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## A Probabilistic Voting Model of Progressive Taxation with Incentive Effects\*

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### Abstract

The purpose of this work is to show under what conditions a marginally progressive income tax emerges as the result of political competition between two parties when labor is elastically supplied and candidates are uncertain about voters' decisions on election day. Assuming a decreasing wage elasticity of labor supply, if we follow Coughlin and Nitzan (1981), only marginal-rate progressive taxes are chosen by both candidates in equilibrium. If, instead, we adopt Lindbeck and Weibull's (1987) probabilistic voting model, the equilibrium tax schedule will be progressive as long as the political power of the rich voter is sufficiently low. The degree of progressivity decreases with population polarization.

*Keywords:* Political economy, progressive taxation, elastic labor supply.

*JEL Classification:* D3, D63, D72, H24.

### 1. Introduction

The question “why do progressive taxes emerge in industrialized countries?” dates from Mirrlees' seminal paper (1971). He showed that marginal-rate progressive tax schedules, like those of industrialized societies, were hardly optimal unless we had a low elasticity of labor supply. Growing literature on the political economy of taxation, inspired by Roberts (1977), Romer (1975) and Meltzer and Richard (1981), questioned whether high marginal taxes could

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be part of a political equilibrium. Although, disincentives effects from taxation were taken into account, restrictions on the tax schedule were imposed making difficult to study tax progressivity. Among those studying tax progressivity, few of them considered disincentive effects from taxation (De Donder and Hindriks, 2003), since most literature was based on the exogenous income hypothesis, see for example Marhuenda and Ortuño-Ortín (1995), Roemer (1999), Carbonell-Nicolau and Klor (2003) and Carbonell-Nicolau and Ok (2007).

Early literature on the political economy of taxation assumed a proportional income tax. Under some conditions on preferences, such as single-crossing, a Condorcet winner (CW) exists and both Downsian candidates play the CW tax rate in equilibrium. In the absence of labor disincentives (an inelastic labor supply), if the median income is below the mean, then the equilibrium marginal tax rate equals 100%. Even if an endogenous labor supply is assumed, taxes are still strictly positive and rise with inequality, defined as the ratio of average income to median income.

Nevertheless, assuming a linear income tax schedule will not help us to understand the fact that, in most industrial economies, marginal tax rates increase with income. The aim of this paper is to understand why there is a democratic demand for income tax progressivity. In order to have a tax schedule that allows for (marginal-rate) progressivity, we need at least three parameters to vote for<sup>1</sup>. One parameter specifies the lump-sum transfer (or level of public good), another the linear tax rate, and the last captures the concavity (regressivity) or convexity (progressivity) of the tax schedule. Thus, we are facing a multidimensional voting model. Conditions for the existence of a CW in models with a multidimensional policy space are known to be very restrictive. For the quadratic tax function, De Donder and Hindriks (2003) and Hindriks (2001) show that it is hard to avoid voting cycles. Approaches other than the direct democracy approach should be considered.

In a citizen-candidate model, Carbonell and Klor (2003) found that, under some conditions, only increasing marginal tax rates are implemented in equilibrium. Roemer (1999) developed the PUNE concept (Party Unanimity Nash Equilibrium). The platform chosen by the party is the outcome of intraparty negotiation among party members. In equilibrium, both parties announce marginal-rate progressive taxes.

In this paper, we adopt the probabilistic voting model introduced by Coughlin and Nitzan (1981) and Coughlin (1992), micro-founded subsequently by Lindbeck and Weibull (1987). When voters cast their ballots in favor of one candidate or another, they consider issues other than economic ones, for instance ideology. Nonetheless, the bigger the difference in economic utility, the higher the probability that a given voter will choose the candidate that brings him the highest (economic) utility. Conditions for the existence of an equilibrium are less restrictive in the Coughlin model. A CW need not exist and, when it does exist, the equilibrium tax schedule need not coincide with the CW tax schedule.

The probabilistic model can be understood as the outcome of a political process where voters choose between candidates probabilistically, with the probability to vote for one can-

didate increasing in the utility difference. The outcome of such a political process, as stressed by Coughlin and Nitzan (1981) in their Theorem 1, involves the maximization of a Nash welfare function. Indeed, in both models, Coughlin (CN) and Lindbeck and Weibull (LW), the equilibrium income tax maximizes some welfare function. In this sense the equilibrium income tax is efficient; it is on the economy's utility frontier. We show which conditions on the welfare function and the labor supply need to be satisfied for a marginal progressive tax to emerge in equilibrium.

For simplicity, we assume quasi-linear preferences, which imply a zero income effect. Therefore, when the tax rate paid by a given group increases, labor supply unambiguously decreases. Labor supply responses will add another mechanism for a given vote to be easier to catch. If the elasticity of labor supply decreases with wage, then we could tax the rich heavily in comparison with the poor, since the former reduce their labor supply to a lesser extent in response to a tax increase. In this respect, there is more scope for tax progressivity than in the fixed (exogenous) income model. It is worth noting that there are few estimates of the elasticity of labor supply by income groups. Saez (2004) finds that families and individuals in the mid-to-upper income bracket do not appear to be sensitive to taxation. Significant elasticities are found, though, at the very top of the income distribution. Whether those elasticities could be explained solely by the evolution of marginal tax rates is not clear, given the heterogeneity in size of responses across time.

The probabilistic voting model would give credible predictions in any of these three scenarios: voters vote probabilistically, candidates are uncertain of vote decisions, or we have interest groups representing voters that compete for influence.

The rest of the paper continues as follows: section 2 presents the model; section 3 draws voters' labor supply as a function of the implemented tax schedule. Preferences over tax schedules by groups are studied in section 4. Section 5 describes the (CN) and (LW) probabilistic voting model and presents the main results of the paper. Section 6 concludes.

## 2. The Model

We develop a static model of political competition between two Downsian parties,  $A$  and  $B$ . Since voters are *ideological*, candidates or parties are uncertain how voters' economic preferences can be translated into party preferences. The (CN) and (LW) model account differently for voters' ideology. We will describe in detail both probabilistic voting models in section 5. Parties announce simultaneously a policy platform  $(t^C, G^C)$  with  $C = A, B$ ; consisting of a vector of marginal income tax rates that finances lump-sum transfers and a public good. Parties choose the policy platform that maximizes their probability of winning and commit to the platform announced. The probability of winning is an increasing function of the voting share. For simplicity, we assume the probability of winning equals aggregate voting share. The party holding the majority of votes wins the election. Once the equilibrium platform is implemented, voters make labor decisions. We solve the model backwards.

