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A Forecast Determination of National Product, Employment, and Price Level in Canada, from an Econometric Model

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Introduction

A PILOT econometric project was begun in Canada in the summer of 1947 under the direction of Lawrence R. Klein. I was one of those who studied and worked under Klein for three hot and busy but delightful months, nearly fifteen years ago. The work was done within the Department of Trade and Commerce, in Ottawa, to which I am eternally grateful, for facilities provided and interest shown. The results displayed here represent my subsequent research plus results due to S. J. May, who did splendid work on the project from 1949 to 1956, and who has since returned to it. Any opinions expressed in what follows on either analytical matters or policy are solely my own, arising out of my personal research, and the Department of Trade and Commerce is not responsible for them.

The purpose of this paper is to present a snapshot of a model at one stage of the Canadian project; to show how this model could be used to determine national product, employment, and price level; and, finally, to show the results obtained with this model on one experimental prediction of the economy for 1958—a critical year in recent economic history.

Model VIII

This model has evolved out of the original model, designed by Klein to suit Canadian conditions and available data. The evolution has included refinements in the underlying hypotheses—the interaction between hypotheses and observed data—and extensive work on re-fashioning basic data into a form to match the economic hypotheses. In this phase of the work, great thanks are due to the Dominion Bureau of Statistics and other government departments and crown

corporations for their help in providing basic series of data. But the processing of these data into the form suggested by theory had to be done within the project, and consumed inordinate amounts of research time. In the field of econometric research one can yearn for the complementarity of highly skilled, resourceful, and painstaking processing of economic data, of the caliber associated with the National Bureau of Economic Research, with the different kinds of specialization required for the econometric research and applications.

In presenting the model I begin with a glossary of the symbols, the shorthand for the economic variables which appear in the model. The variables are presented in the order in which they occur.

GLOSSARY OF SYMBOLS

General notes

1. Symbols for variables which represent flows or stocks of goods and services in real volume terms are in units of billions of constant dollars of 1935–39. (Units for subsequent models have been converted to billions of 1949 dollars.) An exception to this rule is found in the employment and hours-of-work variables, where the units are millions of workers and thousands of hours of work per worker per year. These symbols can be converted to current market or money value by the addition of a subscript, *m*, or by multiplication by an appropriate price variable.

2. Symbols for financial variables (money, securities, international reserves) express the variable in current market or money value. These symbols can be converted to represent real purchasing power by the addition of a subscript, *r*, or by division by an appropriate price variable

3. Superscripts *d* and *s* are used to designate demand and supply functions. They are often implicit rather than explicit in the model. Superscripts or subscripts *a* and *na* are used to designate variables appropriate to the agricultural and nonagricultural private sectors; *g*, the government sector.

4. The unit of time in the model is one year. Thus, flow variables represent a rate of flow per year. Stocks of goods are year-end stocks. Numbers of workers (in a sense, a stock) are the average over a year.

5. The subscript -1 means that the variable is lagged one time period. Its influence is thus delayed one year in this model.

6. Variables marked by an asterisk are endogenous in this model.

Sector A

C_p = aggregate consumer purchases of new perishable goods C_p, services C_s, house rent paid and imputed C_r*

Y_w = disposable wage-salary income*

Y_π = disposable nonwage or property-enterprise income*
$= Y_{\pi p} + Y_{\pi np}$

$Y_{\pi p}$ = disposable nonwage income flowing into household and personal sector*

$Y_{\pi np}$ = disposable nonwage income not paid to persons (mainly undistributed corporation profits)*

u_1 = unexplained random residual associated with behavior equation 1

C_{sd} = consumer purchases of new semidurable goods*

$Y = Y_w + Y_\pi$*

L_h = liquid asset holdings of households (money, federal government securities, deposits in sundry financial institutions)

K_{sd} = household and individual stocks of semidurable goods

C_{hda} = consumer purchases of new household durable goods and automobiles*

$Y_p = Y_w + Y_{\pi p}$

dp = index of consumer credit conditions, reflecting size of minimum downpayment, and time of repayment

De_1 = consumer debt to finance companies plus personal cash loans of chartered banks, small loan companies, licensed money lenders, and credit unions

K_{hda} = household and individual stocks of household durables and automobiles

Sector B

H = total inventory of firms*

GNP^{na} = gross national product generated in the nonagricultural private sector*

GNP = gross national product = GNE*

GNE = gross national expenditure*

F_1' = imports of goods and services, including payments of interest and dividends to foreign owners of domestic capital*

P^+ = index of price level of $GNE + F_1'$*

N_p = number of paid workers employed, private sector*

h = average hours worked per employed worker during time period (one year)*

w_{ph} = average earnings per hour of paid workers employed, private sector*

P = price level of gross national expenditure (*GNE*)*

GI_M = gross investment by firms and nongovernment institutions (private sector) in machinery and equipment

P_1 = price level of imported goods adjusted for import duties and taxes

A_{F1} = shift variable to allow demand for imports equation to adjust to import controls of 1948, 1949, and 1950; in these years A_{F1} is given values of 1.0, 1.0, and 0.5, respectively; in all other years, zero

ΔKS = net inflow of capital for direct investment (K) and for portfolio (S)

P_{F1} = price level of F_1'

N_u = number of unemployed workers*

Sector C

Nh_u = number of unemployed man-hours*

N_1 = labor supply or labor force

N_M = number of personnel in the military, or armed, forces

t = time in years from base = calendar year 1926

Sector D

N_{enp} = number of employers and self-employed (entrepreneurs) plus unpaid family workers

K_{PCM} = stock of producers' fixed capital (plant, construction, machinery, and equipment)*

$\overset{0}{GNP}_{-1}$ = highest previous level of *GNP*

Sector E

T_w = direct taxes on wage-salary incomes*

P = price level of net national expenditure (NNE)*

W_m = total wage bill = $w_{ph}N_p h + w_g N_g + w_M N_M$*

Tr_w = transfer payments to wage-salary incomes from government*

Tr_{c1} = charitable contributions of corporations

Tr_{c2} = bad debt losses of corporations to wage sector

$T_{\pi p}$ = direct taxes on nonwage income of persons plus succession duties on personal wealth*

π_p = nonwage income of personal sector*

G_{ip} = total government interest bill paid to persons

$Tr_{\pi p}$ = transfer payments to nonwage personal incomes from government

Tr_{c3} = bad debt losses of corporations to other firms

$T_{\pi np}$ = direct tax on corporation profits, plus withholding tax, plus portion of trading profits of government business transferred to consolidated revenue of government*

Pr_c = net profit of corporations*

$Tr_{\pi np}$ = transfer payments to nonwage nonpersonal incomes from government

T_{i-s} = indirect taxes less subsidies*

F_{1s} = imports of services

P_C = price level of total consumer spending

C = $C_p + C_{sd} + C_{hda}$ = total consumer spending

GI = gross domestic investment in (producers') new durable capital

= $GI_{PC} + GI_M = GI_{PCM}$

D_{f1} = provision for depreciation and other capital consumption by firms

P_{GI} = price level of GI

π_{di} = payment of interest and dividends to foreign owners of domestic capital

π_{id} = receipts of interest and dividends arising from domestic ownership of foreign capital

G_i = total interest bill of government sector

Sector F

w_g, w_M = average earnings per year of government civilian and military employees

N_g = number of government civilian employees

π = total nonwage or property-enterprise income in *GNP**

D_{f2} = $Tr_{c2} + Tr_{c3}$*

J = capital gains or losses of firms on inventories, arising from price increases or decreases

$\frac{1}{2}R$ = residual error of estimate on income side of national accounts

Pr_f = net profit of all firms*

π_{ip} = interest receipts of resident persons
π_{rp} = rent receipts of resident persons
Pr_{ub} = net profit of unincorporated business*
Pr_{gb} = net profit of government business*
π_{dcp} = dividend payments from corporations to resident persons*
T_d = disposable income of government*
π_{dip} = receipts of dividends from abroad by resident persons
π_{np} = $\pi - \pi_p$*
h_s = standard hours of work per worker per year
D = real depreciation on K_{PCM}

Sector G

GI_d = gross domestic investment in new residential construction (dwellings)
F_2' = exports of goods and services, including receipts of interest and dividends from domestic ownership of foreign capital
GNF^d = $GNE^d + F_1'$, representing demand for all final goods in total economy*
GNF^s = total supply flow of new goods through complete economy*
G_1 = government output or value added = $(w_g N_g + w_M N_M)/P + \pi_{rg}$
π_{rg} = imputed net rent on government buildings and equipment

DESCRIPTION AND DISCUSSION OF MODEL VIII

A macroeconomic structure can be conceived of as the set or matrix of rates at which each aggregative variable in the system is influenced *directly* by all of the other variables in the system. The purpose of an econometric model is to measure this *direct* or *basic* structure, so that it can be used for analysis, prediction, and policy. The common way of proceeding is to express each variable in the system as a function of (or in a functional relationship with) the other variables in the system. In any such equation only those variables are included which deductive theory interacting with observed data indicates to be significant. Thus, the exclusion of a variable from an equation means that its *direct* influence on the variable being explained is believed to be zero. The exclusion of an equation for a variable implies that *no* currently dated variables in the system have any appreciable influence on it. It is accordingly determined outside of the current economic system being studied, and is designated as predetermined—exogenous or lagged. It follows that the predeter-

