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## Working Paper

Gradual versus structural technological change in the transition to a low-emission energy industry: How time-to-build and differing social and individual discount rates influence environmental and technology policies

Dresden discussion paper in economics, No. 09/06

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Suggested citation: Heinzel, Christoph; Winkler, Ralph (2006) : Gradual versus structural technological change in the transition to a low-emission energy industry: How time-to-build and differing social and individual discount rates influence environmental and technology policies, Dresden discussion paper in economics, No. 09/06, <http://hdl.handle.net/10419/22736>

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**Gradual versus structural technological change in  
the transition to a low-emission energy industry**

**How time-to-build and differing social and individual discount  
rates influence environmental and technology policies**

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*Dresden Discussion Paper in Economics No. 09/06*

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Internet:

An electronic version of the paper may be downloaded from the homepage:  
<http://rcswww.urz.tu-dresden.de/wpeconomics/index.htm>

English papers are also available from the SSRN website:  
<http://www.ssrn.com>

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# Gradual versus structural technological change in the transition to a low-emission energy industry

How time-to-build and differing social and individual discount rates influence environmental and technology policies

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## Abstract:

We develop a general equilibrium model to study the transition from an established polluting to a new clean energy technology. Therefore, we consider two distinctive features: (i) the creation of new productive capital exhibits a time-to-build property, and (ii) the social and individual rates of time preference differ. We derive necessary and sufficient conditions for investment in the new and for replacement of the established technology. We show that, in addition to the standard emission externality, a further market failure stemming from the differing discount rates arises, the extent of which positively depends on the time-lag in capital accumulation. Moreover, we show that in a mutually reinforcing way both market failures create less favorable circumstances for the introduction of the new technology compared to the social optimum. The paper thus provides an additional reason why environmental policy should be complemented by technology policy in the transition to a low-emission energy industry.

JEL-Classification: Q48, Q55, H23

Keywords: energy industry, gradual vs. structural technological change, rate of time preference, time-to-build

## Acknowledgements:

Valuable comments on an earlier draft by Stefan Baumgärtner, Marco Lehmann-Waffenschmidt, Normann Lorenz, Maik Schneider and Marcel Thum are gratefully acknowledged. The usual disclaimer applies.

# 1 Introduction

There is some political consensus that the mitigation of anthropogenic climate change, which is caused by the emission of so-called greenhouse gases, in particular CO<sub>2</sub>, is one of the most urgent challenges humanity faces today. In industrialized countries, the biggest source of CO<sub>2</sub> is the combustion of fossil fuels for energy production and transportation. Therefore, it is clear that a successful climate change mitigation strategy has to include a shift from carbon intensive to carbon neutral energy technologies in the long-run.<sup>1</sup>

Yet, the transition to a low-emission energy industry is a time-consuming process, marked by the interplay of different kinds of technological change. For instance, the gradual refinement of established polluting energy technologies via abatement technologies competes with changes of the structural kind via the introduction of new less-polluting energy technologies. While traditional environmental policy is motivated by the external effect caused by emissions and favors market-based instruments, such as taxes or cap-and-trade mechanisms for their internalization, in recent years an ever more detailed knowledge has grown about additional market failures the process of technological transformation comprises. In general, the development and innovation of a new technology is related to a high degree of uncertainty with regard to its success. Its innovation is likely to create knowledge spillovers to other firms which may profit from it by using the new technology as a template for their own R&D. Moreover, the diffusion of new technologies is typically related to dynamic increasing returns stemming from learning-by-using, learning-by-doing or network externalities (Jaffe et al. 2005). The latter provide justification for – possibly supplementary – interventions of technology policy.

The model we develop in this paper provides complementary reasons why successful environmental regulation has to be complemented by technology policy, which do not draw on the standard explanations of uncertain R&D success, technology spillovers and dynamic increasing returns. Therefore, our model, which focuses on the transition from an established polluting to a new clean energy technology, is characterized by two particular time-related features. First, we assume that the creation of new capital goods needs a positive time span  $\sigma$ . That is, there is a time-lag  $\sigma$  between the costs of investment and the benefits of production of new capital goods. Second, we assume that the social rate of time preference is smaller than the private rate of time preference, i.e. individual actors are more impatient to consume than society as a whole. While both, the time-to-build feature and the possible split of social and individual rates of time preference, are known in the environmental economics literature, they have so far, to the best of our knowledge, never been applied in combination.

We further distinguish two kinds of endogenous technological change, gradual and structural. By *gradual* technological change, we understand the refinement of the existing technology via an end-of-pipe abatement technology. By *structural* technological change, we mean the introduction of a new clean technology, which is only to be produced

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<sup>1</sup> In order to comply with the 2 degrees centigrade increase limit of global mean temperature as compared to pre-industrial levels specified by the 1992 UN Framework Convention on Climate Change, the EU Council, for example, recently adopted CO<sub>2</sub> reduction goals of 30% by 2020 and 60–80% by 2050 compared to 1990.

and may replace the established one. The two kinds of technological change mainly differ in two characteristics. First, while abatement immediately reduces the level of emissions, structural change implies that resources must be invested in the accumulation of the specific capital goods for a certain time which could have directly been used in energy production by means of the established technology otherwise. Second, while abatement implies a rise in unit costs of production, structural change may be both more environmentally benign (by inducing less pollution) and economically more efficient (by exhibiting lower unit costs of production).

Our paper contributes to the wide-spanned literature on (induced) technological change and the environment. Closest to our approach are environmental-economic endogenous growth models, such as e.g. Bovenberg and Smulders (1995, 1996) and Tahvonen and Salo (2001), and applied simulation models, such as e.g. Van der Zwaan et al. (2002), Buonanno et al. (2003) and Gerlagh and Lise (2005). While they depart from a more encompassing analytical setting comprising also R&D or learning by doing, allow for different kinds of technological change and analyze both environmental and technology policies, they do not focus on the interplay of gradual and structural technological change and assume a unique rate of time preference. Further contributions on induced technological change, such as e.g. Goulder and Schneider (1999) and Goulder and Mathai (2000), rather concentrate on the effects from environmental policy. The adoption, diffusion and R&D of advanced abatement technologies constitute the focus of the literature on dynamic incentives of environmental policies (for an overview see, for example, Requate 2005a). Denicolo (1999) and Requate (2005b) study particularly the timing of policy measures in a sequential setting. Interestingly, this strand of literature usually refrains from the analysis of specifically dynamic features, such as time preferences or time-lagged processes, or a dynamic analysis,<sup>2</sup> as well as the interplay of different kinds of technological change.

We depart from a modeling framework of time-lagged capital theory which combines different elements developed in a general way by Winkler (2003, 2005).<sup>3</sup> Methodically, we extend it by taking into account abatement, the explicit analysis of market failures and the consideration of the split of the rates of time preference.<sup>4</sup> We derive conditions both for investment in the new technology and for full replacement of the established technology in the long-run stationary state. It turns out that the unit costs of energy of the new technology depend positively on the time-lag  $\sigma$  and the respective rate of time preference. We show that from the split of the rates of time preference a second market failure arises the extent of which positively depends on the time-lag in capital accumulation. We study how the two market failures lead in a mutually reinforcing way to less favorable circumstances in the unregulated market regime compared to the social

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<sup>2</sup> An exception is Parry et al. (2003), who calculate the welfare gains from innovation induced by environmental policy e.g. for different social discount rates.

<sup>3</sup> The idea of time-lagged capital accumulation goes back to the Austrian school of economics (von Böhm-Bawerk [1889]1921) and was revived by the neo-Austrian capital theory in the 1970s (e.g. von Weizsäcker 1971, Hicks 1973, Faber 1979). In the modern macroeconomics real business cycle literature the time-to-build feature became prominent with Kydland and Prescott (1982).

<sup>4</sup> The assumption of the split of the rates of time preference is discussed in detail in section 2.

optimum, both for investment in the new technology and for full replacement in the long-run stationary state. Finally, we consider how the external effects can be internalized by environmental and technology policy.

Although derived from a stylized theoretical model, our results have direct policy implications. First, we provide additional reasons why environmental regulation should be complemented by technology policy. Second, we can substantiate Porter and van der Linde's (1995) claim that "well-designed" environmental regulations exhibit a double dividend. And third, our results give theoretical economic support to existing or planned regulations on subsidizing renewable energies.

The paper is organized as follows. Section 2 introduces the model and discusses the assumption of the split of social and individual rates of time preference. In sections 3 and 4, the intertemporal optimization problem is solved and conditions of investment and replacement are derived for the cases of the social optimum and an unregulated competitive market economy, respectively. Section 5 shows how the two external effects, stemming from the emissions and the split of social and individual rates of time preference, can be implemented via environmental and technology regulation. We discuss our model assumptions and policy implications in section 6. Section 7 concludes.

## 2 The model

In this section, we set up our model, which focuses on the structural change of the energy sector under environmental and technology regulation and encompasses two distinctive features.

First, we assume that the creation of new capital goods needs a positive time span  $\sigma$ . That is, there is a time-lag  $\sigma$  between the costs of investment and the benefits of production of new capital goods. The intuition behind this assumption is that energy plants are not built in a day but need *substantial time* for creation. In general, the time span  $\sigma$  strongly depends on the type of plant produced: while a nuclear power plant may take some ten years to build, a gas cogeneration plant is set up in a year or two. However, in this model we abstract from these differences and only consider an *established* and a *new* energy technology.

Second, we assume that the *social rate of time preference* is smaller than the *private rate of time preference*, i.e. individual actors are more impatient to consume than the society as a whole. The theoretical literature identifies a variety of reasons why the social and the private rate of time preference may differ. In our paper, we focus on the so-called *schizophrenic behavior* approach (e.g. Marglin 1963), i.e. atomistically, individuals maximize personal utility while, in aggregate, individuals prefer a government to care for the overall welfare of society. Moreover, empirical evidence supports the schizophrenic behavior in so far that individuals exhibit higher rates of time preference in private than in social decision contexts.

In the following, the model is introduced in detail.

## 2.1 The model economy

Consider an economy with a *production system* composed of two vertically integrated sectors, the *energy* sector and the *investment* sector. Labor constitutes the only primary input, which is given in a fixed amount of 1 at all times  $t$ .

The *energy* sector comprises two technologies, an established and a new one. The established technology is assumed to be fully set up at the beginning of the planning horizon. As a consequence, we do not explicitly consider capital for the established technology, as the costs of employing and maintaining the capital stock are subsumed in the labor costs, which are normalized to 1. Thus, the established technology generates one unit of energy  $x$  for every unit of labor  $l_1$  employed. In addition, each unit of energy produced gives rise to one unit of an unwanted and harmful joint output  $j$ :

$$x_1(t) = l_1(t) , \quad (1)$$

$$j(t) = x_1(t) = l_1(t) . \quad (2)$$

The joint output can be (partially) disarmed by abatement. If  $a$  is the abatement effort per unit of output (or joint product, respectively),  $G$  denotes the fraction of the joint output  $j$  which is disarmed by abatement.  $G$  is a concave and twice continuously differentiable function with the following properties:

$$G(0) = 0 , \quad G' > 0 , \quad G'' < 0 , \quad \lim_{a \rightarrow \infty} G(a) = 1 . \quad (3)$$

Furthermore,  $G$  satisfies Inada conditions. They ensure that the abatement effort  $a$  along the optimal path is strictly positive and finite as long as  $l_1$  is positive:

$$\lim_{a \rightarrow 0} G'(a) = \infty , \quad \lim_{a \rightarrow \infty} G'(a) = 0 . \quad (4)$$

As a consequence, net emissions  $e$  equal the amount of joint output  $j$  minus abatement:

$$e(t) = x_1(t)(1 - G(a(t))) . \quad (5)$$

The new technology employs  $\lambda$  units of labor together with  $\kappa$  units of the capital good  $k$  to produce one unit of energy:

$$x_2(t) = \min \left[ \frac{l_2(t)}{\lambda} , \frac{k(t)}{\kappa} \right] . \quad (6)$$

In contrast to the established technology, the new one does not produce any unwanted joint output.<sup>5</sup>

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<sup>5</sup> The non-pollution assumption of the new technique is without loss of generality compared to the case of positive but less pollution, because the optimality condition only depends on the difference of the pollution factors. Note further that, if one supposes full employment of the capital stock  $k$  in every period, an efficient labor allocation among the three production processes in the two sectors, an initial capital stock of  $k(0) = 0$  and intertemporal welfare as defined in equations (9) and (10) below, full employment of the capital stock is also *efficient* (Winkler 2005). As a consequence, equation (6) yields  $x_2(t) = \frac{l_2(t)}{\lambda} = \frac{k(t)}{\kappa}$ .



