



Marko Melolinna

Using financial markets information to identify oil supply shocks in a restricted VAR



EUROJÄRJESTELMÄ
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Bank of Finland Research
Discussion Papers
9 • 2008

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The views expressed in this paper are those of the author and do not necessarily reflect the views of the Bank of Finland.

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I am grateful to Juha Kilponen, Antti Ripatti and Jouko Vilmunen for their helpful comments. I would also like to thank Stephen G Donald, Richard J Smith and Harald Uhlig for providing the programming code for their work.

<http://www.bof.fi>

ISBN 978-952-462-434-3
ISSN 0785-3572
(print)

ISBN 978-952-462-435-0
ISSN 1456-6184
(online)

Helsinki 2008

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Bank of Finland Research
Discussion Papers 9/2008

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Abstract

This paper introduces a methodology for identifying oil supply shocks in a restricted VAR system for a small open economy. Financial market information is used to construct an identification scheme that forces the response of the restricted VAR model to an oil shock to be the same as that implied by futures markets. Impulse responses are then calculated by using a bootstrapping procedure for partial identification. The methodology is applied to Finland and Sweden in illustrative examples in a simple 5-variable model. While oil supply shocks have an inflationary effect on domestic inflation in these countries during the past decade or so, the effect on domestic GDP is more ambiguous.

Keywords: oil futures, partial identification, macroeconomic shocks

JEL classification numbers: C01, E32, E44

Rahoitusmarkkinainformaation käyttö öljyn tarjontasokkien identifioimisessa rajoitettua VAR-mallia käyttäen

Suomen Pankin keskustelualoitteita 9/2008

Marko Melolinna
Rahapolitiikka- ja tutkimusosasto

Tiivistelmä

Tässä paperissa esitellään metodologiaa, jonka avulla pieneen avotalouteen kohdistuvia öljyn tarjontasokkeja voidaan identifioida rajoitetulla VAR-mallilla. Tutkimuksessa rakennetaan rahoitusmarkkinainformaation avulla identifikaatio-skeema, joka pakottaa öljysokin vasteen rajoitetussa VAR-mallissa samaksi kuin futuurimarkkinoilla. Impulssivasteet lasketaan tämän jälkeen käyttämällä ns. bootstrapping-menetelmää osittaisidentifikaatiossa. Metodologiaa sovelletaan esimerkkitapauksissa Suomeen ja Ruotsiin yksinkertaisessa viiden muuttujan mallissa. Öljyn tarjontasokeilla on inflatorinen vaikutus kansalliseen inflaatioon näissä maissa noin 10 viime vuoden aikana, mutta vaikutukset kansalliseen BKT:hen ovat epäselvempiä.

Avainsanat: öljyfutuurit, osittaisidentifikaatio, makrotalouden sokit

JEL-luokittelu: C01, E32, E44

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1 Introduction

Due to the sharp rise in the price of crude oil, recent years have seen an increase in the number of papers studying the effects of oil shocks¹ on macroeconomic variables. However, there has been interest in the subject ever since the early 1980's, when the effects of the oil price shocks of the previous decade due to the supply disruptions from OPEC countries came under scrutiny.

The main tools typically used in gauging the effects are either theoretically based macroeconomic models, or VAR models of different specifications. The pioneering paper in the latter category is Hamilton (1983), which finds a strong relationship between oil shocks and real economic variables in seven of the eight US recessions between 1948 and 1980. Support to this is given by Burbridge et al (1984), which identifies – again, using a VAR model – a negative effect of positive oil shocks on economic activity in five major industrial countries.

Since these early efforts at measuring the effects of oil shocks on real economy, there have been differing opinions and explanations offered on the importance of the effects. For example, Hooker (1996) finds few signs of the effects since the early 1970's. Some authors have suggested non-linear oil price specifications, like Jimenez-Rodriguez et al (2005). Most of the results tend to favour a negative non-linear relationship between oil shocks and real variables in major industrial countries.

In recent years, a few studies have questioned the identification of an oil shock. Traditionally, an oil shock in the literature is defined as an oil price shock. However, in reality an oil price shock is an interaction between oil supply and demand shocks, which cannot be easily disentangled. Kilian (2006a and 2006b) finds only limited short-run effects of oil supply shocks – constructed by creating counterfactual production figures for oil-exporting countries experiencing production shortfalls due to wars – on real variables in the US. Kilian (2007) uses a structural VAR model to disentangle demand and supply shocks, finding that, in general, demand shocks tend to have a larger effect on the real economy and inflation than supply shocks.

Anzuini et al (2007) introduce a new methodology for identifying oil supply shocks in a structural VAR. Based on a methodology originally introduced by Faust et al (2004) for studying monetary policy shocks in the US, they determine supply shocks as events in daily futures markets data and are thus able to identify a structural VAR. According to their results, positive oil supply shocks have a stagflationary effect on the US economy and they have contributed significantly to US recessions during the past 30 years.

This methodology has advantages compared to the methodologies traditionally used in the literature to study the effects of oil shocks. First, in the spirit of Kilian (2007), it allows for a way of disentangling supply effects from demand effects through financial market information. Second, unlike most previous studies, it does not rely on recursive Choleski decompositions

¹As a matter of definition, oil shocks are seen as oil *price* shocks in most studies, although an important difference between them and actual oil *supply* shocks is developed below. Throughout this paper, a *positive* oil shock refers to a supply shock that raises the price of oil.

to identify the shocks but instead, the shocks can be identified with financial market data on crude oil futures prices.

This paper relies on the methodology introduced by Faust et al (2004) and Anzuini et al (2007), who have used a full-VAR specification to study the effects of shocks in a large open economy with partial identification. However, I extend the methodology to study the effects of oil supply shocks in a small open economy. This implies using a restricted VAR specification, because some of the domestic variables of a small economy cannot realistically be expected to affect the international price of crude oil. Unlike Anzuini et al (2007), I also use a partial identification scheme introduced by Faust et al (2004).

In an illustrative example, I apply my methodology to a simple five-variable model for both Finland and Sweden. The results show that there is no statistically significant long-term relationship between oil shocks and real variables in these economies. On impact, there is a statistically significant negative effect of positive oil shocks in Finland, but not in Sweden. There is evidence of positive oil supply shocks causing higher inflation, especially in Sweden, whereas in Finland, the effect is more ambiguous. These results are slightly different from the stagflationary effect found by Anzuini et al (2007) in the case of the US.

The paper is organised as follows. Section 2 introduces the methodology with which oil supply shocks are extracted from financial markets information and linked to the VAR model, section 3 deals with the financial market data of oil supply shocks, section 4 introduces the model setup, section 5 presents some results and section 6 concludes. Some technical details about estimation procedures as well as auxiliary results are relegated to the appendices.

2 Financial markets information and oil supply shocks

2.1 Identification of oil supply shocks

Oil supply shocks in the model are identified by using information from oil futures contracts and a restricted VAR model. As regards the restricted VAR, the reduced form residuals are first linked to the structural form disturbances in a standard way.

Consider the reduced form VAR

$$A(L)Y_t = u_t \tag{2.1}$$

where Y_t is a $K \times 1$ vector of endogenous variables, $\sum_{j=0}^p A_j L^j = A(L)$ (where p is the number of lags in the model), $A_0 = I$ and $u_t \sim N(0, \Sigma)$ (where Σ is the covariance matrix of the error term). $A(L)$ is an invertible $K \times K$ matrix and includes certain zero elements to make the system restricted VAR.

Equation (2.1) can be written in form

$$Y_t = B(L)u_t \tag{2.2}$$

where $B(L) = A(L)^{-1} = \sum_{i=0}^{\infty} B_i L^i$. $B(L)$ can be derived recursively from the reduced form A matrices

$$B_i = \sum_{j=1}^i B_{i-j} A_j \quad (2.3)$$

and $B_0 = I_K$. By assuming that the reduced form errors u_t are related to the structural errors ϵ_t as follows

$$u_t = S\epsilon_t \quad (2.4)$$

where S is a full rank $K \times K$ matrix. (2.2) can then be made structural by writing it in terms of the structural shocks

$$Y_t = B(L)S\epsilon_t \quad (2.5)$$

Assume that the first column of S corresponds to the oil supply shock and call it α . The impulse response of all variables in the restricted VAR to the oil shock is then

$$B(L)\alpha = \sum_{j=0}^{\infty} B_j \alpha L^j \quad (2.6)$$

The k th element of the $K \times 1$ vector of lag polynomials $B(L)\alpha$ traces out the response of the k th variable to the oil supply shock. The B s are known from the reduced form estimates through (2.3). Hence, identifying the impulse responses requires picking the K elements of α .