MODEL VIII—JANUARY 1958

Equations	Method of Estimation
A. DEMANDS FOR FINAL GOODS	
(1) $C_p^{\cdot} = .1437 + .4485Y_w + .03945Y_\pi + .5250C_{p,-1}^{\cdot} + u_1$	TFML
(2) $C_{sd} = .05756 + .1123Y + .04695L_{hr,-1} - .1016K_{sd,-1} + u_2$	TFML
(3) $C_{hda} = .02343 + .1499(Y_w + Y_\pi - Y_p^a - Y_{\pi np}) - .06225dp - 1.5104(De_1/L_h)_{-1} - .4759(K_{hda}/Y_p^{na})_{-1} + u_3$	LS
B. DEMANDS FOR FACTORS OF PRODUCTION	
(4) $H^{na} = .1656 + .08124(GNP^{na} + F_1') + .6403H_{-1}^{na} + .5798\Delta P^{+} + u_4$	LS
(5) $N_p^{na}h^{na} = .7139 + .5610GNP^{na} + .1592GNP_{-1}^{na} - 5.4312w_{ph}^{na} + 3.2727P^{\cdot} + u_5$	TFML
(6) $F_1' = .6501 + .1153(GNP^{na} + F_1') + .5821(H^{na} - H_{-1}^{na}) + .5525GI_M + .6847C_{hda} - .3434P_1 - .2155A_{F1} + .3114\frac{\Delta KS}{P_{F1}} + u_6$	LS
1957	*1958*	
(7) $h^{na} = 2.151 - .2425N_u$	$h^{na} = 2.129 - .2425N_u$	
C. LABOR MARKET ADJUSTMENT (SHORT-RUN SUPPLY)		
(8) $w_{ph}^{na} = .08994 + .9029w_{ph,-1}^{na} - .03703Nh_u^{na} + .06117\left(.3 - \frac{10N_u}{N_1 - N_M}\right)_{+\text{ or zero}} + .003941t + u_7$		TFML
D. PRODUCTION FUNCTION AND SUPPLY		
(9) $GNP^{na} = -4.9501 + 1.1579(N_p^{na} + N_{enp}^{na})h^{na} + .1507K_{PCM}^{na} + .3266\overset{0}{GNP}_{-1}^{na} + u_8$		LS
E. TAX AND TRANSFER EQUATIONS		
1957	*1958*	
(10) $T_w = -\frac{2.2148}{P} + .2578\frac{(W_m - W_m^a)}{P}$	$T_w = -\frac{2.1500}{P} + .2375\frac{(W_m - W_m^a)}{P}$	
(11) $PTr_w = 1.699 + 1.1844N_u$ $PTr_{c1} = .033$; $PTr_{c2} = .016$	$PTr_w = 1.946 + 1.2458N_u$ $PTr_{c1} = .030$; $PTr_{c2} = .017$	
(12) $PT_{\pi p} = -.7230 + .2424P(\pi_p + G_{ip})$ $PTr_{\pi p} = .3640$; $PTr_{c3} = .0100$	$PT_{\pi p} = -.7572 + .2368P(\pi_p + G_{ip})$ $PTr_{\pi p} = .3640$; $PTr_{c3} = .0110$	
(13) $T_{\pi np} = \frac{.2560}{P} + .6031(Pr_c - Tr_{c1})$	$T_{\pi np} = \frac{.2560}{P} + .5938(Pr_c - Tr_{c1})$	
$PTr_{\pi np} = .2060$	$PTr_{\pi np} = .2020$	
(14) $T_{i-s} = \frac{m_0}{P} + m_1\left[\left(\frac{P_{F1}}{P}\right)_{\text{Exog.}} F_1' - \frac{F_{1s}}{P}\right] + m_2C_p + m_3\left(\frac{P_C}{P}\right)_{\text{Exog.}} C + m_4\left[\left(\frac{P_C}{P}\right)_{\text{Exog.}} C + \frac{1}{3}\frac{P_M}{P}GI_M\right] + m_5\left[(GNP - D_{f1}) + \frac{P_{GI}}{P}D_{f1} + \frac{\pi_{dim} - \pi_{idm} - G_{im}}{P}\right]$		

(continued)

MODEL VIII—JANUARY 1958 (concluded)

Equations — Method of Estimation

	m_0	m_1	m_2	m_3	m_4	m_5
1957	1.2430	.09788	.01860	.01293	.05700	.01046
1958	1.3360	.09788	.01860	.01293	.05571	.01046

F. ACCOUNTING IDENTITIES AND DEFINITIONS

(1) $W_m = w_{ph}^{na} N_p^{na} h^{na} + w_p^a N_p^a + w_g N_g + w_M N_M$
(2) $Y_w = W_m/P + Tr_w + Tr_{c1} + Tr_{c2} - T_w$
(3) $\pi = GNP - W_m/P - T_{i-s} - D_{f1} - D_{f2} + J^{na} - \frac{1}{2}R$
(4) $D_{f2} = Tr_{c2} + Tr_{c3}$
(5) $\pi = Pr_f + \pi_{ip} + \pi_{rp}$
(6) $Pr_f = Pr_c + Pr_{ub} + Pr_{gb}$
(7) $Pr_{ub}^{na} = n_1 Pr_c$
(8) $Pr_{gb} = n_2 Pr_c$

Note that $Pr_c = (\pi - \pi_{ip} - \pi_{rp} - Pr_{ub}^a)/(1 + n_1 + n_2)$ and that n_1, n_2, and n_3 can vary from year to year.

(9) $\pi_p = \pi_{ip} + \pi_{rp} + \pi_{dcp} + \pi_{dip} + Pr_{ub}$
(10) $\pi_{dcp} = n_3(Pr_c - Tr_{c1})$
(11) $\pi_{np} = \pi - \pi_p$
(12) $Y_{\pi p} = \pi_p + Tr_{\pi p} + Tr_{c3} - T_{\pi p}$
(13) $Y_{\pi np} = \pi_{np} + D_{f2} + Tr_{\pi np} - Tr_{c1} - Tr_{c2} - Tr_{c3} - T_{\pi np}$
(14) $Y_\pi = Y_{\pi p} + Y_{\pi np}$
(15) $T_d = T_w + T_{\pi p} + T_{\pi np} + T_{i-s} - Tr_w - Tr_{\pi p} - Tr_{\pi np} = Y_g$
(16) $N_u = N_1 - N_M - N_g - N_{enp}^a - N_p^a - N_{enp}^{na} - N_p^{na} = N_1^{na} - N_{pe}^{na}$
(17) $Nh_u^{na} = N_1^{na} h_s^{na} - (N_p^{na} + N_{enp}^{na}) h^{na}$
(18) $K_{PCM} = K_{PCM,-1} + GI - D$
(19) $Y = Y_w + Y_\pi$
(20) $GNE = GNF - F_1'$
(21) $GNP = GNP^a + GNP^{na} + G_1$

(22) $P^{\cdot} = \dfrac{(P^{\cdot})(NNE) + P_{GI}(D_{f1} + D_{f2})}{NNE + D_{f1} + D_{f2}}$

(23) $P^{+} = \dfrac{(P)(GNE) + (P_{F1})(F_1')}{GNE + F_1'}$

G. GLOBAL DEMAND, SUPPLY, AND PRICE LEVEL FORMATION

(1) $C_p + C_{sd} + C_{hda} + GI_d + GI + \Delta H^a + H^{na} - H_{-1}^{na} + F_2' - \frac{1}{2}R = GNF^d$
(2) $GNP^{na} + GNP^a + G_1 + F_1'^d = GNP^s + F_1'^s = GNF^s$
(3) $GNF^d = GNF^s$

mined variables exert a one-way influence on the system, whereas the other variables are mutually interacting, determined inside the system, endogenous.

The model to be discussed in this paper—Model VIII—is presented above, with numerical estimates of structure. These were derived by fitting the model to the observed Canadian data of 1926–41 and 1946–56.

A variable which is theoretically endogenous is not always treated as such in a model. This may be because of lack of data on the casual variables which influence it, or in the interest of keeping the model small.

Each equation in Model VIII aims at explaining one endogenous variable in terms of the direct causal influences on it. There are, however, two exceptions. The demand for labor explains the combined demand for employment and hours as man-hours in one equation, with a separate equation to explain the level of hours. Also, there is no separate equation for the price level variable P. This is because the global market for final goods is assumed to reach an average equilibrium within a year; so the equilibrium volume of output and price level are determined jointly by equating global demand with global supply.

