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Part I

THEORETICAL ANALYSIS



Individual Acquisition of Earning Power

Investments in people are time consuming. Each additional period of schooling or job training postpones the time of the individual's receipt of earnings and reduces the span of his working life, if he retires at a fixed age. The deferral of earnings and the possible reduction of earning life are costly. These time costs plus direct money outlays make up the total cost of investment. Because of these costs investment is not undertaken unless it raises the level of the deferred income stream. Hence, at the time it is undertaken, the present value of real earnings streams with and without investment are equal only at a positive discount rate. This rate is the internal rate of return on the investment.

For simplicity the rate of return is often treated as a parameter for the individual. This amounts to assuming that a change in an individual's investment does not change his marginal (hence average) rate of return. Another empirically convenient assumption is that all investment costs are time costs. This assumption is more realistic in such forms of human capital investments as on-the-job training, but less so in others, such as schooling, migration, or investments in health. In calculating schooling costs, an equivalent assumption is that students' direct private costs are exactly offset by their part-

time earnings during the year.¹ Like the preceding one, this assumption is not essential. Detailed information on direct costs can be incorporated into the model to yield a more precise empirical analysis. We forego precision, in order to gain in the simplicity of exposition and analysis.

The first step is to analyze the effects of investments in schooling. This is done by assuming that no further human capital investments are undertaken after completion of schooling and also, at this stage, that the flow of individual earnings is constant throughout the working life. For this the cessation of net investment is a necessary, but not sufficient, condition. Also excluded are economywide changes affecting individual productivity and earnings during the life cycle.

Since changes in earnings are produced by *net* investments in human capital stock, the net concept is used in most of the analysis. In this section, zero depreciation is, in effect, assumed during the school years and zero net investment during the working life. These assumptions are amended in later sections and in empirical interpretations.

In specifying the lengths of earning lives it is first assumed that each additional year of schooling reduces earning life by exactly one year. An alternative, and mathematically simpler, formulation is one in which the span of earning life remains the same in all cases, with more educated people retiring at correspondingly later ages. Empirically, this assumption is more nearly the correct one.²

1. This assumption was defended and used by Hanoeh (1967, pp. 317-320).

2. More educated men retire later. The length of working life is roughly constant. Only after high school does an additional year of schooling reduce earning life somewhat (by less than half a year).

The following table contains estimates of the average "retirement" age and length of working life of men classified by level of schooling. It is based on a March 1970 BLS labor force survey (1970b, Table E, p. A-11). Very similar estimates are produced from data in years before 1970.

Years of Schooling	Estimated Average Re- tirement Age	Estimated Length of Working Life
8	65	47
9-11	66	47
12	67	47
13-15	67	45
16	68	45
17 or more	70	45

(note continued)

When earning life is long, the alternative formulations cannot make much of a difference. What matters is the deferral of earnings: The cost of currently postponing earnings by one year is much more significant than the present cost of reducing earnings by one year, four or five decades hence. An infinite earning life can, of course, be viewed as a special case of the equal-span assumption. The advantage of the latter formulation is both its greater tractability and its flexibility in empirical interpretation.

1.1 THE SCHOOLING MODEL

In calculating the effects of schooling on earnings, it is first assumed that postponement of earnings due to lengthier schooling is tantamount to a reduction of the earning span.

Let

- n = length of working life plus length of schooling
- = length of working life for persons without schooling
- Y_s = annual earnings of an individual with s years of schooling
- V_s = present value of an individual's lifetime earnings at start of schooling
- r = discount rate
- $t = 0, 1, 2, \dots, n$ time, in years
- d = difference in the amount of schooling, in years
- e = base of natural logarithms

Then

$$V_s = Y_s \sum_{t=s+1}^n \left(\frac{1}{1+r} \right)^t,$$

(Note 2 concluded)

Estimates of retirement age are obtained by adding to age 45 the product of participation rates and years beyond the age of 45. The length of working life is the sum of products of participation rates and age intervals.

