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Credit Mismatch and Breakdown

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Abstract: This paper studies the phenomenon of mismatch in a decentralized credit market where borrowers and lenders must engage in costly search to establish credit relationships. Our dynamic general equilibrium framework integrates incentive based informational frictions with a matching process highlighted by (i) borrowers' endogenous market entry and exit decision (entry frictions) and (ii) time and resource costs necessary to locate credit opportunities (search frictions). A key feature of the incentive compatible loan contract negotiated between borrowers and lenders is the interaction of informational frictions (in the form of moral hazard) with entry and search frictions. We find that the removal of entry barriers can eliminate information-based equilibrium credit rationing. More generally, entry and incentive frictions are important in understanding the extent of credit rationing, while entry and search frictions are important for understanding credit market breakdown.

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Time runs a close second to *cash* on every entrepreneur's list of scarce resources. (W. E. Wetzel, Jr., *The Portable MBA in Entrepreneurship*, Wiley & Sons: New York, 1997, p. 185)

1 Introduction

Credit markets are capricious and susceptible to occasional breakdown. A more commonly observed phenomenon that accompanies credit market distress is the mismatch between borrowers and lenders: there is *both* an excess demand for *and* an excess supply of loanable funds.[1] For example, Kanoh and Pumpaisanchai (2006) find the coexistence of excess supply of bank loans to large and medium firms and excess demand by small firms in Japan over the past decade. Moreover, Hwang, Jiang and Wang (2007) document excessive lending to nonperforming chaebols (large conglomerates) in Korea, whereas similar problems have arisen more recently in the U.S. subprime mortage markets in which excessive loans have been directed to low quality borrowers. In the presence of credit mismatch, funds remain under-used despite the existence of potentially profitable but unexploited investment opportunities. This not only generates undesirable social inefficiencies, but also increases the liklihood of credit-market breakdown.

Despite their important macroeconomic consequences and policy implications, these aforementioned credit-market issues have not been thoroughly studied within an integrated theoretical framwork. In particular, most explanations for the turbulence of credit markets that have been advanced in the literature are essentially based on theories of asymmetric information regarding borrowers' types or unobserved actions. Yet such informational asymmetries focus only on the unfulfilled demand for credit. They, by themselves, can neither account for credit mismatch nor the role mismatch plays in determining credit-market participation and tightness (measured by the difficulty of both borrowers and lenders in establishing credit relationships) as well as the likelihood of breakdown.

To explore these issues we consider an environment in which informational frictions (of the type emphasized in the credit rationing literature) coexist with the frictions associated with matching borrowers and lenders in a decentralized loan market.[2] This dynamic matching framework is con-

[1] One may think of the episode delineated in William Shakespeare's *The Merchant of Venice* as an example of credit mismatch. Specifically, the loan provided by the money lender (Shylock, who practices the unpopular "usury") to the merchant (Antonio) remains idle (to back up his friend, Bassanio, in his pursue a highly demanding girl, Portia), leaving potentially productive borrowers unfunded.

[2] While not mutually exclusive, the difference between the two frictions is that the former emphasizes the limits on

sistent with three observations that are prevalent even in well-functioning creit markets. First, there is a continuous flow into and out of the credit market by borrowing firms, mostly small and medium-sized, that face entry barriers (entry frictions). Second, borrowing firms must race against time as they search for funding opportunities while lenders also must invest time and resources to convert their idle funds into active investments (search frictions). Third, part of the costs incurred in identifying viable credit relationships are information-based; lenders may face considerable risk for they cannot be sure of the borrowing firms' investment outcomes (informational frictions in the form of moral hazard).

While locating sources of funding has been recognized as a paramount problem facing exitsting and would-be entrepreneurs, as documented by Blanchflower and Oswald (1998), only recently has it been explicitly recognized that the localized and heterogenous nature of the market for information-intensive loanable funds shares many characteristics of a search market.[3] Our paper contributes to a small but growing literature that advocates a search theoretic framework to model credit market activity. For example, Craig and Haubrich (2006) argues that the credit market may have "as strong a claim to search frictions as the labor market." Complementing the empirical work of Dell'Arriccia and Garibaldi (2005), they document evidence regarding gross credit flows via loan creation and destruction and the entry and exit of banks. They further demonstrate that a simple search theoretic model of lending may capture the gross lending flows evident in the data. Within the search theoretic framework, Den Haan, Ramey, and Watson (2003) illustrate how liquidity shocks are propagated and amplified in credit markets when credit relationships can be renegotiated. While Acemoglu (2001) and Wasmer and Weil (2004) study how credit market imperfections interact with labor market frictions, Silveira and Wright (2006) analyze the match between venture capitalists and entrepreneurs and characterize equilibrium returns. Two studies that most resembles ours are Diamond (1990) and Becsi, Li, and Wang (2005). Diamond characterizes bilateral exchange in credit markets with exogenously fixed matching probabilities and populations of borrowers and lenders. Our earlier work focuses on examining how the entry of heterogeneous borrowers can affect the aggregate composition of borrowers under perfect information. In the present paper, we not only endogenize matching probabilities and market participation, but also include informational frictions. Under this generalized setup, we show that endogenous entry can interact with search,

knowledge and the latter emphasizes the limits on time.

[3]For example, search costs on the part of lenders can be loosely thought of as encompassing the time element of *ex ante* screening in the spirit of Boyd and Prescott (1986). Alternatively, localized creditors tend to specialize in a particular type of lending and must sort through the various heterogenous investment opportunities to identify the most suitable match.

matching and incentive frictions to affect the extent of credit rationing as well as the possibility of credit mismatch and breakdown.

The basic structure we propose is one in which borrowers and lenders choose whether or not to participate in a decentralized loan market where search for bilateral credit relationships is costly. Some participating borrowers and lenders form credit relationships that enable financing of investment projects which yield a productive rate of return. These returns are divided up between the lender in the form of an interest payment and the borrower in the form of residual profits. The incentive frictions that arise from asymmetric information allow the possibility of borrowers to abscond with the borrowed funds subject to a default cost. Hence, when loan contracts are negotiated they must be incentive compatible to overcome this moral hazard problem and credit rationing, where borrowers receive fewer funds than desired, may emerge endogenously. In equilibrium, optimal loan contracts and the extent of credit rationing are determined jointly with market liquidity (aggregate lending), borrower market participation (endogenous entry), and the tightness of the credit market (the excess demand for loans as measured by the ratio of unmatched borrowers to unmatched lenders). Hence, the interaction of entry, search, and informational frictions provides a rich structure with which to capture the flow of credit along both the intensive and extensive margins.

We find that the nature of the optimal incentive-compatible contract in equilibrium varies with aggregate fundamentals as represented by borrower productivity. If productivity falls below a threshold that is determined by default costs, credit markets break down and cease to exist. When productivity begins to exceed this threshold, an active credit market arises but firms will be rationed unless lenders are sufficiently patient. Even high productivity may be associated with rationing if the rate of time preference is sufficiently low or the length of the loan contract period is sufficiently short. Intuitively, a low rate of time preference raises the present discounted value of absconding with borrowed funds and a short contract duration lowers the value of the match for a borrower who must search again after the match is dissolved. Also, we find that credit market tightness and the extent of credit rationing are positively related when entry is exogenous. However, with endogenous entry, there generally does not exist a monotonic relationship between the two. That is, credit may continue to be rationed in a market where it is relatively easy for borrowers to locate lenders if high equilibrium interest rates induce firm exit out of the loanable funds market. Finally, we show that incentive frictions and entry frictions are important in determining the extent of credit rationing, while entry and search frictions are important for understanding the likelihood of credit market breakdown. Moreover, eliminating entry barriers can completely rule-out information-based credit rationing in equilibrium, while search frictions continue to affect whether or not credit markets fail. Intuitively, competition by banks weakens their bargaining position and increases the extent of credit rationing by reducing their ability to dictate loan contract terms.

