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IS IT ALWAYS GOOD TO LET UNIVERSITIES SELECT THEIR STUDENTS?

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Is it always good to let universities select their students?*

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Abstract: We undertake a first step to investigating a reform that has been applied in numerous universities across Europe: the right to select students. We ask to what extent this right will increase the efficiency of the university. While it seems evident that giving universities the right to select students that match best with the human capital of professors should increase efficiency measures in the productivities of students in the labor market, we point to a potentially negative effect. We argue that allowing universities to select the students they prefer can reduce the incentives of the universities to improve the human capital of their professors.

1 Introduction

Following Humboldt's ideals, European post-war universities have for long enjoyed substantial academic freedom in their teaching and research. Funding was largely in fixed terms, and independent of performance. Throughout the last decade, European universities have been involved in a reform process the goal of which has been expressed in the Lisbon agenda: "To make Europe, by 2010, the most competitive and the most dynamic knowledge-based economy in the world".

Many measures have been taken by the EU and different members states with this goal in mind. Among the instruments experimented with are more competitive research funding (through the European Research Council and the National Science Foundations such as the ANR in France or the DFG in Germany), excellence initiatives as triggered by the Schröder Government in 2005 and recently emulated by the "grand emprunt" in France, and the introduction of student fees in some German states and in the UK. The general view is that the governance and financing structure of universities needs to be changed in order to increase the competitiveness of the university system.

*The findings, recommendations, interpretations and conclusions expressed in this paper are those of the authors and not necessarily reflect the view of the Department of Economics of the Universidad del Rosario.

These changes are most notable in the fields of engineering, business and economics, where the success of a university is now increasingly measured by the job market success of the graduates, i.e. by their employment rates and wages. This posits a challenge for many universities that are expected to update their human capital in research and teaching to be able to educate students in a way that they fit the needs of employers. The skills of professors and the university administration naturally does not simply grow and change with the needs of the society, but substantial efforts must be undertaken to be able to provide the teaching that is required in order to produce good graduates.

Consider for instance, the business department of a university in which IT professors have a strong operations research background that they acquired at the time of their graduate education, say in the 80s. The market now demands them to educate students in e-commerce, network economics, in the development of open-source software or e-marketing. This necessarily means that effort must be spent on retraining the faculty, re-directing the research efforts, and, in particular, hiring young people with the right skills. Hiring is a joint effort between faculty and administration, and the administration needs to invest in student services and marketing. Thus staying on top of the requirements implies university-wide investments of time and money.

Besides the array of instruments that have been introduced to steer universities towards providing education and research in line with the needs of the economy, there is also a general tendency to provide universities with more autonomy. In many European economies there are still limits on universities' autonomy in terms of levying tuition fees, but universities frequently have been granted the autonomy to select their own students.

There are hence two general tendencies: a need to undertake efforts in order to update the human capital of universities, and increasing autonomy to select students. We here suggest a simple micro-economic analysis of the interaction of these two tendencies. More precisely we ask to what extent it is good to give universities that need to update their human capital the autonomy to select their students. One could expect that this autonomy is always welfare-enhancing. However, we show that this is not necessarily the case. Rather, we show that there is a complementarity between the two instruments: In the absence of financing constraints for students, giving universities the right to levy tuition fees and select students provides a first-best level of welfare.

However, when tuition fees are unavailable, for instance, because society does not want to exclude liquidity-constrained students, then a trade-off emerges. When universities are allowed to select students, this makes it possible for universities to avoid the private costs of having to educate students they do not like. Thus the places in top universities are taken by top students only, and the ones in less good universities by less good students. The right of selection thus leads to a segmented university system in which professors educate the people they like. This comes at two costs. First, some of the most talented potential students are excluded from the university system all together. Second, universities have no motivation to invest effort in order to adjust to accommodate the most talented students. The welfare comparisons are intricate but we can show that when the proprotion of talent in society is high, giving universities the right to select without the right to levy tuition fees is more likely to be welfare-decreasing.

Our theory identifies a trade-off that is only present when the reform is piecemeal, i.e., when selection by universities is allowed, but no tuition. If, however, tuitions are possible as well, selection of students by universities always improves welfare. There is thus a fundamental complementarity between tuition and selection of students. It should be noted that we do not argue for the introduction of tuitions which in the absence of student grants can have severe efficiency and redistributive effects. We rather argue that allowing universities to select students according to their preferences can make things worse compared to a system in which students are more or less randomly allocated to universities.