Estimates of lengths of working life in eight broad occupational groups, based on 1930–50 Census data, suggested larger differences in the earning spans among occupations. Note, however, that because of occupational mobility, length of stay in an occupational class, even when that is broadly defined, is not coextensive with length of stay in the labor force. Compare Mincer (1958, p. 284, n. 12).

The finding that the length of earning life of more educated men is the same as that of the less educated is not inconsistent with the observed positive relation between schooling and labor force participation at the middle and older ages (Bowen and Finnegan, 1969): A negative relation holds when the more educated are still at school.

when the discounting process is discrete. And, more conveniently, when the process is continuous:

$$V_s = Y_s \int_s^n e^{-rt} dt = \frac{Y_s(e^{-rs} - e^{-rn})}{r}.$$

Similarly, the present value of lifetime earnings of an individual who engages in $s - d$ years of schooling is:

$$V_{s-d} = \frac{Y_{s-d}}{r} (e^{-r(s-d)} - e^{-rn}).$$

The ratio, $k_{s,s-d}$, of annual earnings after s years to earnings after $s - d$ years of schooling is found by letting $V_s = V_{s-d}$:

$$k_{s,s-d} = \frac{Y_s}{Y_{s-d}} = \frac{e^{-r(s-d)} - e^{-rn}}{e^{-rs} - e^{-rn}} = \frac{e^{r(n+d-s)} - 1}{e^{r(n-s)} - 1}. \quad (1.1)$$

It is easily seen that $k_{s,s-d}$ is (1) larger than unity, (2) a positive function of r , (3) a negative function of n . In other words, (1) people with more schooling command higher annual pay; (2) the difference between earnings of individuals due to the difference in investment of d years of schooling is larger the higher the rate of return on schooling; (3) the difference is larger the shorter the general span of working life, since the costs of schooling must be recouped over a *relatively* shorter period.

These conclusions are quite obvious. Less obvious is the finding that $k_{s,s-d}$ is a positive function of s (d fixed); that is, relative income differences between, for example, persons with 10 years and 8 years of schooling are larger than those between individuals with 4 and 2 years of schooling, respectively. However, since the change in $k_{s,s-d}$ with a change in s and n is negligible³ when n is large, it can be, for all practical purposes, treated as a constant, k .

The conclusion that k is constant holds exactly when spans of

$$3. \quad \frac{\partial k}{\partial s} = \frac{r[e^{r(n+d-s)} - e^{r(n-s)}]}{[e^{r(n-s)} - 1]^2} > 0; \quad \frac{\partial k}{\partial s} \rightarrow 0, \text{ when } n \rightarrow \infty;$$

$$\frac{\partial k}{\partial n} = \frac{r[e^{r(n-s)} - e^{r(n+d-s)}]}{[e^{r(n-s)} - 1]^2} > 0; \quad \frac{\partial k}{\partial n} \rightarrow 0, \text{ when } n \rightarrow \infty.$$

Both partial derivatives are numerically very small when r and n are in a wide neighborhood of 0.10 and 40, respectively.

earning life are assumed fixed, regardless of schooling. Redefine n as the fixed span of earning life.

Then

$$V_s = Y_s \int_s^{n+s} e^{-rt} dt = \frac{Y_s}{r} e^{-rs} (1 - e^{-rn});$$

$$V_{s-d} = Y_{s-d} \int_{s-d}^{n+s-d} e^{-rt} dt = \frac{Y_{s-d}}{r} (1 - e^{-rn}) e^{-r(s-d)};$$

and solving for $k_{s,s-d}$ from the equalization of present values, we get:

$$k_{s,s-d} = \frac{Y_s}{Y_{s-d}} = \frac{e^{-r(s-d)}}{e^{-rs}} = e^{rd}. \quad (1.2)$$

Here, in contrast to (1.1) the earnings ratio, k , of incomes differing by d years of schooling does not at all depend on the level of schooling (s) nor, more interestingly, on the length of earning life (n), when that is finite, even if short.