2 The Basic Environment

Time is continuous. There are two types of economic agents, those endowed with resources ("lenders") and those endowed with an "investment" technology which uses those resources to generate a positive return ("borrowers"). Our model focuses on the loanable funds market where available funds provided by a continuum of lenders are channeled to a continuum of potential borrowers through a decentralized credit market. Let N^L and N^B represent the mass of lenders and borrowers in this environment. For convenience, we normalize the measure of lenders to unity. Lenders and borrowers (bilaterally) meet with each other for the purpose of establishing a credit relationship and the ***matching technology*** that brings borrowers and lenders together is given by:

$$m = m_0 M(N_u^L, N_u^B) \tag{1}$$

where m measures flow matches, $m_0 > 0$ indicates the efficacy of credit market matching, and M is strictly increasing and concave, satisfying the constant-returns-to-scale property, the standard Inada conditions, and the boundary conditions $M(0,\cdot) = M(\cdot,0) = 0$. We define N_u^i to be the number of unmatched agents and N_m^i as the number of matched agents of type i where $i = L, B$. By normalizing the mass of lenders to unity, we have: $N_u^L + N_m^L = 1$. A central feature of the loan market highlighted by our model is that market liquidity is determined by credit market tightness. Given the populations of lenders and borrowers, a measure of ***credit market tightness*** in our setup is given by the ratio of unmatched lenders to unmatched borrowers: $\tau \equiv \frac{N_u^B}{N_u^L}$. Intuitively, if τ is high, then there are many potential borrowers relative to lenders with idle funds. Because it is more difficult for borrowers to locate potential lenders under these circumstances, we say that the credit market is "tight" from the perspecticve of borrowers.[4]

Utility generated from consumption is assumed to be linear for both types of agents. Since the focus of this paper is on how credit market frictions and market liquidity affect credit arrangements between borrowers and lenders rather than the intertemporal consumption and saving decisions of

[4] In the appendix, we provide an alternative measure of credit market tightness – the "capital unemployment rate." We show that the capital unemployment rate is a monotone increasing function of τ; thus, our arguments remain valid under this alternative measure.

households, this simplifying assumption is adopted without loss of generality for our purpose. An unmatched lender consumes his flow endowment ω as he searches for borrowers with whom to trade this endowment for the promise of a future payment. A borrower begins the search period with only his investment technology and searches for potential lenders to finance their project. Lenders contact borrowers at a rate of μ, while borrowers contact lenders at a rate of η. Due to asymmetric information about the borrower's behavior, the lender is unsure about whether the borrower will invest in a productive project or abscond (take the money and run). Thus, once a borrower and a lender meet, the lender will set an incentive compatible loan contract to prevent the borrower from absconding with the funds. The contract specifies a gross interest payment, R, and the fraction of available funds actually lent out, $q \leq 1$. When this loan contract is established, the lender gives up a portion of the endowment, $q\omega$, to the borrower and consumes the residual portion $(1-q)\omega$ while waiting for the end of the contract period. In the meantime the borrowed funds are used to implement production which provides the borrower a flow return of $Aq\omega$ where $A > 0$ represents the exogenous level of productivity. The contract period ends when borrowers and lenders are separated at which time the lender pays the borrower an amount $Rq\omega$ (i.e., loan repayment). The exogenous separation rate is given by δ, and hence the length of the contract period is given by $1/\delta$ and the corresponding ***gross interest rate*** is given by δR. After both members of the match become separated, they re-enter the pool of unmatched borrowers and lenders and again search for credit opportunities.

If borrowers in this model know with certainty that the loan will be repaid, our preferences imply that it will be optimal for the lender to set $q = 1$ and lend all of the endowment to the borrower in exchange for the future payment. However, due to unobserved motives, borrowers in our model may choose to default on the loan and abscond without repayment. When this occurs, the defaulter bears two costs. First, we assume that the defaulter is excluded from any future credit transactions. Second, we assume that the borrower must forfeit a real resource cost that is measured as a fraction θ of total loanable funds. This cost is meant to capture the outside penalty of default and may represent legal or institutional features. A legal or monitoring system is considered to be tougher when the default cost higher. This ***moral hazard*** feature is what may cause loanable funds to be ***rationed*** (i.e., $q < 1$). That is, lenders will use this quantity rationing feature of the loan contract so as to insure incentive compatibility and repayment.

We can now characterize the dynamic problem facing borrowers and lenders in our economy. Let J_u and J_m denote the lender's value associated with being in the unmatched (u) and matched

(m) states. These asset values can be expressed as:

$$rJ_u = \omega + \mu\,(J_m - J_u) \tag{2}$$

$$rJ_m = (1-q)\omega + \delta[Rq\omega + (J_u - J_m)] \tag{3}$$

where $r > 0$ is the rate of time preference. Equation (2) says that the flow value associated with an unmatched lender is the flow of consumption from his endowment and arrival rate of borrowers times the net value gained when a loan contract is implemented and the match is formed. Equation (3) says that the flow value associated with a matched lender is the flow of consumption of the residual endowment and the rate at which the contract expires times the interest payment and net value of returning to the unmatched pool.

Similarly, let Π_u and Π_m denote the borrowers's value associated with being in the unmatched and matched states, respectively. Their asset values in the two states are:

$$r\Pi_u = \eta\,(\Pi_m - \Pi_u) \tag{4}$$

$$r\Pi_m = Aq\omega + \delta[-Rq\omega + (\Pi_u - \Pi_m)] \tag{5}$$

Equation (4) simply states that the flow value associated with an unmatched borrower is the rate at which they contact lenders times the net value gained when becoming matched with a lender. Equation (5) says that the flow value associated with a matched borrower is the stream of returns the borrower obtains from implementing the investment project and the value associated with separation which occurs at rate δ. When this occurs, the borrower makes the interest payment $Rq\omega$, gains the state of returning to the unmatched borrowers' pool, and looses the state of being a matched borrower.[5]

Subtracting (2) from (3) gives us the lenders' value of being matched relative to being unmatched as:

$$J_m - J_u = \frac{(\delta R - 1)\,q\omega}{r + \delta + \mu} \tag{6}$$

Notice that equation (6) implies that a necessary condition for an active loan market requires $J_m - J_u > 0$ or $\delta R > 1$. Otherwise, the economy will degenerate into an autarkic state where no credit activity occurs. We will assume that this condition holds.

[5] For simplicity we have assumed that borrowers do not have any assets. More generally, if borrowers also have an asset ω^B, equation (3) must be modified to $r\Pi_u = \omega^B + \eta\,(\Pi_m - \Pi_u)$. If these assets are jointly productive with the endowment of the lender then equation (4) changes to $r\Pi_m = A(q\omega + \omega^B) + \delta[-Rq\omega + (\Pi_u - \Pi_m)]$. Another possibility (that is also beyond the scope of this paper) is that borrower's assets could be used as collateral for loans. For a discussion of these and related issues see, for instance, Hart and Moore (1994).