Economists have only recently begun to analyze the implications of reforms in the University system. Consequently, it seems that we are the first looking at the right-to-select phenomenon. The paper that is closest to ours in terms of the setting (it also regards a matching market in education) is Besley and Ghatak.[1] But they investigate the effects of incentive pay for teachers on a market in which schools match with motivated teachers, while our paper asks to what extent a partial reform, the right of student selection, is welfare enhancing in terms of the matching between universities and students, and in terms of the incentives for universities to adjust to changing demands.

2 The model

2.1 The market for education

There are h potential students and g universities. Students can be of type $s \in \{A, B\}$ with η being the proportion of type A; universities can be of type $u \in \{a, b\}$ with γ_0 being the proportion of type a. The notation is chosen to make clear that there are complementarities between universities and students as explained below, but also a, A and b, B reflect the idea that there is vertical differentiation both in terms of universities and students, with a more productive than b, and A more productive than B.

In the economy we look at, there are more individuals seeking university education than places that can be filled by students. Consequently, universities have bargaining power. Formally, we suppose that

$$\begin{aligned} \eta h &> \gamma_0 g, \\ (1-\eta)\, h &> (1-\gamma_0) g \\ \eta h &< g \end{aligned}$$

[1] Besley, Timothy and Maitresh Ghatak (2006), "Sorting with Motivated Agents: Implications for School Competition and Teacher Incentives", Journal of the European Economic Association, vol 4, 2/3, pp 404-14

so that not all individuals can attend university ($h > g$), there are less universities of type a than individuals of type A, less universities of type b than individuals of type B and there are less type-A students than universities.

Initially, type-a universities are of high productivity and type-b are of low productivity. However, the low-productivity (b) universities can make an effort to update the human capital of their professors, improve their administration etc. Provided they spend a costly effort e, b universities can become highly productive. e is the probability of success of reform or of restructuring the university and it implies a cost $v(e)$ to universities. The productivity of b universities thus becomes stochastic, with probability e, it becomes high (the university transforms itself to an a type), while with probability $1 - e$, they remain b types. Those b universities that successfully transform will be called "lucky", while those who fail to reform are called "unlucky".

Depending on the e chosen in equilibrium, the number of high productivity universities differs from the initial proportion of a universities, γ_0. It is given by

$$\gamma(e) = \gamma_0 + e(1 - \gamma_0).$$

We impose an Inada condition on $v(e)$, namely that $\lim_{e \to \widehat{e}} v'(e) = \infty$, where $\widehat{e} = \frac{\eta h - \gamma_0 g}{(1-\gamma_0)g}$ ($\widehat{e}$ is the effort level needed to have the same number of high productive universities and of type A students.). Evidently, when $e = 0$ in equilibrium, all b universities remain b universities.

We will call type-a and "lucky" type-b universities *productive* universities and type-b "unlucky" universities *unproductive* universities. *A priori* we may have matches between any type of student and a *productive* or an *unproductive* universities. A student of type $s \in \{A, B\}$ that is matched with a productive university will be denoted by $\overline{s} \in \left\{\overline{A}, \overline{B}\right\}$; similarly it will be denoted by $\underline{s} \in \{\underline{A}, \underline{B}\}$ if the match is with an unproductive university.

The assumptions about the composition of universities (a and b) and individuals (A and B) together with the Inada condition could be changed and would yield different equilibria, but we believe this combination of assumptions to be both realistic and to yield interesting implications. In particular, the assumptions guarantee that there will always be some type-B individuals attending universities, and that there is always the possibility of any given type of students attending high or low productivity universities or with out place in a university. Any other set of assumptions we could think of would rule out some of these possibilities.

2.2 Wages

On the labor market, wages can take four different values, $w_i \in \{\omega, w_1, w_2, w_3\}$, with $w_3 > w_2 > w_1 > \omega$. We assume that wages already include the opportunity cost of attending a university (in other words that the opportunity cost of attending a university does not offsets its benefits). As students and universities differ in their productivity, wages differ depending on the student-university match according to the following assumptions:

1. an A student that attends a productive university gets a wage w_3, a B student that attends productive unviersity gets a wage w_2,
2. an A student that attends an unproductive university gets a wage w_2, a B student that attends an unproductive university gets a wage w_1.
3. an individual who not attend university gets a wage ω regardless of his type.

The following table summarizes the labor market outcomes.

