# A labor market with targeted wage offers

József Sákovics*

The University of Edinburgh

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**Abstract**

We model a market for highly skilled workers, such as the academic job market. The outputs of firm-worker matches are heterogeneous and common knowledge. Wage setting is synchronous with search: firms simultaneously make one personalized offer each to the worker of their choice. With large frictions (delay costs), efficient coordination is not possible, but for small frictions efficient matching with Diamond-type monopsony wages is an equilibrium.

## 1 Introduction

We consider labor markets for professionals, who are either finishing their training, or their current performance is public information. They could be doctors, lawyers, MBAs, PhDs, fund managers, athletes, musicians, chefs etc. The common (stylized) characteristic of these markets is that the productivity of workers is both identity and match dependent. This feature not only has implications for the efficiency properties of the match, but also has important consequences for the microstructure of the operation of the decentralized labor

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market. Indeed, in our model – as often in real life[1] – the firms choose who to make a personalized job offer to. This procedure would seem out of place if workers were homogeneous.

Our market is liable to suffer from two types of inefficiencies, caused by market imperfections:[2] the coexistence of unfilled vacancies and qualified job seekers (*frictional* unemployment); and *mismatch*, where workers could be reassigned to different jobs in a way to increase aggregate production.[3]

The recently Nobelized search and matching theory has been the standard – and rather successful – method for the analysis of labor markets, both theoretically and empirically.[4] Our contribution belongs to the family of complete information models within this literature. The sub-field can be split into two camps. One of them uses *ex post* wage setting: first firms and workers meet (according to some well-specified procedure, described via a *matching function*) and once they are matched they negotiate the wages. These models typically exhibit a hold-up-like feature, called the Diamond (1971) paradox: despite the existence of either unemployed workers or unfilled vacancies, the terms of trade (wages) are determined as if the negotiation among the matched parties was taking place in isolation, with no outside opportunities, no matter how inexpensive it is to switch partners. The alternative family of models has *ex ante* wage setting, where the firms commit to wage offers before the matching occurs (see Butters (1977), Montgomery (1991), Peters (1991) and their followers[5]). Here hold up is no longer a problem and the matching process is also more interesting, as now the workers can condition

[1] Even if actually workers apply first, they typically use "blanket" application strategies, which effectively give the relevant choice over to the firms.

[2] Of course, there are many other inefficiencies associated with the labor market, like structural unemployment, discrimination, distortions caused by labor laws etc.. However, these are not caused by the market institution itself and hence are not subjects of this study.

[3] Note that this is a different definition of mismatch from Shimer's (2007), which is closer to structural unemployment, in a multimarket context.

[4] See Rogerson et al. (2005) and the Scientific Background on the Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2010, for surveys.

[5] There is also a small literature, started by McAfee (1993), on competing mechanism designers, where instead of wages, entire mechanisms (for wage determination) are posted by the firms.

their search strategy on the posted wages, what then feeds back into the competition among firms. The principal novelty we introduce[6] is that wages are not determined either before or after the matching. Rather, we have *synchronous* wage setting: each firm with a vacancy has to "address" its offer to a worker, thereby choosing wages and matches at the same time.

We start with the derivation of the unique sub-game perfect equilibrium for the case of two firms (and at least two workers). When both firms would prefer to hire the same worker and the discount factor is low, the equilibrium involves "double mixing:" mixed strategies are used both to select the worker to target and for the wage offered to the better worker. Due to the double mixing, the outcome exhibits both (temporary) frictional unemployment and (permanent) mismatch. Wages for the sought after worker are drawn from an interval. Its lower bound is shown to be her continuation value when both firms have offered to her. The upper bound is the lowest competitive wage.

As the discount factor rises, the upper end of the support of the wage distribution for the top worker does not rise above the lowest competitive wage, despite her increased bargaining power and despite the fact that with a higher discount factor firms are more willing to "poach"; as, if they are unsuccessful, they still have a significant continuation value. The reason for this is that the weaker firm still has the outside option of hiring the weaker worker, which limits how much it is willing to bid for the better worker. The better worker's improved bargaining position manifests itself instead in that the lower bound of her equilibrium wage distribution increases.

When the discount factor is sufficiently high, the equilibrium undergoes a metamorphosis: the weaker firm gives up on trying to compete, and the equilibrium is an efficient matching with monopsony wages. While efficient matching when frictions are still present is remarkable, even more striking is that the equilibrium has a distinct Diamond (1971) paradox flavor: we have a nearly frictionless decentralized market leading to the monopsony prices. The underlying logic is entirely different though, as we explain below, it has nothing to do with the hold-up scenario.

[6]See our discussion of Konishi and Sapozhnikov (2008) below.

Let us return to the efficient strategy profile, where each firm makes an exclusive offer to its corresponding worker and hence wages are the monopsony ones. At first glance, one would think that this could not constitute an equilibrium. If both firms offer zero wages then there seems to exist a profitable deviation where the weaker firm offers $\varepsilon$ to poach its preferred worker. However, outbidding your competitor is not sufficient to obtain the services of a worker. It is also necessary that the worker be willing to accept the highest wage. As it happens, the fact that the worker was willing to accept zero in the putative equilibrium does not imply that she would also accept a deviant offer of $\varepsilon > 0$. The difference is that, in the first case, rejecting the offer would only delay the inevitable, as no other firm would be around to put an upward pressure on the wage. However, following the deviation, rejecting *both* offers would lead to a subgame where there are still two firms left. The continuation value of the top worker following such a double rejection is the lower bound of the mixing distribution, which we show to approach the (lowest) competitive wage as the discount factor tends to 1. This effect would make the incentive to poach disappear exactly at the limit (by the very definition of the competitive wage).

Our model also includes a "vetting" cost, which plays an important role here. This cost is incurred only once, as the first binding offer is made to a worker. As a result, if – following a deviation by the weaker firm – the better worker receives two offers, her continuation value is that of a game with these two firms, where the vetting cost of (only) this worker has already been incurred by both firms. Such a game is biased in favor of the better worker, as firms now need to pay a vetting cost to make an offer to the weaker worker but not if they continue to bid for her. As a result, the upper bound of the wage distribution for the best worker shifts up by the value of the vetting cost. That is, in the continuation game the highest possible wage offers are strictly higher than in the first period. As the collapse of the mixing interval on the upper bound happens here as well, a high enough discount factor leads to a situation where the continuation value of the better worker is higher than the lowest competitive wage, the highest wage the weaker firm is willing to pay her in the first period. Consequently, poaching cannot happen and we end up with the Diamond equilibrium.

It is remarkable that it is exactly the improvement in the workers' bargaining position that leads to an equilibrium with the lowest possible wages. Because the workers are so powerful when there is competition for them, the firms shy away from competition. Workers would benefit from being able to commit to accepting below competitive wages.

The characterization of equilibria becomes exceedingly difficult as the number of firms grows. Nevertheless, we show that the Diamond outcome continues to be an equilibrium for an arbitrary number of firms, if the discount factor is sufficiently high. We can do that because in the continuation following a unilateral deviation by a firm there are always only two firms left – since all the others will have traded according to the equilibrium strategies –, which is exactly the situation we have already characterized.

We also show that to obtain the above result it does not matter the number of vacancies firms have; or whether there are more workers than firms; or whether the workers can hold on to an offer or not.

Finally, we also look at the case where firms can commit not to make a second offer to the same worker. We show that the Diamond equilibrium is no longer possible, as the combination of commitment and lack of direct competition eliminates the high continuation value for a worker who receives two offers. When there are only two firms, the equilibrium is the same as without commitment (and low discount factor), with the only difference that now workers have a zero continuation value, so the support for the wage distribution starts at the worker's outside option, leading to a lower expected wage for the better worker.

## 1.1 A brief review of the closely related literature

The most relevant direct precursor to this contribution is De Fraja and Sákovics (2001). In that paper we allowed for many-to-one matching (together with *ex post* price determination) that potentially created local market conditions that reversed the aggregate ones. We have shown how this could affect the performance of a decentralized market. However, the matching function was exogenously given there. In this paper we endogenize who matches with whom, while maintaining the possibility of market power reversal. In the literature with *ex ante* wage setting mentioned above, not only is there no reversal, but one side of the market sets the conditions of trade and the other chooses who to attempt to trade with. In the current model the same side of the market takes both decisions, thereby changing the nature of competition.

Shi (2001) also presents a model with two-sided heterogeneity, where firms set wages and they can specify the type of worker they would like to hire. The equilibrium is efficient and involves no competition for workers. His model differs from ours in two major respects: First, there is a large number of workers of each skill level. Consequently, targeting a skill level does not imply targeting an individual. Second, there is free entry of firms, leading to zero profits in equilibrium. This makes it easy to discourage poaching.

Bulow and Levin (2006) analyze the special case of our job market where the value of a match is the product of the worker and firm productivities. They consider universal wages: a firm must hire the best worker that shows up for the wage it has advertised. While this is the opposite of targeting, their model provides an interesting benchmark to compare our results to. Their unique (mixed strategy) equilibrium exhibits some mismatch but no frictional unemployment. Wages are not only infra-competitive but they are compressed: the better the worker the farther below competitive his wage is. Importantly, due to the relatively high efficiency of the matching, the firms benefit from the losses of the workers: they earn ultra-competitive profits.