Similarly, subtracting (4) from (5) gives us the borrowers' value of being matched relative to being unmatched as:

$$\Pi_m - \Pi_u = \frac{(A - \delta R)\, q\omega}{r + \delta + \eta} \tag{7}$$

We note that the relative values given by (6) and (7) are aggregate expressions. However, each individual borrower is atomistic and take their unmatched value Π_u as given when evaluating their matched value. If we take this into account, we can rewrite (5) as:

$$\Pi_m = \frac{(A - \delta R)\, q\omega + \delta \Pi_u}{r + \delta} \tag{8}$$

In the presence of the moral hazard problem, a loan contract must be ***incentive compatible*** to eliminate borrowers' default in equilibrium. In the benchmark setup, we follow Sappington (1983), Benerjee and Newman (1993) and Fender and Wang (2003), assuming that the moral hazard problem take the form of absconding. This means that incentive competibility is met if the value associated with being a matched firm is at least as large as the value associated with taking the funds and absconding:

$$(1 - \theta)\frac{q\omega}{r} \leq \Pi_m \tag{9}$$

where the left hand side of (9) gives the present discounted value of absconding as the discounted value of the funds borrowed net of the cost expressed as a fraction θ of the loan and the penalty for defaulting is permanent exclusion from the credit market.[6] Substitution of (7) into this above inequality (9) gives,

$$q\omega \left[(1 - \theta)(\frac{r + \delta}{r}) - (A - \delta R) \right] \leq \delta \Pi_u \tag{10}$$

From (10) we see that an increase in the loan interest rate δR or an increase in the total quantity of the loan $q\omega$ increases the likelihood of absconding. An incentive compatible loan contract is defined as a pair (q, R) such that (10) is satisfied.

When borrowers and lenders meet, they bargain over the terms of the contract. We will assume that the outcome of this bargaining game is consistent with the Nash bargaining solution where the contract (q, R) is designed to maximize the joint surplus of the funds suppliers and demanders $S = (J_m - J_u)^{1/2}(\Pi_m - \Pi_u)^{1/2}$, subject to the incentive compatibility constraint (10), $q \in [0, 1]$ and $R \geq 0$. Using (6) and (7), maximization of the joint surplus implies $\frac{dS}{dq} > 0$ and, from $\frac{dS}{dR} = 0$, one

[6] There are two interesting alternative setups for the default process. First borrowers can use the loanable funds to produce before absconding, and second is where credit market participation is permitted after payment of the default penalty. We consider these alternative specifications later in Section 7.2 and compare the results with our benchmark setup.

obtains:

$$\delta R q\omega = (Aq\omega - r\Pi_u)\Gamma + (1-\Gamma) \tag{11}$$

where $\Gamma \equiv \frac{r+\delta+\mu}{2(r+\delta)+\mu}$ is monotone increasing function of μ. Notice that if the pair (q, R) is not incentive constrained (that is, the incentive compatibility constraint is not binding), then it is always optimal for the lender to loan out his entire endowment: $q = 1$.

3 Characterization of the Loan Contract

We proceed by describing the optimal loan contract in the presence of incentive frictions that cause a moral hazard problem. The optimal contract must be incentive compatible and satisfy the Nash bargaining condition. Both conditions can be represented in an intuitive graphical fashion. The following provides a formal definition of the optimal loan contract.

Definition 1. *An **optimal loan contract** is a pair (R, q) such that*

(i) *$q = 1$ and R solves (11) if (10) does not bind, or otherwise,*

(ii) *$q < 1$ and R solves (11) and (10) with equality.*

We define this latter case as a situation where the optimal loan contract is characterized by credit rationing. Defining $Q \equiv q\omega$, we can then express the optimal loan contract in terms of the amount of rationed funds and the gross interest rate, $(Q, \delta R)$, by rewriting equations (11) and (10) as:

$$\delta R = \left(A - \frac{r\Pi_u}{Q}\right)\Gamma + (1-\Gamma) \tag{12}$$

$$\delta R \leq B + \frac{\delta\Pi_u}{Q} \tag{13}$$

where $B \equiv A - (1-\theta)\frac{r+\delta}{r}$.

The determination of the optimal incentive compatible loan contract can be characterized graphically in $(Q, \delta R)$ space with the origin defined as $Q = 0$ and $\delta R = 1$. We construct the graph in three steps.

First, we plot the surplus maximization condition (12) and call it the ***SM locus***. This locus is upward-sloping and concave with a horizontal intercept $Q_{SM} = \frac{r\Pi_u}{A-1}$ and slope $\frac{\Gamma r\Pi_u}{Q^2}$. An optimal loan contract must be along this SM locus. The slope of the SM locus can be interpreted in terms of the bargaining power of borrowers versus lenders. For example, a steep SM curve, resulting from a faster arrival rate of borrowers, μ, or long loan contract period, low δ, increases the bargaining power of lenders. Hence lenders can demand a higher interest rate (δR) for a given increase in the loan quantity Q and this implies a steeper SM locus.

Second, we plot the incentive compatibility condition (13) with equality in $(Q, \delta R)$ space and call it the ***IC locus***. This locus is downward-sloping and convex to the origin such that $\lim_{Q \longrightarrow 0} \delta R = \infty$ and $\lim_{Q \longrightarrow \infty} \delta R = B$ and has slope $\frac{-\delta \Pi_u}{Q^2}$. Also, for $B < 1$, IC has a horizontal intercept at $Q_{IC} = \frac{\delta \Pi_u}{1-B}$. Any $(Q, \delta R)$ in the area below the IC locus satisfies the incentive compatibility constraint.

Third is the property that an optimal loan contract must always have q as large as possible within the feasible range $[0, 1]$. Formally, we can plot $Q = \omega$ as the upper bound for q. This means that if an optimal loan contract exists, it must be on the part of the SM locus that is below the IC locus and to the left of the $Q = \omega$ locus with the highest q. The existence of an active loanable funds market for all $\delta R > 1$ requires the condition that $A > 1$ and this is satisfied for a sufficiently productive economy.

Figure 1 segments our characterization of the optimal loan contract into three cases, depending on the relative position of the SM and IC loci. Case I indicates a situation when the SM locus is everywhere below IC. Hence all combinations of $(Q, \delta R)$ along the SM locus are incentive compatible and the optimal loan contract is the one with the highest value of q, implying $Q^* = \omega$ and the absence of credit rationing (see point E).

When the IC locus crosses the SM locus as in Case II, the amount of funds available in the economy is central for characterizing the optimal incentive compatible loan contract. When the amount of funds available is low (say, $\omega = \omega^L$), the optimal loan contract represented by $(Q^L, \delta R^L)$ is at point E^L where credit rationing is absent (i.e., $q = 1$). However, when the amount of funds available is high with $\omega = \omega^H$, the incentive compatibility constraint is now binding. As a consequence, there will be a unique optimal loan contract represented by $(Q^H, \delta R^H)$. In this case credit rationing emerges because the highest q attainable is strictly less than one (see point E^H where $Q^H < \omega^H$). Intuitively, loaning out a higher quantity of available funds increases the incentives for the borrower to abscond and leads to a binding incentive compatibility constraint. Since credit rationing only occurs conditionally, depending on a sufficiently large endowment, we will refer to this as a case of "conditional" credit rationing.

Finally, Case III shows that if the IC locus crosses the horizontal axis at a point lower than SM, there exists no incentive compatible combination of $(Q, \delta R)$ and no underwriting of a loan contract. This is the "non-active" credit market outcome and is comparable to a ***credit market breakdown***. After completely characterizing the steady-state equilibrium, we will return to a more detailed analysis of the conditions consistent with each of these possible equilibrium outcomes.

