A Portfolio Theory of Route Choice

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Abstract

Although many individual route choice models have been proposed to incorporate travel time variability as a decision factor, they are typically still deterministic in the sense that the optimal strategy requires choosing one particular route that maximizes utility. In contrast, this study introduces an individual route choice model where choosing a portfolio of routes instead of a single route is the best strategy for a rational traveler who cares about both journey time and lateness when facing stochastic network conditions. The model is then tested with GPS data collected in metropolitan Minneapolis-St. Paul, Minnesota. Our data suggest strong correlation among link speed when analyzing morning commute trips. There is no single dominant route (defined here as a route with the shortest travel time for a 15 day period) in 18% of cases when links travel times are correlated. This paper demonstrates that choosing a portfolio of routes could be the rational choice of a traveler who wants to optimize route decisions under variability.

Transportation planning, route choice, travel behavior, link performance

1 Introduction

Route choice is a daily decision travelers make under variable traffic conditions. Traffic patterns emerge from individual decisions, and each day's collective decisions update the travel experience of all travelers. In the long run, we expect that each traveler will develop an explicit or implicit strategy to guide individual route decisions. Conventional User Equilibrium (UE) models assume that travelers seek to minimize individual travel time with perfect knowledge of network conditions. In equilibrium, "the journey times in all routes actually used are equal and less than those which would be experienced by a single vehicle on any unused route" (Wardrop, 1952). Although this shortest-path (usually measured as shortest travel time path) assumption and the resulting aggregate UE approach is simple, intuitive, and easy to implement (efficient solutions are widely available), it has been criticized for ignoring the heterogeneity in individual preferences among travelers and limitations in their spatial knowledge. Given the stochasticity in network conditions and potential penalties for being late or early, travel time reliability has been widely identified as an important factor in route decisions (e.g. (Bekhor et al., 2006; Brownstone and Small, 2005; Noland and Polak, 2002; Small et al., 2005) among others).

Wardrop's UE principle requires travelers chose the shortest time path. Although several paths may have equal journey time in equilibrium, only one can be chosen by an individual for a given trip. However empirical evidence finds individual travelers chose multiple routes between a given origin-destination pair through repeated choices (Jan et al., 2000). We discuss this in 2.1. While some travelers only had minor deviations, most travelers followed routes that deviated significantly from the shortest time path.

Although many individual route choice models have been proposed to incorporate travel time variability as a decision factor, they are typically still deterministic in the sense that the optimal strategy requires choosing one particular route that maximizes utility.

The Stochastic User Equilibrium (SUE) model adds a random component to the expected travel time. Under SUE, "no user believes he can improve his travel time by unilaterally changing routes" (Daganzo and Sheffi, 1977). The stochasticity is due to either some traveler characteristics not observable by the modeler or other randomness on the network.

The SUE theory however does not explain why some travelers prefer multiple routes over a time period instead of choosing one optimal route. Route choice models based on prospect theory further argue that travelers usually perceive uncertainty in travel cost asymmetrically and human choices usually deviate from what is predicted by expected utility based in empirical studies (Kahneman and Tversky, 1979; Tversky et al., 2005). People are found to underweight high probability events when certainty is not guaranteed (Allais Paradox (Allais, 1979)) and inflate the larger gain when facing alternatives with small probabilities. Parthasarathi (2011) found that traffic network structure variables (such as intersection density, street density, proportion of limited access roads, route complexity, etc.) can also affect travel time perception. Prospect theory has been applied to route choice by investigating the value function and appropriate reference point (Avineri and Prashker, 2004; de Palma and Picard, 2006; Katsikopoulos et al., 2002). However, given an estimated value function, we would expect a pure strategy of choosing the route that minimizes the relative utility when compared to the reference point.

Some researchers approached this multiplicity problem by arguing that travelers are boundedly rational (Lou et al., 2009; Mahmassani and Chang, 1987) and may use one of multiple acceptable routes. Differences in travel cost between these routes and the shortest route are tolerable or not noticeable by travelers. Under this theory, the route in the acceptable set to be chosen will depend on some random events or personal experience. However, no theory is provided to determine the probability of choosing each route. In the context of transit route choice, Spiess and Florian (1989) proposed that the chance of taking a particular transit line among several attractive ones is proportionate to their service frequency. Its applicability to vehicular route choice problem has yet to be explored.

Models such as SUE still treat link travel costs as a deterministic value. In contrast, Watling (2002) assumes network conditions are stochastic and proposes more complicated equilibrium models. Facing such stochasticity, travelers could also change route through a day-to-day learning process, or simply react to previous bad experience. One recent example of that day-to-day dynamics could be the significant link flow oscillation observed after the 2007 I-35 Bridge collapse in Minneapolis, Minnesota (Zhu et al., 2010). However, as travelers accumulate more network knowledge through day-to-day experience, especially for commute trips, deterministic route choice models predict a single optimal route based on the perceived travel time distribution. For example, Mirchandani and Soroush (1987) considered both stochastic link travel time and individual travel time perception error. Although travelers with different risk-taking preferences, thus different utility functions, would take different routes, the final choice for each individual is still deterministic.

To provide such an explanation to the phenomenon that travelers chose multiple routes between a given origin-destination pair through repeated choices, this study introduces an individual route choice model where choosing a portfolio of routes instead of a single route may be the best strategy for a rational traveler trying to satisfy multiple criteria (trading-off journey time and lateness) facing stochastic network conditions. The next section provides empirical evidence of people choosing multiple routes between the same origin and destination, employing GPS data collected in metropolitan Minneapolis-St. Paul, Minnesota. A portfolio theory of route choice is then proposed and tested with the field data. Findings from this paper may inform future travel demand models.

2 Empirical evidence of route portfolios

2.1 GPS data

This study investigates commuters' day-to-day route choices by analyzing a large set of GPS data collected during a 13-week long study targeting behavioral reactions to the I-35W Bridge reopening on September 18th, 2008. Details about this behavioral study and data collection process are provided by Zhu et al. (2010). Participants were randomly selected commuters in the Minneapolis, Saint Paul, Minnesota metropolitan area (Twin Cities). Either a logging Global Positioning System (GPS) devices (QSTARZ BT-Q1000p GPS Travel Recorder powered by DC output from in-vehicle cigarette lighter) or a real-time communicating GPS device (adapted from the system deployed in the Commute Atlanta study ((Rates, 2007)) were installed in the vehicle of study participants. The GPS device is non-intrusive and unlikely to affect the behavior of participants. No instructions were given and participants were free to make travel choices. In all, 190 subjects participated in this study. However, only 143 GPS records were recovered due to the failure of devices (the data from GPS loggers can only be checked at the end of the study. Some of them failed because of power supply problems, such as being disconnected by subjects).