Energy is supposed to be homogeneous. Thus, total energy production  $x$  equals:

$$x(t) = x_1(t) + x_2(t) . \quad (7)$$

The *investment* sector employs one unit of labor to produce one unit of the capital good. It is assumed that it takes a positive time span  $\sigma$  to turn the investment  $i$  into productive capital  $k$ , which, in turn, deteriorates at the constant and exogenously given rate  $\gamma$ . Therefore, the equation of motion for the capital stock  $k$  reads:

$$\frac{dk(t)}{dt} = i(t - \sigma) - \gamma k(t) . \quad (8)$$

Due to the time-lag  $\sigma$ , the equation of motion for the capital stock (8) constitutes a *retarded differential-difference* equation, such that variations of the capital stock  $k$  do not only depend on parameters evaluated at time  $t$ , but also on parameters evaluated at the earlier time  $t - \sigma$ .

## 2.2 Social and individual preferences

We assume a representative consumer who derives instantaneous utility from energy consumption and disutility from net emissions. However, we assume that this representative consumer behaves in a *schizophrenic* way (e.g. Marglin 1963) in so far as, atomistically, she maximizes individual utility while, in aggregate, she prefers a social planner to maximize social welfare. Recently, the terms *homo oeconomicus* for an individual maximizing private utility, and *homo politicus* for an individual who seeks to maximize social welfare have been introduced into the literature (e.g. Faber et al. 1997, Faber et al. 2002, Nyborg 2000).

In our model, the schizophrenic behavior of the representative consumer is reflected by different *intratemporal* weights between utility derived from consumption and disutility derived from net emissions, and different *intertemporal* weights, i.e. rates of time preference, between welfare today compared to welfare tomorrow, depending on whether she acts in a private or a social decision context. For the sake of simplicity, we consider instantaneous welfare to be additively separable in energy consumption  $x$  and net emissions  $e$ . As a consequence, as an individual the representative consumer maximizes

$$W_p = \int_0^{\infty} [U(x(t)) - \alpha D(e(t))] \exp[-\rho_p t] dt , \quad (9)$$

while, at the same time, she wants the social planner to maximize

$$W = \int_0^{\infty} [U(x(t)) - D(e(t))] \exp[-\rho t] dt , \quad (10)$$

where  $U$  and  $D$  are twice differentiable functions with  $U' > 0$ ,  $U'' < 0$  and  $\lim_{x \rightarrow 0} U' = \infty$  and  $D'(0) \geq 0$ ,  $D' > 0$  for any positive amount of emissions  $e$  and  $D'' > 0$ .

Without loss of generality, we assume in the following that the weighting factor  $\alpha = 1$ , as it has no effect on the outcome of our analysis. This holds, because net emissions  $e$

constitute an externality in the unregulated market solution, which is not taken into account by the market mechanism and, thus, the market outcome is independent of the individual valuation of the disutility derived from net emissions. However, we assume that the private rate of time preference  $\rho_p$  is higher than the social rate of time preference  $\rho$ , i.e.  $\rho_p > \rho$ .

From a purely descriptive point of view, our assumption that individuals discount at a higher rate in individual decision contexts compared to social decision contexts is justified, as it is supported by empirical evidence. Lazaro et al. (2001) find in an experimental setting that law students apply higher rates of time preference in individual than in social decision contexts. Moreover, our assumption reflects a typical aspect of sustainability policies. Often, individuals display a higher rate of time preference than policy goals imply at the social level, as formulated, for example, in national sustainability strategies or the 1992 Rio declaration.

The theoretical economic literature identifies a variety of reasons for the social and private rate of time preference to differ (for an overview see, for example, Luckert and Adamowicz 1993). Two lines of argument we briefly want to summarize are that the welfare of future generations has public good properties and that in an uncertain world individual and social risk may differ. The first assumes that the welfare of both present and future generations are part of the individuals' utility functions, i.e. individuals exhibit a certain extent of altruism towards both present and future generations. Depending on the weights given to the welfare of the present and the future generation compared to own welfare, a situation known as the *isolation paradox* can arise (Marglin 1963, Sen 1961, 1967). This situation is structurally equivalent to the well known prisoner's dilemma. Given the choice either to save one unit of consumption today in order to provide  $1 + w$  units of consumption for future generations or to consume the unit themselves, all individuals prefer to consume themselves no matter the decisions of all other individuals. However, all individuals prefer that all individuals save over all individuals consuming. A close cousin of the isolation paradox is the *assurance problem* (e.g. Sen 1967, Runge 1984, Sugden 1984), which only differs from the former in that all individuals prefer saving as long as all other individuals save, too. As Runge (1984) and Sugden (1984) show the assurance problem provides a link between the schizophrenic behavior approach and the assumption of altruistic utility functions.

The second line of argument concentrates on uncertainty (e.g. Sen 1961). If individuals are risk averse, the rate of time preference also includes a *risk premium* addressing the variation of the actual from the expected outcome for postponing consumption today to consumption tomorrow. The argument is that individuals face a higher risk than the society as a whole for postponing consumption into the future via savings or investments. Examples for higher individual risk are the mortality risk an individual faces but not the society as a whole and that societies as a whole may have better risk pooling abilities than the individual.

However, in our model neither altruistic preferences nor uncertainty are explicitly considered. Thus, we rather stick to the schizophrenic behavior explanation for the private and the social discount rate to differ.

### 3 Social optimum

We now derive the optimal plan for the development of the model economy as outlined in section 2. Social welfare is assumed to be given by equation (10). Thus, the social planner solves the following maximization problem:

$$\max_{a(t), i(t)} W = \int_0^{\infty} [U(x(t)) - D(e(t))] \exp[-\rho t] dt , \quad (11a)$$

subject to

$$x(t) = \frac{1 - \frac{\lambda}{\kappa}k(t) - i(t)}{1 + a(t)} + \frac{1}{\kappa}k(t) , \quad (11b)$$

$$e(t) = \left(1 - G(a(t))\right) \left[ \frac{1 - \frac{\lambda}{\kappa}k(t) - i(t)}{1 + a(t)} \right] , \quad (11c)$$

$$\frac{dk(t)}{dt} = i(t - \sigma) - \gamma k(t) , \quad (11d)$$

$$i(t) \geq 0 , \quad (11e)$$

$$l_1(t) \geq 0 , \quad (11f)$$

$$k(0) = 0 , \quad (11g)$$

$$i(t) = \xi(t) = 0, \quad t \in [-\sigma, 0) . \quad (11h)$$

For the dynamics of the model economy it is important that, due to the linearity of the production techniques, two *corner solutions* can occur along the optimal development path. Either, it can be optimal to only use the established technology at all times, which corresponds to  $i(t) = 0 \forall t$ . Or, if investment in the new technology is optimal (i.e.  $i(t) > 0 \forall t$ ), it may eventually be optimal for the new technology to fully replace the established one and thus  $l_1(t) = 0 \forall t \geq t'$ . As a consequence, we have to explicitly check these two corner solutions, apart from the inner solution, in order to characterize the complete dynamics of the model economy.

#### 3.1 Necessary and sufficient conditions for the social optimum

To solve the optimization problem (11), we apply the generalized maximum principle derived in El-Hodiri et al. (1972) for time-lagged optimal control problems. One obtains

the following present-value Hamiltonian  $\mathcal{H}$ :

$$\begin{aligned}
\mathcal{H} = & [U(x(t)) - D(e(t))] \exp[-\rho t] + \\
& q_x(t) \left[ \frac{1 - \frac{\lambda}{\kappa} k(t) - i(t)}{1 + a(t)} + \frac{1}{\kappa} k(t) - x(t) \right] + \\
& q_e(t) \left[ \left(1 - G(a(t))\right) \frac{1 - \frac{\lambda}{\kappa} k(t) - i(t)}{1 + a(t)} - e(t) \right] + \\
& q_k(t + \sigma) i(t) - q_k(t) \gamma k(t) + \\
& q_i(t) i(t) + \\
& q_{l_1}(t) \frac{1 - \frac{\lambda}{\kappa} k(t) - i(t)}{1 + a(t)} ,
\end{aligned} \tag{12}$$

where  $q_k$  denotes the costate variable or shadow price of the capital stock  $k$ , and  $q_x$ ,  $q_e$ ,  $q_i$  and  $q_{l_1}$  denote the Kuhn-Tucker parameters for the (in)equality conditions (11b), (11c), (11e) and (11f). Assuming the Hamiltonian  $\mathcal{H}$  to be continuously differentiable with respect to the control variables  $a$  and  $i$ , the following necessary conditions hold for an optimal solution:

$$q_x(t) = U'(x(t)) \exp[-\rho t] , \tag{13a}$$

$$q_e(t) = -D'(e(t)) \exp[-\rho t] , \tag{13b}$$

$$\frac{q_x(t) l_1(t)}{1 + a(t)} = -q_e(t) l_1(t) \left[ G'(a(t)) + \frac{1 - G(a(t))}{1 + a(t)} \right] + \frac{q_{l_1}(t) l_1(t)}{1 + a(t)} , \tag{13c}$$

$$\frac{q_x(t)}{1 + a(t)} = -q_e(t) \left[ \frac{1 - G(a(t))}{1 + a(t)} \right] + q_k(t + \sigma) + q_i(t) - \frac{q_{l_1}(t)}{1 + a(t)} , \tag{13d}$$

$$\frac{dq_k(t)}{dt} = q_e(t) \frac{\lambda(1 - G(a(t)))}{\kappa(1 + a(t))} - q_x(t) \frac{1 + a(t) - \lambda}{\kappa(1 + a(t))} + q_k(t) \gamma + \frac{q_{l_1}(t) \lambda}{\kappa(1 + a(t))} , \tag{13e}$$

$$q_i(t) \geq 0 , \quad q_i(t) i(t) = 0 , \tag{13f}$$

$$q_{l_1}(t) \geq 0 , \quad q_{l_1}(t) l_1(t) = 0 . \tag{13g}$$

As the maximized Hamiltonian is concave (cf. Appendix A.1), the necessary conditions (13a)–(13g) are also sufficient if, in addition, the following transversality condition holds:

$$\lim_{t \rightarrow \infty} q_k(t) k(t) = 0 . \tag{13h}$$

Due to the *strict* concavity of the maximized Hamiltonian, the optimal solution is also unique.

Conditions (13a) and (13b) state that along the optimal path the shadow price of energy equals the marginal utility of energy and the shadow price of net emissions equals the marginal disutility of net emissions. From condition (13g) we know that  $q_{l_1} l_1 = 0$  holds at all times  $t$ . Hence, the last term in condition (13c) equals 0 and, as long as  $l_1(t) > 0$ , we achieve by inserting conditions (13a) and (13b):

$$U'(x(t)) = D'(e(t)) [G'(a(t)) (1 + a(t)) + 1 - G(a(t))] . \tag{14}$$

This condition expresses that along the optimal path (and as long as condition (11f) is not binding) the utility (in current values) of an additional marginal unit of energy equals the disutility (in current values) of the emissions induced by that marginal unit of energy. Along the optimal path this equation determines the optimal value of the abatement effort  $a$  per unit of output  $x_1$ . In the case that inequality (11f) is binding, and thus  $l_1$  equals 0, condition (13c) reduces to the truism  $0 = 0$ . It is obvious, however, that in the case where the established technique is not used at all, the optimal abatement effort  $a = 0$  as no emissions have to be abated.

As noted above, the optimal system dynamics of the optimization problem (11) splits into three cases, an interior solution and two corner solutions. In Appendix (A.2) we derive the system of functional differential equations for the system dynamics and show that each case exhibits a (different) stationary state. In particular, the stationary state of the interior solution represents a saddle point, i.e. for all sets of initial conditions there exists a unique optimal path which converges towards the stationary state.

We first restrict our attention to the case of an *interior solution*, i.e. both  $q_i(t) = q_{l_1}(t) = 0$ . Together with transversality condition (13h) and inserting conditions (13a) and (13b) condition (13e) can be unambiguously solved:

$$q_k(t) = \int_t^\infty \frac{U'(x(s))(1+a(s)-\lambda) + D'(e(s))\lambda(1-G(a(s)))}{\kappa(1+a(s))} \exp[-\gamma(s-t) - \rho s] ds. \quad (15)$$

Thus, along the optimal path the shadow price for the capital stock equals the net present value of all future welfare gains of one additional marginal unit of the capital good. As capital goods are long lived, they contribute over the whole time horizon increasingly less though due to deterioration. The fraction under the integral equals the marginal instantaneous welfare gain of an additional unit of capital, which comprises two components. The first is the direct welfare gain due to the energy produced. It is positive if the new technology needs less labor input per unit of output than the established one, i.e.  $\lambda < 1 + a$ . The second term is always positive and denotes the welfare gain due to emissions abated by switching from the established to the new production technique.