## 2.1. Preferences

Voters are divided into 3 groups: *poor* ( $P$ ), *middle class* ( $M$ ) and *rich* ( $R$ ). Population size is normalized to one. We assume the proportion of voters in group  $P$  equals the proportion of voters in group  $R$ , which is  $\alpha$ . Thus the proportion of voters in group  $M$  is  $1-2\alpha$ . Groups are differentiated by their wage (ability)  $w_j$ , with  $j=P,M,R$ , such that  $0= w_P < w_M < w_R$ .

Total income of an individual in group  $j$  is  $y_j = w_j l_j$ , where  $l_j$  is labor effort chosen by voter  $j$ . Consumption equals after-tax income,  $c_j = y_j - T(y_j)$ , with  $T(y_j)$  being total tax payment by voter  $j$ .

We denote by  $U_j(c_j, l_j)$  the utility of a member of group  $j$  with consumption  $c_j$ , and labor supply  $l_j \in [0, L]$ , and assume that  $U_j$  is increasing in consumption ( $c_j$ ) and decreasing in labor ( $l_j$ ) which can be seen as labor effort or hours worked per week. For simplicity we assume that the utility function is quasilinear in consumption:  $U_j = u[c_j + v(L - l_j)]$ , and  $U_j$  is well behaved:  $u' > 0$ ,  $u'' < 0$ ,  $v' > 0$ , and  $v'' < 0$ . This utility specification will allow us to make straightforward comparison between outcomes of the two probabilistic voting models. We assume the wage elasticity of labor supply, defined as  $\epsilon_l = \frac{\partial l_j}{\partial w_j} \frac{w_j}{l_j}$ , decreases.

## 2.2. The Tax Schedule

Each group of voters  $j$  pays a marginal tax rate  $t_j$  on income and receives a lump-sum transfer  $G_j$ . All income tax collected finances a public good level,  $G$  and lump-sum transfers  $G_P$  and  $G_M$ , that favor groups  $P$  and  $M$ , respectively. It is assumed that  $G_R = 0$ . From the government budget condition:

$$G(\mathbf{t}, \mathbf{G}) = \sum_{j=P,R} \alpha_j t_j y_j + (1-2\alpha)t_M y_M - \alpha G_P - (1-2\alpha)G_M$$

Provided our normalization of wages, where  $w_P = 0$ , the tax rate paid by group  $P$  is  $t_P = 0$ . This reduces the dimensionality of the economic platform to  $\mathbf{t} = (t_M, t_R) \in T$ , with  $T : [0, 1] \times [0, 1]$  as the set of possible income tax rates and  $\mathbf{G} = (G_M, G_R) \in \mathfrak{R}_+^2$ . The tax schedule will be marginal rate progressive whenever the income tax rate increases with income. This means, in our particular case, that  $t_R - t_M > 0$ . Further conditions should be given to guarantee that indeed  $y_P \leq y_M \leq y_R$ . Given the disincentive effects of taxation we do not expect  $t_M$  or  $t_R$  to be larger than the income tax rates that maximize  $G$ . From the government budget balanced condition, the political struggle takes place between two tax parameters:  $(t_M, t_R)$ , and the lump-sum transfers  $(G_P, G_M)$ , we can express the level of public good as:

$$G(\mathbf{t}, \mathbf{G}) = (1-2\alpha)t_M y_M - \alpha t_R y_R - (\alpha)G_P - (1-2\alpha)G_M \quad (1)$$

Since labor is endogenously supplied, the tax schedule should satisfy the following condition:

$$y_R(\mathbf{t}) \geq y_M(\mathbf{t}) \geq y_P(\mathbf{t}) \quad (2)$$

In general, this condition is easier to satisfy the higher is the wage differential between groups  $R, M$  and  $M, P$ . Note that from preferences' quasi-linear specification, labor supply does not depend on  $G$ . Once the winning platform is in place, voters choose their before tax income, given the parameters of the tax function  $(\mathbf{t}, \mathbf{G})$ , this is equivalent to choosing their labor supply.

### 3. Optimal labor supply

Given the tax parameters  $(\mathbf{t}, \mathbf{G}, G)$ , voter- $j$  decides on consumption and labor supply:

$$\max_{c_j, l_j} U(c_j + v(L - l_j)) \quad \text{s.t.} \quad c_j \leq (1 - t_j)w_j l_j + G + G_j$$

The optimal labor supply:

$$\left( (1 - t_j)w_j - v'(L - l_j) \right) u'(x_j) = 0; \quad u'(x_j) > 0 \Leftrightarrow l_j = L - h\left( (1 - t_j)w_j \right)$$

where  $x_j = c_j + v(L - l_j)$  is consumption plus utility from leisure, and  $h = v'^{-1}$ . From concavity of  $v$  we know that  $h(\cdot)$  is decreasing in its argument. From the quasi-linear specification of  $x_j$  there are no income effects, which implies that  $\partial l_j / \partial t_j < 0$ ,  $\partial l_j / \partial w_j > 0$ , and  $\partial l_j / \partial G = 0$ . Assume all voters supply strictly positive units of labor, that is  $(1 - t_j)w_j - v'(L) > 0$  for all  $j$ , this will be the case, for example for  $v'(L) = 0$ .

Given this optimal labor supply the tax schedule feasibility constraint in (2) can be rewritten as:

$$\begin{aligned} L(w_R - w_M) &\geq w_R h\left( (1 - t_R)w_R \right) - w_M h\left( (1 - t_M)w_M \right) \\ L(w_M - w_P) &\geq w_M h\left( (1 - t_M)w_M \right) - w_P h(w_P) \end{aligned}$$

The above feasibility conditions give an upper bound to  $t_M$  and  $t_R$ ; which are increasing in  $(w_R - w_M)$  and  $(w_M - w_P)$ , respectively.

### 4. Preferences over tax schedules

Next we derive group-specific preferences over  $(\mathbf{t}, \mathbf{G})$ , given  $G(\mathbf{t}, \mathbf{G})$  specified in (1) with pre-tax incomes  $y_M = w_M(L - h(1 - t_M)w_M)$  and  $y_R = w_R(L - h(1 - t_R)w_R)$ .