	A	B
productive	w_3	w_2
unproductive	w_2	w_1
U	ω	ω

Note that from the perspective of universities the wage of its graduates will be stochastic. If the university is of type b type A (B) students will have wages w_3 (w_2) or w_2 (w_1) with probabilities e or $1-e$. We assume that there are complementarities between the high types of students and universities. Thus, the following condition holds:

$$w_3 - w_2 > w_2 - w_1 \tag{1}$$

These assumptions mean three things: (i) students attending a universities have higher expected wages than students attending b universities; (ii) the innate ability of students is more important than the quality of the university; (iii) the ability of students affects the productivity effects of teachers' efforts.

In principle, one could take other assumptions about the composition of universities (type a and b) and students (type A and B) which would yield different equilibria. We believe this combination of assumptions to be both realistic and to yield interesting implications. In particular, the assumptions guarantee that there will always be some type B students attending universities, and that there is always the possibility of any given type of students attending high or low productivity universities or with out place in a university. Any other set of assumptions we could think of would rule out some of these possibilities. To simplify notation we normalize ω to be equal to zero.

2.3 Payoffs

We denote with an upper bar those variables related to a high-productivity university and with a lower bar those related to a low-productivity university; this means that upper bar variables correspond to type-a or lucky type-b universities and lower bar variables correspond to unlucky type-b universities.

Student utility is purely monetary. The utility of an individual of type s and that gets a wage w_i is

$$w_i - t_s$$

where t_s is the fee he has to pay to attend a university of type s. Let $t_s \in \{\overline{t}_s, \underline{t}_s\}$; $\overline{t}_s$ is the tuition fee a student of type s would pay to a high productivity university, and $\underline{t}_s$ is the tuition fee it pays to a low productivity university.

Universities care about the type of students who attend. High-productivity universities prefer students of type A, and low-productivity universities prefer students of type B. The idea behind this assumption is that most professors like to teach people who are similar to themselves. For instance, it is much nicer to teach mathematics to people who like mathematics. We thus assume that there are costs associated with students who do not match the professors' preferences and skills. When a student of type A (B) attends a university a (b) this cost is zero, while the cost associated with a student of the non-matching type is c. Universities also care for the monetary payments they receive so their *ex post* payoff is given by $t_s - v(e)$ or $t_s - c - v(e)$, depending on the type of student they attend.

Note that there is a divergence between the preference ordering (for a given tuition fee) of students and universities for their counterparts in the market. While all students (of type A or B) prefer to attend a-type universities, high (low) productivity universities prefer A (B) students. We denote by $\overline{V}$ the payoff of a high productivity university and by $\underline{V}$ the payoff of a low productivity university.

2.4 Timing and structure of analysis

We study a sequential equilibrium. Universities first chose their effort and then the admission process takes place. The solution is by backwards induction. Clearly, the effort level exerted by a type-b university will depend on the expectation about equilibrium matching in the second period. Since the uncertainty is resolved before individuals choose university, it will only affect universities. Universities will look at the expected value of making the effort given the best response of the other guys

We will study the sorting and effort choice that emerge under three different environments: (i) an unregulated market equilibrium; (ii) a situation in which there are no tuition fees, but universities are allowed to select students, (iii) one in which there are no tuition fees and no selection of students by universities.

3 Equilibrium

The Inada condition on $v(e)$ allows us to keep the analysis to the case where $\gamma(e)g < \eta h$ so that high-productivity universities are always on the short side of the market and have all the bargaining power. *A priori* there are four types of equilibria in the second stage of the game (given e). They are are characterized by the type of student-universities match. The four types of equilibria are shown in Figure 3.

In the first type of equilibrium (i), all type-A get a place in a university, and all individuals who do not get a place are of type B. There is a number $\gamma(e)g$

Type-i equilibrium		Type-ii equilibrium
γg $\overline{A}$ matches $\eta h - \gamma g$ $\underline{A}$ matches	A individuals	γh $\overline{A}$ matches $\eta h - \gamma g$ uneducated
$g - \eta h$ $\underline{B}$ matches $h - g$ uneducated	B individuals	$(1-\gamma)\, g$ $\underline{B}$ matches $(1-\eta)\, h - (1-\gamma)\, g$ uneducated

Type-iii equilibrium		Type-iv equilibrium
$g - (1-\eta)\, h$ $\underline{A}$ matches $h - g$ uneducated	A individuals	$\gamma \eta g$ $\overline{A}$ matches, $(1-\gamma)\, \eta g$ $\underline{A}$ matches $(h-g)\, \eta$ uneducated
γg $\overline{B}$ matches $(1-\eta)\, h - \gamma g$ $\underline{B}$ matches	B individuals	$\gamma\, (1-\eta)\, g$ $\overline{B}$ matches $(1-\gamma)\, (1-\eta)\, g$ $\underline{B}$ matches $(h-g)\, (1-\eta)$ undeducated

Figure 1: Types of equilibria

of $\overline{A}$ matches, a number $\eta h - \gamma(e)g$ of $\underline{A}$ matches and a number $g - \eta h$ of $\underline{B}$ matches; there are also $h - g$ type-B individuals that do not attend university.