Inserting conditions (13a) and (13b) in equation (13d) yields:

$$\frac{U'(x(t)) + D'(e(t))(1 - G(a(t)))}{1 + a(t)} \exp[-\rho t] = q_k(t + \sigma) \quad (16)$$

The equation states that along the optimal path the present value of the welfare loss by investing in one marginal unit of new capital, which is given by the present value welfare gain of the alternative use of one marginal unit of labor in the established production technique (left-hand side), equals the net present value of the sum of all future welfare gains by using the new capital good in production. As the investment needs the time span  $\sigma$  to become productive capital, the sum of all future welfare gains of an investment at time  $t$  is given by the shadow price of capital at time  $t + \sigma$ ,  $q_k(t + \sigma)$ . Note that equation (16) implies that  $p_k$  is always positive along the optimal path and, thus, the second term of the fraction in equation (15) outweighs the first.

### 3.2 Conditions for investment and replacement

So far, however, it is not clear to which of the three possible stationary states the system will tend. In the following, we derive conditions for the exogenous parameters identifying which of the three possible cases for the system dynamics applies. In fact, these conditions determine whether there is any investment in the new technology, and if so, whether the established technology is eventually fully replaced by the new one. We start with the investment condition.

In order to derive a condition which identifies whether investment is optimal, we assume the economy to stay in the no investment corner solution and derive a condition for which the corner solution violates the necessary and sufficient condition for an optimal solution. The following proposition states the result.

**Proposition 1 (Investment condition in the social optimum)**

*Given the optimization problem (11), the new technology is innovated, i.e.  $i(0) > 0$ , if and only if the following condition holds:*

$$1 + a^0 + \frac{1 - G(a^0)}{G'(a^0)} > \lambda + \kappa(\gamma + \rho) \exp[\rho\sigma] , \quad (17)$$

where  $a^0$  is determined by the unique solution of the implicit equation:

$$\frac{U'(1 - a^0)}{D'((1 - a^0)(1 - G(a^0)))} = G'(a^0)(1 + a^0) + 1 - G(a^0) . \quad (18)$$

**Proof:** Assume that it is *optimal not to invest* at all times  $t$ . As a consequence, the economy will remain in the no investment corner solution where no capital is accumulated. Hence,  $i(t) = 0$ ,  $q_i(t) \geq 0 \forall t$  and inequality (11e) is binding. All energy is solely produced by the established production technique which implies that  $x^0 = x_1^0 = 1 - a^0$ ,  $x_2^0 = 0$ ,  $l_1^0 > 0$  and inequality (11f) is not binding (i.e.  $q_{l_1} = 0$ ). The optimal abatement effort  $a^0$  is determined by equation (14) by inserting  $x^0 = 1 - a^0$  and  $e^0 = x^0(1 - G(a^0))$  which yields equation (18). Due to the assumed curvature properties of  $V$ ,  $D$  and  $G$ , there exists a unique solution for  $a^0$ .

In the corner solution  $i(t) = 0$ , we derive the shadow price of capital  $q_k^0(t)$  by inserting equation (14) in equation (15) and solving the integral:

$$q_k^0(t) = D'(e^0) \left[ (1 + a^0 - \lambda)G'(a^0) + 1 - G(a^0) \right] \frac{\exp[-\rho t]}{\kappa(\gamma + \rho)} . \quad (19)$$

Equating conditions (13c), and (13d) and inserting equations (13b) and  $q_k^0(t + \sigma)$  yields the following necessary and sufficient condition for the corner solution to be optimal:

$$D'(e^0)G'(a^0) \exp[-\rho t] - q_i(t) = D'(e^0) \left[ (1 + a^0 - \lambda)G'(a^0) + 1 - G(a^0) \right] \frac{\exp[-\rho(t + \sigma)]}{\kappa(\gamma + \rho)} . \quad (20)$$

Taking into account that  $q_i(t) \geq 0$ , dividing by  $D'(e^0)G'(a^0)$  and rearranging terms yields:

$$1 + a^0 + \frac{1 - G(a^0)}{G'(a^0)} \leq \lambda + \kappa(\gamma + \rho) \exp[\rho\sigma] . \quad (21)$$

Note that condition (21) is independent of  $t$ . This implies that it is optimal not to invest at all times  $t$ , if it is optimal not to invest at time  $t = 0$ . Thus, if condition (21) holds, the optimal solution of the optimization problem (11) is to remain in the no investment corner solution forever.

This, in turn, implies that it is *optimal to invest* in the new technology, if and only if condition (21) does not hold, which is exactly what condition (17) states.

□

**Remark:** Condition (17) for investment in the new technology has an intuitive economic interpretation. In the corner solution without investment the left-hand side corresponds to the unit costs of energy production of the established technology  $UC_{T_1}^0$ , the right-hand side to the unit costs of energy production of the new technology  $UC_{T_2}^0$ . Thus, condition (17) states that for the new technology to be innovated its unit costs of production have to be below those of the established technology, i.e.  $UC_{T_2}^0 < UC_{T_1}^0$ .

The unit costs of production of the first technology are composed of three components, the ‘pure’ labor costs per unit of energy production, the labor costs for abatement per unit and the social costs of unit emissions in terms of labor. The unit costs of production of the non-polluting new technology comprise apart from the ‘pure’ labor costs per unit the costs for building up and maintaining the necessary capital good in terms of labor per unit of output. Obviously, the capital costs per unit of output depend positively on the capital intensity  $\kappa$ , the dynamic characteristics  $\gamma$  and  $\sigma$  of the capital good production, as well as on the time preference rate  $\rho$ . In particular, the longer the time-lag  $\sigma$  and the higher the rate of time preference  $\rho$  the higher are the unit costs of energy of the new technology.

Note that as consumption and emission levels change over time in general the unit costs of energy during transition periods are not constant and are thus not necessarily given by  $UC_{T_1}^0$  and  $UC_{T_2}^0$ .

However, despite the infinite time horizon and the linearity of the two production techniques full replacement of the established technology by the new technology in the long run is not guaranteed, if condition (17) holds. In the following, we deduce conditions for which complete or partial replacement occur in the long run.

Formally, the case of full replacement of the established by the new production technique is given by the full replacement corner solution  $l_1(t) = 0$ . The line of argument to derive a condition for full replacement is similar to the inference of proposition 1. Assuming the economy to be in a long-run stationary state in which the new technology is fully developed and all labor is used up to employ and maintain the capital stock we investigate under which conditions such a full replacement stationary state is consistent with the necessary and sufficient conditions for an optimal solution as given by equations (13a)–(13h). The following proposition states the result.

**Proposition 2 (Full replacement condition in the social optimum)**

Given the optimization problem (11) and that  $U'(x^\infty) - D'(0) \neq 0$  full replacement of the established technology by the new one in the long-run stationary state is consistent with the necessary and sufficient conditions for a social optimum, if and only if the following condition holds:

$$1 + \frac{D'(0)}{U'(x^\infty) - D'(0)} \geq \lambda + \kappa(\gamma + \rho) \exp[\rho\sigma] , \quad (22)$$

where  $x^\infty$  is given by  $x^\infty = \frac{1}{\lambda + \kappa\gamma}$ .

**Proof:** Assume that it is optimal in the long-run stationary state to use the total labor endowment to employ and maintain the capital stock for the new technology, i.e.  $x_2^\infty = \frac{1}{\lambda + \kappa\gamma}$ . Then, all output is solely produced by the new technology, i.e.  $x^\infty = x_2^\infty$ ,  $x_1^\infty = l_1^\infty = 0$ . In addition, no emissions are produced and have to be abated and, thus,  $e^\infty = 0$  and  $a^\infty = 0$ .

Inserting conditions (13a) and (13b) into equation (13e) yields:<sup>6</sup>

$$-\frac{dq_k^\infty(t)}{dt} = \frac{U'(x^\infty)(1 - \lambda) + D'(0)\lambda - q_{l_1}^\infty \lambda}{\kappa} \exp[-\rho t] - q_k(t)\gamma . \quad (23)$$

Together with the transversality condition (13h), equation (23) can be solved to yield:

$$q_k^\infty(t) = \frac{\exp[-\rho t]}{\kappa(\gamma + \rho)} [U'(x^\infty)(1 - \lambda) + D'(0)\lambda - q_{l_1}^\infty \lambda] . \quad (24)$$

By inserting conditions (13a), (13b) and  $q_k^\infty(t + \sigma)$  into equation (13d), and taking into account that  $q_{l_1}^\infty \geq 0$ , we derive condition (22).

□

**Remark:** The economic interpretation of the full replacement condition (22) is analogous to the case of the investment condition (17). Full replacement can only take place, if the costs per unit of output of the new technology in the full replacement stationary state  $UC_{T_2}^\infty$  (right-hand side) are smaller than or equal to the costs of the established technology  $UC_{T_1}^\infty$  (left-hand side). As there are no emissions, there are no labor costs for abatement effort in the full replacement stationary state. Thus, the unit costs of the established technology only consist of the ‘pure’ labor costs plus the social costs, which stem from emissions. In the common case that  $D'(0) = 0$  (which implies that the first marginal unit of emissions does not exhibit any environmental damage and which holds for example, if the disutility function  $D$  is a power function with an exponent greater than 1), the unit costs of the established technology reduce to the ‘pure’ labor costs of production. Note that condition (22) is not well defined, if  $\lim_{x \rightarrow x^\infty} U'(x) = D'(0)$  holds. However, also in this special case full replacement will occur if, in addition, condition (17) holds because the welfare gain of an additional unit of labor assigned to the old

<sup>6</sup> Note that  $q_{l_1}(t)$  is constant in *current values* in the stationary state and, thus,  $q_{l_1}(t) = q_{l_1}^\infty \exp[-\rho t]$  with some constant  $q_{l_1}^\infty \geq 0$  in *present values*.



technology vanishes while the shadow price of capital, which is the net present value of all future welfare gains of an additional unit of capital, remains positive. The unit costs of the new technology are identical in both situations as they do not depend on the level of emissions and its implied disutility.<sup>7</sup> Note that for full replacement to occur conditions (17) and (22) must hold at the same time.

A straightforward corollary from propositions 1 and 2 is that *partial* replacement of the established by the new technology (i.e. the long-run stationary state is an interior solution) takes place, if condition (17) holds but condition (22) is violated.

**Corollary 1 (Partial replacement condition in the social optimum)**

*Given the optimization problem (11) and that  $U'(x^\infty) - D'(0) \neq 0$  partial replacement of the established technology by the new one is optimal in the long-run stationary state, i.e. the long-run stationary state is an interior solution, if and only if the following condition holds:*

$$1 + a^0 + \frac{1 - G(a^0)}{G'(a^0)} > \lambda + \kappa(\gamma + \rho) \exp[\rho\sigma] > 1 + \frac{D'(0)}{U'(x^\infty) - D'(0)}, \quad (25)$$

where  $x^\infty = \frac{1}{\lambda + \kappa\gamma}$  and  $a^0$  is given by the unique solution of the implicit equation (18).

In sum, investment is never optimal if the labor costs per unit of output of the new technology,  $UC_{T_2} = UC_{T_2}^0 = UC_{T_2}^\infty$ , are higher than the labor costs per unit of output of the established technology in the no investment corner solution,  $UC_{T_1}^0$ . If investment is optimal, i.e.  $UC_{T_2} < UC_{T_1}^0$ , then full replacement in the long-run stationary state is optimal if, in addition,  $UC_{T_2} \leq UC_{T_1}^\infty$  holds. Otherwise, i.e. if  $UC_{T_1}^\infty < UC_{T_2} < UC_{T_1}^0$ , the new technology will partially replace the established technology in the optimal long-run stationary state.

## 4 Unregulated competitive market equilibrium

In this section, we assume that the allocation of the model economy is determined by an unregulated market regime. We assume competitive markets for labor, capital and energy in which one representative household and two representative firms interact. We suppose that all markets are cleared at all times and thus supply equals demand. As emissions are free though negatively valued by the household the firms do not account for them in their market decisions. Moreover, as explained in section 2.2, we consider a representative consumer who exhibits different preferences in an individual compared to a social decision context. More precisely, we assume that in the market regime the preferences of the representative consumer are given by equation (9) (with  $\alpha = 1$ ), which differs from equation (10) by a *higher* rate of time preference  $\rho_p$ .

In analogy to the analysis of the social optimum, we derive the conditions for investment in the new technology and for replacement of the established technology in the long-run stationary state. We study the effect of the two external effects the model comprises on these conditions.

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<sup>7</sup> In fact, the unit costs of the new technology are the same among *all* possible stationary states.

