income-expenditure structure after all.

29. The second alternative—namely, a relative change in the investment demand and the investment supply functions—can be expressed by the following inequality:

$$s \gtrless \frac{(\alpha + \pi)k}{1 - kd}$$

The s refers of course to planned savings, whereas the right side defines planned investment. With regard to the effect that such an inequality is likely to have upon the stability of dynamic equilibrium, I have little to add to the insights which the discussion of Harrod's and Domar's work has brought to light during the last few years.[24] Since, as in the previous case, the conclusions must ultimately be derived from behavioral premises, they are again a task for functional rather than for structural analysis. But a brief digression into this area may be of value at this point. It will bring to light the special contribution that structural analysis makes to

[24] See in particular the writings of Sidney Alexander, W. J. Baumol, Joan Robinson, and T. C. Schelling.

"total" analysis. I choose for this purpose the case of potential oversaving, which, in the wake of the Keynesian challenge, was long the center of attention, and which still forms a subject of controversy between Keynesians and certain neoclassicists.[25] The reader will easily be able to apply the argument to potential undersaving.

According to the traditional neoclassical position, an excess of planned savings over planned investment will be adjusted either by a rise in the average capital depth of the system or by a harmless *numéraire* deflation. The link is the depressing effect which potential oversaving is supposed to exert upon the rate of interest, which in its turn is supposed to induce an increase in the demand for capital goods per unit of labor.

The objections to this harmonistic solution concern both links of the argument. First of all, the rate of interest may not fall at all. The initial excess of planned savings over investment must reduce demand for and prices of consumer goods. This is likely to create elastic price expectations all around and increase the demand for cash. But even if the rate of interest were to fall, the situation in the consumer goods market just described would hardly be conducive to an expansion of investment. Thus in either case the result will be a fall in aggregate employment with all the latent dangers of a general "real" deflation.

So far everybody has won and all must have prizes. But this is so only because the two parties argue at cross purposes, implicitly assuming different types of technical structure. The neoclassical argument is correct for a perfectly mobile structure, where absence of specificity reduces the period of constructing additional capital goods to an insignificant length. In this structure the fall in consumer goods output can at once be balanced by a rise in the output of producers goods, leaving the elasticity of price expectations unchanged. Far from obstructing adjustment, the fall in consumer goods output is the very condition for an equilibrating shift of factors from groups z and b to group a. Even if the rate of interest should not fall sufficiently, or the investment elasticity of such a fall should be small, the ensuing general fall of commodity and factor prices (as a result of the "hoarding" of savings, to use old-

[25] See, e.g., the controversy between Domar and E. H. Stern in *American Economic Review*, December 1949, pp. 1160-1172, and, more recently, Harold Pilvin, "Full Capacity vs. Full Employment Growth," *Quarterly Journal of Economics*, November 1953, pp. 545-552, together with R. F. Harrod's "Comment," *ibid.*, pp. 553-559.

fashioned language) need not affect the real magnitudes of the system. When perfect mobility prevails, the downward adjustment of the *numéraire* can take place simultaneously and in the very short run all over the system, leaving expectations again unaffected. The maximum permissible length of this "short run" can be determined. It must not exceed the shortest of the various income periods in the system (i.e. one week, under present institutional arrangements). If it is longer, factor unemployment cumulates and the monetary deflation deteriorates into a depression.

One has only to spell out its implicit conditions to realize that the neoclassical argument is completely unrealistic in an industrial system. But it is equally clear that it is the lags in the adjustment process due to specificity, especially of equipment goods, and the consequent *longue durée* of the construction period that create cumulative unemployment with its detrimental effect upon the elasticity of price expectations. Whatever justification there is for the Harrod-Domar pessimism with regard to the stability of dynamic equilibrium—and there is a good deal—it rests ultimately upon the technical rigidity of an industrial order of production.