The logging GPS devices accurately monitored the travel trajectories of each probe vehicle at a frequency of one point per 25 meters up to 13 weeks, about 3 weeks before the reopening of the bridge and between 8 and 10 weeks after it. The real-time communicating GPS device recorded the position of instrumented vehicles every second. The geographic location and time stamps of each point were documented and projected onto a GIS map for post-processing. The GPS data were then matched to the 2009 Twin Cities Regional Planning network, which has been conflated to real road geometry.

An algorithm was developed and applied to ensure all points have been snapped to the nearest link which:

- is directly connected to the upstream link previously identified;
- is consistent with the travel direction of nearby GPS points; and
- is connected to the downstream link which is also consistent with travel direction of downstream GPS points.

This algorithm rules out the possibility of incorrectly snapping the GPS point to the link in the opposite direction and changing directions mid-link. The high resolution of one point every 25 meters (the real-time communicating GPS provided an even higher resolution) reduces the possibility of holes and keeps discontinuity in identified routes to a minimum. In rare cases of data losses due to the communication difficulties with satellites, the shortest time path was used to connect the different segments of the same trip. This algorithm, combined with accurate GIS files, ensures that the right links will be identified for each trip. It also helps to ensure that the speed estimated from vehicle trajectories will later be assigned to the link through which travelers passed. A visual check was conducted for all trips of two random subjects during the entire study period, and confirms the accuracy of the algorithm.

2.2 Diversity of commute trips

This study focuses on commute trips because 1) A large number of trips could be observed between the same origin and destination; 2) Travelers are likely to gain enough experience through daily commuting to develop a reasonable estimation of the network; 3) People are more concerned about lateness and travel time reliability, for commute trips than for discretionary trips. To keep the problem simple, we only consider home-to-work trips here, although the same analysis could also be applied for work-to-home trips. Home-to-work trips are defined as any trips starting within a 600 m radius from home and ending in a 600 m radius from the workplace during a weekday, without any stop longer than 5 minutes. The threshold of 600 m represents approximately 4 city blocks, which is chosen by observing parking and workplaces for a subset of subjects. To make all trips comparable in the following analysis, minor changes have been made to ensure trips made by the same subjects always start from the same origin node and end at the same destination node. Very few changes resulted, since parking locations at both home and work places are stable for most people.

The reopening of I-35W Mississippi River Bridge during the study period represents a major change of network condition, which may affect people's route choice behavior. To avoid this confounding factor, we only use data collected during the three weeks before the bridge reopening. Since we only focus on the Twin Cities (7 County) area, subjects who live outside of the region are excluded from the study. In total, 657 home-to-work trips made by 95 subjects have been identified. These trips are then compared segment-by-segment using GIS and different home-to-work routes are identified for each subject. Although the problems of route overlapping and trivial alternatives have been discussed by many researchers under various contexts (e.g. (Bovy, 2009; Frejinger and Bierlaire, 2007)), no consensus has been reached for the threshold to define distinct routes. Therefore, a series of threshold values have been tested. Figure 1 summarizes the percentage of subjects with different number of distinct home-to-work routes observed during three weeks.

If routes with any different segments are treated as different routes, then more than three quarters of all subjects used more than 1 route during three weeks. Some subjects traveled on more than 8 different routes. As the threshold of minimum difference in length to define distinct routes increases, home-to-work route choices exhibit less diversity. However, even if more than 30% difference in distance is required to define a different route, about 40% of all subjects followed more than one route during the study period. Therefore, a significant fraction of subjects chose a portfolio of routes for their morning commute trips. Many reasons could help to explain the behavior of choosing multiple commute routes during a period of time. The next section addresses this problem by investigating route decisions of a rational traveler under uncertain network conditions.

3 Portfolio Theory of Route Choice

In his seminal work *Risk, Uncertainty, and Profit*, Frank Knight (1921) established the distinction between risk and uncertainty.

> "... *Uncertainty* must be taken in a sense radically distinct from the familiar notion of *Risk*, from which it has never been properly separated. The term *risk*,

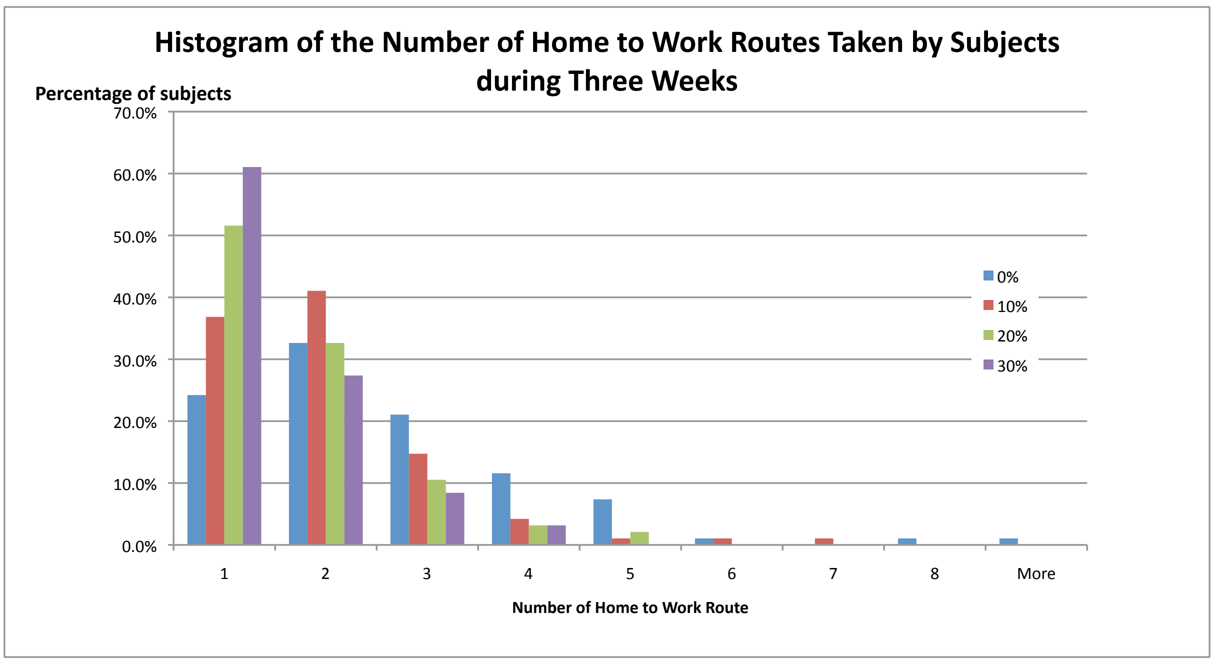

Figure 1: The morning commute route diversity among 95 subject during 3 weeks. Percentage indicates share of distance which may differ without routes still being considered “different”. Data were collected by in-vehicle GPS devices during September, 2008 for a study focusing on route choice behavior before and after the reopening of I-35W Mississippi River Bridge in Minneapolis. Source: Authors

