Documentos de Trabalho

Working Paper Series

"Competitive Targeted Advertising with Price Discrimination"

Rosa-Branca Esteves
Joana Resende

NIPE WP 08/ 2011

NÚCLEO DE INVESTIGAÇÃO EM POLÍTICAS ECONÓMICAS

UNIVERSIDADE DO MINHO

“Competitive Targeted Advertising with Price Discrimination”

Rosa-Branca Esteves
Joana Resende

NIPE[*] WP 08/ 2011

URL:
http://www.eeg.uminho.pt/economia/nipe

[*] NIPE – *Núcleo de Investigação em Políticas Económicas* – is supported by the Portuguese Foundation for Science and Technology through the *Programa Operacional Ciência, Teconologia e Inovação* (POCI 2010) of the *Quadro Comunitário de Apoio III*, which is financed by FEDER and Portuguese funds.

Competitive Targeted Advertising with Price Discrimination*

Rosa-Branca Esteves
Universidade do Minho (EEG) and NIPE
rbranca@eeg.uminho.pt

Joana Resende
Faculdade de Economia do Porto
jresende@fep.up.pt

March 2011
(In progress)

Abstract

This paper investigates the effects of price discrimination by means of targeted advertising in a duopolistic market where the distribution of consumers' preferences is discrete and where advertising plays two major roles. It is used by firms as a way to transmit relevant information to otherwise uninformed consumers, and it is used as a price discrimination device. We compare the firms' optimal marketing mix (advertising and pricing) when they adopt mass advertising/non-discrimination strategies and targeted advertising/price discrimination strategies. If firms are able to adopt targeted advertising strategies, we find that the symmetric price equilibrium is in mixed strategies, while the advertising is chosen deterministically. Our results also unveil that as long as we allow for imperfect substitutability between the goods, firms do not necessarily target more ads to their own market. In particular, firms' optimal marketing mix leads to higher advertising reach in the rival's market than in the firms' own market, provided that advertising costs are sufficiently low in relation to the consumer's reservation value. The comparison of the optimal marketing-mix under mass advertising strategies and targeted advertising strategies reveals that targeted advertising might constitute a tool to dampen price competition. In particular, if advertising costs are sufficiently low in relation to the value of the goods, we obtain that average prices with non-discrimination (mass advertising) are below those with price discrimination and targeted advertising (regardless of the market segment). Accordingly, when (i) goods are imperfect substitutes, (ii) advertising is not too expensive, and (iii) targeted advertising constitutes an effective price discrimination tool, price discrimination through targeted advertising may be detrimental to social welfare since it boosts industry profits at the expense of consumer surplus.

*We are grateful to Jose Luis Moraga, Emmanuel Petrakis, Simon Anderson and participants of the III Conference on the Economics of Advertising and Marketing (IESE, Barcelona 2010). Financial support from the Portuguese Science Foundation is gratefully acknowledged. Any errors are our own responsability.

1 Introduction

In many markets firms need to invest in advertising to inform potential consumers about the existence and price of their new products. The informative view of advertising claims that the primary role of advertising is to transmit information to otherwise uninformed consumers.[1] Until very recently, the scope of targeted advertising was relatively limited and firms tended to privilege an advertising strategy *"above the line"*, as firms' advertising strategies were mostly tailored to *mass* audiences.

However, advertising markets are experiencing a number of deep and fast changes, which are challenging conventional wisdom regarding optimal advertising and pricing strategies. One of the most striking challenges concerns the diffusion and the widespread use of targeted advertising technologies, which allow firms to target their messages to particular market segments (see Bagwell, K. (2005)). Targeting advertising strategies are becoming more and more attractive to firms. One the one hand, the advances in information technologies are enabling firms to send progressively more accurate advertising messages since such information technologies facilitate the process of gathering, storing and processing consumer-specific data. This increases firms' ability to target different marketing strategies, e.g. pricing and advertising, to consumers with different profiles. On the other hand, the prosperity of electronic markets and the breakthrough developments of new media industries are expected to exponentially enhance the reach of targeted advertising messages.

Thus firms are investing more and more in advertising strategies *"below the line"* in order to target advertising messages at individuals according to their needs or preferences. For example, expenses below the line advertising media, such as search-related online advertising are growing exponentially.[2,3]These expenses are expected to growth substantially in the future following the expansion of electronic markets and the development of new targeted advertising technologies (e.g. the advertising platform iAd developed by Apple).[4]

From advertisers' perspective, targeted advertising is indeed a very powerful tool. Not only because it can minimize wasted advertising but mainly because target advertising makes the value of advertising more measurable by allowing firms to adjust their advertising messages to consumers' profiles.

Therefore, apart from its informative role, advertising might be used by firms as a price

[1] In contrast, the persuasive view of advertising holds that the main role of advertising is to increase a consumer's willingness to pay for the advertised product. In this sense, firms may use advertising to alter consumers' tastes and to increase brand loyalty. For a review of models in the persuasive view see Bagwell's (2003) comprehensive survey on the "The Economic Analysis of Advertising."

[2] Examples of search-related online advertising include: Google - AdWords and AdSense; eBay – AdContext; Yahoo!; MSN - adCenter.

[3] According to Evans, in 2007, online advertising expenses already represented about 7% of total advertising expenses in the United States (with online advertising expenses amounting to 21 billion of USD).

[4] On Apple's webpage we may read: *"iAd is a breakthrough mobile advertising platform from Apple. With it, apps can feature rich media ads that combine the emotion of TV with the interactivity of the web. For developers, it means a new, easy-to-implement source of revenue. For advertisers, it creates a new media outlet that offers consumers highly targeted information"*.

discrimination device.

While economists have long been concerned with understanding the competitive and welfare effects of either price discrimination or informative advertising, little theoretical attention has been dedicated to the interaction between informative targeted advertising and price discrimination. This paper is a step understanding of to the understanding of the competitive and welfare effects of price in an informative advertising model.

The main theme of this paper is to investigate the competitive and welfare effects of price discrimination by means of targeted informative advertising. We aim to investigate the social desirability of targeted advertising technologies, investigating how the availability of these technologies may affect firms' optimal marketing mix (pricing and advertising) and the corresponding level of profits, consumers' surplus and social welfare.

To this end, we propose a static game of duopoly competition, in which two firms, A and B are launching two *new* differentiated products and need to invest in advertising to generate awareness. Consumers' heterogeneity is modeled in line with Shilony (1977). Consumers are either loyal (to a specific degree, i.e., $\gamma > 0$) to one firm or the other. Both firms know that half of consumers have a relative preference for A while the remaining have a relative preference for B. As Raju, et al. (1990), γ can be used as a measure of the degree of a consumer's brand loyalty, defined as the minimum difference between the prices of the two competing brands necessary to induce consumers to buy the wrong brand.[5] This means that full informed consumers will buy from the most preferred firm as long as its price is not undercut by more than γ. As in Stahl (1994) a potential consumer cannot be an actual buyer unless firms invest in advertising. By investing in advertising firms endogenously segment the market into captive (partially informed) and selective (fully informed) customers.

Two advertising strategies are studied in the paper: a mass advertising strategy and a targeted advertising strategy. If firms use a mass advertising they choose an intensity of advertising to the entire market and send ads with same content (i.e., all ads have the same price) to the entire market. In this case they are forced to follow a uniform pricing policy. In contrast, under targeted advertising firms choose different levels of advertising to each market segment and ads tailored to different segments quote different prices.

When firms use a mass advertising strategy, and the reservation value is high enough the model yields a symmetric equilibrium in mixed strategies in prices with the advertising component chosen deterministically. In comparison to a full information case (Shilony (1977)) imperfect information boots firm's profits and reduces consumer surplus and welfare. When v is high enough we also find that profits increase with advertising costs increases.

When price discrimination by means of targeted advertising is allowed some new and interesting results arise. The paper shows the price equilibrium is always in mixed strategies and the level of advertising to each segment is chosen deterministically. We show that when the

[5]Even though the paper considers that the market is segmented according to brand loyalty, the model also accommodates other interpretations as search costs, transportation costs and switching costs. In the location interpretation, consumers can purchase costlessly from neighbourhood firms, but incur a transport cost $\gamma > 0$ when buying from more distant firms.

advertising costs are high (or v is low) firms advertise more to its own market. In contrast, we show that when advertising is cheap (and v is high) firms advertise more to the rival's market.

As in other models with price discrimination based on customer recognition we show that a firm charges on average lower prices to the rival's customers than to its own customers (e.g. Thisse and Vives (1988), Fudenberg and Tirole (2000).

The stylised model addressed in this paper brings new insights to the literature on price discrimination based on customer recognition. First, we show that average prices with mass advertising (non-discrimination) are below those with targeted advertising. This is an interesting finding as it challenges the usual finding that price discrimination may reduce all segment prices. Second, we show that price discrimination by means of targeted informative advertising do not necessarily lead to the classic prisoner dilemma result that arises in models with full informed consumers. In this way we show that at least when v is high and advertising is not expensive each firm's profit with perfect targeted advertising and price discrimination is above its non-discrimination counterpart.

Finally, another theme of the paper is to investigate the welfare effects of targeted advertising with price discrimination. We show that at least when advertising costs are not too high in comparison to v, price discrimination by means of targeted advertising suggest boost industry profit at the expense of consumer surplus and welfare. Thus, the paper highlights the importance of taking into account different forms of market competition when public policy tries to evaluate the profit and welfare effects of price discrimination.

Related literature This paper is mainly related to two strands of the literature, namely the literature on competition with informative advertising and the more recent literature on price discrimination based on customer recognition.

Following the seminal work of Butters (1977), a vast literature has investigated competition with informative advertising. In a competitive market for a homogeneous product, where advertising is the sole source of information to uninformed customers, Butters shows that the equilibrium is characterized by price dispersion and that the equilibrium level of advertising is socially optimal. The latter puzzling result was confirmed by Stahl (1994), who extended Butters' model to oligopolistic markets with more general demand curves and advertising technologies. Variations on Butters' model, such as the introduction of product differentiation (Grossman and Shapiro (1984)), or heterogeneity among buyers (Stegeman (1991)), were shown to easily offset this result. They also helped to establish the idea that increased competition stimulated additional advertising—the business stealing effect— while the incapability of the firm to appropriate the social surplus it generates acts as a deterrent to advertising —the nonappropriability of social surplus effect (Tirole (1988)). Our paper is more closely related to oligopoly models of advertising with product differentiation, e.g. Grossman and Shapiro (1984); or Stegeman (1991). These papers have shown that equilibrium level of advertising may not be socially efficient in a context of product differentiation or heterogeneity among consumers (in contrast with the results pointed out by Butters (1977) or Stahl (1994) in the context of a competitive market for a homogeneous good). These papers have also put forward the idea of "business stealing effect"[6] and "the non-appropriability of social surplus effect" (see also Tirole (1988)).[7]

The literature on targeted advertising is relatively recent. This literature is evolving along two major lines. The first strand of literature corresponds to the literature studying the effects of targeted advertising technologies on prices and competition when firms can directly target different consumers.[8] The second strand of literature assumes that firms are not able to directly target their messages to different groups of consumers, taking into consideration the intermediary role played by media.[9] As we assume in this paper that firms have the ability to directly target their advertising messages to specific groups of consumers (which tends to be the case in electronic markets, for example), our paper is more closely related to the first strand of literature, specifically to Iyer, Soberman and Villas-Boas (2005) and Galeotti and Moraga-González (2008).

Iyer et al. (2005) develop a model in which firms face two types of consumers: captive consumers and shoppers (who are not loyal to any of the firms, always buying the less expensive good). They show that targeted advertising leads to higher profits regardless of whether firms have or not the ability to adopt price discrimination strategies. They also conclude that when firms are endowed with a quadratic targeted advertising technology, they always advertise more in the segment of consumers with a strong preference for its own good. This is in contrast with our finding that if v is high enough firms advertise more to the group of consumers with a low preference for its good. As far as concerns the total amount of advertising expenses, this paper concludes that target advertising may either increase or decrease advertising costs when compared with a mass advertising technology.[10]

Galeotti and Moraga-González (2008) studies firms' advertising and pricing strategies in a market with homogeneous good, where market segmentation is based on consumer attributes that are completely unrelated to tastes. The paper compares market outcomes under mass advertising with uniform pricing and targeted advertising with price discrimination. Assuming ex-ante that one market segment is more profitable then the other the paper shows that the possibility of market segmentation may lead to positive profits within an otherwise Bertrand-like setting. In this paper, an increase on advertising costs increases the profitability of market segmentation with firms having unequal sizes. Regarding pricing strategies, in the case of targeted advertising, the paper shows that the price distribution in the less attractive market dominates (in the sense of first order stochastic dominance) the price distribution of the other market. The price distribution under mass advertising is in-between these two distributions

[6] The business stealing effect refers to advertising enhancing effect generated by an increase in the degree of competition.

[7] This effect refers to the advertising deterrence effect generated by incapability of the firm to appropriate the social surplus it generates.

[8] See for example, Iyer, Soberman and Villas-Boas (2005); Galeotti and Moraga-González (2008); Esteban and Hernández (2007); Esteban, Hernández and Moraga-González (2006); Esteves and Guimarães (2008); Chen and Iyer (2002), Roy (2000).

[9] See for example, Chandra (2009); Athey and Gans (2009); Gal-Or and Gal-Or (2005); Gal-Or and Gal-Or (2006); Gal-Or, Gal-Or, May and Spangler (2008).

[10] In particular, if advertising unit costs are high (low), advertising costs are higher (lower) when firms own a targeted than a mass advertising technology.

(again, in the sense of first order stochastic dominance).[11,12]

Finally, the paper is also related to the literature on competitive price discrimination. It is related to those models where, in the terminology of Corts (1998), the market exhibits *best-response asymmetry*.[13] In these models profit will typically decrease when price discrimination is practiced. A useful model for understanding the profit effects of price discrimination in markets with best-response asymmetry is by Thisse and Vives (1988). There are two firms located at the extremes of the segment $[0, 1]$. Consumers are uniformly distributed in the line segment and firms can observe the location (or brand preference) of each individual consumer and price accordingly. The strong (close) market for one firm is the weak (distant) market for the other firm. In this setting they show that price discrimination intensifies competition, *all* prices fall as well as profits. The finding that firms might be worse off when they engage in price discrimination is one of the key differences between monopoly and competitive price discrimination. If we ignore commitment issues, a monopolist is better off when it uses price discrimination. Although with competition price discrimination is a dominant strategy for each firm, for given prices offered by its rival, when all firms follow the same strategy they might find themselves in the classic prisoner's dilemma.[14] Price discrimination based on customer recognition has been examined also by Bester and Petrakis (1996), Chen (1997), Fudenberg and Tirole (2000), and Esteves (2010). In all of these approaches consumers are perfectly informed, there is no role for advertising and profits fall with price discrimination. Esteves (2009a) departs from this hypothesis by assuming that without advertising consumers are uninformed and firms have no demand. However, while in Esteves (2009a) consumers are ex-ante identical regarding their preferences for the firms, here consumers are ex-ante heterogenous. Esteves (2009a) shows that *all* firms might become better off, even when only one of them can engage in price discrimination. By proposing a framework in which advertising is the consumers' sole source of information about a firm/product existence

[11] Esteban and Hernandez (2007) study targeted advertising in an oligopolistic market with vertical differentiation. The authors show that the possibility of targeting advertising with price discrimination may lead to permanent segmentation of the market. From a welfare perspective, targeted advertising is shown to have a positive effect on consumers' surplus and social welfare.

