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4 Are Market Forecasts Rational?

4.1 Introduction

This chapter presents tests of the rationality of both inflation and short-term interest rate forecasts in the bond market. These tests make use of security price data to infer information on market expectations. A closer look at whether market forecasts of inflation and interest rates are rational seems necessary in light of recent work (Pesando 1975; Carlson 1977; Mullineaux 1978; Friedman 1980) that evaluates the inflation and interest rate forecasts from the Livingston and Goldsmith-Nagan surveys. A frequent empirical result in these studies is that the survey forecasts are inconsistent with the restrictions implied by the theory of rational expectations. What conclusions about the behavior of market expectations should we draw from these results?

One view which associates survey forecasts with market forecasts takes these empirical results to be evidence that the market is not exploiting all information in generating its forecasts. The Friedman (1980) study is particularly disturbing in this regard because it uses data from the Goldsmith-Nagan interest rate survey which is made up of interest rate forecasts from actual participants in the market.

An alternative view, Pesando (1975) for example, holds that markets probably do display rationality of expectations. Irrationality in the Livingston and Goldsmith-Nagan survey data would then indicate that these data cannot be used in empirical work to describe market expectations.

The latter view receives support for two reasons. Survey data are frequently believed to be inaccurate reflections of the behavior of market participants and are considered unreliable. More important is a point emphasized in Chapter 2 that is often ignored in discussing the properties of expectations. *Not all market participants need be rational for a market* *to display rational expectations*. The behavior of a market is not necessarily the same as the behavior of the average individual. As long as unexploited profit opportunities are eliminated by some participants in a market, then the market will behave as though expectations are rational despite irrational participants in that market. Therefore, survey forecasts do not necessarily describe the forecasts inherent in market behavior, and the irrationality of survey forecasts does not in itself imply that market forecasts are also irrational.

One purpose of this chapter is to provide indirect evidence on the usefulness of survey data like Livingston's and Goldsmith-Nagan's for describing the expectations reflected by markets. In particular, this chapter contains direct tests of the rationality of the bond market's interest rate and inflation forecasts, tests similar to those found in the studies mentioned in the opening paragraph. Because these tests are designed to use actual price data to infer information on market expectations rather than relying on survey data, they can provide direct information on the rationality of a particular market. They permit a clearer interpretation of results that indicate irrationality in survey forecasts. The empirical work in this chapter thus will shed light not only on the value of these surveys for further research, but also on the rationality of expectations in such markets as those in which bonds are traded.

4.2 Tests of Forecast Rationality

Rationality of expectations requires that

$$E(X_t - X_t^e | \phi_{t-1}) = 0, \qquad (1)$$

where X_t^e is the one-period-ahead forecast of a variable X_t, generated at the end of period $t-1$, and ϕ_{t-1} is the set of information available at the end of $t-1$. This implies that the forecast error, $X_t - X_t^e$, should be uncorrelated with any information or linear combinations of information in ϕ_{t-1}.

This implication is the basis of the tests of rationality found in the studies of survey forecasts mentioned above. Consider the following regression equations where we assume that $E(u_{1t}|\phi_{t-1}) = E(u_{2t}|\phi_{t-1}) = 0$:

$$X_t = b_o + \sum_{i=1}^{k} b_i X_{t-i} + u_{1t}, \qquad (2)$$

$$X_t^e = c_o + \sum_{i=1}^{k} c_i X_{t-i} + u_{2t}. \qquad (3)$$

These equations can be estimated with ordinary least squares (OLS), and under the hypothesis of rational expectations Modigliani and Shiller

(1973) point out that the estimated b_i coefficients should not differ significantly from the estimated c_i coefficients. This null hypothesis that

$$b_i = c_i \text{ for all } i = 0, \ldots, k \tag{4}$$

is subjected to a conventional F test in the survey forecast studies. A more detailed discussion of the rationale behind this test can be found in Chapter 3.

The theory of efficient markets leads to restrictions similar to those in (4) which can also be easily tested. Market efficiency (or, equivalently, rational expectations) implies that securities prices in a capital market should reflect all available information, and hence an expectation assessed by the market should equal the true expectation conditioned on all available information, $E(\ldots|\phi_{t-1})$. To give this concept empirical content, we must specify the relationship between the probability distribution of future prices and current prices. This requires a model which describes how current equilibrium prices are determined. Here, the market is assumed to equate expected, one-period, holding returns across securities, allowing for risk (liquidity) premiums which are constant over time.

In the case of long-term bonds, for example, the one-period return denoted by y_t, is the nominal return from holding the long-term bond from $t - 1$ to t, including both capital gains plus interest payments. The model of market equilibrium implies that the equilibrium return $\tilde{y}_t$ is

$$\tilde{y}_t = E_m(y_t|\phi_{t-1}) = r_{t-1} + d, \tag{5}$$

where

r_{t-1} = the return on a one-period bond from $t - 1$ to t (which of course equals the expected one-period return)—this is just the short-term interest rate,
d = the constant liquidity (risk) premium,
$E_m(\ldots|\phi_{t-1})$ = expectation assessed by the market at $t - 1$.