> as loosely used in everyday speech and in economic discussion, really covers two things which, functionally at least, in their causal relations to the phenomena of economic organization, are categorically different. ... The essential fact is that *risk* means in some cases a quantity susceptible of measurement, while at other times it is something distinctly not of this character; and there are far-reaching and crucial differences in the bearings of the phenomenon depending on which of the two is really present and operating. ... It will appear that a measurable uncertainty, or *risk* proper, as we shall use the term, is so far different from an unmeasurable one that it is not in effect an uncertainty at all. We ... accordingly restrict the term *uncertainty* to cases of the non-quantitive type." (Knight, 1921).

This study investigates the route choice behavior of a rational user who seeks to maximize utility under network variability. A wide spectrum of studies has dealt with risk and uncertainty in route choice, either due to perception errors or stochasticity in network conditions (in most cases travel time). Some researchers followed the expected utility approach which was originally proposed by Bernoulli (1954) (originally in Latin and translated by Dr. Louis Sommer) and later popularized by Von Neumann et al. (1947). For example, Pells (1987) assumes that travelers' utility is a linear combination of the generalized travel cost and a slack time that travelers allocate to avoid arriving late (dubbed a "safety margin" by Knight (1974)). Polak (1987) further defined a safety margin as the difference between the mean arrival time and the work start time. The problem with risk and uncertainty was implicitly addressed since travelers have to reserve a larger safety margin with lower travel time reliability.

In contrast, other researchers insist that travel time reliability has intrinsic value (you still prefer reliability even when you are flexible with arrival time) and should be modeled explicitly. Research work in this direction follows the two-parameter approach (mean-variance in most cases) which was originated by Markowitz (1952a,b) and Hester and Tobin (1967) in portfolio studies and then introduced to transportation by Jackson and Jucker (1982). Given an *a priori* estimate of network conditions (mean travel time and the variance), two-parameter models usually define the objective for a rational traveler as:

$$Min\ U = \alpha E(t) + \tau V(t) + \delta C \tag{1}$$

where $E(t)$ is the expected travel time, the $V(t)$ is the variance of travel time, and C summarizes other generalized costs associated with each route. The relative importance of travel time reliability is captured by the parameter τ. Given an individual whose value of time (measured in α/δ), value of reliability (measured in τ/δ), and perception (or prediction for a specific day) of network conditions are fixed, a deterministic choice would be generated in previous studies.

Although these studies reveal that travelers have strong preference for travel time reliability, they provide limited information about why travelers would choose multiple routes over time under the same condition. Previous studies show that travelers trade-off between travel time and travel time reliability because there is a disutility associated with either arriving too early or arriving too late and because of variability of network conditions. Therefore, we first assume that the objective of travelers is to minimize travel time while keeping travel time variability under a certain threshold. Consider a traveler m who faces N alternative routes whose travel time $t' = \{t_1, t_2, ...t_N\}$ are believed to have expected values $E(t)' = \{E(t_1), E(t_2), ...E(t_N)\}$ and a covariance matrix $\Sigma = (\sigma_{i,j})$. It has to be indicated that route travel time t_n is stochastic, which differs from many previous models such as SUE which assumes deterministic network condition. Here the subscript m is omitted to keep the expression succinct. A rational traveler is to select daily routes according to $p' = \{p_1, p_2, ...p_N\}$ in order to

$$Min\ U = E(p't) \tag{2}$$

subject to:

$$Var(p't) \leq v_c \tag{3}$$

$$\sum_i p_i = 1 \tag{4}$$

$$p_i \in [0,1], \forall i \in N \tag{5}$$

where p_i is the probability of choosing route i on a given day and v_c is the maximum travel time variance the traveler can tolerate. Given a network condition t and a personal preference v_c, the optimal strategy $\hat{p}$ can be derived by solving the problem. If $\hat{p}$ has more than one non-zero member, then the optimal strategy is to choose a route portfolio according to $\hat{p}$ instead of sticking to a single route.

To illustrate the idea, consider the simplest case where the rational traveler faces only two alternative routes: 1 and 2. Figure 2 presents an example of possible distribution of travel time on routes 1 and 2 where the traveler has to trade-off between travel time and travel time reliability.

Depending on the travel time distributions and the tolerance for travel time reliability (or variability), a rational traveler could have a different strategy.

For convenience, assume t_1 and t_2 are independent. Then

$$Var(p't) = p'Var(t)p = p_1^2 Var(t_1) + p_2^2 Var(t_2) = p_1^2 Var(t_1) + (1-p_1)^2 Var(t_2) \tag{6}$$

Here $p_2 = 1 - p_1$ because of equation 4. Therefore, the travel time variance by following strategy p is a quadratic function of p_1. Without losing generality, assume $Var(t_1) \geq Var(t_2)$. Then by evaluating equation 6 on the range $[0,1]$,

$$Var(p't) \in \left[\frac{Var(t_1)Var(t_2)}{Var(t_1)+Var(t_2)}, Var(t_1)\right] \tag{7}$$

and as shown on Figure 3, the minimum is achieved when

$$p_1 = \frac{Var(t_2)}{Var(t_1)+Var(t_2)} \tag{8}$$

Depending on the value of v_c, there are 4 situations.