The closest paper to ours is Konishi and Sapozhnikov (2008). Though they do not have the same motivation, they also present a model with targeted offers – in the context of an abstract assignment problem and assuming a supermodular output matrix. The dynamic variant of the model of Konishi and Sapozhnikov (2008) is cleverly set-up in a way that avoids simultaneous competition in equilibrium. By assuming that offers to a worker are made once and for all and that there is no cost of delay, they are able to construct (pure strategy) equilibria where only a single firm makes an offer in each period. Note that their assumptions amount to giving the last word to the firm moving later, implying that wage competition for a worker cannot occur, as whoever attempts to overbid a follower will be matched by it anyway and hence will not be able to hire that worker. The main point of our model is to draw attention to the intrinsic interest of (endogenous) instantaneous local competition in the dynamic context, which was finessed by Konishi and Sapozhnikov (2008).

Finally, we should mention that there exist centralized models of labor matching markets which involve firms targeting workers and endogenous wages.[7] The pioneer work in this area is Crawford and Knoer (1981). Their model requires that a firm – myopically – always offer to its most preferred worker at the "going" wage vector, thereby enforcing competition and ensuring a competitive outcome.

# 2 The model

There are $M$ firms, each with a single vacancy, and $N \geq M$ workers,[8] each looking for a job. It is common knowledge that the joint output of Firm $i$ and Worker $j$ would be $p_{ij} > 0$. We make no restriction on the output matrix, except that of genericity:

**Assumption 1** *There exists a unique matching of workers to firms that maximizes aggregate output.*

If Firm $i$ hires Worker $j$ at wage $w_{ij}$ then the firm's payoff is $p_{ij} - w_{ij}$, while the worker obtains $w_{ij}$. For convenience, the outside options of both firms and workers are normalized to zero.

The market operates as follows. In period 1, simultaneously and independently, each firm makes a single offer to the worker of their choice. It costs $c > 0$ to approach a worker for the first time. Any subsequent offers to the same worker are free.[9] We assume that $c < \min p_{ij}$,

[7] We consider the large body of models with non-transferable utility too far removed to discuss them in this short overview.

[8] We analyze the case of $N < M$ in Section 5.1.

[9] Think of $c$ as the administrative cost of vetting a worker. We could easily extend the model to endogenize a reason for vetting. Say, there is a small probability that the candidate is not suitable. For small vetting cost, the optimal policy would be to vet candidates with a probability high enough so that an unsuitable candidate

so that it does not discourage any match. The workers who receive (one or more) offers either accept one of those (in which case the firm whose offer has been accepted and the worker exit the market) or reject all offers. In the subsequent periods, the firms with unfilled vacancies keep making offers to the available workers until all vacancies get filled. Firms and workers discount the future by the common discount factor $\delta \in [0, 1)$.

We start with the analysis of the "simple" case of only two firms:

# 3 Duopsony

By Assumption 1 there is a unique efficient matching. There are two possible cases: either both firms weakly prefer their partner in the efficient matching to the other one or not. In the first case firms have no incentive to compete, so we have a unique equilibrium with the efficient matching and zero prices. Therefore, we will concentrate our analysis on the alternative scenario. Let us denote the firm whose preferences agree with efficiency by $H$ and the other one by $L$. Also let the efficient partner of $H$ be denoted by $h$, and the efficient partner of $L$ by $l$. Thus we have that

$$p_{Ll} + p_{Hh} > p_{Lh} + p_{Hl} \tag{1}$$

and

$$p_{Hh} > p_{Hl},\ p_{Lh} > p_{Ll}. \tag{2}$$

We start our analysis by establishing the competitive benchmark: the hypothetical outcome in a centralized, frictionless market. The defining characteristic of such an equilibrium is that – taking the equilibrium wages as given – no firm would strictly prefer to hire a worker different from the one it hires in equilibrium.

would be indifferent to chance getting caught (which would mean a utility loss for him). In that case $c$ would be the expected vetting cost (and with positive probability unsuitable workers would be hired). As long as suitability were independent of productivity, interpreting $N$ as the realized number of workers, there would be no change in the equilibrium strategies.

**Proposition 1** *In all competitive equilibria the matching is efficient. Moreover, in the lowest-wage competitive equilibrium wages are* $w_l^c = 0$ *and* $w_h^c = p_{Lh} - p_{Ll}$.

*Proof:* First, note that both firms must hire a worker in competitive equilibrium. Next, note that no firm will hire a worker who is not included in the efficient matching. To see this, note that otherwise there would be a worker included in the efficient matching who did not get hired. This worker and the firm who hired the worker off the efficient matching would both be better off trading with each other at the wage paid to the worker off the efficient matching. Next, we show that matching must be positively assortative (PAM). Assume to the contrary that Firm $L$ hires Worker $h$ (and thus Firm $H$ hires Worker $l$). For this to be a competitive equilibrium, we would need that no traders would like to switch partners at the going wages: $p_{Ll} - w_{Hl} \leq p_{Lh} - w_{Lh}$ and $p_{Hh} - w_{Lh} \leq p_{Hl} - w_{Hl}$, implying $p_{Hh} - p_{Hl} \leq w_{Lh} - w_{Hl} \leq p_{Lh} - p_{Ll}$, contradicting Assumption 1. Hence we must have PAM in equilibrium. Using the same logic as before, PAM implies $p_{Lh} - p_{Ll} \leq w_{Hh} - w_{Ll} \leq p_{Hh} - p_{Hl}$. Noting that the lowest individually rational salary for Worker $l$ is zero completes the proof. Q.E.D.

Now, the equilibrium in our decentralized market:

**Proposition 2** *The two-firm game has a generically unique subgame-perfect equilibrium (SPE). There exists a well-defined value,* $\underline{w} \in [0, w_h^c + c)$*, such that*

*i) if* $\underline{w} \leq w_h^c$*:* $L$ *with probability* $\Pi_l^L = \frac{p_{Hh} - \delta(p_{Hl} - c) - w_h^c}{p_{Hh} - \delta(p_{Hl} - c)}$*, offers a zero wage to* $l$*, while with the remaining probability it makes an offer to* $h$*, mixing with* $F_h^L(x) = \frac{\Pi_l^L}{1 - \Pi_l^L} \cdot \frac{x}{p_{Hh} - \delta(p_{Hl} - c) - x}$ *over the interval* $[\underline{w}, w_h^c]$*;* $H$ *makes an exclusive offer to* $h$*, offering zero with probability* $Z = \frac{p_{Ll}(1-\delta) + \delta c}{p_{Lh} - \delta(p_{Ll} - c) - \underline{w}}$ *and with the remaining probability mixing with* $F_h^H(x) = \frac{Z}{1 - Z} \cdot \frac{x - \underline{w}}{p_{Lh} - \delta(p_{Ll} - c) - x}$ *over the interval* $[\underline{w}, w_h^c]$*.* $h$ *accepts the highest offer she receives.* $l$ *accepts the offer if he receives it. Any firm that does not hire in the first period, hires* $l$ *for zero in the second.*

*ii) if* $\underline{w} \geq w_h^c$ *: both firms offer zero to their efficient match and these offers are accepted.*

*Proof:* First, note that in equilibrium no firm will make an offer that it knows will be rejected, as both the direct and indirect costs of delay are positive and there is no option value in waiting.

Next, observe that in equilibrium $H$ will bid exclusively for its favorite worker, $h$. To see this, note that either $L$ is bidding exclusively for $l$ and hence $h$ could be hired for free (as $L$ would hire $l$, so $h$ has no credible threat of rejecting) what is the best possible outcome for $H$; or $L$ bids for $h$ with positive probability. If $L$ is bidding for $h$ only, then $H$ could hire $l$ for free, earning $p_{Hl}$, while outbidding[10] $L$ by $\varepsilon$ it could get, $p_{Hh} - \overline{w}_{Lh} - \varepsilon$, where we denote the highest wage $L$ offers to $h$ in equilibrium by $\overline{w}_{Lh}$. As $L$ prefers to bid for $H$, we must have $p_{Lh} - \overline{w}_{Lh} \geq p_{Ll}$. Together with (1) this implies that $p_{Hh} - \overline{w}_{Lh} > p_{Hl}$, so $H$ strictly prefers to bid for $h$. Finally, consider the case where $L$ is mixing over the target of its offer. This would weaken the option of bidding for $l$ – higher wage needs to be paid – and strengthen it for $h$ – as there is not always competition for her.

If $L$ does not bid for $h$, then $H$'s best response is to bid zero for $h$. This can only form part of an equilibrium if any wage that $L$ would be willing to hire $h$ for – namely, $w_{Lh} \leq p_{Lh} - p_{Ll}$ – would be rejected by $h$. We will return later to this possibility. For the moment, let us hypothesize that $L$ bids for $h$ with positive probability.

If $L$ bids for $h$ with positive probability then both $L$ and $H$ must use a mixed strategy for their wage offers to $h$ (recall that the workers go with the more productive firm in case of equal wage offers). Standard arguments imply that both firms must mix on the same support, which we denote by $[\underline{w}, \overline{w}]$, except that $H$ may also bid zero – possibly outside of this interval – in the hope that it is the only bidder (because $L$ is bidding for $l$).[11] It is straightforward to see that the only additional possible mass points in the strategies are at $\underline{w}$ for $L$ (and only if $H$ puts positive probability on zero) and $\overline{w}$ for $H$ (as a mass point there for $L$ could be simply outbid by $H$).