This result is not accidental. Though the proposition cannot be proven in the context of the present paper, it can be stated as a general rule that to endanger the stability of the system, distortions of the income-expenditure structure must influence expectations in a particular manner. Whether they do so or not is largely dependent on the length of the potential adjustment period, which in its turn is directly related to the prevailing technical structure.[26]

In summarizing this discussion of the consequences that possible changes in the strategic variables can have for the stability of dynamic equilibrium, we can distinguish between three levels of progressively restrictive conditions. The optimistic extreme is represented by the traditional neoclassical approach, which disregards all aspects of specificity. There capital depth is treated as a variable, which changes inversely with a highly sensitive rate of interest. Since any shock arising from a change in either the rate of growth or the savings ratio can be absorbed without delay by a change in k, dynamic equilibrium is stable.

An intermediate position is taken by Harrod and Domar. They treat k as a constant; therefore, changes in the rate of growth relative to the savings ratio cannot be absorbed without frictions and pre-

[26] For a more detailed exposition see my paper "On the Mechanistic Approach in Economics," as cited.

carious consequences for expectations. But so long as $s/(\alpha + \pi)$ and therefore k remain constant, dynamic equilibrium once existing is stable, though it may shift from one level of activity to another one.

The most restrictive conditions for dynamic equilibrium have been postulated above. Not only the fraction $s/(\alpha + \pi)$ and thus capital depth, but both the numerator and the denominator of the fraction, that is, the savings ratio as well as the rate of growth, must remain constant in order to prevent destabilizing structural shifts among the groups of production.

Nonneutral Technical Changes

30. The theory of technical change is still a stepchild of economic analysis. The sweeping generalizations of Marx and Schumpeter have not been followed by more detailed macroeconomic investigations. In Keynesian economics technical change figures as no more than one investment variable among others. More recent work, e.g. the writings of Yale Brozen or William Fellner, has by and large taken a microeconomic turn. Therefore, I shall have to introduce my structural analysis with some more general remarks, to determine, first of all, the context in which macroeconomic problems of capital formation arise when technical changes disturb dynamic equilibrium.

In order to narrow the field, the following observations will be confined to *cost-reducing* technical changes, to the exclusion of *want-creating* innovations, or, to use the customary term, "new products." If I read the literature correctly, the latter have so far proved refractory to exact analysis, due mainly to two complications. First, they introduce a simultaneous change in both the supply and the demand function; second, since the product is "new," the change in the supply function cannot be related to any previously existing supply function—the main difference from cost-reducing changes. Thus what follows refers only to "technical progress" in the narrower sense.

In contradistinction to what was discussed earlier, in section 20, here only "nonneutral" technical changes will be considered. Harrod's definition of neutrality, which was adopted above, refers to a proportional rise of output and capital stock, leaving k unchanged. Harrod is fully aware that this definition implies an assumption about the relative demand for factors in the new state as compared with the old. By a neutral advance "the productivity of labor embodied in machines is raised in equal measure with that of those engaged in minding machines."[27] In other words, neutral advances are neither labor-displacing nor capital-displacing.

These considerations yield a convenient definition of nonneutral technical changes. They comprise all changes that cause at least temporary displacement of one or more factors somewhere in the system, though not necessarily in the industry which introduces the change. It is the kind and size of such displacement, and the manner in which the displaced factors can be reabsorbed (what Continental economists have called the "compensation" problem), that are in the center of the structural analysis of nonneutral technical changes, so far as capital formation is concerned.

In the conventional manner I distinguish between labor-displacing and capital-displacing changes. Within each of these two subgroups there are three different types, according to whether, per unit of output, units of both factors are displaced or, whether, with units of one factor displaced, the employment of the other remains constant or even increases. It is highly desirable to find an economic indicator for distinguishing the five possible cases (one case appears in both subgroups). For this purpose, I list in the following table the manner in which each type modifies capital depth, capital intensity, capital productivity, and labor productivity, in the sense in which these terms have been defined in section 11 above.