4 Steady-State Equilibrium

The previous section discussed the properties of incentive compatible optimal loan contracts given the rates by which borrowers and lenders are matched. We now close the model by characterizing the steady state process by which lenders and borrowers meet. This will in turn pin down the equilibrium contact rates by which agents are matched, the steady state population of matched and unmatched borrowers and lenders, and hence equilibrium credit market tightness.

The flow of lenders into the state of being matched is given by μN_u^L and the flow of borrowers into the matched state is given by ηN_u^B. In equilibrium, the flow of funds supplied must be equal to the flow of funds demanded:

$$\mu N_u^L = \eta N_u^B = m \tag{14}$$

Recalling the credit market tightness measure $\tau \equiv \frac{N_u^B}{N_u^L}$, we can use (1) and the constant returns property of the matching technology to rewrite the steady state condition in terms of our tightness measure:

$$\mu = \eta\tau = m_0 M(1, \tau) \tag{15}$$

It is straightforward to show that (15) implies μ is increasing in τ whereas,

$$\eta \equiv \eta(\tau) = m_0 M(\frac{1}{\tau}, 1) \tag{16}$$

is decreasing in τ; moreover, both μ and η are increasing in m_0, $\frac{\mu}{\eta} = \tau$ (independent of m_0), $\lim_{\tau\to 0} \mu(\tau) = 0$ and $\lim_{\tau\to\infty} \eta(\tau) = 0$. This relationship is often referred to as the ***Beveridge curve*** in the search equilibrium literature. For labor markets the curve relates the unemployment rate to the vacancy rate (or establishes a relationship between the associated flow contact rates), while for credit markets we relate the capital unemployment rate to a measure of how much idle funds there are in the system. We are now able to define a steady state loanable funds equilibrium under exogenous and endogenous entry.

Definition 2. *Given credit market tightness τ, a* ***steady-state loanable funds equilibrium with exogenous entry*** *of borrowers is a tuple $(Q^*, \delta R^*, \mu^*, \eta^*)$ satisfying:*

(i) *the optimal incentive compatible loan contract (12) and (13), and*

(ii) *the Beveridge curve relationship (15) and (16).*

Finally, we consider a loanable funds equilibrium with endogenous entry. From (7) and (8), we can eliminate Π_m to obtain the unmatched value facing each potential borrower:

$$\Pi_u = \frac{\eta}{r+\delta+\eta}\frac{(A-\delta R)Q}{r} \tag{17}$$

There is a large mass of potential borrowers. Borrower entry into the loan market is determined by assuming each borrower faces a fixed cost v for setting up the investment technology. By ***equilibrium entry,*** borrowers enter into the unmatched pool of borrowers until their unmatched value is driven down to the entry cost, or,

$$\Pi_u = v \tag{18}$$

Using the Beveridge curve relationship, we can substitute (17) into (18) to yield:

$$\frac{\eta(\tau)}{r+\delta+\eta(\tau)}(AQ-\delta RQ) = rv \tag{19}$$

or, after substituting in (12) and $\frac{\mu}{\eta} = \tau$,

$$Q = \frac{rv}{A-1}\left(1+\tau+\frac{2\,(r+\delta)}{\eta(\tau)}\right) \tag{20}$$

This is referred to as the ex ante zero profit (ZP) locus, which is strictly increasing and strictly concave in (Q, τ) space with a horizontal intercept $\frac{rv}{A-1}$.[7]

Definition 3. *A **steady-state loanable funds equilibrium with endogenous entry** of borrowers is a triplet $(Q^*, \delta R^*, \tau^*)$ that satisfies:*

(i) *the optimal incentive compatible loan contract (12) and (13), and*

(ii) *the Beveridge curve relationship and ex ante zero profit condition given by (20).*

Figure 2 provides an illustration of this steady-state equilibrium with endogenous entry by combining Case II from Figure 1 in the top panel with the ZP locus in the bottom panel (whereby we note that this locus has the same horizontal intercept as the SM locus). Thus, a loanable funds equilibrium is determined in a recursive manner. The optimal incentive compatible contract pins down the equilibrium $(Q^*, \delta R^*)$. Then the ZP locus determines equilibrium entry and hence the market tightness measure τ^* that is consistent with the optimal contract.

[7] A sufficient condition for the concavity of the ZP locus is given by the envelope condition $\frac{(1/\tau)M_{HH}}{M_H} > -2$. For example, this is trivially satisfied when the matching technology $M(\cdot)$ is Cobb-Douglas.

Once this triplet is determined, it is straightforward to derive the steady state populations of matched and unmatched borrowers and lenders. From (16) and (15), we have η^* and μ^*, respectively. Because the inflow of unmatched lenders being matched must equal the outflow of matched lenders being separated: $\mu^* N_u^L = \delta N_m^L$. Substituting the population identity, $N_u^L + N_m^L = 1$, into this equilibrium flow condition gives:

$$N_m^{L*} = \frac{\mu^*}{\delta + \mu^*} = N_m^{B*} \quad \text{and} \quad N_u^{L*} = \frac{\delta}{\delta + \mu^*} \tag{21}$$

From this it is easy to see that the equilibrium number of unmatched borrowers is $N_u^{B*} = \tau^* N_u^{L*}$. It is straightforward to verify that when a loanable funds equilibrium exists, it is unique. Hence, in general, not only does the optimal loan contract determine market tightness and liquidity, but market tightness in turn also affects the optimal loan contract.

5 Equilibrium Credit Rationing and Market Breakdown

We now analyze the properties of the steady-state loanable funds equilibrium. Unless otherwise specified, the following applies to both equilibrium with exogenous and endogenous entry. We identify the conditions that are consistent with existence of the steady state and differentiate between three possible regimes. Specifically, based on the equations underlying Figure 2, we have:

Proposition 1. (*Steady-state Loanable Funds Equilibrium*) *A steady-state loanable funds equilibrium features one of the following three outcomes:*

(i) *If* $(A-1) < (1-\theta)$, *then there does not exist an incentive compatible loan contract and the loan market is non-active (Case III).*

(ii) *If* $(1-\theta) \le (A-1) < 2(1-\theta)$ *and* ω *is sufficiently high, then there exists a credit rationing equilibrium with* $q < 1$ *(Case II).*

(iii) *If* $2(1-\theta) \le (A-1)$ *and* ω *is sufficiently high, then,*

  a. *there exists a credit rationing equilibrium with* $q < 1$ *for* $r < \overline{r} \equiv \frac{(1-\theta)[2\delta+\mu]}{(A-1)-2(1-\theta)}$ *(Case II);*

  b. *the incentive compatibility constraint never binds and the equilibrium loan contract is not rationed for* $r \ge \overline{r}$ *(Case I).*

Proof: See Appendix. □

Proposition 1 outlines the region of the parameter space consistent with the various possible equilibrium outcomes discussed in the previous section. This Proposition has a very intuitive interpretation and can be divided into three cases as illustrated by Figure 1. Loosely, if the productivity of the investment project is sufficiently low relative to the incentive friction, then there will always the incentive to abscond for any loan contract along the SM locus. In this case (Case III), there is no active loanable funds equilibrium and the loan market breaks down. Once productivity begins to exceed a threshold level (Case II), lenders begin channeling loanable funds to borrowers, but the quantity is rationed. Finally, if productivity is sufficiently high, then whether or not there is rationing can be expressed in terms of the rate of time preference. In particular, if the rate of time preference is sufficiently small, then the present discounted value of consumption generated from absconding for the borrower becomes greater than the value of being matched in the loanable funds market. Lenders must continue to ration loans so that the incentive compatibility binds (Case II). If, on the other hand, the rate of time preference is very large, there will never be an incentive for the borrower to abscond and all loan contracts are incentive compatible (Case I).