The indirect utility of voter  $P$ ,

$$V_P(\mathbf{t}, \mathbf{G}) = u(G(\mathbf{t}, \mathbf{G}) + G_P) \tag{2}$$

Remember  $P$  does not pay income taxes. So  $P$  objective is to set  $(t_M, t_R)$  that maximizes the lump-sum transfer  $G_P$ , naturally,  $G_M = G = 0$ .  $G_P$  as a function of tax parameters becomes,

$$G_P(\mathbf{t}) = ((1 - 2\alpha) t_M y_M + \alpha t_R y_R) / \alpha$$

Assume that, <sup>2</sup>,

$$\frac{\partial^2 I_j}{\partial t_j^2} \leq 0, \text{ for } j = M, R$$

It is easy to show that  $G_P$  will be concave in  $(\mathbf{t})$  since provided that,

$$\frac{\partial^2 G}{\partial t_M^2} = (1 - 2\alpha) w_M \left( 2 \frac{\partial I_M}{\partial t_M} + t_M \frac{\partial^2 I_M}{\partial t_M^2} \right) < 0,$$

$$\frac{\partial^2 G}{\partial t_R^2} = \alpha w_R \left( 2 \frac{\partial I_R}{\partial t_R} + t_R \frac{\partial^2 I_R}{\partial t_R^2} \right) < 0, \quad \frac{\partial^2 G}{\partial t_M \partial t_R}$$

The f.o.c. for a maximum,

$$\hat{t}_M: I_M + t_M \frac{\partial I_M}{\partial t_M} = 0 \tag{3}$$

$$\hat{t}_R: I_R + t_R \frac{\partial I_R}{\partial t_R} = 0 \tag{4}$$

The preferred tax schedule of voter  $P$  is the peak of the Laffer curve.

Rearranging terms in (3) and (4), the tax schedule maximizing  $G$  satisfies,

$$|\epsilon_M| = |\epsilon_R| = 1 \tag{5}$$

Where  $\epsilon_j$  is the elasticity of labor supply to changes in the tax rate  $t_j$  with  $j=M,R$ .

$$|\epsilon_j| = \left| \frac{\partial I_j}{\partial t_j} \frac{t_j}{I_j} \right|$$

Whether voter  $P$  prefers a tax schedule that is proportional, marginal rate progressive or regressive depends upon the specific utility function. We assume that  $\partial^2 I_j / \partial t_j^2 \leq 0$ , so  $\epsilon_j$  will decrease with  $t_j^3$ . After some computations  $\epsilon_j = -\frac{t_j}{1-t_j} \epsilon_i$ . Provided that  $\epsilon_i$  is decreasing in  $w_j$ , by assumption,  $\epsilon_j$  will be increasing in  $w_j$ .

These properties together ensures that the tax schedule maximizing  $G_P$ , determined from equation (5), can not be marginal-rate regressive. To show this, consider  $\epsilon_j$  at the proportional tax rate  $t_M = t_R = t$ , since  $\epsilon_j$  is increasing in  $w_j$ , then  $\epsilon_M(t) \leq \epsilon_R(t)$ . If we increase  $t_M$  such that  $t_M > t_R = t$ , being  $\epsilon_j$  decreasing in  $t_j$  then, necessarily  $\epsilon_M(t_M) < \epsilon_M(t) \leq \epsilon_R(t)$ . Which proves that, the preferred tax schedule of voter  $P$  is either proportional ( $t_M = t_R$ ) or progressive ( $t_M < t_R$ ).

Group  $M$  pays income taxes at rate  $t_M$  and receives  $G_M + G$ . Thus, group  $M$  preferred income tax minimizes his tax burden. The indirect utility of a voter  $M$  is given by,

$$V_M(\mathbf{t}, \mathbf{G}) = u\left((1 - t_M)w_M l_M + G(t_M, t_R, G_P, G_M) + G_M\right)$$

The indirect utility  $V_M(\mathbf{t}, \mathbf{G})$  reaches a (global) maximum at  $(t_M, t_R) = (0, \hat{t}_R)$  and

$$G_M = \frac{\alpha}{1 - 2\alpha} \hat{t}_R y_R(\hat{t}_R).$$

Notice that  $\hat{t}_R$  maximizes  $G(\mathbf{t}, \mathbf{G})$  for a given  $t_M$ . Such a tax schedule is marginal rate progressive since  $t_R - t_M = \hat{t}_R > 0$ .

Group  $R$  pays income taxes at rate  $t_R$  and receives the lump-sum transfer  $G$ . Thus group  $R$  preferred income tax minimizes his tax burden. Voter  $R$  indirect utility function is given by,

$$V_R(\mathbf{t}, \mathbf{G}) = u\left((1 - t_R)w_R l_R + G(t_M, t_R, G_P, G_M)\right)$$

Her utility is maximized at the regressive tax schedule:  $t_M = \hat{t}_M$  and  $t_R = 0$ . Remember that  $\hat{t}_M$  maximizes  $G(\mathbf{t}, \mathbf{G})$  for a given  $t_R$ . Naturally, he prefers to set  $G_P = G_M = 0$ .

The following picture shows voters' bliss-points<sup>4</sup>.

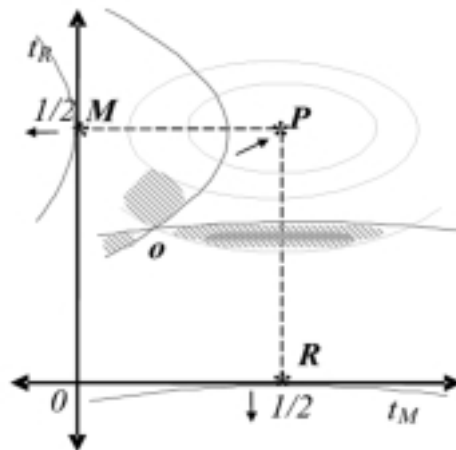


Figure 1. P, M and R's bliss points

It should be noted that no CW winner exists in our voting game, as shown in figure 1. Any point in the rectangle  $ORPM$  can be defeated by a coalition of two groups. The shaded areas in figure 1 represent the alternatives that can defeat alternative “o”, which can be defeated by other alternatives generating a cycle (the voting paradox).

Next section describes how political competition takes place in both probabilistic voting models and studies conditions for tax progressivity in equilibrium.

## 5. The Probabilistic Voting Model

### 5.1. CN Probabilistic voting model

The probabilistic voting model developed by Coughlin relaxes one of the assumptions of the traditional Downsian model: candidates' certainty of voters' reactions in response to their platforms.

In Coughlin and Nitzan (1981) and Coughlin (1992), even after voters have learned the proposals of both candidates in the race, candidates are uncertain how voters will act on election day. This would also be the case if voters' decisions were stochastic in nature.

Two possible interpretations of the Coughlin model are that voters do not vote deterministically but they are still rational: they are more likely to vote for the candidate whose policy platform brings them the highest utility.