We start by hypothesizing that $H$ strictly prefers not to bid zero. In equilibrium, $H$ will obtain the services of $h$, if $L$ either does not bid for her (what happens with probability $\widehat{\Pi}_l^L$) or it offers no more than what $H$ does. If $H$ loses out in the first period, it will hire $l$ in the second

[10] If $h$ prefers $H$ no outbidding is necessary, matching the highest offer is sufficient.

[11] If $h$ receives a single offer (from $H$) then $l$ will accept his offer (from $L$), so in the continuation $h$ will be left alone with $H$, expecting a wage of zero. Consequently, the zero offer would be accepted in equilibrium.

period (for zero, as it will face no competition). When $H$ offers the maximum of the common support, $\overline{w}$, then it wins for sure. As it must be indifferent among all bids in the support of its strategy, the following equality must hold for all $x \in [\underline{w}, \overline{w}]$: $(p_{Hh} - x)\left[\widehat{\Pi}_l^L + \widehat{\Pi}_h^L \widehat{F}_h^L(x)\right] + \widehat{\Pi}_h^L\left[1 - \widehat{F}_h^L(x)\right]\delta\left(p_{Hl} - c\right) = p_{Hh} - \overline{w}$. Rearranging the equation, we obtain

$$\widehat{\Pi}_l^L + \widehat{\Pi}_h^L \widehat{F}_h^L(x) = \frac{p_{Hh} - \delta(p_{Hl} - c) - \overline{w}}{p_{Hh} - \delta(p_{Hl} - c) - x}. \tag{3}$$

Now, observe that $\widehat{F}_h^L(\underline{w})$ must be zero, since a bid of $\underline{w}$ could only win against the same offer by $H$, but the best response of $H$ to a mass point would be never to bid $\underline{w}$, leading to

$$\widehat{\Pi}_l^L = \frac{p_{Hh} - \delta(p_{Hl} - c) - \overline{w}}{p_{Hh} - \delta(p_{Hl} - c) - \underline{w}}. \tag{4}$$

As $L$ could hire $l$ for free, its bid for $h$ is capped at $p_{Lh} - p_{Ll}$. Consequently, by (1), $\overline{w} \leq p_{Lh} - p_{Ll} < p_{Hh} - p_{Hl} < p_{Hh} - \delta p_{Hl}$, so $\widehat{\Pi}_l^L > 0$ : $L$ makes an offer to $l$ with positive probability as well.

Given that $L$ is making an offer to both workers with positive probability, it must be indifferent between making an offer to either of them. As it faces no competition for $l$, it can hire him for zero, leading to $(p_{Lh} - x)\,\widehat{F}_h^H(x) + \left[1 - \widehat{F}_h^H(x)\right]\delta(p_{Ll} - c) = p_{Ll} \Leftrightarrow \widehat{F}_h^H(x) = \frac{p_{Ll}(1-\delta)+\delta c}{p_{Lh} - \delta(p_{Ll} - c) - x}$ for $x \in (\underline{w}, \overline{w})$. Substituting $x = \underline{w}$ we obtain that $\widehat{F}_h^H(\underline{w}) = \frac{p_{Ll}(1-\delta)+\delta c}{p_{Lh} - \delta(p_{Ll} - c) - \underline{w}}$. Note that this value is positive, as $\underline{w} < \overline{w} \leq p_{Lh} - p_{Ll} < p_{Lh} - \delta p_{Ll}$. This would mean that $H$ makes an offer no greater than $\underline{w}$ with positive probability, which rationally can only be an offer of zero, contradicting the hypothesis that it strictly prefers not to offer zero.

We thus know that in equilibrium $H$ weakly prefers to offer zero to $h$. We drop the "hats" of $F$ and $\Pi$ to capture the change in strategy and denote the probability of making an offer of zero by $Z$. As we have seen above, $H$ must mix, so $Z < 1$.

$H$ has to be indifferent between bidding zero (when it only wins if $L$ does not bid for $h$, and otherwise it hires $l$ next period) and $\overline{w}$ (when it wins for sure), so we must have that $p_{Hh}\Pi_l^L + \left(1 - \Pi_l^L\right)\delta(p_{Hl} - c) = p_{Hh} - \overline{w} \Rightarrow$

$$\Pi_l^L = \frac{p_{Hh} - \delta(p_{Hl} - c) - \overline{w}}{p_{Hh} - \delta(p_{Hl} - c)} > 0. \tag{5}$$

By the same token, (3) – without "hats" as we have established that $H$ bids zero with positive probability – must also hold for all $x \in [\underline{w}, \overline{w}]$. Solving for the mixing distribution we have

$$F_h^L(x) = \frac{p_{Hh} - \delta(p_{Hl} - c) - \overline{w}}{\overline{w}} \cdot \frac{x}{p_{Hh} - \delta(p_{Hl} - c) - x} \in (0, 1]. \tag{6}$$

Given that $L$ is making an offer to $l$ with positive probability (see (5)), it must be indifferent between making an offer to either worker. As it faces no competition for $l$, it can hire him for zero, leading to $(p_{Lh} - x)\left(Z + F_h^H(x)(1 - Z)\right) + (1 - Z)\left(1 - F_h^H(x)\right)\delta(p_{Ll} - c) = p_{Ll} \Leftrightarrow$

$$(1 - Z)F_h^H(x) = \frac{p_{Ll}(1 - \delta) + \delta c}{p_{Lh} - \delta(p_{Ll} - c) - x} - Z, \tag{7}$$

for $x \in (\underline{w}, \overline{w})$. If there is no mass point at the upper end of $H$'s strategy, $\lim_{x \to \overline{w}} F_h^H(x) = 1$, then the formula still applies and we obtain that $\overline{w} = p_{Lh} - p_{Ll}$. If there were a mass point, then in order to keep $L$ from overbidding it must be that for all $\varepsilon > 0$, $p_{Lh} - \overline{w} - \varepsilon < p_{Ll}$ $\Leftrightarrow \overline{w} \geq p_{Lh} - p_{Ll}$, which when applied to the formula for $\lim_{x \to \overline{w}} F_h^H(x)$, implies again that $\overline{w} = p_{Lh} - p_{Ll}$ and $F_h^H(\overline{w}) = 1$, therefore no mass point is possible. From (5), substituting in for the upper bound, we obtain that $\Pi_l^L = \frac{p_{Hh} - \delta(p_{Hl} - c) - w_h^c}{p_{Hh} - \delta(p_{Hl} - c)}$.

When $L$ bids the lower bound of its support, it can only win if $H$ is bidding zero. Hence, we have that $(p_{Lh} - \underline{w})\, Z +\ (1 - Z)\delta(p_{Ll} - c) = p_{Ll}$, from which we can solve for $Z = \frac{p_{Ll}(1 - \delta) + \delta c}{p_{Lh} - \delta(p_{Ll} - c) - \underline{w}} \in (0, 1)$. Substituting in (7), we obtain

$$F_h^H(x) = \frac{x - \underline{w}}{w_h^c - \underline{w}} \cdot \frac{p_{Ll} - \delta(p_{Ll} - c)}{p_{Lh} - \delta(p_{Ll} - c) - x}.$$

All we have left to do is to identify the lower bound of the support of the mixed strategies. Observe that – by the single deviation principle – this has to equal the (discounted) expected continuation value of $h$, when she receives two offers[12] and hence expects both firms to be still in the market in the next period.

$$\frac{\underline{w}}{\delta} = \widetilde{Z}\widetilde{\Pi}_h^L \widetilde{F}_h^L(\underline{w})\underline{w} + \int_{\underline{w}}^{\widetilde{w}} x \left[\widetilde{f}_h^H(x)(1 - \widetilde{Z})\left(\widetilde{\Pi}_l^L + \widetilde{\Pi}_h^L \widetilde{F}_h^L(x)\right) + \widetilde{\Pi}_h^L \widetilde{f}_h^L(x)(\widetilde{Z} + (1 - \widetilde{Z})\widetilde{F}_h^H(x))\right] dx. \tag{8}$$

[12] Whenever $L$ offers to $h$, she will receive two offers, so this is the relevant scenario for the determination of the lower bound of $L$'s bidding distribution.

Note that the probability distributions (and $\overline{w}$) carry a tilde. This is because following two bids for $h$, no vetting cost will have to be paid to make a new offer to $h$, tilting the competition in favor of $h$ and slightly modifying the formulas. It is crucial to observe though, that $\underline{w}$ is invariant, as it is only defined following a history where both firms have paid their vetting costs exclusively for $h$.

It is straightforward to see that up to (5) and (6) everything remains the same (except for the substitution of $\widetilde{w}$ for $\overline{w}$) even after a sunk vetting cost for $h$. On the other hand, (7) becomes $(1-\widetilde{Z})\widetilde{F}_h^H(x) = \frac{(p_{Ll}-c)(1-\delta)}{p_{Lh}-\delta(p_{Ll}-c)-x} - \widetilde{Z}$, which in turn implies that $\widetilde{w} = w_h^c + c$, which then leads to $\widetilde{\Pi}_l^L = \frac{p_{Hh}-\delta(p_{Hl}-c)-\widetilde{w}}{p_{Hh}-\delta(p_{Hl}-c)}$ and $\widetilde{\Pi}_h^L = \frac{\widetilde{w}}{p_{Hh}-\delta(p_{Hl}-c)}$. Similarly we have $\widetilde{Z} = \frac{(p_{Ll}-c)(1-\delta)}{p_{Lh}-\delta(p_{Ll}-c)-\underline{w}}$ and $\widetilde{F}_h^H(x) = \frac{x-\underline{w}}{\widetilde{w}-\underline{w}} \cdot \frac{(p_{Ll}-c)(1-\delta)}{p_{Lh}-\delta(p_{Ll}-c)-x}$.