As discussed in Chapter 2 market efficiency implies that

$$E(y_t - \tilde{y}_t|\phi_{t-1}) = E(y_t - r_{t-1} - d|\phi_{t-1}) = 0. \tag{6}$$

If we call the equilibrium return of $\tilde{y}_t$ a "normal" return, then the equation above states that no unexploited profit opportunities exist in the bond market: at today's price, market participants cannot expect to earn a higher-than-normal return by investing in a long-term bond. The efficient markets equation (6) is analogous to an arbitrage condition. Arbitrageurs who are willing to speculate may perceive unexploited profit opportunities and purchase or sell bonds until the price is driven to the point where (6) holds. Thus market efficiency does not require that all participants in the market are rational and use information efficiently.

The average behavior of an individual in the market is not a reliable guide to the market's behavior.

Equation (6) above implies that $y_t - r_{t-1}$ should be uncorrelated with any past available information or linear combinations of this information. A model consistent with (6)—referred to as the efficient-markets model—is

$$y_t = r_{t-1} + d + (X_t - X_t^e)\beta + \epsilon_t, \tag{7}$$

where an e superscript denotes expected values conditional on all past available information (i.e., $X_t^e = E(X_t|\phi_{t-1})$, a one-period-ahead rational forecast), and

X_t = a variable (or vector of variables) relevant to the pricing of long bonds,
β = a coefficient (or vector of coefficients),
ϵ_t = an error process where $E(\epsilon_t|\phi_{t-1}) = 0$ and hence ϵ_t is serially uncorrelated.

The efficient-markets model stresses that only when new information hits the market will y_t differ from $r_{t-1} + d$. As equation (7) makes clear, this is equivalent to the proposition that only unanticipated changes (surprises) in variables can be correlated with $y_t - r_{t-1}$.

The assumption that the coefficient on r_{t-1} equals one in equation (7) has been subjected to empirical test by Fama and Schwert (1977) and Mishkin (1978) and is not rejected. It has been tested also for the 1954–1976 sample period of this chapter. A quarterly bond returns series was regressed on the beginning of period, ninety-day Treasury Bill rate (also at quarterly rates) using weighted least squares to correct for heteroscedasticity. (Mishkin 1978 describes this procedure.) The coefficient on the bill rate was not significantly different from one at the 5 percent level ($t = .51$). In a recent paper, Shiller (1979) has found evidence suggesting that the liquidity premium is correlated with the spread between long rates and short rates. To test this proposition for the 1954–1976 sample period, $y_t - r_{t-1}$ was regressed on this spread, again using weighted least squares to correct for heteroscedasticity. The evidence supporting Shiller's proposition is even weaker in this sample period than in the regression results reported in Mishkin (1978): the coefficient on the spread variable did not differ significantly from zero even at the 10 percent significance level ($t = 1.01$). In addition, as is discussed in Chapter 2, as long as the equilibrium return $\tilde{y}_t$ has small variation relative to other sources of variation in the actual returns, assumptions describing the equilibrium return are not critical to empirical tests of the efficient-markets model. This appears to be the case for the long-term bonds discussed here. For example, using the model of market equilibrium described above, over the 1954–1976 period the variation in

$\tilde{y}_t$ is less than 2 percent of the variation in the actual return stemming from other sources.

It is easy to show that this efficient-markets model is consistent with the expectations hypothesis of the term structure where predictions of future short-term interest rates are optimal forecasts. To be more concrete, if the long-term bond is an n-period security where the liquidity premium is a constant d, the expectations hypothesis of the term structure is approximated by

$$(8) \qquad RL_t = \frac{1}{n} \; E_t(r_t + r_{t+1} + \; . \; . \; . \; + r_{t+n-1}) + d,$$

where

RL_t = the interest rate (yield to maturity) on the long bond,
$E_t = E_m(. \; . \; . \; |\phi_{t-1})$,
n = number of periods until maturity.

When expectations of future short rates are rational, then with some algebraic manipulation the expectations hypothesis described by this equation yields the same implications as equation (7). Note also that the efficient-markets model does not imply causation from $X_t - X_t^e$ to $y_t - r_{t-1}$. It is equally plausible that causation runs in the other direction or that a third factor affects both of these variables simultaneously.

