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Abstract

We show that in a homogeneous-good duopoly market with quality uncertainty and constant unit costs, the Bertrand paradox (i.e., marginal cost pricing) can be avoided.

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1 Introduction

The well-known "Bertrand paradox" (Bertrand, 1883) arises in a static setting when two firms sell a homogeneous good and have identical unit costs. The Bertrand paradox can be resolved within the homogeneous-good framework by introducing capacity constraints (see Maskin (1986) and references therein). The constant unit cost assumption is also crucial, since both decreasing (Dastidar, 1995) and increasing (Vives, 1999) returns eliminate the paradox. Baye and Morgan (1999) further established that bounded monopoly profits are necessary for the emergence of the Bertrand paradox with constant unit costs, while Kaplan and Wettstein (2000) demonstrated in a slightly different setting that unbounded revenues are necessary and sufficient for the emergence of a non-paradoxical mixed-strategy equilibrium. In addition, the influence of different sharing rules (in the case of price ties) on the emergence of the Bertrand paradox has been investigated by Hoernig (2007).

In this paper we provide a new resolution of the Bertrand paradox for homogeneous-good duopolies with constant unit costs by introducing quality uncertainty.[1] We assume that the demand side of the market is given by a representative consumer and that product quality is unknown when the consumer makes his purchase decision. Consumer beliefs

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[1]This is quite distinct from allowing for uncertainty with respect to either demand or entry (as considered, for example, in Reisinger and Ressner (2009) and Janssen and Rasmusen (2002), respectively). In our setting, firms face both known demands and steadfast competition.

with respect to product quality are assumed to be identical across products, so that the goods produced by the duopolists are *ex ante* homogeneous. Given these beliefs, the representative consumer aims to maximize the probability of achieving some threshold level of quality. This approach endogenously generates well-defined utility (and demand) correspondences over goods, and has been justified as consistent with both an intuitive interpretation of expected utility theory (Castagnoli and LiCalzi, 1996) and with a naturalistic view of preferences, in which evolution chooses a survival-maximizing agent (Smith and Tasnádi, 2009).

2 The framework

Formally, a decision-maker ("consumer") is faced with a menu of two goods, x and y, and must choose how much of each to consume, given income m and prices p_x and p_y, respectively. There is a single unobservable characteristic (quality) for which there is a critical threshold: the consumer seeks only to maximize the probability that he consumes k units of this quality. The amounts of the unobservable quality per unit of x and y are independent random variables, denoted C_x and C_y. Hence, the consumer's utility function is given by

$$U\left(x,y\right)=P\left(C_x x+C_y y\geq k\right), \tag{1}$$

and his decision problem can be stated:

$$\begin{aligned} \max_{x,y} \quad & U(x,y) \\ \text{s.t.} \quad & p_x x+p_y y\leq m \\ & x,y\geq 0 \end{aligned} \tag{2}$$

We assume that C_x and C_y are distributed according to the uniform distribution on the interval $[0,1]$. As shown in Smith and Tasnádi (2009), this gives $U\left(x,y\right)$ the following form:

$$U(x,y)=\begin{cases} 0 & \text{if} \quad 0\leq x+y\leq k, \\ 1-\frac{k}{x}+\frac{y}{2x}+\frac{(k-x)^2}{2xy} & \text{if} \quad x+y>k, x\leq k \text{ and } y\leq k, \\ 1+\frac{x}{2y}-\frac{k}{y} & \text{if} \quad x+y>k, x\leq k \text{ and } y>k, \\ 1+\frac{y}{2x}-\frac{k}{x} & \text{if} \quad x+y>k, x>k \text{ and } y\leq k, \\ 1-\frac{k^2}{2xy} & \text{if} \quad x+y>k, x>k \text{ and } y>k. \end{cases}$$

Now, it can be shown that as long as positive utility levels are attainable ($\frac{m}{p_x}>k$ or