Substituting into (8), we have

$$\begin{aligned}
\frac{\underline{w}}{\delta} &= \frac{(p_{Ll}-c)(1-\delta)}{p_{Lh}-\delta(p_{Ll}-c)-\underline{w}} \cdot \frac{p_{Hh}-\delta(p_{Hl}-c)-\widetilde{w}}{p_{Hh}-\delta(p_{Hl}-c)} \cdot \frac{\underline{w}}{p_{Hh}-\delta(p_{Hl}-c)-\underline{w}}\underline{w} + \\
&\int_{\underline{w}}^{\widetilde{w}} \frac{(p_{Ll}-c)(1-\delta)x}{(p_{Lh}-\delta(p_{Ll}-c)-x)^2} \cdot \frac{p_{Hh}-\delta(p_{Hl}-c)-\widetilde{w}}{p_{Hh}-\delta(p_{Hl}-c)-x} dx + \\
&\int_{\underline{w}}^{\widetilde{w}} \frac{p_{Hh}-\delta(p_{Hl}-c)-\widetilde{w}}{(p_{Hh}-\delta(p_{Hl}-c)-x)^2} \cdot \frac{(p_{Ll}-c)(1-\delta)x}{p_{Lh}-\delta(p_{Ll}-c)-x} dx.
\end{aligned} \tag{9}$$

After a bit of work[13], this simplifies to the following equation:

$$\begin{aligned}
0 &= \frac{\underline{w}\,(p_{Hh}-p_{Lh}-\delta(p_{Hl}-p_{Ll}))}{\delta\,(p_{Ll}-c)(1-\delta)\,(p_{Hh}-\delta(p_{Hl}-c)-\widetilde{w})} - \\
&\frac{\underline{w}^2\,(p_{Hh}-p_{Lh}-\delta(p_{Hl}-p_{Ll}))}{(p_{Lh}-\delta(p_{Ll}-c)-\underline{w})\,(p_{Hh}-\delta(p_{Hl}-c))\,(p_{Hh}-\delta(p_{Hl}-c)-\underline{w})} - \\
&\frac{(\widetilde{w}-\underline{w})\,(p_{Lh}-\delta(p_{Ll}-c))}{(p_{Lh}-\delta(p_{Ll}-c)-\underline{w})\,(p_{Lh}-\delta(p_{Ll}-c)-\widetilde{w})} + \\
&\frac{(\widetilde{w}-\underline{w})\,(p_{Hh}-\delta(p_{Hl}-c))}{(p_{Hh}-\delta(p_{Hl}-c)-\underline{w})\,(p_{Hh}-\delta(p_{Hl}-c)-\widetilde{w})} - \\
&\ln\frac{(p_{Lh}-\delta(p_{Ll}-c)-\widetilde{w})\,(p_{Hh}-\delta(p_{Hl}-c)-\underline{w})}{(p_{Hh}-\delta(p_{Hl}-c)-\widetilde{w})\,(p_{Lh}-\delta(p_{Ll}-c)-\underline{w})}.
\end{aligned} \tag{10}$$

The right-hand side of (10) is a continuous function of $\underline{w}$, outside of $[p_{Lh}-\delta(p_{Ll}-c), p_{Hh}-\delta(p_{Hl}-c)]$ where it is not defined. Routine calculations show[14] that it is increasing for

[13]Details are in the Mathematical Appendix.

[14]Details are in the Mathematical Appendix.

$\underline{w} < p_{Lh} - \delta(p_{Ll} - c)$, and that it takes a negative value at $\underline{w} = 0$ and a positive value at $\underline{w} = \widetilde{w}$. Consequently, there is a unique feasible solution.

Finally, note that when $\underline{w} \geq w_h^c$, $L$ has no (strict) incentive to bid for $h$, and as a result we get the efficient equilibrium with zero wages, as discussed above. When $\underline{w} = w_h^c$, both equilibria exist. Q.E.D.

The two possible equilibrium configurations are strikingly different. One displays both frictional unemployment – as, because of $L$'s mixing over targets, $l$ may not receive an offer in the first period – and mismatch – as, because of the mixed wage offers to $h$, $L$ may end up hiring her. It is reminiscent of the equilibrium of *ex ante* wage setting, as analyzed in Bulow and Levin (2006). The other configuration is fully efficient, but leaves zero surplus for the workers. Much along the lines of the equilibrium of *ex post* wage setting. Both outcomes give below competitive expected wages to the better worker.

It is worthwhile to note that the mixed equilibrium when $\underline{w} = w_h^c$ is not the Diamond equilibrium, though it is only singly mixed, as the bids for $h$ become pure strategy bids: $L$ bids $w_h^c$ and $H$ bids zero. As $L$ is still mixing over the target of its offer – and when it bids for $h$ it wins for certain – this equilibrium continues to be inefficient.

Of course, the million-dollar question is: when, if ever, is $\underline{w} \geq w_h^c$? The following corollary gives the almost complete[15] answer.

**Corollary 1** *There exist* $0 < \delta^* \leq \delta^{**} < 1$, *such that*

**i)** *for all* $\delta < \delta^*$ *the unique SPE of the two-firm game is the mixed equilibrium identified in Proposition 2;*

**ii)** *for all* $\delta > \delta^{**}$ *the unique SPE of the two-firm game is efficient, with wages equal to the workers' outside options.*

*Proof:* Note that the equation defining $\underline{w}$, (10), is of the form $g(\underline{w}, \delta, A) = 0$, where $A$ stands for the rest of the parameters. In the range $\underline{w} \in (0, p_{Lh} - \delta(p_{Ll} - c))$, which includes

[15] We conjecture that $\delta^* = \delta^{**}$, but have not been able to prove it yet.

$(0, w_h^c + c)$ as $c < p_{Ll}$, $g$ is continuous in $\delta$, which implies that so is $\underline{w}(\delta)$. Consequently, it is sufficient to show that $\lim_{\delta\to 1} \underline{w}(\delta) > w_h^c$ to prove *ii)*. We actually show that $\lim_{\delta\to 1} \underline{w}(\delta) = w_h^c + c$. To see this, assume to the contrary that $\lim_{\delta\to 1} \underline{w}(\delta) < w_h^c + c$. That would imply that $\lim_{\delta\to 1} \widetilde{Z}(\delta) = 0$. If $H$ never bids zero then it cannot be part of an equilibrium for $L$ to have a mass point on the lower bound of its bidding range for $h$, as such a bid loses with probability one. However, $\widetilde{F}_h^L(\underline{w}) = \frac{p_{Hh}-\delta(p_{Hl}-c)-\widetilde{w}}{\widetilde{w}} \cdot \frac{\underline{w}}{p_{Hh}-\delta(p_{Hl}-c)-\underline{w}}$ does have such a mass point for any $\underline{w} > 0$, which leads to a contradiction.

To see *i)* just note from (8) that $\underline{w}$ is the product of $\delta$ and a number no greater than $\widetilde{w}$. Q.E.D.

That is, the relevant parameter is the discount factor. With patient players we have the efficient equilibrium, with impatient ones the inefficient one. In the situations mentioned in the introduction, we would expect the players to be rather patient, so the prediction favors the Diamond equilibrium.

### 3.1 A numerical example

In order to provide a better feel for the nature of the equilibrium, we provide a numerical example. Let $p_{Hh} = 15,\ p_{Hl} = 10,\ p_{Lh} = 9,\ p_{Ll} = 6,\ c = 1,\ \Rightarrow \overline{w} = 3$.

Efficient surplus: $15 + 6 - 2 \times 1 = 19$.

Mismatch surplus: $10 + 9 - 2 \times 1 = 17$.

The following table displays the main features of the equilibrium for different values of the discount factor:

| $\boldsymbol{\delta}$ | $\Pi_l^L$ | $\mathbf{F}_h^L(\underline{\mathbf{w}})$ | $\mathbf{Z}$ | $\underline{\mathbf{w}}$ | $2\mathbf{Z}\Pi_h^L$ | $\Pi_h^L\left(\left(1-\boldsymbol{\delta}\right)\mathbf{Ll}+c\delta\right)$ |
|---|---|---|---|---|---|---|
| 0 | .80 | 0 | .67 | 0 | .26 | 1.2 |
| .5 | .71 | .27 | .64 | 1.03 | .38 | 1.02 |
| .8 | .62 | .63 | .71 | 2.20 | .54 | .76 |
| .9 | .57 | .89 | .89 | 2.81 | .77 | .65 |
| .92 | .55 | .98 | .97 | 2.96 | .85 | .66 |
| .93 | 1 | – | 1 | 3.04 | 0 | 0 |

The last two columns of the table are lower bounds[16] on the dead-weight loss due to mismatch and frictional unemployment, respectively.

# 4 The general case

The characterization of equilibria for a large number of firms is very complicated. As there are multiple offers received by many workers with positive probability, way too many subgames are possible to allow a clean analysis.