Given a forecasting equation for X_t of the form of equation (2), rationality of expectations implies that

$$(9) \qquad X_t^e = c_o + \sum_{i=1}^{k} c_i X_{t-i},$$

where $c_i = b_i$ for all i because X_t^e must equal the conditional expectation of equation (2). Substituting (9) into (7) we have an efficient-markets model of the following form:

$$(10) \qquad y_t = r_{t-1} + d + \beta\left[X_t - \left(c_o + \sum_{i=1}^{k} c_i X_{t-i}\right)\right] + \epsilon_t.$$

Equations (10) and (2) can then be stacked into one regression system and estimated by nonlinear least-squares as described in Chapter 2, imposing the restrictions in (4) implied by forecast rationality: that $b_i = c_i$ for all i. In the initial estimates of each equation, Goldfeld-Quandt (1965) tests usually indicate the presence of heteroscedasticity, which is corrected for by weighting observations, using a time-trend procedure outlined in Glesjer (1969). The rationality restrictions can now be tested in the efficient-markets framework with the likelihood ratio test described in Chapter 2.

Chapter 3 demonstrates that the tests of the rationality restrictions are equivalent to more common regression tests of the efficient-markets condition in equation (6). However, as we shall see, exploring the efficiency or rationality of the bond market by analyzing the relation of b_i in equation (2) to the c_i in (10) yields insights that the more common tests do not provide.

4.3 Empirical Results

The first set of tests conducted here will scrutinize Friedman's (1980) finding that the survey measures of interest rate forecasts are inconsistent with rationality. Friedman's results were obtained using thirty quarterly observations extending from September 1969 to December 1976. This sample period is used to estimate the equations (10) and (2) system using bond return and Treasury Bill rate data to be described. Friedman's choice of six lagged quarters in the autoregressive specification will be adapted also. An additional test conducted over the longer 1954–1976 sample period will provide more information about the rationality of the bond market's forecasts.

Tests of the rationality of the CPI inflation forecasts will be conducted in a similar manner using the nonlinear efficient-markets procedure. The 1959–1969 sample period used by Pesando, Carlson, and Mullineaux, where so many rejections of rationality have been found, will be used here, in addition to the longer 1954–1976 sample period.

4.3.1 The Data

The sources and definitions of data used in the empirical work are as follows:

y_t = quarterly return from holding a long-term U.S. government bond from the beginning to the end of the quarter. The data were obtained from the Center for Research in Security Prices (CRSP) at the University of Chicago, and are described in Fisher and Lorie (1977) and Mishkin (1978). Note that this return series is calculated from end-of-period price data to avoid the aggregation problem discussed later in this chapter.

r_t = the end of quarter ninety-day Treasury Bill rate at a quarterly rate. Bill rate data were obtained from the Board of Governors of the Federal Reserve Board.

π_t = the CPI inflation rate (quarterly rate) calculated from the change in the log of the CPI (seasonally adjusted) from the last month of the previous quarter to the last month of the current quarter. The CPI was collected from the U.S. Department of Commerce's *Business Statistics* and *Survey of Current Business*.

4.3.2 Results on the Rationality of Interest Rate Forecasts

Table 4.1 provides the tests for the rationality of forecasts in the bond market both in Friedman's 1969–1976 sample period and in the longer

Table 4.1 **Test of Forecast Rationality: Interest Rates**

	Sample Period	
	1969:3–1976:4	1954:1–1976:4
Likelihood ratio statistic	6.55	4.96
Marginal significance level	.364	.549

Note: Likelihood ratio statistic is distributed asymptotically as $\chi^2(6)$. Marginal significance level is the probability of getting that value of the likelihood ratio statistic or higher under the null hypothesis.

1954–1976 sample period, and table 4.2 provides the parameter estimates of the constrained efficient-markets model for both sample periods. The marginal significance levels in table 4.1 are the probability of obtaining that value of χ^2 or higher under the null hypothesis that the rationality constraints are valid. A marginal significance level less than .05 indicates a rejection of the null hypothesis at the 5 percent level and, therefore, a rejection of forecast rationality in the bond market.

As the likelihood ratio statistics in table 4.1 indicate, very little evidence in the bond market data supports irrationality of interest rate forecasts. Not only are there no significant rejections of the rationality restrictions in either Friedman's sample period or the longer 1954–1976 sample period, but the marginal significance levels of table 4.1 are quite high. In addition, the efficient-markets model from which these likelihood ratio statistics have been derived, whose parameter estimates are found in table 4.2, has several attractive properties. The coefficients on the unanticipated movements of the bill rate are significantly different from zero at the 1 percent level, indicating that movements in short-term interest rates are information relevant to the pricing of long-term bonds. As might be expected from the expectations hypothesis of the term structure, the sign of this coefficient is negative, indicating that an unanticipated rise in the bill rate is accompanied by higher long-term rates with a resulting lower bond return. Furthermore, the magnitude of this coefficient is extremely close to that found in another study (Mishkin 1978), where a different measure of short-rate expectations is used.[1]

The failure to reject the rationality of interest rate forecasts in the bond market provides some resolution of how to interpret Friedman's result that the Goldsmith-Nagan survey measures of interest rate forecasts are irrational. This finding suggests that the survey measures of interest rate forecasts are not an accurate description of the actual bond market

1. Note that Mishkin (1978) used Treasury Bill data which are at an annual rate rather than a quarterly rate. The coefficient on the unanticipated bill rate, in that case, must be multiplied by four when compared to the β coefficients in table 4.2.