In the second type (ii), there are type-A and B that do not get a place in a university; all type-A who get a place in a university attend a high-productivity university and all type-B individuals who get a place in a university attend a low-productivity university. There is a number $\gamma(e)g$ of $\overline{A}$ matches, a number $[1 - \gamma(e)]\, g$ of $\underline{B}$ matches. There are $\eta h - \gamma(e)g$ type-A individuals without a place in a university and $(1 - \eta)\, h - (1 - \eta)\, g$ type-B individuals without university.

In the third type (iii), all type-B individuals get a place in a university, and all individuals who do not get a place are of type A. There is a number $\gamma(e)g$ of $\overline{B}$ matches, a number $(1 - \eta)\, h - \gamma(e)g$ of $\underline{B}$ matches and a number $g - (1 - \eta)\, h$ of $\underline{A}$ matches. There are $h - g$ type-A individuals that do not attend university.

Finally, there is a fourth type (iv) in which individuals are randomly matched to universities. There is a number $\gamma(e)\eta g$ of $\overline{A}$ matches, a number $\gamma(e)\,(1 - \eta)\, g$ of $\overline{B}$ matches, a number $[1 - \gamma(e)]\, \eta g$ of $\underline{A}$ matches and a number $[1 - \gamma(e)]\,(1 - \eta)\, g$ of $\underline{B}$ matches. There are also $(h - g)\, \eta$ type-A individuals without a place in a university, and $(h - g)\,(1 - \eta)$ type-B individuals without a place in a university.

3.1 Benchmark: equilibria in the free market

We are interested in comparing the case with autonomy over selection of students vs no selection autonomy, both under the same assumption that no fees can be levied. We here begin the analysis by investigating a setting in which a free market governs the education system. More specifically, students pay fees, $\bar{t}_s$ and $\underline{t}_s$; note that we allow tuition fees to depend on the type (A or B) of student and on whether the university is productive or not. We also assume that when there is shortage of places and several individuals of the same type demand the available places, these places will be assigned randomly.

We now show that in the free market only type-i and type-ii equilibria may take place. Type-iii equilibria can be ruled out since the willingness to pay for attending a university of type-A students will always be higher of type-B students, and random matching as in type-iv will not occur.

The reservation utility of not attending university implies that the maximum tuition fees are given by (recall we normalized $\omega = 0$)

$$\bar{t}_A \leq w_3, \quad \underline{t}_A \leq w_2, \quad \bar{t}_B \leq w_2 \quad \text{and} \quad \underline{t}_B \leq w_1. \tag{2}$$

As universities have all the bargaining power (they are on the short side of the market), these tuition fee levels will obtain in equilibrium.

In type-i equilibrium it must be that

$$\begin{aligned} \underline{t}_A - c &\geq \underline{t}_B, \\ \bar{t}_A &\geq \bar{t}_B - c. \end{aligned} \tag{3}$$

so that low productivity universities prefer type-A students but are willing to take-B type students.

In type-ii equilibrium it must be that

$$\begin{aligned} \underline{t}_B &\geq \underline{t}_A - c, \\ \bar{t}_A &\geq \bar{t}_B - c. \end{aligned}$$

so that high productivity universities prefer type-A students and are not willing to take type-B students.

For type-i equilibrium to take place it must be that

$$\begin{gathered} w_2 - w_1 \geq c, \\ w_3 + c \geq \bar{t}_B. \end{gathered}$$

The second condition is satisfied since from (2) $w_2 \geq \bar{t}_B$.

Accordingly, there exists a $\bar{c}$ such that if $c \geq (<)\bar{c}$ type-ii(i) equilibria obtains. This threshold level of $\bar{c}$ is given by

$$\bar{c} = w_2 - w_1.$$

For further reference it is important to notice that, as long as there are no external restrictions on tuition fees, selection of students by universities is a redundant instrument.