Proposition 1 establishes that credit rationing of productive firms depends on a threshold $\overline{r}$. Next, we consider the underlying changes to this threshold that make credit rationing more likely.

Proposition 2. (*Duration of Loan Contract*) *Suppose that the investment project is sufficiently productive such that* $(A-1) > 2(1-\theta)$. *Then in a steady-state loanable funds equilibrium, an increase in the duration of the loan contract (lower* δ*) leads to an increase in the set of incentive compatible contracts and an increase in the loan market equilibrium interest rate (*δR*). If the latter effect dominates the former, credit rationing is likely to occur.*

Proof: Observe that $\lim_{Q\to\infty} \delta R_{SM}$ and $\lim_{Q\to\infty} \delta R_{IC}$ are decreasing in δ. $\square$

A longer contract duration (low δ) increases the set of incentive compatible contracts since the borrower can enjoy the productive benefits supported by the loanable funds for a greater period of time. This is captured by an upward shift of the IC locus. However, a longer contract duration also makes the match more valuable to the borrower and biases the bargaining power towards lenders. Consequently, the SM locus shifts upwards as well. In Case II, the market equilibrium loan rate increases in both the case where there is no rationing ($\omega{=}\omega^L$) and when there is rationing ($\omega{=}\omega^H$). Whether or not credit rationing is more likely depends upon whether the bargaining effect dominates the incentive compatibility effect. If the bargaining effect dominates, then the a longer contract period both increases rationing and the equilibrium loan rate.

We next study the linkage between the tightness of the credit market and credit rationing in an exogenous entry equilibrium and one where borrower entry is endogenized. In particular, we find:

Proposition 3. (*Market Tightness versus Credit Rationing*)

(i) *In an exogenous entry equilibrium satisfying Definition 2, there is a positive relationship between market tightness (τ) and the likelihood of credit rationing. For all $r > 0$, there exists $\tau < \infty$ sufficiently large such that equilibrium credit rationing will occur.*

(ii) *In an endogenous entry equilibrium satisfying Definition 3, there is no necessary monotonic relationship between credit market tightness and the extent of credit rationing.*

Part (i) follows directly from Proposition 1. From (15) an (exogenous) increase in τ increases the frequency at which lenders meet borrowers, μ. Since $\frac{\partial \overline{r}}{\partial \mu} > 0$ it follows that an increase in market tightness expands the set of feasible rates of time preference consistent with the credit rationing equilibrium. As τ becomes arbitrarily large, $\overline{r} \to \infty$. To see the intuition behind part (i) suppose that the initial steady state equilibrium is given by Case I. In this case, every loan contract that maximizes the joint match surplus of borrowers and lenders is incentive compatible. The Beveridge curve relationship given by (15) implies that an increase in market tightness increases the rate that lenders contact borrowers. This increases the threat point and bargaining power of lenders when negotiating the loan contract. As a result, the SM locus shifts upwards and this shrinks the set of Q and δR combinations consistent with incentive compatibility. If the increase in market tightness is sufficiently large, IC will eventually intersect with the SM locus and credit will begin to be rationed at a higher equilibrium interest rate. Thus, we are more likely to see credit rationed in an illiquid credit market where it is difficult for borrowers to find loan opportunities.

To illustrate part (ii), consider the following comparative steady state analysis of an increase in funds matching efficacy (m_0) that improves matching for both lenders and borrowers in Case II. This is illustrated in Figure 3. Under our funds matching framework, a rise of m_0 raises the effective contact rate of funds suppliers Γ and strengthens their bargaining power. As a consequence, joint surplus maximization grants relatively higher returns to the suppliers, implying an increase in R for each given value Q. That is, the SM locus rotates upwards. From the ZP relationship, an increase in m_o will raise the matching rate $\eta(\tau)$ of borrowers. For a given Q determined by the optimal loan contract, more potential borrowers enter and hence the loan market becomes tighter. That is, the ZP locus twists toward the vertical axis.

For the case of $\omega = \omega^L$ where the incentive compatibility constraint is not binding, the equilibrium loan rate rises as market participation (or tightness) increases. For the case of $\omega = \omega^H$, rationing increases in response to the increased entry of potential borrowers and a higher equilibrium loan rate is required to satisfy incentive compatibility. However, the increased severity of credit rationing reduces potential borrowers' expected profit, thereby decreasing their entry. Due to this latter opposing effect, the net change in the tightness of the loan market is ambiguous. Hence, an observed increase in credit rationing need not imply increased tightness in the credit market.

6 The Role of Market Frictions

We next explicitly consider the impact of search frictions (finite m_0), market entry frictions (v), and incentive frictions (θ) on the structure of optimal lending arrangements and steady state equilibrium in the loanable funds market. Each of these frictions is considered separately so that their relative contributions to explaining credit rationing and market breakdown can be isolated and analyzed.

6.1 Search Frictions

Without search frictions, credit market participants do not have to wait to set up a credit arrangement. When matches are instantaneous, we have

Proposition 4. (*Search Frictions*) *In the absence of search frictions, the equilibrium loan contract and the existence of equilibrium credit rationing are independent of market tightness.*

Proof: The absence of search frictions occurs in the limiting case where $m_0 \longrightarrow \infty$. The Beveridge curve relation implies that while μ and $\eta \longrightarrow \infty$, $\tau \equiv \mu/\eta$ will remain bounded by the constant returns to scale property of the matching technology. Using these and $\lim_{m_0 \longrightarrow \infty} \Gamma = 1$, the steady state equilibrium conditions (12), (13), and (20) are now given by $\delta R = A - \frac{rv}{Q}$, $\delta R \leq B + \frac{\delta v}{Q}$, and $Q = \frac{rv}{A-1}(1+\tau)$, respectively. Since τ no longer appears in the SM and IC loci, the optimal loan contract is independent of market thickness. $\square$

This result says that search frictions are crucial for a linkage between market tightness, the optimal loan contract, and credit rationing. If borrowers and lenders instantaneously match and enter into a lending agreement, their relative bargaining position will not be affected by the tightness of the market. In this situation the only equilibrium outcomes are the conditional (Case II) and non-active (Case III) steady states. Because of the entry costs on the borrower's side, a reduction in search frictions increases the relative ease with which lenders locate borrowers. As in Proposition 3, this causes an upward shift in the SM locus. As the loan equilibrium interest rate rises, lenders must ration in order to keep the contract incentive compatible. The absence of search frictions in general equilibrium can be seen as a limiting case of this sequence of these events.

6.2 Entry Frictions

Here we consider costless entry of borrowers into the credit market. Under these circumstances, the demand for funds is perfectly elastic and we establish:

Proposition 5. (*Entry Frictions*) *In the absence of entry frictions, there does not exist a credit rationing equilibrium. Equilibrium in the loan market is characterized by either an active no rationing loanable funds market or a non-active loan market.*

Proof: The absence of firm entry frictions occurs in the limiting case where $v \to 0$. Since there is now unrestricted borrower entry, the steady state conditions described (12), (13) are now given by $\delta R = \Gamma(A-1)+1$, $\delta R \leq B$. Existence of an active loanable funds market requires $\Gamma(A-1)+1 < B = A-(1-\theta)(\frac{r+\delta}{r})$. Satisfaction of this condition implies that all combinations of $(Q, \delta R)$ along the (horizontal) SM locus is incentive compatible and there is no credit rationing, $q = 1$. $\square$

To obtain intuition behind this result, suppose that the initial steady state equilibrium is given by the active no rationing equilibrium of Case I. In this situation, $B > \Gamma(A-1)+1$. Lower entry frictions encourages the entry of borrowers and this drives down their unmatched value. This rotates the SM locus clockwise as lenders take advantage of their increased bargaining power. At the same time, a lower unmatched value for the borrower must be compensated by a decrease in the loan rate or loan quantity to maintain incentive compatibility. In the limiting case as these costs vanish, the SM and IC loci are horizontal with SM everywhere below IC. No contract that offers an interest rate above B will be incentive compatible and no interest rate above $\Gamma(A-1)+1$ will be consistent with the optimal loan contract. Hence, there will only be an active no rationing equilibrium.