Table 4.2 **Nonlinear Estimates of the Efficient-Markets Model:**

$$y_t = r_{t-1} + d + \beta(r_t - b_0 - \sum_{i=1}^{6} b_i r_{t-i}) + \epsilon_t$$

$$r_t = b_0 + \sum_{i=1}^{6} b_i r_{t-i} + u_t$$

	Sample Period	
	1969:3–1976:4	1954:1–1976:4
d	.0055	−.0018
	(.0091)	(.0032)
β	−3.3613	−3.0950
	(1.1642)	(.4566)
b_0	.0240	.0022
	(.0090)	(.0012)
b_1	.6158	1.0706
	(.1750)	(.0869)
b_2	.0639	−.3123
	(.1913)	(.1287)
b_3	.3159	.2189
	(.1869)	(.1331)
b_4	−.1434	.0296
	(.1872)	(.1348)
b_5	−.3195	−.1473
	(.1911)	(.1324)
b_6	.0463	.0906
	(.1790)	(.0909)

Note: Asymptotic standard errors in parentheses.

forecasts. The value of these survey measures to other empirical work is thus suspect. The view that the bond market could have improved its forecasting behavior by exploiting the information in the past bill rate movements more efficiently is not supported. Of course, these results should not be surprising considering how large a body of evidence (e.g., see Fama 1970) supports efficiency in the bond market.

4.3.3 Results on the Rationality of Inflation Forecasts

The test of the rationality of inflation forecasts in the bond market can be found in table 4.3. The parameter estimates of the constrained efficient-markets model are in table 4.4. The efficient-markets model yields the expected result that an unanticipated rise in inflation is associated with higher long-term rates and lower bond returns, although the coefficients on unanticipated inflation are not as significant as the coefficients on unanticipated interest rate movements. However, the likelihood ratio test rejects the rationality restrictions for the 1959–1969 sample period at the 1 percent significance level—this is the sample period where other studies (Pesando 1975; Carlson 1977; Mullineaux 1978) also find the Livingston price expectations data to be irrational.

Table 4.3 **Test of Forecast Rationality: Inflation**

	Sample Period	
	1959:1–1969:4	1954:1–1976:4
Likelihood ratio statistic	23.77	8.70
Marginal significance level	.001	.191

Note: See table 4.1.

A look at the unconstrained estimates of the autoregressive model of inflation and the efficient-markets model provides a clue to why this rejection of rationality occurs. The sum of the coefficients on the lagged inflation rates in the unconstrained autoregressive model of inflation is positive and greater than one, indicating that a rise in inflation would persist: the $\hat{b}_i$ starting with lag one are $-.06$, .59, .19, $-.03$, .30, and .25. On the other hand, the sum of these autoregressive parameters derived from the unconstrained efficient-markets model is negative, indicating that the bond market expected that a rise in inflation would be reversed:

Table 4.4 **Nonlinear Estimates of the Efficient-Markets Model:**

$$y_t = r_{t-1} + d + \beta(\pi_t - b_0 - \sum_{i=1}^{6} b_i \pi_{t-i}) + \epsilon_t$$

$$\pi_t = b_0 + \sum_{i=1}^{6} b_i \pi_{t-i} + u_t$$

	Sample Period	
	1959:1–1969:4	1954:1–1976:4
d	$-.0036$	$-.0019$
	(.0034)	(.0032)
β	-2.5189	-1.8685
	(1.3319)	(.8436)
b_0	.0003	.0012
	(.0008)	(.0006)
b_1	$-.0464$	.3778
	(.1461)	(.1031)
b_2	.6047	.5173
	(.1210)	(.1100)
b_3	.2626	.2075
	(.1497)	(.1224)
b_4	$-.0477$	$-.1555$
	(.1206)	(.1219)
b_5	.2104	$-.0392$
	(.1147)	(.1100)
b_6	.1233	$-.0374$
	(.1242)	(.1035)

Note: Asymptotic standard errors in parentheses.

the $\hat{c}_i$ starting with lag one are $-.27, .25, 1.04, -.30, -.94$, and -1.60. This discrepancy is what leads to the rejection of the rationality of the bond market's forecasts of inflation, and it should not be all that surprising considering the sample period. The period started with a low inflation rate that then rose to unusually high levels by the end of the period. The fact that this was an unusual period might well cause the rationality restrictions found in table 4.3 to be rejected, even though the bond market would normally have rational inflation forecasts. A similar problem has been found for the rationality of inflation forecasts (represented by forecasts of exchange rate changes) in the German hyperinflation (Frenkel 1977), another unusual inflationary episode. The likelihood ratio test of the rationality of the inflation forecasts in the longer 1954–1976 period provides some evidence for this conjecture. In this period there is no rejection of the rationality restrictions at the 5 percent significance level. The bond market thus appears to have had rational inflation forecasts when a longer time horizon is taken into account.