Types of Technical Change	*Capital Depth*	*Capital Intensity*	*Capital Productivity*	*Labor Productivity*
1. Pure labor-displacing	constant	rising	constant	rising
2. Labor-displacing, capital-displacing	falling	?	rising	rising
3. Labor-displacing, capital-attracting	rising	rising	falling	rising
4. Pure capital-displacing	falling	falling	rising	constant
(5. Capital-displacing, labor-displacing)	falling	?	rising	rising
6. Capital-displacing, labor-attracting	falling	falling	rising	falling

It is at once clear that neither capital depth nor capital intensity is a good criterion, because quite different types of change affect these coefficients in the same manner. This is also true of the two productivity coefficients if only one of them is taken as indicator. If, however, capital productivity is used to characterize the types of labor-displacing changes, and labor productivity to characterize the types of capital-displacing changes, a clear distinction can be

[27] Harrod, *op. cit.*, p. 23.

established.[28] All five cases have empirical significance. Because each type can materialize simultaneously in all three groups, or in any two or in only one, because different types can materialize simultaneously in different groups, and furthermore because capital displacement can refer to fixed as well as to working capital goods, the number of possible models exceeds any manageable range. But in each case the same structural principle is at work, so that the selection of a few simple cases will suffice to formulate the basic problems and to indicate their solution.

I shall concentrate upon pure labor-displacing changes and supplement the results with only brief comments upon type 3, labor-displacing and capital-attracting changes, and type 4, pure capital-displacing changes. The capital change in both cases will be confined to changes in fixed capital. In accord with the general tenor of this paper, only problems related to capital formation will be discussed, to the exclusion of the productivity effects, private or social, and the distributive effects of technical changes.

31. As indicated above, structural problems of capital formation arise in two phases of the innovation process: one at the beginning if the introduction of the new device requires a change in the fixed capital applied; the other when the operation of the new device has displaced some factors of production, the reabsorption of which requires additional real capital. In trying to design models for these problems we are confronted with two alternatives. We can choose as our general frame of reference an economic process in equilibrium, stationary or dynamic, or—an alternative more appropriate for the past history of capitalism—a process in which part of the available resources are idle. Obviously the task of "capital construction" is greatly facilitated in both phases of adjustment if idle capacity exists in the two equipment goods groups. On the other hand, such a frame of reference inevitably involves the whole complex of business cycle analysis. Pursuing this line not only would take us far afield, but would also prevent us from studying the structural issues in isolation. The analysis will therefore be continued within the framework of dynamic equilibrium. Considering the bottlenecks referred to earlier, which nowadays obstruct smooth adjustment to large changes

[28] If depreciation rates should vary inversely with the change in capital stock, new complications would be introduced. They could be taken care of only by explicit reference to the behavior of the depreciation rate. In order not to complicate our analysis, I shall, during the subsequent exposition, assume an unchanged rate of depreciation.

in the rate of change in both developed and underdeveloped countries, the alternative chosen recommends itself also on practical grounds.

These assumptions yield a simple solution of the "construction" problem. It does not arise in a pure labor-displacing technical change because the value—that is, the claim upon factors though not necessarily the physical form of the equipment stock—remains unchanged by definition. This presupposes, of course, that the change-over to the new technique takes place after the old equipment is fully amortized. On the other hand, a construction problem does arise in capital-attracting changes, as does a problem of capital liquidation in capital-displacing changes. For either case the preceding analysis of once-over changes and of a downward shift of the investment function can be used. What was said, for example, about the "construction period" (section 19) and the two types of waste (sections 18 and 28) is fully applicable here.

The problem of "compensation" is more complex. There is, above all, no general agreement that compensation is a "secular-period" problem in the Marshallian sense, or, in other words, that the reabsorption of technologically displaced labor really requires prior capital formation. It is not possible to pursue this question through all its ramifications at this time, and a few remarks must suffice to justify the position taken below.[29]

Starting from a pure labor-displacing device, three short-period solutions of the compensation problem have been suggested since the days when Ricardo deserted the harmonist camp in the chapter "On Machinery." One points to compensating "demand," arising either from the profits of the technical pioneer or, if the improvement is generalized over the whole industry, from the rise in consumer real income resulting from the fall in price of the improved output. Whatever one may think about the cogency of this argument,[30] it certainly implies capital formation in those fields toward which the alleged increase in aggregate demand turns.

[29] For a more systematic treatment see my paper on "Technological Unemployment Reexamined," in *Wirtschaft und Kultursystem*, G. Eisermann, editor, Erlenbach-Zuerich, Eugen Rentsch Verlag, 1955, pp. 229-254.