To show that credit rationing disappears with free entry, suppose that the initial steady state equilibrium is given by the conditional equilibrium of Case II. In this situation, $B < \Gamma(A-1)+1$. Lower entry frictions encourages the entry of borrowers and this drives down their unmatched value. As lenders take advantage of their increased bargaining power, the interest rate consistent with the optimal loan contract rises and the SM locus rotates clockwise. In the limiting case as entry costs vanish, the SM locus is horizontal and the interest rate approaches its maximum value given by $\Gamma(A-1)+1$. At the same time, a lower unmatched value for the borrower must be compensated by a decrease in the loan rate or loan quantity to maintain incentive compatibility and this is captured by a counterclockwise rotation of the IC locus. In the limiting case as these entry costs vanish, the IC locus is horizontal as no contract that offers an interest rate above B will be incentive compatible. Hence, Case II degenerates to a non-active equilibrium where no optimal loan contract will be incentive compatible and the credit market fails to function.

Furthermore, recall that Case I captures an active loan market equilibrium with no credit rationing only if the potential supply of loanable funds (ω) is sufficiently large. If ω is small then there may not be any positive rate of interest consistent with the optimal loan contract. However, as entry costs for borrowers are driven to zero, there will emerge an active non-rationing credit market equilibrium. If, on the other hand, the economy was initially characterized by the conditional Case II, possibly with credit rationing, then a removal of entry barriers will lead to a breakdown of the credit market and non-existence. This situation corresponds to a problem of "over-crowding" in entry of borrowers with limited loanable funds supply.

6.3 Incentive Frictions

Finally, we investigate the role of moral hazard for credit arrangements in the decentralized market. We detail how the equilibrium is affected if the cost of absconding is driven down to zero.

Proposition 6. (*Incentive Frictions*) *In the absence of incentive frictions, there only exists an active, no rationing loanable funds market equilibrium.*

Proof: The absence of incentive frictions corresponds to the limiting case where the costs of absconding as a fraction of total funds, $\theta \to 1$. It is immediate from Proposition 1 that for any given $A > 1$, we can rule out (i) the non-active equilibrium and (ii) the credit rationing equilibrium with $(A-1) \in (1-\theta, 2(1-\theta)) = \emptyset$. In the case where $(A-1) > 2(1-\theta)$, $\overline{r} = \frac{(1-\theta)[2\delta+\mu]}{(A-1)-2(1-\theta)} = 0$. Hence, all $r > 0$ satisfies $r > \overline{r}$ and in this case there exists the active, no rationing equilibrium. $\square$

Proposition 5 says that the moral hazard problem arising from incentive frictions is crucial in explaining the existence of both credit rationing and credit market failure. In the presence of these incentive frictions, search frictions provide a link between market liquidity and credit market tightness and credit rationing. Finally, borrower entry frictions also play a crucial role in explaining credit rationing. While the absence of such frictions does not preclude the non-active equilibrium, it does rule out credit rationing as an equilibrium outcome.

7 Further Discussion

In this section, we examine the duration of incentive compatible loan contracts and consider alternative loan default processes.

7.1 Duration of Loan Contracts

An innovative aspect of this our search model of the credit market is that it incorporates a parameter $\frac{1}{\delta}$ that corresponds to the duration of the loan period). For example, in the previous section we discussed how an increase in the length of the loan contract can affect the incentive compatibility, the interest rate offer, and the possibility of credit rationing. An interesting application of this framework would be to study how the duration of the loan contract affects the interest rate offer and vice versa.

A simple illustration of how the contract's duration affects the loan rate would be to consider the active equilibrium without credit rationing (Case I). Here, the optimal contract interest rate is just given by substituting $Q^* = \omega$ into (12) to get:

$$\delta R^* = [A - \frac{rv}{\omega}]\Gamma + (1 - \Gamma) \tag{22}$$

Notice that $R^* > 0$ as long as $(A - 1) > \frac{rv}{\omega}$, which is satisfied if the supply of loanable funds ω or the productivity of the investment project A is sufficiently large. In this case a decrease in δ, or increase in the length of the loan contract, increases Γ and hence the optimal loan interest rate δR^*. One could interpret result as an upward sloping yield curve in $(\frac{1}{\delta}, \delta R)$ space. While beyond the focus of this paper, it would be of interest in future work to more completely analyze the term structure properties of a search model of credit.

A related issue would be to extend our model to address the endogenous joint determination of the quantity, loan rate, and loan contracting period. One way to approach pinning down $(Q, \delta R, \delta)$ is to have all three objects be the outcome of decentralized bilateral bargaining between lenders and borrowers. In other words, have them be a solution to a Nash bargaining problem (P) which maximizes the joint surplus of the lender and borrower:

$$\max_{q\omega,\, \delta R,\, \delta} \quad (J_m - J_u)^{1/2}(\Pi_m - \Pi_u)^{1/2} \tag{P}$$

$$\text{s.t.} \quad J_m - J_u = \frac{(1-q)\omega + \delta R q\omega - rJ_u}{r+\delta} \quad \text{and} \quad \Pi_m - \Pi_u = \frac{(A - \delta R)q\omega - r\Pi_u}{r+\delta}$$

In addition to the bargaining condition for δR given in (12), we now have an additional first order condition associated with δ that, after simplification, is given by,

$$Rq\omega\left(\frac{1}{J_m - J_u} - \frac{1}{\Pi_m - \Pi_u}\right) = \frac{1}{r+\delta} \tag{23}$$

Because the expression in brackets is equal to zero ($J_m - J_u = \Pi_m - \Pi_u$), this implies an optimal choice of $\delta^* \to \infty$. That is, instantaneous credit transactions and separations maximize the joint surplus of a borrower-lender pair when production is instantaneous at the time of a match. This result delivers an important message. In our admittedly highly stylized framework, where long-term relationships only lower search costs but do not alter incentive frictions, there exists no reason to continue a "long-term" credit relationship. This is because the marginal gain in matched values from continuing a relationship is dominated by the marginal loss in unmatched values (via the bargaining threat points). Undoubtedly, adding more realistic features to the model will provide additional incentive to prolonging credit relationships. For instance, allowing learning and diminishing incentive frictions over the time agents are matched may yield outcomes that favor long-term relationships. Although it is beyond the scope of the current paper to go further along this path, we would like to point out that there is an active literature exploring long-term credit relationships.[8]

7.2 Alternative Default Processes

The benchmark model above assumes a specific form of the default process where (i) borrowers abscond before production and (ii) in addition to the penalty cost θ borrowers who default are permanently excluded from the credit market. We now consider two alternative setups for the default process which relaxes these assumptions. It is straightforward to verify that most of the features and results from our benchmark specification remain valid and we highlight some interesting differences.