Short of a full characterization, what we are really interested in is whether Corollary 1 generalizes to an arbitrary number of firms (and workers). While proving uniqueness has eluded us, we can answer in the affirmative: indeed, the Diamond equilibrium does (only) exist in general for a high enough discount factor.

**Proposition 3** *There exist* $0 < \widehat{\delta} \leq \widehat{\widehat{\delta}} < 1$, *such that the general dynamic game has*

**i)** *a SPE which is efficient, with wages equal to the workers' outside options, if either the efficient match is the preferred match of all firms or* $\delta \geq \widehat{\widehat{\delta}}$*;*

**ii)** *no efficient SPE, if* $\delta < \widehat{\delta}$ *and the efficient match is not the preferred match of all firms.*

*Proof:* For convenience, we relabel the firms so that the efficient matching involves Firm $i$ hiring the worker with the same index. Let us start with i). Consider a deviation by Firm

[16] We ignore the additional inefficiency due to the mixing of offers.

$J$, making an offer to worker $k \neq j$. Since the equilibrium wages are zero, this can be only profitable if it prefers $k$ to $j$: $p_{jk} > p_{jj}$. In order for $k$ to accept, it is not enough to make him a positive offer. Rather, he has to be offered his continuation value in the subgame with Firms $J$ and $K$ and Workers $j$ and $k$ (as, following the equilibrium strategies the rest of the firms will have hired in the first period). Note that, since the putative equilibrium matching is strictly efficient, $p_{jk} > p_{jj}$ implies that $p_{kk} > p_{kj}$ and thus Proposition 2 applies, with Firm $K$ playing the role of $H$. Therefore, by the proof of Corollary 1, the continuation value of Worker $k$ exceeds $p_{jk} - p_{jj}$, as $\delta \to 1$. Thus, for $\delta$ high enough, Firm $J$'s deviation payoff conditional on Worker $k$ accepting is $p_{jk} - (p_{jk} - p_{jj} + \varepsilon) = p_{jj} - \varepsilon$, less than its equilibrium payoff, $p_{jj}$. We still need to check what happens if the deviant offer to Worker $k$ is unacceptable. In that case, Worker $k$ would reject both of his offers. In the continuation, by the proof of Corollary 1, Firm $J$ would end up hiring Worker $j$ for zero, just as in the putative equilibrium, but suffering a delay cost and an extra vetting cost. Hence there exists no profitable deviation for any firm.

If a worker rejected his equilibrium offer, next period he would be faced with the same firm, as all the other firms would have hired. He could not improve on his payoff – as any positive continuation payoff could be slightly undercut by the firm, and it would be in the worker's best interest to accept.

For ii), pick a firm who would prefer to hire a worker, which is not its efficient match. By Corollary 1i), for a discount factor low enough, it could make an acceptable offer to that worker, which would improve on its equilibrium payoff. Q.E.D.

Even in the absence of a uniqueness result, it is arguable that in a situation where the same firms face each other repeatedly, like the job markets we model, they would coordinate on the efficient equilibrium, which maximizes their aggregate welfare.

# 5 Variations

## 5.1 Workers' market

In the main text – for simplicity and realism – we have maintained the assumption that the number of firms did not exceed the number of qualified workers looking for a job. Here we show that the existence of the Diamond equilibrium does not require a firms' market, it exists in a workers' market just as well. As before, the main insight comes from the set-up following a unilateral deviation from the Diamond equilibrium, in this case a single worker (and several firms). The generalization follows the same arguments of Corollary 1 and Proposition 3 from there. As it will become clear the existence results are even stronger, as now we can identify a single threshold in $\delta$. On the other hand, when the productivities are close to each other, we can have an equilibrium, which is still at monopsony wages, but with an inefficient match. Of course, the actual efficiency loss is minimal, since the firms are of similar productivities.

Let us denote the firm that is most productive hiring the worker by $H$ and the second most productive firm by $L$. The corresponding outputs are $p_H$ and $p_L$.

**Proposition 4** *The one-worker-many-firms game has the following set of SPE:*

*i) if $\delta p_L \leq p_L - c$: $L$ with probability $\Pi^L = \frac{p_H - p_L + c}{p_H}$ does not make an offer, while with the remaining probability it mixes its offer with $F^L(x) = \frac{p_H - p_L + c}{p_L - c} \cdot \frac{x}{p_H - x}$ over the interval $[\delta p_L, p_L - c]$; $H$ offers zero with probability $Y = \frac{c}{(1-\delta)p_L}$ and with the remaining probability mixes with $F^H(x) = \frac{x - \delta p_L}{(1-\delta)p_L - c} \cdot \frac{c}{p_L - x}$ over the interval $[\delta p_L, p_L - c]$. The worker accepts the highest offer she receives;*

*ii) if $\delta p_L \geq p_L - c$: $H$, or any other Firm $i$ such that $\delta p_i \geq p_H - c$, makes the only offer, which is zero and is accepted.*

*Proof:* Let us begin the analysis, assuming that there are only two firms. Consider the subgame where both firms have made an offer. If the worker rejects both, in the continuation we have the equivalent of an asymmetric Bertrand competition (with a different tie-breaking rule). This leads to both firms offering $p_L$ with probability one, and the worker taking $H$'s offer.[17] Consequently, the worker's continuation value in this subgame is $\delta p_L$.

Let us return to the main game now. If $L$ does not bid, then $H$'s best response is to bid zero. This can form part of an equilibrium if and only if any wage that $L$ would be willing to pay – namely, $s_L \leq p_L - c$ – would be rejected by the worker.

If $L$ bids with positive probability then both $L$ and $H$ must use a mixed strategy for their wage offers (recall that the workers go with the more productive firm in case of equal wage offers). Standard arguments imply that both firms must mix on the same support, which we denote by $[\underline{s}, \overline{s}]$, except that $H$ may also bid zero – possibly outside of this interval – in the hope that it is the only bidder. It is straightforward to see that the only additional possible mass points in the strategies are at $\underline{s}$ for $L$ (and only if $H$ puts positive probability on zero) and $\overline{s}$ for $H$ (as a mass point there for $L$ could be simply outbid by $H$).

We start by hypothesizing that $H$ strictly prefers not to bid zero. In equilibrium, $H$ will obtain the services of the worker, if $L$ either does not bid for her (what happens with probability $\widehat{\Pi}^L$) or it offers no more than what $H$ does. If $H$ loses out in the first period, it earns zero. When $H$ offers the maximum of the common support, $\overline{s}$, then it wins for sure. As it must be indifferent among all bids in the support of its strategy, the following equality must hold for all $x \in [\underline{s}, \overline{s}]$: $(p_H - x)\left[\widehat{\Pi}^L + \left(1 - \widehat{\Pi}^L\right)\widehat{F}^L(x)\right] = p_H - \overline{s}$. Rearranging the equation, we obtain

$$\widehat{\Pi}^L + \left(1 - \widehat{\Pi}^L\right)\widehat{F}^L(x) = \frac{p_H - \overline{s}}{p_H - x}. \tag{11}$$

Now, observe that $\widehat{F}^L(\underline{s})$ must be zero, since a bid of $\underline{s}$ could never win against $H$, leading to

$$\widehat{\Pi}^L = \frac{p_H - \overline{s}}{p_H - x} > 0. \tag{12}$$

As $L$ is assumed to make an offer with positive probability (12) implies that it must be mixing between making an offer or not, and hence it must be indifferent. Therefore, $(p_L - x)\,\widehat{F}^H(x) - c = 0 \;\Leftrightarrow\; \widehat{F}^H(x) = \frac{c}{p_L - x}$. Substituting $x = \underline{s}$ we obtain that $\widehat{F}^H(\underline{s}) = \frac{c}{p_L - \underline{s}}$. Note that this value is positive, as $\underline{w} < \overline{w} \leq p_L - c$. This would mean that $H$ makes an offer no greater

[17] There is no mixing because the outside options are zero.

than $\underline{w}$ with positive probability, which rationally can only be an offer of zero, contradicting the hypothesis that it strictly prefers not to offer zero.

We thus know that in equilibrium $H$ weakly prefers to offer zero. We drop the "hats" of $F$ and $\Pi$ to capture the change in strategy and denote the probability of making an offer of zero by $Y$. As we have seen above, $H$ must mix, so $Y < 1$.

$H$ has to be indifferent between bidding zero (when it only wins if $L$ does not bid) and $\overline{s}$ (when it wins for sure), so we must have that $p_H \Pi^L = p_H - \overline{s} \Rightarrow$

$$\Pi^L = \frac{p_H - \overline{s}}{p_H} > 0. \tag{13}$$

By the same token, (11) – without "hats" as we have established that $H$ bids zero with positive probability – must also hold for all $x \in [\underline{s}, \overline{s}]$. Solving for the mixing distribution we have

$$F^L(x) = \frac{p_H - \overline{s}}{\overline{s}} \cdot \frac{x}{p_H - x} \in (0, 1]. \tag{14}$$

Given that $L$ is not making an offer with positive probability (see (13)), it must be indifferent between making an offer or not. Thus we have $(p_L - x)\left(Y + F^H(x)(1 - Y)\right) - c = 0 \Leftrightarrow$

$$(1 - Y)F^H(x) = \frac{c}{p_L - x} - Y, \tag{15}$$

for $x \in (\underline{s}, \overline{s})$. If there is no mass point at the upper end of $H$'s strategy, $\lim_{x \to \overline{s}} F^H(x) = 1$, then the formula still applies and we obtain that $\overline{s} = p_L - c$. If there were a mass point, then in order to keep $L$ from overbidding it must be that for all $\varepsilon > 0$, $p_L - \overline{s} - \varepsilon - c < 0$ $\Leftrightarrow$ $\overline{s} \geq p_L - c$, which when applied to the formula for $\lim_{x \to \overline{s}} F^H(x)$, implies again that $\overline{s} = p_L - c$ and $F^H(\overline{s}) = 1$, therefore no mass point is possible. From (13), substituting in for the upper bound, we obtain that $\Pi^L = \frac{p_H - p_L + c}{p_H}$.