Now, define $k_{s,0} = Y_s/Y_0 = k_s$. By (1.2), $k_s = e^{rs}$. In logarithms the formula becomes:

$$\ln Y_s = \ln Y_0 + rs. \quad (1.3)$$

Equation (1.3) exhibits the basic conclusion that percentage increments in earnings are strictly proportional to the absolute differences in the time spent at school, with the rate of return as the coefficient of proportionality. More precisely, equation (1.3) shows the logarithm of earnings to be a strict linear function of time spent at school.

1.2 POST-SCHOOL INVESTMENTS: INDIVIDUAL EARNINGS PROFILES

The "schooling model" represented by equation (1.3) is the most primitive form of a human capital earnings function: Y_s in (1.3) is the level of earnings of persons who do not invest in human capital beyond s years of schooling. Since most individuals continue to develop their skills and earning capacity after completion of schooling, Y_s cannot be directly observed. Instead, an "earnings profile" is observed: the variation of earnings with age during the working life.

We proceed to a human capital analysis of the earnings profile, at first ignoring depreciation phenomena.

After entering the labor force in year j , the worker devotes resources C_j mainly in furthering his job skills and acquiring job-related information, whether in the form of direct dollar outlays or opportunity costs of time devoted to these purposes, on or off the job. His "net" earnings Y_j in year j are obtained, therefore, by deducting C_j dollars from his "gross" earnings or "earnings capacity" E_j , which he would earn if he did not continue to invest in himself.⁴

Accordingly, earnings during the first year of work experience, $j = 0$, are $Y_0 = Y_s - C_0$, where $Y_s (= E_s)$ is the initial earning capacity after completion of s years of schooling.

If investment ceased subsequently, earnings in the next year (and afterward) would be: $Y_1 = Y_s + r_0 C_0$. However, if investment in that year is C_1 , then $Y_1 = Y_s + r_0 C_0 - C_1$. More generally, net earnings in year j are:

$$Y_j = Y_s + \sum_{t=0}^{j-1} r_t C_t - C_j = E_j - C_j. \quad (1.4)$$

The generality of expression (1.4) is evident, since the start of index t is essentially arbitrary. In Becker's original statement of the accounting equation (1.4), Y_0 replaces Y_s ; and in instalments C_t , schooling and post-school investments are not distinguished. In fact, the expression for Y_s , the schooling model, is a special case of (1.4), in which investments are restricted to time costs of schooling and rates of return are the same in all periods. Then, with $C_t = E_t$:

$$E_s = Y_0 + r \sum_{t=1}^s E_{t-1} = Y_0(1 + r)^s, \quad (1.5)$$

which is a discrete approximation of (1.3).

Using equation (1.4) we can proceed to the analysis of variation of earnings over the working life.⁵ On the assumption that working

4. Note that observed earnings, as they are usually reported, would equal "net" earnings if C_j consisted of opportunity costs only. However, direct costs are included in reported earnings. Thus observed earnings overstate "net" earnings, but since direct costs are much smaller than opportunity costs, observed earnings more closely approximate Y_j than E_j .

5. At this point we are abstracting from variations in hours or weeks of labor supplied over the life cycle. Some consideration is given to this factor later.

life starts in the period following the completion of schooling, equation (1.4) points to post-school investments C_j as the variable which traces out the individual "age profile" of earnings. The initial earning capacity Y_s acquired in years of schooling s is taken as constant for a given individual, though it may vary among individuals. Y_s is not readily observed, since most or all individuals are assumed to engage in post-school investment of one form or another.

The variation of earnings with experience is best observed by considering the annual increment of earnings in (1.4):

$$\Delta Y_j = Y_{j+1} - Y_j = r_j C_j - (C_{j+1} - C_j). \quad (1.6)$$

According to (1.6), earnings grow with experience so long as net investment (C_j) is positive and its annual instalments either diminish [$(C_{j+1} - C_j) < 0$] or increase at a rate lower than the rate of return:

$$\text{for } \Delta Y_j > 0, \frac{C_{j+1} - C_j}{C_j} < r_j.$$

Note that if investments increase sharply (at a faster rate than r), net earnings will decline, presumably temporarily. However, gross earnings always increase, so long as investment is positive, since

$$\Delta E_j = r_j C_j. \quad (1.7)$$

If both r_j and investment are the same in all periods ($C_j = C_{j+1}$; $r_j = r$), net and gross earnings grow linearly. Henceforth we shall assume that all $r_j = r$.