We now turn to effort in *type-i equilibrium*. The expected payoffs of a high and low productivity universities are, respectively

$$\begin{aligned} \overline{V} &= \bar{t}_A \\ \underline{V}(e) &= \sigma(e)\left(\underline{t}_A - c\right) + \left[1 - \sigma(e)\right]\underline{t}_B \end{aligned}$$

with

$$\sigma(e) = \frac{\eta h - \gamma(e) g}{g - \gamma(e) g}.$$

$\sigma(e)$ is the probability of having a student of type-A for a low productivity university. These expressions follow from the fact that in type-i equilibrium a high productivity university gets type-A students with probability one, while low-productivity universities get type-A students with a less than one probability.

The utility of a type-a university is

$$V_a = \overline{V}$$

while that of a type-b university is

$$V_b(e) = e\overline{V} + (1 - e)\,\underline{V}(e) - v(e).$$

The FOC for e is

$$\left(\overline{V} - \underline{V}(e)\right) + (1 - e)\,\underline{V}'(e) = v'(e)$$

or

$$\left\{\bar{t}_A - \left[\sigma(\underline{t}_A - c) + (1 - \sigma)(\underline{t}_B)\right]\right\} + (1 - e)\,\sigma'(e)\left[\underline{t}_A - \underline{t}_B - c\right] = v'(e).$$

with

$$\sigma'(e) = -\frac{(1-\gamma_0)(g-\eta h)}{[1-\gamma(e)]^2 g} \leq 0. \tag{4}$$

Using the equilibrium payoffs obtained above and rearranging we can write the FOC for e as

$$w_3 - w_1 - [\sigma - (1-e)\sigma'(e)](w_2 - w_1 - c) = v'(e). \tag{5}$$

Further, in the appendix we show that $[\sigma - (1-e)\sigma'(e)] = 1$ so the FOC is

$$w_3 - w_2 + c = v'(e^{m_i}). \tag{6}$$

The effort in *type-ii equilibrium* can be determined in a similar way. We have that

$$\begin{aligned} \overline{V} &= \bar{t}_A, \\ \underline{V} &= \underline{t}_B. \end{aligned}$$

Type-b universities maximize

$$e\overline{V} + (1-e)\underline{V} - v(e)$$

and, consequently their will be given by

$$\bar{t}_A - \underline{t}_B = v'(e)$$

or, using the equilibrium values of tuition fees

$$w_3 - w_1 = v'(e^{m_{ii}}) \tag{7}$$

To show that only type-i and type-ii equilibrium may take place, note that in type-iii equilibrium it must be that

$$\begin{aligned} w_2 - \bar{t}_B &\geq w_1 - \underline{t}_B \\ w_1 - \underline{t}_B &\geq \omega \\ w_2 - \underline{t}_A &\geq w_3 - \bar{t}_A \\ w_2 - \underline{t}_A &\geq \omega. \end{aligned}$$

There exist fee values such that all of these equations are satisfied; take for example $\bar{t}_A = w_3$, $\underline{t}_A = w_2$, $\bar{t}_B = w_2$ and $\underline{t}_B = w_1$. Since universities have all the bargaining power these will be the equilibrium values of tuition fees. Since there are no type-A students attending high productivity universities $\bar{t}_A$ will not be observed in this type of equilibria; any value for $\bar{t}_A$ which is greater or equal than w_3 will be consistent with this type of equilibrium. Incidentally note that the equilibrium values of tuition fees in type-i and type-iii are the same, so *a priori* for fee levels $\bar{t}_A = w_3, \underline{t}_A = w_2, \bar{t}_B = w_2$ and $\underline{t}_A = w_1$ one cannot know which type of equilibrium will result. The problem can be easily solved since in each case there is one tuition fee level that is not observed and can be set freely by universities; thus if, universities set $\bar{t}_A = w_3$, $\underline{t}_A = w_2$, $\bar{t}_B = w_2 + \varepsilon$ and $\underline{t}_B = w_1$ the equilibrium allocation will be that of type-i. To get type-iii equilibrium it suffices if $\bar{t}_A = w_3 + \varepsilon$, $\underline{t}_A = w_2$, $\bar{t}_B = w_2$ and $\underline{t}_B = w_1$.

In type-iii equilibrium, profits of high-productivity universities are

$$\bar{t}_B - c = w_2 - c$$

and of low-productivity universities are

$$[1 - \widetilde{\sigma}(e)]\, \underline{t}_B + \widetilde{\sigma}(e)(\underline{t}_A - c) = p_B w_1 + p_A\,(w_2 - c)\,.$$

where $\widetilde{\sigma}(e)$ is the probability for a given low productivity university of having a student of type A.