Because these rationality restrictions are generated under the maintained hypothesis that $y_t - r_{t-1}$ is uncorrelated with anticipated movements by X, the rejection may arise from the invalidity of the maintained hypothesis and not from the irrationality of inflation expectations. To explore this possibility, the hypothesis of the rationality of the X_t^e forecasts can be tested along the lines discussed in Chapter 2 without maintaining the hypothesis that $y_t - r_{t-1}$ is uncorrelated with X_t^e. This involves estimating the system

$$X_t = b_o + \sum_{i=1}^{k} b_i X_{t-i} + u_{1t},\tag{11}$$
$$y_t = r_{t-1} + d + \beta\left[X_t - \left(c_o + \sum_{i=1}^{k} c_i X_{t-i}\right)\right] + \delta\left(c_o + \sum_{i=1}^{k} c_i X_{t-i}\right) + \epsilon_t,$$

and testing the null hypothesis that $b_i = c_i$ for all i. Note that this procedure tests $k - 1$ restrictions, one less than in the previous tests. When this test for the rationality of the inflation forecasts is conducted with the same 1959–1969 sample period, the data still strongly reject the rationality restrictions. The resulting likelihood ratio statistic [distributed asymptotically as $\chi^2(5)$] equaled 16.65 with a marginal significance level of .005. This rejection at the 1 percent level adds additional support to the view that inflation forecasts were not rational for this sample period.

The efficient-markets model does not specify whether seasonally adjusted or unadjusted data should be used in these tests. The tests reported in the tables use seasonally adjusted data because they are more comparable with the rationality tests of the Livingston data found in the literature. However, seasonal adjustment of the CPI with the X-11 program tends to "smudge" the data, and thus the tests here have also been conducted with seasonally unadjusted data. The results are similar to those reported in tables 4.3 and 4.4. The likelihood ratio statistic for the 1959:1–1969:4 sample period was 23.25 (marginal significance level of .001), and for the 1954:1–1976:4 sample period it was 12.32 (marginal significance level of .055).

What do these results tell us about the accuracy of the Livingston price expectations data? We must take some care in our interpretation of these results. The Livingston survey does not sample participants in the bond market specifically, but the following conclusion nevertheless seems to be indicated: Because inflation forecasts in the bond market from 1959 to 1969 do not satisfy restrictions implied by rationality, the failure of survey measures to satisfy these restrictions cannot be taken as evidence that they are inaccurate measures of market expectations. Clearly, further research into the rationality of the Livingston price expectations data over longer sample periods is needed before we can pronounce on their accuracy.

4.3.4 Joint Tests of the Rationality of Both Inflation and Interest Rate Forecasts

A further application of these tests relates to the work of Modigliani and Shiller (1973). Modigliani and Shiller's seminal paper postulates that information on both short-term interest rates and inflation would influence the price of long-term bonds, along with the proposition that the autoregressive lag structure on the one-period-ahead short-rate and inflation forecasts would be "rational" in the sense discussed here. They present evidence supporting this position, yet the evidence is incomplete in two ways. First, they do not actually apply formal statistical tests to the proposition of rationality in the autoregressive lag structure. Second, their use of averaged data in the empirical work leads to a potentially severe aggregation problem.

A simple example from Mishkin (1978) illustrates Modigliani and Shiller's argument and why it breaks down with averaged data. Assume that the stochastic process generating the short-term interest rate r_t has an ARIMA (0,1,1) characterization as follows:

$$(1 - L)r_t = (1 - \lambda L)u_t \tag{12}$$

or, equivalently,

$$r_t = \frac{1-\lambda}{1-\lambda L} r_{t-1} + u_t = (1-\lambda) \sum_{i=1}^{\infty} \lambda^i r_{t-i} + u_t, \tag{13}$$

where

L = the lag operator,
u_t = error term with the property that $E(u_t|\phi_{t-1}) = 0$.

Assuming expectations are rational, the market's forecast of r_{t+1} at time t is:

$$(14) \qquad E_t r_{t+1} = \frac{1-\lambda}{1-\lambda L} r_t$$

and since $r_{t+2} = r_{t+1} + u_{t+2} - \lambda u_{t+1}$

$$(15) \qquad E_t r_{t+2} = E_t r_{t+1} = \frac{1-\lambda}{1-\lambda L} r_t$$

and, similarly,

$$(16) \qquad E_t r_{t+j} = \frac{1-\lambda}{1-\lambda L} r_t \text{ for all } j \geq 1.$$

Rewriting equation (8), which characterizes the expectations hypothesis of the term structure, the long bond rate at time t, RL_t, is

$$(17) \qquad RL_t = \frac{1}{n} E_t \left(r_t + r_{t+1} + \ldots + r_{t+n-1}\right) + d.$$

Substituting (14)–(16) into (17) we have

$$(18) \qquad RL_t = \frac{r_t}{n} + \left(\frac{n-1}{n}\right)\left(\frac{1-\lambda}{1-\lambda L}\right) r_t + d = \frac{r_t}{n} + \left(\frac{n-1}{n}\right)(1-\lambda)\sum_{i=0}^{\infty} \lambda^i r_{t-i} + d.$$

Modigliani and Shiller postulate that if the bond market is rational then the lag structure in (13) must be consistent with (18): that is, the λ must be the same in the two equations.