This raises the question why voters do not vote in consonance with their economic preferences. This leads to the second interpretation of the Coughlin model, where voters do indeed vote deterministically but there are other issues apart from economic ones, so they may not vote for the party that promises them the best economic platform. Here voters are ideological.

Candidates use a logit model to infer voters' selection probabilities. In an economy with  $J$  voters, the probability of a voter from group  $j$  voting for candidate  $A$  is equal to the relative utility that  $j$  derives from  $A$ 's platform compared to the utility derived from  $B$ 's platform,

$$\pi_j^A(\mathbf{t}^A, \mathbf{G}^A; \mathbf{t}^B, \mathbf{G}^B) = \frac{V_j(\mathbf{t}^A, \mathbf{G}^A)}{V_j(\mathbf{t}^A, \mathbf{G}^A) + V_j(\mathbf{t}^B, \mathbf{G}^B)}$$

We shall, by convenient abuse of notation, generally denote  $\pi_j^C(\mathbf{t}^C, \mathbf{G}^C; \mathbf{t}^C, \mathbf{G}^{-C}) = \pi_j^C$ , with  $C=A, B$ . Note that voters do not abstain, they either vote for  $A$  or  $B$ , so  $\pi_j^A + \pi_j^B = 1$ . The higher the economic utility of platform  $A$ , the higher the probability that group  $j$  (or a representative voter from group  $j$ ) will vote for  $A$ . Parties are office-motivated. They simultaneously choose the policy platform that maximizes  $\pi^C = \sum_j \pi_j^C$  with  $C=A, B$ . Among the main findings of the probabilistic voting model, we can cite the following:

1. Equilibrium existence and uniqueness. There exists equilibrium (a saddle point to  $\pi^C(\cdot)$ ) as long as  $V_j(\mathbf{t}^C, \mathbf{G}^C)$  is quasiconcave on  $(\mathbf{t}^C, \mathbf{G}^C)$ . Note this is a multidimensional problem and a CW may not exist.
2. Policy convergence. Both candidates face a similar problem  $\pi^A(\mathbf{t}^A, \mathbf{G}^A; \mathbf{t}^B, \mathbf{G}^B)$  for  $A$  and  $1 - \pi^A(\mathbf{t}^A, \mathbf{G}^A; \mathbf{t}^B, \mathbf{G}^B)$  for  $B$ . This implies that they both choose the same policy platform and they have 50% probability of winning.
3. The outcome of the political competition game is the social alternative that maximizes a Nash social welfare function (Theorem 1 of Coughlin and Nitzan, 1981).



Lindbeck and Weibull give a microfundation to (generalize) Coughlin's model. They introduced ideology explicitly. A voter may cast their ballot in favor of a candidate that brings him a lower economic utility if the utility from the non-economic issue outweighs the economic loss. Although voters may indeed vote deterministically, the LW model is called a probabilistic voting model because of its close links with Coughlin's model.

Next we describe and follow LW approach; we will discuss afterwards how our predictions change if we follow Coughlin.

## 5.2. LW Probabilistic voting model

There is a continuum of voters in each group differing in their ideological position, measured as their relative preference from one party over another. In order to combine the economic and ideological side of voters' utility we assume that voter  $i$  in group  $j$  will vote for party  $A$  if the extra "economic" utility he gets from  $A$ 's platform outweighs his ideological preference for  $B$  as opposed to  $A$ . We can capture this preference for a party in parameter  $\sigma_{ij}$ , which represents the location of individual  $i$  from group  $j$  along the real line. If  $\sigma_{ij}$  is positive (negative),  $i$  in group  $j$  prefers  $B$  to  $A$  ( $A$  to  $B$ ) for the same announced platform. Voters with a  $\sigma_{ij}$  of around zero are ideologically neutral; they mainly evaluate the economic benefits they receive from the different platforms proposed by parties.

The utility of a  $ij$ -voter is simply  $V_j(\mathbf{t}^A, \mathbf{G}^A)$  if party  $A$  wins and it is  $V_j(\mathbf{t}^B, \mathbf{G}^B) + \sigma_{ij}$  otherwise.

A voter  $i$  in group  $j$  will vote for  $A$  if:

$$V_j(\mathbf{t}^A, \mathbf{G}^A) > V_j(\mathbf{t}^B, \mathbf{G}^B) + \sigma_{ij} \quad (6)$$

We assume that  $\sigma_{ij}$  has a group-specific cumulative distribution function  $F_j$  with density  $f_j$  and support on the real line. The density function around zero summarizes how many ideologically neutral voters we have in each group. We define the swing voter in group  $j$  as the voter that is indifferent between party  $A$  and party  $B$  given the platforms announced. Let's call it  $\sigma_j$ , from (6),  $\sigma_j = V(\mathbf{t}^A, \mathbf{G}^A) - V(\mathbf{t}^B, \mathbf{G}^B)$ . Voters in group  $j$  with an ideological parameter,  $\sigma_{ij}$ , below (or above)  $\sigma_j$  will vote for party  $A$  (respectively  $B$ ). We assume there is no abstention.

The total share of votes, and therefore the probability of winning of party  $A$  in group  $j$ ,  $\pi_j^A$ :

$$\pi_j^A = n_j \Pr(\sigma_{ij} < \sigma_j) = n_j F_j(\sigma_j).$$

Where  $n_j$  is the proportion of voters in group  $j$ .

Therefore, the probability of winning of party  $A$  is:

$$\pi^A = \alpha F_P(\sigma_P) + (1 - 2\alpha)F_M(\sigma_M) + \alpha F_R(\sigma_R) \quad (7)$$

We next introduce conditions that guarantee existence of a unique pure-strategy Nash equilibrium of the electoral game. These conditions, from Lindbeck and Weibull (1987), Enelow and Hinich (1989) and Coughlin (1992), were unified by Banks and Duggan (2004). In addition, we need to add an extra condition, since in our model,  $F_j$  is not independent of  $j$ .