1. if $v_c \geq Var(t_1)$, all possible strategies p are feasible and the best strategy is to always select the route with smaller expected travel time because 3 is always satisfied;

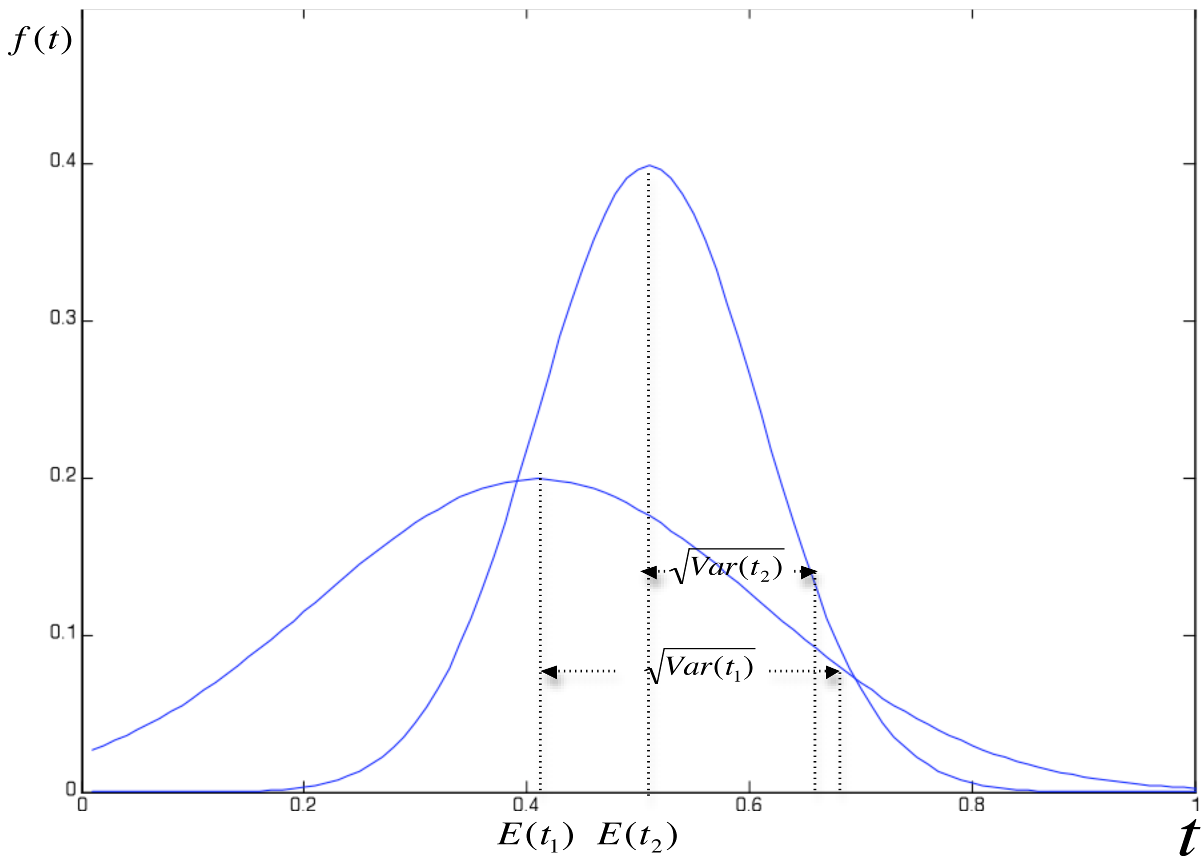

Figure 2: An example of possible distribution of travel time route 1 and route 2. Route 1 has a small expected travel time, but larger travel time variability

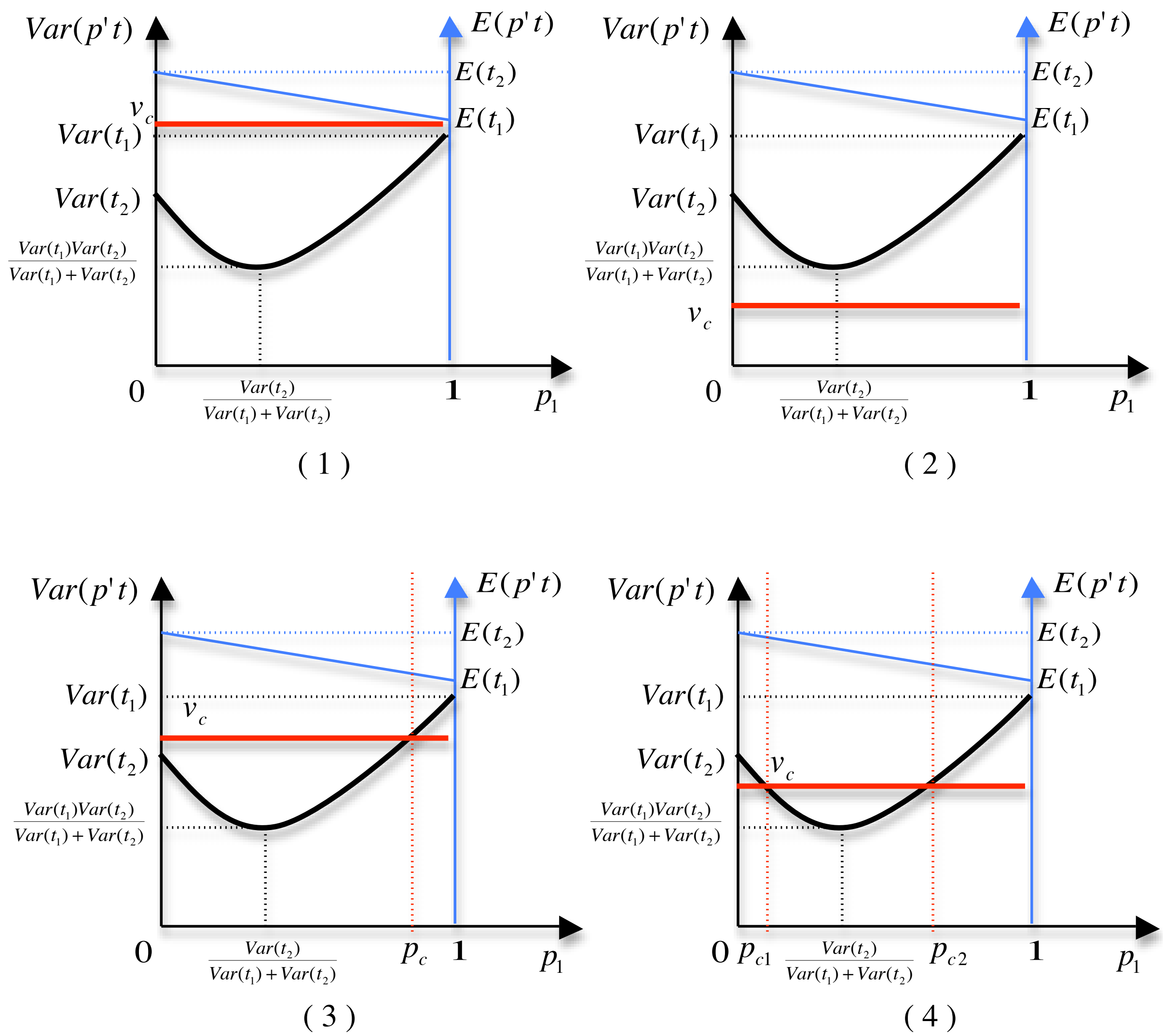

Figure 3: Optimal strategy under different travel time reliability criteria.
(1) Any p_1 is feasible and optimal is achieved at $p_1 = 1$
(2) There is no feasible p_1 that satisfy $Var(p't) < v_c$
(3) p_1 is feasible in $[0, p_c]$and optimal is achieved at $p_1 = p_c$
(4) p_1 is feasible in $[p_c1, p_c2]$and optimal is achieved at $p_1 = p_c2$

