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The optimal composition of government expenditure among transfers, education and public goods*

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Abstract

This paper examines the optimal allocation of tax revenue among a universal transfer payment, a pure public good and expenditure on education. Using a single-period framework, education expenditure raises the productivity of individuals via a human capital production function. The social welfare function is based on individuals' (indirect) utilities. Education creates a substantial fiscal spillover whereby the increase in human capital gives rise to higher labour earnings and thus higher income tax revenue, thereby allowing greater government expenditure on all items than would otherwise be possible. A higher inequality of exogenous ability levels is found to increase all types of expenditure, but only the transfer increases in relative terms. Higher inequality aversion leads to an increase in transfer payments in absolute and relative terms, at the expense of the other two components. However, there is little sensitivity to inequality aversion. An increase in the elasticity of the wage with respect to basic ability leads to lower education spending.

Keywords: government expenditure, public goods, education, transfer payments, social welfare function.

JEL classification: H31, H41, H52, H53, H75

1. Introduction

The aim of this paper is to examine the optimal allocation of government revenue among three types of expenditure. These consist of a universal transfer payment, a pure public good and expenditure on basic education, conditional on the tax rate. These expenditures have differing effects on individuals' labour supplies, which are considered to be endogenous. If the choice is limited to the simpler case of expenditure on a public good and a transfer

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payment, it is known that a more unequal wage rate distribution results in relatively more expenditure being devoted to the transfer payment.[1] The question considered in this paper is how the relationship between the composition of expenditure and basic inequality is affected by the existence of tax-financed education which raises individuals' productivities.[2] Judgements regarding the optimal expenditure composition have to balance the efficiency effects of education expenditure, which influences the general level but not the inequality of wage levels, with the inequality-reducing effect of the unconditional transfer. Education creates a substantial fiscal spillover. Thus, the increase in human capital gives rise to higher labour supply and higher earnings, as a result of the general increase in wage rates. This in turn leads to higher income tax revenue, thereby allowing greater government expenditure on all items than would otherwise be possible. It is also somewhat inequality-increasing. The complex interactions involved mean that the nature of optimal plans are not at all obvious.

In a single-period framework, individuals are assumed to have similar preferences regarding leisure, consumption goods (net income) and the public good, but are endowed with an exogenously given ability level which differs among individuals. The education expenditure is the same for all individuals and combines with the fundamental ability level to raise the productivity of individuals, via a type of human capital production function.

The paper represents an exercise in welfare economics similar to investigations in the optimal tax literature. However, it differs from standard analyses not only because of the introduction of additional types of expenditure, but because emphasis is placed on the composition of expenditure for a given tax rate. The tax rate is therefore assumed to be set by other considerations, perhaps relating to perceptions of 'taxable capacity' and conventional levels. Tax and expenditure decisions are thus treated as being set by different agencies, related only through a budget constraint imposed on expenditure plans.[3] Decisions may perhaps be regarded as involving a two-stage procedure whereby first a judgement is made about the composition of expenditure for a given tax rate, and then secondly a judgement is made about the tax rate, conditional on the optimal expenditure policy being adopted.[4]

It is well-known that standard optimal tax models do not give rise to closed-form solutions, even for quite simple specifications of the tax structure and preferences. However, in the present context it is possible, using a convenient approximation, to obtain explicit solutions for the three expenditure levels, conditional on the tax rate. Nevertheless, the properties of the model are not immediately transparent and for this reason the analysis of comparative statics is reinforced by numerical examples.

There is considerable variation among countries in the composition of government expenditure.[5] The analysis may thus shed some light on the extent to which these differences could arise from cultural factors, such as the extent of inequality aversion of policy makers, or other factors such as tastes or productivity differences.

The framework of analysis is set out in Section 2, which derives individuals' indirect utility functions, expressed in terms of the expenditure levels.

The first-order conditions for maximisation of the welfare function, giving the optimal composition of expenditure, are set out in Section 3: further details of the derivation of results are given in the Appendix. The results depend on a welfare-weighted average of basic ability levels, resulting in nonlinear equations. Section 4 introduces an approximation which allows explicit, or closed-form, solutions to be obtained. Comparative static analyses are carried out in Section 5, which also provides some numerical examples. Brief conclusions are in Section 6.

2. The Framework of Analysis

This section describes the basic framework of anaysis used. The model is static, involving a single period, and individuals' labour supplies respond to tax and expenditure changes. Individual behaviour is described in subsection 2.1. The human capital production function used to generate wage rates is discussed in subsection 2.2, followed in subsection 2.3 by derivation of the government's budget constraint.

2.1. Individual Maximisation

Each individual maximises utility, U, regarded as a function of consumption, c, which, assuming the price index of private goods is normalised to 1, is equivalent to net income, leisure, h, and a tax-financed public good, Q. Assume that U takes the Cobb-Douglas form, where the individual subscript, i, is omitted for convenience:

$$U = c^{\alpha} h^{\beta} Q^{1-\alpha-\beta} \tag{1}$$

This widely-used form has, as seen below, the convenient property in the present context that it leads to linear earnings functions relating earnings to the wage rate, thereby allowing aggregation over all individuals to be carried out relatively easily.[6]

Tax revenue is obtained using a simple proportional income tax at the rate, t, and this is used to finance a basic income (a non-means-tested transfer payment) of b per person. The endowment of time is normalised to 1, so that $h \leqslant 1$, and the wage rate is w. Hence the budget constraint facing the individual is:

$$c + hw(1-t) = w(1-t) + b = M \tag{2}$$

where $M = w(1-t) + b$ is 'full income'.

Using the standard properties of the Cobb-Douglas utility function, and defining $\alpha' = \alpha/(\alpha+\beta)$ and $\beta' = \beta/(\alpha+\beta)$, net income and leisure are:[7]

$$c = \alpha' M \tag{3}$$

$$h = \beta'\left(1 + \frac{b}{w(1-t)}\right) \tag{4}$$

For the individual to work, w must exceed the threshold, $w_{\min}$, needed to ensure h < 1, where:

$$w_{\min} = \frac{b}{1-t}\left(\frac{\beta'}{1-\beta'}\right) \tag{5}$$

Gross earnings, $y = w(1-h)$, are:

$$y = w(1-\beta') - \frac{b\beta'}{1-t} \tag{6}$$

Substituting the optimal values of c and h into the direct utility function gives indirect utility, V, as:

$$V = \left(\alpha'^{\alpha}\beta'^{\beta}\right)\left(w(1-t)\right)^{\alpha}\left(\frac{M}{w(1-t)}\right)^{\alpha+\beta} Q^{1-\alpha-\beta} \tag{7}$$

2.2. Education and Wage Rates

The above results are expressed in terms of the individual's wage rate. However, the wage rate of each individual is itself endogenous because, in addition to financing the transfer payment and expenditure on the public good, tax revenue is used to provide public education. This involves per capita expenditure on education of E. This provides an input into a human capital production function which, along with each individual's exogenous basic ability level, w_0, generates productivity as reflected in the wage rate, w. The production function is also assumed to take the very simple Cobb-Douglas form, similar to that used by Glomm and Ravikumar (1997, p. 188), whereby:

$$w = \gamma w_0^{\theta} E^{1-\theta} \tag{8}$$

Hence the wage is proportional to a weighted average of the individual's basic ability level, w_0, and the per capital expenditure on education, E. The production function does not appear explicitly to allow for any time being spent on education, but the present approach is consistent with an assumption that each individual spends the same amount of time in education, as well as benefiting from the same government expenditure. Furthermore, this time can be regarded as coming out of time otherwise devoted to leisure, and gives the same utility as leisure. It therefore affects only the constant term in (8). The size of this constant term is also affected by the units of measurement and mean of logarithms, μ_{w_0}, of the basic ability level. However, it can be shown that the absolute values of γ and μ_{w_0} do not affect the optimal expenditure shares. For this reason is it most convenient simply to set $\gamma = 1$ in what follows.[8]

The specification is similar in some respects to the kind of human capital production function used in much of the endogenous growth literature, except that it necessarily does not include either private investment by parents in childrens' education or the human capital of parents. For example, the form in Blankenau *et al.* (2007, p. 393) is the same as (8) except that it has the human capital of the previous generation instead of w_0.[9] A further difference is that in much of the growth literature, which involves a general equilibrium context, the labour input into an aggregate production function is defined in terms of 'effective labour' (a proportion of time devoted to labour multiplied by human capital), so that only a single wage rate needs to be determined. In the present partial equilibrium context, there is wage rate heterogeneity and individuals supply simply a number of (unadjusted) hours of labour.