The term "equilibrium" here means that the price and quantity of the solution are taken at the intersection point of the global demand and supply equations. In equation G(1) of the model, GNF^d is a function of P, and GNF^s is also a function of P, through their respective components. Disequilibrium in this context would mean that the solution occurred at some point other than at the intersection of the two global functions. In this case, where $GNF^d \neq GNF^s$ a separate equation would be required for the price level of the form $P^+ = f(P^+_{-1}, GNF^d - GNF^s)$. A *stationary equilibrium* in this system would occur when, if left undisturbed, the system moved to $P^+_{t-1} = P^+_t = P^+_{t+1} \ldots$; $GNF^d_t = GNF^s_t$, etc. Such stationary equilibriums are not likely to last long, because of the many possibilities of changing variables in GNF^d and GNF^s, producing shifts. Model VIII assumes that the *average* condition of the economy over one year is merely an equality of GNF^d and GNF^s for that year only, with a different equilibrium of this kind highly likely in the subsequent year. The model thus expresses a shifting equilibrium, in the sense of the term equilibrium used here. Note that in the labor market for this model, we work with a shifting disequilibrium, indicating the extreme sluggishness of this market.

I return to the point that, in general, each equation aims at explaining one endogenous variable, with the exceptions noted, and can now proceed to count equations and endogenous variables. The equation system contains eight stochastic equations; six equations pieced together by observed ratios and estimated elasticities—B(7) and E(10), E(11), E(12), E(13), and E(14); twenty-three accounting

identities; and three equations expressing the annual average equality of global demand and supply. There are, thus, forty equations appropriate to each year. There must correspondingly be forty endogenous variables used as such in the system. These are marked out in the Glossary above by an asterisk placed directly after the explanation of the symbol.

Most of the identities are peripheral to the basic structure of the model, but are useful for building the tabular results.

With regard to methods of structure estimation, Model VIII was partially estimated by the method of truncated full-information maximum likelihood[1] (TFML), with help from an electronic computer. (Cf. equations 1, 2, 5, 8, Sectors A, B, C.) The remaining behavior equations were estimated by the ordinary least squares (LS) method. Computation resources and time were limited, and it was not possible to calculate the usual goodness-of-fit statistics. But all of the equations used passed reasonably good tests on this score in the preliminary screening tests using LS. For example, in the following equations the LS coefficient of variation and coefficient of correlation (n.a. = not available) for the complete equation are: A(1): 1.39 per cent, 0 999; A(3): 3.28 per cent, 0.999; B(4): 2.58 per cent, 0.998; B(5): 1.45 per cent, n.a.; B(6): 2.90 per cent, 0.997; C(8): 3.25 per cent, n.a.; D(9): 2.60 per cent, n.a. Tax and transfer equations were estimated by appraisal of appropriate elasticities for the individual components, based on past behavior and current changes in the tax-transfer structure.

If the detail of Model VIII is now examined, it is observed, first, that *consumer demand* has been disaggregated into three components. The research on this separation was done by S. J. May. Only the equation for perishables and services retained the explicit habit-persistence effect,[2] which seemed quite reasonable. The demands for the other components were influenced by stocks held (with an implicit habit-persistence influence), liquid asset holdings, debt position, and credit conditions, as well as by incomes.[3] The last term in A(3) reflects the important hypothesis that as real incomes rise, consumers

[1] T. M. Brown, "Simplified Full Maximum Likelihood and Comparative Structural Estimates," *Econometrica*, October 1959, pp. 638–653.

[2] T. M. Brown, "Habit Persistence and Lags in Consumer Behavior," *Econometrica*, July 1952, pp. 355–371.

[3] Cf. C. F. Roos, and V. von Szeliski, "The Demand for Durable Goods," *Econometrica*, April 1943.

raise their sights on their desired or aspiration levels of stocks of consumer durable goods.

The other major final good category for which there should be a set of demand equations is *gross domestic investment.* In this model only the demand for inventories, within this category, is included. For the fixed capital items, the preliminary data of the annual survey of investment intentions were used.[4] It would have been preferable to have also used demand equations for the fixed capital items, along with the survey, but limited resources and time prevented this. In previous and subsequent models, demand equations for plant, construction, and machinery (*GI*) have been used. Investment plans can of course change through the year as economic conditions change, and the initial survey cannot take this into account.[5] An equation should be able to do this. The equation, on the other hand, always has a large random component (exogenous investment), which, to a considerable extent, may be uncovered by the survey. For these reasons the survey and the equation combined may provide an ideal combination.

The *inventory demand* equation expresses the *transaction motive* and the *speculative motive* for holding inventories. The middle causal term expresses the influence of past inventory holdings, producing a certain inertia or implied speed of adjustment with respect to changing the level of inventories. Subsequent results revealed that this term produced too much inertia.

The demand-for-labor equation was tested in a new form in this model—a linear form. Man-hours is the factor of production on the demand side of the equation (the left-hand side), and the main causal variables believed to influence this demand directly are on the right. These causal variables are deduced, from the theory of the firm, to be the expected level of output (GNP, GNP_{-1}), the average price of the factor (w_{ph}), and the price level of the product (P). Previous versions had combined some of these variables into the real wage bill on the left side, with the expected level of output on the right side, expressing essentially the same economic theory in a slightly different equation form. The present equation, reflecting as it does

[4] *Private and Public Investment in Canada, Outlook 1958,* Department of Trade and Commerce.

[5] O. J. Firestone, "Investment Forecasting in Canada," in *Short-Term Economic Forecasting,* Princeton for NBER, 1955.

such fine and close agreement between economic theory and economic statistical data, is to me the most beautiful equation of the model. As a matter of further interest, the elasticity of demand for man-hours with respect to the average wage rate calculated from this equation for 1958 is -0.94, (-0.45 in 1926, -0.61 in 1947), implying that the demand for labor is inelastic. A reduction in wage rates would reduce the total wage bill (direct effect). This result has important implications for employment theory.

In designing the *import demand equation* the theory of the demand of the firm for a factor of production can again be used, taking into account production effects, factor prices, and product prices. Imports separate functionally into industrial materials, capital goods, and consumer durables, with capital goods tending to be prominent in Canadian imports. Hence, production effects in this equation were interpreted by three causal variables: industrial materials and nondurable consumer goods, by $GNP^{na} + F_1'$, which indicated approximately the total flow of nonagricultural goods through the economy; capital goods, by GI_M; and durable consumer goods, by C_{hda}.

The price of the factor of production is accounted for by P_1, but the price of the product (P^+) was not statistically significant, suggesting a low proportion of competitive imports. The inventory variable represents the import content of inventory stocks. A_{F1} is a shift variable to account for the import controls applied in Canada to counter the balance-of-payment difficulties of 1948–50.

The final term in the equation attempted to reflect the impact on imports of the capital inflow for direct investment (K) and portfolio investment (S). This inflow reduced the exchange rate cost of imports, already reflected in P_1, and provided ample international liquidity for imports. (This liquidity was used, for Canada's international reserves held fairly steady through 1950–60, with only a modest upward trend.) In addition, the inflow for direct investment would usually be intended for specific capacity expansion projects, much of the capital goods for which would be imported.

The *demand-for-hours-of-work* equation reflects a long-term downward trend in standard hours of work per week (reflected in the constant term); and the well-known variability of hours over the business cycle. When demand slackens the average production time of the whole plant must decrease; given the degree of complementarity of workers and the downward sluggishness of wage rates,

short-time and layoffs follow. Short-time permits a firm to retain its trained workers and is much preferred to layoffs.

Conversely, as demand steps up and passes the normal capacity level in any plant, it is better to increase the time of operation of plant and workers for the short run than to overcrowd the plant with additional facilities and workers. This also eliminates the need to train extra workers, who may then be lost on the next downswing.

The *labor market adjustment* equation follows closely the original design of Lawrence R. Klein,[6] except that it is now in terms of average hourly earnings, with unemployment in terms of man-hours To this design, May added the third explanatory variable. Without this term the average wage rate is equally sticky upward as well as downward. May's hypothesis was that when demand brought the labor market from a region of near full employment to over-full employment the average wage rate would start to climb at an accelerating rate. The third term in the equation interprets this nonlinearity by giving the equation a corner point at a threshold rate of unemployment of 3 per cent. Should the rate of unemployment be greater than 3 per cent, the third term is negative but is given the value zero. When this rate is less than 3 per cent, the term is positive, and its value is allowed to stand.