While constant or linearly increasing investment C_j is conceivable for some stages of individual work experience, these assumptions cannot be expected to hold over any long periods of the working life. Such assumptions are inconsistent with the theory of optimal allocation of investment in human capital over the life cycle. Rational allocation requires that most of the investment be undertaken at younger ages. Thus schooling, a largely full-time activity, precedes job-training, a largely part-time activity, and the latter diminishes with age, terminating years before retirement.

According to Becker (1964 and 1967) this tendency is due to the following incentives for shifting from learning to earning activities as soon as possible: (1) With finite lifetimes, later investments produce returns over a shorter period; so total benefits are smaller. (2) To the extent that investments in human capital are profitable, their

postponement reduces the present value of net gains. (3) A person's time is an important input in his investment, but the consequence of human capital accumulation is an increase in the value of his time; thus investments at later periods are more costly, because forgone earnings (per hour) increase. However, these incentives would be overridden in the special or temporary cases where productivity in learning grows as fast or faster than productivity in earning.

Should we then not expect an early and quick accumulation of all the desired human capital even before individuals begin their working life? The answer of human capital theory to this question is twofold: Investments are spread out over time because the marginal cost curve of producing them is upward sloping within each period. They decline over time both because marginal benefits decline and because the marginal cost curve shifts upward.

Specifically, the argument (Ben-Porath, 1967; Becker, 1967) visualizes individuals as firms which produce additions (Q) to their own human capital stock (H) by combining their human capital with their own time (T) and with other market resources (R) in a production function:

$$Q = f(H, T, R).$$

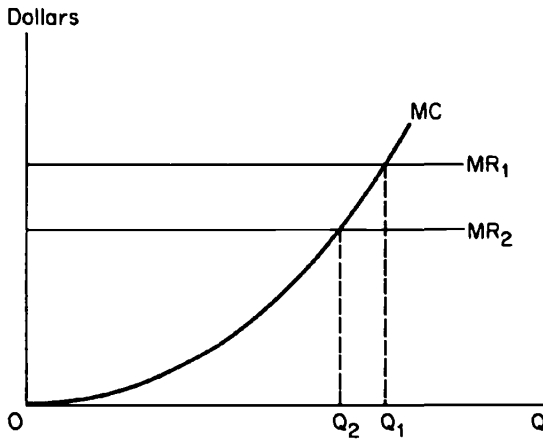
Attempts to increase investments Q within a given period run into diminishing returns: Costs rise with the speed of production. Thus the marginal cost curve in Figure 1.1 is upward sloping.

The marginal revenue obtained by adding a unit of investment to the capital stock is the discounted flow of future increases in earning power. For reasons indicated, the benefits of later investments decline. The MR curve slides downward with increasing age, tracing out a declining pattern of investment over the life cycle.

The decline is reinforced if the MC curve shifts to the left with advancing age. As already mentioned, this is not a logical necessity: MC would remain fixed if earning and learning powers increased at the same rate. A recent attempt by Ben-Porath (1970) to test for such "neutrality" empirically suggests that investments decline over earning life faster than would be predicted by the mere downward slide of MR on a fixed MC curve in Figure 1.1. By implication, marginal costs rise over the life cycle.

Investments, however, need not decline throughout the life cycle. Ben-Porath (1967) has shown that the optimization process

FIGURE 1.1
PRODUCTION OF HUMAN CAPITAL



may lead to an increase in investment during the early stages because of "corner solutions": The initial stock (H_0) may be so small that even an input of all the available time, other resources not being highly substitutable, produces less than the optimal amount of output. As the stock increases, investment output will increase for a while until an optimum is reached with an input of less than the total available time. At this point investments and the time devoted to them begin to decline. The initial period of complete specialization in the production of human capital is devoted to full-time schooling. It is identified by the absence of earnings, a condition which may end before the completion of schooling.