The function in (8) has the property, again in common with many studies in the context of endogenous growth modes, that it does not specify a productivity externality. That is, individuals do not receive any benefit, in the form of a higher wage rate, from the fact that other households may have more human capital. For example, complementarities in production between low-skilled and high-skilled individuals may give rise to externalities which raise the wage received by the low-skilled individuals.[10] A specification allowing for a slightly different kind of externality is used by Angelopoulos *et al.* (2007, p. 6), and papers cited therein, where individual human capital accumulation depends on, among other things, arithmetic mean human capital. Of course, there are other types of externality arising from education which are non-economic in nature, involving social and cultural values, which are often used to justify tax-financed public education, along with market failure arguments concerning capital markets.

However, the present model contains a very important spillover effect. The tax-financed public education investment gives rise to higher incomes and hence a higher tax base. This means that the relatively low-ability individuals face a lower tax rate and receive a higher transfer payment than they would in the absence of the tax-financed investment. This is reflected in a higher social evaluation function (discussed below) which allows for both equity and efficiency considerations.

2.3. The Government Budget Constraint

All government expenditure is assumed to be financed from the proportional tax. Hence the budget constraint can be written, where n is the number of individuals and p is the constant cost per unit of producing the public good, as:

$$b + E + \frac{pQ}{n} = t\bar{y} \tag{9}$$

Aggregation over (6), and letting $F_1(w_{\min})$ and $F(w_{\min})$ denote respectively the proportion of total wage (rates) and the proportion of people with $w < w_{\min}$, gives arithmetic mean earnings, $\bar{y}$, as:

$$\bar{y} = \bar{w}(1-\beta')\{1-F_1(w_{\min})\} - \frac{b\beta'}{1-t}\{1-F(w_{\min})\} \tag{10}$$

where $\bar{w}$ is the arithmetic mean wage rate. The expression in (10) is nonlinear since $w_{\min}$ depends on the tax parameters. However, on the assumption that there are relatively few non-workers, the terms F_1 ($w_{\min}$) and F ($w_{\min}$) can be neglected, giving the linear form:

$$\bar{y} = (1-\beta')\bar{w} - \beta'\left(\frac{b}{1-t}\right) \tag{11}$$

Substituting into (9) and rearranging gives the transfer payment expressed in terms of E and Q, reflecting the fact that only two of the policy variables can be chosen independently. Thus:

$$b = \frac{t\bar{w}(1-\beta') - (E + pQ/n)}{1+\beta' t/(1-t)} \tag{12}$$

The average wage rate can be expressed in terms of basic abilities and public expenditure on education. Using (8):

$$\bar{w} = E^{1-\theta}\left(\frac{1}{n}\sum_{i=1}^{n} w_{0,i}^{\theta}\right) \tag{13}$$

and writing $\bar{w}_\theta = \frac{1}{n}\sum_{i=1}^{n} w_{0,i}^{\theta}$ as the moment of order θ (about the origin):

$$\bar{w} = E^{1-\theta}\bar{w}_\theta \tag{14}$$

This can be substituted into (12) to give:

$$b = \frac{tE^{1-\theta}\bar{w}_\theta(1-\beta') - (E + pQ/n)}{1+\beta' t/(1-t)} \tag{15}$$

The government budget constraint therefore involves a loss of one degree of freedom in the choice of policy instruments.

3. The Optimal Composition of Expenditure

The social planner is modelled as selecting values of Q, E and b, for an exogenously given tax rate, in order to maximise a social welfare function, W, of the additive form:

$$W = \sum_{i=1}^{n} W(V_i) \tag{16}$$

subject to the government's budget constraint in (9), and where indirect utility can be written as $V_i = V(Q, E, b|t)$. Form the Lagrangean:

$$\mathcal{L} = \sum_{i=1}^{n} W(V_i) + \lambda\left[t\bar{y} - b - E - pQ/n\right] \tag{17}$$

The first-order conditions are thus:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial b} &= \sum_i \frac{\partial W}{\partial V_i}\frac{\partial V_i}{\partial b} + \lambda\left[t\frac{\partial \bar{y}}{\partial b} - 1\right] = 0 \\
\frac{\partial \mathcal{L}}{\partial Q} &= \sum_i \frac{\partial W}{\partial V_i}\frac{\partial V_i}{\partial Q} + \lambda\left[t\frac{\partial \bar{y}}{\partial Q} - \frac{p}{n}\right] = 0 \\
\frac{\partial \mathcal{L}}{\partial E} &= \sum_i \frac{\partial W}{\partial V_i}\frac{\partial V_i}{\partial E} + \lambda\left[t\frac{\partial \bar{y}}{\partial E} - 1\right] = 0 \\
\frac{\partial \mathcal{L}}{\partial \lambda} &= t\bar{y} - b - E - pQ/n = 0
\end{aligned} \tag{18}$$

Write $\frac{\partial W}{\partial V_i}\frac{\partial V_i}{\partial b} = \upsilon_i$, the marginal 'social' value attached to an increase in i's income. Furthermore:

$$\frac{\partial W}{\partial V_i}\frac{\partial V_i}{\partial Q} = \upsilon_i \frac{\partial V_i/\partial Q}{\partial V_i/\partial b} \tag{19}$$

$$\frac{\partial W}{\partial V_i}\frac{\partial V_i}{\partial E} = \upsilon_i \frac{\partial V_i/\partial E}{\partial V_i/\partial b} \tag{20}$$

From (10) the partial derivates in (18) are $\partial\bar{y}/\partial b = \beta'/(1-t)$ and $\partial\bar{y}/\partial Q = 0$ and $\partial\bar{y}/\partial E = (1-\beta')(1-\theta)\,\bar{w}_\theta E^{-\theta}$. Therefore, the first three first-order conditions above can be rewritten:

$$\sum_i \upsilon_i = \lambda\left(1 - \beta'\frac{t}{1-t}\right) \tag{21}$$

$$\sum_i \upsilon_i \frac{\partial V_i/\partial Q}{\partial V_i/\partial b} = \lambda\frac{p}{n} \tag{22}$$

$$\sum_i \upsilon_i \frac{\partial V_i/\partial E}{\partial V_i/\partial b} = \lambda\left\{1 - t(1-\beta')(1-\theta)\bar{w}_\theta E^{-\theta}\right\} \tag{23}$$