[30] The proposition that aggregate demand for commodities rises, in the manner postulated, above aggregate supply and thus raises demand for labor to the equilibrium level is highly dubious. Since the displacement of workers also initially reduces demand, a compensating demand of pioneers or consumers seems required to restore equilibrium between supply and demand within the smaller flow of production from which the displaced workers are eliminated.

The second solution hinges upon the variability of factor proportions. In its most extreme version the argument asserts that any amount of idle labor can always be employed on the *existing* capital stock, if only wages adjust to the declining marginal productivity. Now under conditions of factor specificity, as they prevail in an industrial system, the argument cannot refer to the physical form of the existing capital stock, but only to its value. In other words, compensation is then a long-period problem, depending upon the prior transformation of the existing stock of capital goods into a physically different one. Such transformation will not by itself create any friction if the change-over coincides with the moment when the old equipment must be replaced anyhow. But while firms can plan "construction" in this manner, there is no mechanism which assures such a happy coincidence in the case of compensation. If the two points of time differ, compensation is again conditional upon the formation of new real capital.

The third solution is Ricardo's own: employment of displaced workers in occupations which do not require fixed capital, such as menial services and—happy age!—warfare. The wide range of "services" offered in any depression is an indication that this solution is not without practical relevance. The general trend of modern industrialism toward "tertiary" occupations at the expense of "secondary" ones may indeed provide a safety valve for *secular* technological unemployment of a steady nature. But it is unlikely to absorb the shocks which arise from large discontinuous innovations of a labor-displacing character.

However, the adherents of short-period compensation may point to the fact that many of these discontinuous innovations are at the same time capital-attracting. In this case the compensation issue, far from creating an adjustment problem, seems to alleviate the difficulties bound up with the initial construction phase. By creating another need for capital formation, compensation of technological unemployment offers work to the additional stock of primary equipment which had to be built to make the initial capital expansion possible, and may thus preclude "waste." The argument certainly deserves consideration. But it is decisive only when the capital depth of the initial investment approximates that of the compensation investment, and even then only over the period during which the compensating equipment must be built. After this second construction period the waste problem again appears, though somewhat mitigated by the need for larger replacements, which have now to maintain two capital stocks.

From all these observations it appears that large, sudden, and highly productive innovations of a labor-displacing nature do pose a problem of capital formation (as capital-displacing changes pose a problem of capital liquidation). Therefore, the results obtained above for the length of the construction period and for waste can again be utilized. But they now require an important modification, because nonneutral technical changes alter permanently the relative scarcity of labor and equipment. This is equivalent to saying that the factors have to be reshuffled among the three groups before the new equilibrium can be attained. This new shift in the group ratios is much more complicated than the adjustment to changes in the over-all rate of growth, discussed above (sections 22-29).

32. Limitations of space permit the detailed exposition of such a shift for only the simplest case, namely, a pure labor-displacing improvement occurring in the consumer goods group. Given a system in dynamic equilibrium as described in equation 20, and also the ratio of workers displaced in group z to workers originally employed there, how will the group ratios, which describe the new equilibrium after compensation, differ from the original? To aid the comparison it is assumed that the supply of money is kept constant, so that output prices adjust to the fall in unit costs which results from the reduction of labor costs.

The principal link between the two equilibria is a systematic rise in the capital depth of the critical group z. This rise in capital depth can be derived from the more obvious rise in capital intensity, which is only another way of saying that less labor is now applied per unit of capital.[31]

The general relationship between capital intensity c (see section 11) and k can be established as follows:

$$c = \frac{F}{n_v} = \frac{ok}{o - f} = \frac{ok}{o - kdo} = \frac{k}{1 - kd}$$

or

$$k = \frac{c}{1 + cd}$$

[31] It may be appropriate to stress once more the fact that both capital depth and capital intensity are understood here in value terms.

Denoting the displacement ratio as defined above by δ_z, we have

$$c_{z_1} = \frac{c_{z_0}}{1 - \delta_z}$$

where the subscripts 0 and 1 refer to the original and subsequent equilibrium respectively. Therefore,

$$(22) \qquad k_{z_1} = \frac{c_{z_0}}{1 - \delta_z + c_{z_0} d_{z_0}} = \frac{k_{z_0}}{1 - \delta_z (1 - k_{z_0} d_{z_0})}$$

Since the technical change is confined to group z, no change in capital depth occurs in groups a and b, so that $k_{b_1} = k_{b_0}$ and $k_{a_1} = k_{a_0}$. By simply substituting $k_{z_1} = k_{z_0}$ in equation 20, the group ratios in the new equilibrium can be established.