2. if $v_c < \frac{Var(t_1)Var(t_2)}{Var(t_1)+Var(t_2)}$, there is no solution to the problem because no strategy p can satisfy 3, in which case the traveler needs to adapt v_c or not travel;

3. if $v_c \in [Var(t_2), Var(t_1)]$, a feasible strategy p must satisfy $p_1 \in [0, p_c]$, while p_c is the strategy when $Var(p't)$ equals v_c. Therefore, if $E(t_1) < E(t_2)$, the best strategy should be to always select route 1. However, when $E(t_1) > E(t_2)$, the optimal is achieved by selecting route 1 by p_c of the time and route 2 by $1 - p_c$ of the time. A route portfolio serves better the objective than a strategy of always choosing a single route. When $E(t_1) = E(t_2)$, the traveler is indifferent.

4. if $v_c \in [\frac{Var(t_1)Var(t_2)}{Var(t_1)+Var(t_2)}, Var(t_2))$, the feasible strategy is depicted by $[p_{c1}, p_{c2}]$ and the best strategy is to always select a route portfolio. The minimum expected travel time is achieved on either p_{c1} or p_{c2}, depending on which route has smaller mean travel time.

Therefore, under some circumstances (such as that of cases 3 and 4), the proposed model predicts that choosing a route portfolio over time represents a better strategy compared to that of always choosing a single route. The independence of travel time on alternatives is not a required condition, but only helps to simplify the presentation. Actually, when the decision maker holds a belief of travel time correlation (captured by Σ), the only difference is that Equation 6 becomes

$$Var(p't) = p'\Sigma p = p_1^2 Var(t_1) + p_2^2 Var(t_2) + 2p_1p_2Cov(t_1, t_2) \tag{9}$$

The conclusion may differ depending on the new quadratic curve depicted by 9. Under certain conditions, a route portfolio could become a dominant strategy.

The results could be further extended to the case of route choice when facing N alternatives. Because the covariance matrix Σ is positive semi-definite, the feasible set defined by Equation 3 is convex. The objective function is a linear combination of expected travel time of all alternatives, so the optimal solution will always fall on the boundary of the feasible set. As long as the optimal solution is achieved at a point other than such corner points that one of the $p_i = 1$ and other members of $\hat{p}$ equal zero, a route portfolio becomes a dominant strategy. Moreover, since the feasible set defined by Equation 3 is convex, the objective function Equation 2 could be a non-linear function as long as it is also a convex function. By following the same reasoning as presented in this section, situations under which a route portfolio dominates a single-route strategy can be derived. The math is likely to be more complex.

As a further extension, other criteria regarding travel time reliability are also applicable. For example, travelers might prefer that the travel is less than 5 minutes longer than the average 95% of the time. Given the travel time co-variance matrix of alternative routes, this constraint can be easily translated into forms similar to Equation 3 ($Pr([t - E(t)] \leq 5) \geq 0.95) \Leftrightarrow Var(t) \leq \left(\frac{5}{1.645}\right)^2$) if we assume t is normal. This too is likely to result in a route portfolio being preferred in circumstances similar to cases 3 and 4.

Although the proposed route portfolio theory may generate similar aggregate travel demand, it differs from conventional User Equilibrium or Stochastic User Equilibrium models through several fundamental behavior assumptions. Both UE and SUE models assume deterministic network conditions. However, given the same travel demand, travel time could still fluctuate significantly for reasons such as signal control, freeway bottleneck activation when demand is close to capacity, etc. When the travel time fluctuation becomes small, the reliability constraint imposed by 3 is no longer binding. The proposed route portfolio theory collapses to UE models. In contrast, SUE model assumes deterministic travel time on each route, but with an individual specific perception error ξ. For our one OD pair, two routes case, the perceived travel time on two alternative routes becomes $T_1 = t_1 + \xi_1$ and $T_2 = t_2 + \xi_2$. The individual error term $\xi = (\xi_1, \xi_2)$ follows uncertain distribution among the population. For one individual, once this perception error is known (e.g. a value is drawn from the population distribution), the traveler would choose the one with shorter travel time T. For the entire population, ξ follows, for example, Multi Variate Normal distribution. Following the standard Probit SUE model, the probability of choosing alternative 1 becomes:

$$P_1 = \Phi\left(\frac{t_2 - t_1}{\sqrt{\sigma^2}}\right) \tag{10}$$

where σ^2 represents the variance of the normal distributed error $\xi_2 - \xi_1$. However, this aggregate route choice probability across the population differs from an individual mixed strategy predicted by the proposed Route Portfolio Theory.

We expect that Portfolio Theory and SUE appropriately calibrated would both give the same aggregate results (averaged over many simulation runs in the case of portfolio theory). However, portfolio theory gives individual travelers different routes on different days (all else equal), while SUE gives each traveler the same route probabilistically. This difference is important for (1) modeling traveler learning behavior, in Portfolio Theory, travelers explicitly learn about some alternatives as they are actually experienced, in SUE, travelers cannot know about the routes that are not traveled on, and some unreasonable assumptions are required about travelers possessing perfect information about alternatives never chosen, (2) modeling the behavior of individuals which is important for air quality, pricing, and many other applications, (3) understanding the underlying logic of traveler behavior. Future research is required to estimate individual preferences. The objective is not necessarily to have a better aggregate assignment, it is to have a better disaggregate route choice. This will begin to matter as HOT lanes and other differentiated pricing schemes on roads are deployed, where assuming the same value of time for all travelers would result in mis-estimation of benefits.

4 Field study

The portfolio theory of route choice we propose shows when the strategy of randomly choosing a route among alternatives is superior to the strategy of always using one route, given travelers' belief of network conditions. The posterior outcome of perceived average travel time could differ from the perceived expected travel time based on prior information. In the long term, however, especially for commute trips where enough experience has been gained to develop consistent perceptions of network conditions, we anticipate convergence between prior and posterior estimates. To check the theoretical reasoning, this section tests the proposed model against field data.