A procedure for solving these equations, along with the budget constraint, is presented in the Appendix. This section therefore simply states the main results. First it is necessary to define several terms. Letting $\upsilon_i' = \upsilon_i / \sum_i \upsilon_i$, the term $\tilde{w}_\theta$ is defined as:

$$\tilde{w}_\theta = \sum_i \upsilon_i' w_{0,i}^\theta \tag{24}$$

This is a welfare-weighted average of the $w_{0,i}^\theta$. Furthermore, define $\hat{w}_\theta$ as another weighted average:

$$\hat{w}_\theta = \tilde{w}_\theta \left(1 - t\left(1 - \beta'\right)\right) + t\left(1 - \beta'\right)\bar{w}_\theta \tag{25}$$

This allows pQ to be written in terms of E as:[11]

$$\frac{pQ}{n} = \Psi\left(E^{1-\theta}\hat{w}_\theta - E\right) \tag{26}$$

where:

$$\Psi = \frac{\left(1 - \alpha - \beta\right)}{1 + 2\beta t / \left(1 - t\left(1 + \beta'\right)\right)} \tag{27}$$

A corresponding solution for b in terms of E can be obtained as:

$$b = \frac{1 - t}{1 - t\left(1 - \beta'\right)}\left[\left\{t\left(1 - \beta'\right)\bar{w}_\theta - \Psi\hat{w}_\theta\right\}E^{1-\theta} + E\left(\Psi - 1\right)\right] \tag{28}$$

Finally, the solution for E is:

$$E = \left[\left(1 - \theta\right)\frac{\left(1 - \alpha - \beta\right)\left(1 - \beta'\right)H\Psi^{-1} - \beta\left\{t\left(1 - \beta'\right)\bar{w}_\theta - \Psi\hat{w}_\theta\right\}}{\left(1 - \alpha - \beta\right)\left(\Psi^{-1} - 1\right) + \left(1 - \theta\right)\beta\left(\Psi - 1\right)}\right]^{1/\theta} \tag{29}$$

where H is:

$$H = \tilde{w}_\theta\left\{1 - t\left(1 + \beta'\right)\right\} + t\bar{w}_\theta \tag{30}$$

Hence, for a given tax rate and the various parameters of utility functions and the education production function, it is possible to solve for the optimal values of the policy variables by first using (29). In the Appendix, expressions for the optimal value of Q in terms of E, and of b in terms of E and Q are derived: these can be used to obtain Q and b. Finally, writing total government expenditure as $T = pQ / n + b + E$, gives, after simplification:[12]

$$T = \frac{E^{1-\theta}\left(t\bar{w}_\theta\left(1 - \beta'\right)\left(1 - t\right) + \beta' t\Psi\hat{w}_\theta\right) + \beta' tE\left(1 - \Psi\right)}{1 - t\left(1 - \beta'\right)} \tag{31}$$

The ratio of each expenditure component to total expenditure can then be obtained. It is easily seen that the resulting complex expressions are independent of the population size.

However, these expressions do not represent 'closed form' solutions because of their nonlinearity and the complex form taken by $\tilde{w}_\theta$, which depends on the welfare weights and thus the form of the social welfare function used. Further progress can be made using an explicit form for W and an approximation for $\tilde{w}_\theta$ which allows explicit solutions to be obtained, making the properties of the model clearer. This is examined in the following section.

4. The Welfare Weights

In examining the welfare-weighted average, $\tilde{w}_\theta$, it is clear that the term $\upsilon_i = \partial W/\partial V_i \; \partial V_i/\partial b$, and hence the weight υ_i', is highly complex even for simple forms of W. For example, suppose the social planner maximizes an additive social welfare function with iso-elastic weights applied to the V_i such that:

$$\begin{aligned} W &= \frac{1}{1-\varepsilon}\sum_{i=1}^{n} V_i^{1-\varepsilon} && \varepsilon \neq 1, \varepsilon > 0 \\ &= \log y && \varepsilon = 1 \end{aligned} \tag{32}$$

where ε is the degree of concavity of the weighting function and represents the degree of constant relative inequality aversion of the planner. Hence $\partial W/\partial V_i = V_i^{-\varepsilon}$ and substituing for V from (1), along with $\partial V_i/\partial b$ from (3) gives an awkward expression for $\partial W/\partial V_i \; \partial V_i/\partial b$.

It can be shown that, if b is relatively small (in comparison with average earnings), a reasonable approximation to $\tilde{w}_\theta$ can be based on an equally distributed equivalent measure. In general for the variable, x, the 'equally distributed equivalent' value, x_e, is the value which, if obtained by everyone, gives the same welfare, defined in terms of x, as the actual distribution. This concept is associated with the Atkinson measure of inequality, A, which is expressed as the proportional difference between the arithmetic mean and the equally distributed equivalent value, so that:

$$A = 1 - \frac{x_e}{\bar{x}} \tag{33}$$

Using an iso-elastic welfare function, such as (32), expressed in terms of x, x_e is:

$$x_e = \left(\frac{1}{n}\sum_{i=1}^{n} x_i^{1-\varepsilon}\right)^{1/(1-\varepsilon)} \tag{34}$$

Furthermore, if x is lognormally distributed as $\Lambda(\mu, \sigma^2)$, then $x^{1-\varepsilon}$ is lognormally distributed as $\Lambda((1-\varepsilon)\mu, (1-\varepsilon)^2\sigma^2)$ and using the result for the arithmetic mean of a lognormal variable (see Aitchison and Brown, 1957), (34) becomes:

$$x_e = \exp\left(\mu + (1-\varepsilon)\frac{\sigma^2}{2}\right) \tag{35}$$

Hence if w_0 is assumed to be lognormally distributed as $\Lambda(\mu_{w0}, \sigma^2_{w0})$, then:

$$\bar{w}_\theta = \exp\left(\theta\mu_{w_0} + \frac{1}{2}\theta^2\sigma^2_{w_0}\right) \tag{36}$$

and:

$$\tilde{w}_\theta = \exp\left(\theta\mu_{w_0} + \frac{1}{2}(1-\varepsilon)\theta^2\sigma^2_{w_0}\right) \tag{37}$$

This allows the implications of alternative degrees of inequality aversion to be examined.

Using these results it is possible to show, after much manipulation, that the mean of logarithms, μ_{w0}, cancels from the expenditure ratios, *E/T, (pQ/n)/T* and *b/T*. It is clearly convenient that the ratios do not depend on the unit of measurement, though of course the absolute values do depend on μ_{w0}.

5. Numerical Examples

Previous sections have obtained explicit results regarding the optimal values of expenditure components. However, the complexity of the expressions means that the comparative static properties are not transparent. The present section therefore uses numerical examples to reinforce discussion of the model's properties. First, suitable parameter values for calibration are outlined, after which the effects of varying a range of parameters are examined. These include the elasticity of the wage rate with respect to basic ability, θ, the inequality of the basic ability level, σ^2_{w0}, the degree of inequality aversion of the judge, ε, and the exogenously fixed tax rate, t.

5.1 Calibration of The Model

In selecting basic parameter values it is necessary first to ensure that they give rise to sensible values of the proportion of time devoted to labour supply. Using preference parameters of $\alpha = 0.5$ and $\beta = 0.4$, along with a tax rate of $t = 0.35$, the proportion of time devoted to labour supply, at the arithmetic mean wage rate, is around 0.58. Furthermore, it is necessary to consider values of θ which, along with other parameters, give rise to positive values of the transfer payment, b, and reasonable absolute and relative values of expenditure. In the following numerical examples, θ is allowed to vary between 0.75 and 1: the latter corresponds to the case where education expenditure has no role in generating ability levels.[13]

As a benchmark case, individuals' basic ability levels, w_θ, are assumed to follow a lognormal distribution with variance of logarithms, $\sigma^2_{w_0} = 0.36$ and mean of logarithms, $\mu_{w_0} = 3$, remembering that the latter affects only absolute values of expenditure.