#### 4.1 The household's market decisions

The household is assumed to own the two firms and the total endowment 1 of labor in the economy. In line with the standard literature on capital accumulation and growth (e.g. Barro and Sala-i-Martin 1995: chap. 2), we assume that also capital assets are owned by the household. Thus, the household chooses between selling labor to the firms at the market price of labor  $w$  or to invest labor in the accumulation of capital  $k$ , which the household rents to the firms at the market price of capital  $r$ . In addition, the household buys energy  $x$  at the market price of energy  $p$ . As the household cannot incur debts, the following budget constraint has to hold at all times  $t$ :

$$p(t)x(t) = w(t)(1 - i(t)) + r(t)k(t) + \pi_1(t) + \pi_2(t) , \quad (26)$$

where  $\pi_1$  and  $\pi_2$  denote the profits of firm 1 and 2. In addition, capital can be accumulated according to equation (8).

The household is assumed to maximize its intertemporal welfare (9), i.e. the household solves the following maximization problem:

$$\max_i \int_0^\infty [U(x(t)) - D(e(t))] \exp[-\rho_p t] dt , \quad (27a)$$

subject to

$$p(t)x(t) = w(t)(1 - i(t)) + r(t)k(t) + \pi_1(t) + \pi_2(t) , \quad (27b)$$

$$\frac{dk(t)}{dt} = i(t - \sigma) - \gamma k(t) , \quad (27c)$$

$$i(t) \geq 0 , \quad (27d)$$

$$k(0) = 0 , \quad (27e)$$

$$i(t) = \xi(t) = 0, \quad t \in [-\sigma, 0) . \quad (27f)$$

Thus, the present value Hamiltonian  $\mathcal{H}^H$  reads:

$$\mathcal{H}^H = [V(x(t)) - D(e(t))] \exp[-\rho_p t] + \quad (28a)$$

$$q_b(t) [w(t)(1 - i(t)) + r(t)k(t) - p(t)x(t)] + \quad (28b)$$

$$q_k(t + \sigma)i(t) - q_k(t)\gamma k(t) + \quad (28c)$$

$$q_i(t)i(t) , \quad (28d)$$

where  $q_k$  denotes the costate variable or shadow price of the capital stock  $k$ , and  $q_b$  and  $q_i$  denote the Kuhn-Tucker parameters for the (in)equality conditions (27b) and (27d). The strict concavity of the Hamiltonian  $\mathcal{H}^H$  can be shown following a similar line of argument as in Appendix A.1.

Assuming that the Hamiltonian  $\mathcal{H}^H$  is continuously differentiable with respect to the control variable  $i$  the following necessary conditions hold for an optimal solution:

$$q_b(t)p(t) = U'(x(t)) \exp[-\rho_p t] , \quad (29a)$$

$$q_b(t)w(t) = q_k(t + \sigma) + q_i(t) , \quad (29b)$$

$$-\frac{dq_k(t)}{dt} = q_b(t)r(t) - q_k(t)\gamma , \quad (29c)$$

$$q_i(t) \geq 0 , \quad q_i(t)i(t) = 0 . \quad (29d)$$

Due to the concavity of the Hamiltonian, the necessary conditions (29a)–(29d) are also sufficient if in addition a transversality condition analogous to condition (13h) holds. Moreover, the strict concavity of the Hamiltonian ensures a unique solution. Then, together with condition (29a) condition (29c) can be unambiguously solved:

$$q_k(t) = \exp[\gamma t] \int_t^\infty q_b(s)r(s) \exp[-\gamma s] ds . \quad (30)$$

## 4.2 The firms' market decisions

The firms are assumed to maximize their profits in the competitive market equilibrium taking prices as given. Firm 1 produces energy according to the first production technology described by equations (1) and (11c). Thus, the profit  $\pi_1$  at time  $t$  is given by:

$$\pi_1(t) = p(t)l_1(t) - w(t)(1 + a(t))l_1(t) . \quad (31)$$

Firm 1 chooses  $l_1$  and  $a$  such as to maximize the net present value of all future profits which is equivalent to maximize the profit  $\pi_1$  at all times  $t$ . As the negative externality of emissions is not accounted for in the unregulated market economy, abatement effort  $a$  is a pure cost to the firm. As a consequence, a necessary condition for profit maximization for firm 1 is:

$$a(t) = 0 . \quad (32)$$

As  $\pi_1$  is linear in  $l_1$ ,  $\pi_1$  is non negative for any  $l_1 > 0$  as long as output prices exceed input prices. The labor demand of firm 1 is given by the following correspondence:

$$l_1(t) \begin{cases} = \infty & , \text{ if } p(t) > w(t) \\ \in [0, \infty) & , \text{ if } p(t) = w(t) \\ = 0 & , \text{ if } p(t) < w(t) \end{cases} . \quad (33)$$

Firm 2 produces energy according to the second production technology described by equation (6). Thus, the profits  $\pi_2$  at time  $t$  equal:

$$\pi_2(t) = \frac{1}{\kappa} p(t)k(t) - \frac{\lambda}{\kappa} w(t)k(t) - r(t)k(t) , \quad (34)$$

which is a linear function of  $k$ . As a consequence, profits  $\pi_2$  are non negative for any  $k > 0$  as long as the value of outputs exceeds the value of inputs. Analogously to firm 1, firm 2 demands as much capital as possible together with  $\frac{\lambda}{\kappa}k$  units of labor, if the value of the output exceeds the value of the inputs. Thus, the demand of firm 2 is given by the following correspondence:

$$k(t) \begin{cases} = \infty \quad \wedge \quad l_2(t) = \frac{\lambda}{\kappa}k(t) & , \text{ if } p(t) > \lambda w(t) + \kappa r(t) \\ \in [0, \infty) \quad \wedge \quad l_2(t) = \frac{\lambda}{\kappa}k(t) & , \text{ if } p(t) = \lambda w(t) + \kappa r(t) \\ = 0 \quad \wedge \quad l_2(t) = 0 & , \text{ if } p(t) < \lambda w(t) + \kappa r(t) \end{cases} . \quad (35)$$

### 4.3 Necessary and sufficient condition for the market equilibrium

In the market equilibrium supply equals demand on all markets. As in the social optimum, the market solution may exhibit two corner solutions. This is either the case, if the household does not invest in capital at all times or if the total labor endowment is used to employ and maintain the capital stock. In the former case, firm 2 is unable to operate, while in the latter case firm 1 is driven out of the market.

First, we analyze the interior market equilibrium where both firms operate. From the demand correspondences (33) and (35) of firm 1 and firm 2 we know that for any positive and finite amount of  $l_1$ ,  $l_2$  and  $k$  the following conditions hold:

$$\frac{w(t)}{p(t)} = 1, \quad \frac{r(t)}{p(t)} = \frac{1}{\kappa} \left( 1 + \lambda \frac{w(t)}{p(t)} \right) = \frac{1 - \lambda}{\kappa}. \quad (36)$$

Solving equation (29a) for  $q_b$  and taking into account conditions (36), we achieve for the shadow price of capital  $q_k$

$$q_k(t) = \frac{1 - \lambda}{\kappa} \exp[\gamma t] \int_t^\infty U'(x(s)) \exp[-(\gamma + \rho_p)s] ds, \quad (37)$$

and the following necessary and sufficient condition for an interior market equilibrium:

$$U'(x(t)) \exp[-\rho_p t] = q_k(t + \sigma). \quad (38)$$

Analogously to the corresponding condition (16) in the social optimum, equation (38) states that along the optimal path the present value of the household's welfare loss by investing in one marginal unit of new capital, given by the present value welfare gain of the alternative use of one marginal unit of labor in the established production technique (left-hand side), equals the net present value of the sum of all future welfare gains by using the new capital good in production. As investment needs the time span  $\sigma$  to become productive capital, the sum of all future welfare gains of an investment at time  $t$  is given by the shadow price of capital at time  $t + \sigma$ ,  $q_k(t + \sigma)$ . Both costs and benefits of investment are smaller in the market equilibrium compared to the social optimum. However, in order to decide how the external effects influence the conditions of investment and replacement we have to check them explicitly.

### 4.4 Conditions for investment and replacement

In order to derive an investment condition similar to condition (17) in the social optimum, we assume again the economy to stay in the no investment corner solution. Given the stationary state with no investment in capital at all times, we derive a condition on the exogenous parameters for which the corner solution violates the necessary and sufficient condition (38) for an unregulated market solution. The following proposition states this condition.

**Proposition 3 (Investment condition in the competitive market equilibrium)**

Given the optimization problem (27) of the representative household and the profit functions (31) and (34) of firm 1 and firm 2, the new technology is innovated, i.e.  $i(0) > 0$ , if and only if the following condition holds:

$$1 > \lambda + \kappa(\gamma + \rho_p) \exp[\rho_p \sigma] . \quad (39)$$

**Proof:** Assume that it is *optimal not to invest* at all times  $t$ . As a consequence, the economy will remain in the no investment corner solution where no capital is accumulated. Hence,  $i(t) = 0$   $q_i(t) \geq 0 \forall t$  and the inequality (29d) is binding. All energy is solely produced by the established production technique (i.e.  $x^0 = x_1^0 = 1$ ,  $x_2^0 = 0$ ).

From the demand correspondences (33) and (35) we know that

$$\frac{w(t)}{p(t)} = 1 , \quad \frac{r(t)}{p(t)} \geq \frac{1}{\kappa} \left( 1 - \lambda \frac{w(t)}{p(t)} \right) = \frac{1 - \lambda}{\kappa} . \quad (40)$$

Solving equation (29a) for  $q_b$  and inserting it, together with conditions (40), in equation (30) yields the following inequality for the shadow price of capital:

$$q_k^0(t) \geq \frac{1 - \lambda}{\kappa(\gamma + \rho_p)} U'(1) \exp[-\rho_p t] . \quad (41)$$

Inserting  $q_b(t)$  and  $q_k^0(t + \sigma)$  into equation (29b) and taking into account that  $q_i(t) \geq 0$  yields the following necessary and sufficient condition for the corner solution to be a market equilibrium:

$$U'(1) \exp[-\rho_p t] \geq \frac{1 - \lambda}{\kappa(\gamma + \rho_p)} U'(1) \exp[-\rho_p(t + \sigma)] . \quad (42)$$

Dividing by  $U'(1) \exp[-\rho_p t]$  and rearranging terms yields that it is *optimal to invest* in the new technology, if and only if condition (39) holds.

□

**Remark:** Condition (39) displays the unit costs of energy production of the established and the new technology, respectively, in the competitive market equilibrium. Again, the new technology has to necessarily display lower unit costs of production than the established technology in order to be innovated. As the social costs of pollution are not accounted for in the unregulated market regime firm 1 has no incentive to abate. Thus, the unit costs of energy of the established technology reduce to the ‘pure’ costs of production and are thus *lower* than socially optimal. The unit costs of energy of the new technology display the same composition as at the social optimum. As they now depend on  $\rho_p > \rho$  they *exceed* the socially optimal unit costs of energy of the new technology. Thus, in the unregulated market regime higher (compared to the social optimum) unit costs of energy of the new technology must stay below lower units costs of energy of the established technology for the new technology to be innovated.

As there is no abatement investment in the new technology according to condition (39) always implies the full replacement of the initially established technology in the long run and thus excludes partial replacement. The following proposition states these results.

**Proposition 4 (Full replacement in the competitive market equilibrium)**

*Given the optimization problem (27) of the representative household, the profit functions (31) and (34) of firm 1 and firm 2, full replacement of the established technology by the new one in the long-run stationary state is consistent with the necessary and sufficient conditions for a competitive market equilibrium, if and only if the following condition holds:*

$$1 \geq \lambda + \kappa(\gamma + \rho_p) \exp[\rho_p \sigma] . \quad (43)$$

*In particular, this implies that partial replacement of the established technology by the new one cannot occur in the unregulated market regime.*

**Proof:** Assume that it is optimal in the long-run stationary state to use the total labor endowment to employ and maintain the capital stock for the new technology, i.e.  $l_1^\infty = 0$ ,  $l_2^\infty = \frac{\lambda}{\lambda + \kappa\gamma}$ . Then, all output is solely produced by the new technology, i.e.  $x^\infty = x_2^\infty = \frac{1}{\lambda + \kappa\gamma}$ ,  $x_1^\infty = l_1^\infty = 0$ .

From the demand correspondences (33) and (35) we know that

$$\frac{w(t)}{p(t)} \geq 1 , \quad \frac{r(t)}{p(t)} = \frac{1}{\kappa} \left( 1 - \lambda \frac{w(t)}{p(t)} \right) . \quad (44)$$

Solving equation (29a) for  $q_b$  and inserting, together with conditions (44) in equation (30) yields for the shadow price of capital:

$$q_k^\infty(t) = \frac{1 - \lambda w^\infty}{\kappa(\gamma + \rho_p)} U'(x^\infty) \exp[-\rho_p t] , \quad (45)$$

where  $w^\infty = \frac{w(t)}{p(t)}$  evaluated at the full replacement stationary state, and is thus a constant. Inserting  $q_b(t)$  and  $q_k^\infty(t + \sigma)$  into equation (29b) we derive the following condition:

$$w^\infty U'(x^\infty) \exp[-\rho_p t] = \frac{1 - \lambda w^\infty}{\kappa(\gamma + \rho_p)} U'(x^\infty) \exp[-\rho_p(t + \sigma)] . \quad (46)$$

Dividing by  $\frac{U'(x^\infty)}{\kappa(\gamma + \rho_p)} \exp[-\rho_p(t + \sigma)]$ , taking into account that  $w^\infty \geq 1$  and rearranging terms yields condition (43).