4.1 Predicting route travel time

In order to evaluate whether a rational decision maker guided by our theoretical model is likely to choose a portfolio of routes due to concerns of travel time reliability, travel time distributions of different routes are required. Ideally, we can obtain this information by observing day-to-day route choices during a period so long that we can collect enough samples for each route to empirically establish its travel time distribution. However, this is infeasible due to limited resources for most studies, especially for those subjects with very diverse route choices. Moreover, relying exclusively on direct observations also limits our ability to extend our analysis to the general population and to inform travel demand modeling efforts. Instead, empirical models are built in this study to predict route travel time distributions.

The large number of GIS equipped vehicles are used as probe vehicles for the purpose of measuring travel speed on the network in this study. The speed with which the probe vehicle traversed a link along its trajectory could be estimated by comparing the spatial and temporal distances between points at each end of the link. The average link speed could be estimated from all probe vehicles passing this link during a defined time period. The long study period allows us a large number of observations not only on freeway links, but also on major arterial links and local streets in downtown (see Figure 4).

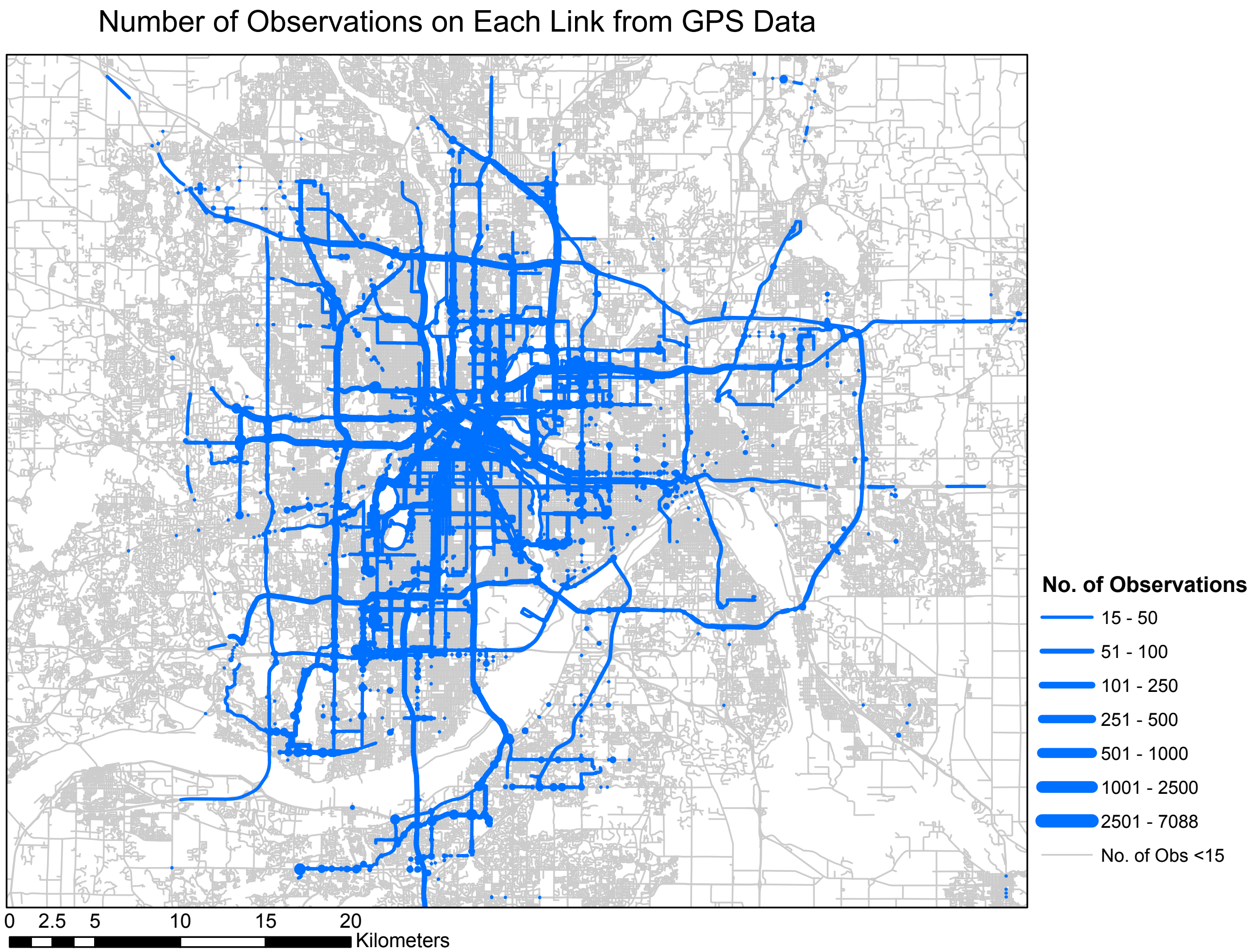

Figure 4: The number of speed observations on each link during the entire study period

Speed samples on arterial roads in the outer suburbs are generally low. However, road density in those areas were low and the traffic was unlikely to vary much due to scattered demand. Therefore, speed on roads with insufficient samples were assumed constant through the study and equal to the average speed on all the links of the same functional class defined by the US Census Bureau in their TIGER files (Marx, 1990). [1] More details about GPS data processing and link speed estimation are provided by Zhu et al. (2010).

4.2 Normality test

Given the mean and variance of link speed, we can simulate route travel time by generating random link speed and then summing up link travel time for all links along a specific route. Although route travel time has been widely assumed to follow a normal distribution in previous research (e.g. (Liu et al., 2004; Ryuichi and Mohamed, 1997)), this assumption is

[1]The data can be downloaded from http://www.datafinder.org.

empirically tested here using GPS observations. Figure 5 provides the normal probability plot of home-to-work travel time observations for one subject over 13 weeks. The y-axis represents route travel time in seconds, while the x-axis represents the Z-score of corresponding points ordered from small to large. The normality of data is established if a straight line can be fit to the points. Three points in red are clearly outliers according to the plot. According to the original data, the unusually long travel time in these three cases are due to stops near the destination. These stops could be due to activities such as searching for parking, visiting a coffee shop, or making a phone call along the route. However, GPS data alone cannot detect the causes and future research is required to define those trips.