[12] Other papers studying targeted advertising include Brahim, Lahmandi-Ayed and Laussel (2010), Roy (2000), Gal-Or and Gal-Or (2006); Gal-Or, Gal-Or, May and Spangler (2008). Brahim et al (2010) develop an extended version of Grossman and Shapiro (1984) with targeted advertising, concluding that firms prefer to target their natural market when advertising costs are low, while they cross-advertise if advertising costs are high. The authors also show that targeted advertising not always has a positive effect on firms' advertising profits.

Roy (2000) studies optimal advertising choices when firms can target consumers on the basis of their address (i.e. their location on a Hotelling framework). Gal-Or and Gal-Or study the competitive effects of targeted advertising when a single media content distributor delivers advertising messages on the behalf of firms. Gal-Or, Gal-Or, May and Spangler (2008) deal with the issue of imperfect advertising tailoring, studying to which extent an advertiser should allocate resources to increase the quality of its targeting.

[13] The market exhibits best response asymmetry when one firm's "strong" market is the other's "weak" market. In the literature of price discrimination, a market is designated as "strong" if in comparison to uniform pricing a firm wishes to increase its price there. The market is said to be "weak" if the reverse happens.

[14] Esteves (2009b) extends the Thisse and Vives model to a two-dimensional differentiation model and shows that price discrimination might not necessarily lead to the prisoner's dilemma result. This happens when firms observe the location of consumers in the less differentiation dimension and price discriminate accordingly while they remain ignorant about their location in the more differentiated dimension.

and its price, the paper shows that price discrimination by means of targeted advertising may boost industry profit at the expense of consumer surplus and welfare.

The rest of this paper is organized as follows. Section 2 describes the main ingredients of the model. Section 3 analyses the benchmark case, in which firms are endowed with a mass advertising technology that forces firms to adopt a uniform pricing policy. Section 4 analyses the equilibrium advertising and pricing strategies when firms are endowed with a targeted advertising technology and price discrimination is permitted. Section 5 stresses the competitive effects of perfect targeted advertising. Section 6 focus on the impact of targeted advertising on social welfare and, finally, Section 7 sets up the main conclusions.

2 Model Assumptions

Consider a market in which two firms, A and B, are launching two new products, A and B (respectively). With no loss of generality, we suppose each firm produces its product at zero marginal cost.[15]

On the demand side, there is a large number of potential buyers, with mass normalized to one. Each consumer wishes to buy at most a single unit of either good A or B. Goods are differentiated and consumers have heterogeneous preferences in relation to them. In particular, we suppose that the set of potential buyers is composed of two distinct segments of equal mass: market segment a and market segment b. The market segments differ one from the other, as a result of consumers' heterogeneity regarding the relative intrinsic value of good A and good B (within each group, consumers' preferences are assumed to be homogeneous). In the case of segment a, consumers prefer product A over product B by a degree equal to $\gamma > 0$: the reservation price of buying good A is equal to $v > 0$, while the reservation price of buying good B is equal to $v - \gamma > 0$. In contrast, consumers belonging to segment b intrinsically prefer product B over product A, whose reservation prices (for consumers in group B) are respectively given by $v - \gamma > 0$ and $v > 0$. The reservation prices (v and $v - \gamma$) are assumed to be sufficiently high so that the duopoly equilibrium exhibits competition, i.e. $v > p_i + \gamma$, $\forall i = A, B$.

In relation to the parameter γ, notice that, as in Shilony (1977), Raju, et al. (1990) and Esteves (2009), γ can be used as a measure of the degree of a consumer's preference for the most preferred product. It can also be defined as the minimum difference between the prices of the two competing products necessary to induce consumers to buy the least preferred product. This means, for instance, that consumers with a preference for product i will buy product i as long as its price is not undercut by more than γ by firm j.[16]

Consumers are initially uninformed about the existence and the price of the goods. As in Stahl (1994) a potential consumer cannot be an actual buyer unless firms invest in advertising.[17]

[15] The assumption of zero marginal costs can be relaxed without altering the basic nature of the results derived throughout the model.

[16] Raju, et al (1990) present also the case with asymmetric loyalties towards the two firms.

[17] Implicitly we are assuming that for new products search costs are prohibitively high.

The advertising messages of each firm contain truthful[18] and complete information about the existence of its product and price. After firms have sent their ads independently (i.e., advertising reach is independent for each firm), in each segment of the market (a and b), there are, in principle, four different mutually exclusive and exhaustive categories of consumers. Some consumers are *captive* to a given firm (either firm A or firm B), because they are are only aware of the existence of one firm. Other consumers received ads from both firms and are *selective*. This latter group of consumers has complete information and will buy from the firm that offers them the highest surplus. Finally, the remaining consumers are *uninformed* and excluded from the market as they haven't received any ad from any of the firms.

The game is static and proceeds as follows. Firms choose advertising intensities and prices simultaneously and non-cooperatively. Under mass advertising firms choose an intensity of advertising to the entire market and all ads announce the same price. With targeted advertising firms choose different levels of advertising to each market segment and ads tailored to different segments quote different prices.

Advertising technology:

Advertising is a costly activity for firms. The advertising technology is exogenously given and it is the same for both firms. Two advertising technologies will be used throughout this paper, a mass advertising technology and a targeted advertising technology.

When firms use a mass advertising technology they are also forced to follow a uniform pricing policy which means that all the ads quote the same price. In this case, the problem of firm i consists in choosing an optimal advertising reach, ϕ_i, and the corresponding uniform pricing strategy p_i, $i = A, B$. The cost of reaching a fraction ϕ of consumers is given by the function $A(\phi) = \lambda\eta(\phi)$. Following the literature on informative advertising (e.g. Butters (1977) and Tirole (1988)), it is assumed that the cost of reaching consumers increases at an increasing rate, which formally can be written as $A_\phi > 0$ and $A_{\phi\phi} \geq 0$.[19] The latter condition means that it is increasingly more expensive to inform an additional customer or likewise to reach a higher proportion of customers.[20] It is also assumed that there are no fixed costs in advertising, i.e. $A(0) = 0$. Finally, in order to make advertising viable, it is assumed that $A_\phi(0) < v$. The quadratic technology proposed in Tirole (1988) is not based upon an underlying technology of message production. However, it has the advantage of being extremely simple to manipulate algebraically. It is given by $A(\phi) = \lambda\eta(\phi)$, where $\eta(\phi) = \phi^2$. As in the present model there is a large number of buyers, normalized to one, λ can be identified with the cost per ad. In what

[18] This is guaranteed by the FTC regulation that prohibits advertisers from making false and deceptive statements about their products (see www.ftc.gov/bcp/conline/pubs/buspubs/ad-faqs.htm).

[19] Subscripts denote partial derivatives.

[20] Several justifications support this assumption. One justification has to do with the advertising technology itself. In other words, if ads are sent randomly at a fixed cost per ad, then the probability of reaching a consumer not yet informed decreases with the amount already advertised (e.g. the Butters' urn technology and the Constant Reach Independent Readership (CRIR) technology proposed by Grossman and Shapiro (1984)). Thus, as the number of ads sent increases it becomes increasingly costly to inform a consumer who has not yet received an ad from the firm. Other justifications are the existence of different predispositions to view ads on the part of the target population and the possibility of media saturation.

follows, whenever a functional form is needed, we will use the quadratic technology.

When firms are able to use targeted advertising, firms can target ads to specific segments of the market. In the context of our model, this amounts to say that each firm may send two types of ads: ads targeted to the group of consumers who intrinsically prefer its own product and ads targeted to the group of consumers who intrinsically prefer the rival's product. Within each segment messages are randomly distributed among consumers. On other words, within each group some consumers may receive one ad, more than one or none. In addition, targeted advertising can also be used as a tool for price discrimination since ads targeted to different segments of the market may quote different prices. In this case, the problem of firm i becomes more complex as it must choose an optimal advertising reach and an optimal price strategy to each segment of the market.

With respect to advertising, the problem of firm i consists in an advertising intensity for each segment of the market, ϕ_i^o targeted to its own market and ϕ_i^r targeted to the rival's market. Ads targeted to each segment will have different prices, respectively p_i^o and p_i^r, $i = A, B$. The parameter $\phi_i^o \in [0, 1]$ can be interpreted as the share of consumers in segment i that have received firm i's ads targeted to its own customers. Similarly, $\phi_i^r \in [0, 1]$ can be interpreted as the share of consumers who received ads from firm i, despite preferring the product offered by the rival's firm. The cost of reaching a fraction ϕ_i^k of consumers, $k = \{o, r\}$, is given by the strictly convex function $A(\phi_i^k) = \lambda\eta\left(\phi_i^k\right)$. It is also assumed that $A(0) = 0$ and $A_{\phi_i^k}(0) < v$. In what follows, whenever a functional form is needed, we will use the advertising technology proposed in Tirole (1988).

3 Mass advertising

This section investigates optimal pricing and advertising strategies when firms are endowed with a mass advertising technology. In this case, firms follow a uniform pricing policy as they are unable to target ads to specific segments of the market.

Accordingly, there are two components to a firm's strategy: Firm i must choose its advertising level (denoted by ϕ_i), as well as its price (denoted by p_i). We start the analysis by investigating consumers' decisions. In each segment of the market (segment a and segment b), it is possible to find four mutually exclusive and exhaustive categories of consumers: *(i)* firm A's captive consumers; *(ii)* firm B's captive consumers; *(iii)* selective consumers; and *(iv)* uninformed consumers.

The latter are uninformed about the existence of both products and therefore they simply do not buy any of the available goods. In contrast, selective consumers are fully informed and they shop for the better bargain. For example, selective consumers in segment a compare the net utility of purchasing good A at price p_A, $v - p_A$, with the net utility of purchasing good B at price p_B, $v - \gamma - p_B$. Such selective consumers buy good A if and only if $p_A < p_B + \gamma$. Analogously, selective consumers in segment b compare goods' relative utility and they buy good A if and only if $p_A - \gamma < p_B$. Reversing the previous inequalities one gets the conditions under which different types of selective consumers buy good B. Additionally, each firm has a group of captive consumers who buy its product as long as $p_i < v$ if the consumer is of type i or $p_i + \gamma < v$ if the consumer is not of type i.

The relative size of each group of consumers (captive, selective or uninformed consumers) depends on firms' decisions with respect to advertising. After firms have sent their ads independently, a proportion ϕ_i and ϕ_j of customers is reached by firm i and j advertising, respectively. Hence, the potential demand of firm i is made of a group of captive (locked-in) customers, namely $\phi_i \left(1 - \phi_j\right)$, and a group of selective customers, namely $\phi_i \phi_j$, $i = A, B$; $j = A, B$.

In the light of this, when $p_i < v - \gamma$, $\forall i = A, B$, the demand for good i when firms are endowed with a mass advertising technology, D_i, is equal to:

$$D_i = \underset{\text{captive}}{\phi_i \left(1 - \phi_j\right)} + \phi_i \phi_j \left[\underset{\text{selective type } i}{\frac{1}{2} \Pr\left(p_i - \gamma < p_j\right)} + \underset{\text{selective type } j}{\frac{1}{2} \Pr\left(p_i + \gamma < p_j\right)} \right] \tag{1}$$

Firm i's expected profit is equal to

$$E\pi_i = p_i D_i - A\left(\phi_i\right) \tag{2}$$

Given the rival's strategies, ϕ_j and p_j, the problem of firm i consists in choosing the advertising intensity, ϕ_i, and the pricing policy, p_i that maximize its profit, $p_i D_i - A\left(\phi_i\right)$.

3.1 Equilibrium Analysis

In this section, we study equilibrium price and advertising strategies, concentrating the analysis on the symmetric Nash equilibrium. Proposition 1 points out the conditions under which a symmetric price equilibrium in pure strategies exists.

Proposition 1 *(i) When $v < 2\gamma$ then a pure strategy equilibrium exists with $p_i = p_j = v$.*

(ii) When $2\gamma < v < 3\gamma$, there is a symmetric price equilibrium in pure strategies with $p_i = p_j = v - \gamma$, as long as $\phi_j < 1 - \frac{\gamma}{v-\gamma}$.

(iii) When $v > 3\gamma$ then there is no pure strategy equilibrium in prices. There is however a mixed strategy Nash equilibrium in prices.

Proof. See the Appendix.

In what follows we characterize the symmetric Nash equilibrium in prices. Firs we study cases *(i)* and *(ii)*, in which firms do not compete for selective consumers (monopoly cases). Then, we address case *(iii)*, in which firms compete for selective consumers (duopolistic competition).

3.1.1 Monopoly cases

In this section there are two relevant symmetric cases where firms act as local monopolists. The first case corresponds to situation *(i)* in Proposition 1 in which firms charge a price equal to v, as they exclusively serve the segment of the market with a strong preference for its good. In this case, firms charge a price equal to v in order to extract all the surplus from its customers and, as a result, some consumers stay out of the market after being exposed to advertising.

The second case corresponds to situation *(ii)* in Proposition 1. In this case firms charge a price equal to $v - \gamma$, and we observe that both firms serve all captive consumers, sharing evenly selective consumers. Accordingly, in this case, all informed consumers can enter the market. Firms have the monopoly provision of the good to its captive consumers and they choose not to compete for the selective consumers.

Case 1: $(p_i = v, p_j = v)$

Consider first the case where $v < 2\gamma$. In line with Shilony (1977) if $v \leq 2\gamma$, we obtain that, in equilibrium, both sellers charge a price v and so each firms acts as a monopolist in its strong market segment (corresponding to the segment of consumers whose reservation price for the good offered by the firm is equal to v).

It is therefore straightforward to obtain that firm i's equilibrium profit is

$$\pi_i = \frac{1}{2}v\left(\phi_i(1-\phi_j) + \phi_i\phi_j\right) - A\left(\phi_i\right) = \frac{1}{2}v\phi_i - A\left(\phi_i\right)$$

The equilibrium level of advertising is obtained maximizing π_i in order to ϕ_i. From the *FOC* we obtain

$$\frac{1}{2}v = A_\phi(\phi_i)$$

and $A_\phi(0) > \frac{1}{2}v$.

Equilibrium profits are in this case equal to

$$\pi_i^* = \phi^* A_\phi\left(\phi^*\right) - A\left(\phi^*\right).$$

For the quadratic technology for instance we would obtain

$$\phi^* = \frac{v}{4\lambda} \text{ with } v < 4\lambda$$

and thus

$$\pi_i^* = \lambda\left(\frac{v}{4\lambda}\right)^2.$$

Given the firms' price and advertising choices, when $v < 2\gamma$, total welfare is equal to:

$$\begin{aligned} W &= v\left[\phi^*\left(1-\phi^*\right) + \left(\phi^*\right)^2\right] - 2A\left(\phi^*\right) \\ &= v\phi^* - 2A\left(\phi^*\right) \end{aligned}$$

Since industry profit is equal to

$$\pi_{ind} = v\phi^* - 2A\left(\phi^*\right),$$

it follows that expected consumer surplus is equal to zero. As a matter of fact, in this case, the price quoted by firms corresponds to the reservation price of consumers with a strong preference for their good (v) and firms extract all the surplus from their customers (in addition, there are some consumers who stay out of the market after being exposed to advertising).