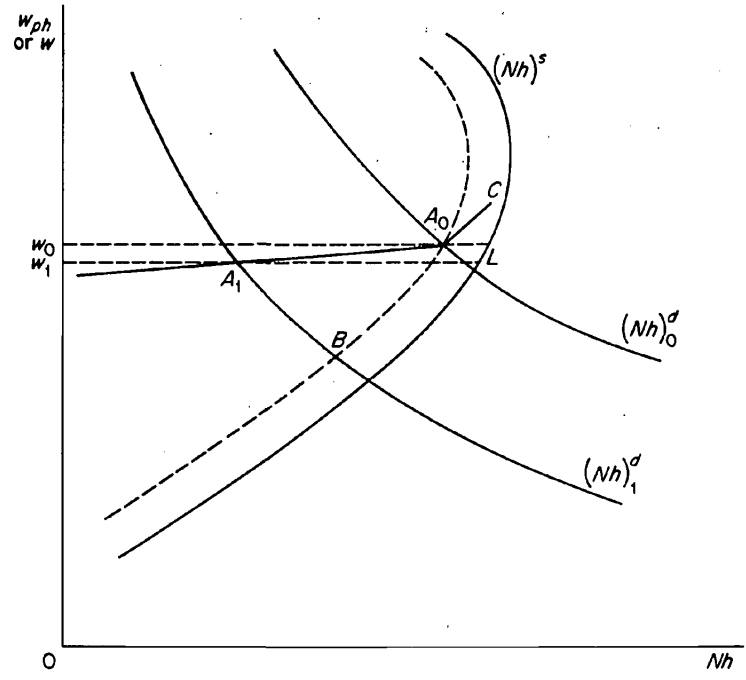
The interpretation of C(8) in terms of labor market demand and supply curves is shown in the chart. $(Nh)^s$ is the long-run supply of labor, which I assume to be a function of the real wage. (A change in prices would shift this function.) Assume that a stationary equilibrium has formed at A_0 at 3 per cent unemployment. If now demand drops back to $(Nh)_1^d$, equation C(8) says that the market position will not move directly to B, but will move to A_1, because of wage viscidity. At A_1 excess supply is A_1L; given time this may gradually drag the market position down to B. Should demand increase, still in the short run, to $(Nh)_0^d$ and beyond, the market would move along the line A_1A_0, and then along A_0C as May's term comes into play.

A_1A_0C is the short-run, very elastic supply curve of labor posited by Keynes,[7] and clarified by Klein.[8] It follows that C(8) is the Keynesian short-run supply curve of labor.

[6] *Economic Fluctuations in the United States, 1921–1941*, New York, 1950, pp. 51 and 121. Klein's original design for the Canadian model included the first, second, and fourth terms of C(8), in terms of number of people unemployed.

[7] John M. Keynes, *The General Theory of Employment, Interest and Money*, London, 1947 (reprint), p. 8.

[8] Lawrence R. Klein, *The Keynesian Revolution*, New York, 1947, p. 74.



Equation D(9) shows the combination of the major factors of production into a *production function.* When the levels of the factors have been determined by demand and supply in their markets, this technological relation gives the global output, and hence supply, of final goods. The production function presents a difficult estimation problem, since it shifts upward from time to time as the *joint* or *total productivity* of the factors of production increases. This gain in productivity arises from the discovery of new knowledge and its application to production. If it could be assumed that such gains occur regularly over time, a time trend could be added to the equation to shift it upward a systematic amount each year. This is the easiest and perhaps the best solution. But if the shifts occur in a way which is not regular with respect to time, the time trend gives an imperfect result.

In Model VIII an hypothesis was tested which, it was hoped, would let the statistical data decide when productivity shifts occurred. The hypothesis was that the level of the production function depends on the highest previous level of output obtained, $\overset{0}{GNP}_{-1}$. This hypothesis is defective in that it does not take into account changing quantities of factor inputs in the highest previous levels. However,

it should allow for the broad changes in total productivity as they occur. Also, it assumes that knowledge once gained is never lost; hence, that the production function ratchets upward and that its level never slips downward.

The resulting parameter estimates seemed reasonable and the fit was good. The elasticity of production with respect to labor in this equation was 0.55 for 1958, and 1 07 for 1926. In 1926 the effective constant term is -1.42. By 1958, the function had been shifted upward by the lagged term and the growing stock of capital; consequently, it had an effective short-run constant term of $+5.96$. Further research and experience are needed to reveal whether this is a reasonable portrayal of reality. The use of a linear form for this function may prove to be a serious shortcoming.

The *tax and transfer equations* were built from elasticity estimates and assumptions for the individual major taxes and transfer payments. It was necessary here to attempt to portray the drastic changes in fiscal policy which the government had quickly put into effect to stem the swelling tide of recession. It is felt that the estimates for this sector need considerable refinement.

In the aggregate, i.e., in the equation on the *global market for final goods*, it is assumed that equilibrium, in the sense discussed above, is reached within yearly periods, a condition which might be much less likely for quarterly or monthly time periods. The equations of GNF^d and GNF^s in this "market" permitted the determination of both *GNF* and the equilibrating variable. The latter was the price level P^+, which included P, the unknown deflator variable used throughout the model.

Technique of the Forecast

Known or estimated predetermined data for 1957, and predicted predetermined data for 1958 were assembled. The 1957 data were inserted in the model, and a solution vector of endogenous variables Y_{c57} was calculated. Through this solution the model determined national income, employment, and price level, taking into account simultaneously the estimated economic structure and all of the interactions and feedback effects among the endogenous variables.

The model was solved simultaneously through the elimination of variables by substitution, until finally an equation in only one endogenous variable was obtained. In the procedure used for this model the final equation was a fourth-degree polynomial in N_p^{na}. (Note the

various nonlinearities throughout the model.) That root of the quartic which was close to the current economic situation was localized, and a solution for it was made by Horner's method. This was followed by a back solution for all of Y_{c57}, and a final check.

This solution vector was now compared with the best available estimates of the corresponding observed data for 1957, Y_{57}. The discrepancies ($Y_{57} - Y_{c57} = u_{s57}$) were assumed to be due to random causes, unless they appeared to represent some known structural change. In the latter case an alteration of the structure involved might be made, and a new 1957 solution derived.[9] The approach to such alteration may be outlined as follows. Let the original estimated model of direct economic structure be

(1) $Y_t = F(Y_t, Z_t) + u_t$, where Y_t, Z_t and u_t are respectively the vectors of endogenous, predetermined, and direct structural residuals of the model. F is a vector of functions. The structural reduced form of the system is derived by solving (1) for Y, with u omitted, giving

(2) $Y_c = R(Z)$; $Y = Y_c + u_s$ = the observed values of Y.

Where u_{s57} indicated that structure revision was needed for say the ith and jth equations of (1), u_{i57} and u_{j57} were calculated from (1). These values were then added to the structure of (1) as a vector $e = (0, 0, \ldots, 0, u_{i57}, 0, u_{j57}, 0, 0, \ldots, 0)$, thereby producing a revised structure.

(3) $Y = F(Y, Z) + e + u^r$.

(3a) $Y = F^r(Y, Z) + u^r$.

This structure was now solved again for 1957, omitting the residuals u^r_{57}, to obtain a revised structural reduced form

(4) $Y_{F57} = R^r(Z_{57})$. The structure (3a) was likewise used to compute the forecast solution vector

(5) $Y_{F58} = R^r(Z_{58})$.

It is Y_{F57} and Y_{F58}, as derived from (4) and (5), that are used in Tables 2–5, which follow.

This solution vector could now be compared with Y_{57} or Y_{c57}. If the latter is used, only the movements of the *systematic* part of the model can be considered, i.e., only purely economic movements based on the systematic structure of the model. If on the other hand a comparison is made of Y_{F58} with $Y_{57} = Y_{c57} + u_{s57}$, the systematic

[9] See Lawrence R. Klein, and A. S. Goldberger, *An Econometric Model of the United States, 1929–1952*, Amsterdam, 1955, pp. 77–78, for a systematic way of making many of these adjustments.

movement from 1957 to 1958 is mixed with one random vector. Such mixing has, in the past, produced anomalous economic predictions, when a large random element in $u_{s,t-1}$ has been in the same direction as the corresponding systematic movement in $Y_{Ft} - Y_{c,t-1}$.

Ideally, what we would like to forecast is $Y_{58} = Y_{F58} + u_{s58}$, and compare it with $Y_{57} = Y_{c57} + u_{s57}$, showing movements $Y_{i58} - Y_{i57}$ or, in percentage form, $(Y_{i58}/Y_{i57} - 1) \times 100$. This comparison involves *two* random disturbances. Do we help our cause any by comparing Y_{F58} with $Y_{57} = Y_{c57} + u_{s57}$, involving only *one* of these two random components? Is there some advantage in knowing that our forecast contains only the movements evolving from the systematic structure?[10]

These questions require more study, but the forecast with Model VIII presented below consists of a comparison of Y_{F58} with Y_{c57}. Both of these solutions involved one or two adjustments of structure based on a study of disturbances and other very current information. Inventory demand was the main case in point, where the lag effect and the price effect seemed to give the equation too much trend inertia. Improved structure specification is clearly called for here.

Forecast Results

THE EXTERNAL CAUSES

Once the structure of the model has been decided upon, the keystone of the forecast becomes the *exogenous demands*. The period of forecast was one of uncertainty, and I made three sets of assumptions about these demands, and tested them all. One set involved large declines; another, small declines; and the third took an intermediate position. This intermediate forecast seemed the most plausible to me in January 1958, and is the one presented here. The main components of this part of the forecast are in Table 1.

In this table the assumption of deteriorating world economic conditions can be seen, as well as domestic prospects for fixed investment. At the same time, a part of government fiscal policy can be seen as coming to the rescue with an increased spending program, and with increased mortgage money for housing.