□

**Remark:** At first sight it might be puzzling that condition (39) is a strict inequality while condition (43) also allows for the equality sign to hold. The interpretation is, however, straightforward. Condition (43) states the requirements for a full replacement

stationary state to be consistent with the necessary and sufficient conditions for a market equilibrium. However, from the strict inequality (39) we know that starting with a vanishing capital stock  $k(0) = 0$  there is no investment at all times, if the equality sign in (43) holds. Nevertheless, in the hypothetical situation that the economy would already start with the full replacement capital stock  $k^\infty = \frac{\kappa}{\lambda + \kappa\gamma}$  and that, in addition, condition (43) holds with equality the economy would stay in the full replacement market equilibrium forever.

In sum, in the unregulated market economy the new technology has to exhibit lower costs per unit of output than the ‘*pure*’ labor costs of the established technology to be innovated. This holds as the social costs of emissions which are an inevitable joint output of the old production technique are not accounted for in the market equilibrium. Moreover, the unit costs of the new technology are higher in the unregulated market equilibrium compared to the social optimum. This holds as the ‘costs’ of the period of waiting between the investment and the new capital good becoming productive are higher, due to the higher rate of time preference  $\rho_p$  of individual actors compared to the social rate of time preference  $\rho$ . Thus, in a mutually reinforcing way the two market failures imply that the new technology might not be innovated in the competitive market equilibrium, although innovation would be socially optimal.

## 5 Competitive market equilibrium with emission tax and investment subsidy

In this section, we consider how the social optimum can be implemented. In general, two independent instruments are needed to implement the social optimum corresponding to the two externalities arising in the model. In fact, we study the introduction of an emission tax  $\tau_e$  to internalize the external effect from emissions, and of an investment subsidy  $\tau_i$  to internalize the second externality from the individuals’ higher time preference. We assume the emission tax to be a tax per unit of emissions, directly collected from firm 1, and the investment subsidy to be a subsidy per unit of investment, paid directly to the household.

### 5.1 The household’s and firms’ market decisions under regulation

The emission tax and the investment subsidy alter the profit function of firm 1 and the household’s maximization problem, respectively. Thus, we have to reconsider the corresponding decisions in a regulated market regime.

Given a per unit tax  $\tau_e$  per unit of emissions, the profit function of firm 1 alters as follows:

$$\pi_1(t) = p(t)l_1(t) - w(t)(1 + a(t))l_1(t) - \tau_e(t)(1 - G(a(t)))l_1(t) . \quad (47)$$

Firm 1 chooses both labor  $l_1$  and abatement effort  $a$  such as to maximize the net present value of all future profits which is equivalent to maximizing the profit  $\pi_1$  at all times  $t$ .

A necessary condition for profit maximization is

$$\frac{\partial \pi_1(t)}{\partial a(t)} = -l_1(t)w(t) + \tau_e(t)G'(a(t))l_1(t) = 0, \quad (48)$$

which is an implicit equation for the unique optimal abatement effort  $a^*(t)$  as long as  $l_1(t) > 0$ . However, if  $l_1(t) = 0$ , the optimal abatement effort  $a^*(t) = 0$  as in this case there are no emissions which have to be abated. Again, the profit function  $\pi_1(t)$  is linear in the labor demand  $l_1(t)$ . Thus, the demand for  $l_1(t)$  is given by the following correspondence:

$$l_1(t) \begin{cases} = \infty & , \text{ if } p(t) > w(t)(1 + a(t)) + \tau_e(t)(1 - G(a(t))) \\ \in [0, \infty) & , \text{ if } p(t) = w(t)(1 + a(t)) + \tau_e(t)(1 - G(a(t))) \\ = 0 & , \text{ if } p(t) < w(t)(1 + a(t)) + \tau_e(t)(1 - G(a(t))) \end{cases}, \quad (49)$$

where the optimal abatement effort  $a$  is given by the solution of the implicit equation  $\tau_e(t)G'(a(t)) = w(t)$  if  $l_1(t) > 0$ , and  $a(t) = 0$  if  $l_1(t) = 0$ .

With the investment subsidy  $\tau_i(t)$  paid per unit of investment  $i$ , the household's budget constraint equals:<sup>8</sup>

$$p(t)x(t) = w(t)(1 - i(t)) - \tau_i(t)i(t) + r(t)k(t) + \pi_1(t) + \pi_2(t). \quad (50)$$

Thus, the necessary and sufficient condition (29b) is replaced by:

$$q_b(t)(w(t) + \tau_i(t)) = q_k(t + \sigma) + q_i(t). \quad (51)$$

Neither the emission tax  $\tau_e$  nor the innovation subsidy  $\tau_i$  directly affect firm 2. As a consequence, the market decisions of firm 2 remain unaltered.

## 5.2 Necessary and sufficient condition for the regulated market equilibrium

Given the adjusted equations (47) and (51), which replace equations (33) and (29b) of section 4 in the case that an emission tax  $\tau_e$  and an investment subsidy  $\tau_i$  are enacted, we analyze how the interior market equilibrium changes.

From conditions (48), (49) and (35) we derive the following conditions for an interior market equilibrium where both firms operate (i.e.  $l_1(t) > 0$ ,  $i(t) > 0$ ):

$$1 = \frac{\tau_e(t)}{p(t)} [G'(a(t))(1 + a(t)) + 1 - G(a(t))], \quad (52)$$

$$\frac{w(t)}{p(t)} = \frac{1 - \frac{\tau_e(t)}{p(t)}(1 - G(a(t)))}{1 + a(t)}, \quad (53)$$

$$\frac{r(t)}{p(t)} = \frac{1 + a(t) - \lambda + \lambda \frac{\tau_e(t)}{p(t)}(1 - G(a(t)))}{\kappa(1 + a(t))}. \quad (54)$$

<sup>8</sup> For the sake of consistency, a positive  $\tau_e$  ( $\tau_i$ ) denotes a tax and a negative  $\tau_e$  ( $\tau_i$ ) denotes a subsidy.



Solving equation (29a) for  $q_b$  and taking into account conditions (54) we achieve for the shadow price of capital  $q_k$ :

$$q_k(t) = \frac{\exp[\gamma t]}{\kappa} \int_t^\infty \frac{1 + a(s) - \lambda + \lambda \frac{\tau_e(s)}{p(s)} (1 - G(a(s)))}{1 + a(s)} U'(x(s)) \exp[-(\gamma + \rho_p)s] ds. \quad (55)$$

Inserting  $q_b$  and equation (53) into equation (51) yields

$$\frac{1 - \frac{\tau_e(t)}{p(t)} (1 - G(a(t)))}{1 + a(t)} U'(x(t)) \exp[-\rho_p t] = q_k(t + \sigma) - \frac{\tau_i(t)}{p(t)} U'(x(t)) \exp[-\rho_p t], \quad (56)$$

which together with equation (52) determines the interior market equilibrium for a given emission tax  $\tau_e$  and investment subsidy  $\tau_i$ . Note that equations (52) and (56) determine the market equilibrium only in terms of relative prices. Thus, one price can freely be chosen as a numeraire.

Choosing the price of energy  $p$  as numeraire we calculate the optimal emission tax and the optimal investment subsidy. Comparing equation (52) with the corresponding condition (14) in the social optimum we achieve for the optimal emission tax  $\tau_e^{opt}$ :

$$\frac{\tau_e(t)^{opt}}{p(t)} = \frac{D'(e(t))}{U'(x(t))}. \quad (57)$$

For condition (56) and the corresponding condition (17) in the social optimum to coincide the investment subsidy  $\tau_i^{opt}$  has to be set to:

$$\begin{aligned} \frac{\tau_i(t)^{opt}}{p(t)} = & - \frac{\exp[-\gamma(t+\sigma)]}{\kappa U'(x(t))} \int_{t+\sigma}^\infty \frac{U'(x(s))(1 + a(s) - \lambda) + D'(e(s))\lambda(1 - G(a(s)))}{1 + a(s)} \\ & \times \exp[-\gamma s] (\exp[-\rho(s-t)] - \exp[-\rho_p(s-t)]) ds. \end{aligned} \quad (58)$$

As expected, if the two instruments are set in such a way that the market equilibrium is identical to the social optimum,  $\tau_e^{opt}$  is always positive (i.e. emissions are taxed) and  $\tau_i^{opt}$  is always negative (i.e. investment is subsidized).

In the following, we consider how the conditions for investment and replacement, i.e. the two corner solutions, change compared to the unregulated market economy when an emission tax  $\tau_e$  is raised from firm 1 and an investment subsidy  $\tau_i$  is paid to the household. We show that setting  $\tau_e$  and  $\tau_i$  as defined in equations (57) and (58) also implements the social optimum in case of the corner solutions.

### 5.3 Conditions for investment and replacement

Again, we first consider the case that the economy stays in the no investment corner solution. We derive a condition for positive investment to be a market equilibrium in the regulated market regime with emission tax  $\tau_e$  and investment subsidy  $\tau_i$ . The following proposition states this condition.

**Proposition 5 (Investment condition in the regulated market regime)**

Given the optimization problem (27) of the household with the adjusted budget constraint (50), the profit functions (47) and (34) of firm 1 and firm 2, and the emission tax  $\frac{\tau_e(t)}{p(t)}$  and the investment subsidy  $\frac{\tau_i(t)}{p(t)}$ , the new technology is innovated in the market equilibrium, i.e.  $i(0) > 0$ , if and only if the following condition holds:

$$1 + a^0 + \frac{1 - G(a^0)}{G'(a^0)} > \lambda + \left[ 1 + \frac{\tau_i^0}{\tau_e^0 G'(a^0)} \right] \kappa(\gamma + \rho_p) \exp[\rho_p \sigma] , \quad (59)$$

where  $\tau_e^0 = \frac{\tau_e(t)}{p(t)}$ ,  $\tau_i^0 = \frac{\tau_i(t)}{p(t)}$  evaluated at the no investment stationary state and  $a^0$  is determined by the unique solution of the implicit equation:

$$1 = \tau_e^0 (G'(a^0)(1 + a^0) + 1 - G(a^0)) . \quad (60)$$

Condition (59) for the market equilibrium is identical to the corresponding condition for the social optimum (17), if  $\tau_e^0$  and  $\tau_i^0$  are set as follows:

$$\tau_e^0 = \frac{D'(e^0)}{U'(x^0)} > 0 , \quad (61)$$

$$\tau_i^0 = \frac{D'(e^0) [(1 + a^0 - \lambda)G'(a^0) + 1 - G(a^0)]}{\kappa U'(x^0)} \left( \frac{\exp[-\rho_p \sigma]}{\gamma + \rho_p} - \frac{\exp[-\rho \sigma]}{\gamma + \rho} \right) < 0 , \quad (62)$$

where  $x^0 = 1 - a^0$  and  $e^0 = (1 - a^0)(1 - G(a^0))$ .

**Proof:** The proof is analogous to the proof of proposition 3. Assume that it is *optimal not to invest* at all times  $t$ . As a consequence, the economy will remain in the no investment corner solution where no capital is accumulated. Hence,  $i(t) = 0$ ,  $q_i(t) \geq 0 \forall t$  and the inequality (29d) is binding. All energy is solely produced by the established production technique (i.e.  $x^0 = x_1^0 = 1 - a^0$ ,  $x_2^0 = 0$ ). We know from conditions (48), (49) and (35):

$$1 = \tau_e^0 (G'(a^0)(1 + a^0) + 1 - G(a^0)) , \quad (63a)$$

$$\frac{w(t)}{p(t)} = \tau_e^0 G'(a^0) , \quad (63b)$$

$$\frac{r(t)}{p(t)} \geq \frac{\tau_e^0 [(1 + a^0 - \lambda)G'(a^0) + 1 - G(a^0)]}{\kappa} . \quad (63c)$$

Equation (63a) determines the profit maximizing abatement effort  $a^0$  of firm 1. Solving equation (29a) for  $q_b$  and inserting, together with conditions (63c), in equation (29c) yields the following inequality for the shadow price of capital:

$$q_k^0(t) \geq \frac{\tau_e^0 [(1 + a^0 - \lambda)G'(a^0) + 1 - G(a^0)]}{\kappa(\gamma + \rho_p)} U'(x^0) \exp[-\rho_p t] . \quad (64)$$

Inserting equation (63b),  $q_b$  and  $q_k^0$  into equation (51) and taking into account that  $q_i(t) \geq 0$  we derive:

$$\tau_e^0 G'(a^0) U'(x^0) \geq \frac{\tau_e^0 [(1 + a^0 - \lambda)G'(a^0) + 1 - G(a^0)]}{\kappa(\gamma + \rho_p)} U'(x^0) \exp[-\rho_p \sigma] - \tau_i^0 U'(x^0) . \quad (65)$$

Dividing by  $\tau_e^0 G'(a^0) U'(x^0)$  and rearranging terms yields that the *no* investment corner solution is a market equilibrium, if:

$$\lambda + \left[ 1 + \frac{\tau_i^0}{\tau_e^0 G'(a^0)} \right] \kappa(\gamma + \rho_p) \exp[\rho_p \sigma] \geq 1 + a^0 + \frac{1 - G(a^0)}{G'(a^0)}. \quad (66)$$

That, in turn, implies that in the regulated market equilibrium there *is* investment in the new technology, if and only if condition (59) holds.