In type-i equilibrium profits of high productivity universities are

$$w_3$$

and those of low productivity universities are

$$\sigma(e)\,(w_2 - c) + [1 - \sigma(e)]\, w_1.$$

Clearly profits of high productivity university are higher in type-i than in type-iii equilibrium. If in equilibrium of type-iii, high-productivity universities want to achieve higher profits they can do so by deviating from $\bar{t}_A = w_3 + \varepsilon$, $\bar{t}_B = w_2$ to $\bar{t}_A = w_3$,$\bar{t}_B = w_2 + \varepsilon$.

A final remark concerns effort as a function of c. We know that when c increases, we go from a type-i equilibrium to a type-ii equilibrium. In type-i equilibrium effort increases with c[2]; in type-ii equilibrium effort does not change with c (see 7). Since the FOC for effort in type-i equilibrium converges to that of type-ii equilibrium we have that effort weakly increases with c; i.e., it increases up to $\bar{c}$, and then is constant.

3.2 No tuition fees but selection of students by teachers

Suppose now that no (substantial) tuition fees can be levied, but universities can decide whether they accept a particular student. First notice, that it can be readily shown that all potential students will want to attend a university, all prefer highly productive universities, but only those of type A will get a place in universities of type a, while low productivity universities will only accept students of type B. Equilibrium sorting is thus as in the type-ii equilibrium discussed above. There will be $\gamma(e)g\ \overline{A}$ matches, $[1 - \gamma(e)]\, g\ \underline{B}$ matches. Additionally, $\eta h - \gamma(e)g$ type-A and $(1 - \eta)\, h - [1 - \gamma(e)]\, g$ type-B individuals will not attend university.

[2]From (6)

$$\frac{\partial e^{Mi}}{\partial c} = \frac{1}{v''(e^{Mi})} > 0.$$

In this case

$$\overline{V} = \underline{V} = 0$$

and

$$V_a = V_b = 0$$

The most important result of course is also very easy to show: effort will be nil, no university has an incentive to update their human capital, because this involves a cost of effort without reaping any gain, given that no pay for tuition can be asked from the student.

3.3 No tuition fees and no selection

In this setting, we have random assignment of students to universities, because regardless of their type, all students prefer a universities to b universities. The equilibrium assignment is thus as in the type-iv equilibrium discussed above. Consequently the expected payoffs of high productivity and low productivity universities are, respectively

$$\begin{aligned}\overline{V} &= -(1-\eta)\,c \\ \underline{V} &= -\eta c\end{aligned}$$

The payoffs of a university of type a and of a university of type b are, respectively

$$\begin{aligned}V_a &= \overline{V} \\ V_b &= e\overline{V} + (1-e)\,\underline{V} - v(e)\end{aligned}$$

In contrast to the case discussed above, the optimal effort is not trivial, and determined by

$$\begin{aligned}\frac{\partial V_b}{\partial e} &= \overline{V} - \underline{V} - v'(e^{ns}) \leq 0 \\ &= (2\eta - 1)c - v'(e^{ns}) \leq 0\end{aligned}$$

There will be a strictly positive effort (*i.e.* $e^{ns} > 0$) if

$$\eta \geq \frac{1}{2}.$$

Otherwise effort will be equal to zero. Moreover effort is non-decreasing in c. Note that in the case in which $\eta > \frac{1}{2}$ effort will be higher when universities are not allowed to charge for tuition than when they are allowed.

4 Welfare comparisons

4.1 First best

The allocation of students to universities in the first-best depends on the size of c. Clearly it is optimal to fill places in high productivity universities with A

students. Whether we have places in low productivity universities filled with A or B students depends on whether c is higher or smaller than $w_2 - w_1$. If $c < w_2 - w_1$the first-best allocation is type-i equilibrium. If $c > w_2 - w_1$ having type-A students in a low productivity university is too costly, and the first-best allocation is type-ii equilibrium. Notice that first-best effort is in line with the outcome in the free market equilibrium.