When $L$ bids the lower bound of its support, it can only win if $H$ is bidding zero. Hence, we have that $(p_L - \underline{s})Y - c = 0$, from which we can solve for $Y = \frac{c}{p_L - \underline{s}} \in (0, 1)$. Substituting in (15), we obtain

$$F^H(x) = \frac{x - \underline{s}}{p_L - c - \underline{s}} \cdot \frac{c}{p_L - x}.$$

All we have left to do is to identify the lower bound of the support of the mixed strategies. Observe that – by the single deviation principle – this has to equal the (discounted) expected continuation value of the worker when she receives two offers[18] and hence expects both firms to be still in the market in the next period. We have already established that this value is $\delta p_L$. When $p_L - c \leq \delta p_L$, it is not profitable for $L$ to make a bid when $H$ is bidding for the worker. However, we also have to consider the case that $H$ is not bidding. By the same token as above, when $p_H - c \leq \delta p_L$, it is not profitable for $H$ to bid when $L$ is bidding for the worker. Thus, when $p_H - c \leq \delta p_L$, we have both equilibria.

Let us consider now the case with more than two firms. We proceed in three steps. First, we show that the above equilibria continue to be equilibria. Second, we show that no equilibrium exists with more than two firms bidding with positive probability. Finally, we check whether the firms bidding can be different from $H$ and $L$.

Note that in the two-firm equilibrium $L$ always expects zero net profit. When $p_L - c \geq \delta p_L$, by making a bid that $L$ also makes in equilibrium, any firm with a lower productivity can only fare worse than $L$. By making a bid below $\delta p_L$ the entrant would win with probability $Y\Pi^L$ and it would need to offer at least $\delta p_i$ to be accepted. This leads to an expected gross profit of $\frac{p_H - p_L + c}{p_H} \cdot \frac{c}{(1-\delta)p_L}(1-\delta)p_i = \frac{p_H - p_L + c}{p_H} \cdot \frac{p_i}{p_L} \cdot c < c$. When $p_L - c \leq \delta p_L$, $p_i - c \leq \delta p_i$ so there is no room for a profitable bid for the worker.

Next note that $H$ can guarantee itself $p_H - p_L$, the amount it makes in the two-firm equilibrium (for low $\delta$). Any other player who bids, must expect to recover the vetting cost, $c$. Thus, if we had more than two bidders, the worker should expect a lower wage than with two bidders, what is clearly impossible.

It is straightforward to see that if the two firms bidding were not $H$ and $L$ then the one left out could outbid the intruder and expect strictly more than $c$. Finally, as we have seen before, Firm $i$ could be the only bidder as long as $p_H - c \leq \delta p_i$ . Q.E.D.

[18] Whenever $L$ makes an offer, the worker will receive two offers, so this is the relevant scenario for the determination of the lower bound of $L$'s bidding distribution.

## 5.2 Multiple vacancies per firm

In the main text we have made the simplifying assumption that each firm has a single vacancy. As shown by Kojima (2007), this assumption is crucial for the results of Bulow and Levin (2006), without it, competitive wages can be part of an equilibrium. Nonetheless, we can show that in our model the assumption is indeed without loss of generality.

**Corollary 2** *Firms having multiple vacancies would not alter Proposition 3.*

*Proof:* First note that no firm would try to compete with itself for a worker. So any deviation from the Diamond equilibrium must involve a firm poaching a worker which in equilibrium it would not hire. If such a deviation occurs, just as in the main model, all the other workers will be hired, so in the continuation there will only be two vacancies of different firms left. Q.E.D.

## 5.3 Holding on to an offer

In the main text we have assumed that workers had to respond to each offer immediately. This is not very realistic, so here we demonstrate that the assumption is actually reducing the number of equilibria, so it cannot be the reason for the existence of the Diamond equilibrium.

**Corollary 3** *Workers having several periods to ponder an offer would not destroy the Diamond equilibrium.*

*Proof:* We will show that the continuation value of a worker rejecting two offers can only improve with the workers' option to hold on to an offer. As a result, the incentives for a firm to deviate from the Diamond equilibrium can only decrease. Recall, that in the continuation there are only two workers who receive offers. One of them has no competition for him, so he has no incentive to wait. The other worker is supposed to accept the highest offer in equilibrium If she decides to hold on to it, she must be better off doing that, increasing her expected payoff. Q.E.D.

The intuition for this result is simple: the only reason to hold on to an offer (rather than accept it right away) is the hope of receiving a better offer in the future. This can only improve a worker's payoff. It does not happen on the equilibrium path as there are no suitors left, while the effect off the equilibrium path only strengthens the equilibrium.

## 5.4 Full commitment to wage offers

An alternative model of targeted wages is one where the firms make a single take-it-or-leave-it offer to the worker of their choice, which she has to accept within $t$ periods. The firm is committed both not to make another offer to the same worker (ever) and not to approach another worker while its offer is on the table.[19]

We start with a general result that equilibria with full commitment must involve simultaneous competition.

**Proposition 5** *With TIOLI offers, in any SPE some worker must receive two simultaneous offers with positive probability.*

*Proof:* Assume to the contrary, that there exists an SPE where each worker receives a maximum of one offer on the equilibrium path. Then all these offers would have to be simultaneous, as they would be accepted immediately and hence any delay in making them would be suboptimal. If all offers are simultaneous and one per worker, then they must be zero. But then there is an incentive to deviate and bid $\varepsilon$ for a better worker. The firm whose worker is "poached" cannot react, while the others hire their equilibrium worker, so the worker would be compelled to accept. Q.E.D.

Note that Proposition 5 rules out both Diamond-type equilibria and the sequential-move equilibria of Konishi and Sapozhnikov (2008), where firms make offers one after the other. This shows that the assumption that leads to their results is the absence of discounting and not the non-explosive nature of the offers.

[19] Konishi and Sapozhnikov (2008) make this assumption, with $t = \infty$ (and $\delta = 1$).

In order to get a better feel for what equilibria with full commitment look like, we discuss the case of a duopsony. When $t$ is zero (exploding offers) then the equilibrium is the same as in the case without commitment (and low $\delta$), except that the mixing interval starts from zero, as the continuation value of a worker is zero, since the offer explodes and next period she would face a monopsony situation. When $t > 0$, the better firm would sometimes (for $\delta$ high enough) prefer to wait and see what the other firm has offered to the better worker, as matching that offer it would hire the worker for sure. However, anticipating this, the worker would accept the first offer she received, thereby bringing trade forward by one period. Consequently, $t > 0$ does not affect equilibrium behavior.

As the only change is the zero lower bound for the mixing interval, the expected wage of the better worker is lower with commitment than without it (as long as in the absence of commitment the mixed equilibrium would prevail). However, the mismatch probability is increased: note that the weaker firm before had a mass point at $\underline{w}$. With that offer it won if and only if the better firm bid zero. Now this same mass is distributed over $(0, \underline{w}]$, while the better firm redistributes the mass he had on $(\underline{w}, \overline{w}]$ on to $(0, \overline{w}]$. As a result, the weaker firm sometimes will win when it bids in $(0, \underline{w}]$, and it will win more often than before when it bids in $(\underline{w}, \overline{w}]$. Consequently, the weaker firm and the weaker worker expect the same as without commitment, the better worker is clearly worse off, while the effect on the better firm is ambiguous.

With more firms, the situation is less clear cut. If with positive probability there was competition for a worker in the second period, she would consider "sitting" on her offer (when $t > 0$). Of course, to keep the first period offer being mixed – otherwise there would be no reason to wait and see what the offer was going to be – we would need competition with positive probability in the first period as well. An additional factor is that a firm may decide to wait, not in order to learn the realization of a mixed wage offer, but to learn the realization of mixed targeting: a low productivity firm may want to wait and see if there was a coordination failure, leaving some high productivity worker without suitors.

# 6 Conclusion

This paper is about the nature of endogenous competition, where agents on one side of the market have to decide which agents on the other side to compete for. In the presence of heterogeneity, efficient matching often requires the absence of direct competition, but the latter would lead to monopoly rents, making the incentives to compete too strong to resist. So, what can be done to drive such a market towards efficiency? The surprising answer is to differentially increase the bargaining power of the passive side of the market: a local monopsonist retains all of her bargaining power in equilibrium, but if she becomes the target of a "raider" – off the equilibrium path – the ensuing price competition drives the raider's profits down. Thus, paradoxically, the increased bargaining power has an adverse effect on the passive side of the market, as it scares off the competition for them. The beauty of the model is that nothing untoward is required to achieve the above effect: all we need is to empower the bid takers to reject all their bids and send the game to the next period. The vetting cost is only needed to ensure that the efficient equilibrium appear for traders with finite patience.