First, following Hart and Moore (1998), borrowers may have the ability to divert returns as well as assets. In this case production occurs before absconding and the incentive compatibility condition given by (9) becomes:

$$(1-\theta)\frac{Aq\omega}{r} \leq \Pi_m$$

In this case, equation (13) is the same except for $B^* = A[1-(1-\theta)\frac{r+\delta}{r}] < B$. This implies that the IC locus is everywhere below the original IC curve derived above. Hence, the set of incentive compatible contracts shrinks and credit rationing becomes more likely relative to our benchmark case.

[8] For instance, Hart and Moore (1998) investigate mechanisms and incentives to renegotiate debt contracts in the face of credit market imperfections and incomplete contracting.

Second, the Diamond (1990) assumption that the penalty of default is exclusion from the credit market forever may be viewed as too harsh in light of modern bankruptcy laws. If instead credit market participation is allowed after payment of the default penalty, the incentive compatibility constraint becomes

$$(1-\theta)\frac{q\omega}{r} + \Pi_u \leq \Pi_m$$

In this case, the IC locus is upward-sloping as $\delta R \leq B - \frac{r\Pi_u}{Q}$ and the uniqueness of the steady state equilibrium is not guaranteed. In particular, it is possible to have coexistence of a credit rationing equilibrium and an inactive equilibrium.

8 Concluding Remarks

This paper has presented a simple search-theoretic model of the credit market. The model focuses on endogenous entry and moral hazard as particularly important factors in determining the fortunes of entrepreneurs. Our analysis describes the optimal incentive compatible loan contracts and equilibria that emerge in such an environment. While we tie the extent of mismatch to the tightness of credit markets, we find that mismatch and tightness are somewhat of a "red-herring" for understanding the extent of credit rationing; endogenous entry of borrowers can break the conventionally positive relationship between credit market tightness and credit rationing. We also establish the following regarding the interrelationship between incentive and matching frictions. Incentive frictions continue to be necessary in accounting for credit rationing and market breakdown. Given these incentive frictions, both entry and search frictions are important for understanding the possibility of market breakdown, where credit markets cease to function, while entry frictions are important for determining the extent of credit rationing.

There are several other possibilities for future work. First, it would be interesting to re-examine the optimal loan contract and the likelihood of market breakdown in the incomplete contract framework, as in Hart and Moore (1998). To do so, one would need to modify the moral hazard problem. In particular, rather than absconding, a borrower may "consume" part of the loan without converting that portion into a productive investment project. This loan-eating tendency measures the degree of shirking, though it cannot be perfectly monitored or verified by a court of law (due, for example, to a concurrent change in the state of nature). Then allowing renegotiations of the repayment schedule between the borrower and the lender would not only create more reinforcing forces between search/entry and incentive frictions, but also make the loan contract more "performance based," depending on the realized rate of returns on the investment project. The latter consideration may, in turn, lead to a greater likelihood of a long-term borrowing-lending relationship

Second, one might argue there is too much "randomness" in our matching model and that this randomness exaggerates the moral hazard problem and diminishes the benefit of long-term relationships. One way to address this issue is to adopt the directed-search price-posting game developed by Peters (1991). Specifically, there are two segregated submarkets: market one similar to the environment in the present paper and market two with lenders requiring borrowers to provide costly credit documentation prior to granting loans that partially mitigate the incentive frictions. Since all borrowers are identical *ex ante*, each lender in each segregated submarket posts for all borrowers the flow interest rate and the duration of the loan contract to maximize the expected value subject to a no-arbitrage condition that ensures all borrowers receive equal value *ex ante*. As a consequence, the loan contracts are generally different between the two submarkets, and free mobility of borrowers results in different matching probabilities and hence different measures of tightness within the two credit markets.

Finally, by incorporating an intertemporal role for financial markets in allowing consumers to smooth consumption over time via endogenous saving decisions, the model can be naturally extended to explicitly analyze the linkage between credit markets, rationing, and aggregate economic activity. In particular, our framework may provide a novel approach to understanding how impulses arising from aggregate and financial market shocks are propagated through a decentralized credit channel via search and entry frictions to explain aggregate fluctuations.

Appendix

Measure of Credit Market Tightness

An alternative measure of credit market tightness is given by the ratio of unmatched borrowers to the total pool of borrowers or the "capital unemployment rate": $\kappa = \frac{N_u^B}{N_u^B + N_m^B}$. Since $N_u^B = \tau N_u^L$ and $N_m^B = N_m^L = \frac{\mu}{\delta} N_u^L$, we have:

$$\kappa = \frac{\tau}{\tau + \mu/\delta} = \frac{\tau}{\tau + m_0 M(1,\tau)/\delta} = \frac{1}{1 + \frac{m_0}{\delta} M(\frac{1}{\tau}, 1)} \equiv \kappa(\tau)$$

It is easily verified that κ is monotonically increasing in τ.

Proof of Proposition 1

(i) Consider the case where $(A-1) < (1-\theta)$. This implies that $B \equiv A - (1-\theta)(\frac{r+\delta}{r}) < 1$ so that the IC locus has a horizontal intercept at $Q_{IC} = \frac{\delta \Pi_u}{1-B}$. Suppose there exists an active loanable funds equilibrium. This implies: $\frac{\delta \Pi_u}{1-B} = Q_{IC} < Q_{SM} = \frac{r\Pi_u}{A-1}$, or, $r(1-B) < \delta(A-1)$, or by manipulating,

$$r[(1-\theta) - (A-1)] < \delta[(A-1) - (1-\theta)] \tag{A1}$$

Since the right hand side of this expression is negative while the left-hand side is positive, we have a contradiction. Hence, no active loanable funds equilibrium exists. (Case III)

(ii) Consider the case where $(1-\theta) \leq (A-1) \leq 2(1-\theta)$. From the SM locus given by (12) notice that $\lim_{Q\to\infty} \delta R_{SM} = \Gamma(A-1) + 1$. Similarly, from the IC locus given by (13), $\lim_{Q\to\infty} \delta R_{IC} = B$. If $B < 1$, then from (A1) we know $Q_{IC} \leq Q_{SM}$ and there exists a loanable funds equilibrium. Since the SM locus must cross the IC locus, we have the credit rationing case where $q < 1$ for ω sufficiently large. For $B > 1$, we need to verify that, $\lim_{Q\to\infty} \delta R_{SM} > \lim_{Q\to\infty} \delta R_{IC}$, or, $\Gamma(A-1) + 1 > B$, which can be rewritten as: $[\frac{r+\delta+\mu}{2(r+\delta)+\mu}](A-1) + 1 > A - (1-\theta)(\frac{r+\delta}{r})$, or,

$$r[(A-1) - 2(1-\theta)] < (1-\theta)[2\delta + \mu] \tag{A2}$$

Since the right-hand side of (A2) is non-positive and the right-hand side is strictly positive, this condition holds. Thus, there is a unique credit rationing equilibrium where the SM locus intersects the IC locus (Case II).