Utilitarian welfare is given by

$$W = h\Big[\eta U_A + (1-\eta)\,U_B\Big] + g\Big[\gamma_0 V_a + (1-\gamma_0)\,V_b\Big]$$

In the free market, since universities are on the short side of the market and have all the bargaining power students have zero net utility ($U_A = U_B = \omega = 0$) and total welfare is given by the sum of profits of the universities. Welfare is then given by

$$W = h\Big\{\gamma_0 \overline{V} + (1-\gamma_0)\left[e\overline{V} + (1-e)\,\underline{V}(e) - v(e)\right]\Big\}$$

If $c < w_2 - w_1$,in type-i equilibrium, welfare is

$$\begin{aligned} W^{m_i} &= h\Bigg\{\gamma_0 w_3 + (1-\gamma_0)\Big[e^{m_i} w_3 + (1-e)\,\sigma(e^{m_i})\,(w_2 - c) \\ &\qquad + (1-e^{m_i})\,[1-\sigma(e)]\,w_1 - v(e^{m_i})\Big]\Bigg\} \\ &= h\Bigg\{\gamma(e^{m_i})w_3 + [1-\gamma(e^{m_i})]\Big[\sigma(e^{m_i})\,(w_2 - c) + [1-\sigma(e^{m_i})]\,w_1\Big] \\ &\qquad - (1-\gamma_0)\,v(e^{m_i})\Bigg\} \end{aligned}$$

with

$$\sigma(e) = \frac{\eta h - \gamma(e) g}{g - \gamma(e) g}.$$

If $c > w_2 - w_1$,in type-ii equilibrium, welfare is

$$\begin{aligned} W^{m_{ii}} &= h\Big\{\gamma_0 w_3 + (1-\gamma_0)\,[e^{m_{ii}} w_3 + (1-e^{m_{ii}})\,w_1 - v(e^{m_{ii}})]\Big\} \\ &= h\Big\{\gamma(e^{m_{ii}})w_3 + (1-\gamma_0)\,(1-e^{m_{ii}})\,w_1 - (1-\gamma_0)\,v(e^{m_{ii}})\Big\} \end{aligned}$$

In the selection, no-tuition fee equilibrium welfare is given by the sum of utilities of individuals, which is tantamount to the expected wages. Universities receive no tuition fees, make zero effort and do not receive students of the type which is costly to receive ($V_a = V_b = 0$):

$$W^s = \gamma_0 g w_3 + (1-\gamma_0)\,g w_1$$

In the no-selection, no-tuition fee equilibrium welfare takes into account the utility of the universities who incur effort costs and may be matched with students they do not like.

$$\begin{aligned} W^{ns}(e^{ns}) &= g\Big\{\gamma(e^{ns})\left[\eta w_3 + (1-\eta)(w_2 - c)\right] \\ &\quad + \left[1-\gamma(e^{ns})\right]\left[\eta(w_2 - c) + (1-\eta) w_1\right] - (1-\gamma_0) v(e^{ns})\Big\} \end{aligned}$$

4.2 Comparison of welfare: selection vs no selection

First, suppose $\eta < \frac{1}{2}$ so that effort of type b universities is zero. In that case $\gamma(0) = \gamma_0$ and we have that

$$\begin{aligned} W^s &= g\Big\{\gamma_0 w_3 + (1-\gamma_0) w_1\Big\}, \\ W^{ns}(0) &= g\Big\{\gamma_0\Big[\eta w_3 + (1-\eta)(w_2 - c)\Big] \\ &\quad + (1-\gamma_0)\left[\eta(w_2 - c) + (1-\eta) w_1\right]\Big\}. \end{aligned}$$

We can then write a condition under which student selection by universities increases welfare (under the assumption that no tuition fees are levied):

$$\begin{aligned} W^s &> W^{ns}(0) \\ &\Longleftrightarrow \\ \gamma_0(1-\eta) w_3 + (1-\gamma_0)\eta w_1 &> \left[\gamma_0(1-\eta) + (1-\gamma_0)\eta\right](w_2 - c). \end{aligned}$$

Evidently whether or not welfare is higher under selection of students depends on the parameters. To see this note that by assumption $w_3 > w_2 - c$, so that $(1-\eta) w_3 > (1-\eta)(w_2 - c)$. In this case, if $c > w_2 - w_1$, then $W^s > W^{ns}(0)$. Hence, $c < w_2 - w_1$ is a necessary condition for $W^s < W^{ns}(0)$. Writing the welfare difference between selection and no selection as below leads to the first proposition:

$$\begin{aligned} W^s - W^{ns}(0) &= \gamma_0(1-\eta)(w_3 - w_2) \\ &\quad - (1-\gamma_0)\eta(w_2 - w_1) + \left[\gamma_0(1-\eta) - (1-\gamma_0)\eta\right] c. \end{aligned}$$

Proposition 1 *When $\eta < \frac{1}{2}$, i.e., the majority of students is of type B, the welfare is always higher under selection when $c > w_2 - w_1$. If $c < w_2 - w_1$, welfare is more likely to be higher under selection than under no selection the bigger are $w_3 - w_2$ or c, and the smaller are $w_2 - w_1$ or η.*

The Proposition reflects a simple tradeoff. When the majority of students is of type B, universities do not make adjustment efforts. Under selection, there are type-A students that do not get a university education. Taking these A students into a b university would create unit costs c for the b university but increase wages by $w_2 - w_1$. So, if $c > w_2 - w_1$, welfare is maximized under selection, while otherwise it depends on the parameters.