Conditions On  $F_j$  and aggregate  $V_j$ :

- 1) C1.  $F_j$  is continuous and strictly increasing.
- 2) C2. Aggregate concavity holds, for any  $\mathbf{t}^C$ ,  $\pi^C(\mathbf{t}^A, \mathbf{t}^B)$  is strictly concave on  $\mathbf{t}^C$ ,  $C = A, B$ :

$$\pi^A = (\mathbf{t}^A, \mathbf{G}^A; \mathbf{t}^B, \mathbf{G}^B) = \sum_{j=PM,R} n_j F_j \left( V_j(\mathbf{t}^A, \mathbf{G}^A) - V_j(\mathbf{t}^B, \mathbf{G}^B) \right)$$

$$\pi^B = (\mathbf{t}^A, \mathbf{G}^A; \mathbf{t}^B, \mathbf{G}^B) = 1 - \sum_{j=PM,R} n_j F_j \left( V_j(\mathbf{t}^A, \mathbf{G}^A) - V_j(\mathbf{t}^B, \mathbf{G}^B) \right)$$

- 3) C3. Laussel and Le Breton (2002). This guarantees that there is no profitable deviation at the political equilibrium of this game.

$\forall_j f_j$  is symmetric around zero and  $f_j(0) > 0$ .

The main difference between CN and LW is how voters' ideology determines their voting decisions. It is easy to show that CN is as if a voter  $i$  in group  $j$  with ideological parameter  $\sigma_{ij}$  favors party  $A$  whenever  $(1 - \sigma_{ij})V_j(\mathbf{t}^A, \mathbf{G}^A) > \sigma_{ij}V_j(\mathbf{t}^B, \mathbf{G}^B)$  and  $\sigma_{ij}$  is distributed uniformly on the interval  $[0, 1]$ .

Next we present some results for the LW game and compare to CN results afterwards. We assume for LW that  $u(x_j)$  is logarithmic with  $x_j = c_j + v(L - l_j)$ .

Without loss of generality we write the probability of winning of party  $A$  simply as  $\pi(\mathbf{t}, \mathbf{G}; \mathbf{t}^B, \mathbf{G}^B)$ , where  $(\mathbf{t}, \mathbf{G})$  is the platform chosen by party  $A$ , and  $(\mathbf{t}^B, \mathbf{G}^B)$  the platform chosen by  $B$ .

**Lemma 1.** Assume conditions C1, C2 and C3 are satisfied. The bliss points of group  $P$ ,  $M$  and  $R$  are never part of a political equilibrium.

**Proof:**

- a) The bliss point of  $P$  is not an equilibrium. If it were an equilibrium, then for a given,

$$\left( \mathbf{t}^B, \mathbf{G}^B \right), \frac{\partial \pi(\mathbf{t}^A, \mathbf{t}^B)}{\partial \mathbf{t}_M} d\mathbf{t}_M + \frac{\partial \pi(\mathbf{t}^A, \mathbf{t}^B)}{\partial \mathbf{t}_R} d\mathbf{t}_R \Big|_{\mathbf{t}_M = \hat{\mathbf{t}}_M, \mathbf{t}_R = \hat{\mathbf{t}}_R} = 0 \text{ for } d\mathbf{t}_M, d\mathbf{t}_R < 0.$$

We next show that  $\pi(\mathbf{t}, \mathbf{G}; \mathbf{t}^B, \mathbf{G}^B)$  actually increases as we move from  $(\hat{\mathbf{t}}_M, \hat{\mathbf{t}}_R)$  to lower tax rates.

$$\frac{\partial \pi(\mathbf{t}, \mathbf{G}; \mathbf{t}^B, \mathbf{G}^B)}{\partial t_M} \Big|_{t_M = \hat{t}_M, t_R = \hat{t}_R} = (1 - 2\alpha) f_M(\sigma_M) \left( \frac{1}{x_M} \frac{\partial x_M}{\partial t_M} \right) \Big|_{t_M = \hat{t}_M} < 0$$

$$\frac{\partial \pi(\mathbf{t}, \mathbf{G}; \mathbf{t}^B, \mathbf{G}^B)}{\partial t_M} \Big|_{t_M = \hat{t}_M, t_R = \hat{t}_R} = (1 - 2\alpha) f_M(\sigma_M) \left( \frac{1}{x_M} \frac{\partial x_M}{\partial t_M} \right) \Big|_{t_M = \hat{t}_M} < 0$$

$$\frac{\partial \pi(\mathbf{t}, \mathbf{G}; \mathbf{t}^B, \mathbf{G}^B)}{\partial t_R} \Big|_{t_M = \hat{t}_M, t_R = \hat{t}_R} = \alpha f_R(\sigma_R) \left( \frac{1}{x_R} \frac{\partial x_R}{\partial t_R} \right) \Big|_{t_R = \hat{t}_R} < 0$$

which proves that the income tax preferred by group P is not an equilibrium.

b) The bliss point of M is not an equilibrium.

$$\frac{\partial \pi(\mathbf{t}^A, \mathbf{G}^A; \mathbf{t}^B, \mathbf{G}^B)}{\partial t_M} \Big|_{t_M = 0, t_R = \hat{t}_R} = (1 - 2\alpha) f_M(\sigma_M) \left( \frac{1}{x_M} \frac{\partial x_M}{\partial t_M} \right) \Big|_{t_M = 0} > 0$$

$$\frac{\partial \pi(\mathbf{t}^A, \mathbf{G}^A; \mathbf{t}^B, \mathbf{G}^B)}{\partial t_R} \Big|_{t_M = 0, t_R = \hat{t}_R} = \alpha f_R(\sigma_R) \left( \frac{1}{x_R} \frac{\partial x_R}{\partial t_R} \right) \Big|_{t_R = \hat{t}_R} < 0$$

which proves that an increase in  $t_M$  and a decrease in  $t_R$ , from M's preferred platform, improves party A chances of winning. Therefore, the income tax preferred by group M can not be part of equilibrium.

c) The bliss-point of R is not an equilibrium.

$$\frac{\partial \pi(\mathbf{t}^A, \mathbf{G}^A; \mathbf{t}^B, \mathbf{G}^B)}{\partial t_M} \Big|_{t_M = \hat{t}_M, t_R = 0} = (1 - 2\alpha) f_M(\sigma_M) \left( \frac{1}{x_M} \frac{\partial x_M}{\partial t_M} \right) \Big|_{t_M = \hat{t}_M} < 0$$

$$\frac{\partial \pi(\mathbf{t}^A, \mathbf{G}^A; \mathbf{t}^B, \mathbf{G}^B)}{\partial t_R} \Big|_{t_M = \hat{t}_M, t_R = 0} = \alpha f_R(\sigma_R) \left( \frac{1}{x_R} \frac{\partial x_R}{\partial t_R} \right) \Big|_{t_R = 0} > 0$$

which proves that the income tax preferred by group R is not an equilibrium.

The above Lemma proves that, under this political process, the outcome will never correspond with any group's ideal income tax schedule. This is because the probabilistic model implies some compromise among voters. This explains the fact that none of the ideal income tax schedules constitutes an equilibrium. The fact that the probabilistic model in a multidimensional space picks a policy that is different from a voter ideal has already been stressed in Casamatta, Cremer and Pestieau (2006).