It is now possible to determine the capital requirements upon which compensation depends. The new group ratios in combination with the respective capital depth coefficients yield the new ratios among the capital stocks of the three groups, from which the absolute capital increments can be calculated for any given absolute level of the original output. Our previous investigations supply the tools for determining the length of the construction period required for compensation and the size of capital waste occurring, if any. As a matter of fact, since innovations are the prime cause of uneven changes in the rate of growth, it is in this context that these tools prove their usefulness.

There is, however, an additional obstacle to short-run compensation. It is immediately clear that, in view of the change in capital intensity and capital depth in group z, all workers originally displaced cannot be re-employed there. Some of them must be shifted to groups a and b to produce, and subsequently to replace, the addition to the capital stock required in group z. A measure can be devised for the required shift which at the same time expresses the resistance that labor specificity offers to short-run adjustment. Such a measure is the shift ratio γ_z, that is, the number of workers displaced in group z that must be shifted to groups a and b relative to the total number of workers originally displaced in group z. It can be determined as follows:[32]

[32] The term N symbolizes here the number of workers, labor being treated in the interest of simplification as the sole income-receiving factor. This term is supposed to be uniquely related in all groups to the value input of labor per period. In this interpretation N is a stock magnitude equivalent to F, with a depreciation rate equal to N_v/n_v. (For this interpretation see section 6).

$$\gamma_z = \frac{N_{z_0} - N_{z_1}}{N_{z_0}\delta_z} = \frac{n_{z_0} - n_{z_1}}{n_{z_0}\delta_z}$$

since $n_z = z - f_z = (1 - k_z d_z)z$, we obtain

$$\gamma_z = \left[1 - \frac{1 - k_{z_1}d_{z_1}}{1 - k_{z_0}d_{z_0}}\right]\frac{1}{\delta_z}$$

According to our previous assumption (footnote 28), the rates of depreciation are not supposed to change. Therefore, by substituting for k_{z_1} the expression obtained in equation 22, we have

$$\gamma_z = \frac{k_{z_0}d_{z_0}}{1 - \delta_z(1 - k_{z_0}d_{z_0})} \tag{23}$$

If instead of being purely labor-displacing, the technical change is at the same time capital-attracting, then

$$k_{z_1} = \frac{k_{z_0}(1 + \beta_z)}{1 - \delta_z(1 - k_{z_0}d_{z_0}) + \beta_z k_{z_0}d_{z_0}} \tag{24}$$

where β_z expresses the increase of the capital stock in group z as the ratio of the increment to the original value of the stock. From this we obtain

$$\gamma_z(\beta_z) = \frac{k_{z_0}d_{z_0}(\delta_z + \beta_z)}{[1 - \delta_z + k_{z_0}d_{z_0}(\delta_z + \beta_z)]\delta_z} \tag{25}$$

33. The general principle expounded in the foregoing section is applicable to all cases of factor-displacing innovations. However, in most cases additional considerations must be taken into account, of which two are briefly indicated here.

First, whenever labor-displacing innovations are introduced in one or both equipment goods groups, technology in the consumer goods group remaining unchanged, one might expect that the value capital depth coefficients would change in the innovating groups. In fact this is not so, because the value of the *input* "equipment" adjusts itself in the new equilibrium to the increased productivity—that is, to the value of its *output*, which itself is equipment. Therefore, both the value of the capital stock and that of output must fall in the same proportion, leaving the quotient k unchanged. On the other hand, the price fall of secondary equipment because of the technical change in group a and/or group b must affect the value of the capital stock in group z, reducing there the value capital depth coefficient although the physical-technical combination of the factors has not itself changed.

Second, in applying our procedure to capital-displacing improvements, we must remember that such changes *reduce* both value capital intensity and value capital depth in the innovating groups. Thus the effect is just the opposite of that which arises from labor-displacing changes. This enables us to treat capital-displacing changes as labor-attracting changes. To give an example, for a pure capital-displacing change in the consumer goods group we have only to substitute $1/(1-\delta)$ for $1-\delta$ in equation 22. We then obtain the critical coefficient k_z, with the help of which the shift ratio and the new group ratios can be derived in the manner described.