In calibrating this type of model it is difficult to obtain obvious empirical counterparts. For example it may be thought that estimates of labour supply elasticities could be used, but there is no single elasticity from which calibration can be based. The approach adopted with this kind of exercise is to ensure that the orders of magnitude of certain important variables make sense, such as the labour supplied by the individual with arithmetic mean wage rate, and typical values of expenditure on consumption. Extensive sensitivity analyses were carried out to ensure that results presented are fully representative and do not arise from pathological cases. With the benchmark parameters mentioned here, the absolute values of expenditure on education, public goods, and the transfer payment in all reported results have reasonable magnitudes, in relation to approximate observed values.[14]

5.2. Comparative Statics

Consider first the influence of σ_w^2 on the optimal composition of government expenditures. From the human capital production function (8) the variance of logarithms of wage rates is $\sigma_w^2 = \theta^2\sigma_{w0}^2$ and is not affected by E, which is the same for everyone. It is, however, influenced by θ, which also governs the extent to which education expenditure contributes to human capital, $1-\theta$. In the following discussion, it is convenient to define the term $G = pQ$ as the expenditure on the public good.

A higher value of σ_{w0}^2 has two effects. First, ceteris paribus, it increases the arithmetic mean value of w and hence average earnings.

$$\tilde{w} = \exp\left(\mu_w + \sigma_w^2\right) = \exp\left(\theta\mu_{w_0} + (1-\theta)\log E + 0.5\theta^2\sigma_{w_0}^2\right) \tag{38}$$

This gives rise to a higher total revenue, which allows all types of expenditure to increase. The higher E also reinforces the effect on average earnings, though it is moderated somewhat by the increase in b. Secondly, an inequality averse planner makes the expenditure pattern more redistributive, so that b increases relative to G/n and E. Hence optimal $(G/n)/T$ and E/T fall, despite their absolute increase, while b/T increases. However, all these effects are relatively small given that σ_{w0}^2 is only a small component of the arithmetic mean wage.

The way in which optimal shares vary as σ_{w0}^2 increases is shown in Figure 1. The top panel illustrates variations in the share of expenditure on education, E/T, with changes in σ_{w0}^2; the middle panel shows how the ratio of optimal public goods expenditure per person to total expenditure, $(G/n)/T$, varies; and the lower panel shows how the share of transfer payments varies with σ_{w0}^2. In each case the variations are obtained for an inequality aversion coefficient of $\varepsilon = 0.8$, and profiles are shown for four values of the parameter, θ. The effects of variations in σ_{w0}^2 are seen to be similar for all values of θ. That is, profiles of optimal expenditure components and shares, as σ_{w0}^2 varies, are simply shifted upwards or downwards by increases or decreases in θ.