**Proposition 1.** Assume C1, C2 and C3 are satisfied. Assume that for all  $t_R \leq \hat{t}_R$  and  $t_M \leq \hat{t}_M$  the feasibility constraint in (2) is satisfied. There exists a unique interior equilibrium. By symmetry of the game, in equilibrium we have policy coincidence,  $\mathbf{t}^A = \mathbf{t}^B = \mathbf{t}$  and  $\mathbf{G}^A = \mathbf{G}^B = \mathbf{G}$  with  $t_M < \hat{t}_M$  and  $t_R < \hat{t}_R$ .

**Proof.** Uniqueness comes from the fact that we maximize a strictly concave function (C2) under a convex set. By symmetry of the model if  $(\mathbf{t}^A, \mathbf{G}^A; \mathbf{t}^B, \mathbf{G}^B)$  is an equilibrium so it is  $(\mathbf{t}^B, \mathbf{G}^B; \mathbf{t}^A, \mathbf{G}^A)$ , and from uniqueness  $\mathbf{t}^A = \mathbf{t}^B = \mathbf{t}$  and  $\mathbf{G}^A = \mathbf{G}^B = \mathbf{G}$ . If (2) is not binding, parties choose  $(\mathbf{t}^C, \mathbf{G}^C)$  that maximize a weighted sum of voters utilities (Lindbeck

and Weibull, 1987). Thus, no tax  $t_j > \hat{t}_j$  with  $j=M, R$ ; will be chosen in equilibrium since it is Pareto dominated. From Lemma 1, we know that the equilibrium tax schedule is different from  $(\hat{t}_M, \hat{t}_R)$ , the bliss-point for group  $P$ . Note that the feasibility constraint in (2) can be omitted (will not be binding) as long as  $\tilde{t}_R \geq \hat{t}_j$  and  $\tilde{t}_M \geq \hat{t}_M$  where  $\tilde{t}_R$  is the lowest possible  $t_R$  satisfying (2) (remember that  $y_j$  is decreasing in  $t_j$ ):  $y_R(\tilde{t}_R) - y_M(0) = 0$ . Similarly for  $t_M$ ,  $\tilde{t}_M$ :  $y_M(\tilde{t}) - y_P = 0$ . Moreover  $\tilde{t}_R, \hat{t}_M$  are higher the higher is  $w_R$  and the lower is  $w_P$ .

From policy coincidence, note that the outcome of the LW game, from the maximization of (6), is equivalent to the result of the maximization of an Utilitarian Social Welfare Function  $S^{LW}$ , with weights (or *political power*) on voters  $P$ ,  $M$ , and  $R$  utility given by  $\alpha f_P(0)$ ,  $(1-2\alpha) f_M(0)$  and  $\alpha f_R(0)$ , respectively.

$$S^{LW}(\mathbf{t}, \mathbf{G}) = \alpha f_P V_P(\mathbf{t}, \mathbf{G}) + (1-2\alpha) f_M V_M(\mathbf{t}, \mathbf{G}) + \alpha f_R V_R(\mathbf{t}, \mathbf{G})$$

Where  $f_j = f_j(0)$ , for  $j = P, M, R$ .

Since we are interested in tax progressivity, next we develop conditions under which a progressive income tax emerges as an outcome of the LW game. Note that our assumption regarding the elasticity of labor supply (we assume that  $\varepsilon_l$  decreases with  $w$ ) facilitates the implementation of a progressive income tax, since group  $R$ 's labor *response* to changes in the marginal tax rate they pay is lower than that of group  $M$ . Moreover, the decreasing marginal utility of consumption (net of labor disincentives) also facilitates the emergence of a progressive income tax schedule, since it increases the political power of groups  $P$  and  $M$  compared to group  $R$ . Despite all this a proportional or even marginal-rate regressive income tax may arise in equilibrium if the proportion of ideologically neutral voters in group  $R$ ,  $f_R$  is sufficiently high.

The equilibrium income tax satisfies the following first-order conditions,

$$\Phi(t_M, t_R) \left(1 - |\varepsilon_M(t_M)|\right) x_M(t_M, t_R) - f_M = 0$$

$$\Phi(t_M, t_R) \left(1 - |\varepsilon_R(t_R)|\right) x_R(t_M, t_R) - f_R = 0$$

$$\text{Where } \Phi(t_M, t_R) = \frac{\alpha f_P}{x_P(t_M, t_R)} + \frac{(1-2\alpha) f_M}{x_M(t_M, t_R)} + \frac{\alpha f_R}{x_R(t_M, t_R)}$$

The tax schedule will be marginal-rate progressive if, with a proportional tax schedule, there is a profitable deviation to a more progressive schedule (with a higher  $t_R$  or/and lower  $t_M$ ). From the uniqueness of the equilibrium, this would imply that the equilibrium income tax schedule can not be proportional or regressive since there would not be a profitable deviation from moving toward a regressive tax (lower  $t_R$ ). The condition is,

$$\frac{(1 - |\varepsilon_M(t)| x_M(t, t))}{(1 - |\varepsilon_R(t)| x_R(t, t))} < \frac{f_M}{f_R}$$

Since  $\varepsilon_j$  is increasing in  $w_j$  the expression  $(1+\varepsilon_M(t)/1+\varepsilon_R(t)) < 1$ . Since  $x_j$  is increasing in  $w_j$  ( $\partial x_j/\partial w_j = (1-t_j)l_j > 0$ ), which implies that  $x_M(t,t)/x_R(t,t) < 1$ . If  $f_M \geq f_R$ , only marginal-rate progressive taxes will emerge in equilibrium (note that this is stronger than needed).

**Proposition 2.** *The equilibrium income tax is marginal-rate progressive as long as the inequality below holds,*

$$\frac{(1-|\varepsilon_M(t)|)x_M(t,t)}{(1-|\varepsilon_R(t)|)x_R(t,t)} < \frac{f_M}{f_R} \quad (8)$$

**Proof.** *If, from the proportional tax, a progressive tax is a profitable deviation for Party A, then  $[(\Phi(t,t)(1-|\varepsilon_M(t)|)x_M(t,t)-f_M)dt_M + (\Phi(t,t)(1-|\varepsilon_R(t)|)x_R(t,t)-f_R)dt_R] > 0$  for  $dt_M < 0$  and  $dt_R > 0$ . Dividing both sides by the RHS of the expression in brackets and rearranging terms we find the condition for progressivity (8) in the LW game.*

We now follow the approach of Coughlin (1992), from Coughlin and Nitzan (1981) we know that the outcome of the electoral competition game is the social alternative that maximizes a Nash social welfare function. For simplicity, we assume that  $V_j(t_M, t_R) = x_j(t_M, t_R)$ .