For the quadratic technology

$$W = \pi_{ind} = v\left(\frac{v}{4\lambda}\right) - 2\lambda\left(\frac{v}{4\lambda}\right)^2 = \frac{1}{8}\frac{v^2}{\lambda}$$

and

$$CS = W - 2\pi = 0.$$

Case 2: $(p_i = v - \gamma, p_j = v - \gamma)$

In this case, firms charge a price corresponding to the reservation price of consumers with a weakest prference for their good (equal to $v - \gamma$). In the light of this pricing strategy, firms sell all their captive consumers and they share the selective consumers. Here firm i's equilibrium profit is

$$\pi_i = (v-\gamma)\left(\phi_i(1-\phi_j) + \frac{1}{2}\phi_i\phi_j\right) - A(\phi_i) = \frac{(v-\gamma)}{2}\phi_i\left(2-\phi_j\right) - A(\phi_i)$$

The equilibrium level of advertising is obtained maximizing π_i in order to ϕ_i, from which we obtain

$$\frac{(v-\gamma)\left(2-\phi_j\right)}{2} = A_\phi(\phi_i)$$

Given the symmetry of the problem it must be the case that $\phi_i = \phi_j = \phi$. Thus, the equilibrium level of advertising is given by

$$\frac{(v-\gamma)(2-\phi^*)}{2} = A_\phi(\phi^*)$$

as long as $\phi^* < 1 - \frac{\gamma}{v-\gamma}$.

Equilibrium profits are in this case equal to

$$\pi_i^* = \phi^* A_\phi(\phi^*) - A(\phi^*).$$

Given the firms' price and advertising choices, when $2\gamma < v < 3\gamma$ and $\phi^* < 1 - \frac{\gamma}{v-\gamma}$, total welfare is equal to:

$$\begin{aligned} W &= v\left[2\phi^*(1-\phi^*) + (\phi^*)^2\right] - \gamma\left[\phi^*(1-\phi^*)\right] - 2A(\phi^*) \\ &= (v-\gamma)\phi^*(1-\phi^*) + v\phi^* - 2A(\phi^*) \end{aligned}$$

Since industry profit is equal to

$$\pi_{ind} = (v-\gamma)(2-\phi^*)\phi^* - 2A(\phi^*),$$

consumers surplus is

$$W-\pi_{ind}=\gamma\phi^{*}>0$$

In this case, consumer surplus is positive. Captive consumer who buy their least preferred version of the good have no surplus but consumers who buy their most preferred good (density equal to $(1-\phi)\,\phi+\phi^2$) gain the differential between the reservation price they attribute to their most preferred good (equal to v) and the price they actually pay for it (equal to $v-\gamma$)..

For the quadratic technology we would obtain

$$\phi^{*}=\frac{2\,(v-\gamma)}{v+4\lambda-\gamma}$$

As $v>\gamma$, it is always true that $\phi^{*}>0$. We only need to impose that $\phi<1$ from which one obtains:

$$v<4\lambda+\gamma \tag{3}$$

According to Proposition 1, the equilibrium candidate $p^{*}=v-\gamma$; $\phi^{*}=\frac{2(v-\gamma)}{v+4\lambda-\gamma}$ only constitutes a Nash equilibrium if $\phi^{*}<1-\frac{\gamma}{v-\gamma}$, leading to the following additional condition:

$$\frac{4\lambda+\gamma-\sqrt{(4\lambda+\gamma)^{2}-32\lambda\gamma}}{2}<v<\frac{4\lambda+\gamma+\sqrt{(4\lambda+\gamma)^{2}-32\lambda\gamma}}{2}<4\lambda+\gamma.$$

Thus, for a quadratic advertising technology, an equilibrium corresponding to case *(ii)* in Proposition 1 occurs when:

$$\frac{4\lambda+\gamma-\sqrt{(4\lambda+\gamma)^{2}-32\lambda\gamma}}{2}<v<\min\left\{3\gamma,\frac{4\lambda+\gamma+\sqrt{(4\lambda+\gamma)^{2}-32\lambda\gamma}}{2}\right\},$$

with $(4\lambda+\gamma)^{2}>32\lambda\gamma$.

Under these conditions, total welfare is equal to:

$$\begin{aligned}W &= (v-\gamma)\,\phi^{*}\,(1-\phi^{*})+v\phi^{*}-2\phi^{*2}\\ &= \frac{2\,(v-\gamma)\,[(\gamma-4)\,(v-\gamma)+4\lambda\,(2v-\gamma)]}{(v+4\lambda-\gamma)^{2}}\end{aligned}$$

Consumer surplus is given by:

$$W-\pi_{ind}=2\gamma\frac{v-\gamma}{v+4\lambda-\gamma}.$$

3.1.2 Duopolistic Competition

In line with the mainstream literature on advertising, we obtain that, when v is sufficiently large, there is no price equilibrium in pure strategies.

Lemma 1. *When $v>2\gamma$ firms have always incentives to serve all captive consumers. Hence, after being exposed to advertising all informed consumers can enter the market. When $v>3\gamma$ firms compete to attract selective consumers.*

From lemma 1 we can establish the next lemma.

Lemma 2. *In order to be able to serve all captive consumers, for each firm i,we must observe $p_{i,\max} \leq v - \gamma$.*

From Proposition 1 we already know that when $v > 3\gamma$ an equilibrium in pure strategies fails to exist. The intuition for the inexistence of a pure strategy price equilibrium is when v is high enough or γ is low enough, the existence of a positive fraction of selective consumers with a preference for the rival firm creates a tension between the firm's incentives to price low in order to attract this latter set of customers and firm's incentives to price high as a way to extract rents from the group of its captive and selective loyal customers. Therefore, each firm follows a mixed pricing strategy as an attempt to prevent the rival from systematically predicting its price, which in turn makes undercutting less likely. In this section, we investigate the mixed strategy Nash equilibrium in prices when firms compete for selective consumers, with $p_{\max} < v - \gamma$.

Proposition 2 bellow characterizes this equilibrium, pointing out the conditions under which such equilibrium exits. In the Appendix we prove by construction that there is a symmetric mixed strategy equilibrium in prices in which firms compete for selective consumers, with $p_{\max} < v - \gamma$. Suppose that firm i selects a price randomly from the cdf $F_i(p)$. In a symmetric mixed strategy equilibrium, both firms follow the same pricing strategy, thus, for the sake of simplicity write $F_i(p) = F_j(p) = F(p)$. Suppose further that the support of the equilibrium prices is $[p_{\min}, p_{\max}]$ with $p_{\max} < v - \gamma$. When firm i charges price p, there are three relevant outcomes. Firstly, p can be low enough to make all selective consumers interested in buying good i. This event occurs with probability $1 - F(p + \gamma)$. Secondly, p can be such that each firm sells to the group of selective consumers with a preference for its product. This event occurs with probability $[F(p + \gamma) - F(p - \gamma)]$. Finally, p can be so high that firm i is unable to attract any selective consumers, only serving the group of captive consumers.

Hence, for a given ϕ_i and ϕ_j, when firm i charges price $p < v - \gamma$, firm i's expected profit, denoted $E\pi_i$, is

$$E\pi_i = p\phi_i \left(1 - \phi_j\right) + p\phi_i\phi_j \left[1 - \frac{1}{2}F_j(p + \gamma) - \frac{1}{2}F_j(p - \gamma)\right] - A(\phi_i).$$

In the mixed strategy nash equilibrium, for a given ϕ_i and ϕ_j, each firm expected profit satisfies:

$$E\pi_i = p\phi_i \left(1 - \phi_j\right) + p\phi_i\phi_j \left[1 - \frac{1}{2}F_j(p + \gamma) - \frac{1}{2}F_j(p - \gamma)\right] - A(\phi_i) = k - A(\phi_i).$$

or equivalently:

$$p\phi_i \left(1 - \phi_j\right) + p\phi_i\phi_j \left[1 - \frac{1}{2}F_j(p + \gamma) - \frac{1}{2}F_j(p - \gamma)\right] = k.$$

Proposition 2. *In the benchmark case, with a mass advertising technology and no price discrimination, as long as* $p_{\max} < v - \gamma$

(i) each firm's price is randomly chosen from the cdf given by

$$F^m(p) = \left\{ \begin{array}{ccc} 0 & if & p < p_{\min} \\ 1 - \frac{2}{(\phi^m)^2}\left(\frac{k^m}{p+\gamma} - \phi^m\left(1-\phi^m\right)\right) & if & p_{\min} \leqq p \leqq p_{\max} - \gamma \\ 2 - \frac{2}{(\phi^m)^2}\left(\frac{k^m}{p-\gamma} - \phi^m\left(1-\phi^m\right)\right) & if & p_{\max} - \gamma \leqq p < p_{\max} \\ 1 & if & p \geqq p_{\max} \end{array} \right\}$$

with

$$p_{\min} = p_{\max} - 2\gamma \text{ and } p_{\max} = \frac{2k^m}{\phi^m\left(2-\phi^m\right)} + \gamma < v - \gamma.$$

For the mixed strategy price equilibrium in which firms compete for selective consumers to exist, it must be the case that $p_{\max} < v - \gamma$*, implying:*

$$v > \gamma\left(\frac{2-\phi^m}{\phi^m}\right)\left(1 + \sqrt{1 + \left(\frac{\phi^m}{2-\phi^m}\right)^2}\right) + 2\gamma$$

From $k^m = \frac{\phi^m}{2}\left(p_{\max} - \gamma\right)\left(2 - \phi^m\right)$ *we obtain that:*

$$k^m = \frac{\gamma}{2}\left(2-\phi^m\right)^2\left(1 + \sqrt{1 + \left(\frac{\phi^m}{2-\phi^m}\right)^2}\right).$$

(ii) Each firm chooses an advertising reach $\phi^m \in [0,1]$*, implicitly given by:*

$$\frac{1}{2}\left(p_{\max} - \gamma\right)\left(2-\phi^m\right) = A_\phi\left(\phi^m\right) \tag{4}$$

(iii) Each firm earns an overall expected profit equal to

$$E\pi^m = \phi^m A_\phi\left(\phi^m\right) - A\left(\phi^m\right) \tag{5}$$

or, equivalently,

$$E\pi^m = \frac{\gamma}{2}\left(2-\phi^m\right)^2\left(1 + \sqrt{1 + \left(\frac{\phi^m}{2-\phi^m}\right)^2}\right) - A\left(\phi^m\right).$$

Proof. See the Appendix.

Proposition 3. *Each firm serves its group of selective customers with probability given by* $q^m \in [0,1]$*:*

$$q^m = 1 - \frac{8\left(k^m\right)^2}{\left(\phi^m\right)^4}\left[\ln\left(\frac{\left(p_{\max}-\gamma\right)^2}{p_{\max}\left(p_{\max}-2\gamma\right)}\right) - \frac{1}{\left(p_{\max}-\gamma\right)^2}\right].$$

Proof. See the Appendix.

Quadratic Technology:

Next we study the firms' advertising and pricing strategies when the advertising technology is the quadratic one. This technology has been also used in other papers studying targeted advertising, e.g. Iyer and Villas-Boas (2005) and Galeotti et al (2008). Although this technology is not based upon an underlying technology of message production, it has the advantage of being extremely simple to manipulate algebraically. It is given by $A(\phi) = \lambda\eta(\phi)$, where $\eta(\phi) = \phi^2$. As in the present model there is a large number of buyers, normalized to one, λ can be identified with the cost per ad. Under these assumptions, we now investigate the firms' advertising and pricing strategies.

For the quadratic technology $A_\phi(\phi) = 2\lambda\phi = \lambda\phi^2$ and $A_\phi(\phi) = 2\lambda\phi$. From (4) it follows that if $p_{\max} < v - \gamma$ the equilibrium level of advertising is implicitly given by:

$$\frac{\gamma}{2\phi}\left((2-\phi)^2\left(1+\sqrt{1+\left(\frac{\phi}{2-\phi}\right)^2}\right)\right) = 2\lambda\phi$$

When v is sufficiently high such that $p_{\max} < v-\gamma$ and the advertising technology is the quadratic one the equilibrium level of advertising is given by

$$\phi^m = \frac{2\gamma^2 + 16\lambda\gamma - 8\lambda\sqrt{8\lambda\gamma+\gamma^2}}{\gamma^2 + 8\lambda\gamma - 16\lambda^2}. \tag{6}$$

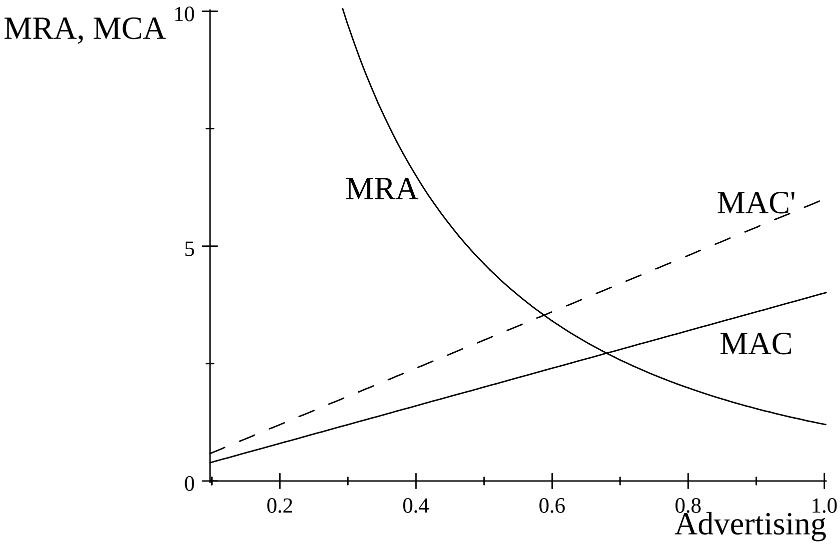
In the context of the advertising technology $\phi A_\phi(\phi) = 2A(\phi)$. Thus

$$E\pi^m = \frac{1}{2}\phi^m A_\phi(\phi^m) = \lambda(\phi^m)^2 > 0.$$

Proposition 4. *When v is high enough that is $p_{\max} < v-\gamma$ and the advertising technology is the quadratic one, i.e. $A(\phi) = \lambda\phi^2$, if $\lambda > \frac{\gamma(\sqrt{2}+1)}{4}$ then $\phi^m \in (0,1)$.*

Proof. See the Appendix.

In the following figure, we illustrate firms' optimal advertising reach when firms are endowed with a quadratic mass advertising technology.



Mass advertising technology: ϕ^m.