$\frac{m}{p_y} > k$), the optimal solution to (2) is given by $(x^*, y^*) \in$

$$\begin{cases} \left\{\left(\frac{m}{2p_x}, \frac{m}{2p_y}\right)\right\} & \text{if } \frac{m}{2p_x} > k \text{ and } p_x \geq p_y; \\ \left\{\left(0, \frac{m}{p_y}\right)\right\} & \text{if } \frac{m}{2p_x} < k \text{ and } p_x > p_y; \\ \left\{\lambda\left(\frac{m}{2p_x}, \frac{m}{2p_y}\right) + (1-\lambda)\left(0, \frac{m}{p_y}\right),\ \lambda \in [0,1]\right\} & \text{if } \frac{m}{2p_x} = k \text{ and } p_x > p_y; \\ \left\{\left(\frac{m}{2p_x}, \frac{m}{2p_y}\right)\right\} & \text{if } \frac{m}{2p_y} > k \text{ and } p_x < p_y; \\ \left\{\left(\frac{m}{p_x}, 0\right)\right\} & \text{if } \frac{m}{2p_y} < k \text{ and } p_x < p_y; \\ \left\{\lambda\left(\frac{m}{2p_x}, \frac{m}{2p_y}\right) + (1-\lambda)\left(\frac{m}{p_x}, 0\right),\ \lambda \in [0,1]\right\} & \text{if } \frac{m}{2p_y} = k \text{ and } p_x < p_y; \\ \left\{\left(0, \frac{m}{p_y}\right), \left(\frac{m}{p_x}, 0\right)\right\} & \text{if } \frac{m}{2p_x} < k \text{ and } p_x = p_y; \\ \left\{\left(\frac{m}{p_y} - \lambda, \lambda\right),\ \lambda \in [0, \frac{m}{p_x}]\right\} & \text{if } \frac{m}{2p_x} = k \text{ and } p_x = p_y. \end{cases}$$

These demands are set-valued in four cases. For simplicity, we resolve this indeterminacy by assuming that the consumer spends his money equally between the two products whenever possible. However, this is not possible if $\frac{m}{2p_x} < k$ and $p_x = p_y$. In this case, we assume that the consumer randomizes between the two corner solutions by choosing each with probability 1/2.[2] We shall denote the demand function for good x by $D_x(p_x, p_y)$ and for good y by $D_y(p_x, p_y)$.

We assume two duopolists in the market, firms x and y, setting respective prices p_x and p_y. The firms have linear cost functions with respective positive unit costs c_x and c_y. Thus, firm i's profit function is given by

$$\Pi_i(p_x, p_y) = D_i(p_x, p_y)(p_i - c_i),$$

where $i = x, y$. If there is no equilibrium in pure strategies, we will consider an ε-equilibrium as a solution of our price-setting game whenever such exists. In what follows we assume that $\frac{m}{c_x} > k$ or $\frac{m}{c_y} > k$, which ensures that at least one firm can be viable on the market.

3 Resolving the Bertrand paradox

In analyzing the game described in Section 2, we consider four sub-cases.

Proposition 1 *If $\frac{m}{2c_x} \geq k$ and $\frac{m}{2c_y} \geq k$, then there exists a unique Nash equilibrium in which both firms set price $p^* = \frac{m}{2k} \geq \max\{c_x, c_y\}$.*

Proof. If $p_x \geq \frac{m}{k}$ and $p_y \geq \frac{m}{k}$, then the consumer cannot achieve the threshold k with positive probability, and therefore will not consume anything at all. Hence, at least one firm, say firm x, sets a price less than $\frac{m}{k}$, which in turn implies that firm y will not set price $p_y \geq \frac{m}{k}$, since otherwise firm x would capture the entire market.

There cannot be an equilibrium with $(p_x, p_y) \in \left(\frac{m}{2k}, \frac{m}{k}\right]^2$, since each firm benefits by unilaterally undercutting its respective opponent. Moreover, if one firm sets its price not above $p^* = \frac{m}{2k}$ while the other firm sets a price $p > p^*$, then the low-price firm will

[2] Resolving indeterminacy in this way guarantees the existence of an equilibrium in pure strategies in Proposition 1. Otherwise, there would exist many ε-equilibria in pure strategies close to the solution given in Proposition 1.

capture the entire market. Hence, in equilibrium we must have $p_x \leq p^*$ and $p_y \leq p^*$. In this price region, however, the demand functions of the firms become functions of only their own prices. Moreover, since the two demand functions are hyperbolic, the firms cannot increase their revenues by lowering their prices below p^*. Doing so, however, would increase their demands and thus their costs. If $\frac{m}{2c_x} > k$ and $\frac{m}{2c_y} > k$, each firm will make positive profit at price p^*, and therefore each firm will remain in the market. Thus, when $\frac{m}{2c_x} > k$ and $\frac{m}{2c_y} > k$, (p^*, p^*) is the unique Nash equilibrium of the price-setting game.

The case in which $\frac{m}{2c_x} = k$ or $\frac{m}{2c_y} = k$ deserves additional scrutiny, in order to show that (p^*, p^*) is still the unique equilibrium. This situation implies that at least one firm (say x) makes zero profit. Therefore, x could choose to stay out of the market by switching to a sufficiently high price. But then y (now serving the entire market) would have an incentive to raise its price in order to reduce its demand (and hence its costs). In turn, this implies that firm x would like to re-enter the market once again by slightly undercutting the new price p_y. Therefore, (p_x^*, p_y^*) is the unique Nash equilibrium. ■

We note that this outcome is in some sense a resolution to the Bertrand Paradox, in that we have duopolists competing in price but the equilibrium price is greater than marginal cost. This also holds for the case of identical unit costs, as assumed in the standard Bertrand setting. Intuitively–as the above proof should make clear–this result obtains because each firm chooses the smallest price that ensures the other firm will not drive it from the market.

It should also be noted that if the unit cost of at least one firm is sufficiently high relative to the consumer's quality threshold, then the usual Bertrand results obtain. We show this in the following three propositions.

Proposition 2 *If* $\frac{m}{2c_x} \geq k > \frac{m}{2c_y}$*, then firm* x *will drive firm* y *out of the market by setting a price slightly below* c_y*, while firm* y *sets price* c_y*.*

Proof. The proof is similar to that of Proposition 1. In addition, note that firm y would make a negative profit at price $p^* = \frac{m}{2k}$. In particular, it will drop out of the price war when the price falls below c_y. ■

The following proposition follows immediately from Proposition 2 by interchanging the roles played by firms x and y.

Proposition 3 *If* $\frac{m}{2c_y} \geq k > \frac{m}{2c_x}$*, then firm* y *will drive firm* x *out of the market by setting a price slightly below* c_x*, while firm* x *sets price* c_x*.*

The fourth case of $\frac{m}{2c_x} < k$ and $\frac{m}{2c_y} < k$ boils down to the usual Bertrand game.

Proposition 4 *Assuming that* $\frac{m}{2c_x} < k$ *and* $\frac{m}{2c_y} < k$*,*

- *if* $c_x = c_y$*, then both firms set price* $p^* = c_x = c_y$*, and*
- *if* $c_x \neq c_y$*, then the low-cost firm drives the high-cost firm out of the market by setting a price slightly below the high-cost firm's unit cost, while the high-cost firm sets its price equal to its unit cost.*

Proof. Once again the proof is similar to that of Proposition 1. Now–in contrast to Propositions 2 and 3–both firms are constrained by their unit costs in the price war. Hence, the high-cost firm drops out of the market if $c_x \neq c_y$. ■

4 Conclusion

We have formulated a model of duopolistic price competition in which demand is endogenously derived from a few simple assumptions about consumer preference for quality. Given certain parameter restrictions, we have shown that *ex ante* uncertainty about product quality will lead consumers to choose positive quantities of both goods, even when prices are unequal and expected quality is the same across products. These conditions thus make it possible for both firms to remain in the market, and to sell at prices that exceed marginal cost. This raises the obvious implication that information about product quality–even if such information is not product-specific–might be an important strategic variable in oligopolistic settings.

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