The party's objective function is then,

$$S^{\text{CN}}(\mathbf{t}, \mathbf{G}) = \alpha \ln x_P(\mathbf{t}, \mathbf{G}) + (1-2\alpha) \ln x_M(\mathbf{t}, \mathbf{G}) + \alpha \ln x_R(\mathbf{t}, \mathbf{G})$$

In this game the political power of the poor and the rich is the same, the first prefers a progressive (or proportional) income tax and the last a regressive tax, while voter  $M$  unambiguously prefers a progressive income tax, hence only marginally-rate progressive taxes emerge in equilibrium.

**Proposition 3.** *If the elasticity of labor supply,  $\varepsilon_i$ , decreases with  $w$ , only marginal-rate progressive taxes emerge in equilibrium.*

**Proof.** *Note that the CN game is equivalent to the LW game for  $F_j$  independent of  $j$  (this was previously stressed by Banks et al., 2004). In such a case  $f_j = f$ , i.e.  $f_M = f_R$ . As proved in Proposition 2, this is a sufficient condition for marginal-rate progressive taxes.*

When voters choose a candidate (party) probabilistically the best response for both candidates is to announce a marginal-rate progressive tax schedule. This is because when competing in elections, they try to attract *swing* voters: the ones that, through a slight increase in consumption, are much more likely to vote for the party. The probability that a group votes for a given party, say  $A$ , is concave in their consumption level (being  $\ln V_j$  a proxy of the probability of them voting for the party). Then, voters in group  $P$  and  $M$  are more attractive than voters in group  $R$ , since they increase faster the probability to vote for the party that benefits them. Under our assumption of decreasing wage elasticity of labor supply, the preferred tax schedule of  $P$  is either proportional or progressive. This guarantees that a move from regressivity or proportionality toward progressivity is profitable. It captures more votes by swing voters, since the marginal gain in increased consumption for group  $M$  is higher than for group  $R$ .

## 6. Comparative Statics

We wonder at this point how the degree of progressivity changes as a result of changes in the parameters of the model. Assume (2) is satisfied at the solution of  $S^{CN}:(\mathbf{t}^*, \mathbf{G}^*)$ .

**Proposition 4. 1.** *In the CN game, an increase in the polarization of the population, measured by  $\alpha$ , decreases the progressivity degree.*

**2.** *In the LW game; an increase in  $f_M$  ( $f_R$ ) decreases the equilibrium tax rate  $t_M$  ( $t_R$ ). The lump-sum transfer to the poor,  $G_P$ , rises as  $f_P$  increases.*

**Proof.** *At the equilibrium tax schedule,*

$$\frac{\partial \pi}{\partial t_M} = \alpha \left( \frac{1}{x_P} \frac{\partial x_P}{\partial t_M} + \frac{1}{x_R} \frac{\partial x_R}{\partial t_M} \right) + (1-2\alpha) \left( \frac{1}{x_M} \frac{\partial x_M}{\partial t_M} \right) = 0 \quad (9)$$

$$\frac{\partial \pi}{\partial t_R} = \alpha \left( \frac{1}{x_P} \frac{\partial x_P}{\partial t_R} + \frac{1}{x_R} \frac{\partial x_R}{\partial t_R} \right) + (1-2\alpha) \left( \frac{1}{x_M} \frac{\partial x_M}{\partial t_R} \right) = 0$$

*We know that for  $t_M^* = 0$ ,  $\partial x_M / \partial t_M < 0$ . So,*

$$\left( \frac{1}{x_P} \frac{\partial x_P}{\partial t_M} + \frac{1}{x_R} \frac{\partial x_R}{\partial t_M} \right) > 0$$

*to satisfy equation (9). While for*

$$t_R^* < \hat{t}_R, \frac{\partial x_M}{\partial t_R} > 0$$

*So, necessarily*

$$\left( \frac{1}{x_P} \frac{\partial x_P}{\partial t_R} + \frac{1}{x_R} \frac{\partial x_R}{\partial t_R} \right) > 0$$

*to satisfy equation (9). Consider now a different economy with  $\alpha' > \alpha$ . Concerning the second equation in the first-order condition in (9), at  $(t_M^*, t_R^*)$  it can be easily shown that a higher  $\alpha$  gives a higher weight to the negative part and a lower weight to the positive part  $\partial x_M / \partial t_R$ . Then at  $(t_M^*, t_R^*)$ ,*

$$\frac{\partial \pi}{\partial t_R} = \alpha' \left( \frac{1}{x_P} \frac{\partial x_P}{\partial t_R} + \frac{1}{x_R} \frac{\partial x_R}{\partial t_R} \right) + (1-2\alpha') \left( \frac{1}{x_M} \frac{\partial x_M}{\partial t_R} \right) < 0$$

*From concavity of  $\pi(\cdot)$ , this implies that the equilibrium tax rate  $t_R$  at the economy  $\alpha'$  is lower than  $t_R^*(\alpha)$ , i.e.  $t_R^*(\alpha') < t_R^*(\alpha)$ . Following the same logic, at  $(t_M^*, t_R^*)$ ,  $\partial \pi / \partial t_M > 0$  for  $\alpha' > \alpha$ ,*

$$\frac{\partial \pi}{\partial t_M} = \alpha' \left( \frac{1}{x_P} \frac{\partial x_P}{\partial t_M} + \frac{1}{x_R} \frac{\partial x_R}{\partial t_M} \right) + (1-2\alpha') \left( \frac{1}{x_M} \frac{\partial x_M}{\partial t_M} \right) > 0$$

*From concavity of  $\pi(\cdot)$ , this implies that the equilibrium tax rate  $t_M$  at the economy  $\alpha'$  is higher than  $t_M^*(\alpha)$ , i.e.  $t_M^*(\alpha') > t_M^*(\alpha)$ . Finally,*

$$t_R^*(\alpha') < t_R^*(\alpha) \text{ and } t_M^*(\alpha') > t_M^*(\alpha) \Rightarrow t_R^*(\alpha') - t_M^*(\alpha') < t_R^*(\alpha) - t_M^*(\alpha)$$