The total assumed decline of 5.3 per cent in exogenous demands was very serious from the point of view of output and employment. With a growing labor force and productivity, the economy must

[10] See Klein and Goldberger, pp. 76–77, for an alternative interpretation of such a comparison.

TABLE 1

FORECAST OF VOLUME CHANGES IN EXOGENOUS DEMANDS, 1956–58
(percentage changes)

	1956–57	1957–58
G = government spending	+1.0	+2.9
$GI + GI_d$ = gross private investment in fixed capital	+5.1	−10.3
GI_d = residential construction	−12.5	+10.7
GI_{PC} = nonresidential construction	+21.8	−12.8
GI_M = machinery and equipment	−1.9	−15.6
F' = exports of goods and services	−1.0	−6.5
Total exogenous impact	+1.8	−5.3

grow at a rate of 4 or 5 per cent to maintain full employment at current hours of work. Of course the built-in stabilizers of the modern economy, reflected in the tax and transfer equations of the model, would ameliorate this decline to some extent. But something more than these would be needed to overcome the serious fall in exogenous demands.

Late in 1957, as the economic situation worsened, various changes in the government fiscal program were put into effect. Various tax rates were decreased, and transfer payment rates were increased. Presumably, this was a second wave of attack against the recession. The impact of these changes can be observed in the tax and transfer equations of Model VIII for 1957 and 1958. One assumes that the objective on this front was to increase the disposable incomes of the lower income groups because of their need and because their marginal propensity to consume is highest. Consumer demand is the largest single employment-producing demand. Income tax exemptions were increased, and rates were reduced in the lower brackets. On the transfer side, allowances to war veterans and dependents were increased, old age pension rates were stepped up by something like 37.5 per cent, and family allowance rates were increased about 4 per cent. Unemployment insurance was extended to cover a longer period of unemployment.

Taxes were also reduced on nonwage incomes through the reduction of personal income tax rates; through extension of the lower range of corporation profits covered by a lower tax rate; and through elimination of the estate tax on smaller estates. On the indirect tax

side, a special excise tax on automobiles was reduced, to stimulate sales and employment in this industry.

The exogenous demands and government policy constitute the main external or exogenous influences on the economy for a forecast period. The lagged variables cause inertia in the short run, but of course generate trends or cycles over the longer run. In the forecast solution there is a struggle between declining exogenous demands on one side, and the expansionist government policies and the inertias of the model on the other. To the outcome of this struggle I now turn.

THE INTERNAL OR ENDOGENOUS EFFECTS

The results of the forecast solution are presented in Tables 2, 3, and 4 which follow directly. Table 2 shows the forecast outcome with respect to the national accounts. Table 3 gives the all-important prediction of the results in the labor market. Table 3 also shows the changes in the stock of producers' durable capital, movements in the price level of GNE, and finally the government surplus or deficit and its main components. In Table 4, the outputs of the three major producing sectors of the model—private nonagriculture, agriculture, and government—are shown. Then, a section is devoted to the disposable incomes of the wage, nonwage, and government sectors of the economy, all of which add up to GNP. Finally, the predicted savings ratio is presented.

It is not necessary to discuss here all of the detail of these tables. What will be attempted is to draw the broad picture of the results which are found in the tables and the causal analysis given by the model.

The labor market

To go first to the heart of the forecast, we examine the outcome in the labor market in Table 3. The battle between declining exogenous demands on one side and the inertias, automatic stabilizers, and direct government policy on the other is there revealed to have been lost. Unemployment increased from 5.3 per cent of the civilian labor force (311,000 people) to 7.4 per cent (448,000 people), an increase of 44.1 per cent in the number of unemployed. At the same time, the model predicted an increase in the average wage rate of 3.0 per cent. This was produced mainly by the inertia of past upward shifts in the

TABLE 2

ECONOMETRIC PROJECTIONS OF NATIONAL PRODUCT AT MARKET PRICES, MODEL VIII, 1958
(millions of current dollars)

	Model Solution 1957	*Forecast for 1958*	*Percentage Change 1957 and 1958*		
			Value	Volume	Price
INCOME					
Salaries, wages, and supplementary labor income	15,107	15,261	+1.0	+0.2	+0.8
Military pay and allowance[a]	475	499	+5.1	+4.3	+0.8
Corporation profits	2,803	2,461	−12.2	−12.9	+0.8
Other property enterprise or non-wage income	5,110	5,137	+0.5	−0.3	+0.8
Net national income at factor cost	23,495	23,358	−0.6	−1.4	+0.8
Indirect taxes less subsidies	3,799	3,773	−0.7	−1.5	+0.8
Depreciation allowances and similar business costs[a]	3,393	3,664	+8.0	+8.0	0.0
Gross national product	30,687	30,795	+0.4		
GNP less inventory profit	30,617	30,895	+0.9	+0.1	+0.8
EXPENDITURE					
Personal expenditure on consumer goods and services	19,261	20,047	+4.1	+2.5	+1.5
Government expenditure on goods and services[a]	5,545	5,789	+4.4	+2.9	+1.5
Gross home investment in durable assets:[a]	7,311	6,624	−9.4	−10.3	
Housing	1,386	1,533	+10.6	+10.6	0.0
Plant and construction	3,233	2,819	−12.8	−12.8	0.0
Machinery and equipment	2,692	2,272	−15.6	−15.6	0.0
Total producers' durables	5,925	5,091	−14.1	−14.1	0.0
Change in inventories (book value)	+104	−380			
Nonagricultural business[b]	251	−239			
Agriculture and grain[a,b]	−217	−41			
Capital gain or inventory profit[a]	70	−100			
Exports of goods and services[a]	6,379	5,844	−8.4	−6.5	−2.0
Imports of goods and services	7,913	7,129	−9.9	−9.9	0.0
Gross national expenditure	30,687	30,795	+0.4		
GNE less inventory profit	30,617	30,895	+0.9	+0.1	+0.8

[a] Exogenous.
[b] Value of physical change.

TABLE 3

LABOR MARKET, PRICE LEVELS, AND OTHER PROJECTED MATERIAL, MODEL VIII, 1957–58

	Unit	Model Solution 1957	Forecast for 1958	Percentage Change 1957 to 1958
Labor market[a]				
Civilian labor force (LFS)[b]	1,000's	5,914	6,015	+1.7
Total civilian employment (LFS)		5,603	5,567	−0.7
Employment of paid workers, nonfarm		3,897	3,855	−1.1
Unemployment		311	448	+44.1
Unemployment as per cent of civilian labor force (LFS)	%	5.3	7.4	
Average hourly earnings of paid workers, private nonagricultural sector				
Current market value	Current $	1.558	1.605	+3.0
Real value	1935–39 $	.708	.724	+2.3
Average hours worked per week, private nonagricultural sector	1 hour	39.8	39.2	−1.5
Total man-hours worked, private nonagricultural sector	Millions of man-hours	9,195	8,930	−2.9
Real output per man-hour, private nonagricultural sector	1935–39 dollars	1.310	1.340	+2.3
Stock of durable producers' capital in private nonagricultural sector	Millions of 1935–39 dollars	17,228	18,486	+7.3
Prices	1935–39			
Price index of *GNP*	average	223.4	225.1	+0.8
Price index of gross national supply flow (*GNE* + imports)	equals 100.0	225.4	226.7	+0.6
Price index of net national product		220.0	221.7	+0.8
Government	Millions of			
Disposable income (government tax revenue less transfer payments)	current dollars	5,349	4,610	−13.8
Expenditure on goods and services[b]		5,545	5,789	+4.4
Surplus (+) or deficit (−) (on transactions relating to the national accounts)		−196	−1,179	

LFS = Labor Force Survey concepts, which exclude Yukon and Northwest Territories.

[a] Includes remote areas, Yukon and Northwest Territories.
[b] Exogenous.

TABLE 4

ECONOMETRIC PROJECTIONS OF PRODUCTION, DISPOSABLE INCOME, AND THE SAVINGS RATIO, MODEL VIII, 1958

	Model Solution 1957	*Forecast for 1958*	*Percentage Change 1957 to 1958*		
			Value	Volume	Price
PRODUCTION BY SECTORS (millions of 1935–39 dollars)					
Gross output, private nonagriculture	12,041	11,961		−0.7	
Gross output, agriculture[a]	693	775		+11.8	
Output of government services[a]	973	987		+1.4	
Gross national product	13,707	13,723		+0.1	
DISPOSABLE INCOME (millions of current dollars)					
Wage and salary	16,020	16,630	+3.8	+3.0	+0.8
Personal, nonwage	4,874	4,951	+1.6	+0.8	+0.8
Total personal sector	20,894	21,581	+3.3	+2.5	+0.8
Total nonwage					
Net of depreciation	5,951	5,919	−0.5	−1.3	+0.8
Gross of depreciation	9,318	9,555	+2.5		
Private sector					
Net	21,971	22,549	+2.6	+1.8	+0.8
Gross	25,338	26,185	+3.3		
Government	5,349	4,610	−13.8	−14.5	+0.8
Taxes less transfer payments					
Wage and salary	−393	−828			
Nonwage	1,943	1,667			
Indirect taxes less subsidies	3,799	3,771	−0.7	−1.5	+0.8
Gross national product (private sector plus government)	30,687	30,795	+0.4		
SAVINGS RATIO (personal disposable income)					
	0.0782	0.0711			

[a] Exogenous.

short-run supply curve of labor, reflecting a trend which arose out of the postwar strengthening of the trade unions, and the postwar demand inflation.