The downward sloping curve MRA is the marginal revenue of advertising with mass advertising i.e., the left hand-hand side of equation (4). The MRA is plotted for the case where $\gamma = 1$ and for v high enough i.e., for $v > p_{\max} + \gamma$. The upward sloping curves are the marginal cost of advertising for the quadratic technology for the special case where $\lambda = 2$ (MCA) and $\lambda = 3$ (MCA'). Shifts in the latter curve are only due to changes in λ. We can see that an increase (decrease) in marginal advertising costs makes the curve move upwards (downwards) and gives rise to less (more) advertising in equilibrium.[21] The intersection between the MRA and a MCA curve provides the equilibrium level of advertising with mass advertising. When $\lambda = 2$, the equilibrium level of advertising for each firm is $\phi^m = \frac{16\sqrt{17}-34}{47}$ which is approximately equal to 0.68. In this case $k^m = \frac{22032-4352\sqrt{17}}{2209}$ which is approximately equal to 1.85.

$$p_{\max} = \frac{2k^m}{\phi^m(2-\phi^m)} + 1 = \sqrt{17} + 1 < v - 1$$

which implies that $v > \sqrt{17} + 2$. Equilibrium profit is in this case equal to

$$E\pi^m = 2\left(\frac{16\sqrt{17}-34}{47}\right)^2 \cong 0.925\,36.$$

In this case $q_m = 0.7694$.

When $\lambda = 3$ and $\gamma = 1$ the equilibrium level of advertising for each firm is $\phi^m = \frac{10}{17}$ which is approximately equal to 0.588. In this case $E\pi^m = 3\left(\frac{10}{17}\right)^2 = 1.038\,1$ and $p_{\max} \simeq 6$, thus $v > 7$. Firms share the selective group of consumers with probability equal to $q^m = 0.763$.

3.2 Welfare Analysis

Next we compute total welfare with mass advertising. Recall that customers' gross benefit of buying a certain good can be given by $v-$"expected disutility cost", where the latter is equal

[21] To be precise, because $A_{\phi\lambda} > 0$ and $A_{\phi\phi} \geq 0$, static comparative analysis shows that $\frac{\partial\phi^m}{\partial\lambda} < 0$.

to γ, when the consumer buys the least preferred good; and zero, when the consumer buys the most preferred good.

In the social optimal solution, consumers would buy from the most preferred firm, in order to obtain a gross benefit of v (and minimize the expected disutility cost). However, in a symmetric equilibrium such outcome can only be reproduced if firms *(i)* share equally the group of selective consumers; and *(ii)* firms' captive consumers are those with a stronger preference for its own good . Otherwise, there is always a group of consumers who buy their least preferred good, thereby supporting a kind of disutility cost, i.e. γ.

Accordingly, total welfare can be represented as:

$$W^m = v\left[1-(1-\phi^m)^2\right] - EDC^m - 2A(\phi), \tag{7}$$

where EDC^m stands for the expected disutility costs in the mass advertising/no discrimination case with $p_{\max} < v-\gamma$,

$$EDC^m = \gamma\left[\underbrace{\phi^m(1-\phi^m)}_{\text{captive}} + \underbrace{\frac{1}{2}(1-q^m)}_{\text{selective}}\right] \tag{8}$$

The function EDC^m reflects the fact that market outcomes in the context of mass advertising/ no discrimination may not be fully efficient as some consumers may buy from the least preferred firm. The function EDC^m has two components. The first component refers to the disutility supported by captive consumers who are only aware of the least preferred product, i.e. $\phi^m(1-\phi^m)\gamma$. The second component refers to the disutility supported by selective consumers who bought the least preferred version of the good as a result of firms' pricing decisions. Recalling that q^m represents the probability of demand sharing with mass advertising, we may say that, under non discrimination, all selective consumers buy efficiently with probability equal to q^m. With probability $1-q^m$, half of selective consumers buy inefficiently and the remaining buy efficiently. Accordingly, the expected disutility cost supported by selective consumers is simply: $\frac{1}{2}(1-q^m)\gamma$.

Plugging (8) in (7), total welfare can be re-written as:

$$W^m = v\left[1-(1-\phi^m)^2\right] - \gamma\left[\phi^m(1-\phi^m) + \frac{1}{2}(1-q^m)\right] - 2A(\phi^m). \tag{9}$$

For the numerical example where $\gamma = 1, \lambda = 2, q^m = 0.7694, \phi^m = \frac{16\sqrt{17}-34}{47}$ we that:

$$W^m = 0.89773v - 2.1835. \tag{10}$$

As for this numerical example we should observe that $v > \sqrt{17}+2$ it follows that $W^m \gtrsim 3.313$. Industry profits are in this case equal to:

$$\pi^m_{ind} = 4\left(\frac{16\sqrt{17}-34}{47}\right)^2 \simeq 1.8507.$$

Thus, expected consumer surplus is equal to:

$$ECS^m = W^m - \pi^m_{ind} \tag{11}$$

Thus, $ECS^m \gtrsim 1.462$.

For the case where $\lambda = 3$, $\gamma = 1$ we have $\phi^m = \frac{10}{17}$, $E\pi^m = 3\left(\frac{10}{17}\right)^2 = 1.038\,1$ and $p_{\max} = 6$, thus $v > 7$. Firms share the selective group of consumers with probability equal to $q^m = 0.763$. Total welfare equals:

$$W^m = 0.83045v - 2.436\,8. \tag{12}$$

As $v > 7$ then $W^m \gtrsim 3.376\,4$. Industry profits are

$$\pi^m_{ind} = 2.076$$

$$ECS^m = W^m - \pi^m_{ind} \gtrsim 1.3004.$$

Comparing equations (10) and (12), we conclude that, as expected, for a given reservation price v, social welfare is larger when $\lambda = 2$ than when $\lambda = 3$. The same is true for consumers' surplus. For example, if for comparison purposes, we take $v = 7$, we observe that $W^m_{\lambda=2} = 4.100\,6$; and $ECS^m_{\lambda=2} = W^m - \pi^m_{ind} = 2.249\,9$, with $ECS^m_{\lambda=2} > ECS^m_{\lambda=3}$. However, firms do not necessarily benefit from having a more efficient advertising technology. In fact, for the values of the parameters considered in the example above, we observe that: $\pi^m_{\lambda=2} < \pi^m_{\lambda=3}$. The rationale for this result lies on the competition-dampening effects associated with a less efficient advertising technology. By making advertising more expansive, a increase in λ, reduces firms' advertising intensity (e.g. in the context of our examples $\phi^m_{\lambda=2} > \phi^m_{\lambda=3}$), with a direct negative effect on profits. However, the decline in firms' advertising intensities diminishes consumers' awareness of the product, shrinking the group of selective consumers. As a result, firms prefer to focus on the group of captive consumers, relaxing price competition (with a positive effect on profits). For the values of the parameters we considered in this example, we observe that the anti-competitive effect following an increase of λ more than compensates the direct negative effects stemming from an increase in advertising marginal costs.

4 Targeted advertising and price discrimination

This section investigates firms' advertising and pricing decisions when firms have the possibility to target ads to specific segments of the market and therefore firms may use advertising strategies as a tool for price discrimination. Now firm i's strategy is to choose the levels of advertising to be targeted to its own and to the rival's market (ϕ^o_i and ϕ^r_i, respectively) and the prices to be quoted to each group of consumers (p^o_i and p^r_i, respectively).

In this paper, we assume that firms' targeting ability is perfect, i.e.:

$$\begin{aligned} \Pr(\text{fall in } i \quad &| \quad \text{targeted to } i) = 1 \\ \Pr(\text{fall in } i \quad &| \quad \text{targeted to } j) = 0, \end{aligned}$$

which means that all messages targeted to group $i, j = A, B$ fall in the targeted segment and therefore there is no leakage of ads between groups.

The following table clarifies consumers' information regarding prices contingent on firms' advertising strategies.

p_i^k	% Type$-a$ consumers who know p_i^k	% Type$-b$ consumers who know p_i^k
p_A^o	ϕ_A^o	0
p_A^r	0	ϕ_A^r
p_B^o	0	ϕ_B^o
p_B^r	ϕ_B^r	0

Since there is no leakage of ads between different market segments, type$-a$ consumers are only aware of p_A^o and p_B^r, since the remaining pricing strategies, p_A^r and p_B^o are quoted in the ads that are targeted to type$-b$ consumers.

Again, in each segment of the market potential consumers can be divided into captive, selective and uninformed consumers. In market segment i, after firms have sent their ads independently, a proportion ϕ_i^o and ϕ_j^r of customers is reached, respectively, by firm i and j advertising. Thus, the firm i's demand in this segment of the market is made of a group of captive (locked-in) customers, namely $\phi_i^o\left(1-\phi_j^r\right)$, and a group of selective customers, namely $\phi_i^o\phi_j^r$, $i = A, B$; $j = B, A$.

In the light of this, firm $A's$ sales in segment a (at price p_A^o) are equal to:

$$D_A^o = \left[\underbrace{\frac{1}{2}\phi_A^o\left(1-\phi_B^r\right)}_{\text{captive}} + \underbrace{\frac{1}{2}\phi_A^o\phi_B^r \Pr(p_A^o < p_B^r + \gamma)}_{\text{type}-a\text{ selective}}\right]$$

and firm $A's$ sales in segment b (at price p_A^r) are equal to:

$$D_A^r = \left[\underbrace{\frac{1}{2}\phi_A^r\left(1-\phi_B^o\right)}_{\text{captive}} + \underbrace{\frac{1}{2}\phi_A^r\phi_B^o \Pr(p_A^r + \gamma < p_B^o)}_{\text{type}-b\text{ selective}}\right]$$

Similarly, firm B's sales in segment a (at price p_B^r) are given by

$$D_B^r = \left[\underbrace{\frac{1}{2}\phi_B^r\left(1-\phi_A^o\right)}_{\text{captive}} + \underbrace{\frac{1}{2}\phi_B^r\phi_A^o \Pr(p_A^o > p_B^r + \gamma)}_{\text{type}-a\text{ selective}}\right]$$

and firm B's sales in segment b (at price p_B^o) are equal to:

$$D_B^o = \left[\underbrace{\frac{1}{2}\phi_B^o\left(1-\phi_A^r\right)}_{\text{captive}} + \underbrace{\frac{1}{2}\phi_B^o\phi_A^r \Pr(p_A^r + \gamma > p_B^o)}_{\text{type}-b\text{ selective}}\right]$$

As there is no leakage, segments are totally independent. For a given strategy of the rival firm, firm i's expected profit conditional on ads and prices targeted to segment $k = o, r$, is equal to

$$E\pi_i^k = p_i^k D_i^k - A\left(\phi_i^k\right),\ i = A, B;\ \text{and } k = o, r.$$

Recall that, even when targeting is perfect, there always exist captive and selective consumers in each segment of the market. Look first at segment a. Within this segment, captive consumers to firm A are only aware of p_A^o, while the captive consumers to firm B are only aware of p_B^r. The selective consumers in segment a are aware of p_A^o as well as p_B^r. The same applies *mutatis mutandis* to segment i. Therefore, firm i's expected profit in its own segment, denoted $E\pi_i^o$, is given by:

$$E\pi_i^o = p_i^o \frac{\phi_i^o}{2}\left[\left(1-\phi_j^r\right) + \phi_j^r \Pr(p_i^o < p_j^r + \gamma)\right] - A\left(\phi_i^o\right), \tag{13}$$

with $i = A, B;$ and $j = B, A$.

Similarly, firm i's expected profit in the rival's market, denoted $E\pi_i^r$, is given by:

$$E\pi_i^r = p_i^r \frac{\phi_i^r}{2}\left[(1-\phi_j^o) + \phi_j^o \Pr\left(p_i^r + \gamma < p_j^o\right)\right] - A\left(\phi_i^r\right). \tag{14}$$

In each segment k, given firm j's advertising and pricing strategy, firm i chooses the advertising level (ϕ_i^k) and the targeted price (p_i^k) in order to maximize its expected profit defined by (13), in the case of its own market segment, and (14), in the case of the rival's market segment.

Proposition 5. *There is no price equilibrium in pure strategies.*

Proof. See the Appendix.

Proposition 6. *When target is perfect there is a symmetric Nash equilibrium in which:*

(i) In its own market segment each firm i, $i = A, B$ chooses a price randomly from the distribution $F_i^o(p)$ given by

$$F_i^o(p) = \left\{ \begin{array}{ccc} 0 & if & p \leq p_{j\,\min}^r + \gamma \\ \frac{1}{\phi_i^{o*}}\left[1 - \frac{(v-\gamma)(1-\phi_i^{o*})}{p-\gamma}\right] & if & p_{j\,\min}^r + \gamma \leq p \leq v \\ 1 & if & p \geq v \end{array} \right\}$$

where $p_{j\,\min}^r = (v-\gamma)\left(1-\phi_i^o\right)$. The advertising level ϕ_i^{o*} is implicitly given by

$$\frac{1}{2}v - \phi_i^{o*}\left(v-\gamma\right) = A_{\phi_i^o}\left(\phi_i^{o*}\right) \tag{15}$$

or equivalently,

$$p_{i\,\min}^o - \frac{1}{2}v = A_{\phi_i^o}\left(\phi_i^{o*}\right)$$

with $A_{\phi_i^o}(0) < \frac{1}{2}v$. Equilibrium profit in its own market is:

$$E\pi_i^{*o} = \phi_i^{*o} A_{\phi_i^{*o}}\left(\phi_i^{*o}\right) + \frac{1}{2}\left(\phi_i^{*o}\right)^2\left(v-\gamma\right) - A\left(\phi_A^{*o}\right). \tag{16}$$

(ii) In the rival's market segment each firm chooses a price randomly from the distribution $F_i^r(p)$ *given by*

$$F_i^r(p) = \left\{ \begin{array}{ccc} 0 & if & p \leq p^r_{j\min} \\ \frac{1}{\phi_i^{r*}}\left(1 - \frac{v\left(1-\phi_j^{o*}\right)+\gamma\phi_j^{o*}}{p+\gamma}\right) & if & p^r_{j\min} \leq p \leq v-\gamma \\ 1 & if & p \geq v-\gamma \end{array} \right\}$$

The advertising level ϕ_i^{r*} *is implicitly given by*

$$\frac{1}{2}(v-\gamma) - \frac{1}{2}\phi_j^{o*}(v-\gamma) = A_{\phi_i^r}\left(\phi_i^{r^*}\right), \tag{17}$$

or, equivalently,

$$\frac{1}{2}p^r_{j\min} = A_{\phi_i^r}\left(\phi_i^{r^*}\right), \tag{18}$$

where ϕ_j^{o*} *solves condition (15) and* $A_{\phi_i^r}(0) < \frac{1}{2}\left(1-\phi_j^{o*}\right)(v-\gamma)$. *Equilibrium profit in the rival's market is:*

$$E\pi_i^{*r} = \phi_i^{*r}A_{\phi_i^{*r}}\left(\phi_i^{*r}\right) - A\left(\phi_i^{*r}\right). \tag{19}$$

Proof. See the Appendix.