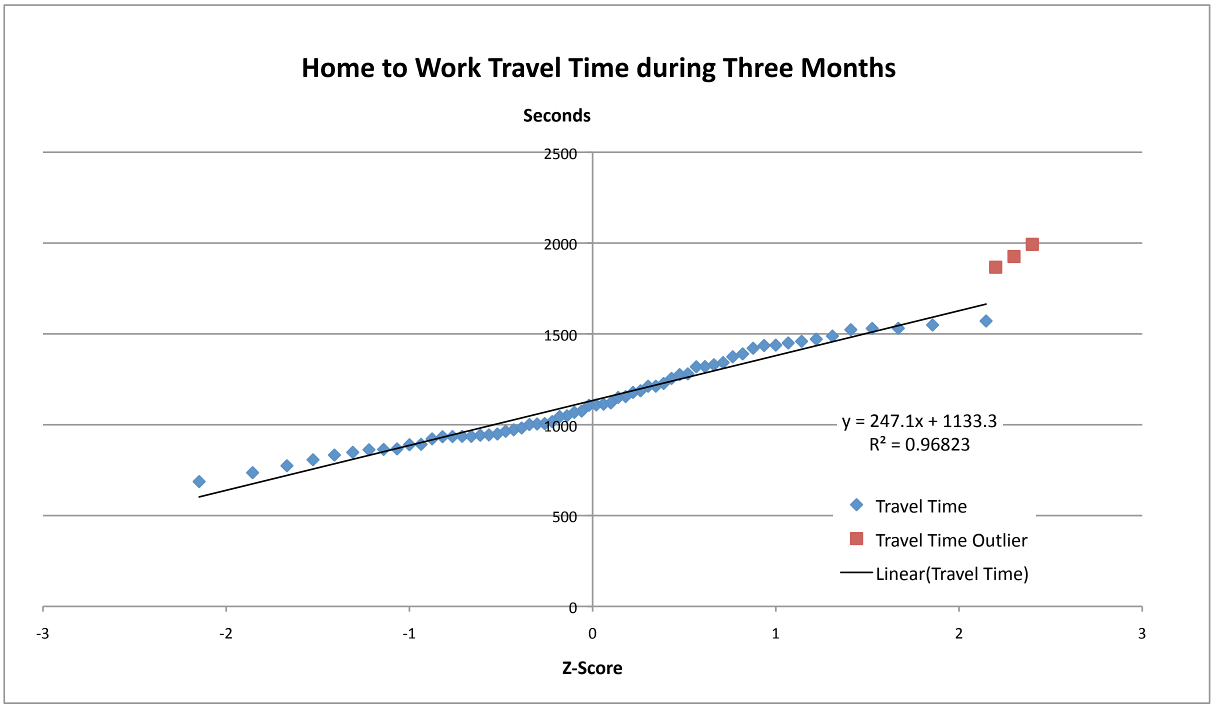

Figure 5: Normal probability plot of home-to-work travel time observations for one subject during 13 weeks

Although the normal probability plot provides an intuitive illustration of how well a normal distribution fits the data, a more robust statistical test is required. We apply the Shapiro-Wilk test to all subjects and choose 0.05 as the critical value to reject normal assumption. In total, the assumption that route travel time follows a normal distribution has been rejected for 22 out of all 95 subjects (23%). Future research efforts for better detecting side trips can help to exclude outliers in current commute time data set and the normal assumption could be more convincingly supported. Given that the majority of evidence do not reject the normal assumption, we assume route travel time follows a normal distribution in the following sections.

4.3 Testing link travel times for independence or correlation

In order to simulate route travel time from random draws of link speed, assumptions about speed interdependency among different links on the network have to be made. Some studies conveniently assumed link travel time are independent and identically distributed (IID), which implies no correlation across links. However, there is clearly speed correlation across links, presumably due in part to exogenous factors like weather, holidays, etc. affecting overall demand and in part to congestion (recurring and non-recurring) causing link interactions. Although many previous studies investigate the short-term spatiotemporal pattern of traffic flow (e.g. Kalman filtering approach by Whittaker et al. (1997)), it is hard to extend these models to a real network where the number of variables to be estimated become prohibitive. Few studies address spatiotemporal speed patterns on a large network (Kamarianakis and Prastacos, 2005). To simplify the analysis, this study evaluates assumptions of both no correlation and perfect correlation across links. Reality is probably somewhere in between. Perfect correlation is modeled using a single random number to draw travel time from a link's travel time distribution (a normal distribution as per above, given the link's mean and standard deviation) on all links on the route, IID is modeled using a different random number for each link.

Link travel time for 15 days is simulated and travel time on all routes are evaluated, under both IID and perfect correlation assumptions for link speed. The simulated mean route travel time and its standard deviation are then compared to those derived directly from GPS data. Figures 6 and 7 compare the observed mean travel time and simulated travel time under two extreme assumptions about link speed correlation. Both graphs show systematic bias in travel time estimation. Our simulation model underestimates travel time for most cases. As previously indicated, some GPS observations provide unusually long route travel time due to stops for various reasons. These trips inflate average route travel time derived from GPS data. Future research should address this issue.

To reduce the impacts of extreme values, median travel time instead of mean travel time is used in the following analysis. A regression has been conducted to test the following assumption:

$$T_{GPS} = \alpha_T T_{IID} + \beta_T T_{Corr} \tag{11}$$

Where T_{GPS} is the observed median overall route travel time; T_{IID} and T_{Corr} are simulated route travel time under IID and perfect correlation assumptions, respectively. Only routes with more than 5 GPS observations are used. By enforcing constant term as 0, we obtain a high R^2 value of 0.95 for our regression, implying a good fit for data. According to the regression model, α_T equals 0.03 and β_T equals 1.17. The difference in coefficients suggests that high speed correlation exists across links. As all the trips in this analysis are morning commute trips, recurrent congestion during the morning peak hour may help to explain the strong correlation in link speed. Ideally, we would expect $\alpha_T + \beta_T = 1$. Deviation from this assumption also suggests that systematic bias exists in our model.

$$\sigma_{GPS} = \alpha_\sigma \sigma_{IID} + \beta_\sigma \sigma_{Corr} \tag{12}$$

A similar regression model is applied for travel time standard deviation. An R^2 value of 0.46 is obtained. Estimated α_σ and β_σ are 3.71 and -0.17, respectively. Because of the limited