By setting  $\tau_e^0 = \frac{D'(e^0)}{U'(x^0)}$ , condition (63a) which determines the profit maximizing abatement effort  $a^0$  becomes identical to equation (18) which determines the socially optimal abatement level. Furthermore, inserting  $\tau_e^0$  and  $\tau_i^0$  from equations (61) and (62) into condition (59) yields (after some tedious calculations) the investment condition in the social optimum (17).

□

**Remark:** Condition (59) displays the unit costs of energy production of the established and of the new technology, respectively, in the no investment market equilibrium when an emission tax  $\tau_e$  is imposed and an innovation subsidy  $\tau_i$  is paid. By implementing a cost on the output of emissions  $e$  the imposition of the emission tax  $\tau_e$  induces a positive abatement effort  $a$  of firm 1 and provides for the social cost of emissions to be taken into account in the unit costs of production of the established technology. By setting  $\tau_e^0$  equal to the ratio between marginal damage from environmental degradation and marginal benefit from consumption the unit costs of production of the established technology are raised to their socially optimal level and thus the external effect from the emissions is internalized. However, it is obvious from condition (59) that an emission tax does not suffice for the market equilibrium to resemble the socially optimal outcome. In addition, an investment subsidy has to be paid, lowering the unit costs of energy production for the new technology to their level at the social optimum.

Note that conditions (61) and (62) are identical to the corresponding conditions (57) and (58) for an interior market equilibrium evaluated at the no investment corner solution.

We will now derive the conditions for which full replacement of the established by the new technology is a market equilibrium in the long run given the state imposes an emission tax  $\tau_e$  and pays an investment subsidy  $\tau_i$ .

**Proposition 6 (Full replacement condition in the regulated market regime)**  
*Given the optimization problem (27) of the household with the adjusted budget constraint (50), the profit functions (47) and (34) of firm 1 and firm 2, the emission tax  $\frac{\tau_e(t)}{p(t)}$  and the investment subsidy  $\frac{\tau_i(t)}{p(t)}$ , full replacement of the established technology by the new one in the long-run stationary state is consistent with the necessary and sufficient conditions for a regulated market equilibrium, if and only if the following condition holds:*

$$1 + \frac{\tau_e^\infty}{1 - \tau_e^\infty} \geq \lambda + \left[ 1 + \frac{\tau_i^\infty}{1 - \tau_e^\infty} \right] \kappa(\gamma + \rho_p) \exp[\rho_p \sigma], \quad (67)$$

where  $\tau_e^\infty = \frac{\tau_e(t)}{p(t)}$ ,  $\tau_i^\infty = \frac{\tau_i(t)}{p(t)}$  evaluated at the long-run stationary state.

Condition (67) for the market equilibrium is identical to the corresponding condition for the social optimum (22), if  $\tau_e^\infty$  and  $\tau_i^\infty$  are set as follows:

$$\tau_e^\infty = \frac{D'(0)}{U'(x^\infty)} \geq 0, \quad (68)$$

$$\tau_i^\infty = \frac{U'(x^\infty)(1 - \lambda) + D'(0)\lambda}{\kappa U'(x^\infty)} \left( \frac{\exp[-\rho_p \sigma]}{\gamma + \rho_p} - \frac{\exp[-\rho \sigma]}{\gamma + \rho} \right) < 0, \quad (69)$$

where  $x^\infty = \frac{1}{\lambda + \kappa \gamma}$ .

**Proof:** Assume that using the total labor endowment to employ and maintain the capital stock for the new technology in the long-run stationary state is a market equilibrium, i.e.  $l_1^\infty = 0$ ,  $i^\infty > 0$  and  $q_i^\infty = 0$ . Then, all output is solely produced by the new technology, i.e.  $x^\infty = x_2^\infty = \frac{1}{\lambda + \kappa \gamma}$  and  $x_1^\infty = l_1^\infty = 0$ . In addition, no emissions are produced and have to be abated and thus  $e^\infty = 0$  and  $a^\infty = 0$ . For this case, we know from the demand correspondences (49) and (35) of firm 1 and firm 2:

$$\frac{w(t)}{p(t)} \leq 1 - \frac{\tau_e(t)}{p(t)}, \quad (70a)$$

$$\frac{r(t)}{p(t)} = \frac{1}{\kappa} \left( 1 - \lambda \frac{w(t)}{p(t)} \right). \quad (70b)$$

Solving equation (29a) for  $q_b$  and inserting it, together with condition (70b), in equation (29c), yields for the the shadow price of capital:

$$q_k^\infty(t) = \frac{1 - \lambda w^\infty}{\kappa(\gamma + \rho_p)} U'(x^\infty) \exp[-\rho_p t], \quad (71)$$

where  $w^\infty = \frac{w(t)}{p(t)}$  evaluated at the full replacement stationary state and thus is a constant.

Inserting  $q_b$ ,  $q_k$  and inequality (70a) into equation (51), and taking into account that  $q_i(t) = 0$  we derive the following condition:

$$(1 - \tau_e^\infty)(\lambda + \kappa(\gamma + \rho_p) \exp[\rho_p \sigma]) \leq 1 - \tau_i^\infty \kappa(\gamma + \rho_p) \exp[\rho_p \sigma]. \quad (72)$$

Dividing by  $(1 - \tau_e^\infty)$  and rearranging terms yields condition (67).

Furthermore, inserting  $\tau_e^\infty$  and  $\tau_i^\infty$  from equations (68) and (69) into condition (67) yields (after some tedious calculations) the full replacement condition in the social optimum (22).

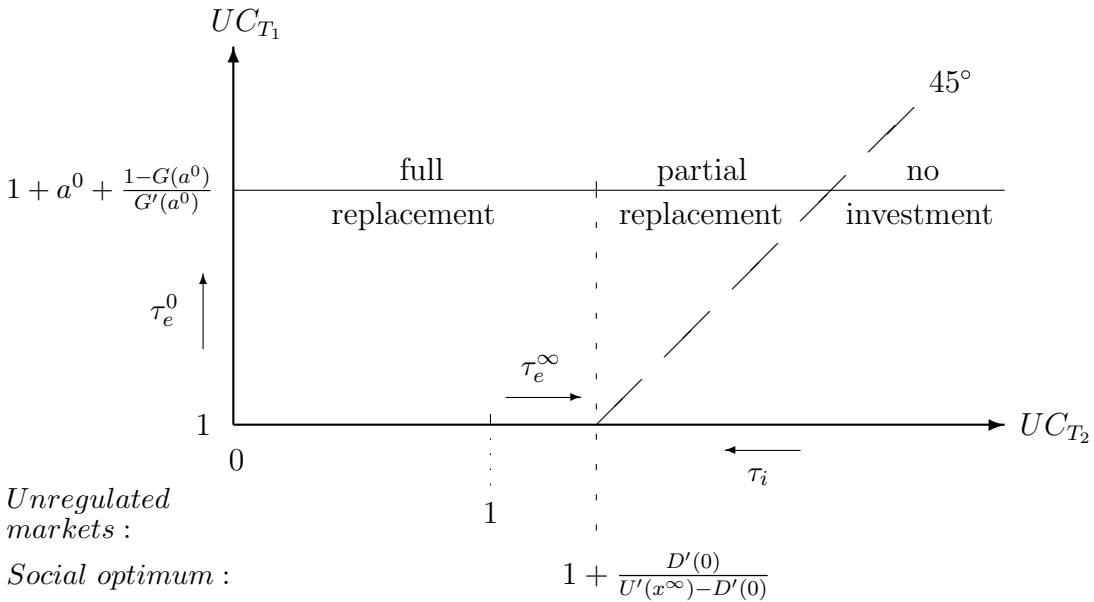
□

**Remark:** Note that although in the case of full replacement the external effect from the emissions vanishes the emission tax has to be raised as long as  $D'(0) > 0$  for the market equilibrium to resemble the social optimum. If  $D'(0) = 0$ , then the optimal tax in the full replacement stationary state is given by  $\tau_e^\infty = 0$ . However, the optimal investment subsidy  $\tau_i^\infty$  has to be non-zero in any case.

As in the case of the social optimum, conditions (59) and (67) have to hold simultaneously for full replacement to occur in the regulated market regime in the long run. Moreover, if the emission tax  $\tau_e$  and the investment subsidy  $\tau_i$  are such that condition (59) is always fulfilled but condition (67) is always violated, the economy exhibits a market equilibrium where both technologies are used, i.e. there is a partial replacement of the established by the new technique.

## 6 Discussion

Before we discuss our results with respect to model assumptions and policy implications, we briefly summarize our findings graphically. Figure 1 displays the conditions of investment and full and partial replacement in the cases of the unregulated market regime and the social optimum as well as the effects from the application of the policy instruments, the emission tax and the investment subsidy.



**Figure 1:** Full, partial and no investment in the unregulated market equilibrium and the social optimum

For the emission externality, our model resembles the standard environmental economic result that there is no incentive to abate in an unregulated market economy where the social costs of emissions are not taken into account by the polluting firm. As a consequence, the unit costs of the established technology  $UC_{T_1}$  equal its ‘pure’ labor costs per output unit of 1, which is *below* the socially optimal level of  $1 + a^0 + \frac{1-G(a^0)}{G'(a^0)}$ .

In addition to the standard setting we assume in our model that the new energy producing technology exhibits a time-lag  $\sigma$  for the accumulation of the specific capital good and that social and individual rates of time preferences differ. This has implications for the unit costs of energy of the new technology  $UC_{T_2}$ . Apart from the labor costs per output unit  $\lambda$ , they comprise the capital costs per output unit which depend, in particular, positively on the time-lag  $\sigma$  and the rate of time preference. Thus, the schizophrenic behavior assumption implies that  $UC_{T_2}$  in the unregulated market equilibrium is *above* its socially optimal level.

The new technology is only innovated, if the unit costs of energy of the new technology are below those of the established technology. Thus, for investment in the new technology to be (individually) optimal in the unregulated market regime, a higher  $UC_{T_2}$  has to stay below a lower  $UC_{T_1}$  compared to the social optimum. Thus, investment is hampered by the combined effect of both externalities.

Figure 1 shows the double effect the levy of the emission tax  $\tau_e$  has on the conditions of investment and replacement, respectively. While  $\tau_e^0$  raises the unit costs of energy of the established technology to its socially optimal level  $1 + a^0 + \frac{1-G(a^0)}{G'(a^0)}$ , and thereby creates, in particular, the possibility of an interior solution with both technologies operating in the long-run stationary state,  $\tau_e^\infty$  moves the point until which full replacement takes place to  $1 + \frac{D'(0)}{U'(x^\infty) - D'(0)}$ . The social optimum can, in general, only be implemented if in addition an investment subsidy  $\tau_i$  is paid which reduces the unit costs of the new technology to their socially optimal level.

## 6.1 Model assumptions

The most crucial assumption for our results to hold is that the social and the individual rate of time preference differ. The line of argument we draw upon is schizophrenic behavior, i.e. the household applies different preferences depending on whether the decision context is of private or social interest. This assumption has been discussed in detail in section 2.2.

In fact, the second distinctive feature of our model that the accumulation of the specific capital good for the new technology exhibits a substantial time-lag  $\sigma$  is not necessary for the second externality to occur. However, the externality imposed by the differing discount rates is amplified by the time-lag  $\sigma$ . Thus, the second externality gains higher importance the higher is  $\sigma$ , as it can easily be seen from the term  $\rho_p \exp[\rho_p \sigma]$  in the unit costs of the new technology. As time-to-build is of considerable length in energy plant construction (ranging from two years for gas cogeneration plants up to some ten years for the construction of a nuclear power plant), the second externality is of particular importance in the context of the technological transition of energy systems. In our opinion, this importance justifies the additional mathematical obstacles incurred by time-lagged equations of motion.