From Proposition 6, it follows that firm $i's$ decisions in relation to advertising reach in its own market, ϕ_i^o, is independent of firm $j's$ advertising intensity in this same market, ϕ_j^r. However the reverse is not true, since from condition (**??**) it follows that $\phi_i^{r^*}$ decreases with ϕ_j^{o*} (strategic substitutability). As ϕ_j^{o*} increases, there is a decrease in firm i's advertising reach targeted to the rival's market (segment j). The rationale for this is the following: as ϕ_j^{o*}, there is a reduction on the minimum price charged by firm j, (as the number of firm i's captive consumers become smaller). Accordingly, this market becomes less interesting to firm i, who reduces its advertising intensity ϕ_i^r, after observing an increase of ϕ_j^{o*}.

Proposition 6 also unveils that whether a firm advertises more on its own market or the rival's market depends on the values of the parameters v and γ.

From (15) it follows that $p^r_{j\min} + \left(\gamma - \frac{1}{2}v\right) = A_{\phi_i^o}\left(\phi_i^{o*}\right)$. For a given ϕ the right hand side of equations (15) and (17) is the same. Thus, it is straightforward to obtain that firms advertise more to its own market than to the rival's market when $p^r_{j\min} > v - 2\gamma$, otherwise the reverse happens. They choose the same intensity of advertising to both markets when $p^r_{j\min} = v - 2\gamma$.

Proposition 7. *Regardless the advertising technology considered, when price discrimination is permitted and target advertising is perfect*:

(i) When advertising is costless, firms do not select full market coverage in its own market.

(ii) When the advertising costs are high, i.e., if λ *is such that* $\phi_i^{o*} < \frac{\gamma}{v-\gamma}$, *firms advertise more to its own market*; *when advertising is cheap, i.e., if* λ is such that $\phi_i^{o*} > \frac{\gamma}{v-\gamma}$, *they advertise more to the rival's market*; *firms choose the same level of advertising to both market segments when* λ is such that $\phi_i^{o*} = \frac{\gamma}{v-\gamma}$.

Proof. See the Appendix.

Corollary 1. *When $\phi_i^{o*} < \phi_i^{r*}$, the cdf $F_i^r(p)$ has a mass point at $v-\gamma$, with $F_i^r(v-\gamma) = \frac{\phi_i^{o*}}{\phi_i^{r*}}\left(1-\frac{\gamma}{v}\right) < 1$.*

Proof. See the Appendix.

In the equilibrium derived above it can be said that firm i uses a "Hi-Lo" pricing strategy in the rival's market. To squeeze more surplus from its captive customers, it charges the highest price $v-\gamma$, with probability $m_i^r = 1-\frac{\phi_i^{o*}}{\phi_i^{r*}}\left(1-\frac{\gamma}{v}\right)$. However, in order win the selective customers it quotes occasionally a low price.

When $\phi_i^o < \frac{\gamma}{v-\gamma}$, the firm advertises more in its own market. In this case, we may say that firms are adopting a very restrictive advertising strategy and thereby they prefer to concentrate on its own market as a way to relax price competition. As expected, the larger γ in relation to v (corresponding to cases in which either products are very differentiated or consumers exhibit very heterogeneous tastes towards each product), the higher the likelihood of observing this type of competition dampening effects. In contrast, if the value of γ is low, then the condition $\phi_i^o < \frac{\gamma}{v-\gamma}$ becomes more restrictive and it becomes more likely to observe firms advertising more to the rival's market than to its own market. In this case, product differentiation is less important and firms have strong incentives to compete for selective consumers. As a result firms prefer to target more ads to the rival's market than to its own market.

Quadratic advertising technology: Next we study the firms' advertising and pricing strategies for the quadratic technology.

Proposition 8. *With the quadratic technology the equilibrium level of advertising to firm i's own market is*

$$\phi^{o*} = \frac{v}{2(v-\gamma)+4\lambda}$$

and the equilibrium level of advertising to the rival's market is

$$\phi^{r*} = \frac{(v-\gamma)(v+4\lambda-2\gamma)}{8\lambda(v+2\lambda-\gamma)}.$$

For the quadratic technology $A(\phi) = \lambda\phi^2$ and $A_\phi(\phi) = 2\lambda\phi$. Thus $\phi A_\phi(\phi) = 2\ A(\phi)$. Equilibrium profit in each market is

$$E\pi_i^{*o} = \phi_i^{*o} A_{\phi_i^{*o}}(\phi_i^{*o}) + \frac{1}{2}(\phi_i^{*o})^2(v-\gamma) - A(\phi_A^{*o}) = A(\phi_i^{*o}) + \frac{1}{2}(\phi_A^{*o})^2(v-\gamma)$$

and

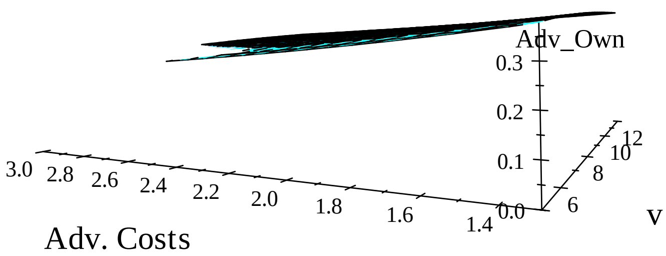
$$E\pi_i^{*r} = \frac{1}{2}\phi_i^{*r} A_{\phi_i^{*r}}(\phi_i^{*r}) = A(\phi_i^{*r}) \tag{20}$$

Corollary 2. *For the quadratic technology it follows that when $v > 2\gamma$:*
(i) it is always true that $\phi_i^{o*} \in (0,1)$.
(ii) $\phi_i^{r*} \in (0,1)$ iff $\lambda > \frac{\sqrt{(5v-9\gamma)(v-\gamma)}-(v-\gamma)}{8}$

Proof. See the Appendix.

Proposition 9. *With the quadratic technology firms advertise more to their rival's market than to their own market, i.e.,* $\phi^{r*} \geq \phi^{o*}$ *when* $v \geq \frac{1}{2}\left(3\gamma + \sqrt{\gamma^2 + 16\lambda\gamma}\right)$. *The reverse happens when* $2\gamma < v < \frac{1}{2}\left(3\gamma + \sqrt{\gamma^2 + 16\lambda\gamma}\right)$.

Proof. See the Appendix.



Advertising Intensities in each segment

Figure 2 shows that when $\gamma = 1$, $v = 5$ and $\lambda = 3$ $v = \frac{1}{2}\left(3\gamma + \sqrt{\gamma^2 + 16\lambda\gamma}\right)$ and thus $\phi^{r*} = \phi^{o*}$. When v is high and λ is low $\phi^{r*} > \phi^{o*}$.

5 Competitive effects of perfect targeted advertising

This section investigates how targeted advertising and price discrimination affects the equilibrium outcomes—i.e., advertising intensities, prices and profits.

Proposition 10 *With perfect targeting each wins the group of selective customers in its own market with probability equal to* $\tau \in [0,1]$ *where*

$$\tau = \frac{v-\gamma}{\phi^{r*}v}\left(1 - \frac{1}{\phi^{o*}}\right)\frac{\phi^{o*}(v-\gamma)-v}{\gamma^2 p^r_{\min}}\left(\left(v - p^o_{\min}\right)\gamma + v\left(\gamma - p^o_{\min}\right)\ln\left(\frac{p^o_{\min}}{p^r_{\min}}\frac{v-\gamma}{v}\right)\right).$$

Proof: See the Appendix.

Effects on Prices Figure 3 plots $F_i^m(p)$ (bold), $F_i^r(p)$ (dash) and $F_i^o(p)$ (solid) for the case where $\gamma = 1, \lambda = 2$ and $v = 7$. For this numerical example we observe that $F^m > F^k$, $k = o, r$, thus F^k is stochastically larger than F^m implying that in this case average prices with perfect targeting and price discrimination are above average prices with mass advertising and non-discrimination. This is an interesting finding because a common finding in the literature on competitive price discrimination is that uniform prices are above the discriminatory ones (e.g. Thisse and Vives (1988), Fudenberg and Tirole (2000). The intuition is that targeted advertising may act to soften price competition.

Corollary 3. *Regardless the advertising technology considered, as advertising becomes cheaper, the minimum price in the equilibrium support of both cdf* $F_i^r(p)$ and $F_i^o(p)$ *decreases. When advertising becomes costless* $p^o_{\min} > \gamma$.

Note that when $\lambda \to 0$, $\phi^o = \frac{v}{2(v-\gamma)}$ and $p^o_{i\,\min} = \frac{1}{2}v > \gamma$ which is true as $v > 2\gamma$.

Effects on profits Firm i's profit with perfect targeted advertising is $E\pi^t = E\pi_i^{*r} + E\pi_i^{*o}$. For the quadratic technology we have

$$E\pi_i^{*o} = A\left(\phi_i^{*o}\right) + \frac{1}{2}\left(\phi_A^{*o}\right)^2 (v-\gamma), \tag{21}$$

and

$$E\pi_i^{*r} = A\left(\phi_i^{*r}\right). \tag{22}$$

Each firm profit with targeted advertising is

$$E\pi^t = A\left(\phi_i^{*o}\right) + \frac{1}{2}\left(\phi_A^{*o}\right)^2 (v-\gamma) + A\left(\phi_i^{*r}\right)$$

Using the fact that with the quadratic technology $\phi^{o*} = \frac{v}{2(v-\gamma)+4\lambda}$ and $\phi^{r*} = \frac{(v-\gamma)(v+4\lambda-2\gamma)}{8\lambda(v+2\lambda-\gamma)}$ we have that:

$$E\pi^{*t} = \underbrace{\left(\lambda + \frac{v-\gamma}{2}\right)\left(\frac{v}{2(v-\gamma)+4\lambda}\right)^2}_{E\pi_i^{*o}} + \underbrace{\lambda\left(\frac{(v-\gamma)(v+4\lambda-2\gamma)}{8\lambda(v+2\lambda-\gamma)}\right)^2}_{E\pi_i^{*r}}.$$

With mass advertising and with the quadratic technology where $\phi^m = \frac{\frac{2}{4\lambda}\sqrt{8\lambda\gamma+\gamma^2}}{1+\frac{1}{4\lambda}\sqrt{8\lambda\gamma+\gamma^2}}$, and $\lambda > \frac{\gamma(\sqrt{2}+1)}{4}$, if we take into account that v is high enough to guarantee that $p_{\max} < v-\gamma$, equilibrium profit is:

$$E\pi^m = \frac{1}{2}\phi^m A_\phi\left(\phi^m\right) = A\left(\phi^m\right) = \lambda\left(\frac{\frac{2}{4\lambda}\sqrt{8\lambda\gamma+\gamma^2}}{1+\frac{1}{4\lambda}\sqrt{8\lambda\gamma+\gamma^2}}\right)^2. \tag{23}$$

Figure 4 plots equilibrium profit with mass (green) and targeted advertising (blue) when $\gamma = 1$ and assuming that v and λ are such that all the conditions for the equilibrium proposed in proposition 2 ($p_{\max} < v - \gamma$) are met.

We can see that when v is large enough in comparison to λ equilibrium profit with perfect targeted advertising is above equilibrium profit with mass advertising.

For the numerical example where $\gamma = 1, \lambda = 2$ and $v = 7$ it follows that $\phi^{o*} = \frac{7}{20}$ and $\phi^{r*} = \frac{39}{80}$. In this case we obtain $E\pi_i^{*o} = 0.612$ and $E\pi_i^{*r} = 0.475$. Thus, $E\pi^t = 1.087$. In this case, $E\pi^m = 0.92536$. For the numerical example where $\gamma = 1, \lambda = 3$ and $v = 8$ it follows that $\phi^{o*} = \frac{4}{13}, \phi^{r*} = \frac{21}{52}$, $p^r_{B\min} = \frac{63}{13}, p^o_{\min} = \frac{76}{13}$. Thus, $E\pi_i^{*o} = 0.615$ and $E\pi_i^{*r} = 0.489$. Thus, $E\pi^t = 1.104$, while in this case $E\pi^m = 1.0381$.

These numerical examples show that if we take into account that with mass advertising $v > 3\gamma$ and v is high enough such that $p_{\max} < v - \gamma$ then if advertising costs are not too high targeted advertising and price discrimination can boost firms' profits in relation to the mass advertising/no discrimination case. This is an interesting finding because it shows that discrimination by means of targeted informative advertising do not necessarily lead to the classic prisoner dilemma result that arises in models with full informed consumers (Thisse and Vives (1988), Fudenberg and Tirole (2000)). In this way we show that at least when v is high and advertising is not too expensive each firm's profit with perfect targeted advertising and price discrimination is above its non-discrimination counterpart.

It is also interesting to note that while with mass advertising while with mass advertising the firm's profit increases with increases in the advertising costs the reverse happens with targeted advertising.

Effects on market segmentation

$$\begin{aligned}
\phi^{o*}(1-\phi^{r*}) &= \left[\left(\frac{v}{2(v-\gamma)+4\lambda}\right)\left(1-\frac{(v-\gamma)(v+4\lambda-2\gamma)}{8\lambda(v+2\lambda-\gamma)}\right)\right]_{\gamma=1} \\
\phi^{o*}\phi^{r*} &= \left[\left(\frac{v}{2(v-\gamma)+4\lambda}\right)\left(\frac{(v-\gamma)(v+4\lambda-2\gamma)}{8\lambda(v+2\lambda-\gamma)}\right)\right]_{\gamma=1} \\
\phi^{r*}(1-\phi^{o*}) &= \left[\left(\frac{(v-\gamma)(v+4\lambda-2\gamma)}{8\lambda(v+2\lambda-\gamma)}\right)\left(1-\frac{v}{2(v-\gamma)+4\lambda}\right)\right]_{\gamma=1}
\end{aligned}$$

Non-informed consumers With targeted advertising the number of consumer who stay out of the market is given by

$$NI^t = (1-\phi^{o*})(1-\phi^{r*})$$

while with mass advertising this number of consumers

$$NI^m = (1-\phi^m)^2.$$

With the quadratic technology we have

$$NI^t = \left(1-\frac{v}{2(v-\gamma)+4\lambda}\right)\left(1-\frac{(v-\gamma)(v+4\lambda-2\gamma)}{8\lambda(v+2\lambda-\gamma)}\right)$$

$$NI^m = \left(1-\frac{\frac{2}{4\lambda}\sqrt{8\lambda\gamma+\gamma^2}}{1+\frac{1}{4\lambda}\sqrt{8\lambda\gamma+\gamma^2}}\right)^2$$

6 Welfare Issues

This section compares aggregate welfare with mass and targeted advertising. As before total welfare can be written as $v-$"expected disutility cost"$-$advertising costs. In the social optimal solution selective consumers should buy from the preferred firm. With targeted advertising this only happens when a firm wins the group of selective consumers in its own market, which happens with probability τ. Similarly, in what concerns captive consumers, we observe that in the social optimal solution, captive consumers would buy from its most preferred firm. However, at equilibrium, this is nor necessarily the case as some consumers might be captive to their least preferred brand, which introduces an additional source of inefficiency as these consumers buy inefficiently. Accordingly, with perfect targeted advertising overall welfare is given by:

$$W^t = v\left[1-(1-\phi^o)(1-\phi^r)\right] - \gamma\left[\phi^r(1-\phi^o) + \phi^o\phi^r(1-\tau)\right] - 2A(\phi^o) - 2A(\phi^r). \tag{24}$$

We have seen that with mass advertising aggregate welfare:

$$W^m = v\left[1-(1-\phi^m)^2\right] - \gamma\left[\phi^m(1-\phi^m) + \frac{1}{2}(\phi^m)^2(1-q^m)\right] - 2A(\phi^m). \tag{25}$$

Expected consumer surplus which is equal to

$$ECS = W - \pi_{ind}.$$

Table plots industry profit, consumer surplus and welfare for the case where $\gamma = 1$. When $\lambda = 2$ and $v = 7$ with mass advertising $q^m = 0.7694$ and $\phi^m = \frac{16\sqrt{17}-34}{47}$. In this case, with targeted advertising, $\phi^{o*} = \frac{7}{20}$, $\phi^{r*} = \frac{39}{80}$ and $\tau = 0.31529$. When where $\gamma = 1, \lambda = 2, v = 8$, $\phi^m = \frac{16\sqrt{17}-34}{47}$ and $q^m = 0.7694$ with mass advertising, while with targeted advertising $\phi^{o*} = \frac{4}{11}, \phi^{r*} = \frac{49}{88}$, $\tau = 0.29228$. Finally, when where $\gamma = 1, \lambda = 3, v = 8$, $\phi^m = \frac{10}{17}$ and $q^m = 0.786$ with mass advertising, while with targeted advertising $\phi^{o*} = \frac{4}{13}, \phi^{r*} = \frac{21}{52}$, $\tau = 0.33934$.