Average hours of work (nonfarm) declined by 1.5 per cent, as the

economy ground to a halt and unemployment grew. Productivity (real output per man-hour) increased by 2.3 per cent as the production function shifted upward under the influence of its trend and of the growing stock of producers' capital. The latter increased at the spectacular rate of 7.3 per cent, despite the slowing down of investment.

Global real output

The next most vital area of the forecast is the level of total output. This volume remained substantially unchanged (Table 2). The explanation provided by the model from the demand side is as follows. The demand for investment in producers' durables fell heavily (−14.1 per cent), with a substantial decline also in exports (−6.5 per cent). The decline in inventory investment (from investment to extensive disinvestment) was the final major contributor to recession.

Consumer demand was one of the main strengths opposing the decline. By far the largest single component of total demand for final goods, it increased by 2.5 per cent (volume), under the influence of previous standards of consumption, the built-in fiscal stabilizers, and the expansionist fiscal policy of government. Personal disposable income increased by 2.5 per cent in purchasing power, and the savings ratio (personal) in the model declined from 0.0782 to 0.0711 (Table 4).

Government spending was a second force standing against the recession, and advanced 2.9 per cent. A third spearhead of the attack was the provision by the government of considerable extra funds to its Central Mortgage and Housing Corporation for providing insured mortgages and direct mortgage loans for new residential construction. This component of final demand increased strongly, by 10.6 per cent (Table 2).

A final defense against the recession came from another kind of built-in stabilizer, the level of imports. With the investment program falling off, and an assumed decline in the capital inflow as conditions also deteriorated in the United States, imports fell rapidly in the model—by 9.9 per cent.

The net outcome of the struggle between the forces of decline and the forces of defense and attack against the decline was an even balance. The level of total final demand (*ex post*) barely changed. Corresponding to this, on the supply side of the economy, an increase in nonfarm productivity of 2.3 per cent was partly offset by a decline

in hours of 1.5 per cent, but the residual effect fell on employment. Demand for paid nonfarm workers fell by 1.1 per cent (*ex post*).

As a result of this prediction of almost no change in output, but of rising labor costs, the model predicted that total nonwage income would decline by 4.0 per cent in money terms, with the heaviest impact in this sector on residual profits (Table 2).

The price level

No attempt was made in this forecast to analyze the shifts in the global demand and supply functions as functions of the global price level. But the forecast solution suggests the following hypothesis. There was only a slight shift in the demand curve, of unknown direction. The supply curve was shifted forward by the gain in productivity (+2.3 per cent) and backward by the increase in average wage rates (+3.0 per cent). The price level increased by 0.8 per cent, suggesting that the net shift was a slight backward shift of supply. The "inflation" would then be produced by cost rather than demand forces. This hypothesis is reinforced by a calculation, from Table 3, that labor cost per unit of output (average wage divided by productivity) increased by 0.7 per cent.

SUMMING UP

Within the framework of analysis of the model, the battle of the recession was nearly won, but was in fact lost. The evidence reveals a valiant attack on a variety of fronts, plus some well-placed defenses. The combined tactics raised the deficit of all three levels of government by $1 billion. But the model indicates that it would take more demand for final goods, or a shortening of hours of work, to close the employment gap. This demand could be carefully aimed at the industries of greatest unemployment. The government deficit need not increase if government policy created further expansion, since government revenues become increasingly elastic with respect to output at full-employment and boom levels of output.

But one problem still remains. Expansion of demand in the model would undoubtedly cause the price level to increase. By the time the economy reached full employment the price level might have increased by 3 or 4 per cent. This is not too great a price to pay for full employment, but it typifies a troublesome defect in modern Western economies. Apparently, it is becoming impossible to reach full

employment without inflation.[11] This inflationary problem may inhibit some Western governments from pursuing full-employment policies steadfastly, until complete victory is won. The problem seems to stem from the ability of large firms and associations of all kinds, including labor unions and professional bodies, to administer their supply prices, instead of having these determined by the impersonal demand and supply forces of perfectly competitive markets. The solution of this problem is of vital importance to the Western democracies.

A Brief Post-Mortem

In the above analysis we were discussing the picture of the economy as projected into 1958 by the model. It is now possible to look at the outcome in the real economy, as it actually happened four years ago.

TABLE 5

COMPARISON OF MODEL VIII FORECAST OF 1958 WITH SUBSEQUENT OBSERVATIONS

	Percentage Changes, 1957 to 1958	
	Predicted	First Revised Observation[a]
Input of selected exogenous data		
G	+2.9	+3.8
$GI + GI_d$	−10.3	−7.8
F_2'	−6.5	+0.9
Total exogenous demand impact	−5.3	−1.4
$N_1 - N_M$	+1.7	+2.2
Output of selected endogenous variables		
N_p^{na}	−1.1	−0.2
$N_p + N_{enp} + N_g$	−0.7	−0.4
N_u	+44.1	+57.6
w_{ph}^{na}	+3.0	+2.4
GNP	+0.1	+0.5
C	+2.5	+2.6
F_1'	−9.9	−6.3
P	+0.8	+1.8
Y_{pm}	+3.3	+7.1
π_m	−4.0	+4.3

[a] *National Accounts, Income and Expenditure, 1958*, Ottawa, Dominion Bureau of Statistics, 1959. Underlying values are in 1949 constant-dollar units.

[11] T. M. Brown, "Unemployment or Inflation—Economic Dilemma of the West," *Queen's Quarterly*, Summer 1961.

In Table 5 a comparison between forecast and observed outcome is made for selected items.

How good or how bad was this forecast? What guidance can it give us for future forecast work? In comparing the forecast with what subsequently happened we must recall that, in the forecast, we are presenting movements of the systematic part of the model only, whereas in our observations of the real world we are studying movements of a systematic and random component combined.

In the first place, I was overly pessimistic in predicting exogenous demands, and made a considerable error in the total exogenous impact. The exogenous variables are by their nature difficult to predict.[12] This is because they emanate from noneconomic areas of causation containing their own laws of behavior and random components. However, this error could have been gradually corrected in the first quarter and first half of the forecast year as more information became available.

Taking into account this error in the exogenous forecast, the model might be expected to predict a greater decline in employment than occurred, and a smaller increase in the total volume of output. Unemployment, however, actually increased more than the model predicted, because the labor force grew at a higher rate than was predicted.

Many of the other errors in the forecast can be largely traced to errors in the exogenous demands. For example, imports would have been closer to the mark but for this error. The error in the price level also reflects the weakness in the assumptions about exogenous demands. On the global supply side, wage rates did not rise as much as the model predicted, but neither did productivity (one estimate put it at +1.1 per cent). Hence, the model may have been about right on the supply side, but too weak on the demand side of the aggregate market for final goods.

In the case of personal disposable income it appears that the rather large error cannot be attributed mainly to the exogenous input. It is likely that here the tax and transfer equations are in need of structural correction through more research and more collaboration with experts in the tax and transfer field.

The error in total nonwage income reflects the volatility of this

[12] H. Theil, *Economic Forecasts and Policy*, Amsterdam, 1958, Chap. 7, observes that "erroneous estimates of exogenous changes were by far the most important sources of the errors of the unconditional forecasts."

variable relative to the level of activity and the error in the average wage rate.

Conclusions

This experimental forecast as well as considerable other experience make me believe that useful models of income determination can be derived by econometric measurement. The model described in this paper has many imperfections, but surely all of these could be corrected by adequate research resources. The inventory equation and the tax and transfer equations especially, including the indirect tax equation, need much more study. Also aggregate production and supply in the model should be reformulated from a GNP to a gross domestic product basis. Solution of the model with the correct exogenous and lagged data would point up many other errors and weaknesses.

In addition to research along these and other lines to improve the structure specification and estimates, we should also be thinking about how to measure the *very current* structure of an economy. If economic structure is slowly evolving, the estimates from a time series may not always be able to sort out the time trends appropriately. Some structure estimates may then be only an average of a changing structure. But we might be able to measure the very current structure of an economy by analyzing a stratified sample of households and of firms which keep complete economic records, and passing these monthly, without analysis, to the Dominion Bureau of Statistics or the United States Department of Commerce. Analysis of these records both as time series and as cross-section samples may reveal the clue to the use of current cross-section samples for estimating the *current* "time series" structure.[13]

Also needed is more excellent data designed to match our hypotheses. In matters of basic economic measurement and in the suggestion of hypotheses to be tested, arising out of study of these data, the National Bureau of Economic Research can be a powerful ally to econometric development in the United States (as I am sure it already is). This is because of its wealth of experience and skill in these vital areas.