Note that the r_t and RL_t are end-of-period variables so this proposition is necessarily valid only for end-of-period data. Indeed, it does not hold for averaged data. To see this, take the case where the short rate is a random walk: that is, $\lambda = 0$ in (12). Working (1960) has shown that a variable that is a random walk will, if it is averaged, have an ARIMA (0,1,1) time-series process with the correlation coefficient at lag one equal to .25. The appearance of the moving average term when the data is averaged is really quite intuitive. If a variable is a random walk, then a rise in its average value from the first period to the second is more likely if its value at the end of the second period is higher than its average for the second period. Then the average for the third and following periods is likely to be higher than the average in the second period. This is exactly what we would find for an ARIMA (0,1,1) time-series process.

With the random walk characterization of the short rate,

$$E_t r_{t+j} = r_t \text{ for } j \geq 1 . \tag{19}$$

It is easy to see that the expectations hypothesis implies that the long rate will be a random walk as well. Using Working's result, averages of both these variables should have the following ARIMA (0,1,1) characterization:

$$(1 - L) r_t^a = (1 + .268L) u_t , \tag{20}$$

$$(1 - L) RL_t^a = (1 + .268L) u_t , \tag{21}$$

where

r_t^a = the average value of r over the period $t - 1$ to t,
RL_t^a = the average value of RL over the period $t - 1$ to t.

Using the expectations hypothesis equation (17) where r and RL are replaced by r^a and RL^a, equation (20) implies that the averaged value of the long rate has the following time-series process

$$(1 - L) RL_t^a = \left(1 + \frac{.268}{n} L\right) u_t . \tag{22}$$

This time-series process is different from (21) and is obviously incorrect. Indeed, for large n it will be very close to a random walk.

The above example thus indicates that, if the data are averaged, equation (17) cannot be used to derive the lag weights of short rates in a long equation. Modigliani and Shiller's evidence on the rationality of the term structure proceeds with exactly this derivation with averaged data, and then comparing these lag weights with those actually estimated from a long rate equation. Yet as the example shows, this procedure is not valid.

The efficient-markets model discussed in this paper leads to a formal statistical test of the Modigliani-Shiller results using end-of-period data. Including both short-term interest rate and inflation movements as relevant information to the pricing of long-term bonds as is done by Modigliani and Shiller, we can write the efficient-markets model as

$$y_t = r_{t-1} + d + \beta_r (r_t - r_t^e) + \beta (\pi_t - \pi_t^e) + \epsilon_t , \tag{23}$$

where π_t = CPI inflation rate.
The autoregressive models for r and π are

$$r_t = k_r + \sum_{i=1}^{k} d_i r_{t-i} + \sum_{i=1}^{k} e_i \pi_{t-i} + u_{1t} , \tag{24}$$

$$\pi_t = k_\pi + \sum_{i=1}^{k} f_i r_{t-i} + \sum_{i=1}^{k} g_i \pi_{t-i} + u_{2t} ,$$

and using these autoregressive models to derive expectations,

(25) $$y_t = r_{t-1} + d + \beta_r\left[r_t - \left(k_r + \sum_{i=1}^{k} d_i\pi_{t-i} + \sum_{i=1}^{h} e_i\pi_{t-i}\right)\right] + \beta_\pi\left[\pi_t - \left(k_\pi + \sum_{i=1}^{k} f_i r_{t-i} + \sum_{i=1}^{k} g_i\pi_{t-i}\right)\right] + \epsilon_t.$$

The equations of (24) and (25) can then be estimated jointly as before and tests of the rationality restrictions can be conducted with the likelihood ratio test. These tests then provide direct information on the Modigliani and Shiller rationality proposition.

These tests and estimates of the efficient-markets model can be found in tables 4.5 and 4.6. The term-structure equation in the MPS (MIT-Penn-SSRC) Quarterly Econometric Model and the Modigliani and Shiller paper both use a sample period extending from 1954:4 to 1966:4 and an eighteen-quarter lag on short rates and inflation estimated with a third-order Almon lag. Therefore both the 1954:4–1966:4 and the 1954:1–1976:4 sample period, as well as the Modigliani-Shiller procedure for estimating the lag structure, are used in the rationality tests conducted here.