Table 1. Profits, Consumer Surplus and Welfare

	$\lambda = 2, v = 7$	$\lambda = 2, v = 8$	$\lambda = 3, v = 8$
$E\pi^t_{ind}$	2.174	2.694	2.208
$E\pi^m_{ind}$	1.851	1.851	2.076
W^t	3.514	4.361	3.821
W^m	4.162	5.06	4.289
ECS^t	1.34	1.667	1.613
ECS^m	2.311	3.209	2.212

The numerical examples show that targeted advertising and price discrimination can boost industry profit at the expense of social welfare and consumer welfare. This result is therefore in stark contrast with the general presumption of Chen (2005), according to whom "price discrimination under customer recognition ... is by and large unlikely to raise significant antitrust concerns. In fact, as the economics literature suggests, such pricing practices in oligopoly markets often intensify competition and potentially benefit consumers." (p. 123). Thus the paper highlights the importance of taking into account different forms of market competition when try to evaluate the welfare effects of price discrimination based on customer recognition.

7 Conclusions

This paper has investigated the effects of price discrimination by means of targeted advertising in a duopolistic market where advertising plays two major roles: it is used by firms as a way to transmit relevant information to otherwise uninformed consumers, and it is used as a price discrimination device.

Two advertising and pricing strategies were studied in the paper: (i) a mass advertising/non-discrimination strategy and a targeted advertising/price discrimination strategy.

Under mass advertising firms choose an intensity of advertising to the entire market and all ads announce the same price. With a mass advertising strategy, if the reservation value is high enough the model yields a symmetric equilibrium in mixed strategies in prices with the advertising component chosen deterministically.

When price discrimination by means of targeted advertising is used firms choose different levels of advertising to each market segment and ads tailored to different segments quote different prices. The paper has shown that the price equilibrium is always in mixed strategies and the level of advertising to each segment is chosen deterministically. We have also shown that when the advertising costs are high (or v is low) firms advertise more to its own market. In contrast, when advertising is cheap (and v is high) firms advertise more to the rival's market.

As in other models with price discrimination based on customer recognition we showed that a firm charges on average lower prices to the rival's customers than to its own customers (e.g. Thisse and Vives (1988), Fudenberg and Tirole (2000).

The stylised model addressed in this paper has shown that new insights may arise in the literature on price discrimination based on customer recognition if firms need to invest in advertise to generate demand. First, we showed that average prices with mass advertising (non-discrimination) are below those with targeted advertising. This is an interesting finding as it challenges the usual finding that price discrimination may reduce all segment prices. Second, we showed that price discrimination by means of targeted informative advertising do not necessarily lead to the classic prisoner dilemma result that arises in models with full informed consumers. In this way we showed that at least when v is high and advertising is not expensive each firm's profit with perfect targeted advertising and price discrimination is above its non-discrimination counterpart.

Finally, another theme of the paper was to investigate the welfare effects of targeted advertising with price discrimination in comparison to the mass advertising/non-discrimination case. We showed that at least when advertising costs are not too high in comparison to v, price discrimination by means of targeted advertising can boost industry profit at the expense of consumer surplus and welfare. Thus, the paper has highlighted the importance of taking into account different forms of market competition when public policy tries to evaluate the profit and welfare effects of price discrimination.

Appendix

Proof of Proposition 1. Look first at case *(i)*. Suppose (v, v) is an equilibrium in pure strategies. Then

$$\pi_i = \frac{1}{2}v\left(\phi_i(1-\phi_j)+\phi_i\phi_j\right).$$

Any price greater than v is not part of an equilibrium strategy since such price is higher than consumers' reservation price for the most preferred good and therefore no consumer would be interested in buying the good and firms would not make any profit.

Any price lower than v but greater than $v-\gamma$ gives firm i the same market share but reduces its profit and so it is dominated by v. If firm i deviates and chooses price $v-\gamma$ it gets the remaining group of captive consumers and its profit from deviation is

$$\pi_i^d = (v-\gamma-\varepsilon)\left(\phi_i(1-\phi_j)+\phi_i\phi_j\right)$$

The firm has no incentive to deviate to $v-\gamma$ as long as

$$(v-\gamma-\varepsilon)\left(\phi_i(1-\phi_j)+\phi_i\phi_j\right) < \frac{1}{2}v\left(\phi_i(1-\phi_j)+\phi_i\phi_j\right),$$

which requires:

$$\varepsilon > \frac{v-2\gamma}{2}$$

It is straightforward to see that the deviation is not profitable if $v < 2\gamma$. In contrast it is profitable if $v > 2\gamma$ as such an ε always exists. Therefore, when $v < 2\gamma$, (v, v) is a Nash equilibrium in pure strategies.

(ii) Look next in the case where $v > 2\gamma$ and investigate in which conditions $(v-\gamma, v-\gamma)$ is an equilibrium in pure strategies. In this case

$$\pi_i = (v-\gamma)\left(\phi_i(1-\phi_j)+\frac{1}{2}\phi_i\phi_j\right)$$

If firm i deviates to a higher price it must be to price v. In this case, deviation profits are equal to

$$\pi_i^d = \frac{1}{2}v\left(\phi_i(1-\phi_j)+\phi_i\phi_j\right) = \frac{1}{2}v\phi_i$$

Firm i has an incentive to deviate iff

$$\frac{1}{2}v\phi_i > (v-\gamma)\left(\phi_i(1-\phi_j)+\frac{1}{2}\phi_i\phi_j\right),$$

which holds as long as:

$$\phi_j > 1-\frac{\gamma}{v-\gamma}.$$

This means that for $(v-\gamma, v-\gamma)$ to be a price equilibrium in pure strategies:

$$0 < 1 - \frac{\gamma}{v-\gamma} < \phi_j < 1$$

Note that for $v > 2\gamma$ is is always true that $0 < 1 - \frac{\gamma}{v-\gamma} < 1$. Thus firms have no incentive to deviate to a higher price as long as $\phi_j < 1 - \frac{\gamma}{v-\gamma}$. Otherwise the deviation to a higher price is profitable.

Consider next a deviation to a lower price as a way to capture the group of selective consumers who prefer the rival. The deviating price must satisfy: $p_i^d = v - 2\gamma - \varepsilon$. The deviation is profitable iff

$$(v - 2\gamma - \varepsilon)\left(\phi_i(1-\phi_j) + \phi_i\phi_j\right) > (v-\gamma)\left(\phi_i(1-\phi_j) + \frac{1}{2}\phi_i\phi_j\right)$$

which requires:

$$0 < \frac{2\gamma}{v-\gamma} + \frac{2\varepsilon}{v-\gamma} < \phi_j < 1.$$

As $\varepsilon \to 0$ to ensure that $\frac{2\gamma}{v-\gamma} + \frac{2\varepsilon}{v-\gamma} < \phi_j < 1$ it must be the case that $v > 3\gamma$. Thus, there is no equilibrium in pure strategies in $(v-\gamma, v-\gamma)$ if $v > 3\gamma$.■

Proof of Proposition 2. Next we prove that there is an equilibrium in mixed strategies in prices for an interior solution in the advertising equilibrium levels. Suppose that firm i selects a price randomly from the cdf $F_i(p)$. To simplify we will only focus the analysis in a symmetric mixed strategy equilibrium in which both firms follow the same pricing strategy. For the sake of simplicity write $F_i(p) = F_j(p) = F(p)$. Suppose further that the support of the equilibrium prices is $[p_{\min}, p_{\max}]$, with $p_i < v - \gamma$. When firm i chooses any price that belongs to the equilibrium support of prices, and firm j uses the cdf $F(p)$, firm i's expected profit is always equal to a constant, which is denoted k minus advertising costs. When firm i charges price $p_i < v - \gamma$, firm i's expected profit, denoted $E\pi_i$, is

$$E\pi_i = p\phi_i\left(1-\phi_j\right) + p\phi_i\phi_j\left[1 - \frac{1}{2}F_j\left(p+\gamma\right) - \frac{1}{2}F_j\left(p-\gamma\right)\right] - A(\phi_i)$$

In a MSNE we must observe that:

$$p_i\left[\phi_i\left(1-\phi_j\right) + \phi_i\phi_j\left[1 - \frac{1}{2}F_j\left(p_i+\gamma\right) - \frac{1}{2}F_j(p_i-\gamma)\right]\right] - A\left(\phi_i\right) = k - A\left(\phi_i\right),$$

from which we obtain

$$F_j\left(p_i+\gamma\right) + F_j(p_i-\gamma) = 2 - \frac{2k}{\phi_i\phi_j p_i} + \frac{2\left(1-\phi_j\right)}{\phi_j} \tag{26}$$

Suppose that

$$\begin{aligned} p_1 \text{ is such that } p_1 - \gamma &= p_{\min} \\ p_2 \text{ is such that } p_2 + \gamma &= p_{\max} \end{aligned}$$

Then,

$$\forall p \leq p_1,\ F\left(p-\gamma\right)=0$$

$$\forall p \geq p_2,\ F\left(p+\gamma\right)=1.$$

Using (26) it follows that

$$\forall p \leq p_1 \Rightarrow F_j\left(p_i+\gamma\right)=2-\frac{2k}{\phi_i\phi_j p_i}+\frac{2\left(1-\phi_j\right)}{\phi_j}$$

and

$$\forall p \geq p_2 \Rightarrow F_j(p_i-\gamma)=1-\frac{2k}{\phi_i\phi_j p_i}+\frac{2\left(1-\phi_j\right)}{\phi_j}$$

Thus,

$$\forall p \leq p_1 \Rightarrow F\left(p\right)=2-\frac{2k}{\phi_i\phi_j\left(p-\gamma\right)}+\frac{2\left(1-\phi_j\right)}{\phi_j}$$

Similarly,

$$\forall p \geq p_2 \Rightarrow F(p)=1-\frac{2k}{\phi_i\phi_j\left(p+\gamma\right)}+\frac{2\left(1-\phi_j\right)}{\phi_j}$$

Now show that $p_1=p_2$. Suppose first that $p_2<p_1$. Then, $\forall p\in[p_2,p_1]$ it follows $F\left(p-\gamma\right)=0$ and $F\left(p+\gamma\right)=1$ thus

$$p\left[\phi_i\left(1-\phi_j\right)+\frac{1}{2}\phi_i\phi_j\right]=k$$

Assume now that $p_2>p_1$ and take $p\in[p_1,p_2]$ s.t. (26) holds.

$$\begin{aligned}\exists\widetilde{p}\text{ s.t. }\widetilde{p}-\gamma &= p_L<p_1\\ \widetilde{p}+\gamma &= p_H>p_2\end{aligned}$$

Since $p_L<p_1$ and $p_H>p_2$, it follows that

$$F\left(\widetilde{p}\right)=2-\frac{2}{\phi_i\phi_j}\left(\frac{k}{\left(\widetilde{p}-\gamma\right)}-\phi_i\left(1-\phi_j\right)\right)$$

and

$$F(\widetilde{p})=1-\frac{2}{\phi_i\phi_j}\left(\frac{k}{\widetilde{p}+\gamma}-\phi_i\left(1-\phi_j\right)\right).$$

From the continuity of F it must be true that

$$2-\frac{2}{\phi_i\phi_j}\left(\frac{k}{\left(\widetilde{p}-\gamma\right)}-\phi_i\left(1-\phi_j\right)\right)=1-\frac{2}{\phi_i\phi_j}\left(\frac{k}{\widetilde{p}+\gamma}-\phi_i\left(1-\phi_j\right)\right),$$

from which it follows that there is a unique positive value of $\widetilde{p}$ given by:

$$\widetilde{p}=\sqrt{\frac{4k\gamma}{\phi_i\phi_j}+\gamma^2}$$

Since this must hold $\forall p\in[p_1,p_2]$ and they cannot all be equal it must be the case that $p_1=p_2$. Since $p_1=p_{\min}+\gamma$ and $p_2=p_{\max}-\gamma$ it follows that $p_{\min}+\gamma=p_{\max}-\gamma$ or equivalently $p_{\max}-p_{\min}=2\gamma$.

Let p be the price of firm i, then given that $p_{\max}-p_{\min}=2\gamma$, it follows that for any $p<v-\gamma$, $F(p)$ is equal to

$$F(p)=\left\{\begin{array}{ccc} 0 & if & p<p_{\min} \\ 1-\frac{2}{\phi_i\phi_j}\left(\frac{k}{p+\gamma}-\phi_i\left(1-\phi_j\right)\right) & if & p_{\min}\leqq p\leqq p_{\max}-\gamma \\ 2-\frac{2}{\phi_i\phi_j}\left(\frac{k}{(p-\gamma)}-\phi_i\left(1-\phi_j\right)\right) & if & p_{\max}-\gamma\leqq p<p_{\max} \\ 1 & if & p\geqq p_{\max} \end{array}\right\} \tag{27}$$

From $F\left(p_{\min}\right)=0$ and $F\left(p_{\max}\right)=1$ it follows that

$$1-\frac{2}{\phi_i\phi_j}\left(\frac{k}{p_{\min}+\gamma}-\phi_i\left(1-\phi_j\right)\right)=0 \Longleftrightarrow \frac{2k}{\phi_i\phi_j+2\phi_i\left(1-\phi_j\right)}-\gamma=p_{\min}$$

and

$$2-\frac{2}{\phi_i\phi_j}\left(\frac{k}{\left(p_{\max}-\gamma\right)}-\phi_i\left(1-\phi_j\right)\right)=1$$

Thus we obtain that:

$$p_{\max}\in\min\left\{\frac{2k}{\phi_i\phi_j+2\phi_i\left(1-\phi_j\right)}+\gamma, v-\gamma\right\} \tag{28}$$

By continuity, for $p=p_{\max}-\gamma=\frac{2k}{\phi_i\phi_j+2\phi_i\left(1-\phi_j\right)}$, it must be true that:

$$\frac{2\phi_i\phi_j\left(k^m\right)^2}{\left[\phi_i\phi_j+2\phi_i\left(1-\phi_j\right)\right]^2}-2\gamma k^m-\frac{\phi_i\phi_j}{2}\gamma^2=0$$

and the unique positive solution for k is equal to:

$$k^m=\frac{\gamma\phi_i}{2\phi_j}\left(2-\phi_j\right)^2\left(1+\sqrt{1+\left(\frac{\phi_j}{2-\phi_j}\right)^2}\right).$$

Thus,

$$p_{\max}=\gamma\frac{2-\phi_j}{\phi_j}\left(1+\sqrt{1+\left(\frac{\phi_j}{2-\phi_j}\right)^2}\right)+\gamma$$

and

$$p_{\min}=\gamma\left(\frac{2-\phi_j}{\phi_j}\right)\left(1+\sqrt{1+\left(\frac{\phi_j}{2-\phi_j}\right)^2}\right)-\gamma$$

From $p_{\max}<v-\gamma$ we must observe that

$$v>\gamma\left(\frac{2-\phi_j}{\phi_j}\right)\left(1+\sqrt{1+\left(\frac{\phi_j}{2-\phi_j}\right)^2}\right)+2\gamma.$$

From (28) we obtain that

$$k^m=\frac{1}{2}\phi_i\left(p_{\max}-\gamma\right)\left(2-\phi_j\right)$$

and so

$$k^m = \frac{\gamma \phi_i}{2\phi_j} \left(2 - \phi_j\right)^2 \left(1 + \sqrt{1 + \left(\frac{\phi_j}{2 - \phi_j}\right)^2}\right) \tag{29}$$

The expected profit of firm i, $E\pi_i = k^m - A(\phi_i)$ is equal to

$$E\pi_i = \frac{1}{2}\phi_i \left(p_{\max} - \gamma\right) \left(2 - \phi_j\right) - A(\phi_i),$$

or equivalently:

$$E\pi_i = \frac{\gamma \phi_i}{2\phi_j} \left(2 - \phi_j\right)^2 \left(1 + \sqrt{1 + \left(\frac{\phi_j}{2 - \phi_j}\right)^2}\right) - A(\phi_i)$$

It is important to stress that when the level of firm i's advertising tends to zero $p_{\max}$ would be above $v - \gamma$, and so, as expected, k^m would not be given by expression (29). Instead, as firm i's advertising tends to zero, firm i's expected profits would necessarily tend to zero.

For $0 < \phi_i < 1$ (interior solution), each firm's advertising equilibrium level with mass advertising is obtained by maximizing $E\pi_i$ in order to ϕ_i. From the first order condition, the interior solution is given by[22] $\frac{\partial k^m}{\partial \phi_i} = A_{\phi_i}(\phi_i)$ which under symmetry writes as

$$\frac{1}{2}\left(p^m_{\max} - \gamma\right)\left(2 - \phi^m\right) = A_\phi\left(\phi^m\right) \tag{30}$$

Each firm expected equilibrium profit is

$$E\pi^m = \phi^m A_\phi\left(\phi^m\right) - A\left(\phi^m\right). \blacksquare \tag{31}$$

Proof of Proposition 3 Let $q \in [0,1]$ represent the probability with which each firm serves its group of selective customers. Because the model is symmetric both firms have the same support of prices. Then q can be written as:

$$q = 1 - 2\int_{p_{\min}+\gamma}^{p_{\max}} \left(\int_{p_{\min}}^{p_A - \gamma} f(p_B)\, dp_B\right) f(p_A)\, dp_A =$$

from which we obtain:

$$q^m = 1 + \frac{8\left(k^m\right)^2}{\left(\phi^m\right)^4}\left[\frac{1}{\left(p_{\max} - \gamma\right)^2} - \ln\left(\frac{\left(p_{\max} - \gamma\right)^2}{p_{\max}\left(p_{\max} - 2\gamma\right)}\right)\right].$$

$\blacksquare$

[22] Note that the second order condition is satisfied as it is given by $-A_{\phi\phi}\left(\phi^m\right)$, which is always true, by assumption.

Proof of Proposition 4. When v is sufficiently high such that $p_{\max} < v - \gamma$ and the advertising technology is the quadratic one the equilibrium level of advertising is given by

$$\phi^m = \frac{2\gamma^2 + 16\lambda\gamma - 8\lambda\sqrt{8\lambda\gamma + \gamma^2}}{\gamma^2 + 8\lambda\gamma - 16\lambda^2},$$

with $\gamma^2 + 8\lambda\gamma - 16\lambda^2 < 0$ for $\lambda > \frac{\gamma(\sqrt{2}+1)}{4}$. Then, for ϕ^m to be positive, the numerator of ϕ^m must also be negative. This is always the case when $\lambda > \frac{\gamma(\sqrt{2}+1)}{4}$. In fact, the polynomial $2\gamma^2 + 16\lambda\gamma - 8\lambda\sqrt{8\lambda\gamma + \gamma^2}$ has three roots, $-\frac{1}{8}\gamma$, $-\frac{\gamma(\sqrt{2}-1)}{4}$; and $\frac{\gamma(\sqrt{2}+1)}{4}$. Furthermore: $\lim_{\lambda\to+\infty}\left(2\gamma^2 + 16\lambda\gamma - 8\lambda\sqrt{8\lambda\gamma + \gamma^2}\right) = -\infty$. Accordingly, for $\lambda > \frac{\gamma(\sqrt{2}+1)}{4}$, both the numerator and the denominator of ϕ^m are negative and the ratio is positive.

For the condition $\phi^m < 1$ to hold, it must be the case that:

$$\frac{2\gamma^2 + 16\lambda\gamma - 8\lambda\sqrt{8\lambda\gamma + \gamma^2}}{\gamma^2 + 8\lambda\gamma - 16\lambda^2} < 1.$$

For $\lambda > \frac{\gamma(\sqrt{2}+1)}{4}$ the previous condition is equivalent to:

$$2\gamma^2 + 16\lambda\gamma - 8\lambda\sqrt{8\lambda\gamma + \gamma^2} > \gamma^2 + 8\lambda\gamma - 16\lambda^2,$$

which is always the case for $\lambda > \frac{\gamma(\sqrt{2}+1)}{4}$, given that the roots of the polynomial $\gamma^2+8\lambda\gamma+16\lambda^2 - 8\lambda\sqrt{8\lambda\gamma + \gamma^2}$ are $-\frac{1}{4}\gamma\left(\sqrt{2}-1\right)$ and $\frac{1}{4}\gamma\left(\sqrt{2}+1\right)$, with $\lim_{\lambda\to+\infty}\left(\gamma^2 + 8\lambda\gamma + 16\lambda^2 - 8\lambda\sqrt{8\lambda\gamma + \gamma^2}\right) = +\infty$.

■

Proof of Proposition 5. (Introduce proof)

■

Proof of Proposition 6. Next we prove that there is an equilibrium in mixed strategies in prices for an interior pure strategy equilibrium in advertising. Suppose that, in segment a, firm A selects a price randomly from the cdf $F_A^o(p)$ while firm B selects a price randomly from the cdf $F_B^r(p)$. We concentrate on symmetric MSNE in prices, in which $F_A^o(p) = F_B^o(p) = F^o(p)$, and $F_B^r(p) = F_A^r(p) = F^r(p)$. Accordingly, for the sake of simplicity, we restrict our attention to firms' decisions in segment A, obtaining $F_A^o(p) = F^o(p)$ and $F_B^r(p) = F^r(p)$.

Given firm A's pricing and advertising strategies to segment a, p_A^o and ϕ_A^o, resp., firm A's expected profit in segment A, denoted $E\pi_A^o$ is equal to

$$E\pi_A^o = p_A^o\left[\frac{1}{2}\phi_A^o\left(1-\phi_B^r\right) + \frac{1}{2}\phi_A^o\phi_B^r\Pr(p_A^o < p_B^r + \gamma)\right] - A\left(\phi_A^o\right)$$

or equivalently,

$$E\pi_A = p_A^o\left\{\frac{1}{2}\phi_A^o\left(1-\phi_B^r\right) + \frac{1}{2}\phi_A^o\phi_B^r\left[1 - F_B^r(p_A^o - \gamma)\right]\right\} - A\left(\phi_A^o\right).$$

Similarly firm B's expected profit in segment a, denoted $E\pi_B^r$, is

$$E\pi_B^r = \frac{1}{2}p_B^r\left[\phi_B^r(1-\phi_A^o) + \phi_B^r\phi_A^o \Pr\left(p_B^r + \gamma < p_A^o\right)\right] - A\left(\phi_B^r\right)$$

or,

$$E\pi_B^r = \frac{1}{2}p_B^r\left\{\phi_B^r(1-\phi_A^o) + \phi_B^r\phi_A^o\left[1 - F_A^o\left(p_B^r + \gamma\right)\right]\right\} - A\left(\phi_B^r\right).$$

Note that the minimum price firm B is willing to charge even if it is assured of getting the entire segment of selective customers in market A should satisfy the following condition

$$\frac{1}{2}p_{B\min}^r\left[\phi_B^r\phi_A^o + \phi_B^r(1-\phi_A^o)\right] = \frac{1}{2}\left(v-\gamma\right)\phi_B^r(1-\phi_A^o)$$

from which we obtain:

$$p_{B\min}^r = \left(v-\gamma\right)\left(1-\phi_A^o\right), \tag{32}$$

which corresponds to the expression of $p_{B\min}^r$ pointed out in the Proposition.

Given that firm B would never want to price below $p_{B\min}^r$, we observe that it is a dominated strategy for firm A to price below $p_{B\min}^r + \gamma$. Thus, the support of equilibrium prices for firm A is $[p_{B\min}^r + \gamma, v]$ while for firm B is $[p_{B\min}^r, v-\gamma]$

As usual in a MSNE each firm must be indifferent between charging any price in the support of equilibrium prices.

For firm B we must observe that for any $p_{B\min}^r \leq p_B^r \leq v - \gamma:$

$$\begin{aligned} & \frac{1}{2}p_B^r\left\{\phi_B^r(1-\phi_A^o) + \phi_B^r\phi_A^o\left[1 - F_A^o\left(p_B^r + \gamma\right)\right]\right\} - A\left(\phi_B^r\right) \\ = \; & \frac{1}{2}\left(v-\gamma\right)\phi_B^r(1-\phi_A^o) - A\left(\phi_B^r\right) \end{aligned}$$

which simplifies to

$$F_A^o\left(p+\gamma\right) = \frac{1}{\phi_A^o}\left[1 - \frac{\left(v-\gamma\right)\left(1-\phi_A^o\right)}{p}\right]$$

or, equivalently to

$$F_A^o\left(p\right) = \frac{1}{\phi_A^o}\left[1 - \frac{\left(v-\gamma\right)\left(1-\phi_A^o\right)}{p-\gamma}\right]$$

where $F_A^o\left(v\right) = 1$. Note also that $F_A^o\left(p_{B\min} + \gamma\right) = 0.$ In this way

$$F_A^o\left(p\right) = \left\{\begin{array}{ccc} 0 & if & p \leq p_{B\min}^r + \gamma \\ \frac{1}{\phi_A^o}\left[1 - \frac{(v-\gamma)(1-\phi_A^o)}{p-\gamma}\right] & if & p_{B\min}^r + \gamma \leq p \leq v \\ 1 & if & p \geq v \end{array}\right\},$$

which corresponds to the specifcation of $F_i^o\left(p\right)$ presented in the Proposition.

Analogously, for firm A we must observe that for any $p_{B\min}^r + \gamma \leq p_A^o \leq v:$

$$\begin{aligned} E\pi_A^o &= p_A^o\left\{\frac{1}{2}\phi_A^o\left(1-\phi_B^r\right) + \frac{1}{2}\phi_A^o\phi_B^r\left[1 - F_B^r(p_A^o - \gamma)\right]\right\} - A\left(\phi_A^o\right) \\ &= \left(p_{B\min}^r + \gamma\right)\left(\frac{1}{2}\phi_A^o\left(1-\phi_B^r\right) + \frac{1}{2}\phi_A^o\phi_B^r\right) - A\left(\phi_A^o\right) \end{aligned}$$

or

$$
\begin{aligned}
& p_A^o\left[\frac{1}{2}\phi_A^o\left(1-\phi_B^r\right)+\frac{1}{2}\phi_A^o\phi_B^r\left[1-F_B^r(p_A^o-\gamma)\right]\right]-A\left(\phi_A^o\right) \\
= & \frac{1}{2}\phi_A^o\left(p_{B\min}^r+\gamma\right)-A\left(\phi_A^o\right)
\end{aligned}
$$

This simplifies to

$$F_B^r(p-\gamma)=\frac{1}{\phi_B^r}\left(1-\frac{p_{B\min}^r+\gamma}{p}\right)$$

or,

$$F_B^r(p)=\frac{1}{\phi_B^r}\left(1-\frac{p_{B\min}^r+\gamma}{p+\gamma}\right)$$

$$F_B^r(p)=\frac{1}{\phi_B^r}\left[1-\frac{v\left(1-\phi_A^o\right)+\gamma\phi_A^o}{p+\gamma}\right]$$

As expected note that $F_B^r(p_{B\min})=0$. Thus, the corresponding distribution is

$$F_B^r(p)=\left\{\begin{array}{ccc} 0 & if & p\leq p_{B\min}^r \\ \frac{1}{\phi_B^r}\left(1-\frac{v\left(1-\phi_A^o\right)+\gamma\phi_A^o}{p+\gamma}\right) & if & p_{B\min}^r\leq p\leq v-\gamma \\ 1 & if & p\geq v-\gamma \end{array}\right\},$$

which corresponds to the expression of $F_i^r(p)$ pointed out in the Proposition.