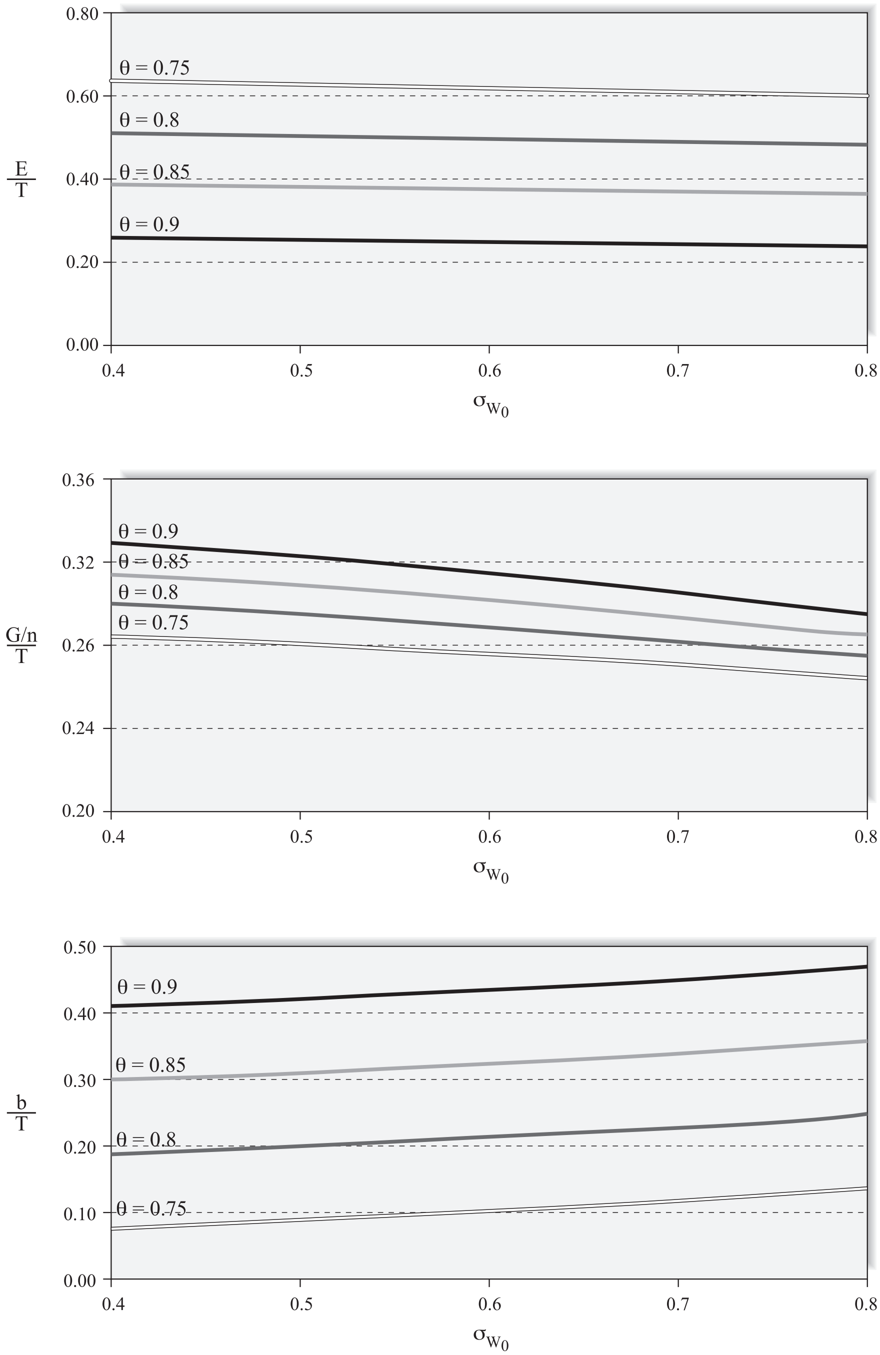


Figure 1. The Optimal Composition and Variations in the Variance of Initial Wage

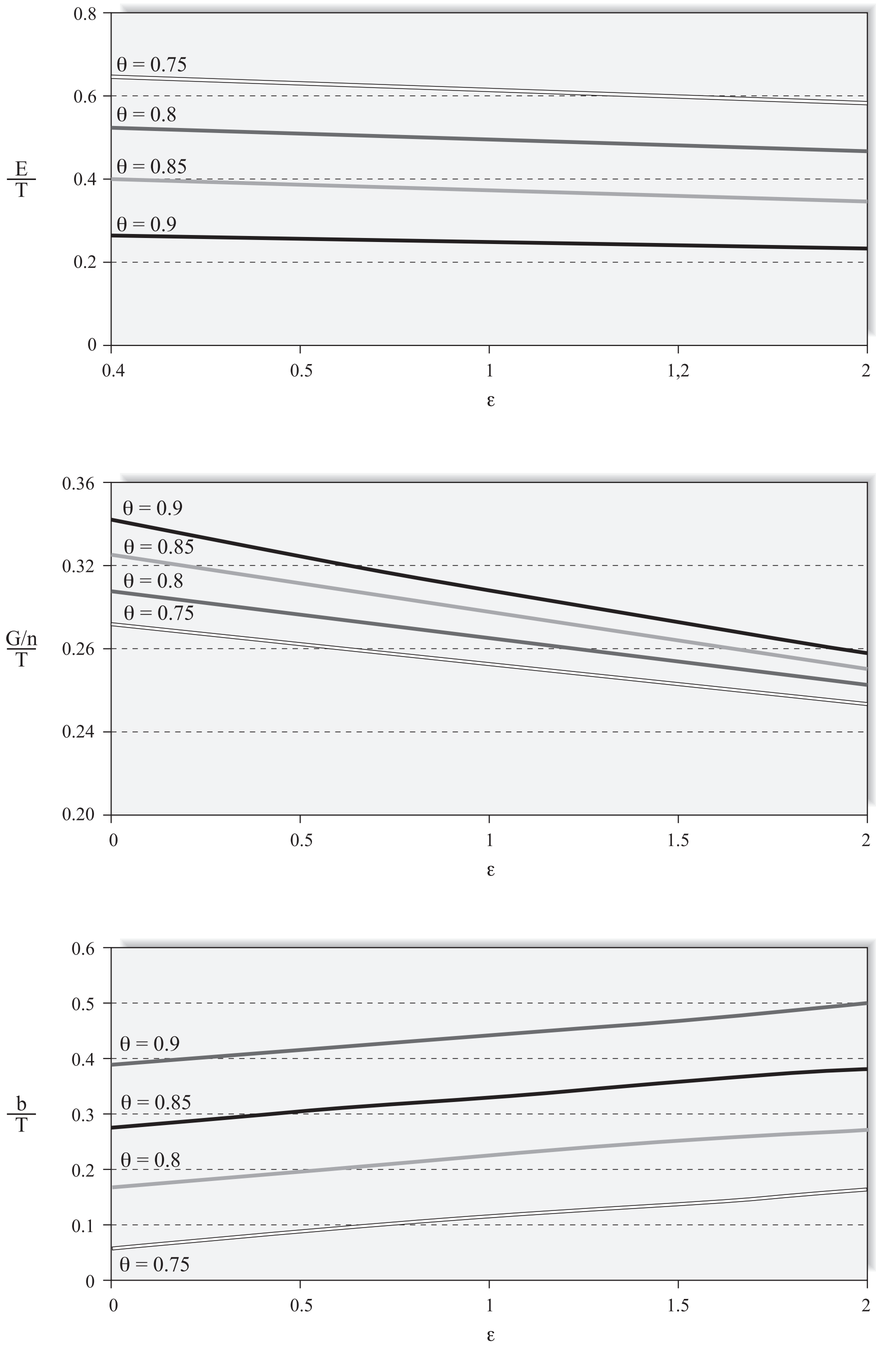


Figure 2. The Optimal Composition and Variations in Inequality Aversion

An increase in inequality aversion, ε, does not have a similar effect to that of an increase in basic inequality, σ^2_{w0}, because the latter affects arithmetic mean earnings and hence tax revenue. The increase in ε increases the difference between $\tilde{w}_\theta$ and $\bar{w}_\theta$. The desire for a more redistributive policy on the part of the planner leads to an increase in b, in absolute and relative terms, at the expense of E and Q. Thus b and b/T increase while Q, E, $(G/n)/T$ and E/T all fall slightly. Again, these effects are small and are similar for all values of θ. The three panels of Figure 2 show in turn the relationship between ε and the optimal E/T, $(G/n)/T$ and b/T, again for diffferent values of θ. In each case the variation in the budget share, as inequality aversion increases, is small. Hence different planners may have very different degrees of inequality aversion and yet would display little disagreement concerning optimal expenditure shares. It is clear that the value of θ, influencing the role of education in generating ability levels, is much more important. This is discussed next.

The effect of an increase in θ is to produce an investment effect. This arises from the fact that a higher θ implies that education is less effective in raising the average level of productivity. This leads to the planner reducing E in absolute terms. In the limit, a value of $\theta = 1$ means that education plays no role in determining w, so that there is no point in investing in education and optimal E falls to zero. Hence the investment effect of variations in θ can be expected to be quite substantial. The consequent reduction in education spending, in absolute and relative terms as θ increases, leaves more tax revenue available for spending on transfer payments and the public good, despite the associate reduction in average earnings. However, the reduction in average earnings via the term $(1 - \theta)\log E$ is modified to a small extent by the effect of higher θ in raising $\bar{w}$ for given E, via the terms $\theta\mu_{w_0}$ and $0.5\theta^2\sigma^2_{w_0}$ in equation (38). The last term is similar to the effect of increasing inequality, while the effect on $\bar{w}$ operating via μ_{w_0} is known to have no effect on relative orders of magnitude. Figure 3 shows the variation in optimal E/T, $(G/n)/T$ and b/T with θ, while Figure 4 shows the variation in the absolute levels of E, G/n and b.