The likelihood ratio tests in table 4.5 confirm Modigliani and Shiller's results. The restrictions implied by rationality in both the inflation and interest rate forecasts are not rejected at the 5 percent significance level and again the marginal significance levels are high. Seasonally unadjusted CPI inflation data rather than the seasonally adjusted data again leads to results like those reported in tables 4.5 and 4.6. The likelihood ratio statistic with the unadjusted data for the 1954:4–1966:4 period is 17.00 (marginal significance of .074) and for 1954:1–1976:4 it is 11.89 (marginal significance level of .292). Thus Modigliani and Shiller's contention that the term structure of interest rates displays rationality is supported in these tests, a finding we should have expected considering the results of the previous tests in this chapter and in Sargent (1979).

4.4 Conclusion

This chapter provides an answer to the question, Are market forecasts rational? Empirical tests conducted here, with one exception, indicate

Table 4.5 Modigliani-Shiller Tests of Forecast Rationality

	Sample Period	
	1954:4–1966:4	1954:1–1976:4
Likelihood ratio statistic	13.87	12.90
Marginal significance level	.179	.230

Note: Likelihood ratio statistic is distributed asymptotically as $\chi^2(10)$.

Table 4.6 **Nonlinear Estimates of the Modigliani-Shiller Efficient-Markets Model**

$$y_t = r_{t-1} + d + \beta_r\left(r_t - k_r - \sum_{i=1}^{18} d_i\, r_{t-i} - \sum_{i=1}^{18} e_i\, \pi_{t-i}\right) + \beta_\pi\left(\pi_t - k_\pi - \sum_{i=1}^{18} f_i r_{t-i} - \sum_{i=1}^{18} g_i \pi_{t-i}\right) + \epsilon_t$$

$$r_t = k_r + \sum_{i=1}^{18} d_i\, r_{t-i} + \sum_{i=1}^{18} e_i\, \pi_{t-i} + u_{1t}$$

$$\pi_t = k_\pi + \sum_{i=1}^{18} f_i\, r_{t-i} + \sum_{i=1}^{18} g_i\, \pi_{t-i} + u_{2t}$$

Sample Period 1954:4–1966:4

$d = -.0021$ (.0032) $\beta_r = -10.9928$ (2.4208) $\beta_\pi = -1.8397$ (.9752)

$k_r =$ .0001 (.0010)				$k_\pi =$ .0009 (.0027)			
$d_1 =$ .8908 (.1072)	$d_{10} =$ .0438 (.0229)	$e_1 =$ .0512 (.1879)	$e_{10} = -.0528$ (.0517)	$f_1 =$ .1579 (.0730)	$f_{10} = -.0303$ (.0164)	$g_1 = -.3029$ (.1407)	$g_{10} = -.0618$ (.0352)
$d_2 = -.0640$ (.0743)	$d_{11} =$ .0431 (.0228)	$e_2 =$ .0515 (.1659)	$e_{11} = -.0456$ (.0478)	$f_2 =$ .1236 (.0502)	$f_{11} = -.0215$ (.0170)	$g_2 =$ .1312 (.1222)	$g_{11} = -.0744$ (.0322)
$d_3 = -.0372$ (.0463)	$d_{12} =$ .0401 (.0247)	$e_3 =$ .0132 (.1094)	$e_{12} = -.0373$ (.0524)	$f_3 =$ .0665 (.0325)	$f_{12} = -.0123$ (.0187)	$g_3 =$ .1103 (.0830)	$g_{12} = -.0800$ (.0344)
$d_4 = -.0147$ (.0308)	$d_{13} =$ .0351 (.0271)	$e_4 = -.0158$ (.0897)	$e_{13} = -.0289$ (.0588)	$f_4 =$ .0237 (.0232)	$f_{13} = -.0042$ (.0206)	$g_4 =$ .0859 (.0657)	$g_{13} = -.0775$ (.0381)
$d_5 =$ .0038 (.0267)	$d_{14} =$ .0284 (.0287)	$e_5 = -.0366$ (.0903)	$e_{14} = -.0214$ (.0605)	$f_5 = -.0066$ (.0205)	$f_{14} = -.0008$ (.0217)	$g_5 =$ .0593 (.0621)	$g_{14} = -.0654$ (.0392)
$d_6 =$ .0184 (.0277)	$d_{15} =$ .0201 (.0288)	$e_6 = -.0502$ (.0921)	$e_{15} = -.0159$ (.0541)	$f_6 = -.0261$ (.0204)	$f_{15} = -.0013$ (.0217)	$g_6 =$ .0317 (.0614)	$g_{15} = -.0424$ (.0356)
$d_7 =$ .0295 (.0282)	$d_{16} =$ .0105 (.0285)	$e_7 = -.0576$ (.0878)	$e_{16} = -.0133$ (.0437)	$f_7 = -.0365$ (.0200)	$f_{16} = -.0047$ (.0215)	$g_7 =$ .0045 (.0581)	$g_{16} = -.0073$ (.0296)
$d_8 =$ .0372 (.0270)	$d_{17} = -.0001$ (.0324)	$e_8 = -.0598$ (.0773)	$e_{17} = -.0147$ (.0576)	$f_8 = -.0395$ (.0188)	$f_{17} = -.0188$ (.0248)	$g_8 = -.0211$ (.0514)	$g_{17} = -.0413$ (.0374)
$d_9 =$ .0419 (.0248)	$d_{18} = -.0114$ (.0468)	$e_9 = -.0579$ (.0636)	$e_{18} = -.0210$ (.1144)	$f_9 = -.0369$ (.0172)	$f_{18} = -.0427$ (.0358)	$g_9 = -.0436$ (.0428)	$g_{18} = -.1047$ (.0718)