The optimization process described above applies explicitly (Ben-Porath, 1967) to gross investments in human capital. Note, however, that the predicted decline in gross investment applies a fortiori to net investment if depreciation is constant or increases with age.

Two major conclusions can be drawn from the Ben-Porath analysis:

1. The higher the marginal revenue curve and the lower the marginal cost curve (cf. Figure 1.1), the larger the investment in human capital in any given period. Marginal revenue is higher the lower the discount rate and the depreciation rate, and the longer the expected length of working life. Marginal cost is lower, the greater the learning

ability of the individual. Since the nature and conditions of individuals which these factors describe change rather slowly, the size of single-period investments is likely to be an index of lifetime investments. Longer schooling is likely to be followed by greater post-school investment, and generally, the serial correlation of instalments of investment is likely to be positive.

2. While the preceding inference is significant for a distributional analysis, the major implication of Ben-Porath's optimization analysis for the individual investment profile is that investment costs can be expected to decline after the schooling stage. As a result, both gross and net earnings slope upward during the positive net investment period. Moreover, the age profile of gross earnings is concave from below. From (1.7), we have the second difference:

$$\Delta^2 E_j = r\Delta C_j < 0, \quad (1.8)$$

since $\Delta C_j < 0$. Net earnings need not be concave throughout. The profile is concave if the decline of investments (C_j) is a nonincreasing function of j , i.e., if

$$\Delta^2 Y_j = r\Delta C_j - \Delta^2 C_j < 0. \quad (1.9)$$

If investments decline at a strongly increasing rate for a while, so that the inequality sign is reversed, age profiles may rise at an accelerating rate for a while; but eventually they become concave as net investment terminates.

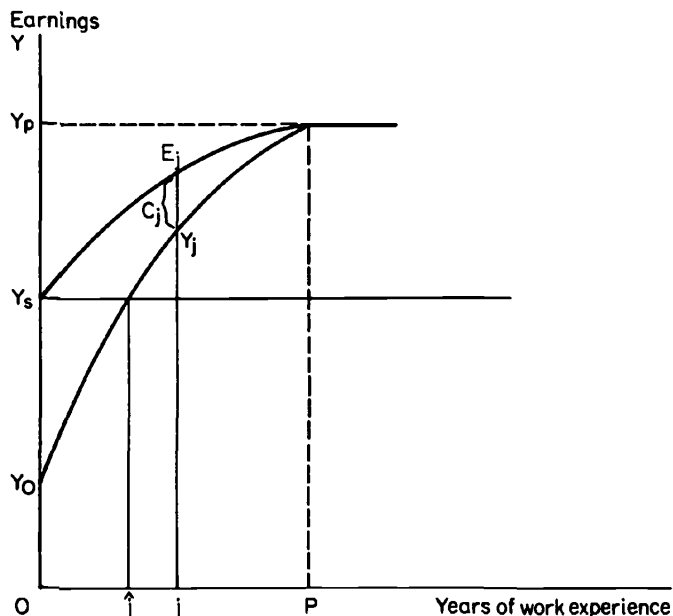
The profile of net earnings has a steeper slope than gross earnings, since $\Delta Y_j = \Delta E_j - \Delta C_j$, and $\Delta C_j < 0$. The peak of both gross and net earnings is reached when positive net investments equal zero.⁶

Figure 1.2 indicates the shape of gross earnings E_j and net earnings Y_j during the post-school investment period OP . Of particular interest are the initial earnings capacity Y_s and peak earnings Y_p . The former, Y_s , is the earnings concept used in the schooling model. Its estimate is particularly useful for the empirical analyses in this study. Estimates of Y_s and of Y_p would make possible quick and simple methods of estimating rates of return and amounts of investment costs.