To identify the oil supply shocks through α , I use information contained in the futures contracts in correspondence of events classified as oil supply shocks. There are two steps in the identification procedure: (a) deriving the response of the expected oil prices from the futures and (b) imposing the equality between the restricted VAR impulse response of the oil prices to the oil shock and the response measured through the futures. The next three sections briefly describe first (b), then an identification issue related to (b), and finally (a).

2.2 Matching responses of oil prices

Assume that, in the case of no uncertainty, the response of the oil price identified from the futures markets at time $t + h$ to an oil price shock at time t is r_h , $h = 0, 1, \dots, K - 1$. Hence

$$r_h = B_{h,oil}\alpha \quad (2.7)$$

where $B_{h,oil}$ is the row of B_h corresponding to the oil price (in this case, the first row). Stacking all these equations for $h = 0, \dots, K - 1$

$$r = R\alpha \tag{2.8}$$

where the rows of R are the relevant row vectors $B_{h,oil}$ and the elements of r are the corresponding elements of r_h . The $B_{h,oil}$ are derived from the reduced form model according to (2.3). The response of oil prices to an oil supply shock, r_h , can be obtained by using information contained in the futures (specified below).

The above system has K unknowns (the elements of α) in K equations. Its solution, under the condition that R is of rank K , is

$$\alpha = R^{-1}r \tag{2.9}$$

2.3 Partial identification

In the above discussion, r and R are treated as if they were known with certainty. In reality, uncertainty in both r and R must be taken into account for inference. Specifically, if R is not full rank, then the system cannot be identified with certainty. When I test the rank of the R matrix in the example cases, it turns out the full rank assumption of equation (2.9) fails (see discussion in section 5.2 for details). Thus the system is only partially identified. The reason for this is also intuitively clear: the response of the oil price variable is very similar at different horizons ($B_{h,oil} \approx B_{h+1,oil}$), so after imposing the impulse response in the VAR to the shock at horizon h , one gets very little additional identifying power from also imposing the response at $h + 1$, $h + 2$ and so on.

Partial identification does not doom inference, but proper care must be taken when identifying the model. In particular, the most striking implication of partial identification is that point estimates of the impulse responses must be given up and only confidence intervals can be considered.

To see how these confidence intervals are constructed, consider a scalar parameter f , which could be, for example, the impulse response of a particular variable to an oil shock at a particular horizon. Calling all the reduced form parameters of the VAR θ , f is a function of θ and α ; $f(\theta, \alpha)$.

The vector α as described above is the contemporaneous effect of an oil supply shock on each variable in the restricted VAR. Economic reasoning and other considerations should allow us to make some restrictions on the sign and magnitude of the elements of α , and so to restrict the parameter space for α to be in some set A^+ . These restrictions are detailed below.

The key step in forming a confidence interval for f is to form a confidence interval for α from the restrictions that α must lie in A^+ and that $R\alpha = r$, taking into account the uncertainty in r and R , and without relying on assumptions about the rank of R . The construction of this confidence interval follows the work of Stock and Wright (2000) and is discussed in detail in Appendix 1. I construct a confidence interval for α with about 70% coverage this way (ie, slightly less than two standard deviations), and call this set A .

Next consider forming a confidence interval for f conditional on the point estimate of the reduced form parameters, $\widehat{\theta}$. Under full identification, this would be associated with a unique estimate of f . Under partial identification, there is a range of $f(\widehat{\theta}, \alpha)$, consistent with the α vectors that are included in A . Thus the confidence interval is

$$\left[\inf_{\alpha \in A} f(\widehat{\theta}, \alpha), \sup_{\alpha \in A} f(\widehat{\theta}, \alpha) \right]$$

This confidence interval needs to be extended to a situation where uncertainty in α and θ is taken into account. For any fixed α , the model is identified, and a conventional bootstrap (described in Appendix 1) can be used to construct a 85% confidence interval for $f(\theta, \alpha)$. Let this confidence interval be $[cl(\alpha), cu(\alpha)]$. Next, form the outer envelope of all of these intervals across all α s in A , as $[\inf_{\alpha \in A} cl(\alpha), \sup_{\alpha \in A} cu(\alpha)]$. This confidence interval has asymptotic coverage of at least 70%, from the Bonferroni inequality, because asymptotically, (i) the true α is included in A with probability 85% and (ii) the bootstrap confidence interval has 85% coverage for any fixed α . The technique is conservative in that coverage may be asymptotically higher than 70 percent.²

2.4 Measuring oil price shocks using futures

This section develops the claim, taken as given above, that the impulse response of the oil price to oil supply shocks can be measured directly from the crude oil futures market.

An oil futures contract for date $t + h$ is a bet on the oil spot price s on date $t + h$ (where h is the number of months forward from date t). Parties to the h -period contract agree at time t on a price f_{t+h} for oil to be delivered at $t + h$. Standard no-arbitrage condition implies that

$$0 = E_t[m_{t+h}(s_{t+h} - f_{t+h})] \quad (2.10)$$

where m is the stochastic pricing kernel. This can be rewritten as

$$f_{t+h} = E_t s_{t+h} + \frac{cov(s_{t+h}, m_{t+h})}{E_t(m_{t+h})} \quad (2.11)$$

which states that the futures price is equal to the expected future spot price plus a risk term. The focus here will be on the change in oil futures prices $\Delta_{dt} f_{t+h}$ on the day dt of events classified as oil supply shocks. Hence, as long as the risk term in equation (2.11) does not change on the day of the event, we can write

$$\Delta_{dt} f_{t+h} = f_{t+h}^{dt} - f_{t+h}^{dt-1} = E_{dt} s_{t+h} - E_{dt-1} s_{t+h} \equiv \Delta_{dt}^e s_{t+h} \quad (2.12)$$

²For example, even when the true α is not in A , the confidence interval may contain the true f .

where $\Delta_{dt}^e s_{t+h}$ is the change in the expectations about the spot price at $t+h$ due to the unanticipated event that has hit the market at date d_t .

In the restricted VAR, the expected oil price at $t+h$, conditional on information in the dataset at t is

$$E_t s_{t+h} = \sum_{i=0}^{\infty} B_{h+i, oil} S \epsilon_{t-i} \quad (2.13)$$

The change in expectations on day t for the price of oil at $t+h$ is due to changes in shocks on day t , $\Delta_{dt}^e \epsilon_t$, given that all the past ϵ s ($\epsilon_{t-1}, \epsilon_{t-2}, \dots$) are known at the beginning of day t . In order to single out the changes in expectations due to the oil shock $\epsilon_{oil,t}$, and assuming again that the risk premium does not change, (2.12) can be written

$$\Delta_{dt} f_{t+h} = B_{h, oil} \alpha \Delta_{dt}^e \epsilon_{oil,t} + B_{h, oil} S^* \Delta_{dt}^e \epsilon_t \quad (2.14)$$

where matrix S^* is equal to S with the first column replaced by zeros and α is the first column of S . The second term can be assumed to be zero: news do not lead markets to reassess views of the other shocks as the shocks are orthogonal. Then

$$\Delta_{dt} f_{t+h} = B_{h, oil} \alpha \Delta_{dt}^e \epsilon_{oil,t} \quad (2.15)$$

Combining equations (2.7) and (2.15)

$$\Delta_{dt} f_{t+h} = r_h \Delta_{dt}^e \epsilon_{oil,t} \quad (2.16)$$

where $r_h = B_{h, oil} \alpha$ is the impulse response of the oil price to the oil shock at horizon h . Since this equation holds for every h , including 0, when $\Delta_{dt} f_t / r_0 = \Delta_{dt} s_t / r_0$, the unobserved error term ϵ can be substituted out. This yields

$$\Delta_{dt} f_{t+h} = \frac{r_h}{r_0} \Delta_{dt} s_t \quad (2.17)$$

This equation measures the proportionality of change in the futures price compared to the spot price on day t when the shock occurs. The factor of proportionality is the same for each shock, while, of course, the magnitude of different shocks can be different. This factor of proportionality is estimated from the futures contract data for each h and then used in (2.7) to obtain the estimated \hat{r}_h for the identification strategy.

The above steps allow for recovering the point estimate of α in equation (2.9) and, thus, the identification of the model.

3 Extracting the shocks from financial markets data

The key to the identification strategy described above is identifying individual oil supply shocks – or events – in the financial markets. This is carried out based on information gathered from The Monthly Energy Chronology, compiled by The United States Energy Information Administration.³

The identification of the shocks requires that daily surprise price changes in the crude oil spot price and the crude oil futures price at different horizons are collected from the days when the oil supply shocks took place.⁴ This allows for a regression that produces the factors of proportionality for equation (2.17).

The definition of an oil supply shock builds on Anzuini et al (2007). These shocks include new information that becomes available to the market, and this information has direct consequences for the future amount of crude oil produced. These kind of news include, for example, OPEC production decisions, outbreaks of war and terrorist attacks. There are also shocks that have more direct effects on the amount of crude oil made available in the global oil markets, rather than the actual production of oil. These include, for example, announcements regarding the United States strategic petroleum reserves (SPR) and adverse weather conditions which may hinder the transfer of crude oil to oil refineries. Nevertheless, the price responses of these shocks can be expected to be similar to actual production shocks, so they are also included in the analysis.

Overall, 140 shocks⁵ were identified between 1996 and 2006. The time period considered was determined by data availability issues related to the Energy Chronology as well as the fact that the model sample in the example cases (described below) is close to this time period. The shocks were cross-checked with Bloomberg news service to have had a significant effect on oil price movements on the day they took place. This cross-checking is a unique feature and should help ensure the shocks were actually significant factors in market movements of the day.

The percentage distribution of different kinds of shocks is set out in Table 3.1. Of course, the list is not unambiguous and different news events may be listed by other sources. Nevertheless, I maintain it is a good representative sample of oil supply shocks to have taken place during the time period considered.

³This chronology lists daily events that have affected the price of crude oil and is available on the website www.eia.doe.gov.

⁴The price data used is that of the West Texas Intermediate (WTI) crude oil, which is the North American benchmark crude oil type.

⁵List of the shocks is available from the author on request.

Table 3.1 **Types of identified oil supply shocks**

Type of shock	Percentage of all shocks
OPEC meetings and statements by OPEC officials	44
Political and UN statements related to Iran nuclear programme and Iraq food-for-oil programme	18
Involuntary supply changes due to accidents and terrorist activity	9
Military operations and unrest in the Middle East region	8
Voluntary short-term supply changes by non-OPEC nations	6
Industrial action	4
Adverse weather conditions	4
Yukos-affair in Russia	4
US SPR changes	3
Major new oil discoveries	1

By far the biggest group of shocks is OPEC production decisions and other OPEC statements. This is not surprising; these decisions take place at regular intervals and can in some sense be compared with central bank policy decisions in monetary policy models. Other oil supply shocks that feature regularly are related to political and military conflicts in the Middle East region.

Even though my list of oil supply shocks differs slightly from that of Anzuini et al (2007) (mainly due to the cross-checking and the different time horizon used), conclusions about the shocks are largely very similar. The shocks have had a large effect on the spot price of oil; on average, the surprise change in the spot oil price has been about 3%, and the largest changes have been over 10% in absolute value (Figure 3.1).

The scatter plot of logged oil spot price changes due to the shocks vis-a-vis oil futures price changes at different horizons shows that the linearity assumption implicit in equation (2.14) is satisfied; the futures prices change in linear proportion to the size of the spot price change (Figure 3.2).

To derive r (see equation (2.8)), the event-day changes in futures contracts prices for horizons 1–4 months are regressed on the oil spot price change on that day. The impulse responses are taken as the OLS point estimates of these regressions. They are listed in Table 3.2, along with their standard errors in parentheses. According to the results, the effect of the shocks gradually diminishes to about 70% of the impact effect. The standard errors prove that all effects are strongly significant within the 4-month period.

Figure 3.1 Ordered daily surprise changes in logged spot price

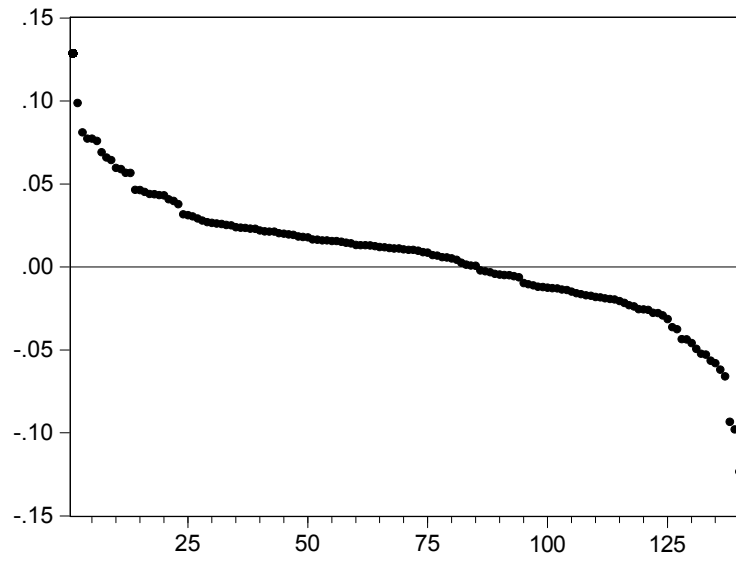


Figure 3.2 Oil spot price surprise changes and futures price changes (in logarithms)

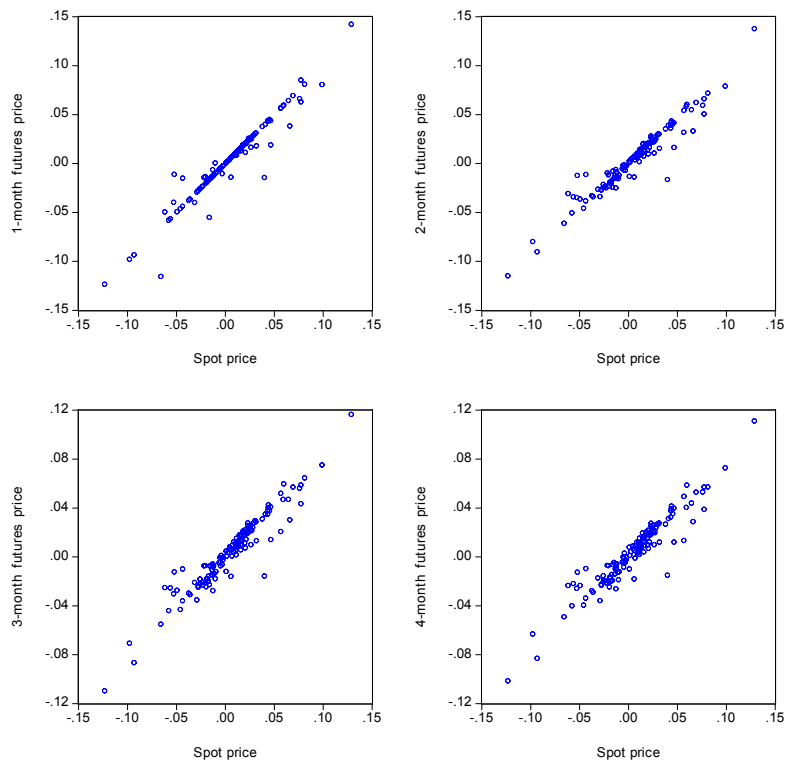


Table 3.2 also lists different specifications of the events. In particular, one may assume that the OPEC decisions aren't entirely exogenous, and the fact that markets have preordained expectations on their outcome could contaminate the price data on the actual event-day. However, the second column that excludes the OPEC decisions shows that the results are very similar to those obtained with the full event set. The same can be said for an event set that only includes price increases. Anzuini et al (2007) also lists other specifications⁶ of the event set, but the conclusion stays the same. Thus, the results are very robust to different event sets, and there seems to be no reason to abandon using the full original event set in the analysis.

This approach to the identification of the system requires that futures markets provide an efficient forecast of the change in the time path of the oil price, or at least, that risk premia in oil futures do not change. Following Anzuini et al (2007), I test the assumption that at horizons 1–4 month-ahead oil futures provide efficient forecasts for subsequent oil spot price changes by regressing the log of average oil price (the variable included in the restricted VAR) on the log of the forecast for month t implied by oil futures at month $t-1, \dots, t-4$. The test (with 95% confidence interval) that the slope coefficient is equal to 1 is supported in every case (Table 3.3) and all estimates of the intercepts are not different from zero, but a joint test fails to reject the assumption of the intercepts being equal to zero and the slopes being equal to 1. This result is similar to that in Anzuini et al (2007). Yet, as long as the non-zero intercept is related to a constant risk premium, the identification scheme is valid. It would only be undermined by a varying risk premium, and this possibility is limited due to the short time period of the shock (one day). Therefore, a constant risk premium is assumed and thus the identification scheme is deemed to be valid for the analysis.

Table 3.2 Impulse responses of oil price to oil supply shocks. OLS estimates, standard errors in parantheses. The regression is the percentage change in the futures price contract at $t+h$ on the surprise change in the spot price.

h	1996-2006	Positive shocks	Non-OPEC shocks
1	0.95 (0.02)	0.93 (0.02)	0.95 (0.03)
2	0.84 (0.02)	0.85 (0.03)	0.83 (0.03)
3	0.76 (0.02)	0.77 (0.03)	0.74 (0.03)
4	0.71 (0.02)	0.72 (0.03)	0.69 (0.02)
obs	140	85	93

⁶Anzuini et al (2007) also consider the question of the oil price data being contaminated by releases of important macroeconomic data on the event day and find that the results stay similar to the full event set. For the purposes of my analysis, I take these results as given.

Table 3.3 **Forecast efficiency tests for oil price futures.**
OLS estimates, standard errors in parantheses.
The regression is the log spot price at date t+h on the
log futures price contract at date t expiring h
months later.

h	constant (α)	slope (β)	p-value ($\beta = 1$)	p-value ($\alpha = 0$)	p-value ($\beta = 1$ and $\alpha = 0$)
1	0.05 (0.05)	0.99 (0.02)	0.42	0.35	0.40
2	0.11 (0.07)	0.97 (0.02)	0.18	0.11	0.06
3	0.15 (0.08)	0.96 (0.03)	0.13	0.07	0.01
4	0.17 (0.09)	0.96 (0.03)	0.16	0.07	0.01

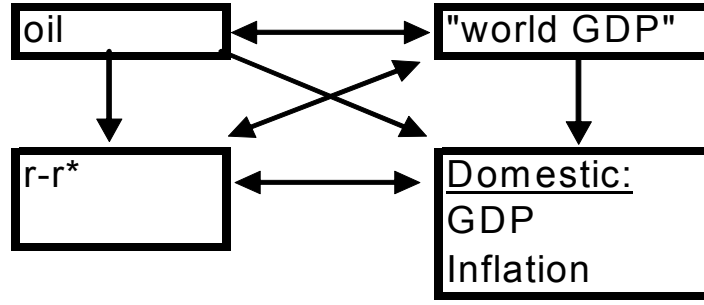
4 Restricted VAR setup for a small open economy

The methodology described above closely follows that used by Anzuini et al (2007) for the US economy. The difference is that as I am modelling a small open economy, where some of the domestic variables cannot realistically be expected to affect the international price of crude oil, a full-VAR specification is no longer valid. Furthermore, the proposed specification of variables used in the VAR differs from that of Anzuini et al (2007) as well as earlier studies.

The basic idea is to divide the variables in the model into two categories: international and domestic. Variables defined as international can affect all the other variables contemporaneously in the model, whereas domestic variables are not allowed to affect international variables. A simple 5-variable setup is illustrated in Figure 4.1. The 5 variables in the model are the price of oil (oil), a measure of global economic activity ('world GDP'), short-term interest rate differential between the domestic currency and the currency in which the oil price is quoted (US dollar) ($r-r^*$), a measure of domestic economic activity (GDP) and domestic inflation (Inflation).

The channels of contemporaneous effects are marked with directional arrows in Figure 4.1. The price of oil is allowed to affect all the other variables, as is the global economic activity. This is intuitive, since these two variables can be expected to affect domestic variables of small countries through, for example, trade channels. On the other hand, domestic variables of a small economy cannot be expected to have a significant effect on global GDP or the price of oil, since by definition their weight in the global economy is small.

Figure 4.1 A 5-variable restricted VAR setup



The key variable linking the international and domestic variables in the model is the interest rate differential. It has contemporaneous links to both international and domestic variables since it includes an important international variable (interest rate of the biggest economy in the world) and a domestic variable (domestic interest rate). In the benchmark case, interest rate differential is not allowed to have a contemporaneous effect on oil price. This is slightly contentious, but it is supported, for example, by Frankel (2006), who finds that interest rates do not have a statistically significant effect on oil prices in the US between 1950 and 2005. Furthermore, it would probably be counter-intuitive to suggest that by changing the monetary policy stance, the Federal Reserve could have an instant impact on the global price of oil. The decision taken in my model specification is also congruent with the literature dealing with monetary policy shocks in VAR models, which typically assume that other variables react to shocks with a lag.

As a consequence of these restrictions on the contemporaneous channels, estimating the model with OLS as a full VAR model is no longer asymptotically efficient. Instead, estimation is carried out as restricted VAR, using Feasible Generalised Least Squares (FGLS). The resulting A matrix in equation (2.1) for each lag p ($p > 1$) is

$$\begin{bmatrix} a_{p,11} & a_{p,12} & 0 & 0 & 0 \\ a_{p,21} & a_{p,22} & a_{p,23} & 0 & 0 \\ a_{p,31} & a_{p,32} & a_{p,33} & a_{p,34} & a_{p,35} \\ a_{p,41} & a_{p,42} & a_{p,43} & a_{p,44} & a_{p,45} \\ a_{p,51} & a_{p,52} & a_{p,53} & a_{p,54} & a_{p,55} \end{bmatrix} \begin{bmatrix} oil_{t-p} \\ wgdpt_{t-p} \\ rd_{t-p} \\ hicpt_{t-p} \\ gdpm_{t-p} \end{bmatrix} \quad (4.1)$$

and as a consequence the R matrix (see equation (2.8)) is of the following form

$$\begin{bmatrix} r_0 \\ r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix} = \begin{bmatrix} b_{0,11} & 0 & 0 & 0 & 0 \\ b_{1,11} & b_{1,12} & 0 & 0 & 0 \\ b_{2,11} & b_{2,12} & b_{2,13} & 0 & 0 \\ b_{3,11} & b_{3,12} & b_{3,13} & b_{3,14} & b_{3,15} \\ b_{4,11} & b_{4,12} & b_{4,13} & b_{4,14} & b_{4,15} \end{bmatrix} \begin{bmatrix} s_{11,oil} \\ s_{21,wgdp} \\ s_{31,rd} \\ s_{41,hicp} \\ s_{51,gdpm} \end{bmatrix} \quad (4.2)$$

where the 5 variables are ordered, from the first row to the last, as oil price (*oil*), global economic activity (*wgdp*), interest rate differential (*rd*), domestic inflation (*hicp*) and domestic economic activity (*gdpm*), the r vector is on the left hand-side and the α vector is the second term on the right hand side. The different rows of R reveal the ‘coefficient’ effects of the original oil shock through the B matrices (see equation (2.7)) back to the oil price at different horizons (measured here in months). At period zero, when the oil supply shock takes place, the only effect naturally occurs through the coefficient on the oil price variable itself ($b_{0,11}$). At period 1, there is also an effect from the *wgdp* variable, and in period 2, from the *rd* variable. The domestic variables’ effect occurs at periods 3 and 4.

5 Applications: Finland and Sweden

The methodology described in the previous section is applied in illustrative examples to Finland and Sweden, both of which can be described as small open economies; the value of GDP of both countries is less than 1% of global GDP, and foreign trade, calculated as the sum of the value of imports and exports of goods and services, was over 80% of GDP in Finland and over 90% of GDP in Sweden in 2006.

5.1 Data and estimation

The dataset in both cases includes short-term (3 month) interest rate differential between the home country and the United States, the price of crude oil in US dollars (WTI quality) and a composite measure of industrial production in the OECD countries, published by the OECD. This industrial production measure represents global economic activity. Obviously such a measure has various shortcomings; it only measures a fraction of GDP as services are not included, and it does not cover non-OECD countries. However, no other readily available measure of monthly global economic activity exists, so I include this one with the obvious caveats.

As far as domestic variables are concerned, for both countries inflation is measured by the Harmonised Index of Consumer Prices, and GDP is measured by a monthly GDP indicator. The latter is especially useful, since it covers, at least in principle, services as well as industrial activity, and is thus superior to production indicators, like, for example, monthly industrial production traditionally used in this type of literature.

Due to data availability issues both samples are relatively short; for Finland, the sample covers the years 1994–2006 and for Sweden, 1993–2006. With monthly data, this means that the sample size is over 150 for both cases. The size of the sample does not allow for examining shorter sub-samples, which would have been interesting especially in the case of Finland, since the monetary policy regime of the country changed with the inception of the European Monetary Union at the start of 1999. On the other hand, the sample

period is well suited to investigating the effects of recent oil supply shocks, which presumably might differ from those experienced in the 1970's and 1980's.

All variables are in logs, except the interest rate differential, which is in percentage points. As is the convention in much of the literature in the field, raw data in levels without seasonal or other adjustments is used. Seasonal effects are captured by dummy variables. Enough cointegration between the variables to render the models valid is implicitly assumed, but error correction models are not considered (see Sims, 1990). As mentioned above, the method of estimation is feasible generalised least squares.

The models in both cases are affected by a strong autocorrelation of the residuals, which is also confirmed by the Portmanteau and LM tests for autocorrelation. This autocorrelation cannot be corrected by including lags of 12 months or more, which is a usual remedy for the problem. In any case, including a large amount of lags in the model isn't viable due to the short sample period. Based on Akaike Information Criterion and the need to preserve as parsimonious a model as possible, the lag length of 4 was chosen for both models.

5.2 Results

5.2.1 Results with partial identification

As indicated in the methodology description above, the rank of the R matrix needs to be tested to ascertain whether full identification is possible. For both cases, it is clear that R is reduced rank (for details see Appendix 2). Thus, point estimates of impulse responses need to be given up and inference needs to be based on confidence intervals constructed as described above and in Appendix 1.

Impulse responses constructed from the S-sets for Finland and Sweden are presented in Figures 5.1 and 5.2, respectively. These responses have been restricted on the sign and size of their impact (ie, the impulse responses at period zero) so that the sign is the same as that of the FGLS point estimates, and the impact response confidence band includes the impact response vector (ie α).⁷ This keeps the confidence intervals of the impulse responses bounded whilst keeping the restrictions much less strict than those traditionally applied in a recursive decomposition. As suggested by Faust et al (2004), this makes the methodology more plausible than traditional recursive methods.

⁷In practice, the impact responses of all variables are restricted between zero and (in absolute value) the largest integer that allows the point estimate to be included in the range. This is a fairly weak assumption and conforms to that made, for example, in Anzuini et al (2007).

Figure 5.1 Impulse responses for Finland using partial identification and 70% confidence intervals

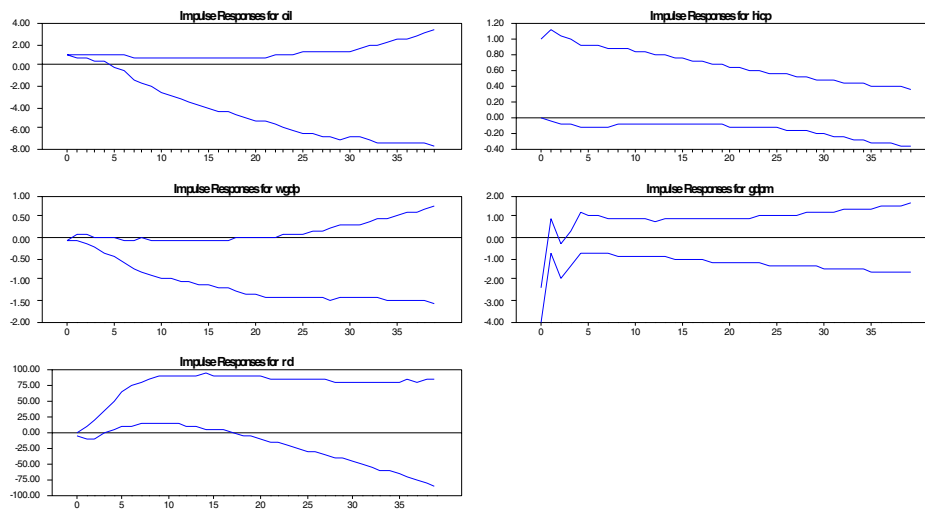
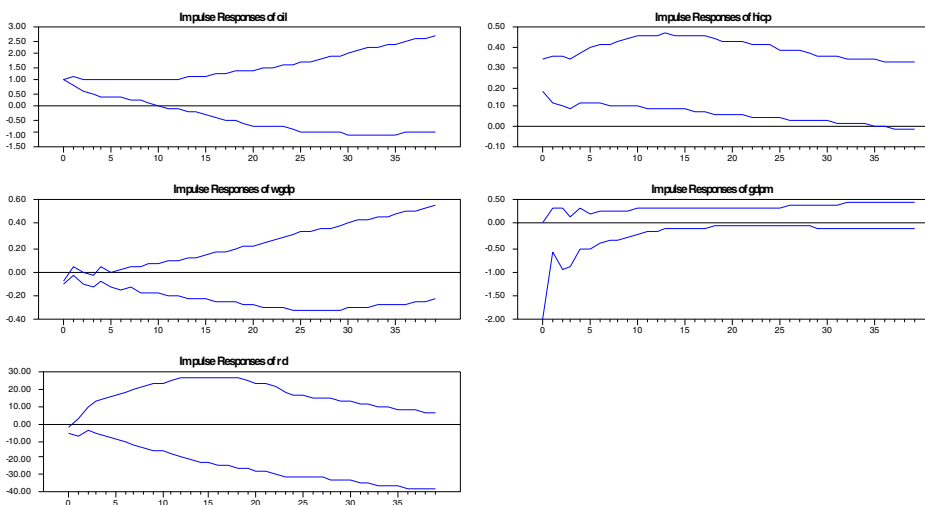


Figure 5.2 Impulse responses for Sweden using partial identification and 70% confidence intervals



Confidence intervals are wide as expected, especially in the case of Finland. However, they do allow for some inference. The results are largely similar for these two countries. Both simulations indicate that the reaction of global real activity to an oil supply shock is initially slightly negative, but not permanently or significantly so. The reaction of the interest rate differential is slightly positive in Finland, implying that monetary policy has been quicker to react to oil price changes caused by oil supply shocks in Finland than in the US.

However, this result suffer from the very large range of the confidence intervals. For Sweden, no statistically significant conclusions about the interest rate variable can be made.

As regards the domestic variables, the reaction of headline inflation is slightly stronger for Sweden than for Finland. In fact, for the latter, the reaction isn't even statistically significant for the benchmark specification. This is most probably due to certain country-specific factors (changes in indirect taxes and increased competition in services) that have had a strong effect on inflation in Finland in recent years. This may have rendered the Finnish headline inflation data difficult to interpret for oil supply shock purposes.

The reaction of the domestic GDP indicator is negative on impact for Finland, but quickly returns to zero. In fact, the impact response is puzzlingly large (between 2–4% in absolute value to a 1% oil shock) in this specification. For Sweden, the profile of the impulse response also hints at a more negative response on impact than at longer horizons, although the response is never statistically significantly different from zero. These results are in contrast with, for example, Anzuini et al (2007), who find that oil supply shocks have a stagflationary effect on the US economy during the past 40 years. However, there are various reasons why the effects of oil supply shocks might have been weaker during the past decade or so, which would help to explain my results. This issue is explored further below.

5.2.2 Complementary results with a Monte Carlo experiment

To complement the partial identification analysis, traditional point estimates were also computed. This was carried out through Monte Carlo integration (for technical details see Appendix 3), and uncertainty in both the R matrix and its rank as well as the r vector was taken into account. Impact responses obtained from 10,000 accepted draws for Finland and Sweden are presented in Appendix 4, figures A4.1, A4.2, A4.3 and A4.4, with 68% confidence intervals. The first two graphs present the benchmark case, where the signs of the impact responses were restricted as described above for partial identification.⁸ For comparison, the next two graphs present a second case where the signs of the impact responses were not restricted, but the absolute size was restricted to be the same as in the benchmark case.

The results of the benchmark case for Monte Carlo integration are qualitatively close to those obtained with partial identification, although understandably the confidence intervals are much narrower. In the second case, the results are much more ambiguous, which illustrates the importance of the restrictions chosen. Certain restrictions, however, are intuitive. In particular, the domestic inflation variable includes the price of petrol (very closely correlated with the price of crude oil) by construction and can thus be expected to have a positive response on impact to a positive oil shock.

⁸The restrictions on the impact response of the domestic GDP are slightly different in the Monte Carlo experiment, as the lower limit is restricted to be -1 (ie, a 1% oil shock causes at most a 1% GDP response).

5.2.3 Response of domestic GDP

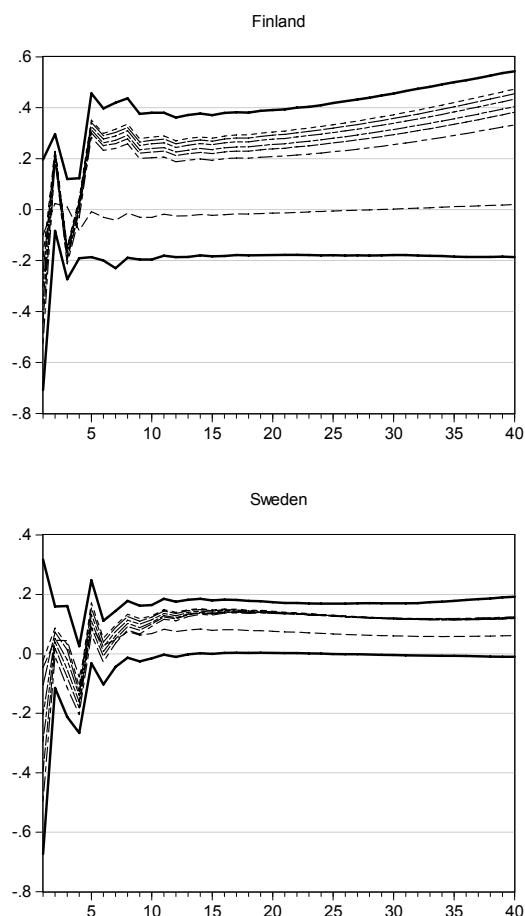
Perhaps the most interesting question pertains to the robustness of the domestic GDP impulse responses under different sign restrictions. In theory, a positive oil price shock can be expected to have a negative effect on the real economy of an oil-importing country. This is due to various different channels. The first, supply-side channel, is caused by the fact that oil is a factor of production, and the price rise will have a negative effect on firms' output and employment. The second, demand-side channel is due to the deterioration in the terms of trade and income redistribution to the oil-exporting economies, and this will tend to suppress demand in the oil-importing economy. There are also 'confidence' effects, which can affect investment and consumption decisions, as well as wealth effects due to stock market responses, but these effects are obviously very difficult to measure or predict.

To gauge the effects of different specifications of the restrictions on the impact responses, using Monte Carlo integration, the impulse responses for the domestic GDP were calculated by allowing the impact response of this variable to vary. In particular, while the impact responses of the other variables were forced to have the same sign as their FGLS point estimates, the impact response of the domestic GDP was allowed to move from the space $\{-1,0\}$ to $\{-1,1\}$ at 0.2-unit steps. This kind of robustness check is, of course, arbitrary in its definitions, but the idea here is merely to catch a representative range of GDP impulse responses under different restrictions. This shows how sensitive the results can be to these restrictions, which is in line with the findings of Uhlig (2005) for monetary policy shocks. The results are presented in Figure 5.3 and – together with those detailed in the previous section – they clearly indicate how ambiguous the effect on the domestic GDP is for both cases.

These results on the ambiguity of the GDP response to an oil supply shock are in line with those of Blanchard and Gali (2007), who, using a structural VAR model, find no statistically significant effect since 1984 in a number of European countries (France, Italy and Germany). Also, Cunado et al (2003) find that an asymmetric oil price measure affects real variables in most European countries, but in Finland, the causation is relatively weak.⁹

⁹One factor that may counteract the negative effects of higher oil prices in Finland is the importance of Russia as an export destination for Finnish goods; they accounted for about 10% of the value Finland's total exports in the year 2006. Being one of the most important oil-exporting economies in the world, Russia has benefited from the recent rise in the oil price and this presumably may have had spillover effects. For Sweden, however, Russia is not an important export destination (about 2% of total exports in 2006). Another, less important, factor might be the existence of oil refining industry in both countries. This effect is likely to be very small, since the proportion of the sector of total exports (less than 4% in both countries) and of value added (about 0.5% in Finland and about 0.2% in Sweden) is small. Furthermore, it is obvious that the parameter of interest in oil refining is the refining margin between the price of the end product and the price of crude oil, and not just the price of crude oil.

Figure 5.3 **The impulse responses of domestic GDP to a positive oil supply shock using Monte Carlo integration and alternative impact response restrictions**



Several reasons have been suggested in the literature for the lack of a clear negative effect of oil supply disruptions. Blanchard and Gali (2007) find that, in particular, this phenomenon is due to three factors. First, the increased real wage flexibility in most industrialised economies in recent years will have mitigated the real effects of oil supply shocks. Second, increased credibility of monetary policy has helped make the effects of shocks smaller on both inflation and output and third, the importance of oil for more service-based economies has declined in recent years.

Another factor blurring the relationship between oil supply shocks and real economic variables in oil-importing economies is the difficulty of disentangling demand and supply shocks in the oil price. This issue has been studied by, for example, Barsky and Kilian (2004) and Kilian (2007). Some of the conclusions of these papers are that oil demand shocks tend to have a larger effect on real variables than supply shocks, and that oil supply shocks caused by OPEC production decisions aren't necessarily exogenous. In particular, the latter proposition suggests that strong global GDP growth tends to strengthen the

OPEC cartel, whereas weak global GDP growth and the consequent weak demand for oil tends to weaken the cartel as it invites cartel members to cheat and flood the market with output to protect their revenues. This implies that trying to elicit the responses to pure exogenous oil supply shocks could be difficult, even with the detailed event-based methods used in this study.

Of course, the way the experiment of Figure 5.3 is set out implies that the effect in Finland and Sweden is ambiguous even though the impulse response of the world GDP, measured by the OECD industrial production, is in fact negative at certain horizons. This could at least in part be due to the problems of measuring the global real economic activity by using industrial production as its proxy, but still, it makes the response of the domestic GDP variables all the more puzzling. Nevertheless, these results give support to the ambiguous effects of oil shocks on real economic activity found in several studies during recent years. Clearly, however, understanding the effects of oil supply shocks on real economic activity is an open issue requiring further research.

6 Conclusions

This paper presents a unique way of identifying the effects of oil supply shocks in a small open economy. More specifically, I define oil supply shocks as events and measure their effects on crude oil futures' prices in financial markets at different maturities. The relative size of these shocks is then forced to equal the relative size of an orthogonal oil shock in a simple 5-variable restricted VAR system for a small open economy at different horizons. This allows for constructing an identification scheme that reveals the impulse responses of the variables in the model. Thus, this paper presents one way of moving away from the conventional recursive Choleski type decomposition approach in identifying the effects of a certain type of shock on certain macroeconomic variables. As ever in this kind of literature, the objective is not to find the perfect identification scheme, but rather to provide one tool that can be used for this type of analysis.

The main contribution of this paper is to propose how to apply this methodology to a small open economy. This takes place in a restricted VAR system, since not all the variables in the model are allowed to have a contemporaneous effect on all the other variables. Specifically, domestic variables (in this case, inflation and GDP) of a small open economy do not have an effect on variables like global crude oil price or global GDP. This is intuitive, but the efficient estimation method (FGLS) is different from a normal VAR system (OLS).

The identification scheme relies on parameter estimates, so mere point estimates cannot be used for analysis. This is confirmed by a rank test for the identification matrix, which finds this matrix to be short rank. This leads to a partial identification bootstrapping procedure being used for inference, and I have to resort to confidence intervals point estimates of the impulse responses cannot be computed. For comparison, I also report results using more traditional Monte Carlo integration techniques.

The proposed methodology is applied to two illustrative example cases, Finland and Sweden. Even though the confidence intervals in the bootstrapping procedure are large, some conclusions can be drawn. These conclusions are also qualitatively backed up by the Monte Carlo integration procedure. Results imply that the effect of an oil supply shock that raises the price of crude oil is positive on inflation, even though not statistically so in the benchmark case for Finland. However, the effect for domestic GDP is ambiguous for both countries, although the impact response for Finland is puzzlingly large. This supports the view that finding a long-lasting negative effect of recent oil price hikes on GDP in industrialised oil-importing economies in recent years is very difficult.

References

- Anzuini, A – Pagano, P – Pisani, M (2007) **Oil supply news in a VAR: Information from financial markets**. Banca D'Italia Temi di discussione del Servizio Studi, Number 632.
- Barsky, R – Kilian, L (2004) **Oil and the macroeconomy since the 1970s**. National Bureau of Economic Research Working Paper No. 10855, October.
- Blanchard, O J – Gali, J (2007) **The macroeconomic effects of oil shocks: Why are the 2000s so different from the 1970s?** National Bureau of Economic Research, Working Paper 13368.
- Burbridge, J – Harrison, A (1984) **Testing for the effects of oil-price rises using vector autoregressions**. International Economic Review 25, No. 2, 459–484.
- Cragg, J G – Donald, S G (1997) **Inferring the rank of a matrix**. Journal of Econometrics 76, 223–250.
- Cunado, J – de Gracia, F P (2003) **Do oil price shocks matter? Evidence for some European countries**. Energy Economics 25, 137–154.
- Enders, W (1995) **Applied Econometric Time Series**. John Wiley and Sons, Inc.
- Faust, J – Swanson, E T – Wright, J H (2004) **Identifying VARs based on high frequency futures data**. Journal of Monetary Economics 51, 1107–1131.
- Greene, W H (2003) **Econometric Analysis**. Pearson Education, Inc.
- Hamilton, J D (1983) **Oil and the macroeconomy since world war II**. The Journal of Political Economy 91, No. 2, 228–248.
- Hooker, M A (1996) **What happened to the oil price-macroeconomy relationship?** Journal of Monetary Economics 38, 195–213.
- Jimenez-Rodriguez, R – Sanchez, M (2005) **Oil price shocks and real GDP growth: Empirical evidence for some OECD countries**. Applied Economics 37, 201–228.
- Kilian, L (2006a) **Exogenous oil supply shocks: How big are they and how much do they matter for the U.S. economy?** Mimeo, University of Michigan, October 9.
- Kilian, L (2006b) **Not all oil price shocks are alike: Disentangling demand and supply shocks in the crude oil market**. Mimeo, University of Michigan and CEPR, November 9.
- Kilian, L (2007) **Not all oil price shocks are alike: Disentangling demand and supply shocks in the crude oil market**. Mimeo, University of Michigan, February 23.

- Kleibergen, F – Paap, R (2006) **Generalized reduced rank tests using the singular value decomposition.** *Journal of Econometrics* 133, 97–126.
- Lutkepohl, H (2005) **New Introduction to Multiple Time Series Analysis.** Springer-Verlag.
- Robin, J-M – Smith, R J (2000) **Tests of rank.** *Econometric Theory* 16, 151–175.
- Runkle, D E (1987) **Vector autoregressions and reality.** *Journal of Business and Economic Statistics* 5, 437–442.
- Sims, C – Stock, J H – Watson, M W (1990) **Inference for multivariate time series models with trend.** *Econometrica* 58, 113–144.
- Stock, J – Wright, J (2000) **GMM with weak identification.** *Econometrica* 68, 1055–1096.

Appendix 1

Partial identification¹⁰

Here I describe how to construct the confidence interval A for the vector α when A^+ denotes the parameter space for α , the restrictions $R\alpha = r$ must be satisfied, R is estimated by \widehat{R} , r is estimated by \widehat{r} , R may be rank deficient, $T^{1/2}(\text{vec}(\widehat{R}) - \text{vec}(R)) \rightarrow_d N(0, V_R)$ and $T^{1/2}(\widehat{r} - r) \rightarrow_d N(0, V_r)$. Consider the GMM objective function

$$S(\alpha) = T(\widehat{R}\alpha - \widehat{r})'[(\alpha' \otimes I_K)\widehat{V}_R(\alpha \otimes I_K) + \widehat{V}_r]^{-1}(R\alpha - r) \quad (\text{A1.1})$$

In standard GMM terminology, this is the continuous updating GMM objective function. The estimator $\widehat{\alpha}$ that minimises this objective function is not consistent for the true α because of the rank deficiency of the matrix R . However, $S(\alpha_0)$ has a χ^2 null distribution regardless of the rank of R where α_0 denotes the true value of the vector α . Accordingly, the confidence interval

$$A = \{\alpha \in A^+ : S(\alpha) \leq F_{\chi^2}\}$$

is a confidence interval for α with asymptotic coverage 85%, regardless of the rank of R , where F_{χ^2} denotes 85th percentile of a χ^2 distribution (with degrees of freedom equal to the number of elements in r). This confidence interval is therefore immune to the rank deficiency of R .

The use of such confidence intervals in models that are not fully identified was proposed by Stock and Wright (2000), where they are referred to as S-sets. If the matrix R is rank deficient, then there exists a subspace of vectors α that are observationally equivalent to α_0 . Any vector in this subspace must be included in A with probability 85%, asymptotically. Any other vector α will be excluded from A with probability 1, asymptotically.

Concretely, I proceed by forming a grid of about 6.7 million points in A^+ . For each point in this grid, I calculate the objective function in (A1.1). If this is above the critical value, I ignore the point and move on to the next point in the grid. On the other hand, if $S(\alpha)$ is below the critical value, I include that value of α in the confidence interval A . For each such accepted α , I compute the lower and upper bounds of the bootstrap confidence intervals for all the parameters of interest (which are the impulse responses of the variables at different horizons), conditional on that α . Each bootstrap replication includes calculating a new θ from the bootstrap sample while holding α fixed. I then construct the confidence intervals from 500 replications using the Runkle (1987) bootstrap method. Having cycled through all the points in the grid, my confidence intervals for the impulse responses are given by the smallest and largest values of these percentiles.¹¹

¹⁰This section draws heavily on Faust et al (2004).

¹¹I have used RATS to carry out the procedure, and the code is available upon request.

Appendix 2

Testing the rank of the R matrix

Several tests have been suggested in the literature to test for the rank of a stochastic matrix (for example, Cragg and Donald, 1997, and Kleibergen and Paap, 2006). However, few of these are suited to testing a matrix whose covariance matrix isn't full rank, which is the case with the R matrix. A test that is robust to this specification is introduced by Robin and Smith (2000) and used in this paper.¹²

Robin and Smith (2000) introduce a test for the null hypothesis that the rank of matrix R is r^* ; $H_0 : rk(R) = r^*$, versus the alternative hypothesis $H_1 : rk(R) > r^*$. The Wald form of the relevant test statistic is

$$CRT_{r^*}^W = T \sum_{i=r^*+1}^q L_i \quad (\text{A2.1})$$

where T is the number of observations, r^* is the rank that is tested, q is the number of columns in the matrix (in this case 5), and L_i are the ordered estimators of the characteristic roots derived from $\widehat{\Sigma}\widehat{R}\widehat{\Psi}\widehat{R}'$, $\widehat{L}_1 \geq \dots \geq \widehat{L}_q$, which solve the equation

$$\left| \widehat{R}\widehat{\Psi}\widehat{R} - L\widehat{\Sigma}^{-1} \right| = 0 \quad (\text{A2.2})$$

In this case, the test is performed without loss of generality so that both Σ and Ψ are I_q matrices. The test is performed sequentially starting at rank zero, and if the null hypothesis is rejected, the rank to be tested is increased by one until the null is accepted.

The limiting distribution of CRT , when $r^* < q$, is described by

$$\sum_{i=1}^{t^*} L_i^{r^*} Z_i^2 \quad (\text{A2.3})$$

where $t^* \leq \min\{s, (q - r^*)(q - r^*)\}$, where s is the rank of the covariance matrix of R , Ω , and $L_1^{r^*} \geq \dots \geq L_{t^*}^{r^*}$ are the nonzero ordered characteristic roots of the matrix

$$(D_{q-r^*} \otimes C_{p-r^*})' \Omega (D_{q-r^*} \otimes C_{p-r^*}) \quad (\text{A2.4})$$

$\{Z_i\}_{i=1}^{t^*}$ in equation (A2.3) are random independent standard normal variates and the D_{q-r^*} and C_{p-r^*} matrices are the last $q - r^*$ and $p - r^*$ columns, respectively, of matrices D and C that collect as columns the characteristic vectors associated with τ^2 of

¹²I thank Pentti Saikkonen for suggesting this test statistic.

$$|R\Psi R' - \tau^2 \Sigma^{-1}| = 0 \tag{A2.5}$$

for C and

$$|R'\Sigma R - \tau^2 \Psi^{-1}| = 0 \tag{A2.6}$$

for D .

Results of the test for both Finland and Sweden in the benchmark are reported in the table below. Rank deficiency is very clear cut for both cases, as the rank implied for R is 2. Thus full identification of the model is not possible and partial identification must be carried out.

Table A2.1 **Robin-Smith rank test**

rank	Finland		Sweden	
	test value	p-value	test value	p-value
0	302.2	0.00	270.7	0.00
1	64.6	0.00	60.6	0.00
2	3.4	0.95	3.4	0.95
3	1.4	0.85	2.1	0.56
4	0.2	0.69	0.7	0.40

Appendix 3

Monte Carlo Integration using pure-sign restriction approach

The strategy used in the Monte Carlo integration is based on that of Uhlig (2005) with a pure-sign restriction approach.

Let b and S be the OLS estimates of full VAR coefficient vector β and covariance matrix of residuals Σ , respectively. It can then be shown that Σ is Normal-inverse Wishart with

$$\Sigma^{-1} \sim \text{Wishart} [(TS)^{-1}, T - P] \quad (\text{A3.1})$$

where T is the number of observations and P is the number of explanatory variables, and that

$$\beta \sim N [b, \Sigma \otimes (X'X)^{-1}] \quad (\text{A3.2})$$

where X is the matrix of dependent variables.

In each draw, the covariance matrix from the full VAR model is drawn from the inverse Wishart distribution, and this is then used in the restricted VAR model to calculate the difference between the FGLS point estimates and the draw. To achieve this, it helps to simplify the covariance matrix in (A3.2) to

$$(F_{\Sigma} \otimes F_{XX})(F_{\Sigma} \otimes F_{XX})' = F_{XX}VF_{\Sigma}' \quad (\text{A3.3})$$

where $F_{\Sigma}F_{\Sigma}' = \Sigma$ and $F_{XX}F_{XX}' = (X'X)^{-1}$ and V is a PxK matrix of Normal draws. For the decomposition of F_{XX} and F_{Σ} , Choleski decomposition is used.¹³ Equation (A3.3) thus produces a random PxK matrix, which is then added to the matrix of the point estimates to complete the coefficients for the draw.

The identification scheme is then carried out as detailed above, for each draw. Certain restrictions are then set on the impact responses, and draws that fulfil these restrictions are stored, whilst others are discarded. The impulse responses and their confidence intervals are then calculated based on the accepted draws.

¹³The results are robust to any kind of decomposition.

Appendix 4

Results with Monte Carlo integration

Figure A4.1 Impulse responses for Finland using Monte Carlo integration and sign restrictions

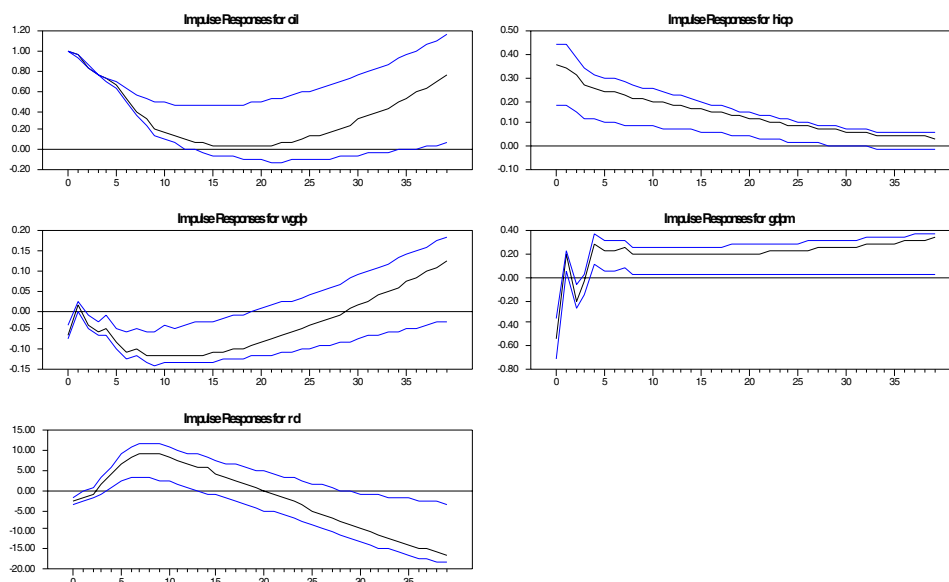


Figure A4.2 Impulse responses for Sweden using Monte Carlo integration and sign restrictions

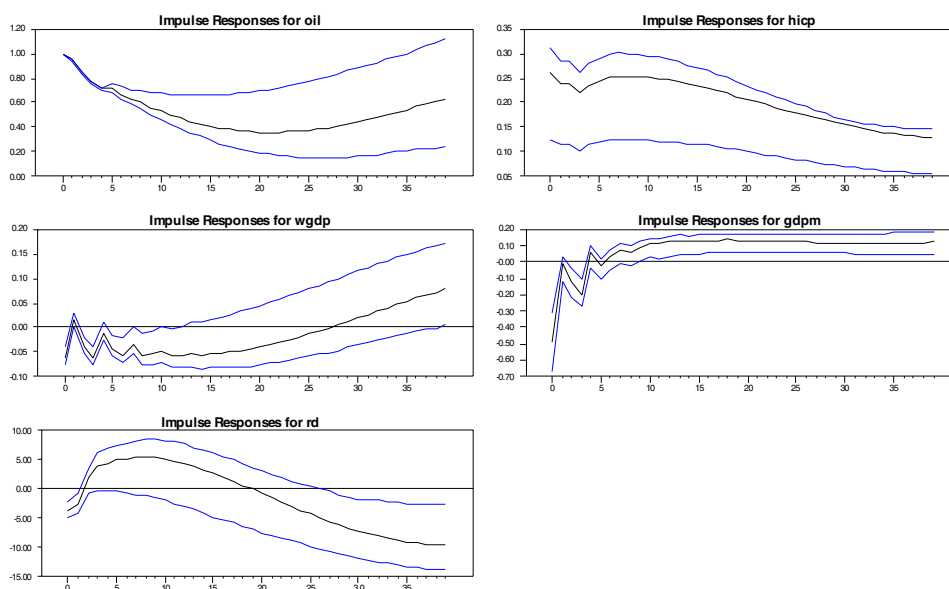


Figure A4.3 Impulse responses for Finland using Monte Carlo integration

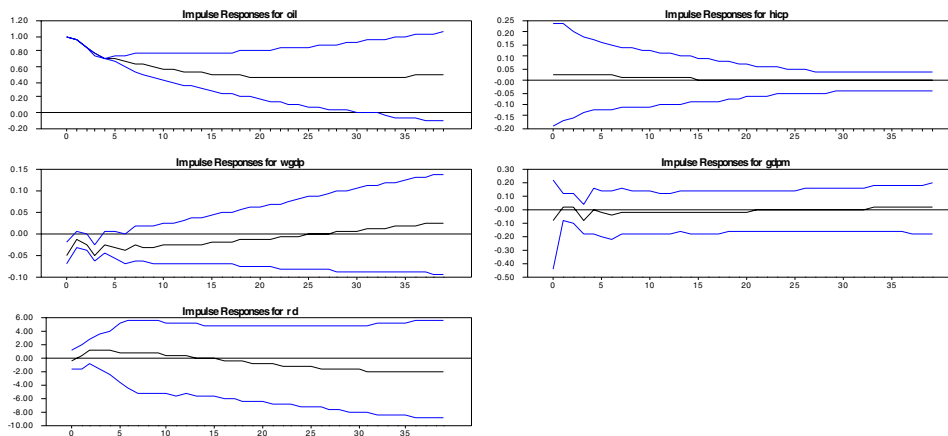
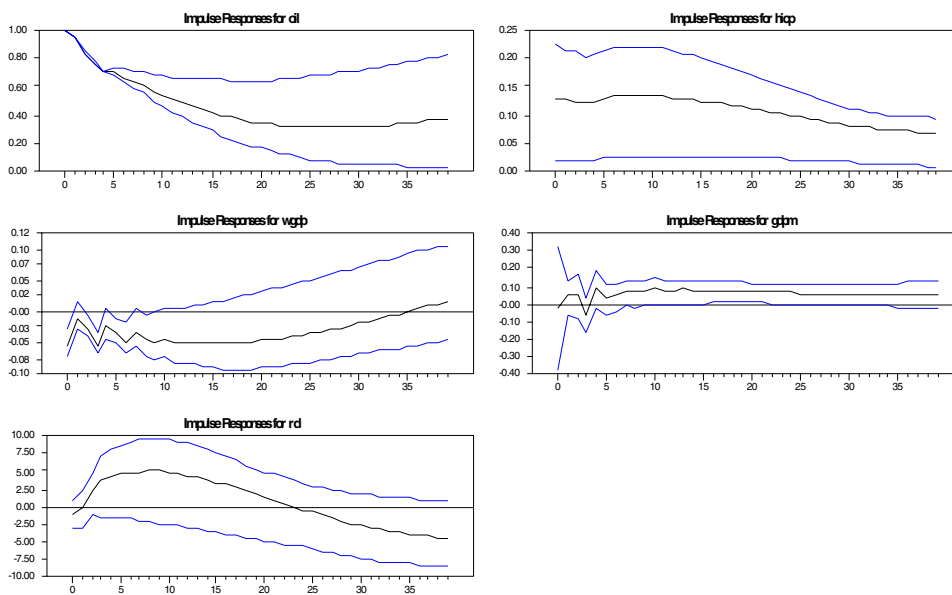


Figure A4.4 Impulse responses for Sweden using Monte Carlo integration



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