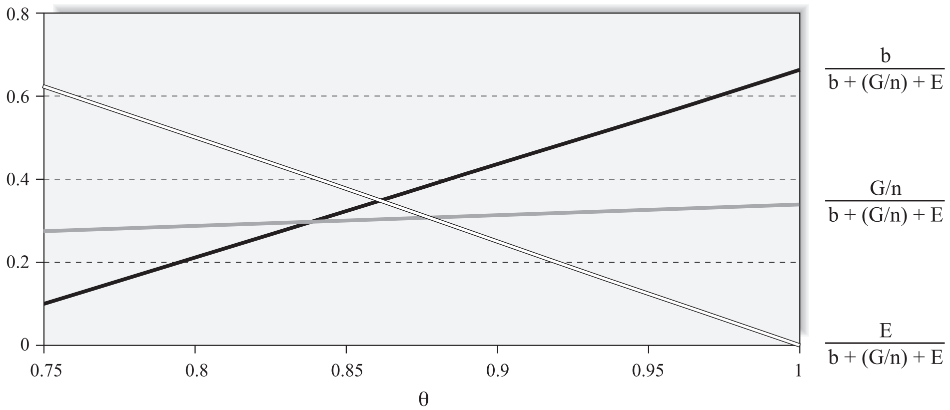


Figure 3. The Optimal Composition and Variations in the Variance of Initial Wage

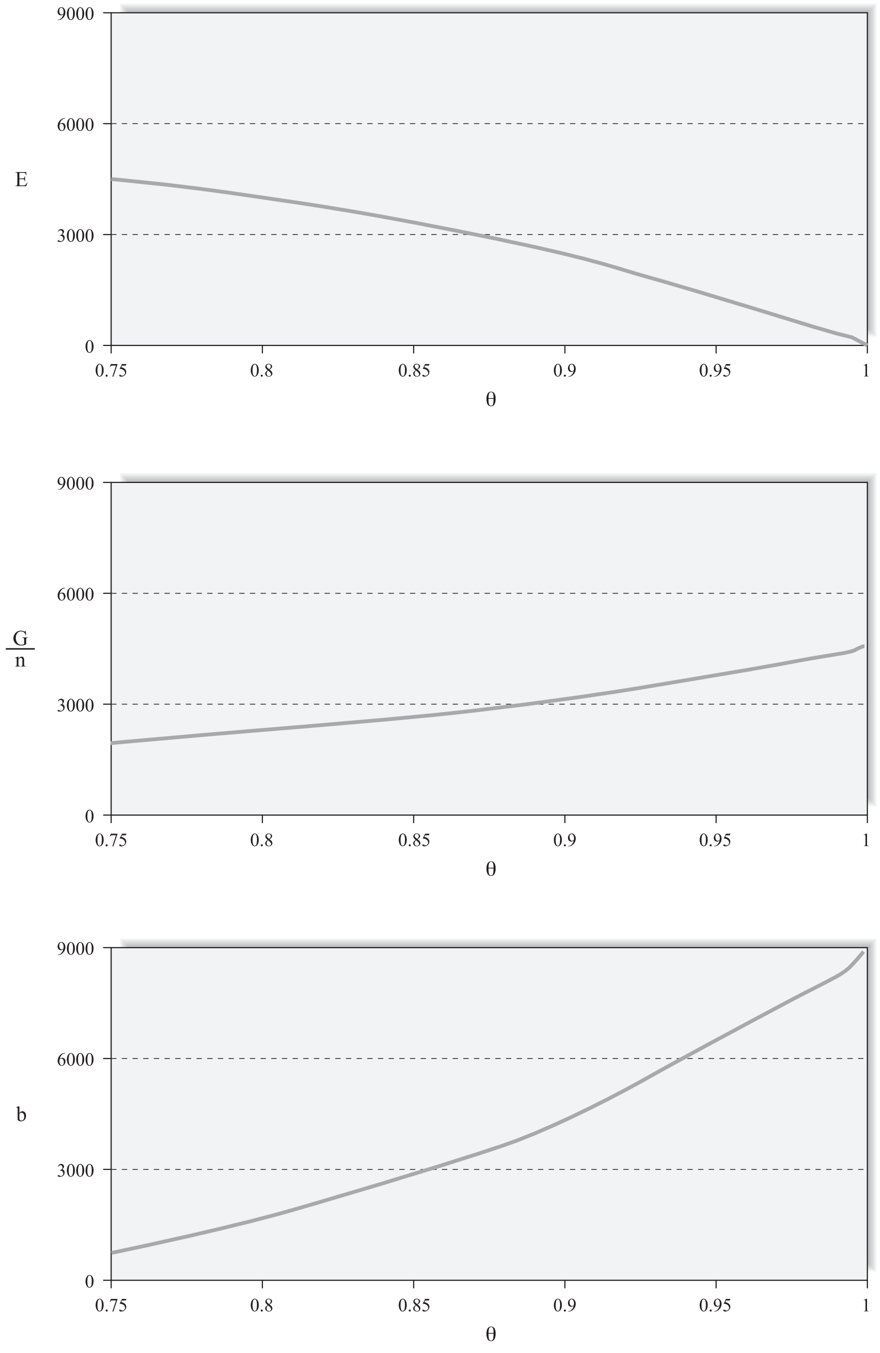


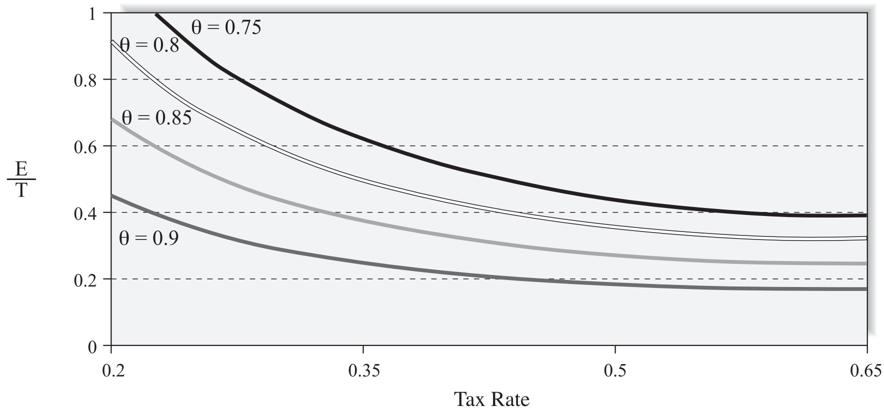
Figure 4. The Optimal Composition and Variations in the Efficiency Parameter θ

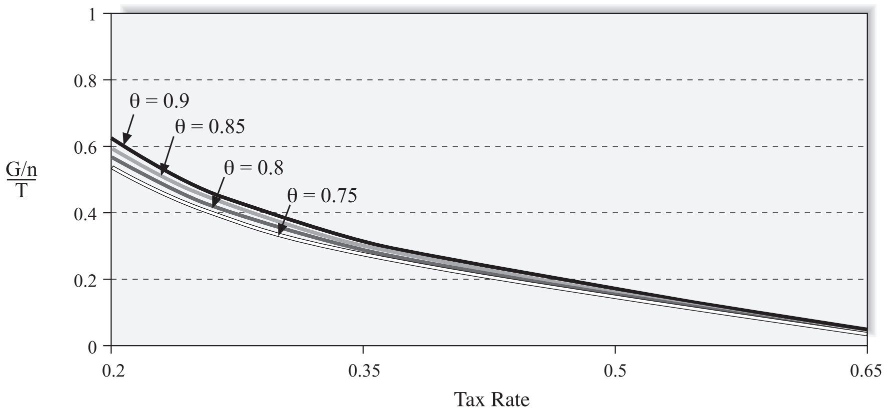
These diagrams show that the overall effect of increasing θ is to reduce optimal E and E/T while raising G, b, G/T and b/T. In view of the significant investment effect of variations in θ, this parameter has a much larger effect on policy variables than inequality aversion or basic inequality. The ratios appear to be approximately linearly related to θ, but this property is far from evident from the corresponding analytical expressions.

The above discussion relates to changes in parameters and their effects on optimal expenditure patterns for a given tax rate, which is assumed to be set exogenously. Clearly, a minimum value of t is necessary in order to ensure that the transfer payment is positive, given that some non-transfer expenditure is carried out, otherwise the inequality-reducing benefit becomes the equivalent of a regressive poll tax. It is useful to consider how the optimal composition varies as the tax rate varies. First, the incentive effect of an increasing tax rate, ceteris paribus, reduces average earnings. Initially, the increasing 'tax rate' effect of a higher rate outweighs the falling 'tax base' effect, so that total revenue rises. However, for higher tax rates, further increases lead to a reduction in total revenue as the falling tax base effect dominates. This kind of effect is of course familiar. Hence an inequality averse planner would raise the transfer payment as the tax rate is increased – but only to the extend that this is made possible by the revenue effect. It has been mentioned that increasing the tax rate has a disincentive effect on labour supply, thereby reducing average earnings. The return from investing in education is therefore lower, so an increase in t has the effect of lowering the optimal value of E. The effect of increasing b and reducing E means that eventually as the tax rate is increased, Q can in principle turn negative: such a 'public bad' does not make sense in the present framework, thereby imposing an upper limit on the tax rate. The relationship between the tax rate and the optimal share of expenditure on education, public goods and transfer payments are shown in the three panels of Figure 5. In each case variations are illustrated for four different values of the parameter, θ, in the education production function.

6. Conclusions

This paper has examined the optimal allocation of tax revenue among a universal transfer payment, a pure public good and expenditure on education. Using a single-period framework, individuals were assumed to have similar preferences but to be endowed with an exogenously given heterogeneous ability level. The education expenditure raises the productivity of individuals, via a type of human capital production function. The social welfare function was based on individuals' (indirect) utilities, and reflecting a degree of relative inequality aversion. Judgements regarding optimal expenditure therefore have to balance the efficiency effects of education expenditure (which does not reduce wage rate inequality), with the inequality reducing effect of the transfer. In this framework, education creates a substantial fiscal spillover whereby the increase in human capital gives rise to higher labour supply earnings and thus higher income tax revenue, thereby allowing greater government expenditure on all items than would otherwise be possible. Explicit solutions were obtained for the expenditure levels. However, in view of the complexity of the expressions, comparative static properties of the model were reinforced by numerical examples.