The main purpose of this paper was to isolate the effects of targeted offers on the market outcome, even at the cost of some loss of realism. It is important therefore to note that the robust result is that the efficient matching obtains and does so at below competitive wages, not that there is no wage dispersion. It is easy to extend the model so that the wage vector is increasing in productivity. Just assume that the workers' outside options are increasing with their productivity. As long as the outside option grows at a lower rate than productivity, such a change would not affect the main conclusions. Similarly, a moderate level of uncertainty about the level of outside options would lead to higher wages.

Finally note that other frictions, like (small) uncertainty about productivities, or non-pecuniary preferences on part of the workers, would not destroy the efficient equilibrium. Their effect would be the same on the two-firm continuation game as in the main game, leaving the incentives to deviate unaffected. The vetting cost is very special in this sense as it has a different effect on and off the equilibrium path.

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# 7 Mathematical Appendix

## 7.1 Intermediate steps to get to (10) from (9):

Dividing across by the common factor in (9), we have

$$\begin{aligned}
&\frac{\underline{w}}{\delta\left(p_{Ll}-c\right)\left(1-\delta\right)\left(p_{Hh}-\delta(p_{Hl}-c)-\widetilde{w}\right)} \\
= \;& \frac{1}{p_{Lh}-\delta(p_{Ll}-c)-\underline{w}}\cdot\frac{\underline{w}}{p_{Hh}-\delta(p_{Hl}-c)}\cdot\frac{\underline{w}}{p_{Hh}-\delta(p_{Hl}-c)-\underline{w}}+ \\
&\int_{\underline{w}}^{\widetilde{w}}\frac{x}{\left(p_{Hh}-\delta(p_{Hl}-c)-x\right)\left(p_{Lh}-\delta(p_{Ll}-c)-x\right)^2}dx+ \\
&\int_{\underline{w}}^{\widetilde{w}}\frac{x}{\left(p_{Hh}-\delta(p_{Hl}-c)-x\right)^2\left(p_{Lh}-\delta(p_{Ll}-c)-x\right)}dx.
\end{aligned}$$

Using that $\int\frac{x}{(a-x)^2(b-x)}dx=\frac{b\ln\frac{a-x}{b-x}}{(a-b)^2}-\frac{a}{(a-b)(a-x)}$ the equation becomes

$$\begin{aligned}
&\frac{\underline{w}}{\delta(p_{Ll}-c)(1-\delta)(p_{Hh}-\delta(p_{Hl}-c)-\widetilde{w})}-\frac{\underline{w}^2}{(p_{Lh}-\delta(p_{Ll}-c)-\underline{w})(p_{Hh}-\delta(p_{Hl}-c))(p_{Hh}-\delta(p_{Hl}-c)-\underline{w})}= \\
&\frac{p_{Hh}-\delta(p_{Hl}-c)}{(p_{Hh}-p_{Lh}-\delta(p_{Hl}-p_{Ll}))^2}\left(\ln\frac{p_{Lh}-\delta(p_{Ll}-c)-\widetilde{w}}{p_{Hh}-\delta(p_{Hl}-c)-\widetilde{w}}-\ln\frac{p_{Lh}-\delta(p_{Ll}-c)-\underline{w}}{p_{Hh}-\delta(p_{Hl}-c)-\underline{w}}\right)+ \\
&\frac{p_{Lh}-\delta(p_{Ll}-c)}{p_{Lh}-p_{Hh}+\delta(p_{Hl}-p_{Ll})}\cdot\left(\frac{1}{p_{Lh}-\delta(p_{Ll}-c)-\underline{w}}-\frac{1}{p_{Lh}-\delta(p_{Ll}-c)-\widetilde{w}}\right)+ \\
&\frac{p_{Lh}-\delta(p_{Ll}-c)}{(p_{Hh}-p_{Lh}-\delta(p_{Hl}-p_{Ll}))^2}\left(\ln\frac{p_{Hh}-\delta(p_{Hl}-c)-\widetilde{w}}{p_{Lh}-\delta(p_{Ll}-c)-\widetilde{w}}-\ln\frac{p_{Hh}-\delta(p_{Hl}-c)-\underline{w}}{p_{Lh}-\delta(p_{Ll}-c)-\underline{w}}\right)+ \\
&\frac{p_{Hh}-\delta(p_{Hl}-c)}{p_{Hh}-p_{Lh}-\delta(p_{Hl}-p_{Ll})}\cdot\left(\frac{1}{p_{Hh}-\delta(p_{Hl}-c)-\underline{w}}-\frac{1}{p_{Hh}-\delta(p_{Hl}-c)-\widetilde{w}}\right)= \\
&\frac{p_{Hh}-\delta(p_{Hl}-c)}{(p_{Hh}-p_{Lh}-\delta(p_{Hl}-p_{Ll}))^2}\ln\frac{(p_{Lh}-\delta(p_{Ll}-c)-\widetilde{w})(p_{Hh}-\delta(p_{Hl}-c)-\underline{w})}{(p_{Hh}-\delta(p_{Hl}-c)-\widetilde{w})(p_{Lh}-\delta(p_{Ll}-c)-\underline{w})}+ \\
&\frac{p_{Lh}-\delta(p_{Ll}-c)}{p_{Lh}-p_{Hh}+\delta(p_{Hl}-p_{Ll})}\cdot\frac{\underline{w}-\widetilde{w}}{(p_{Lh}-\delta(p_{Ll}-c)-\underline{w})(p_{Lh}-\delta(p_{Ll}-c)-\widetilde{w})}+ \\
&\frac{p_{Lh}-\delta(p_{Ll}-c)}{(p_{Hh}-p_{Lh}-\delta(p_{Hl}-p_{Ll}))^2}\ln\frac{(p_{Hh}-\delta(p_{Hl}-c)-\widetilde{w})(p_{Lh}-\delta(p_{Ll}-c)-\underline{w})}{(p_{Lh}-\delta(p_{Ll}-c)-\widetilde{w})(p_{Hh}-\delta(p_{Hl}-c)-\underline{w})}+ \\
&\frac{p_{Hh}-\delta(p_{Hl}-c)}{p_{Hh}-p_{Lh}-\delta(p_{Hl}-p_{Ll})}\cdot\frac{\underline{w}-\widetilde{w}}{(p_{Hh}-\delta(p_{Hl}-c)-\underline{w})(p_{Hh}-\delta(p_{Hl}-c)-\widetilde{w})}= \\
&\frac{1}{p_{Hh}-p_{Lh}-\delta(p_{Hl}-p_{Ll})}\ln\frac{(p_{Lh}-\delta(p_{Ll}-c)-\widetilde{w})(p_{Hh}-\delta(p_{Hl}-c)-\underline{w})}{(p_{Hh}-\delta(p_{Hl}-c)-\widetilde{w})(p_{Lh}-\delta(p_{Ll}-c)-\underline{w})}- \\
&\frac{\underline{w}-\widetilde{w}}{p_{Hh}-p_{Lh}-\delta(p_{Hl}-p_{Ll})}\left(\frac{p_{Lh}-\delta(p_{Ll}-c)}{(p_{Lh}-\delta(p_{Ll}-c)-\underline{w})(p_{Lh}-\delta(p_{Ll}-c)-\widetilde{w})}-\frac{p_{Hh}-\delta(p_{Hl}-c)}{(p_{Hh}-\delta(p_{Hl}-c)-\underline{w})(p_{Hh}-\delta(p_{Hl}-c)-\widetilde{w})}\right).
\end{aligned}$$

Moving everything to the LHS and multiplying across by $p_{Hh} - p_{Lh} - \delta(p_{Hl} - p_{Ll})$, we obtain (10).

## 7.2 RHS of (10) is increasing on $(-\infty, p_{Lh} - p_{Ll} + c)$

To enable Scientific Workplace, we eliminate the subindices, denoting $p_{Hh}$ by $H$, $p_{Lh}$ by $h$, $p_{Ll}$ by $L$ and $p_{Hl}$ by $l$.