In the case of $\eta > \frac{1}{2}$, welfare comparison becomes more involved, because it also depends on e.

Proposition 2 *When $\eta > \frac{1}{2}$,i.e., the majority of students is of type A so that welfare also depends on e.*

Now whether selection is welfare enhancing or not depends on the comparison of

$$\begin{aligned} W^s &= g\left\{\gamma_0 w_3 + (1-\gamma_0)w_1\right\} \\ W^{ns}(e^{ns}) &= g\left\{\gamma(e^{ns})\left[\eta w_3 + (1-\eta)(w_2-c)\right]\right. \\ &\quad \left. + \left[1-\gamma(e^{ns})\right]\left[\eta(w_2-c) + (1-\eta)w_1\right]\right\} - v(e^{ns}). \end{aligned}$$

with $e^{ns} > 0$. Since W^{ns} is increasing in e not allowing universities to select their own students will other thinsg equal increase the welfare of society. Put differently, when there are many A, i.e., talented students, it is more likely that no selection is better, provided that there are no tuition fees.

Suppose now that the government decides on whether to allow selection or not only taking into account net monetary benefits of education. In particular the government disregards the effort costs of adjustment of universities. In this case the government compares

$$\begin{aligned} \widetilde{W}^s &= g\left\{\gamma_0 w_3 + (1-\gamma_0)w_1\right\} \\ \widetilde{W}^{ns}(e^{ns}) &= g\left\{\gamma(e^{ns})\left[\eta w_3 + (1-\eta)(w_2-c)\right]\right. \\ &\quad \left. + \left[1-\gamma(e^{ns})\right]\left[\eta(w_2-c) + (1-\eta)w_1\right]\right\}. \end{aligned}$$

In the case in which $\eta < \frac{1}{2}$ the same comparison as above is valid. In the case in which $\eta > \frac{1}{2}$ the main difference is that $\widetilde{W}^{ns} > W^{ns}$. Consequently the government will have a stronger tendency towards not allowing selection than when full welfare is taken into account.

4.3 Concluding remarks

We have suggested a simple model which shows that partial reforms of the university system may decrease welfare. We are not arguing that tuitions should be introduced, because they involve the risk of excluding certain groups of students from education. However, in the absence of tuition fees, allowing universities to select their students may distort the incentives to modernize universities, in terms of updating human capital of professors or of creating more professional management structures. It thus seems that the policy alternative is between keeping the old bureaucratic system or a market-based system in which there are tuitions and the right of universities to select students. This tradeoff can be made more favorable for the market solution when accompanies by means-tested subsidies. This paper is not dwelling on these important issues, but rather points to the pitfalls of partial reforms.

A Proof of $\sigma(e) - (1-e)\,\sigma'(e) = 1$

$$\sigma(e) - (1-e)\,\sigma'(e) = \sigma(e)\left[1 - (1-e) \times \frac{\sigma'(e)}{\sigma(e)}\right]$$

Note that

$$\begin{aligned} 1 - (1-e)\,\frac{\sigma'(e)}{\sigma(e)} &= 1 - (1-e)\,\frac{\gamma'(e)\,(\eta h - g)}{[1-\gamma(e)]^2\,g}\,\frac{[1-\gamma(e)]\,g}{\eta h - \gamma g} \\ &= 1 - (1-e)\,\frac{\gamma'(e)}{1-\gamma(e)}\,\frac{(\eta h - g)}{(\eta h - \gamma(e) g)} \end{aligned}$$

since $(1-e)\,\gamma'(e) = 1 - \gamma(e)$ we get

$$\begin{aligned} 1 - (1-e)\,\frac{\sigma'(e)}{\sigma(e)} &= 1 - \frac{\eta h - g}{\eta h - \gamma(e) g} \\ &= \frac{\eta h - \gamma(e) g - \eta h + g}{\eta h - \gamma(e) g} = \frac{g - \gamma(e) g}{\eta h - \gamma(e) g} = \frac{1}{\sigma(e)}. \end{aligned}$$

Consequently, $\sigma(e) - (1-e)\,\sigma'(e) = 1$.