During the early years of experience, earnings of continuing in-

6. I abstract from depreciation and from changing hours of work.

FIGURE 1.2
EARNINGS PROFILES



vestors are smaller than the Y_s earnings that can be obtained after s years of schooling without further investments. But earnings of investors continue to grow and, before long, exceed Y_s . In Figure 1.2, \hat{j} is the overtaking year of experience. Knowledge of \hat{j} permits one to read off the value Y_s from the profile of observed earnings Y_j . It turns out that \hat{j} is an early stage of experience, and its upper limit can be estimated from equation (1.4):

$$Y_j = Y_s + r \sum_{t=0}^{j-1} C_t - C_j = Y_s, \quad \text{when} \quad \sum_{t=0}^{j-1} C_t = C_j.$$

If instalments C_t from $t = 0$ to $t = \hat{j}$ are equal, then $r\hat{j}C_j = C_j$; so $\hat{j} = 1/r$. If C_t declines, \hat{j} is reached sooner; therefore, assume C_t is not increasing. Then,

$$\hat{j} \leq \frac{1}{r}. \tag{1.10}$$

To illustrate, if r exceeds 10 per cent, it takes less than a decade for the trained person to overtake the untrained one, if both start their working life with the same initial earning capacity.

Even for the rough estimate of Y_s by (1.10) it is necessary to know the value of r_p , the rate of return to post-school investment. If r_p is known, Y_s can be more precisely determined, since at the start of working life the present value of the constant earnings stream Y_s must equal the present value of the observed earnings profile Y_j , with r_p as the discount rate. If, as is perforce assumed in empirical calculations of rates of return to "education," the rate of return to post-school investment (r_p) equals r_s , the rate of return to schooling, the conventionally calculated rates can be applied to estimate Y_s . In turn, estimates of Y_s at two levels s_1 and s_2 make it possible to apply a check of internal consistency to the hypothesized equality $r_s = r_p$, since, by (1.3) $\ln Y_{s_2} - \ln Y_{s_1} = r_s(s_2 - s_1)$. Further applications of the "overtaking" or "crossover" point of the earnings profile to short-cut estimation of changes in rates of return and to distributional analysis are elaborated in Part II of this study.

At the end of the net investment period,

$$Y_p = Y_s + r_p \sum_{t=0}^p C_t. \quad (1.11)$$

The total volume of post-school investment costs $\sum_{t=0}^p C_t$ can be estimated, once r_p , hence Y_s , are known,⁷ since

$$\sum_{t=0}^p C_t = \frac{Y_p - Y_s}{r_p}. \quad (1.12)$$

Similarly, the costs of rising from schooling level s_1 to level s_2 are:

$$\sum_{s_1}^{s_2} C_s = \frac{Y_{s_2} - Y_{s_1}}{r_s}. \quad (1.13)$$

The above analysis of dollar profiles of earnings is easily translated into an analysis of logarithmic earnings profiles. This is not only useful but necessary, for two reasons: (1) Relative (percentage) variation in earnings is of major interest in the study of income inequality; and (2) for empirical analysis, post-school investments must be expressed in the same "time" units as schooling. Indeed, the conversion of investment costs into time-equivalent values trans-

7. In Figure 1.2, total post-school investment costs are given by the area $Y_0 Y_s Y_p$.

forms the earnings equation (1.4) into its logarithmic version. This is accomplished by the following device:⁸

Let k_j be the ratio of investment costs C_j to gross earnings E_j in period j . This ratio can be viewed as the fraction of time (or "time-equivalent," if investment costs include direct outlays as well as time costs) the worker devotes to the improvement of his earning power. His net earnings in year j are, therefore, smaller by this fraction than they would be if he did not invest during year j :

$$C_j = k_j E_j,$$

and

$$E_j = E_{j-1} + rC_{j-1} = E_{j-1}(1 + rk_{j-1}).$$

By recursion, therefore:

$$E_j = E_0 \prod_{t=0}^{j-1} (1 + r_t k_t).$$

Assuming $k \leq 1$ and r relatively small, this is approximately:

$$\ln E_j = \ln E_0 + \sum_{t=0}^{j-1} r_t k_t; \quad (1.14)$$

and since $Y_j = E_j(1 - k_j)$, we get

$$\ln Y_j = \ln E_0 + \sum_{t=0}^{j-1} r_t k_t + \ln(1 - k_j). \quad (1.15)$$

The assumption that $k_j = 1$ during the school years shows (1.15) to be an expansion of the schooling model:

$$\ln Y_j = \ln E_0 + r_s s + r_p \sum_{t=0}^{j-1} k_t + \ln(1 - k_j). \quad (1.16)$$