With excellent models derived by painstaking, large-scale research

[13] Pioneering work in the use of cross-section data in econometrics has been done by Klein and his associates. See Lawrence R. Klein (ed.), *Contributions of Survey Methods to Economics*, New York, 1954; and Klein and Goldberger, *op. cit.* (above, note 9).

we can devise and test rational and accurate alternative policies for maintaining economic health—especially full employment and adequate rates of development and growth.[14]

The economic costs of unemployment and inadequate growth in any country are represented by the value of production lost because of these shortcomings. In Canada in the circumstances of 1958 it would have been necessary to increase real output by about 5 per cent to restore full employment. Roughly $1.5 billion were lost that year as a result of our failure to keep healthy. The United States in similar circumstances in the same year probably lost about $21 billion for the same reason. In both countries the *human* costs of the unemployment to workers and their families must be counted as infinite.

The cost of good quantitative research of all kinds aimed at policy problems would be small indeed in comparison to the above costs of economic ill health. On purely economic grounds investment in these fields should have a very high yield.

But in the world of today something greater than domestic costs and gains is at stake. Our governments must show the world that it is possible for the "mixed enterprise" economy to give high regard and value to the individual, to provide a high degree of general welfare, *and to run and grow smoothly*. To me, the best instrument we have been able to devise to help us regulate the over-all behavior of such an economy by indirect means is the econometric model. It is the only instrument which tries to put everything together, and to study the whole economy as an operating unity.

Let us develop these models to their full potentialities, and use them.

COMMENT

CARL F. CHRIST, Johns Hopkins University

This morning's session, with a paper on a Canadian model by T. M. Brown and a paper on a United States model by Lawrence R. Klein, stands as a testimony to the achievements of economists who have persevered in building and testing and improving aggregate econometric models, including Brown and especially Klein. Both of these models represent real progress, and have led to quite good

[14] Cf. Lawrence R. Klein, "The Use of Econometric Models as a Guide to Economic Policy," *Econometrica*, April 1947.

forecasts. Klein's model, which is two years later than Brown's, incorporates several excellent new features. For example: (1) accounting identities are expressed in money terms, as they should be, and the model is designed to explain both real magnitudes and price levels for the various income and expenditure flows; (2) anticipations data are used at several points; (3) inventory investment is related to new and unfilled orders; (4) a relation between actual and capacity output is introduced.

We are particularly fortunate to have Brown describe the Canadian model, fitted to 1926–41 and 1946–56, for although he and his associates in the Dominion Department of Trade and Commerce have been working on models of Canada for some fifteen years, rather little material has appeared concerning these models. Fragmentary reports have suggested that the work is progressing well. Brown's paper gives us a chance to see for ourselves that this is so, to learn a good deal about the models used, and to offer some comments in a constructive spirit with the same good cause in mind.

Quantitative Results

In January 1958, the model was used, together with forecasts of the exogenous variables, to make unconditional forecasts of the performance of the Canadian economy in 1958. Brown denotes by Y_{C58} in equation A(2) the forecast for 1958 of the variable Y made by the reduced form of the estimated model, on the assumption that the residuals for 1958 are zero. If there is high serial correlation of disturbances, then the forecast may be improved by adding to certain estimated structural equations the corresponding estimated structural residual calculated for the preceding year, to get a revised estimated structure, and then solving this to get a revised forecast. This is what Brown does, and he denotes his actual revised forecast for Y for 1958 by Y_{F58} in equation B(5).

In comparing his forecasts for 1958 with the actual performance of the Canadian economy (as estimated in the national accounts early in 1959) he compares the actual and the forecast *percentage changes*, namely $100(Y_{58} - Y_{57})/Y_{57}$ and $100(Y_{F58} - Y_{F57})/Y_{F57}$ (see his Table 5). The *difference* between these two statistics is a rough indicator of the accuracy of forecast, but it is hard to interpret because it is *not* a simple function of the actual forecasting error $Y_{F58} - Y_{58}$, unless $Y_{F57} = Y_{57}$ (in which case it would be the percentage error of the 1958 forecast referred to a 1957 base, i.e.

$100(Y_{F58} - Y_{58})/Y_{57}$). For the major aggregate variables, real GNP and total civilian employment ($N_{pe} + N_g$), these *differences* were 0.4 per cent and 0.3 per cent, respectively, i.e., the accuracy was extremely good. The GNP deflator and the wage rate were quite well forecast, too, with differences of 1 per cent or less. Certain subcategories of total income and expenditure had differences of up to 8 per cent. Unfortunately for those who want a more detailed assessment, the paper presented the actual and forecast values of only ten of the forty endogenous variables; so the results for the other thirty are obscure.

Four of the eight stochastic equations were estimated by the truncated full-information maximum-likelihood method (TFML), and four by least squares (actually least squares was used for all equations first). For four of the equations we are given the multiple correlation coefficient R obtained from the least squares fit. The four values are all at least 0.997, remarkably high.

Statistical Presentation and Evaluation

For *all* of the stochastic equations, it would be desirable to show the multiple correlation coefficient R. For equations fitted by the TFML method, it would be desirable to show also an analogous measure of goodness of fit, such as the square root of the expression

$$1 - \left(\frac{\text{standard error of estimate}}{\text{standard deviation of "explained" variable}}\right)^2.$$

For each of the estimated coefficients, it would be desirable to show the estimated standard deviation, so that observers could form an impression (whether by classical or Bayesian methods) of the reliability of the estimates of individual coefficients.

In addition to the eight stochastic equations (which can be identified by the presence of a random disturbance variable, $u_1, \ldots, u_8$) there are six equations that have numerically estimated coefficients but no disturbances; these are five of the six tax and transfer equations and the equation relating hours worked per week to the level of unemployment. It would be helpful to know how the numerical values of the coefficients in these six equations were obtained.

Forecasting results for this or similar Canadian models for 1959, 1960, and 1961 would be most welcome.

Brown points out that a large share of his forecasting error is due

to errors in forecasting the exogenous variables. It has become common practice in presenting the forecasts made by an econometric model to show both the unconditional forecasts (as Brown did), and also the conditional forecasts given the *ex post* correct values of the exogenous variables (as Brown did not). Had he done so, it would be possible for readers to see how large the forecasting errors would have been in the absence of any errors in exogenous variables, and thus to evaluate the model itself, apart from the forecasting process as a whole, which depends on both the model's and one's own ability to forecast the exogenous variables.

Economic Content of the Model

One of the most conspicuous features of the model which calls for revision is that all investment expenditures except inventory investment are treated as exogenous. Brown tells us that in previous and subsequent models demand equations for plant, construction, and machinery have been used. It would be enlightening to have a report on the alternatives tried and the results obtained with the more important ones.

A number of small questions arise about the treatment of income and wealth in the consumption equations A(1), A(2), and A(3). Why are Y_w and Y_π (labor and property income) separated in the perishables equation, but not in the semidurables and durables equations? Why are Y_p^a and $Y_{\pi np}$ (agricultural income and property income of nonpersons) excluded from the durables equation but not from the perishables and semidurables equations? Why is the stock of consumer capital goods divided by income in the durables equation and not in the semidurables equation?

The model is linear for the most part, but there are several clever nonlinearities. Some of them occur when it is desired to take into account that total man-hours worked is a product of employment and the number of hours worked by the average worker per year, or that the total money wage bill is the hourly wage rate times the number of man-hours of labor used. Others occur with respect to price level variables, when a relation between real and money magnitudes is wanted. Another, simple but ingenious, is in the wage adjustment equation, C(8), which asserts the hypothesis that wage rates become more sensitive to the level of unemployment when that level falls below 3 per cent of the labor force. This is because unemployment

enters in two terms in equation C(8), the term containing Nh_u^{na} and the term containing N_u; and N_u responds to changes in unemployment only when the level is below 3 per cent.

The discussion at this conference showed that Brown's treatment of prices in his *use* of the model is somewhat different, and more intelligent, than the explicit statement of the model's equations and the list of endogenous and exogenous variables indicates. The model indicates that the only endogenous price level variables are P, P', and P^+, being respectively the deflators of net and gross national product and of GNP plus imports. The prices of consumer goods, investment goods, and the like appear to be exogenous. Such a procedure would not make sense if adhered to rigidly, for then the levels chosen for these "exogenous" price variables would have to be permitted to affect in an important way the general price level variables P, P', and P^+. Actually, in using the model to make forecasts, Brown and his colleagues relate the value of the consumption deflator to the general price level by means of a separate informal equation, and if I understand correctly they do the same with certain other price level variables. Thus, the equilibrium condition [equation G(3)] equating aggregate demand and supply determines the general price level P, and the other price level variables are determined in relation to that. Therefore, in effect, the number of endogenous variables in the model is forty *plus* the number of price variables thus explained, and the number of equations in the model is forty *plus* the number of price-explaining relationships thus employed.

This means that the treatment of prices in this Canadian model is rather similar to their treatment in the model presented here by Klein, for Klein expresses each of his endogenous sector price levels by means of an explicit equation in terms of either the GNP deflator or the wage rate. The main differences on this score between the two models are that Klein's accounting identities are in money terms, as they should be, while Brown's are in real terms (with the exception of the wage bill identity), and Klein shows the influence of relative prices on the demands for certain classes of goods and services. Brown's model would be improved by following this example, and also by showing explicitly the relationships used among price variables that are handled in an endogenous manner for forecasting purposes.