$$\frac{x(H-h-\delta(l-L))}{\delta(L-c)(1-\delta)(H-\delta(l-c)-c-h+L)} - \frac{x^2(H-h-\delta(l-L))}{(h-\delta(L-c)-x)(H-\delta(l-c)-x)(H-\delta(l-c))} -$$
$$(h-L+c-x)\left(\frac{h-\delta(L-c)}{(h-\delta(L-c)-x)(1-\delta)(L-c)} - \frac{H-\delta(l-c)}{(H-\delta(l-c)-x)(H-\delta(l-c)-h+L-c)}\right) -$$
$$\ln\frac{(h-\delta(L-c)-h+L-c)(H-\delta(l-c)-x)}{(H-\delta(l-c)-h+L-c)(h-\delta(L-c)-x)}.$$

$$\frac{d\left(\frac{x(H-h-\delta(l-L))}{\delta(L-c)(1-\delta)(H-\delta(l-c)-c-h+L)}\right)}{dx} = -\frac{1}{\delta(\delta-1)(L-c)}\frac{H-h+L\delta-l\delta}{H+L-c-h+c\delta-l\delta}$$

$$\frac{d\left(-\frac{x^2(H-h-\delta(l-L))}{(h-\delta(L-c)-x)(H-\delta(l-c)-x)(H-\delta(l-c))}\right)}{dx} = \frac{x}{H+c\delta-l\delta}\frac{H-h+L\delta-l\delta}{(H-x+c\delta-l\delta)^2(h-x-L\delta+c\delta)^2}$$
$$\left(\begin{array}{c} Hx-2Hh-2c^2\delta^2+hx+2HL\delta-2Hc\delta-Lx\delta-2ch\delta+ \\ 2hl\delta+2cx\delta-lx\delta+2Lc\delta^2-2Ll\delta^2+2cl\delta^2 \end{array}\right)$$

$$\frac{d\left(-(h-L+c-x)\left(\frac{h-\delta(L-c)}{(h-\delta(L-c)-x)(1-\delta)(L-c)} - \frac{H-\delta(l-c)}{(H-\delta(l-c)-x)(H-\delta(l-c)-h+L-c)}\right)\right)}{dx} =$$
$$\frac{H-h+L\delta-l\delta}{(H-x+c\delta-l\delta)^2(h-x-L\delta+c\delta)^2}$$
$$\left(c^2\delta^2+Hh-x^2-HL\delta+Hc\delta+ch\delta-hl\delta-Lc\delta^2+Ll\delta^2-cl\delta^2\right)$$

$$\frac{d\left(-\ln\frac{(h-\delta(L-c)-h+L-c)(H-\delta(l-c)-x)}{(H-\delta(l-c)-h+L-c)(h-\delta(L-c)-x)}\right)}{dx} = -\frac{H-h+L\delta-l\delta}{(H-x+c\delta-l\delta)(h-x-L\delta+c\delta)}.$$

Putting the terms together and dividing by $H-h+L\delta-l\delta > 0$, (recall that $H-h > l-L$ by (1)):

$$-\frac{1}{\delta(\delta-1)(L-c)}\frac{1}{H+L-c-h+c\delta-l\delta} + \frac{x}{H+c\delta-l\delta}\frac{1}{(H-x+c\delta-l\delta)^2(h-x-L\delta+c\delta)^2}$$
$$\left(\begin{array}{c} Hx-2Hh-2c^2\delta^2+hx+2HL\delta-2Hc\delta-Lx\delta-2ch\delta+2hl\delta+ \\ 2cx\delta-lx\delta+2Lc\delta^2-2Ll\delta^2+2cl\delta^2 \end{array}\right)$$
$$+\frac{c^2\delta^2+Hh-x^2-HL\delta+Hc\delta+ch\delta-hl\delta-Lc\delta^2+Ll\delta^2-cl\delta^2}{(H-x+c\delta-l\delta)^2(h-x-L\delta+c\delta)^2} - \frac{1}{(H-x+c\delta-l\delta)(h-x-L\delta+c\delta)}.$$

Note that the last term is decreasing in $x$. Therefore we can bound it from below by substituting the largest possible $x = h-L+c$. The last term then becomes

$-\frac{1}{(H+L-c-h+c\delta-l\delta)(1-\delta)(L-c)}$. Adding it to the first term, we have $\frac{1}{\delta(L-c)(H+L-c-h+c\delta-l\delta)}$. This is positive as long as $H+L-c-h+c\delta-l\delta > 0$, which holds by (1) and the fact that $c <$

$\min\{H,L,h,l\}$. We can multiply the rest of the terms by $(H-x+c\delta-l\delta)^2\,(h-x-L\delta+c\delta)^2$ :

$$\frac{x}{H+c\delta-l\delta}\cdot\left[2\delta^2(L-c)(c-l)+2\delta(H(L-c)+h(l-c))+x\delta(2c-l-L)+x(h+H)-2hH\right]+$$
$$\delta^2(L-c)(l-c)-\delta(H(L-c)+h(l-c))+Hh-x^2=$$
$$\left[\delta^2(L-c)(l-c)-\delta(H(L-c)+h(l-c))+Hh\right]\left[1-\frac{2x}{H+c\delta-l\delta}\right]+$$
$$x^2\left[\frac{\delta(2c-l-L)+h+H}{H+c\delta-l\delta}-1\right]=(h-\delta(L-c))(H-\delta(l-c)-2x+\frac{x^2}{H+c\delta-l\delta})$$

The first term is positive, the second is positive if $x<H-\delta(l-c)$. Finally, note that $H-\delta(l-c)>H-l+c>h-L+c$, by (1) and (2). Q.E.D.

## 7.3 RHS of (10) is negative at $x=0$.

$$\left[\begin{array}{c}\frac{x(H-h-\delta(l-L))}{\delta(L-c)(1-\delta)(H-\delta(l-c)-c-h+L)}-\frac{x^2(H-h-\delta(l-L))}{(h-\delta(L-c)-x)(H-\delta(l-c)-x)(H-\delta(l-c))}-\\(h-L+c-x)\left(\frac{h-\delta(L-c)}{(h-\delta(L-c)-x)(1-\delta)(L-c)}-\frac{H-\delta(l-c)}{(H-\delta(l-c)-x)(H-\delta(l-c)-h+L-c)}\right)-\\\ln\frac{(h-\delta(L-c)-h+L-c)(H-\delta(l-c)-x)}{(H-\delta(l-c)-h+L-c)(h-\delta(L-c)-x)}\end{array}\right]_{x=0}=$$

$$\left(\frac{1}{(\delta-1)(L-c)}+\frac{1}{H+L-c-h+\delta(c-l)}\right)(c-L+h)-\ln\left(-\frac{H+\delta(c-l)}{h-\delta(L-c)}\frac{c-L+\delta(L-c)}{H+L-c-h+\delta(c-l)}\right).$$

Now recall that $\ln y\geq 1-1/y$. Hence, the above no more than

$$\left(\frac{1}{(\delta-1)(L-c)}+\frac{1}{H+L-c-h+\delta(c-l)}\right)(c-L+h)-\left(1+\frac{h-\delta(L-c)}{H+\delta(c-l)}\frac{H+L-c-h+\delta(c-l)}{(\delta-1)(L-c)}\right)=$$
$$\frac{c-L+h}{(\delta-1)(L-c)}-\frac{H+\delta(c-l)}{H+L-c-h+\delta(c-l)}-\left(\frac{h-\delta(L-c)}{H+\delta(c-l)}\frac{H+L-c-h+\delta(c-l)}{(\delta-1)(L-c)}\right).$$

Multiplying across by $(L-c)\,(1-\delta)>0$ we get

$$L-c-h-\frac{(H+\delta(c-l))(L-c)(1-\delta)}{H+L-c-h+\delta(c-l)}+\frac{h-\delta(L-c)}{H+\delta(c-l)}\left(H+L-c-h+\delta\,(c-l)\right)=$$
$$(L-c)\,(1-\delta)\left[1-\frac{H+\delta(c-l)}{H+L-c-h+\delta(c-l)}\right]+\frac{h-\delta(L-c)}{H+\delta(c-l)}\,(L-c-h)=$$
$$(L-c)\,(1-\delta)\frac{L-c-h}{H+L-c-h+\delta(c-l)}+\frac{h-\delta(L-c)}{H+\delta(c-l)}\,(L-c-h)=$$
$$(L-c-h)\left[\frac{(L-c)(1-\delta)}{H+L-c-h+\delta(c-l)}+\frac{h-\delta(L-c)}{H+\delta(c-l)}\right],$$

the first term is clearly negative, while the second is positive. Q.E.D.

## 7.4 RHS of (10) is positive at $x=p_{Lh}-p_{Ll}+c$.

$$\left[\begin{array}{c}\frac{x(H-h-\delta(l-L))}{\delta(L-c)(1-\delta)(H-\delta(l-c)-c-h+L)}-\frac{x^2(H-h-\delta(l-L))}{(h-\delta(L-c)-x)(H-\delta(l-c)-x)(H-\delta(l-c))}-\\(h-L+c-x)\left(\frac{h-\delta(L-c)}{(h-\delta(L-c)-x)(1-\delta)(L-c)}-\frac{H-\delta(l-c)}{(H-\delta(l-c)-x)(H-\delta(l-c)-h+L-c)}\right)-\\\ln\frac{(h-\delta(L-c)-h+L-c)(H-\delta(l-c)-x)}{(H-\delta(l-c)-h+L-c)(h-\delta(L-c)-x)}\end{array}\right]_{x=h-L+c}=$$

$\frac{1}{H+\delta(c-l)}\frac{(c-L+h)^2}{c-L+\delta(L-c)}\frac{H-h+\delta(L-l)}{H+L-c-h+\delta(c-l)} - \frac{1}{\delta(\delta-1)(L-c)}(c-L+h)\frac{H-h+\delta(L-l)}{H+L-c-h+\delta(c-l)}$.
Dividing by the common positive term $\frac{(c-L+h)(H-h+\delta(L-l))}{H(h-l\delta)+c(\delta-1)-L(h-l)}$ :
$\frac{1}{\delta(\delta-1)} \cdot \frac{1}{c-L} + \frac{1}{H+\delta(c-l)}\frac{c-L+h}{(\delta-1)(L-c)}$. Multiplying by $(L-c)(1-\delta)$ :
$\frac{1}{\delta} - \frac{c-L+h}{H+\delta(c-l)} = \frac{H+\delta(c-l)-\delta(c-L+h)}{(H+\delta(c-l))\delta} = \frac{H-\delta(l-L+h)}{(H+\delta(c-l))\delta} > 0$. Q.E.D.