The assumption that r_j is the same for all post-school investments simplifies matters. Let

$$K_j = \sum_{t=0}^{j-1} k_t,$$

the cumulative amount of "time" expended in post-school investments before year j . Then

$$\ln E_j = \ln E_0 + r_s s + r_p K_j = \ln Y_s + r_p K_j. \quad (1.17)$$

8. This device was applied by Becker and Chiswick (1966) to schooling investment. Here it is extended to cover post-school investments.

If $r_s = r_p$, we have, denoting $h_j = (s + K_j)$, the simplest generalization of the schooling model:

$$\ln E_j = \ln E_0 + rh_j. \quad (1.18)$$

When the investment period is completed, K_p is total "time" devoted to post-school investment. It can be calculated from (1.17), if r_p is known:

$$K_p = \frac{\ln Y_p - \ln Y_s}{r_p}. \quad (1.19)$$

The earnings profiles under these assumptions provide information on the number of "years" of post-school "training," a statistic that is impossible to obtain in surveys of workers or firms, and one that is bound to be greatly underestimated if it is based on reported apprenticeship periods or other formal training programs in firms.⁹

The shape of the log-earnings profile is upward sloping so long as $k_j > 0$. Its rate of growth and concavity are given by the first and second derivative of (1.15) with the same conclusions as in the dollar profiles, with k_j replacing C_j .

Note that the decline in k_j with experience follows a fortiori from the assumption of declining dollar values of post-school investments, and that consequently concavity in the logarithmic profile is to be expected more frequently, that is, even when the dollar profile is linear or S-shaped.

In the foregoing analysis it was assumed that (1) net investment is never negative, that is, the formulation abstracts from depreciation phenomena; and (2) changes in earnings over the life cycle represent changes in earning capacity rather than changes in hours of work supplied to the labor market (including the hours spent in on-the-job training).

The first assumption is not seriously misleading in the life-cycle context if the second is maintained: As Chart 4.4 in Part II shows, "full-time" earnings or wage rates reach a peak and remain on a plateau until men reach an age near retirement. On average (the data are mean earnings classified by years of education), net investment may be viewed as non-negative through most of the working life. Still, the finiteness of life, the increasing incidence of illness at older

9. Cf. discussion in Mincer (1962). Of course K_p includes forms of investment other than post-school training. Information and job mobility are examples.

ages, and the secular progress of knowledge, which makes older education and skill vintages obsolescent, are compelling facts suggesting that as age advances, effects of depreciation eventually begin to outstrip gross investment.

To accommodate these phenomena the formulation is amended by positing a rate δ_t at which the human capital stock H_t depreciates in time period t . Then

$$E_t = E_{t-1} + rC_{t-1}^* - \delta_{t-1}E_{t-1}, \quad (1.20)$$

where C_t^* denotes gross investment, as C_t denoted net investment. Letting the gross investment ratio $k_t^* = C_t^*/E_t$, we get:

$$\frac{E_t}{E_{t-1}} = 1 + rk_{t-1}^* - \delta_{t-1} = 1 + rk_{t-1};$$

thus $k_t = [k_t^* - (\delta_t/r)]$, and

$$\ln E_t = \ln E_{t-1} + \ln (1 + rk_{t-1}^* - \delta_{t-1})$$

by recursion, and assuming $(rk_t^* - \delta_t)$ is small, we have:

$$\ln E_t = \ln E_0 + \sum_{j=0}^{t-1} (rk_j^* - \delta_j), \quad (1.21)$$

and

$$\ln Y_t = \ln E_t + \ln (1 - k_t^*) \quad (1.22)$$

as an amendment to (1.14). It is clear that the peak of earning capacity E_t is reached when $k_t = 0$, i.e., when $k_t^* = \delta_t/r$; call it $k^*(E \text{ max})$. It is also clear that observed wage rates reach a peak some time thereafter, since from (1.22):

$$\ln Y_t = \ln Y_{t-1} + \ln (1 + rk_{t-1}^* - \delta_{t-1}) + \ln (1 - k_t^*) - \ln (1 - k_{t-1}^*) = 0 \quad (1.23)$$

only when $rk_{t-1}^* < \delta_{t-1}$, i.e., when net investment is negative. It can be shown ¹⁰ that if δ is fixed and if the gross investment ratio k_t^* declines

10. From (1.23) Y_t reaches a maximum when, approximately

$$rk_{t-1}^* - \delta = k_t^* - k_{t-1}^*.$$

Then

$$r[k_{t-1}^* - k^*(E \text{ max})] = k_t^* - k_{t-1}^*$$

and

$$\hat{t} = \frac{k^*(E \text{ max}) - k_{t-1}^*}{k_{t-1}^* - k_t^*} = \frac{1}{r}.$$

linearly over time, the (unobservable) peak of earning capacity precedes the (observed) peak of wage rates by $\hat{t} = 1/r$, that is, by about a decade or even earlier if the rate of decline of k_t^* diminishes over the life cycle. Note, also, that while the net investment period terminates before peak earnings (wage rates) are observed, the gross investment period continues beyond it.

In a few recent human capital analyses in which depreciation is taken into account, the rate is assumed to be fixed purely for mathematical convenience.¹¹ Yet, the depreciation rate on human capital is likely to be related to age, experience, and size and vintage of stock. If descriptions in developmental psychology can serve as a guide, the life-cycle pattern of δ_t after the individual matures is plausibly described as flat and very low, beginning to rise in the fifties.¹²

To the extent that hours of work vary over the life cycle, the profile of annual earnings is affected. Under conditions of certainty, for example, individual wealth can be considered fixed, while the cost of time grows with experience until peak earning capacity is reached. If so, the growth and decline of earning capacity is likely to induce a corresponding pattern of hours of work supplied to the market. Hence, the growth of observed *annual* earnings leads to overestimates of investments in human capital or of rates of return. Hours of

11. Cf. Johnson (1970), Rosen (1974), and Koeune (1972).

12. Health statistics show the proportions of workers with some limitations of work activities during the year to be rising slowly to 13 per cent of those in the 45–54 age range, and accelerating to 55 per cent at age 75. However, in a survey of the psychological literature, Birren (1968, pp. 180–181) states: "Except for individuals with cumulative injuries or problems of health, worker performance up to age 60 should be little influenced by *physiological* changes in aging." In discussing age changes in *learning capacity*, the same author states: "There has been a general tendency since the work of E. L. Thorndike in the 1920's to advance continually the age at which subjects in learning research are regarded as aged. At the present time there is little evidence to suggest that there is an intrinsic age difference in learning capacity over the employed years, i.e. up to age 60."

Psychologists note, of course, that it is difficult empirically to isolate *intrinsic* age patterns in productivity, that is, changes that are not affected by the individual's adaptation, such as health care and training—gross investment, in our terminology. Hence, their observations of time changes in "productive capacity" often show systematic differences when individuals are stratified by education, social background, ability measures, and so forth. [See Mincer (1957, Chap. 1, n. 1).] To the extent that these patterns reflect differential patterns of "adaptation," the analysis of human capital investment behavior is likely to contribute to an understanding of these findings, rather than conversely.

work may peak before observed wage rates because, as noted above (cf. note 10), capacity wage rates decline before observed wage rates do, given human capital depreciation.¹³

Variation in hours (weeks) worked is taken account of in the empirical analyses. The analysis of the relation between hours of work and human capital investments is not theoretically integrated into the present model. Though the problem is discussed in several places below, its fuller development is relegated to a future study.

13. Recent analyses of optimal allocation of consumption and work over the life cycle by Becker and Ghez (1967 and 1972) suggest that hours of work are likely to peak before earning capacity, a fortiori before observed wage rates decline.