(iii) Consider now $(A-1) > 2(1-\theta)$. Solving for r in (A2) gives, $r < \frac{(1-\theta)(2\delta+\mu)}{(A-1)-2(1-\theta)} \equiv \overline{r} > 0$. This condition is sufficient to guarantee that $\lim_{Q\to\infty} \delta R_{SM} > \lim_{Q\to\infty} \delta R_{IC}$ and the existence of a credit rationing equilibrium (Case II). However, if $r \geq \overline{r}$, then $\lim_{Q\to\infty} \delta R_{SM} \leq \lim_{Q\to\infty} \delta R_{IC}$ and every loan contract along the SM locus is incentive compatible. The incentive compatibility constraint does not bind in this case and there is no rationing (Case I). $\square$

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Chart 1: The Structure of the Economy

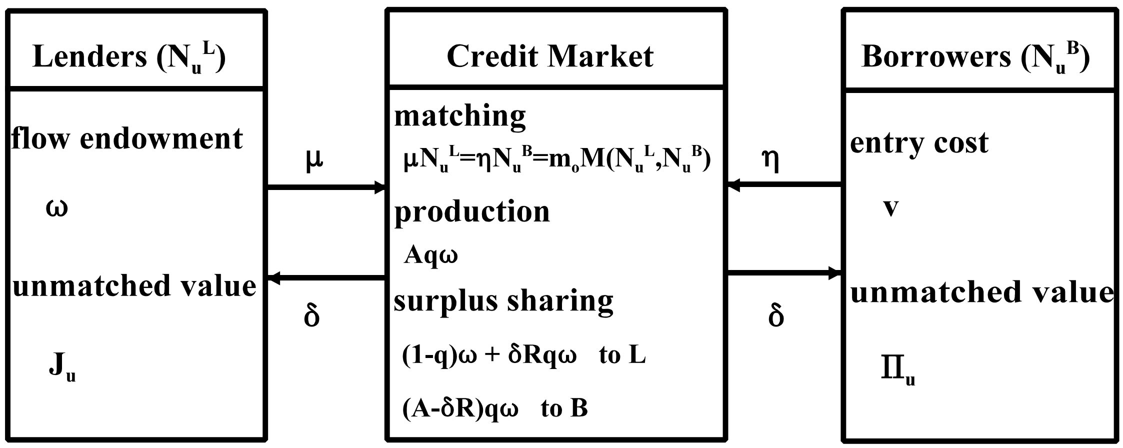
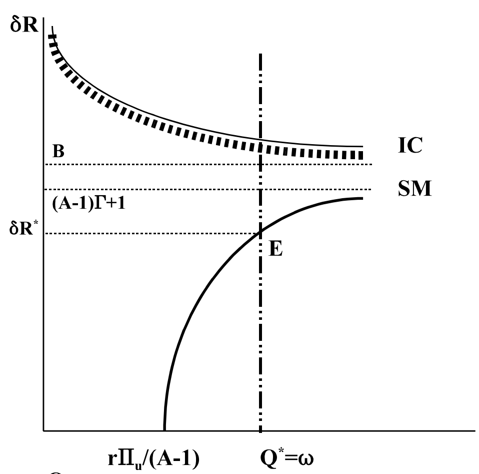
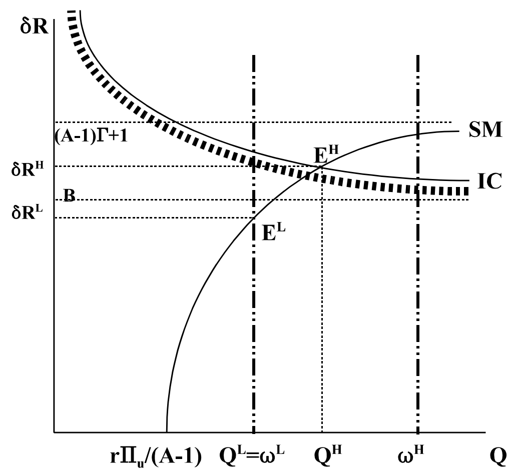


Figure 1: Optimal Incentive Compatible Loan Contract (Q = ωq and δR)

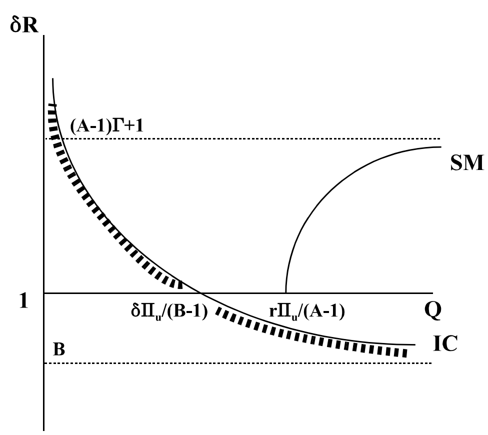
Case I: $(A-1)>2(1-\theta)$ **and** $r \geq \bar{r}$



Case II: $(A-1)>2(1-\theta)$ **and** $r<\bar{r}$
or $(1-\theta)\leq(A-1)\leq 2(1-\theta)$



Case III: $(A-1)<(1-\theta)$



Notes: There are three cases, depending on the relative position of the SM and IC loci:

1. Case I — incentive compatibility constraint never binds;
2. Case II — incentive compatibility constraint binds only when funds available are high and in that case, the amount of loan is rationed;
3. Case III — optimal incentive compatible loan contract does not exist.

Figure 2: Steady-State Loanable Funds Equilibrium

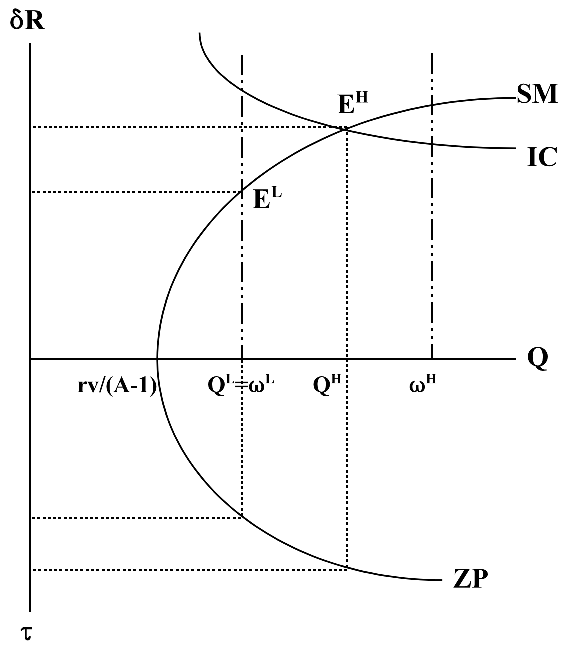


Figure 3: Equilibrium Responses to a Reduction in Search Frictions (Higher m_0)

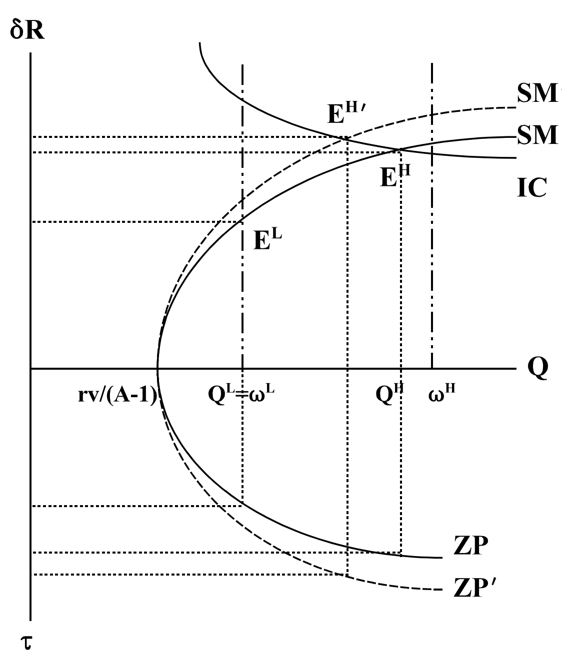


Figure 3: Multiple Equilibria When Defaulters May Re-enter the Credit Market