By modeling the energy producing technologies as linear and linear limitational functions, respectively, we assume very specific forms of technology for the production of energy. The rationale for this specific functional form is to account for rigidities in energy production due to technical and thermodynamic constraints. However, for the sake

of simplicity fuel inputs are not considered explicitly but subsumed under the one non-producible production input labor. Moreover, we want to concentrate on substitution effects *between* (the established and the new) production technologies and therefore exclude substitution possibilities *within* the individual energy technologies. From a more technical point of view, it is the linearity of the production functions which gives rise to the corner solutions which we exploit to derive conditions for investment, and partial and full replacement.

Finally, for the sake of a tractable model we abstract from a series of peculiarities of the economics of the energy industry. First, the energy industry exhibits oscillatory demand fluctuations on different time-scales (for example day/night-time, summer/winter, etc.). As different energy technologies exhibit different turn-on/turn-off costs and rigidities, a mix of energy technologies is in general preferable over ‘energy monocultures’ (for example, hydroelectric power and gas cogeneration plants can easily be turned-on and switched off and are therefore well suited to supply energy for peak demands while nuclear power plants are extremely costly and time-consuming to load adjustments and are thus preferably used to supply the base load).

Second, in contrast to our assumption of a perfectly competitive market the energy industry rather exhibits a oligopolistic market structure. As it is well known from the industrial organization literature, unregulated oligopolistic market regimes lead in general to market failures in the sense that the market outcome is not a social optimum. We abstract from these additional market failures to concentrate on the externalities imposed by emissions and differing private and social discount rates.

## 6.2 Policy implications

The main result of our paper is that for the transition towards a low-emission energy industry the imposition of an energy tax is in general not sufficient to achieve the social optimum. Although this result was derived from a highly stylized theoretical model, there are direct policy implications to be drawn which are relevant for the regulation of energy markets in order to induce a socially optimal transition towards a cleaner energy system.

First, Jaffe et al. (2005) argue that the development and innovation of a new technology is generally related to a high degree of uncertainty with regard to its success. Furthermore, its innovation is likely to create knowledge spillovers to other firms, and the diffusion of new technologies is typically related to dynamic increasing returns stemming from learning-by-using, learning-by-doing or network externalities. Thus, the process of technological change involves a series of (potential) market failures which provide reasons for state interventions of technology policy. In our model, we derive a similar result without taking into consideration the uncertain nature of R&D and by abstracting from dynamic effects, such as learning-by-doing or network externalities. In fact, in our model it is the time-consuming nature of bringing a new technology into use combined with the split of the social and individual rates of time preference which leads to an additional externality. Thus, we provide a complementary justification for technology policies.

Second, Porter and van der Linde (1995) argue that “well-designed” environmental

regulations may exhibit a double dividend in the sense that they achieve both less pollution due to higher environmental standards, taxes, etc., and higher competitiveness by increasing the productivity with which resources are used. With the findings of our model we can substantiate what constitutes a “well-designed” environmental regulation. We distinguish between gradual and structural technological change which we identify in the context of the transition to a low-emission energy industry with the installation of an end-of-pipe abatement technology or with a switch to a clean energy technology, respectively. It is obvious from the formulae of the unit costs of production for the established and the new technology that gradual technological change always induces additional costs to the existing technology, while structural technological change may exhibit higher, equal or lower unit costs of production compared to the pure labor costs of the established technology. In the latter case, the new technology offers in fact a double dividend in the sense of the Porter hypothesis (no emissions and lower unit costs). Yet, the standard environmental policy of solely imposing an emission tax favors gradual technological change and does not sufficiently account for structural change as compared to the social optimum. These findings suggest that for a well-designed environmental regulation the standard environmental policy has to be supplemented by technology policy. This holds even more, if we consider in addition the dynamic effects of R&D and innovation, as discussed by Jaffe et al. (2005).

Third, our findings provide theoretical economic support for policies that subsidize clean energy technologies, such as the German “Erneuerbare-Energien-Gesetz” (Renewable Energy Sources Act), first enacted in April 2000 and amended in August 2004. The stated purpose of the act is to increase the share of renewable energy technologies to at least 12.5% in 2010 and 20% in 2020. Moreover, the act aims at a sustainable provision of energy, a reduction of insecure markets for fossil fuels and a promotion of technological progress in renewable energy technology. According to the act, suppliers of renewable energy get unit subsidies which depend on the energy technology and the size of the plant. Moreover, the subsidies are decreasing over time to give incentives for productivity increases. The subsidy rates are adjusted regularly (last adjustment in August 2004). At least the basic idea of subsidizing renewable energies of the Renewable Energy Sources Act is supported by the combined findings of Jaffe et al. (2005), Porter and van der Linde (1995) and our own results. However, if both the instrument of feed-in tariffs employed by the Renewable Energy Sources Act and their respective levels are efficient is beyond the scope of our paper.

## 7 Conclusion

In this paper, we have studied the relationship between gradual and structural technological change in the transition to a low-emission energy industry. Our findings are driven by the combined effect of a standard emission externality and the additional external effect, stemming from the split of private and social rates of time preference, and the time-to-build feature of capital accumulation. We have derived the ratio of the unit costs of energy of the two technologies as the decisive criterion whether investment and



partial or full replacement of the new technology occur. Our results imply that environmental policy has to be supplemented by technology policy in order to achieve the social optimum in a market regime. Moreover, in the case that only an emission tax is enacted the investment decision is biased in favor of gradual technological change compared to the social optimum.

In our analysis we abstract from a series of peculiarities of the economics of technological change and power systems. This allows us to show that another market failure may arise in technological transition processes stemming from the split of social and individual rates of time preference, which is fostered by the time-to-build feature of the specific capital goods and, thus, particularly relevant in the energy sector.

Therefore, our investigation provides an additional reason why it may be beneficial, in terms of economic efficiency, to combine instruments of environmental and technology policy to achieve a socially optimal outcome, which does not draw on the standard explanations of uncertain R&D success and technology spillovers. Apart from that, the analysis shows that structural change may exhibit a double dividend in the sense of the Porter hypothesis being both economically more efficient and more environmentally benign. In fact, our model can substantiate Porter and van der Linde's (1995) claim of "well-designed" environmental regulations. Thus, our findings provide some general theoretical support for policies which subsidize renewable energies.

Moreover, the results constructively contribute to the Kyoto conflict between the United States and the European Union. While the European Union strongly promotes emission regulation policies, the United States rather advocate technology policy. Our findings indicate that the correct question to ask is not which of the two kinds of policy is the most beneficial in terms of social welfare but rather which combination of the two is to be applied.

Of course, the analysis provides only a theoretical indication. It is up to further empirical research to investigate how social and individual preferences actually differ in the case of essential and desired goods, such as energy, the production of which is necessarily linked to the by-production of a harmful joint output. This is particularly important to derive concrete levels of investment subsidies for technology policies. Moreover, we did not take into account the oligopolistic market structures which are common in energy markets and the integration of which into our model constitutes a fruitful agenda for further theoretical research.

## Appendix

### A.1 Concavity of the Hamiltonian along the optimal path

In the following, we show that the maximized Hamiltonian  $\mathcal{H}^0$ , which is the Hamiltonian  $\mathcal{H}$  as defined in equation (12) in which the optimal paths for  $a$  and  $i$  are substituted, is jointly concave in the variables  $x$ ,  $e$ , and  $k$  along the optimal path. Although we cannot solve the optimal paths for  $a$  and  $i$ , we can eliminate them by employing the necessary conditions for an optimal solution.

The Hamiltonian (12) can be written as:

$$\begin{aligned} \mathcal{H} = & [V(x(t)) - D(e(t))] \exp[-\rho t] + q_x(t) \left[ \frac{1}{\kappa} k(t) - x(t) \right] - q_e(t)e(t) + \\ & + q_k(t + \sigma)i(t) - q_k(t)\gamma k(t) + q_i(t)i(t) + q_{l_1} \frac{1 - \frac{\lambda}{\kappa} k(t) - i(t)}{1 + a(t)} + \\ & \frac{1 - \frac{\lambda}{\kappa} k(t) - i(t)}{1 + a(t)} [q_x(t) + q_e(t)(1 - G(a(t)))] , \end{aligned} \quad (\text{A.1})$$

From the necessary condition (13d), we know that

$$\frac{q_x(t) + q_e(t)(1 - G(a(t)))}{1 + a(t)} = q_k(t + \sigma) + q_i(t) - \frac{q_{l_1}(t)}{1 + a(t)} . \quad (\text{A.2})$$

Inserting equation (A.2) into equation (A.1) yields the maximized Hamiltonian  $\mathcal{H}^0$ , in which the control variables  $a$  and  $i$  are eliminated:

$$\begin{aligned} \mathcal{H}^0 = & [V(x(t)) - D(e(t))] \exp[-\rho t] + q_x(t) \left[ \frac{1}{\kappa} k(t) - x(t) \right] - q_e(t)e(t) \\ & + q_k(t + \sigma) \left[ 1 - \frac{\lambda}{\kappa} k(t) \right] - q_k(t)\gamma k(t) . \end{aligned} \quad (\text{A.3})$$

Obviously,  $\mathcal{H}^0$  is concave, as it is the sum of concave functions.

### A.2 Optimal transition dynamics and stationary states

The optimal system dynamics of the optimization problem (11) splits into three cases. The first case corresponds to the corner solution  $i(t) = 0 \forall t$ . In this case, there is no system dynamics at all. The system will remain in a stationary state where the labor endowment is used up by energy production via the established technology and abatement.

In the second case, the optimal system dynamics is an interior solution, i.e. along the optimal path  $i(t) > 0 \forall t$  and  $l_1(t) > 0 \forall t$  holds. Then, the system dynamics is governed by the following system of differential equations

$$\begin{aligned} \frac{di(t)}{dt} = & \Phi_1(t) \left[ (\gamma + \rho)D'(t)G'(t) + \frac{\exp[-\rho\sigma]}{\kappa} (\lambda D'(t+\sigma)G'(t+\sigma) + U'(t+\sigma)) \right] + \\ & + \Phi_2(t) [i(t-\sigma) - \gamma k(t)] , \end{aligned} \quad (\text{A.4a})$$

$$\begin{aligned} \frac{da(t)}{dt} = & \Phi_3(t) \left[ (\gamma + \rho)D'(t)G'(t) + \frac{\exp[-\rho\sigma]}{\kappa} (\lambda D'(t+\sigma)G'(t+\sigma) + U'(t+\sigma)) \right] + \\ & + \Phi_4(t) [i(t-\sigma) - \gamma k(t)] , \end{aligned} \quad (\text{A.4b})$$

$$\frac{dk(t)}{dt} = i(t-\sigma) - \gamma k(t) , \quad (\text{A.4c})$$

where  $\Phi_n(t)$  ( $n = 1, \dots, 4$ ) are functions of  $i(t)$ ,  $a(t)$  and  $k(t)$  as shown in Appendix A.3. As  $\frac{di(t)}{dt}$ ,  $\frac{da(t)}{dt}$  and  $\frac{dk(t)}{dt}$  also depend on *advanced* (i.e. at a later time) and on *retarded* (i.e. at an earlier time) variables, equations (A.4) form a system of *functional differential equations*.<sup>9</sup>

Although this system is not analytically soluble in general (not even in the linear approximation around the stationary state), we show in Appendix A.3 that the unique stationary state, which is given by the following implicit equations

$$U'(x^*) = D'(e^*) [G'(a^*) (1 + a^*) + 1 - G(a^*)] , \quad (\text{A.5a})$$

$$\gamma + \rho = \exp[-\rho\sigma] \frac{\lambda D'(e^*) G'(a^*) + U'(x^*)}{\kappa D'(e^*) G'(a^*)} , \quad (\text{A.5b})$$

$$i^* = \gamma k^* , \quad (\text{A.5c})$$

is a saddle point. Hence, for all sets of initial conditions there is a unique optimal path which converges towards the stationary state. In general, these optimal paths are oscillatory and exponentially damped.<sup>10</sup>

In the third case, which corresponds to the corner solution  $l_1(t) = 0$ , the established technology will eventually be fully replaced by the new technology, and all labor is used to employ and maintain the capital stock  $k$ . Thus, if the restriction  $l_1(t) \geq 0$  is binding, there exists a direct link between the capital stock  $k$  and the investment  $i$ :

$$k(t) = \frac{\kappa}{\lambda} (1 - i(t)) . \quad (\text{A.6})$$

Differentiating with respect to time  $t$  and inserting into the equation of motion for the capital stock (11d), yields the following linear first-order differential-difference equation of retarded type, which governs the system dynamics:

$$\frac{di(t)}{dt} + \gamma i(t) + \frac{\lambda}{\kappa} i(t - \sigma) = \gamma . \quad (\text{A.7})$$

The solution to this equation is analyzed in detail in Winkler et al. (2005). In general, the optimal paths converge oscillatorily and exponentially damped towards the stationary state, which is given by

$$i^* = \frac{\kappa\gamma}{\lambda + \kappa\gamma} , \quad k^* = \frac{\kappa}{\lambda + \kappa\gamma} . \quad (\text{A.8})$$

### A.3 Saddle point stability of the interior solution

To show the saddle point property of the stationary state in the case of an interior solution, we investigate the following general maximization problem

$$\max \int_0^{\infty} F(i(t), a(t), k(t)) \exp[-\rho t] dt \quad (\text{A.9a})$$

<sup>9</sup> For an introduction to functional differential equations see Asea and Zak (1999: section 2) and Gandolfo (1996: chapter 27). A detailed exposition for linear functional differential equations is given in Bellman and Cooke (1963) and Hale (1977).

