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Differential Taxation and Corporate Futures-Hedging

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Differential Taxation and Corporate Futures-Hedging

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Abstract:

Using a two-moment decision model this paper analyzes corporate hedging behavior in the presence of unified and differential income taxation. We start with the well-known result that risk-taking may increase when income tax rates increase and, therefore, the incentive for hedging reduces. We demonstrate that pure hedging is differently affected by taxation than speculative hedging is. Analysing tax-sensitivity of the corporate hedge shows that a higher risk in the first place may reduce the tax-induced incentive to revise a futures position.

JEL-Classification: F21, F31, H20

Keywords: taxation; hedging; mean-variance model; unified and differential taxation; Roy preference function

1. Introduction

Our paper analyzes optimum corporate hedging by futures in the presence of full loss-offset income taxation. A two-moment decision model[1] is presented which is consistent with expected utility maximization in order to explicitly derive the effect of risk in the case of unified and differential taxation. The optimal hedging strategy of the risk averse management is developed and comparative statics are presented. In detail we ask the following questions: What is the impact of taxation on corporate futures demand? Does a change in risk and return affect the magnitude of this impact?

The literature discusses many reasons explaining why corporate hedging occurs. One line of argument points out that hedging is a tool to reduce expected taxes, and hence increasing expected profits.[2] Furthermore, the phenomenon that by introducing taxation risk-taking can be encouraged is well-known in the literature.[3] The aim of our investigation is to focus on cross effects, because bearing risk for tax reasons is not independent from the incentive to take risks in the first place. The same holds for the hedging behavior. In addition, our note studies differential taxation of profits from corporate sales and profits from corporate hedging activity.[4]

Our study proceeds as follows: Section 2 presents the futures-hedging de-

[1]See, e.g., Schneeweiß, 1967, Sinn, 1980, Meyer, 1987, Battermann, Broll, and Wahl, 2002, and Broll, Wahl, and Wong, 2006.

[2]See Froot, Scharfstein, and Stein, 1993.

[3]See, e.g., Mossin, 1968, Bamberg and Richter, 1984, Sandmo, 1989, and Konrad and Richter, 1995.

[4]See, e.g., Zilcha and Eldor, 2004. In reality, there exist differential taxation systems in many countries, i.e., we observe different tax rates for corporate income tax, personal income tax, consumption tax, property and wealth taxes, etc. See OECD, 2001, 2007.

cision problem and introduces a generalized form of Roy's two-moment preference function which is representative for our purpose.[5] Section 3 demonstrates the relationship between income taxation and corporate futures-hedging under unified and differential taxation of the return on sales and the gains from hedge. Our paper concludes with some remarks.

2. The Decision Model

Consider a risk averse corporate management which seeks to hedge the risky return $\tilde{r}$ on sales s by selling futures contracts on $\tilde{r}$ up to the amount h. The futures rate is denoted by f.

The random corporate income is then given by

$$\tilde{y} = \tilde{r}s + (f - \tilde{r})h. \tag{1}$$

Expected income and the income's variance follow from equation (1):

$$\mu = E(\tilde{y}) = E(\tilde{r})(s - h) + fh, \tag{2}$$

$$\sigma^2 = V(\tilde{r})(s - h)^2, \tag{3}$$

where E and V denote the expectation and variance operator, respectively. From equations (2) and (3) we obtain the opportunity line. This line contains the feasible combinations of μ and σ. The line is linear and its slope represents the price of risk reduction:

$$\mu = fs + \frac{E(\tilde{r}) - f}{D(\tilde{r})}\,\sigma, \tag{4}$$

where D represents the standard deviation operator.

[5]See Roy (1952).

Corporate management maximizes a two-moment function $V(\mu, \sigma)$ which exhibits convex indifference curves in (σ, μ)-space. Hence, the decision problem reads:

$$\max_{\mu,\sigma} V(\mu, \sigma) \tag{5}$$

subject to opportunity line (4).

Optimum hedging volume h^* has to satisfy equality between the price of risk reduction and the marginal rate of substition of income risk σ for expected income μ:

$$\frac{E(\tilde{r}) - f}{D(\tilde{r})} = \frac{d\mu}{d\sigma}\Big|_{h=h^*}. \tag{6}$$

We wish to focus our investigation on the case in which optimum hedging volume can be explicitly solved and can also be separated additively in a pure and a speculative part (see the hedging equation (8) below). A representative preference function which is rational under normality regarding the expected utility hypotheses and also satisfies the above mentioned separability is given by the generalized Roy function (Roy, 1952, and Schneeweiß, 1967). For convenience, let us use the following monotone transformation of this function:

$$V(\mu, \sigma) = \frac{\mu^2}{1 + \alpha\,\sigma^2}, \;\; \alpha > 0. \tag{7}$$

Remark: Pure and speculative parts in the hedge occur with preference functions of the form $V(\mu, \sigma) = J(\mu)/R(\sigma)$, given some weak conditions for the functions J and R. The preference function (7) is representative for this type of preference functions. The enumeration of all utility functions $u(y)$ implying the above form under the expected utility hypotheses and normality are given in Schneeweiß, 1967 (p. 154). In addition, Meyer, 1987 (p. 426, Property 6), reveals the importance of relative risk aversion regarding the impact of proportional changes in μ and σ upon a general functional form of the rate of substitition in equation (6) between the two parameters.

In what follows we further assume that the random return on sales is normally distributed with expected return μ_r and standard deviation of return σ_r.

With the above preference function V the optimality condition (6) implies:

$$h^* = s + \frac{f - \mu_r}{\alpha \sigma_r^2 f s}. \tag{8}$$

Note that the full-hedge theorem holds.[6] In the optimum futures contracts have to cover pure hedge s and speculative hedge $h^* - s$. The sign of the risk premium $(\mu_r - f)$ determines whether or not we obtain a short or long position in optimum speculative demand $\Delta^* = s - h^*$.

Furthermore, $\partial \ln |\Delta^*| / \partial \ln s = -1$, that is to say, a given percentage increase in sales reduces speculative demand for futures contracts by the same percentage, when there is non-zero risk premium. Notice that the underlying utility function exhibits increasing absolute and relative risk aversion in the Arrow-Pratt sense for the relevant range of income, whereas the elasticity of absolute risk aversion with respect to income is unity (Schneeweiß, 1967, p. 153).

[6]See, e.g., Wahl and Broll, 2006.

3. Taxation and Hedging

In reality, corporate income is subject to taxation. What is the impact of taxation on corporate futures-hedging? Does a change in risk and return affect the magnitude of this impact? In the following we analyze these questions in detail.

3.1 Unified Taxation

Consider unified taxation of sales returns and futures returns. Let t denote the income tax rate which is valid for both types of returns. Hence the expectation of after-tax income becomes $(1-t)\mu$ and the standard deviation of after-tax income becomes $(1-t)\sigma$. The given preference function (7) yield the following marginal rate of substitution of after-tax income risk for after-tax expected income: $\alpha(1-t)^2\mu\sigma/[1+\alpha(1-t)^2\sigma^2]$.

Corporate Hedge

The optimality condition (6) adjusted for unified taxation requires the hedging policy to be:

$$h_t^* = s + \frac{1}{(1-t)^2}\frac{f-\mu_r}{\alpha\sigma_r^2 fs}. \tag{9}$$

First, the full-hedge theorem holds. Second, optimum pure hedge is independent of taxation, optimum speculative hedge has to be tax-adjusted, if there is a risk premium. If there is no risk premium, unified taxation does not alter optimum hedging volume, that is to say, $h_t^* = h^*$. Like under certainty the full-hedge case neutralizes the impact of a unified taxation scheme because marginal rate of substitution remains constant.

Comparative Statics

Result 1 (Unified taxation and hedging) Speculative demand for futures contracts, i.e., $|s - h_t^*|$, increases with the unified tax rate. Furthermore, the tax-sensitivity of futures demand increases.

Proof Let $\Delta_t = s - h_t$ denote tax-adjusted speculative demand for futures contracts. From (9) it follows that $\partial|\Delta_t^*|/\partial t > 0$ and $\partial^2|\Delta_t^*|/\partial t^2 > 0$.

Intuition is well-known and runs as follows: Increasing the unified tax rate reduces expected income but also reduces risk. Since the marginal rate of substitution of risk for return reacts negatively to an increase in taxation and the price of risk reduction is fixed corporate management is willing to take more risk, i.e. to decrease the hedge rate.

Result 2 (Tax-sensitivity and risk) The increase in the speculative demand for futures contracts due to an increase in the unified tax rate lessens when return on sales becomes more risky.

Proof From (9) we get $\partial^2|\Delta_t^*|/\partial t \partial \sigma_r < 0$.

Here, the following intuition holds: Increasing return risk holding expected income constant makes speculative demand less tax-sensitive since higher potential losses are less utility decreasing than higher potential gains are, when taxation enhances. Note that we consider a full loss-offset taxation scheme.

3.2 Differential Taxation

Let us now consider differential taxation of sales returns and futures returns. Let t_s and t_h denote the income tax rate for the return on sales and for the gains from hedge, respectively. Hence the expected after-tax income with differential taxation, μ_{t_s,t_h}, becomes $\mu_{t_s,t_h} = \mu_r[(1-t_s)s - (1-t_h)h] + (1-t_h)fh$. The variance of the after-tax income, $\sigma^2_{t_s,t_h}$, reads $\sigma^2_{t_s,t_h} = \sigma^2_r[(1-t_s)s - (1-t_h)h]^2$. Marginal rate of substitution of after-tax income risk for after-tax expected income becomes: $\alpha\mu_{t_s,t_h}\sigma_{t_s,t_h}/(1+\alpha\sigma^2_{t_s,t_h})$.

Corporate Hedge

The optimality condition (6) adjusted for differential taxation implies for optimum hedging:

$$h^*_{t_s,t_h} = s\,\frac{1-t_s}{1-t_h} + \frac{1}{(1-t_s)(1-t_h)}\,\frac{f-\mu_r}{\alpha\sigma^2_r\,fs}. \tag{10}$$

Note that there is no full-hedge theorem since $\text{sgn}(\mu_r - f) = \text{sgn}[(1-t_s)s - (1-t_h)h] \neq \text{sgn}(s-h)$. In contrast to the unified taxation scheme of equation (9) the full-hedge case does not neutralize the impact of taxation on hedging. Furthermore, both tax rates have an impact on the pure as well as the speculative part of the amount of hedging.

Comparative Statics

Result 3 (Differential taxation and hedging) