



Mikael Juselius

# Cointegration implications of linear rational expectation models



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**Suomen Pankki  
Bank of Finland  
PO Box 160  
FI-00101 HELSINKI  
Finland  
☎ +358 10 8311**

**<http://www.bof.fi>**



Mikael Juselius\*

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The views expressed in this paper are those of the author and do not necessarily reflect the views of the Bank of Finland.

\* University of Helsinki, Department of Economics, PO Box 17, FI-00014 Helsinki. E-mail: [mikael.juselius@helsinki.fi](mailto:mikael.juselius@helsinki.fi).

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# Cointegration implications of linear rational expectation models

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Mikael Juselius  
Monetary Policy and Research Department

## Abstract

This paper derives the cointegration spaces that are implied by linear rational expectations models when data are  $I(1)$ . The cointegration implications are easy to calculate and can be readily applied to test if the models are consistent with the long-run properties of the data. However, the restrictions on cointegration only form a subset of all the cross-equation restrictions that the models place on data. The approach is particularly useful in separating potentially data-consistent models from the remaining models within a large model family. Moreover, the approach provides useful information on the empirical shock structure of the data.

Keywords: rational expectations, cointegration

JEL classification numbers: C52

# Lineaaristen rationaalisten odotusten mallien implikoimat yhteisintegroituvuusrelaatiot

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Mikael Juselius  
Rahapolitiikka- ja tutkimusosasto

## Tiivistelmä

Tässä tutkimuksessa kehitetään menetelmä, jonka avulla lineaaristen rationaalisten odotusten (LRE) mallien implikoimat yhteisintegroituvuusrelaatiot voidaan helposti johtaa. Yhteisintegroituvuusrelaatioiden avulla voidaan testata, ovatko LRE-mallien ja aineiston pitkän ajan ominaisuudet yhdenmukaisia keskenään. Toisaalta yhteisintegroituvuusavaruuden rajoitteet muodostavat vain osan kaikista rajoitteista, jotka LRE-malli implikoi. Siten yhteisintegroituvuusrelaatiot antavat välttämättömän, mutta eivät riittävää ehtoa mallin voimassaololle. Menetelmä on erityisen käyttökelpoinen silloin, kun halutaan erottaa mahdolliset aineiston kanssa yhdenmukaiset mallit muista malliperheen jäsenistä. Lisäksi menetelmän avulla voidaan vertailla LRE-mallin ja empiirisen mallin pitkän ajan sokkirakenteita keskenään.

Avainsanat: rationaaliset odotukset, yhteisintegroituvuus

JEL-luokittelu: C52

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# 1 Introduction

Rational expectations (RE) models are often difficult to estimate empirically. There are several reasons for this. For instance, rational expectations models imply cross equation restrictions that demand complicated and computationally burdensome econometric techniques, such as the powerful methods derived by Hansen and Sargent (1980, 1981, 1991) and Johansen and Swensen (1999, 2004). Moreover, rational expectations models often provide sharp predictions about long-run steady state behavior but may be less detailed with respect to short-run dynamics.<sup>1</sup> However, the short-run dynamics are also heavily constrained by the cross equation restrictions of the theoretical models, and can thereby cause rejections of potentially useful models.

Estimates of rational expectations models also rely on various assumptions regarding the degree of integration of the variables. It is often the case that these assumptions do not conform with the corresponding statistical properties of the data, essentially invalidating the approach at the outset. For example, most empirical specifications assume stationary variables even though statistical unit-roots are commonly found in actual data.<sup>2</sup>

The preceding discussion suggests that there is need for a formal evaluation method of rational expectations models that; (i) are relatively easy to compute and apply, (ii) take into account the statistical properties of the data, for example statistical unit-roots, and (iii) do not place heavy restrictions on the short-run dynamics of the data.

This paper suggest an approach that solves all of the conditions (i)–(iii) for a very broad class of linear rational expectations models. In particular, linear rational expectations models have implications for cointegration when data is non-stationary. The restrictions on cointegration form a subset of the complete set of restrictions on data that are implied by the theory model. Hence, the cointegration implications serve as a necessary, but not sufficient, condition for the model to hold. The cointegration implications ensure consistency with the long-run properties of the data, but places *no* restrictions on the short-run dynamics. Moreover, the cointegration implications turn out to be very easy to derive and apply.

The cointegration implications *cannot*, by themselves, distinguish between forward looking rational expectation models and backward looking models. Such distinctions can only be made by considering the complete set of cross equation restrictions.

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<sup>1</sup>This is well known and has led to practitioners to augment the models by, for example by adding lags, in order to get more realistic dynamics. Examples can be found in Fuhrer (2000) and Gali and Gertler (1999).

<sup>2</sup>This paper takes a pragmatic view with respect to unit-roots and treats them as statistical properties of data that need to be taken into account. The unit-roots are typically dependent on sample length's or different choices of variable measures and deterministic terms. However, due to potential spurious correlation and non-standard asymptotic inference, the problem cannot be ignored. The problems of ignoring statistical unit-roots are discussed by Johansen (2006).

These ideas are illustrated in an empirical application to various versions of the new Keynesian policy model (NKM) on US quarterly data.<sup>3</sup> This application demonstrates the usefulness of the present approach. In particular, the analysis suggests that a version of the NKM, with a particular shock structure and marginal costs approximated by the output gap, is more consistent with the long-run properties of the data than the other versions of the NKM.

The paper is organized as follows. The next section introduces the class of exact linear rational expectations model that are considered. The cointegration implications of this class, both for its exact and inexact versions, are derived and discussed. Section 3 discusses alternative specifications of the NKM, introduces the data, the statistical model, and investigates the cointegration implications of the model. Section 4 concludes.

## 2 The linear rational expectations model

This section introduces the linear rational expectations model and derives its cointegration implications for alternative interpretations of the disturbance terms. In particular, both exact and inexact versions of the model are considered.

The general form of the linear rational expectations models that are considered here is

$$E_t A_f(L^{-1}) A_b(L) y_t + B(L) x_t + \Phi D_t = 0 \quad (2.1)$$

$$C(L) x_t = \varepsilon_t \quad (2.2)$$

where  $z = (y_t', x_t')$  is a  $p$  dimensional column vector,  $y_t$  is a  $r$  dimensional column vector of endogenous variables,  $x_t$  is a  $p-r$  dimensional column vector of exogenous variables,  $E_t$  is a shorthand for  $E_t[\cdot | \Omega_t]$  and  $\Omega_t$  is the agents information set at time  $t$ ,  $A_f(z)$  and  $A_b(z)$  are  $r \times r$  matrix polynomials of the  $m_1$ :th and  $m_2$ :th degree with zeros inside and outside the unit circle respectively,  $B(z)$  is a  $r \times (p-r)$  matrix polynomial of the  $m_3$ :th degree with zeros outside the unit circle,  $\Phi D_t$  collects the deterministic terms, and  $\varepsilon_t$  is vector white noise. It is also assumed that both  $A_f(z)$  and  $A_b(z)$  have square summable inverses. These assumptions guarantee that  $A_f(z)$  has a one sided polynomial expansion of degree  $m_1$  in the negative powers, while  $A_b(z)$  and  $B(z)$  has corresponding expansions in the positive powers of degrees  $m_2$  and  $m_3$  respectively. Thus,  $A_f(L^{-1}) A_b(L)$  and  $B(L)$  can be represented as

$$A(L) = \sum_{i=-m_1}^{m_2} a_{-i} L^i, \quad B(L) = \sum_{i=0}^{m_3} b_{-i} L^i \quad (2.3)$$

---

<sup>3</sup>Derivations and discussions of this model can be found in Clarida et al (1999), Gali and Gertler (1999), and Woodford (2003), among others.

where  $A(L) = A_f(L^{-1})A_b(L)$  and  $A(1)$  is of full rank.<sup>4</sup> Note that the non-stationary elements of  $y_t$  is completely determined by the  $x_t$  process since  $A(1)$  is of full rank.

The  $(p - r) \times (p - r)$  matrix polynomial  $C(z)$  is assumed to satisfy the assumption that  $C(z) = 0$  implies either  $|z| > 0$  or  $z = 1$ . These assumptions allow unit-roots in the exogenous vector process  $x_t$  but excludes the possibility of seasonal unit-roots. Also, since the focus here is on cointegration properties of (2.1), it is assumed that all elements  $x_{i,t}$  of  $x_t$  are difference stationary, ie  $x_{i,t} \sim I(1)$  for  $i = 1, \dots, p - r$ , and not internally cointegrated.<sup>5</sup>

The first equation of the model can be thought of as an Euler equation that arise through optimizing behavior on behalf of the agents, given quadratic objective functions and linear constraints. Alternatively, (2.1) can be seen as a linearized version of a non-linear Euler equation, given that the latter is not 'too' concave or convex.<sup>6</sup> Two examples illustrate the model.

*Example 1 (term structure of interest rates)*

Let  $p_{1,t}$  be the one-period interest rate at time  $t$  and let  $p_{n,t}$  be the  $n$ -period interest rate at  $t$ . Following Hansen and Sargent (1991) a rational expectations hypothesis of the term structure takes the form

$$E_t \frac{1}{n} (p_{1,t} + p_{1,t+1} + \dots + p_{1,t+n-1}) = p_{n,t} \quad (2.4)$$

This model is a special case of (2.1)–(2.2) where  $y_t = p_{1,t}$ ,  $x_t = p_{n,t}$ ,  $A_f(L^{-1}) = (1 + L^{-1} + L^{-2} + \dots + L^{-n+1})/n$ ,  $A_b(L) = 1$ ,  $B(L) = -1$ ,  $\Phi D_t = 0$ . If we choose  $C(L) = 1 - L$ , then  $p_{n,t}$  is a random walk process which implies that  $p_{1,t}$  is a unit root process as well by virtue of (2.4). Despite the simple structure of (2.4), this model cannot be formulated within the framework of Johansen and Swensen (1999). The reason is that the term structure equation contain negative powers of the lag operator of higher order than one.

*Example 2 (new Keynesian model)*

Consider the following model<sup>7</sup>

$$\bar{y}_t = \phi_1 E_t \bar{y}_{t+1} + \phi_2 \bar{y}_{t-1} - \phi_3 (i_t - E_t \pi_{t+1}) \quad (2.5)$$

$$\pi_t = \phi_4 E_t \pi_{t+1} + \phi_5 \pi_{t-1} + \phi_6 \bar{x}_t \quad (2.6)$$

$$i_t = \phi_7 i_{t-1} + (1 - \phi_7) (\phi_8 (E_t \pi_{t+1} - \pi_t^*) + \phi_9 \bar{y}_t) \quad (2.7)$$

---

<sup>4</sup>The assumptions that ensure  $A(1)$  to be of full rank are not crucial to the cointegration implications derived below. However, if the condition is violated,  $y_t$  can be non-stationary independently of the  $x_t$  process. In such cases, the assumed order of integration of the  $x_t$  process below must be reconsidered the ensure consistency of the model. An example of this is provided in Section 3.1.2.

<sup>5</sup>This assumption is made out of convenience and to avoid cumbersome notation. It would be easy to include stationary  $x_{i,t}$ , or stationary linear combinations of the  $x_{i,t}$ 's, since they do not change the cointegration space implied by the model.

<sup>6</sup>Since, the  $x_t$  process is assumed to be  $I(1)$  the model must be linearized around stochastic trends as in Altug (1989) and Ireland (2004). In fact, these linearizations provide the cointegration relationships in the general case.

<sup>7</sup>Constant terms representing the sums of steady state values are ignored to ease the exposure.

where  $\bar{y}_t = y_t^r - y_t^f$  is the gap between real output  $y_t^r$  and the flexible price output level  $y_t^f$ ,  $i_t$  is the nominal interest rate,  $\pi_t$  is the inflation rate,  $\bar{x}_t$  is a measure of marginal costs, and  $\pi_t^*$  is the target level of inflation set by the central bank. The parameters  $\phi_i$  are functions of the structural parameters of the theory. The first equation is an optimizing IS curve, the second is a new Keynesian Phillips curve (NKPC), and the last equation is a Taylor type policy rule combined with interest rate smoothing behavior.

For particular choices of parameter values, marginal cost measures, shock structures, and policy targets (2.5)–(2.7) correspond to the ‘benchmark’ new Keynesian models of Clarida et al (1999) and Woodford (2003), among others. The model also (roughly) includes more recent specification, such as Ireland (2004) and Bekaert et al (2005), where unobservable shocks enter the model.

The model given by (2.5)–(2.7) is a special case of (2.1)–(2.2) if we set  $y_t = (\bar{y}_t, \pi_t, i_t)'$ ,  $x_t = (\bar{x}_t, \pi_t^*)'$ ,

$$A(L) = \begin{pmatrix} 1 - \phi_1 L^{-1} - \phi_2 L & -\phi_3 L^{-1} & \phi_3 \\ 0 & 1 - \phi_4 L^{-1} - \phi_5 L & 0 \\ -\phi_9(1 - \phi_7) & -\phi_8(1 - \phi_7)L^{-1} & -\phi_7 L \end{pmatrix} \quad (2.8)$$

and

$$B(L) = \begin{pmatrix} 0 & 0 \\ -\phi_6 & 0 \\ 0 & \phi_8(1 - \phi_7) \end{pmatrix}$$

The linear rational expectations model (2.1)–(2.2) is similar to that considered by Hansen and Sargent (1980, 1981). Given  $\Omega_t$ , (2.1) is exact in the sense that it will fit perfectly. Hansen and Sargent indicate two ways of introducing a disturbance term into the equation and thereby making it suitable for econometric evaluation.

The first option, following Hansen and Sargent (1981), is to assume that the investigator observes a information set,  $\Lambda_t$ , that consists of (at least) current and past  $y$ :s and  $x$ :s but is smaller than the agents information set  $\Omega_t$ , ie  $\Lambda_t \subset \Omega_t$ . In addition,  $\Omega_t$  includes additional information that is relevant for forecasting  $x_t$ . In this case, one get a omitted information interpretation of the disturbance term. This is an example of an exact linear rational expectations model.

The other option is to partition  $x_t = (x'_{1,t}, x'_{2,t})'$  where only  $x_{1,t}$  is observable to the investigator. In this case one gets a missing variables interpretation of the disturbance term and the rational expectations model is said to be inexact.

In the next subsections we derive the cointegration implications for both the exact and the inexact case of the linear rational expectations model.

## 2.1 Cointegration implications of the exact model

In the subsequent discussion we set  $\Phi D_t = 0$  without loss of generality.<sup>8</sup> The assumptions so far ensures that  $x_t \sim I(1)$  and  $y_t \sim I(1)$ . It is very easy to work out the cointegration implications for the exact model. If the model is true and  $\Lambda_t$  is observable, then the  $r \times p$  dimensional matrix

$$\tilde{A} = -(A(1), B(1)) \quad (2.9)$$

defines the implied cointegration space of the theoretical model. Since  $A(1)$  is of full rank, equation (2.9) defines  $r$  cointegration relations. That (2.9) defines the theoretical cointegration space can be shown by applying a similar argument as that of CS87. Evaluate the matrix polynomials of (2.1) at unity and subtract the results from both sides of the equation to get

$$E_t(A(L) - A(1))y_t + (B(L) - B(1))x_t = -A(1)y_t - B(1)x_t \quad (2.10)$$

Noticing that

$$\begin{aligned} A(L) - A(1) &= \sum_{i=-m_1}^{m_2} a_{-i}(L^i - 1) \\ &= \sum_{i=1}^{m_1} a_i(L^{-i} - 1) - \sum_{i=1}^{m_2} a_{-i}(1 - L^i) \\ &= \sum_{i=1}^{m_1} a_i^* \Delta^+ - \sum_{i=1}^{m_2} a_{-i}^* \Delta \end{aligned}$$

where  $\Delta^+ = (L^{-1} - 1)$ ,  $\Delta = (1 - L)$ ,  $a_i^* = \sum_{j=i}^{m_1} a_j$ ,  $a_{-i}^* = \sum_{j=-i}^{m_2} a_{-j}$ , and that a similar decomposition can be made for  $B(L) - B(1)$  by defining the parameters  $b_{-i}^* = \sum_{j=i}^{m_3} b_{-j}$ , (2.10) can be rewritten as

$$E_t \sum_{i=1}^{m_1} a_i^* \Delta^+ y_t - \sum_{i=1}^{m_2} a_{-i}^* \Delta y_t - \sum_{i=1}^{m_3} b_{-i}^* \Delta x_t = -A(1)y_t - B(1)x_t \quad (2.11)$$

where the left hand side is stationary since  $y_t \sim I(1)$  and  $x_t \sim I(1)$ . This implies that the right hand side is stationary as well, and hence, that (2.9) defines the cointegration space implied by the model.

It is important to understand that, together with the model in (2.1)–(2.2), (2.9) has several testable implications. In particular, the theory implies a specific structure of the cointegration space, a given cointegration rank, and a fixed number of common trends. All of these predictions can and should be used to distinguish the subset of models that are (long-run) data consistent from those that are not.

The preceding is readily illustrated by using the examples from above.

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<sup>8</sup>However, when specifying an empirical model within which the cointegration implications can be tested one needs to carefully consider the deterministic terms. For instance, It is often the case that the deterministic terms need to be restricted in order to avoid undesired behavior such as quadratic growth etc.

*Example 1 (continued)*

Evaluating the polynomial of Example 1 in the unit point yields  $A_f(1) = \frac{(1+1+\dots+1)}{n} = \frac{n}{n} = 1$ . Thus, (2.9) implies  $\tilde{A} = (-1, 1)$  as expected.

*Example 2 (continued)*

Evaluating the polynomial in (2.8) of Example 2 in the unit point and combining with the matrix  $B(L) = B = B(1)$  yields

$$\tilde{A} = - \begin{pmatrix} 1 - \phi_1 - \phi_2 & -\phi_3 & \phi_3 & 0 & 0 \\ 0 & 1 - \phi_4 - \phi_5 & 0 & -\phi_6 & 0 \\ -\phi_9(1 - \phi_7) & -\phi_8(1 - \phi_7) & -\phi_7 & 0 & \phi_8(1 - \phi_7) \end{pmatrix}$$

Hence, in this case the model implies three cointegration vectors and two common stochastic trend. It was assumed in Section 2 that all elements of  $x_t$  are  $I(1)$  and not cointegrated. In Example 2 this implies that the two stochastic trends are associated with marginal costs and the central bank inflation target. It is easy to modify the model setup to accommodate other, perhaps more standard shock structures as well. We return to this issue in Section 3.

## 2.2 Cointegration implications of the inexact model

The preceding section assumed that observations on the vector processes  $y_t$  and  $x_t$  were available. However, suppose that only a part of the exogenous process  $x_t$ , a  $p_1 = p - r - p_2$  dimensional vector  $x_{1,t}$  say, is observable. In this case we would like to derive the cointegration implications of the theory model for the observable process  $(y'_t, x'_{1,t})'$ . To achieve this, note that the cointegration space under complete information on  $x_t$  is given by (2.9). If  $B(1) = (B_1(1), B_2(1))$  is partitioned correspondingly to  $x_t = (x'_{1,t}, x'_{2,t})'$ , the stationary linear combinations can be written as

$$(A(1)y_t + B_1(1)x_{1,t} + B_2(1)x_{2,t}) \sim I(0) \quad (2.12)$$

where  $B_1(1)$  is a  $r \times p_1$  matrix and  $B_2(1)$  is a  $r \times p_2$  matrix that describes how the unobserved stochastic trends enter into the structural cointegration space. Let  $0 < b = \text{rank}(B_2(1)) \leq \min\{r, p_2\}$ . The problem is to find the cointegration sub-space that only involves stationary linear combinations of the observable variables. This can be achieved by premultiplying the term inside the parenthesis in (2.12) by a  $(r - b) \times r$  matrix  $\Theta$  of full row rank that satisfies  $\Theta B_2(1) = 0$ . Unfortunately,  $\Theta$  is not unique and there is no way to guarantee a representation of the cointegration space that makes economic sense. Nevertheless, it allows us to derive testable cointegration implications of the model

Finding a candidate for  $\Theta$  is computationally very easy. One way is to perform elementary operations on  $B_2(1)$ , summarized by some  $r \times r$  matrix  $\Xi$ , such that

$$\Xi B_2(1) = \begin{pmatrix} 0 & 0 \\ 0 & I_b \end{pmatrix}$$

The candidate for  $\Theta$  is then obtained by collecting the first  $r - b$  rows of  $\Xi$ . To see this, partition  $\Xi$  as  $\Xi = (\Xi'_1, \Xi'_2)'$ , where  $\Xi_1$  collects the first  $r - b$  rows, and premultiply (2.12) by it to get

$$\begin{pmatrix} \Xi_1 A(1)y_t + \Xi_1 B_1(1)x_{1,t} \\ \Xi_2 A(1)y_t + \Xi_2 B_1(1)x_{1,t} + x_{2,t} \end{pmatrix} \sim I(0) \quad (2.13)$$

which demonstrates that  $\Xi_1$  is a candidate for  $\Theta$ .

Given a matrix  $\Theta$  that satisfies the properties above, the sub-cointegration space for the observable part of the process is defined by

$$\tilde{A}_\Theta = -(\Theta A(1), \Theta B_1(1)) \quad (2.14)$$

Thus, in general, there will be a reduction in the number of cointegration vectors as a result of the (non-stationary) unobservables. A model that includes more unobservable  $I(1)$  variables than there are endogenous variables will in general imply no cointegration.

A question of considerable theoretical interest is how to specify the processes that govern the unobservable variables. For example, is there a stochastic productivity trend in output as in Ireland (2004) or a time varying central bank inflation target as in Kozicki and Tinsley (2005)? The rotation of the cointegration space in (2.13) can be used to provide empirical answers to such questions. A test of cointegration rank on the observable variables combined with stationarity tests of the  $x_{1,t}$  variables will reveal the number of unobserved stochastic trends. Moreover, the  $b \times (p - p_2)$  matrix  $-(\Xi_2 A(1), \Xi_2 B_1(1))$  provides a representation of the  $b$  unobserved stochastic trends in terms of the observable variables, as can be seen from (2.13). This representation shows how the unobserved stochastic trends are propagated in the observed part of the system and can therefore be used to empirically distinguished between different structures of the unobserved shocks.

*Example 2 (continued)*

Following Ireland (2004), let  $\bar{x}_t = \bar{y}_t$ ,  $\pi_t^* = \pi^* = 0$ , and  $y_t^f = y_{t-1}^f + \varepsilon_{y,t}$ , where  $\varepsilon_{y,t}$  is white noise technology shock.<sup>9</sup> Hence,  $y_t = (y_t^r, \pi_i, i_t)'$ ,  $x_{1,t} = \{\emptyset\}$ , and  $x_{2,t} = (y_t^f)'$ . With these choices we get

$$A(1) = \begin{pmatrix} 1 - \phi_1 - \phi_2 & -\phi_3 & \phi_3 \\ -\phi_6 & 1 - \phi_4 - \phi_5 & 0 \\ -\phi_9(1 - \phi_7) & -\phi_8(1 - \phi_7) & -\phi_7 \end{pmatrix} \quad (2.15)$$

and

$$B_{21}(1) = \begin{pmatrix} \phi_1 + \phi_2 - 1 \\ \phi_6 \\ \phi_9(1 - \phi_7) \end{pmatrix}$$

---

<sup>9</sup>Ireland04 also assumes stationary preference, cost push, and policy shocks. These are ignored here since they do not affect cointegration. He also uses a slightly different policy rule than that of (2.7).

A representation of the cointegration space is obtained by

$$\tilde{A}_\Theta = -\Theta A(1) = - \begin{pmatrix} 1 & 0 & \frac{1-\phi_1-\phi_2}{\phi_9(1-\phi_7)} \\ 0 & 1 & \frac{-\phi_6}{\phi_9(1-\phi_7)} \end{pmatrix} A(1)$$

which implies two cointegration relations and one common trend given by the unobserved flexible price level of output.

### 3 Empirical application

This section illustrates the approach of Section 2 on different alternative empirical specifications of the new Keynesian model presented in Example 2. For this purpose, a sample of US quarterly data starting in 1983:1 and ending in 2006:4 is used. First, the different empirical versions of the NKM are discussed. This is followed by a discussion of the data and the statistical model. Finally the cointegration properties of the different models are discussed. The focus in this section is on illustrating the approach and, thus, some empirical aspects, for instance sensitivity analyzes with respect choices of variable measures or sample lengths, are not considered fully here.

#### 3.1 Alternative specifications of the NKM

The main advantage of the methods developed in Section 2 is that they can readily be used to distinguish between the potentially data consistent model variants when there are many candidates within a model family, such as the new Keynesian model. Here, alternative versions of the NKM are considered that are distinguished from each other with respect to different variable measures and shock structures. In particular, I consider two popular measures for marginal costs, the output gap and labor's share, and two alternatives for the stochastic trends, a technology shock and permanent changes to the central bank inflation target. The models for the different measures of marginal costs are treated separately in order to facilitate the exposition.

##### 3.1.1 The NKM with output gap as marginal costs

Consider the NKM of Example 2 in Section 2. Let marginal costs equal the flexible price output gap, ie  $\bar{x}_t = \bar{y}_t = y_t^r - y_t^f$ , and assume that the flexible price level of output is approximately equal to some measure of potential output,



$y_t^n$ , ie  $y_t^f \approx y_t^n$ .<sup>10</sup> With these changes, the model can be written as

$$y_t^r = y_t^n + \phi_1 E_t (y_{t+1}^r - y_{t+1}^n) + \phi_2 (y_{t-1}^r - y_{t-1}^n) - \phi_3 (i_t - E_t \pi_{t+1}) \quad (3.1)$$

$$\pi_t = \phi_4 E_t \pi_{t+1} + \phi_5 \pi_{t-1} + \phi_6 (y_t^r - y_t^n) \quad (3.2)$$

$$i_t = \phi_7 i_{t-1} + (1 - \phi_7) (\phi_8 (E_t \pi_{t+1} - \pi_t^*) + \phi_9 (y_t^r - y_t^n)) \quad (3.3)$$

where the variables have the same interpretation as before. There are two possible sources of stochastic trends within this specification of the system, a technology shock originating in  $y_t^f$  as in Ireland (2004), and a time varying central bank inflation target,  $\pi_t^*$ , as in Kozicki and Tinsley (2005).<sup>11</sup>

We first consider the exact rational expectations model variants that can be obtained from this system. Since the true central bank inflation target is unknown to the public, it cannot be a source for the stochastic trends in the exact model. Thus, the only possibility to introduce a stochastic trend in the exact version of (3.1)–(3.3) is through potential output. But this implies that real output shares this stochastic trend, and hence, that the output gap must be stationary. Since there are no other sources of stochastic trends, both the nominal interest rate and inflation must be stationary as well. Hence, given the data vector  $z_t = (y_t^r, i_t, \pi_t, y_t^n)'$ , the exact version of (3.1)–(3.3) implies three cointegration vectors corresponding to the stationary output gap, the stationary inflation rate, and the stationary nominal interest rate.

A more interesting variant of (3.1)–(3.3) is obtained by assuming that there are permanent changes in the unobserved central bank inflation target. In this case, it is no longer certain that the output gap is stationary since real output can contain the stochastic trend in the central bank inflation target as well as the stochastic technology trend. In fact, which variables share the stochastic trends in  $y_t^n$  and  $\pi_t^*$  are now dependent on the particular parameter values in (3.1)–(3.3).<sup>12</sup> The cointegration space of the inexact variant of (3.1)–(3.3) is easy to calculate given  $y_t = (y_t^r, \pi_t, i_t)'$ ,  $x_{1,t} = (y_t^n)$ ,  $x_{2,t} = (\pi_t^*)$

$$A(1) = \begin{pmatrix} 1 - \phi_1 - \phi_2 & -\phi_3 & \phi_3 \\ -\phi_6 & 1 - \phi_4 - \phi_5 & 0 \\ -\phi_9(1 - \phi_7) & -\phi_8(1 - \phi_7) & -\phi_7 \end{pmatrix}, B_1(1) = \begin{pmatrix} \phi_1 + \phi_2 - 1 \\ \phi_6 \\ \phi_9(1 - \phi_7) \end{pmatrix}$$

and

$$B_2(1) = \begin{pmatrix} 0 \\ 0 \\ \phi_8(1 - \phi_7) \end{pmatrix}$$

From  $B_2(1)$  it is directly seen that

$$\Theta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

---

<sup>10</sup>This latter assumption has been severely questioned in the literature, for example by Gali and Gertler (1999). Since the empirical analysis here is intended merely to illustrate the method of Section 2, this assumption is maintained throughout for convenience.

<sup>11</sup>These are by no means the only possibilities if unobserved preference and cost push shocks are included in the model as in Ireland (2004).

<sup>12</sup>For example, a parameter choice that does not violate  $\det(A(1)) \neq 0$  and yields a stationary output gap is  $\phi_4 + \phi_5 = 1$  and  $\phi_6 \neq 0$ .

satisfies the requirements of Section 2.2. Thus, the sub-cointegration space in terms of the observable variables  $z = (y_t^r, \pi_t, i_t, y_t^n)'$  have the representation

$$\tilde{A}_\Theta = \begin{pmatrix} 1 - \phi_1 - \phi_2 & -\phi_3 & \phi_3 & \phi_1 + \phi_2 - 1 \\ -\phi_6 & 1 - \phi_4 - \phi_5 & 0 & \phi_6 \end{pmatrix} \quad (3.4)$$

implying two cointegration vectors and two common trends. This cointegration space can be interpreted as describing an optimizing IS-curve and a new Keynesian Phillips curve, where the nominal interest rate is exogenously given. The output gap is stationary if  $\phi_4 + \phi_5 = 1$  and  $\phi_6 \neq 0$ . The real interest rate is stationary if  $\phi_1 + \phi_2 = 1$  and  $\phi_3 \neq 0$ .

### 3.1.2 The NKM with labor's share as marginal costs

The other popular measure of marginal costs is labor's share. Let  $s_t$  represent this measure. The NKM can then be written as

$$y_t^r = y_t^n + \phi_1 E_t (y_{t+1}^r - y_{t+1}^n) + \phi_2 (y_{t-1}^r - y_{t-1}^n) - \phi_3 (i_t - E_t \pi_{t+1}) \quad (3.5)$$

$$\pi_t = \phi_4 E_t \pi_{t+1} + \phi_5 \pi_{t-1} + \phi_6 s_t \quad (3.6)$$

$$i_t = \phi_7 i_{t-1} + (1 - \phi_7) (\phi_8 (E_t \pi_{t+1} - \pi_t^*) + \phi_9 (y_t^r - y_t^n) + \phi_{10} s_t) \quad (3.7)$$

where  $s_t$  has been added to the policy rule for consistency. To make things interesting, both  $y_t^n$  and  $s_t$  are assumed to have separate stochastic trends. This implies that we can have both exact and inexact versions of the model that are interesting.

The exact model is obtained by assuming a constant central bank inflation target,  $\pi_t^* = \pi^* = 0$ . Given  $y_t = (y_t^r, \pi_t, i_t)'$  and  $x_t = (y_t^n, s_t)'$ , the cointegration space of the exact version of (3.5)–(3.7) is given by

$$\tilde{A} = \begin{pmatrix} 1 - \phi_1 - \phi_2 & -\phi_3 & \phi_3 & \phi_1 + \phi_2 - 1 & 0 \\ 0 & 1 - \phi_4 - \phi_5 & 0 & 0 & -\phi_6 \\ -\phi_9(1 - \phi_7) & -\phi_8(1 - \phi_7) & -\phi_7 & \phi_9(1 - \phi_7) & -\phi_{10}(1 - \phi_7) \end{pmatrix} \quad (3.8)$$

implying three cointegration vectors and two common stochastic trends. It is interesting to note that, with marginal costs equal to the exogenously given labor's share, the restriction  $\phi_4 + \phi_5 = 1$  implies that  $A(1)$  is singular. This leads to a difficult problem if labor's share is  $I(1)$ , since in that case the model has the unreasonable implication that inflation is  $I(2)$ .<sup>13</sup>

As before, the inexact model is obtained by assuming that  $\pi_t^* \sim I(1)$ . The cointegration implications of this variant of (3.5)–(3.7) are very easy to derive given (3.8) and the discussion in Section 3.1.1. Since,  $y_t = (y_t^r, \pi_t, i_t)'$ ,  $x_{1,t} = (y_t^n, s_t)'$ , and  $x_{2,t} = (\pi_t^*)$ ,  $B_2(1)$  is the same as in Section 3.1.1 implying

<sup>13</sup>To see this, note that given  $\phi_4 + \phi_5 = 1$  and by subtracting  $(\phi_4 + \phi_5)\pi_t$  from both sides of (2.6) the equation can be written as  $\Delta\pi_t = \frac{\phi_4}{\phi_5} E_t \Delta\pi_{t+1} + \phi_6 \bar{x}_t$ . Juselius (2006) shows that it is difficult to reject a unit-root for labor's share measure even over moderately long samples. Thus, from an empirical point of view it would be hard to reconcile the use of this measure with the restriction  $\phi_5 + \phi_6 = 1$ .

Model	Type	$y_t$	$x_{1,t}$	$x_{2,t}$	$MC$	$r + p_1$	CI-rank
1	E	$(y_t^r, \pi_t, i_t)'$	$y_t^n$	–	$y_t^r - y_t^n$	4	3
2	I	$(y_t^r, \pi_t, i_t)'$	$y_t^n$	$i_t^*$	$y_t^r - y_t^n$	4	2
3	E	$(y_t^r, \pi_t, i_t)'$	$(y_t^n, s_t)'$	–	$s_t$	5	3
4	I	$(y_t^r, \pi_t, i_t)'$	$(y_t^n, s_t)'$	$i_t^*$	$s_t$	5	2

Table 1: Versions of the NKM implied by different assumptions on the stochastic trends and measures of marginal costs. The column labeled ‘type’ indicates if the rational expectations model is exact or inexact. The column ‘ $MC$ ’ indicates which measure is used for marginal costs, the column ‘ $r + p_1$ ’ provides the number of observables, and ‘CI-rank’ gives the implied cointegration rank.

that the same  $\Theta$  can also be used. Thus, the cointegration space in terms of the observable variables  $z = (y_t^r, \pi_t, i_t, y_t^n, s_t)'$  is given by

$$\tilde{A}_\Theta = \begin{pmatrix} 1 - \phi_1 - \phi_2 & -\phi_3 & \phi_3 & \phi_1 + \phi_2 - 1 & 0 \\ 0 & 1 - \phi_4 - \phi_5 & 0 & 0 & -\phi_6 \end{pmatrix} \quad (3.9)$$

implying two cointegration vectors and three stochastic trends. This cointegration space can be interpreted as before, with  $i_t$  exogenously given.

Table 1 summarizes and labels the four different model variants of Section 3.1.1 and Section 3.1.2.

## 3.2 Data

The discussion in the preceding section indicates that data on real output, the inflation rate, the short term interest rate, labor’s share, and potential output are needed in order to test the different model specifications in Table 1. I use (the log of) seasonally *unadjusted* nominal GDP, deflated by a chain index for the GDP deflator, obtained from the Bureau of Economic Analysis (BEA), as a measure of real output. The inflation rate is calculated from the consumer price index (all categories) which is obtained from the Bureau of Labor Statistics (BLS). The short-term interest rate is the 3-month secondary market yield on US Treasury bills obtained from the Federal Reserve Board’s historical data. The measure of Labor’s share is constructed from non-farm business data and obtained from the BLS. Finally, the measure of potential output is Hodrick and Prescott filtered real GDP (using scale parameter 1600). The quarterly sample is 1983:1–2006:4 or 96 observations of each variable.<sup>14</sup> Figure 1 shows the output gap and labor’s share. Figure 2 shows the inflation rate and the short term interest rate.

<sup>14</sup>It is possible to obtain a much longer sample than that used here. Unfortunately, there is strong evidence of a structural break in the cointegration space in the beginning of the 1980’s (see Juselius, 2006). Since the objective here is to illustrate the method proposed in Section 2, only on the most recent structurally stable sample is considered.

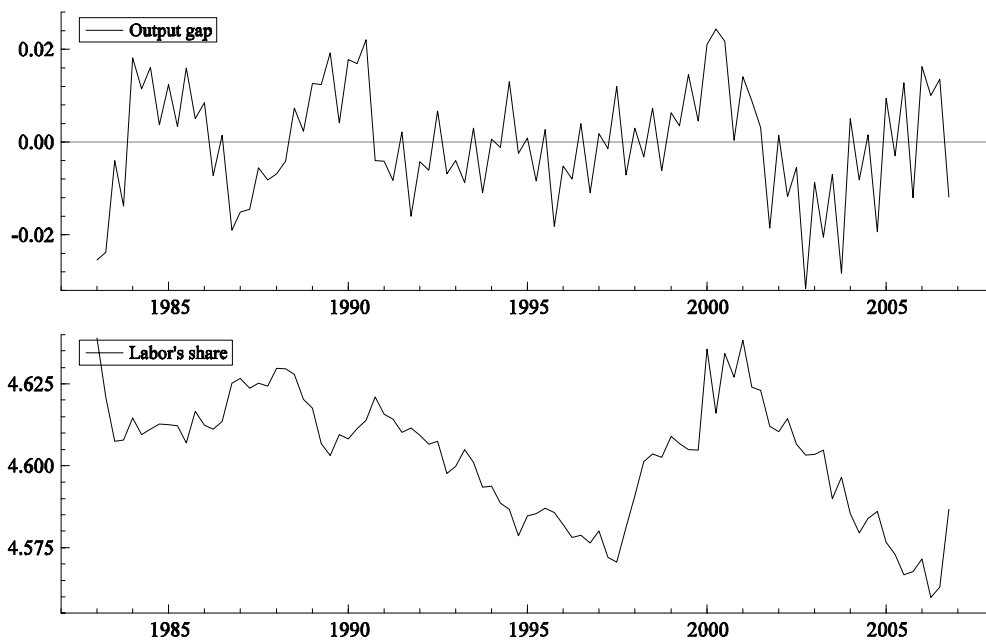


Figure 1: The output gap (seasonally adjusted in order to facilitate the exposure) and non-farm business labor's share

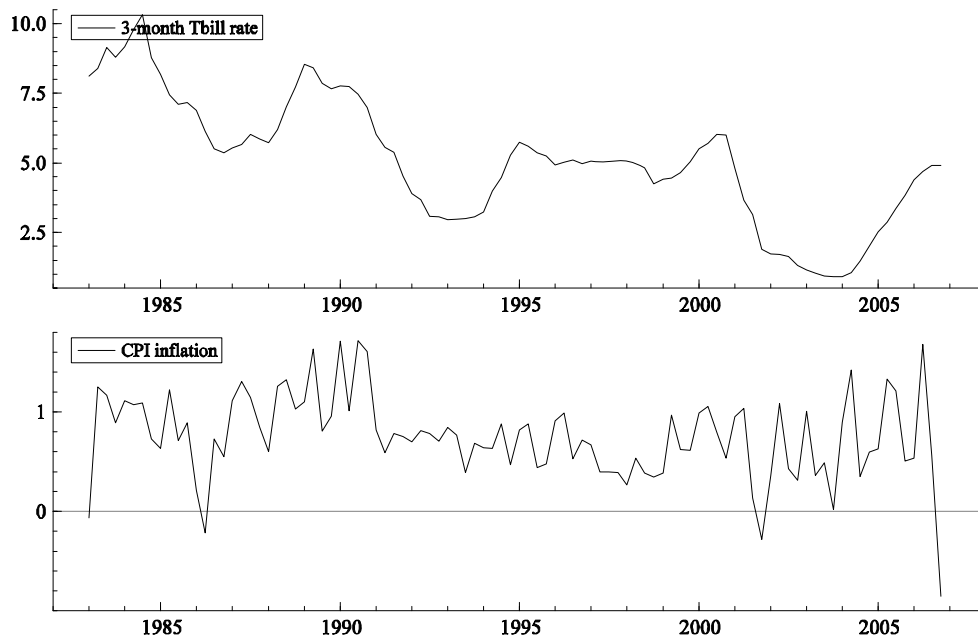


Figure 2: CPI inflation and short term interest rates

### 3.3 Statistical model

The linear rational expectations model presented in Section 2 has several testable implications for the long-run properties of the data, such as the number of common trends and the structure of the cointegration space. It seems natural to choose a model within which these implications can be tested in a consistent way.

One such alternative is provided by the the  $p$ -dimensional cointegrated VAR-model with  $k$  lags

$$\Delta X_t = \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \Pi X_{t-1} + \Psi D_t + \varepsilon_t \quad (3.10)$$

where the  $p$ -dimensional vector process  $X_t$  is assumed to be at most  $I(1)$ ,  $D_t$  is a  $p \times f$  matrix that collects the deterministic components, and  $\varepsilon_t \sim N_p(0, \Sigma)$ . The parameter matrices are  $\Gamma_i$ ,  $\Pi$ ,  $\Psi$ , and  $\Sigma$  respectively. Cointegration can be tested as a reduced rank hypothesis on the  $\Pi$  matrix. If the rank,  $r$ , of  $\Pi$  is equal to  $p$ , then  $X_t$  is stationary, ie  $X_t \sim I(0)$ . If  $0 < r < p$ , then  $X_t \sim I(1)$  is cointegrated with  $r$  cointegration vectors and  $p - r$  common trends. In this case,  $\Pi = \alpha\beta'$ , where  $\alpha$  and  $\beta$  are two  $(p \times r)$  matrices of full column rank. The cointegration space is spanned by  $\beta'$ . If  $r = 0$  then  $X_t \sim I(1)$  and the process is not cointegrated.

An important special case is obtained when  $0 < r < p$  and a deterministic linear trend is restricted to the cointegration space. The reason for restricting the linear trend is that (3.10) implies quadratic trends in  $X_t$  otherwise. If the trend is restricted,  $\Pi X_{t-1}$  in (3.10) can be written as  $\alpha\tilde{\beta}'\tilde{X}_{t-1}$ , where  $\tilde{\beta} = (\beta', \kappa)'$ ,  $\kappa$  is a  $r$ -dimensional vector, and  $\tilde{X}_{t-1} = (X'_{t-1}, t)'$ .

The test for the reduced rank of  $\Pi$ , known as the trace test, was developed by Johansen (1991). The null hypothesis of the trace test is that the rank of  $\Pi$  is less or equal to  $r$ . Hence, the natural testing sequence from a statistical point of view is to start by testing  $r = 0$  and then successively increasing the rank by one until the first non-rejection.

Given  $\Pi = \alpha\beta'$ , general linear hypotheses on  $\beta$  can be tested in the form

$$\mathcal{H}_\beta : \beta = (H_1\varphi_1, \dots, H_r\varphi_r) \quad (3.11)$$

where  $H_i(p \times (p - m_i))$  imposes  $m_i$  restrictions on  $\beta_i$ , and  $\varphi_i((p - m_i) \times 1)$  consists of  $p - m_i$  freely varying parameters. The likelihood ratio test of the hypotheses is asymptotically  $\chi^2$ . The  $\alpha$ -vectors can also be restricted in a similar way. Of special interest is the case where one or several rows in  $\alpha$  consist of zeros. A variable with a zero row in  $\alpha$  is said to be weakly exogenous. In many cases, some or all of the exogenous variables  $x_t$  in (2.1)–(2.2) will be weakly exogenous.<sup>15</sup> The properties of the model are investigated in Johansen (1995).

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<sup>15</sup>If we allow cointegration among the exogenous variables in (2.2), weak exogeneity will not necessarily hold.

Rank	$X_{12,t}$		$X_{34,t}$	
	$\lambda_i$	$p$ -value	$\lambda_i$	$p$ -value
0	0.52	0.00	0.54	0.00
1	0.22	0.04	0.25	0.02
2	0.07	<b>0.53</b>	0.12	<b>0.17</b>
3	0.01	<b>0.29</b>	0.11	<b>0.14</b>
4	–	–	0.01	<b>0.33</b>

Table 2: The reduced rank hypothesis of the different models and samples. The  $\lambda_i$  are the eigenvalues corresponding to the estimates of the cointegration vectors. Bold values indicate non-rejection at the 5% significance level.

### 3.4 Cointegration implications of the NKM

Inspection of Table 1 of Section 3.1 reveals that models 1 and 2, where marginal costs are measured by the output gap, share the same set of observable variables. Thus, only one statistical model is needed in order to tests both versions. A similar observation can be made for models 3 and 4 of the table. The data vector that nests models 1 and 2 is  $X_{12,t} = (y_t^r, \pi_t, i_t, y_t^n)'$  and the data vector that nests models 3 and 4 is obtained by adding labor's share to  $X_{12,t}$ , ie  $X_{34,t} = (y_t^r, \pi_t, i_t, y_t^n, s_t)'$ .

Initial modeling of  $X_{12,t}$  suggested that two lags were needed to account for the variation in the data. In addition, one dummy variable was included corresponding to a series of interest rate cuts in the fourth quarter of 1984. Given these choices, there were no significant misspecification in the model and the long-run parameters were structurally stable.<sup>16</sup> Virtually identical results were obtained for  $X_{34,t}$  expect that an extra dummy variable for the first quarter of 2000 was added to account for an outlier in labor's share.

With these choices of lags and dummy variables, cointegrated VAR models were fitted to  $X_{12,t}$  and  $X_{34,t}$ . The results from testing the reduced rank hypothesis of the two statistical models reported in Table 2. The tests indicate that appropriate choice of cointegration rank is two in both models, although the choice of rank equal to one was also borderline accepted. These results have the important implication that the exact model variants 1 and 3 in Table 1 *cannot* be consistent with the long-run structure of the data. However, the inexact models 2 and 3 are still viable options. If this is the case, we should expect that the nominal interest rate is weakly exogenous, in addition to the exogenous variables of the theoretical models.

The results of testing the null hypotheses of stationarity and weak exogeneity in the variables are reported in Table 3. As can be seen form the table, stationarity *is rejected* in all variables. Moreover, weak exogeneity *cannot be rejected* in the nominal interest rate, the potential output measure, and labor's share. These results are supportive of the two inexact models. In particular, the assumptions of a non-stationary technology shock and a

<sup>16</sup>The tests used to investigate long-run structural stability of the models include constancy tests of the estimated cointegration space, constancy of the log-likelihood, and constancy of the canonical correlations  $\lambda_i$ , described in Dennis (2006). The details are available upon request.

$X_t$	Test	$y_t^r$	$\pi_t$	$i_t$	$y_t^n$	$s_t$
$X_{12,t}$	stat	0.00	0.00	0.00	0.00	–
	exo	0.00	0.00	<b>0.93</b>	<b>0.38</b>	–
$X_{34,t}$	stat	0.00	0.00	0.00	0.00	0.00
	exo	0.00	0.00	<b>0.97</b>	<b>0.38</b>	<b>0.11</b>

Table 3: Tests for stationarity (stat) and weak exogeneity (exo). The numbers are p-values of the null hypothesis. Bold values indicate non-rejection at the 5% significance level. The ‘Model’ column indicate to which model version of the NKM the tests apply (see Table 1).

$\mathcal{H}$	Restrictions	$\tilde{\beta}$	LR, $\chi^2(df)$	p-value
$X_{12,t} = (y_t^r, \pi_t, i_t, y_t^n)'$				
$\mathcal{H}_{IS,1}$	–	$\tilde{\beta}_1 = (1 - \phi_1 - \phi_2, -\phi_3, \phi_3, \phi_1 + \phi_2 + 1)'$	1.49(1)	<b>0.22</b>
$\mathcal{H}_{IS,2}$	$\phi_1 + \phi_2 = 1$	$\tilde{\beta}_3 = (0, -\phi_3, \phi_3, 0)'$	17.73(2)	0.00
$\mathcal{H}_{PC,1}$	–	$\tilde{\beta}_2 = (-\phi_6, 1 - \phi_4 - \phi_5, 0, \phi_6)'$	7.38(1)	0.01
$\mathcal{H}_{PC,2}$	$\phi_4 + \phi_5 = 1$	$\tilde{\beta}_3 = (-\phi_6, 0, 0, \phi_6)'$	11.04(2)	0.00
$\mathcal{H}_{IS,1} \cap \mathcal{H}_{PC,1}$	–	See (3.4) in the text.	10.52(2)	0.01
$X_{34,t} = (y_t^r, \pi_t, i_t, y_t^n, s_t)'$				
$\mathcal{H}_{IS,1}$	–	$\tilde{\beta}_1 = (1 - \phi_1 - \phi_2, -\phi_3, \phi_3, \phi_1 + \phi_2 + 1, 0)'$	0.83(2)	<b>0.66</b>
$\mathcal{H}_{IS,2}$	$\phi_1 + \phi_2 = 1$	$\tilde{\beta}_3 = (0, -\phi_3, \phi_3, 0, 0)'$	17.35(3)	0.00
$\mathcal{H}_{PC,1}$	–	$\tilde{\beta}_2 = (0, 1 - \phi_4 - \phi_5, 0, 0, -\phi_6)'$	14.11(2)	0.00
$\mathcal{H}_{IS,1} \cap \mathcal{H}_{PC,1}$	–	See (3.9) in the text.	18.81(4)	0.00

Table 4: Structural test on the cointegration spaces implied by models 2 and 4 in Table 1.  $\mathcal{H}_{\cdot}$  indicates that the null hypothesis of the restrictions implied by either the optimizing IS curve (index *IS*) or the new Keynesian Phillips curve (index *PC*) is being tested. The second index indicates if there are restrictions on the parameters.

non-stationary central bank inflation target are consistent with the long-run variation in the data.

The cointegration spaces (3.4) and (3.9) of models 2 and 4 in Table 1 have testable restrictions since they are over identifying. These restrictions are tested in Table 4, where the individual cointegration vectors, corresponding to the optimizing IS curve and the new Keynesian Phillips curve, are first tested separately and then jointly.

Several interesting results emerge from Table 4. The tests of  $\mathcal{H}_{IS,1}$  indicate that the long run behavior of the data is roughly consistent with an optimizing IS curve when the coefficients are allowed to vary freely. However, the restriction  $\phi_1 + \phi_2 = 1$  tested in  $\mathcal{H}_{IS,2}$  is rejected implying a non-stationary real interest rate. The results of the new Keynesian Phillips curve are weaker. The Phillips curve in  $\mathcal{H}_{PC,1}$  is rejected on a 5% significance level, but not on a 1% significance level, when the output gap is used to measure marginal costs. The restriction  $\phi_4 + \phi_5 = 1$  is rejected implying that the output gap

is non-stationary.<sup>17</sup> The NKPC is rejected when labor's share is used as a measure of marginal costs.<sup>18</sup> The test of the cointegration space implied by model 2 in Table 1,  $\mathcal{H}_{IS,1} \cap \mathcal{H}_{PC,1}$ , is rejected on a 5% significance level but not on a 1% significance level. The joint test of the cointegration space implied by model 4 is rejected.

The analysis of the new Keynesian model in this section demonstrates the applicability of the approach suggested in this paper, particularly when there are several model candidates within a family of models. Taken together, the evidence in this section indicate that out of the four NKMs that were considered (see Table 1), only model 2 is (roughly) consistent with the long-run properties of the data. However, it should be remembered that the empirical analysis in this paper was primarily intended to illustrate the present approach. Thus, a more careful analysis that takes other specifications of the NKM into account, for instance with respect different variable measures or open economy considerations, is need in order to properly evaluate the new Keynesian framework. Evidence from other countries would also be helpful in this respect.

## 4 Conclusion

This paper derived the cointegration implications of both exact and inexact linear rational expectations models when data is non-stationary. These implications are easy to calculate and can readily be used to test the long-run data consistency of the model. The approach is particularly useful in distinguishing the potentially data relevant models from the remaining models within a large model family. Moreover, the approach offers useful information on the shock structure of the data which can be used to further advance the models. The approach was illustrated by an application to various versions of the new Keynesian model.

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<sup>17</sup>This result indicates that the persistence in the business cycles is sufficiently high that 25 years of quarterly data is not enough to reject a unit-root in the output gap. If a longer sample is considered, 1960:1–2006:4 say, then the output gap becomes stationary. Another interpretation of the result is that potential output is poor proxy for the flexible price level of output.

<sup>18</sup>Note, that in this case it is unnecessary to test the restriction  $\phi_4 + \phi_5 = 1$  since the labor's share was found to be non-stationary. In fact, labor's share does not become stationary even if the sample is extended to 1960:1–2006:4.



## References

- Altug, S (1989) **Time-to-Build and Aggregate Fluctuations: Some New Evidence.** *International Economic Review* 30 (4), 889–920.
- Bekaert, G – Cho, S – Moreno, A (2005) **New-Keynesian Macroeconomics and the Term Structure.** School of Economics and Business Administration, University of Navarra, Faculty Working Papers 04/05.
- Campbell, J Y – Shiller, R J (1987) **Cointegration and Tests of Present Value Models.** *The Journal of Political Economy* 95 (5), 1062–1088.
- Clarida, R – Gali, J – Gertler, M (1999) **The Science of Monetary Policy: A New Keynesian Perspective.** *Journal of Economic Literature* 37 (4), 1661–1707.
- Dennis, J (2006) **CATS in RATS, Cointegration Analysis of Time Series.** Estima, Evanston, Illinois, USA.
- Fuhrer, J C (2000) **Habit Formation in Consumption and Its Implications for Monetary-Policy Models.** *The American Economic Review* 90 (3), 367–390.
- Gali, J – Gertler, M (1999) **Inflation dynamics: A structural econometric analysis.** *Journal of Monetary Economics* 44, 195–222.
- Hansen, L P – Sargent, T J (1980) **Formulating and estimating dynamic linear rational expectations models.** *Journal of Economic Dynamics and Control* 2 (2), 7–46.
- Hansen, L P – Sargent, T J (1981) **Exact linear rational expectations models: specification and estimation.** Federal Reserve Bank of Minneapolis Staff Report number 71.
- Hansen, L P – Sargent, T J (1991) **Rational Expectations Econometrics.** Westview Press, Boulder.
- Hodrick, R J – Prescott, E C (1997) **Postwar U.S. Business Cycles: An Empirical Investigation.** *Journal of Money, Credit and Banking* 29 (1), 1–16.
- Ireland, P (2004) **Technology Shocks in the New Keynesian Model.** *The Review of Economics and Statistics* 86 (4), 923–936.
- Johansen, S (1991) **Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models.** *Econometrica* 59, 1551–1580.
- Johansen, S (1995) **Likelihood-Based Inference in Cointegrated Vector Auto-Regressive Models.** Oxford University Press.

Johansen, S (2006) **Post Walrasian Macroeconomics**. Cambridge University Press, Cambridge, Ch. Confronting the economic model with the data, 287–300.

Johansen, S – Swensen, A R (1999) **Testing exact rational expectations in cointegrated vector autoregressive models**. *Journal of Econometrics* 93, 73–91.

Johansen, S – Swensen, A R (2004) **More on testing exact rational expectations in cointegrated vector autoregressive models: Restricted constant and linear term**. *Econometrics Journal* 7, 389–397.

Juselius, M (2006) **Testing the New Keynesian Model on US and European Data**. HECER working paper.

Kozicki, S – Tinsley, P (2005) **Permanent and transitory policy shocks in an empirical macro model with asymmetric information**. *Journal of Economic Dynamics and Control* 29 (11), 1985–2015.

Woodford, M (2003) **Interest and Prices: Foundations of a Theory of Monetary Policy**. Princeton University Press.

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Suomen Pankki  
Bank of Finland  
P.O.Box 160  
**FI-00101** HELSINKI  
Finland



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