The model as it stands, with forty equations and forty endogenous

variables, is rather hard to apprehend. Its central structure may become more visible if certain reducing or simplifying operations are performed, which make no change in its central content. First, the endogenous variable T_d appears only in equation F(15), so both it and that equation can be dropped without changing the content of the remainder of the model. Second, there are two endogenous variables that are essentially exogenous, since each is determined in an equation that contains no other endogenous variables; they are D_{f2} in equation F(4) and K_{PCM} in equation F(18); hence, these two equations can be dropped and the two variables regarded as exogenous. Third, the six endogenous tax and transfer variables can be expressed in terms of the variables on which they depend, and thus they and their six equations can be removed by substitution. Fourth, the three profit variables other than corporate profits can all be expressed in terms of the latter, and so they and their three defining equations can also be eliminated by substitution. The same is true of some half-dozen variables representing different kinds of income flows; after they are eliminated, property income (π) and disposable labor and property income, Y_w and Y_π, remain. Similarly, the price levels P^- and P^+ can be expressed in terms of P, the net national product deflator. Continuing such a process, one can obtain a model of sixteen equations, eight stochastic and eight nonstochastic, in sixteen endogenous variables, as follows:

Equations (*stochastic)	*Endogenous Variables Corresponding*
* 3 consumption demands, A(1–3)	3 consumption variables
* Import demand, B(6)	Imports
* Inventory demand, B(4)	Inventory investment
* Labor demand, B(5)	Employment
* Wage equation, C(8)	Wage rate
* Production function, D(9)	Gross private nonfarm product
Hours equation, B(7)	Average hours per worker
6 definitions	Wage bill, property income, disposable labor and property incomes, corporate profits, unemployment
Aggregate equilibrium condition, G(3)	Price level

All these equations are interrelated in an essential way, i.e., there is no way of breaking them up into two subsets such that one of the subsets alone determines the values of a corresponding subset of the endogenous variables in a manner logically independent of the determination of the remaining variables. This is as it should be.

Monetary policy and behavior variables have too limited a role in the equations of this model. They enter through lagged real balances of households ($L_{hr,-1}$) and consumer credit terms (dp) in two of the consumption equations, and through capital inflow (ΔKS) in the import equation. And they affect the model by influencing investment in plant and equipment and housing, which the model treats as exogenous. More explicit treatment of money market variables is desirable if Brown's discussion of the roles of demand inflation and cost inflation is to be evaluated adequately.

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For rather more than a decade, T. M. Brown worked on the official econometric model of the Canadian economy in Ottawa. He has published several contributions to the theory of estimation and forecasting, and some empirical results, but so far as I know this paper represents the first description of the Canadian model as it was developed by Brown and his associates which has been prepared for publication. Since the work that has been done is of high quality, Canadians will not be alone in their pleasure that it will now be available.

In these comments I shall confine myself to brief remarks on:

1. uses to which the models have been put in the service of the Canadian government
2. selected characteristics of the model that has been presented
3. certain statements and conjectures in the paper.

Uses to Which Models Have Been Put in Canadian Government Service

The Economics Branch of the Department of Trade and Commerce has the principal responsibility within the government service in Ottawa for providing aggregate forecasts of forthcoming developments in the Canadian economy. The methods of forecasting are diverse. Substantial use is made of the regular survey of investment

intentions, though the results of that survey are not used without adjustment. Considerable application is made of the analysis of reference cycles, diffusion indexes of movements of deseasonalized data, and similar devices. The definitions of the national accounts and other related identities are used as a framework within which to integrate informed estimates. The econometric model, deriving its forecasts of exogenous variables from other work in the branch, is used as an alternative or supplemental means of organizing the information available into forecasts of the main components of national income, expenditure, employment, and prices.

Users of the model have found it particularly helpful in assessing quickly the likely effects on the main economic aggregates of changes in particular elements of federal government expenditure and tax policies. It has also been found to be particularly useful, I am told, in assessing the impact of the opposing demand and supply forces upon the price level.

Some Characteristics of the Model

While the model has been useful in assessing the effects of particular changes in government fiscal policy, it is not constructed with a view to exhibiting explicitly the requirements of government policies of various kinds. In using the model it has not been the practice to specify explicit targets for national income, employment, prices, and the like, and then seek to determine what policy measures would be required in the circumstances to ensure that these targets will be achieved. Some key variables upon which the government might be expected to operate directly do not appear, at least not explicitly, in the model. For example, the stock of money as such does not appear in the model. The rate of interest does not appear explicitly in the model; it may be reflected in the index of consumer credit conditions (*dp*), though the definition of that variable does not indicate that it is. The rate of exchange does not appear explicitly in the model, though it is reflected in the price of imported goods.

It is important to emphasize, as Brown does, how strategic is the role played by the exogenous variables in the model. I refer not to lagged variables, but to unlagged ones. The model is designed to forecast the gross national output and expenditure and its main components among other items. The exogenous expenditures, however, account for some 58 per cent of gross national expenditure (GNE)

and 48 per cent of GNE plus imports. I do not refer to this fact as a criticism, but it *is* an outstanding characteristic of the model and it shows that as of the date of the model at any rate, formal econometric techniques of this particular kind were being used to predict only about one-half of the total of GNE and this, on the whole, the more stable (and hence more predictable) half.

The absence of equations to explain the demand for new fixed capital helps to explain the relatively small role played by financial variables in the model. In the equations dealing with the demand for consumers' goods, financial variables appear. An explicit place has been found for the increment in the inflow of capital from abroad for direct and for portfolio investment in the import equation. But there are financial variables that presumably are important and that have no place in the model. In any event, all financial variables that appear are exogenous. Consequently, the model is not able to handle questions relating to the impact of monetary, debt management, and exchange rate policies upon the economy. These have been important issues of policy in Canada and will continue to be. Perhaps others will feel with me that in building upon Brown's econometric work we should seek to give much more empirical content to our discussions of financial policies and our predictions of their outcomes.

I shall only take time to comment upon one equation in the model, namely, the demand for imports. This equation has a larger number (seven) of independent variables than any other equation in the model. The interpretation given to the equation by Brown is plausible. I would point out, though, that the first four independent variables in the equation have special relations with each other and with the dependent variable. The first independent variable is nonagricultural GNP plus the dependent variable. The second, the increase in nonagricultural inventories, is a component of the first and itself has a substantial import content. The third, investment in machinery and equipment, is a component of the first, and something over half of it is composed of imports of machinery and equipment. The fourth is a component of the first and also has a substantial import content. Thus, the dependent variable or a part of it appears as a component of each of the first four independent variables; and the second, third, and fourth independent variables are components of the first. The equation was estimated by the least squares procedure. I should be curious to see the variances of the estimates of the coefficients of the first four variables of this equation.

Selected Statements and Conjectures in the Paper

I confess that the brief discussion of the determination of prices left me confused. I refer in particular to the paragraph which reads as follows: "In the aggregate or i.e. in the equation on the *global market for final goods*, it is assumed that equilibrium, in the sense discussed above, is reached within yearly periods, a condition which might be much less likely for quarterly or monthly time periods. The equations of GNF^d and GNF^s in this 'market' permitted the determination of both *GNF* and the equilibrating variable. The latter was the price level P^+, which included P, the unknown deflator variable used throughout the model."

It seems to me that the equation which determines the price level, equation G(3), is an identity, not an equilibrium condition. In what sense is it thought that an equilibrium price is reached within yearly periods? Is it an equilibrium price in the sense that the market is cleared? Surely not, for inventory accumulation is not zero. Is it an equilibrium price in the sense that its value is expected to persist into and through the next period? Surely not, for then it would only be necessary to forecast next year's price as equal to this year's. Does it make sense in general to speak of the average of the prices of the year as an equilibrium price? Would it not be better *not* to refer to the price level as an equilibrium price but rather to refer to it simply as the level of prices indicated by the equations and identities?

There are two observations in the paper concerning the importance of cost-push inflation. On page 82 it is suggested that the forecast implied that the expected rise in prices would result from forces on the side of cost rather than on the side of demand. On page 84 it is suggested that the forecast erred in underestimating the strength of demand forces and that the greater increase in prices than was forecast was to be attributed to this strength on the demand side of the market. Would Brown now feel that in 1958 the rise in prices in Canada would be fairly described as showing the effects both of cost push and demand pull?

One final comment: The author has indicated that forecasts of endogenous variables may be wide of the mark because of errors in the forecasts of exogenous variables, and he has given qualitative illustrations of this from the experience he reports. It is also possible that errors in forecasts of endogenous variables may be small even with poor forecasts of the exogenous variables because of unsatis-

factory specification and estimation of the structural equations. It would have been particularly illuminating if Brown had had the resources to permit him to calculate the prediction his model would have yielded of the endogenous variables had the realized values of the exogenous variables been used rather than their projected values.