Table 4.6 (continued)

Sample Period 1954:1–1976:4							
		$d = -.0013$	$\beta_r = -12.1804$	$\beta_\pi = -1.3112$			
		(.0032)	(1.9664)	(.8497)			
$k_r =$ .0007				$k_\pi = -.0016$			
(.0005)				(.0013)			
$d_1 =$ .8013	$d_{10} =$ .0326	$e_1 =$ .1427	$e_{10} = -.0409$	$f_1 =$ .1277	$f_{10} = -.0184$	$g_1 = -.0298$	$g_{10} = -.0195$
(.0841)	(.0178)	(.1545)	(.0340)	(.0563)	(.0113)	(.1075)	(.0219)
$d_2 = -.0520$	$d_{11} =$ .0389	$e_2 =$ .0605	$e_{11} = -.0406$	$f_2 =$ .0435	$f_{11} = -.0040$	$g_2 =$ .2530	$g_{11} = -.0233$
(.0544)	(.0183)	(.1158)	(.0266)	(.0362)	(.0115)	(.0776)	(.0186)
$d_3 = -.0410$	$d_{12} =$ .0431	$e_3 =$ .0334	$e_{12} = -.0388$	$f_3 =$ .0051	$f_{12} =$ .0106	$g_3 =$ .1889	$g_{12} = -.0228$
(.0328)	(.0192)	(.0693)	(.0220)	(.0215)	(.0121)	(.0457)	(.0180)
$d_4 = -.0296$	$d_{13} =$ .0449	$e_4 =$ .0114	$e_{13} = -.0362$	$f_4 = -.0217$	$f_{13} =$ .0239	$g_4 =$ .1348	$g_{13} = -.0185$
(.0208)	(.0197)	(.0437)	(.0219)	(.0132)	(.0125)	(.0281)	(.0195)
$d_5 = -.0179$	$d_{14} =$ .0440	$e_5 = -.0061$	$e_{14} = -.0331$	$f_5 = -.0383$	$f_{14} =$ .0345	$g_5 =$ .0900	$g_{14} = -.0113$
(.0179)	(.0189)	(.0398)	(.0241)	(.0113)	(.0121)	(.0255)	(.0210)
$d_6 = -.0063$	$d_{15} =$ .0401	$e_6 = -.0195$	$e_{15} = -.0300$	$f_6 = -.0461$	$f_{15} =$ .0409	$g_6 =$ .0537	$g_{15} = -.0019$
(.0190)	(.0173)	(.0448)	(.0254)	(.0122)	(.0111)	(.0288)	(.0211)
$d_7 =$ .0049	$d_{16} =$ .0328	$e_7 = -.0292$	$e_{16} = -.0273$	$f_7 = -.0466$	$f_{16} =$ .0418	$g_7 =$ .0253	$g_{16} =$ .0092
(.0197)	(.0191)	(.0479)	(.0254)	(.0128)	(.0124)	(.0306)	(.0207)
$d_8 =$ .0153	$d_{17} =$ .0218	$e_8 = -.0357$	$e_{17} = -.0256$	$f_8 = -.0412$	$f_{17} =$ .0355	$g_8 =$ .0042	$g_{17} =$ .0211
(.0192)	(.0301)	(.0466)	(.0292)	(.0124)	(.0198)	(.0295)	(.0252)
$d_9 =$ .0247	$d_{18} =$ .0069	$e_9 = -.0394$	$e_{18} = -.0252$	$f_9 = -.0313$	$f_{18} =$ .0208	$g_9 = -.0105$	$g_{18} =$ .0332
(.0183)	(.0509)	(.0415)	(.0460)	(.0117)	(.0337)	(.0261)	(.0408)

Note: The $d_2 - d_{18}$, $e_2 - e_{18}$, $f_2 - f_{18}$, and $g_2 - g_{18}$ have each been estimated with a third-order polynomial with no fore- or endpoint constraints. Asymptotic standard errors in parentheses.

that for the bond market the answer is yes. Bond market data provides no evidence that interest rate forecasts are irrational. Thus evidence of irrationality in the Goldsmith-Nagan survey of interest rate expectations can be interpreted as casting doubt on the accuracy of this survey measure for describing market expectations. The accuracy of the Livingston price expectations data, however, is still an open question since irrationality has been found in both the bond market and survey data for the 1959–1969 period. This issue cannot be resolved without further empirical research on the rationality of this survey data over longer sample periods.