5. *Application: "Ideal" and "Real" Models*

34. As stated in the Introduction, this paper is concerned only with the structural part of the theory of dynamic processes. Within this area two main problems have been studied: (1) the minimum capital requirements for various types of economic growth, and (2) the optimum paths that the system must follow in order to readjust the dislocations that different types of growth inflict upon a pre-existing state of stationary or dynamic equilibrium, "minimum" and "optimum" to be related to minimum waste of resources and/or maximum speed of adjustment. A final question remains to be answered: Are the results of such structural analysis useful in helping to solve the empirical growth problems which arise in advanced as well as in backward economic systems?

From the outset it is readily admitted that, in an empirical science, the effort spent upon the construction of "models" can be ultimately justified only by what they contribute to the understanding of real phenomena. In order to pass this test our structural models need elaboration in at least two directions. First, the level of abstraction will have to be reduced below the one chosen in the foregoing exposition, which is appropriate to pure theory only. Second, and more important, we have so far been concerned almost exclusively with "structural" relations and movements, that is, with the impersonal conditions for the absorption of dynamic shocks. But, as was stressed in the beginning, no practical economic problem can even be posed, not to say solved, without due regard to the personal forces as manifested in the motivations and behavior patterns of the actors. To formulate the general principle in the terminology established above: Only when combined with functional analysis can structural analysis be "applied."

However, even when supplemented by a study of the appropriate motivations and behavior patterns, structural analysis of the type performed above will never yield a "real" model, that is, a simplified image of any actual growth process. We have to remember that we have not been concerned with the *descriptive* analysis of structural relations and movements as they occur in empirical systems in historical time, but with *normative* analysis, that is, with the structural requirements for the optimal achievement of a postulated goal, say, equilibrating growth. Were we to extend our analysis to the functional conditions required to hold the system to the structurally required path, we should be able to fill the lacunae in our "ideal" models, but reality, that is, actual behavior and the real structure that emanates from it, would still escape us.

This gulf between "ideal" and "real" models is perhaps less wide in a collectivist system. There the actual dispositions of the planning authority reflect what, under the aspect of the chosen "holistic" goal, are regarded as required structure and required behavior. Any deviation of the "real" from the "ideal" behavior is then treated as illegitimate, and as a subject for the penal code rather than for economic study. No such concurrence prevails in a free market system, where the real order of the whole is not based on holistic decisions but is the result of the independent decisions of the "particles." If we want to construct models of the ensuing real processes, we have to study the actual motivations and behavior patterns that prevail in the actual structure under observation, and to derive from them the actual paths of adjustment to change. Obviously this task is beyond the reach of the purely deductive method with the help of which our "ideal" models are established; it can be accomplished only with the help of inductive procedures which, in particular, should tell us what the actual behavior patterns are in a given situation.

Thus our results have, in principle, nothing to contribute to the description of actual growth processes, and even less to the prediction of their future course. However, by establishing what, relative to certain postulated goals, are the most economical forms of growth, they yield the "efficiency norms" by which the performance of empirical growth processes must be judged. They present an image of "perfect" growth, and thus point up and locate the structural and functional deficiencies of any empirical system under observation. By disclosing at the same time the structural relations and functional forces most appropriate to optimum performance, our "ideal" models offer guidance for the improvement of the real processes, and are thus the scientific foundation for economic policy. A few remarks on some practical issues to which our findings can be applied in this manner are to bring these observations to a close.

35. It is not claimed that the models of once-over and continuous growth described above exhaust the possible range of dynamic processes. Above all, they are concerned only with "exogenous" shocks, to the exclusion of those endogenous changes which, following Frisch's example, modern econometrics has placed in the center of its dynamic investigations. But the number of practically relevant dynamic processes is few, if attention is focused upon the formal properties of rise and fall, continuity and discontinuity, proportionality and disproportionality.

Each of the models discussed above can be associated with one or more characteristic growth phenomena which have appeared during the era of world-wide industrialization. Our "ideal" model of once-over growth has some sort of empirical replica in the processes by which a "stationary" preindustrial system moves off dead center into "development." It is hardly necessary to stress the essential difference between the model underlying the analysis in Part 2 and the real structure of any underdeveloped country: the almost complete absence in the latter of groups *b* and *a*. But this very difference emphasizes the strategic position of real capital formation, that is, the nature and duration of the processes by which real capital is built.

How easily this aspect of the developmental process can be lost sight of is apparent in an otherwise most interesting attempt at utilizing structural analysis for practical purposes. I refer to H. W. Singer's essay on "The Mechanics of Economic Development."[33] There the Harrod-Domar formula for dynamic equilibrium is used to determine for certain parametric values the relationship between the rate of development, the rate of population increase, capital productivity, and savings. On the basis of what he regards as plausible parameters, Singer concludes that autonomous development not supported by capital imports is practically impossible.

I come to the same conclusion, but for different reasons. For

[33] *Indian Economic Review*, Vol. 1, No. 2, 1952, esp. pp. 15-18.

Singer the main obstacle lies in the size of the capital depth coefficient for developmental investment, which he puts at 5. With this assumption, a savings ratio of more than 16 per cent would be required to sustain a population increase of 1.25 per cent per year and an annual increase of real income of 2 per cent. But unless most of the new investment consists of "social overhead capital" like transportation, irrigation, etc., a capital depth coefficient of 5 seems much too high. If development were to concentrate on manufacturing projects with an average depth coefficient of slightly under 2, the developmental goal might be reached with a savings ratio of only 6 per cent, which Singer regards as a feasible level. In other words, with this one change in the empirical parameters, autonomous development does not seem to encounter any obstacles.

It is against such optimism that our structural analysis guards. The Harrod-Domar formula (equation 16) tells us what rate of growth can be sustained with a given savings ratio, *once the primary real capital necessary for such growth has been formed*. It does not reveal anything about the period of construction, which separates the moment when consumption shrinks because of saving from the moment when real capital has expanded sufficiently for additional consumer goods to reach the market. We saw above that this period of construction is positively correlated with the period of maturation in the field of equipment goods production, and negatively correlated with the originally available stock of real capital. In both respects the typical underdeveloped country is placed most unfavorably, and the "waiting period" during which savings depress the standard of living may extend over many years, unless capital imports alleviate the situation. Therefore, a savings ratio that can be regarded as tolerable once development has actually started, may prove far too high to move the system off dead center.

The next step would then lead to an analysis of the structural consequences of importing real capital from abroad. They obviously differ according to whether the imports consist of primary equipment or secondary equipment. The latter case seems typical for the early stages of industrialization. But there is at least one example—the Soviet Union—in which emphasis was placed from the beginning upon primary equipment. In either case the construction period is drastically reduced. The short-run benefits to the consumer are greatest with imports consisting of secondary equipment. They are pure gains, which do not require even temporary sacrifices in the standard of living, if the real capital can be borrowed under such conditions that the current productivity increase exceeds interest and amortization, and if an equivalent of the current service on the foreign debt is physically adapted to the demand for exports. But even if the equipment imports have to be paid for by consumer goods exports, foreign trade is a much speedier method of physical transformation than domestic investment.

36. Scattered remarks about the "maturity" issue have already pointed to the potential use of our structural models for the secular growth problems of developed countries. As was stated before, whereas capital formation creates a difficult *bottleneck* in the early stages of industrialization, in the later stages it is the need for interindustry *shifts of resources* and the threat of *capital waste* that, in addition to the bottlenecks related to compensation, delay and deflect the "ideal" adjustment. From this it follows that real growth in developed countries is so closely interrelated with fluctuations that, at least for the past, secular and cyclical problems cannot be studied fruitfully in isolation.

This impression is strengthened by some earlier considerations (section 28) which suggest that the model of once-over growth, superimposed upon the model of continuous growth, might be usefully applied to the analysis of a "strong" boom. To build a structural framework for cyclical analysis one must of course go further. The model of a discontinuous change in a positive rate of growth must be refined on the basis of what can be known of models of non-neutral technical changes. An even more complicated superimposition of change processes—introducing analysis of noncompensated factor displacement—seems required for the study of "weak" booms.