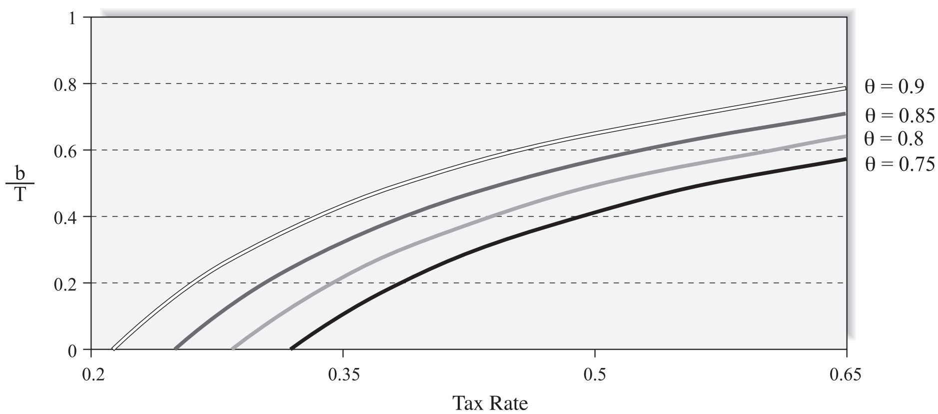


Figure 5. Optimal Expenditure Shared and Variations in the Tax Rate

A higher inequality of exogenous ability levels was found to increase average earnings. The higher total revenue allows all types of expenditure to increase. Secondly, higher inequality aversion makes the expenditure pattern more redistributive, so that the transfer payment increases relative to public good and education expenditure, despite their absolute increase. However, these effects are relatively small.

The desire for a more redistributive policy leads to an increase in transfer payments in absolute and relative terms, at the expense of the other two components. The variation in budget shares, as inequality aversion increases, is small. Hence different independent judges may have very different degrees of inequality aversion and yet would display little disagreement concerning optimal expenditure shares.

The effect of an increase in the coeffient on basic ability in the human capital production function (the elasticity of the wage with respect to basic ability) is to produce an investment effect. First, it implies that education is less effective in raising the average level of productivity, which leads to the planner reducing education spending in absolute terms. Ultimately, as the coefficient approaches unity, education plays no role, so the investment effect of such variations are expected to be substantial. The consequent reduction in education spending, in absolute and relative terms, leaves more tax revenue available for spending on transfer payments and the public good, despite the associate reduction in average earnings. In view of the significant investment effect of variations in this elasticity, this parameter has a much larger effect on policy variables than inequality aversion or basic inequality.

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Notes

1. See Creedy and Moslehi (2009). In a standard optimal tax model, where income tax finances a universal transfer payment, it is well-known that a higher inequality of ability (and hence wage rates) is associated with choice of a higher tax rate, and hence a more redistributive policy. The same is true of majority voting models, in the tradition of early work of Meltzer and Richard (1981). See the survey by Borck (2007).
2. To avoid other issues associated with educational choice, private education is not modelled here.
3. Tridimas (2001, p. 308) suggests that, ‘This is less restrictive than it first appears, since in practice governments are often constrained in the policy instruments that they may vary at any one time’.
4. Such a sequential approach is used in voting models of government expenditure, in view of the complexities involved in voting over more than one dimension. On such two stage voting in a public choice context, see McCaleb (1985) who emphasises uncertainties involved during the first stage.
5. Some comparisons are given in Creedy and Solmaz (2009).
6. It is shown in subsection (3) that with this form of utility function the expenditure shares do not depend on population size. Furthermore, the shares are independent of units, and thus the average wage.
7. In the standard case, expenditure on any item is the product of full income and the ratio of the relevant coefficient to the sum of coefficients. In the present context, the term $Q^{1-\alpha-\beta}$ must be treated effectively as a constant term in the utility function, because Q does not have a corresponding consumer price (only a ‘tax price’). Thus, for example in (3), full income is multiplied by $\alpha / (\alpha + \beta)$ rather than α.
8. Allowing for individuals to choose an amount of time spent in education, and supposing that this does not give direct utility, gives rise to a highly nonlinear solution for the supply of labour.
9. In Capolupo (2000, p. 168), human capital accumulation is determined purely by government education expenditure and an efficiency term.
10. This type of externality is explored at length in Creedy (1995), in the different context of the choice of higher education (considered as a private investment good).
11. In the special case where inequality aversion is zero, $\tilde{w}_\theta = \overline{w}_\theta = \hat{w}_\theta$ and it can be seen that $Q = 0$ when $\overline{w} = E$.
12. Total tax revenue is $t\overline{y}$, and comparing this with $pG/n + b + E$ provides a useful independent check on computations, such as those reported below.
13. Strictly, results are undefined for $\theta = 1$, so the upper limit in practice is set at 0.999.
14. As mentioned earlier, an independent check on total tax revenue, resulting from arithmetic mean earnings, and the sum of expenditure on education, public goods and the transfer payment, confirmed both the accuracy of the analytical expression and the computations.
15. Unfortunately, H is not a weighted average of the two terms involving w.
16. Alternatively it is possible to write $E^{-\theta} = (1 - \alpha - \beta)(\Psi^{-1} - 1) + (1 - \theta)\beta(\Psi - 1) / (1 - \theta)(A_1\tilde{w}_\theta + A_2\tilde{w}_\theta)$, where $A_1 = (1 - \alpha - \beta)(1 - \beta')\Psi^{-1}(1 - t(1 + \beta')) + \beta t\Psi(1 - t(1 - \beta'))$ and $A_2 = (1 - \alpha - \beta)(1 - \beta')\Psi^{-1}t - \beta t(1 - \beta') + \beta t^2\Psi(1 - \beta')$. Unfortunately $A_1 + A_2 \neq 1$.

Resumen

Este trabajo analiza la asignación óptima de los ingresos fiscales entre una transferencia universal, un bien público puro y el gasto público en educación. Utilizando un marco temporal de un único periodo, el gasto en educación incrementa la productividad de los individuos modificando el capital humano en la función de producción. La función de bienestar social se basa en las utilidades indirectas de los individuos. La educación crea externalidades fiscales importantes, por lo que el incremento del capital humano incrementa los salarios y, por tanto, la recaudación fiscal, lo que permite incrementar el gasto gubernamental en general. Una mayor desigualdad en los niveles de habilidad exógenos incrementa todos los tipos de gasto, pero solo la transferencia universal aumenta en términos relativos. Una mayor aversión a la desigualdad lleva a incrementar las transferencias en términos absolutos y relativos, a costa de los otros dos componentes. En todo caso, la sensibilidad a la aversión a la desigualdad es pequeña. Un incremento de la elasticidad de los salarios respecto a las habilidades básicas lleva a reducciones del gasto educativo.

Palabras clave: gasto público, bienes públicos, educación, transferencias, bienestar social.

Clasificación JEL: H31, H41, H52, H53, H75

Appendix: Solving the First-Order Conditions

This appendix derives explicit (closed-form) solutions for the policy variables, Q, b and E, conditional on the income tax t.