<sup>10</sup> The system of functional differential equations (A.4) may also exhibit so called *limit-cycles*, i.e. the optimal paths oscillate around the stationary state without converging towards or diverging from it (e.g. Feichtinger et al. 1994, Asea and Zak 1999, Liski et al. 2001 and Wirl 1995, 1999, 2002).

subject to

$$\dot{k} = i(t-\sigma) - \gamma k(t) , \quad (\text{A.9b})$$

$$i(t) = \xi(t) = 0, \quad t \in [-\sigma, 0) , \quad (\text{A.9c})$$

which is equivalent to the optimization problem (11) in the case of an interior solution with

$$F = V \left( \frac{1 - \frac{\lambda}{\kappa} k(t) - i(t)}{1 + a(t)} + \frac{k(t)}{\kappa} \right) - D \left( (1 - G(a(t))) \frac{1 - \frac{\lambda}{\kappa} k(t) - i(t)}{1 + a(t)} \right) . \quad (\text{A.10})$$

the corresponding present-value Hamiltonian reads

$$\mathcal{H} = F(t) \exp[-\rho t] + q(t+\sigma)i(t) - q(t)\gamma k(t) , \quad (\text{A.11})$$

where  $q$  denotes the shadow price for the state variable  $k$ .

If the maximized Hamiltonian (A.11) is strictly concave, which is assumed in the following, the following conditions are necessary and sufficient for an optimal solution:<sup>11</sup>

$$q(t+\sigma) = -F_i(t) \exp[-\rho t] , \quad (\text{A.12a})$$

$$F_a(t) = 0 , \quad (\text{A.12b})$$

$$\dot{q}(t) = -F_k(t) \exp[-\rho t] + \gamma q(t) , \quad (\text{A.12c})$$

$$\lim_{t \rightarrow \infty} q(t)k(t) = 0 . \quad (\text{A.12d})$$

Differentiating equations (A.12a) and (A.12b) with respect to time  $t$ , inserting (A.12a), (A.12c) and (A.9b) into the resulting equations, and solving for  $\frac{di(t)}{dt}$ ,  $\frac{da(t)}{dt}$  and  $\frac{dk(t)}{dt}$  yields the following set of functional differential equations:

$$\begin{aligned} \frac{di(t)}{dt} &= \frac{F_{aa}(t)}{\Delta F(t)} [(\gamma + \rho)F_i(t) + \exp[-\rho\sigma]F_k(t+\sigma)] + \\ &+ \frac{F_{ia}(t)F_{ak}(t) - F_{aa}(t)F_{ik}(t)}{\Delta F(t)} [i(t-\sigma) - \gamma k(t)] , \end{aligned} \quad (\text{A.13a})$$

$$\begin{aligned} \frac{da(t)}{dt} &= \frac{F_{ia}(t)}{\Delta F(t)} [(\gamma + \rho)F_i(t) + \exp[-\rho\sigma]F_k(t+\sigma)] + \\ &+ \frac{F_{ia}(t)F_{ik}(t) - F_{ii}(t)F_{ak}(t)}{\Delta F(t)} [i(t-\sigma) - \gamma k(t)] , \end{aligned} \quad (\text{A.13b})$$

$$\frac{dk(t)}{dt} = i(t-\sigma) - \gamma k(t) , \quad (\text{A.13c})$$

where  $\Delta F(t) = F_{ii}(t)F_{aa}(t) - F_{ia}(t)^2$ . Introducing the following abbreviations

$$\Phi_1(t) = \frac{F_{aa}(t)}{\Delta F(t)} , \quad \Phi_2(t) = \frac{F_{ia}(t)F_{ak}(t) - F_{aa}(t)F_{ik}(t)}{\Delta F(t)} ,$$

$$\Phi_3(t) = -\frac{F_{ia}(t)}{\Delta F(t)} , \quad \Phi_4(t) = \frac{F_{ia}(t)F_{ik}(t) - F_{ii}(t)F_{ak}(t)}{\Delta F(t)} ,$$

<sup>11</sup> In the following, partial derivatives are denoted by subscripts and for presentational convenience only the time argument is stated explicitly.

and inserting  $F_i(t)$  and  $F_k(t+\sigma)$ , yields the system of differential equations (A.4).

In the stationary state,  $\frac{di(t)}{dt} = \frac{da(t)}{dt} = \frac{dk(t)}{dt} = 0$  holds. Thus, the unique stationary state  $(i^*, a^*, k^*)$  is determined by the three implicit equations:

$$\gamma + \rho = -\exp[-\rho\sigma] \frac{F_i(i^*, a^*, k^*)}{F_k(i^*, a^*, k^*)}, \quad (\text{A.14})$$

$$0 = F_a(i^*, a^*, k^*), \quad (\text{A.15})$$

$$i^* = \gamma k^*. \quad (\text{A.16})$$

Inserting  $F_i$ ,  $F_a$  and  $F_k$ , yields the equations (A.5).

To investigate the stability properties of optimization problem (A.9) in a neighborhood around the stationary state  $(i^*, a^*, k^*)$ , we linearize the system of functional differential equations (A.13) around the stationary state. Therefore, we first introduce new variables  $\hat{i}(t) = i(t) - i^*$ ,  $\hat{a}(t) = a(t) - a^*$  and  $\hat{k}(t) = k(t) - k^*$ . Applying the first order Taylor approximation of the system (A.13) around the stationary state  $(i^*, a^*, k^*)$ , yields

$$\begin{aligned} \frac{d\hat{i}(t)}{dt} \approx & \Phi_1 \exp[-\rho\sigma] \left[ F_{ik}i(t+\sigma) - \frac{F_{ii}F_k}{F_i}i(t) + F_{ak}a(t+\sigma) - \frac{F_{ia}F_k}{F_i}a(t) + F_{kk}k(t+\sigma) - \right. \\ & \left. - \frac{F_{ik}F_k}{F_i}k(t) \right] + \Phi_2 [u(t-\sigma) - \gamma k(t)], \end{aligned} \quad (\text{A.17a})$$

$$\begin{aligned} \frac{d\hat{a}(t)}{dt} \approx & \Phi_3 \exp[-\rho\sigma] \left[ F_{ik}i(t+\sigma) - \frac{F_{ii}F_k}{F_i}i(t) + F_{ak}a(t+\sigma) - \frac{F_{ia}F_k}{F_i}a(t) + F_{kk}k(t+\sigma) - \right. \\ & \left. - \frac{F_{ik}F_k}{F_i}k(t) \right] + \Phi_4 [u(t-\sigma) - \gamma k(t)], \end{aligned} \quad (\text{A.17b})$$

$$\frac{d\hat{k}(t)}{dt} \approx u(t-\sigma) - \gamma k(t), \quad (\text{A.17c})$$

where all functions are evaluated at the stationary state  $(i^*, a^*, k^*)$ . Similar to the case of ordinary linear first-order differential equations, the elementary solutions for  $\hat{i}$ ,  $\hat{a}$  and  $\hat{k}$  are exponential functions, and the general solution is given by the superposition of the elementary solutions

$$\hat{i}(t) \approx \sum_n i_n \exp[z_n t], \quad \hat{a}(t) \approx \sum_n a_n \exp[z_n t], \quad \hat{k}(t) \approx \sum_n k_n \exp[z_n t], \quad (\text{A.18})$$

where the  $i_n$ ,  $a_n$  and  $k_n$  are constants, which can (at least in principle) be unambiguously determined by the set of initial conditions and the transversality condition. The *eigenvalues*  $z_n$  are the roots of the *characteristic polynomial*  $Q(z)$ . The characteristic polynomial  $Q(z)$  for the system of differential-difference equations (A.17) is given by the determinant of the Jacobian of (A.17) minus the identity matrix times  $z$

$$Q(z) = \begin{vmatrix} A_{11} - z & A_{12} & A_{13} \\ A_{21} & A_{22} - z & A_{23} \\ A_{31} & A_{32} & A_{33} - z \end{vmatrix}, \quad (\text{A.19a})$$

where

$$A_{11} = \Phi_1 \{ F_{ik} (\exp[\sigma(z - \rho)] - \exp[-\sigma z]) + F_{ii}(\gamma + \rho) \} + \frac{F_{ia}F_{ak}}{\Delta F} \exp[-\sigma z] , \quad (\text{A.19b})$$

$$A_{12} = \Phi_1 \{ F_{ak} \exp[\sigma(z - \rho)] + F_{ia}(\gamma + \rho) \} , \quad (\text{A.19c})$$

$$A_{13} = \Phi_1 \{ F_{kk} \exp[\sigma(z - \rho)] + F_{ik}(2\gamma + \rho) \} - \frac{F_{ia}F_{ak}}{\Delta F} \gamma , \quad (\text{A.19d})$$

$$A_{21} = \Phi_2 \{ F_{ik} (\exp[\sigma(z - \rho)] - \exp[-\sigma z]) + F_{ii}(\gamma + \rho) \} - \frac{F_{ii}F_{ak}}{\Delta F} \exp[-\sigma z] , \quad (\text{A.19e})$$

$$A_{22} = \Phi_2 \{ F_{ak} \exp[\sigma(z - \rho)] + F_{ia}(\gamma + \rho) \} , \quad (\text{A.19f})$$

$$A_{23} = \Phi_2 \{ F_{kk} \exp[\sigma(z - \rho)] + F_{ik}(2\gamma + \rho) \} + \frac{F_{ii}F_{ak}}{\Delta F} \gamma , \quad (\text{A.19g})$$

$$A_{31} = \exp[-\sigma z] , \quad (\text{A.19h})$$

$$A_{32} = 0 , \quad (\text{A.19i})$$

$$A_{33} = -\gamma . \quad (\text{A.19j})$$

Thus, one obtains for the characteristic polynomial  $Q(z)$ :

$$\begin{aligned} Q(z) = & -(z - \gamma - \rho)(z + \gamma) + \Phi_2 \{ (z - \gamma - \rho) \exp[-\sigma z] - (z + \gamma) \exp[\sigma(z - \rho)] \} + \\ & + \frac{F_{aa}F_{kk} - F_{ak}^2}{\Delta F} \exp[-\sigma \rho] . \end{aligned} \quad (\text{A.20})$$

$Q(z)$  is a *quasi-polynomial*, which exhibits an infinite number of complex roots. To determine whether the stationary state is a saddle point we need to know the signs of the real parts of the characteristic roots. In fact, we show that the characteristic polynomial  $Q(z)$  has an infinite number of roots with negative real part and an infinite number of roots with positive real part and, thus, the stationary state is a saddle point.

First, note that the characteristic roots are symmetric around  $\rho/2$ , i.e. if  $z^0$  is a characteristic root, then  $\rho - z^0$  is also a characteristic root (one can easily verify that  $Q(z^0) = Q(\rho - z^0)$ ). Second, we introduce the new variable  $y = \sigma z$  and multiply  $Q$  with  $\sigma^2 \exp[y]$

$$\begin{aligned} Q(y) = & -(y - \sigma\gamma - \sigma\rho)(y + \sigma\gamma) \exp[y] - \sigma\Phi_2 \{ (y - \sigma\gamma - \sigma\rho) - (y + \sigma\gamma) \exp[2y - \sigma\rho] \} + \\ & + \sigma^2 \frac{F_{aa}F_{kk} - F_{ak}^2}{\Delta F} \exp[y - \sigma\rho] , \end{aligned} \quad (\text{A.21})$$

in order to apply Theorem 13.1 of Bellman and Cooke (1963: 441). As  $Q(y)$  has no *principal term*, i.e. a term where the highest power of  $y$  and the highest exponential term appear jointly,<sup>12</sup>  $Q(y)$  has “an unbounded number of zeros with arbitrarily large positive real part” (ibid). But, as the characteristic roots are symmetric around  $\rho/2$ , this also implies an unbounded number of roots with arbitrarily large negative real part.

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<sup>12</sup> In this case the principal term would be a term with  $y^2 \exp[2y]$ .

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