*Progressivity falls as the population becomes more polarized.*

2. In the LW model applying the implicit function theorem to the first order conditions we study how  $t_M$  changes in response to a change in  $f_M$ ,

$$\frac{\partial t_M}{\partial f_M} = - \left( \frac{\partial \Phi}{\partial f_M} (1 - |\varepsilon_M(t_M)|)^{x_M(t, G)} - 1 \right) / D$$

$$\text{where } D = \left( \frac{\partial \Phi}{\partial t_M} (1 - |\varepsilon_M(t_M)|)^{x_M(t, G)} + \Phi \frac{\partial \varepsilon_M}{\partial t_M} x_M(t, G) + \Phi (1 - |\varepsilon_M(t_M)|) \frac{\partial x_M}{\partial t_M} \right) < 0$$

as a result of the concavity of  $\pi(\mathbf{t}, \mathbf{G})$ . Substituting

$$\frac{\partial \Phi}{\partial f_M} = \frac{(1 - 2\alpha)}{x_M(t_M, t_R)}$$

in the above equation:

$$\text{sgn} \left( \frac{\partial t_M}{\partial f_M} \right) = \text{sgn} \left( (1 - 2\alpha) (1 - |\varepsilon_M(t_M)|) - 1 \right) < 0$$

Similarly for  $t_R$ ,

$$\frac{\partial t_R}{\partial f_R} = - \left( \frac{\partial \Phi}{\partial f_R} (1 - |\varepsilon_R(t_R)|)^{x_R(t_M, t_R)} - 1 \right) / D,$$

$$\text{sgn} \left( \frac{\partial t_R}{\partial f_R} \right) = \text{sgn} \left( \alpha (1 - |\varepsilon_R(t_R)|) - 1 \right) < 0.$$

*The degree of progressivity decreases with  $f_R$ , since  $\partial \Phi / \partial f_R = \alpha / x_R(t_M, t_R) > 0$  and more generally  $\text{sgn}(\partial t_M / \partial f_j) = \text{sgn}(\partial \Phi / \partial f_j) > 0$ ,  $j = P, R$ . Finally the lump-sum transfer level  $G_P$  increases with  $f_P$ , the political power of the group whose preferred tax schedule coincides with the peak of the Laffer curve. Note that since  $G_M$  and  $G$  does not benefit group  $P$ , both decrease with  $f_P$ .*

The previous result states that the degree of progressivity decreases with the political power of group  $M$ , the group whose preferred tax is maximal progressivity. Note that this happens irrespective of our assumption on the elasticity of labor supply. In CN the size of a group measures his political power, thus regardless of how population is distributed among groups, tax progressivity increases as the size of group  $M$  increases (or population polarization decreases). This is the case in both models. Finally, in the LW model the lump-sum transfer,  $G_P$ , increases as the political power of group  $P$  rises.

## 7. Conclusions

With this simple model, we wanted to show that even with the preferences specification and only substitution effects from taxation, only marginal-rate progressive taxes will constitute a political equilibrium in the CN model. Our assumption regarding the elasticity of labor

supply (that  $\varepsilon_l$  decreases with  $w$ ), which is satisfied by several familiar utility specifications, makes it easier to introduce a marginal-rate progressive tax and makes middle-class voters ( $M$ ) more likely to become swing voters. We could say that provided that we wanted to show that despite there is only substitution effects from taxation, only marginal rate progressive taxes will emerge as the political equilibrium for the CN game. Our assumption on the elasticity of labor supply (that  $\varepsilon_l$  is decreasing in marginal wage) is crucial for our result. It facilitates the implementation of marginal progressive taxes in both models (CN and LW) with respect to the fixed or exogenous income case. Indeed the condition on Proposition 1 is harder to satisfy at  $\varepsilon_l = 0$ :  $x_M(t, t)/x_R(t, t) < f_M/f_R$  (condition in Proposition 1 for progressivity at the fixed income case). In this context, labor disincentives make the wealthy group cheaper to tax than the middle class.

The second point to highlight is that inequality is not one-dimensional. We also have to think about the polarization of population groups, political power and about wage differences. Tax progressivity decreases with greater population polarization in CN model and, by extension, progressivity increases as the middle classes' political power grows in the LW game. A larger degree of marginal progressivity is expected in societies with a stronger middle class.

## Notes

1. Note that if we restrict the policy space to tax functions ordered by Lorenz dominance, a single parameter is enough to describe whether the tax schedule is marginal-rate progressive. In this case, after-tax income can be represented as  $x_i = (y_i)^{1-\tau} (\bar{y})^\tau$ , where  $\bar{y}$  is common to all agents (it is determined so that average post-tax income equals per-capita income),  $y_i$  is pre-tax income; and  $\tau$  is the tax parameter, the tax schedule is progressive ( $0 \leq \tau \leq 1$ ), or regressive if ( $\tau \leq 0$ ). See for instance Bénabou (2000).
2. Utility functions satisfying this assumption and the assumption on the elasticity of labor supply are:  $v(l_j) = -1/2 (l_j)^2$ ,  $v(L - l_j) = \sqrt{(L - l_j)}$ .
3. Given that there are only substitution effects from taxation and by assumption the second derivative of  $l_j$  with respect to  $t_j$  is negative,  $\varepsilon_j$  will be decreasing in  $t_j$ :

$$\frac{\partial \varepsilon_j}{\partial t_j} = \frac{\partial^2 l_j}{\partial t_j^2} t_j + \frac{\partial l_j}{\partial t_j} \left( \frac{1_j - \frac{\partial l_j}{\partial t_j} t_j}{(l_j)^2} \right) < 0.$$

4. The plot was made for the particular utility function  $U_j = c_j - 1/2 l_j^2$ . For this utility function  $\hat{t}_M = \hat{t}_R = 1/2$ .

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## Resumen

El propósito de este trabajo es mostrar bajo qué condiciones un impuesto sobre la renta de tipo marginal progresivo surge como consecuencia de la competencia entre dos partidos políticos, cuando la oferta de trabajo es elástica y los candidatos no conocen con certeza las decisiones de voto de los electores. Suponiendo que la elasticidad renta de la oferta de trabajo es decreciente, si seguimos a Coughlin y Nitzan (1981), ambos candidatos escogerán únicamente impuestos de tipo marginal progresivo en equilibrio. Si, adoptamos más bien el modelo probabilístico de Lindbeck y Weibull (1987), el esquema impositivo de equilibrio será progresivo si el poder político del votante con mayor renta es suficientemente bajo. El grado de progresividad disminuye con la polarización de la población.

*Palabras clave:* economía política, impuestos progresivos, oferta de trabajo elástica.

*Clasificación JEL:* D3, D63, D72, H24.