Firm $A's$ expected profit when charging any price in the support of equilibrium prices is equal to:

$$E\pi_A=\frac{1}{2}\phi_A^o\left(v-\left(v-\gamma\right)\phi_A^o\right)-A\left(\phi_A^o\right).$$

From the previous expression it follows that $E\pi_A$ depends on ϕ_A^o. The profit-maximizing advertising intensity is obtained from the condition $\frac{\partial E\pi_A}{\partial\phi_A^o}=0$, since $\frac{\partial^2 E\pi_A}{\partial\phi_A^{o2}}<0$. Accordingly, the optimal advertising intensity is implicitly given by the condition:

$$\underbrace{\frac{1}{2}v-\phi_A\left(v-\gamma\right)}_{MRA^o}=A_{\phi_A^o}\left(\phi_A^o\right),$$

which can also be written as:

$$p_{A\min}^o-\frac{1}{2v}=A_{\phi_A^o}\left(\phi_A^o\right)$$

Given firm B's expected profit in the MSNE given by

$$E\pi_B=\frac{1}{2}\left(v-\gamma\right)\phi_B^r(1-\phi_A^o)-A\left(\phi_B^r\right),$$

the following first order condition defines firm B's optimal advertising level in segment a

$$\frac{\partial E\pi_B}{\partial\phi_B^r}=0\Rightarrow\frac{1}{2}\left(v-\gamma\right)\left(1-\phi_A^o\right)=A_{\phi_B^r}\left(\phi_B^r\right),$$

since the second order condition $\frac{\partial^2 E\pi_A}{\partial\phi_A^{o2}}<0$ is always met. This condition may be alternatively written as

$$\frac{1}{2}p_{j\min}^r=A_{\phi_i^r}\left(\phi_i^{r^*}\right). \tag{33}$$

Firm $i's$ profit conditional on ads targeted to its own market and to the rival's market is:

$$E\pi_i^{*o} = \phi_i^{*o} A_{\phi_i^{*o}} \left(\phi_i^{*o}\right) + \frac{1}{2} \left(\phi_A^{*o}\right)^2 (v - \gamma) - A\left(\phi_A^{*o}\right) \tag{34}$$

$$E\pi_i^{*r} = \phi_i^{*r} A_{\phi_i^{*r}} \left(\phi_i^{*r}\right) - A\left(\phi_i^{*r}\right). \blacksquare \tag{35}$$

Proof of Proposition 7. The first part of proposition 7 follows directly from the equilibrium condition with respect to ϕ_i^{o*}. When advertising is costless, this condition writes as:

$$\underbrace{\frac{1}{2}v - \phi_A (v - \gamma)}_{MRA^o} = 0,$$

yielding $\phi_A^o = \frac{v}{2(v-\gamma)}$, which is always smaller than 1 for $v > 2\gamma$. The second part of the propostion follows directly from plugging the equilibrium value of $p_{\min}^r$ (computed in the Proof of Proposition 6) into the conditions $p_{\min}^r \lesseqqgtr v - 2\gamma$.$\blacksquare$

Proof of Corollary 1. From Proposition 2, it follows that $F_i^r(v-\gamma) = \frac{\phi^{o*}}{\phi^{r*}}\left(1 - \frac{\gamma}{v}\right)$, which is always smaller than 1 when $\phi_i^{o*} < \phi^{r*}$.$\blacksquare$

Proof of Corollary 2. From $\phi_i^{o*} = \frac{v}{2(v-\gamma)+4\lambda}$ it is straightforward to see it is always true that $\phi_i^{o*} > 0$ and that $\phi_i^{o*} < 1$ as long as $v > 2\gamma$.

From $\phi_j^{r*} = \frac{(v-\gamma)(v+4\lambda-2\gamma)}{8\lambda(v+2\lambda-\gamma)}$ it follows that $\phi_j^{r*} > 0$ iff $v > 2\gamma$. Look next in which circumstances we obtain $\phi_j^{r*} < 1$. From

$$\frac{(v-\gamma)(v+4\lambda-2\gamma)}{8\lambda(v+2\lambda-\gamma)} < 1$$

we obtain

$$\frac{\left(v^2 - 4v\lambda - 3v\gamma - 16\lambda^2 + 4\lambda\gamma + 2\gamma^2\right)}{8\lambda(v+2\lambda-\gamma)} < 0$$

which is true when $\lambda > \frac{\sqrt{(5v-9\gamma)(v-\gamma)}-(v-\gamma)}{8}$. The root exists only when $5v - 9\gamma > 0$ or when $v > \frac{9\gamma}{5}$, which is true under the assumption that $v > 2\gamma$.$\blacksquare$

Proof of Proposition 9. From Proposition 8 we obtain the optimal values of $\phi^{o*} = \frac{v}{2(v-\gamma)+4\lambda}$ and $\phi^{r*} = \frac{(v-\gamma)(v+4\lambda-2\gamma)}{8\lambda(v+2\lambda-\gamma)}$. The firm advertises more in its own market if:

$$\phi^{o*} - \phi^{r*} = \frac{v}{2(v-\gamma)+4\lambda} - \frac{(v-\gamma)(v+4\lambda-2\gamma)}{8\lambda(v+2\lambda-\gamma)} \geq 0,$$

This is equivalent to:

$$-\frac{1}{8}\frac{-3v\gamma + 2\gamma^2 - 4\lambda\gamma + v^2}{\lambda(v+2\lambda-\gamma)} \geq 0$$

or $-3v\gamma+2\gamma^2-4\lambda\gamma+v^2<0$.Solving for v, we obtain that $\phi^{o*}-\phi^{r*}\geq 0$ if

$$v\in\left[\frac{3}{2}\gamma-\frac{1}{2}\sqrt{\gamma^2+16\lambda\gamma},\frac{3}{2}\gamma+\frac{1}{2}\sqrt{\gamma^2+16\lambda\gamma}\right]$$

However, it can be easily seen that $\frac{3}{2}\gamma-\frac{1}{2}\sqrt{\gamma^2+16\lambda\gamma}<2\gamma$. Since we have imposed $v>2\gamma$ in order to concentrate on interior solutions in the advertising market, under $v>2\gamma$, we have that $\phi^{o*}-\phi^{r*}>0$ if

$$2\gamma<v<\frac{1}{2}\left(3\gamma+\sqrt{\gamma^2+16\lambda\gamma}\right)$$

and $\phi^{o*}-\phi^{r*}<0$, if $v\geq\frac{1}{2}\left(3\gamma+\sqrt{\gamma^2+16\lambda\gamma}\right)>2\gamma$.■

Proof of Proposition 10. *Each firm serves its group of selective customers at p^o with probability given by $\tau\in[0,1]$:*

$$\tau=\int_{p^R_{\min}}^{v-\gamma}\left(\int_{p^o_{\min}}^{p_B+\gamma}f\left(p_A\right)dp_A\right)f\left(p_B\right)dp_B,$$

or equivalently:

$$\tau=\int_{p^R_{\min}}^{v-\gamma}\left(\int_{p^o_{\min}}^{p_B+\gamma}-\frac{\left(v-\gamma\right)\left(x-1\right)}{x\left(p-\gamma\right)^2}dp\right)\left(\frac{x\gamma-v\left(x-1\right)}{y\left(p+\gamma\right)^2}\right)dp_B,$$

which is equivalent to:

$$\tau=\frac{v-\gamma}{\phi^{r*}v}\left(1-\frac{1}{\phi^{o*}}\right)\frac{\phi^{o*}\left(v-\gamma\right)-v}{\gamma^2p^r_{\min}}\left(\left(v-p^o_{\min}\right)\gamma+v\left(\gamma-p^o_{\min}\right)\ln\left(\frac{p^o_{\min}}{p^r_{\min}}\frac{v-\gamma}{v}\right)\right),$$

since $p^o_{\min}=p^r_{\min}+\gamma$.■

References

Athey, S., and Gans, J. (2009), "The Impact of Targeting Technology on Advertising Markets and Media Competition." *mimeo.*

Brahim,N., Lahmandi-Ayed, R., and Laussel,D. (2010), "Is targeted advertising always benefficial?" *mimeo.*

Bagwell, K. (2005), "The Economic Analysis of Advertising." In Armstrong, M. and R. Porter, eds., *The Handbook of Industrial Organization*, Vol.3, Amsterdam: North-Holland.

Bester, H. and Petrakis, E. (1996), "Coupons and Oligopolistic Price Discrimination." *International Journal of Industrial Organization,* 14, 227-242.

Butters, G. (1977), "Equilibrium Distribution of Sales and Advertising Prices." *Review of Economic Studies*, 44, 465-491.

Chandra, A. (2009), "Targeted Advertising: The Role of Subscriber Characteristics in Media Markets." *Journal of Industrial Economics,* 57(1), 58-84.

Chen, Y., and Iyer, G. (2002), "Consumer Addressability and Customized Pricing." *Marketing Science,* 21, 197-208.

Chen, Y. (2005), "Oligopoly Price Discrimination by Purchase History." In: *The Pros and Cons of Price Discrimination*, The Swedish Competition Authority, Stockholm, pp. 101-130.

Esteban, L., and Hernandez J.M. (2007). "Strategic Targeted Advertising and Market Fragmentation." *Economics Bulletin*, 12 (10), 1-12.

Esteban, L., Hernandez, J.M. and Moraga-Gonzalez, J.L. (2006). "Customer Directed Advertising and Product Quality." *Journal of Economics & Management Strategy,* 15(4), 943-968.

Esteves, R.B. (2009a), "Customer Poaching and Advertising." *Journal of Industrial Economics,* 57(1), 112-146.

Esteves, R.B. (2009b), "Price Discrimination with Partial Information: Does it pay-off?" *Economics Letters*, 105, 28-31

Esteves, R.B. (2010), "Pricing with Customer Recognition." forthcoming *International Journal of Industrial Organization.*

Esteves, R.B., and Guimaraes, P. (2008), "Price Discrimination and Targeted Advertising: A Welfare Analysis." *The IUP Journal of Applied Economics, IUP*, 0(5), 41-47.

Fudenberg, D. and Tirole, J. (2000), "Customer Poaching and Brand Switching." *RAND Journal of Economics,* 31, 634-657.

Gal-Or, E., Gal-Or, M., May, J. and Spangler, W. (2006), "Targeted Advertising Strategies on Television." *Management Science,* 713-725.

Gal-Or, E. and Gal-Or, M. (2005) "Customized Advertising on Television." *Marketing Science*, 241-253.

Galeotti A., and Moraga-Gonzalez, J. L., (2008), "Segmentation, Advertising and Prices." *International Journal of Industrial Organization* , 26(5), 1106-1119.

Grossman, G., and Shapiro, C. (1984), "Informative Advertising with Differentiated Products." *Review of Economic Studies*, 51, 63-81.

Iyer, G., Soberman, D. and Villas-Boas, M. (2005), "The Targeting of Advertising." *Marketing Science,* 24, 461-476.

Narasimhan, C. (1988), "Competitive Promotional Strategies." *Journal of Business*, 61, 427-449.

Raju, J., Srinivasan, V. and Rajiv, L. (1990), "The Effects of Brand Loyalty on Competitive Price Promotional Strategies." *Management Science,* 36, 276-304.

Roy, S. (2000), "Strategic Fragmentation of a Market." *International Journal of Industrial Organization,* 18, 1279-1290.

Shilony, Y. (1977), “Mixed Pricing in Oligopoly,” *Journal of Economic Theory,* 14, 373-388.

Stahl, D. (1994), “Oligopolistic Pricing and Advertising.” *Journal of Economic Theory*, 64, 162-177.

Stegeman, M. (1991), “Advertising in Competitive Markets.” *American Economic Review*, 81, 210-223.

Thisse, J., and Vives, X. (1988), “On the Strategic Choice of Spatial Price Policy.” *American Economic Review,* 78, 122-137.

Tirole, J. (1988), *The Theory of Industrial Organization.* Cambridge. The MIT Press.

Varian, H. (1980), “A Model of Sales.” *American Economic Review*, 70, 651-659.

Villas-Boas, M. (1999), “Dynamic Competition with Customer Recognition.” *RAND Journal of Economics,* 30, 604-631.

Most Recent Working Paper

NIPE WP 08/2011	**Esteves, Rosa Branca e Joana Resende,** "Competitive Targeted Advertising with Price Discrimination", 2011
NIPE WP 07/2011	**Bastos, Paulo, Natália P. Monteiro e Odd Rune Straume** "The effect of private versus public ownership on labour earnings", 2011
NIPE WP 06/2011	**Castro, Vítor e Rodrigo Martins,** "Mayors' Reelection Choice and the Economy: Evidence from Portugal", 2011
NIPE WP 05/2011	**Brekke, Kurt R. Luigi Siciliani e Odd Rune Straume,** "Quality competition with profit constraints: Do non-profit firms provide higher quality than for-profit firms?", 2011
NIPE WP 04/2011	**Baleiras, Rui Nuno** "Collective efficiency strategies: a policy instrument for the competitiveness of low-density territories", 2011
NIPE WP 03/2011	**Machado, C. Sofia, Miguel Portela** "Age and opportunities for promotion", 2011
NIPE WP 02/2011	**Machado, C. Sofia, Miguel Portela** " Hours of work and retirement behavior", 2011
NIPE WP 01/2011	**Amado, Cristina, Timo Teräsvirta** "Modelling Volatility by Variance Decomposition", 2011
NIPE WP 37/2010	**Aguiar-Conraria, Luís, Fernando Alexandre e Manuel Correia de Pinho,** "O euro e o crescimento da economia portuguesa: uma análise contrafactual", 2010
NIPE WP 36/2010	**Aguiar-Conraria, Luís, e Maria Joana Soares,** "Business Cycle Synchronization and the Euro: a Wavelet Analysis", 2010
NIPE WP 35/2010	**Lommerud, Kjell Erik, Frode Meland e Odd Rune Straume** "North-South technology transfer in unionised multinationals", 2010
NIPE WP 34/2010	**Kizilca, Kemal, João Cerejeira, Miguel Portela e Carla Sá,** "Minimum wage, fringe benefits, overtime payments and the gender wage gap", 2010
NIPE WP 33/2010	**Martins, Rodrigo, e Francisco José Veiga ,** " Economic Voting in Portuguese Municipal Elections ", 2010
NIPE WP 32/2010	**Gabriel, Vasco J. e Pataaree Sangduan,** "An Efficient Test of Fiscal Sustainability", 2010
NIPE WP 31/2010	**Batini, Nicoletta, Vasco J. Gabriel, Paul Levine e Joseph Pearlman,** "A Floating versus Managed Exchange Rate Regime in a DSGE Model of India", 2010
NIPE WP 30/2010	**Gabriel, Vasco J. e Luis F. Martins,** "The cost channel reconsidered: a comment using an identification-robust approach", 2010
NIPE WP 29/2010	**Gabriel, Vasco J., Paul Levine, Joseph Pearlman e Bo Yang,** "An Estimated DSGE Model of the Indian Economy", 2010
NIPE WP 28/2010	**Gabriel, Vasco J. e Luis F. Martins,** "Cointegration Tests Under Multiple Regime Shifts: An Application to the Stock Price-Dividend Relationship", 2010
NIPE WP 27/2010	**Nogueira, Carlos Gama e Linda Gonçalves Veiga,** " Economic determinants of citizens' support for the European Union", 2010
NIPE WP 26/2010	**Castro, Vitor e Ricardo M. Sousa**, "How Do Central Banks React to Wealth Composition and Asset Prices?", 2010
NIPE WP 25/2010	**Agnello, Luca e Ricardo M. Sousa,** "Fiscal Policy and Asset Prices", 2010
NIPE WP 24/2010	**Castro, V.,** "The duration of business cycle expansions and contractions: Are there change-points in duration dependence?", 2010
NIPE WP 23/2010	**Aguiar-Conraria e Maria Joana Soares,** "The Continuous Wavelet Transform: A Primer*", 2010
NIPE WP 22/2010	**Alexandre, Fernando, Pedro Bação, João Cerejeira e Miguel Portela,** "Manufacturing employment and exchange rates in the Portuguese economy: the role of openness, technology and labour market rigidity", 2010
NIPE WP 21/2010	**Aguiar-Conraria, Luís, Manuel M. F. Martins e Maria Joana Soares,** "The yield curve and the macro-economy across time and frequencies", 2010
NIPE WP 20/2010	**Kurt Richard Brekke, Tor Helge Holmås e Odd Rune Straume,** "Margins and Market Shares: Pharmacy Incentives for Generic Substitution", 2010