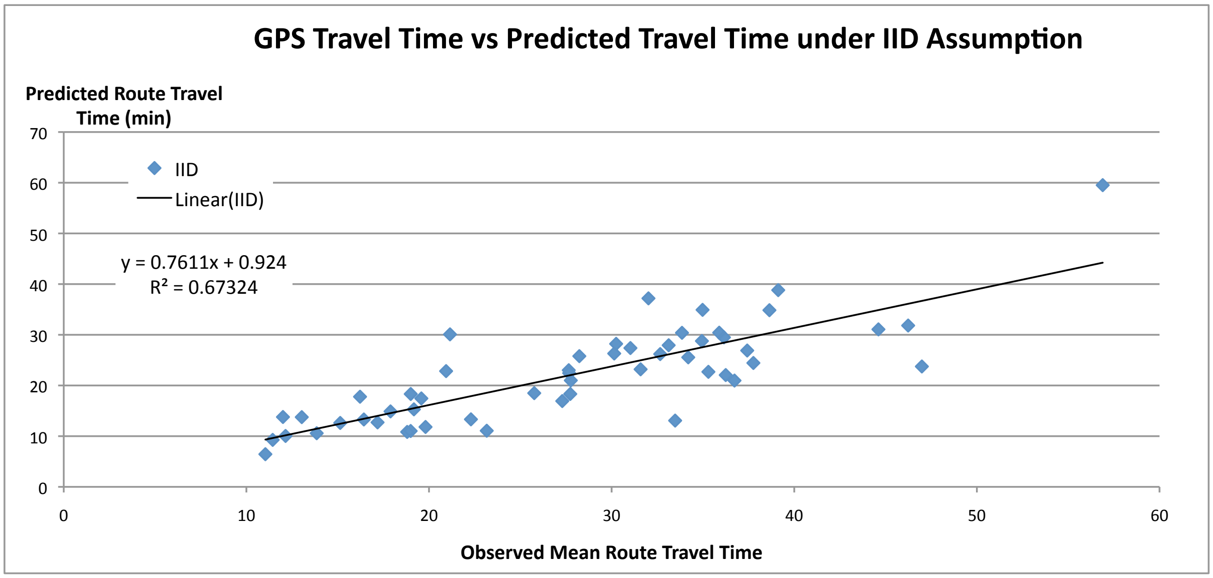

Figure 6: Comparison between observed mean route travel time and predicted mean route travel time under the IID assumption

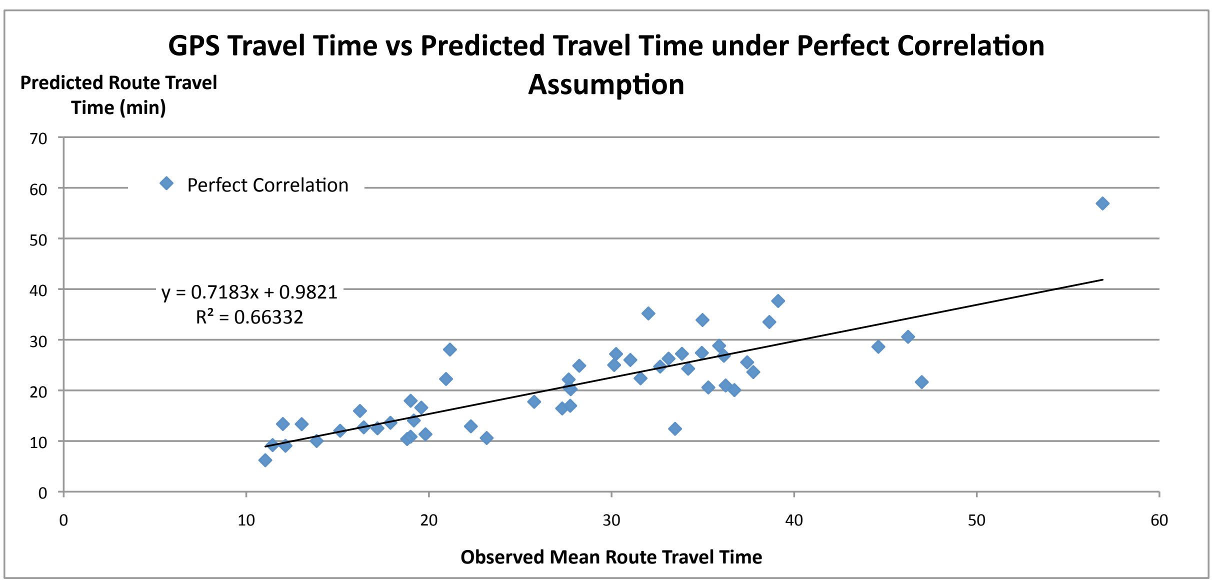

Figure 7: Comparison between observed mean route travel time and predicted mean route travel time under the perfect correlation assumption

number of commute trips during three weeks, prediction of travel time standard deviation is less successful compared to that of mean travel time. Should more data be available, reliable prediction of travel time variance can be obtained by following the same process.

4.4 Mixed strategy in route choice

Many reasons explain the choice of multiple commute routes during a period of time. This study adds one more explanation by assuming that some travelers seek to minimize travel time while maintaining an appropriate level of travel time reliability. In this study, 60 subjects used more than 1 morning commute route during the study period. We then compare the predicted travel time of the two most frequently used alternatives suggested by GPS data. Under the IID assumption, random numbers will be drawn separately for each link from a standard-normal distribution and a normal distributed link travel time will be calculated using the mean link travel time and the variance previously derived from GPS data. A minimal speed of 8 mile/hour is assumed to truncate the extremely long travel time. Path travel time can then be obtained by summing link travel time along the path. Totally 15 random draws are conducted to simulate random commute time for 15 days and the mean and variance of travel time for each path can be calculated and compared. Similarly, the same process is followed to derive path travel time and variance in the case of perfect correlation except for that the same random number is used for all links in each day so that link travel time are perfectly correlated across the network.

Under the IID assumption, there is no single dominant route (travel time always shorter during 15 days) in 38% of cases. It drops to 18% under perfect correlation conditions. If we define a dominant route as the those which possess both shorter travel time and smaller travel time variance, 16 out of 60 subjects do not have a dominant route under IID conditions (12 under the perfect correlation condition). Thus, it is possible for these people to choose a portfolio of routes in order to trade-off between travel time and travel time reliability, as illustrated by our theoretical model. However, better designed experiments controlling for more confounding factors are required to establish a causal relationship between the process of seeking travel time reliability and the choice of a route portfolio.

5 Conclusions

Many travelers use multiple routes to connect the same origin and destination on different days. This paper demonstrates that choosing a portfolio of routes could be the rational choice of a traveler with multiple criteria (e.g. minimizing journey time subject to avoiding frequent lateness) who wants to optimize route decisions under variability. This result can be extended to the choice among N alternatives and by following a more sophisticated objective function. By testing the proposed model against field data, this paper advances our understanding about travelers' route choice behavior. Specifically, this study provides an additional explanation for the stochasticity in individual route choice decisions.

Field study based on GPS data suggest that route travel time can be predicted from estimates of link travel time, which are readily available through a variety of data collection technologies. However, it is not appropriate to assume full independence across links. Our data suggest strong correlation among link speeds when analyzing morning commute trips.

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