Partial Derivatives of Indirect Utility

First, it is necessary to obtain expressions for the ratios of partial derivatives of the indirect utility function. Substituting for $w = \gamma w_0^{\theta} E^{1-\theta}$ and $M = w(1 - t) + b$ into (7), the indirect utility function can be written, again omitting i subscripts for convenience, as:

$$V = \left(\alpha'^{\alpha}\beta'^{\beta}\right)\left(w_0^{\theta}E^{1-\theta}(1-t)\right)^{-\beta}\left(w_0^{\theta}E^{1-\theta}(1-t)+b\right)^{\alpha+\beta}Q^{1-\alpha-\beta} \tag{1}$$

The three partial derivatives of V with respect to Q, b and E are:

$$\frac{\partial V}{\partial Q} = \frac{(1-\alpha-\beta)V}{Q} \tag{2}$$

$$\frac{\partial V}{\partial b} = \frac{(\alpha+\beta)V}{M} \tag{3}$$

$$\frac{\partial V}{\partial E} = \frac{(\alpha+\beta)(1-\theta)(1-t)wV}{EM} - \frac{\beta(1-\theta)V}{E} = \frac{(1-\theta)V}{E}\left(\frac{(\alpha+\beta)(1-t)w}{M} - \beta\right) \tag{4}$$

Hence the required ratios are:

$$\frac{\partial V/\partial Q}{\partial V/\partial b} = \frac{(1-\alpha-\beta)M}{(\alpha+\beta)Q} \tag{5}$$

and:

$$\frac{\partial V/\partial E}{\partial V/\partial b} = \frac{(1-\theta)M}{E}\left(\frac{w(1-t)}{M}-\beta'\right) = \frac{(1-\theta)}{E}\left\{w(1-t)(1-\beta')-\beta' b\right\} \tag{6}$$

Substituting (6) and (5) into the relevant first-order conditions gives:

$$\sum_i \frac{\upsilon(1-\alpha-\beta)M}{(\alpha+\beta)Q} = \lambda\frac{1}{n} \tag{7}$$

$$\sum_i \frac{\upsilon(1-\theta)}{E}\left\{w(1-t)(1-\beta')-\beta' b\right\} = \lambda\left\{1-tE^{-\theta}\bar{w}_\theta(1-\beta')(1-\theta)\right\} \tag{8}$$

Solutions for Optimal *Q* and *b*

Consider first the two-dimensional choice of b and Q, for given E. This is equivalent to the requirement that a social indifference curve is tangential to the budget constraint, which is obtained by dividing (7) by (21). Writing $\upsilon_i' = \upsilon_i/\Sigma_i\,\upsilon_i$ gives:

$$\sum_i \upsilon'\left\{w(1-t)+b\right\} = \left(\frac{\alpha+\beta}{1-\alpha-\beta}\right)\frac{pQ/n}{\left(1-\beta'\frac{t}{1-t}\right)} \tag{9}$$

However, using (8) to substitute for w, the left hand side of (9) becomes:

$$\sum_i \upsilon'\left\{w(1-t)+b\right\} = b+(1-t)E^{1-\theta}\sum_i \upsilon' w_0^\theta \tag{10}$$

Let $\tilde{w}_\theta = \Sigma_i\,\upsilon_i'\,w_{0,i}^\theta$, a welfare-weighted average of w_0^θ (the weighted moment of order θ about the origin). Hence, rearrangement of (9) gives:

$$b = \left(\frac{\alpha+\beta}{1-\alpha-\beta}\right)\frac{pQ/n}{\left(1-\beta'\frac{t}{1-t}\right)} - (1-t)E^{1-\theta}\tilde{w}_\theta \tag{11}$$

This result expressed the basic income in terms of the expenditure on the public good and on education per person. Using the government budget constraint, that is setting (11) equal to (15) and rearranging, gives:

$$\frac{pQ}{n} = \frac{E^{1-\theta}\left\{\tilde{w}_\theta + t(1-\beta')(\bar{w}_\theta-\tilde{w}_\theta)\right\}-E}{1+\frac{\alpha+\beta}{1-\alpha-\beta}\left(\frac{1-t(1-\beta')}{1-t(1+\beta')}\right)} \tag{12}$$

This result can be simplified as follows. Denoting the denominator by Ψ^{-1}, it can be seen that Ψ can be expressed as:

$$\Psi = \frac{(1-\alpha-\beta)}{\left(1+\frac{2\beta t}{1-t(1+\beta')}\right)} \tag{13}$$

Furthermore, define $\hat{w}_\theta$ as the weighted average:

$$\hat{w}_\theta = \tilde{w}_\theta\left(1-t\left(1-\beta'\right)\right)+t\left(1-\beta'\right)\bar{w}_\theta \tag{14}$$

This allows (12) to be written more succinctly as:

$$\frac{pQ}{n} = \Psi\left(E^{1-\theta}\hat{w}_\theta - E\right) \tag{15}$$

This expresses pQ/n in terms of E only. A corresponding solution for b in terms of E can be obtained by substituting for pQ/n, using (15), into (11), whereby:

$$b = \frac{1-t}{1-t\left(1-\beta'\right)}\left[\left\{t\left(1-\beta'\right)\bar{w}_\theta - \Psi\hat{w}_\theta\right\}E^{1-\theta} + E(\Psi-1)\right] \tag{16}$$

It therefore remains to solve for E. This is described in the following subsection.

Solving for Optimal *E*

Consider, for given Q, the choice between b and E, involving a tangency solution between a social indifference curve and the budget constraint, when taking the two dimensions b and E. Thus, divide (8) by (21), and again use (8) to express wage rates in terms of basic abilities and education expenditure, to get:

$$\frac{1-\theta}{E}\left[(1-t)\left(1-\beta'\right)E^{1-\theta}\tilde{w}_\theta - \beta' b\right] = \frac{1-tE^{-\theta}\bar{w}_\theta\left(1-\beta'\right)(1-\theta)}{1-\beta'\frac{t}{1-t}} \tag{17}$$

Further rearrangement gives:

$$(1-t)\left(1-\beta'\right)(1-\theta)E^{-\theta}\left[\tilde{w}_\theta + t\left\{\bar{w}_\theta - \left(1+\beta'\right)\tilde{w}_\theta\right\}\right] = (1-t)+(1-\theta)\beta'\left\{1-t\left(1+\beta'\right)\right\}\frac{b}{E} \tag{18}$$

However, from the result for b in terms of E in (16), the required ratio in (18) becomes:

$$\frac{b}{E} = \frac{(1-t)\left[\left\{t\left(1-\beta'\right)\bar{w}_\theta - \Psi\hat{w}_\theta\right\}E^{-\theta} + (\Psi-1)\right]}{1-t\left(1-\beta'\right)} \tag{19}$$

Hence, collecting terms in $E^{-\theta}$ and rearranging gives:

$$E^{\theta} = (1-\theta)\frac{(1-\beta')\{1-t(1-\beta')\}H - \beta'\{1-t(1+\beta')\}\{t(1-\beta')\bar{w}_{\theta} - \Psi\hat{w}_{\theta}\}}{\{1-t(1-\beta')\} + (1-\theta)\beta'(\Psi-1)\{1-t(1+\beta')\}} \tag{20}$$

Where H is:[15]

$$H = \tilde{w}_{\theta}\{1-t(1+\beta')\} + t\bar{w}_{\theta} \tag{21}$$

Taking right hand side of (20), dividing numerator and denominator by $1 - t\,(1 + \beta')$, and using the fact that:

$$\frac{1-t(1-\beta')}{1-t(1+\beta')} = \frac{(1-\alpha-\beta)(\Psi^{-1}-1)}{\alpha+\beta} \tag{22}$$

the solution for E is given by the result in (29).[16] The resulting value of E can then be substituted into (15) to obtain the optimal value of Q. Finally, these values of E and Q can be used in (11) to obtain the basic income, b.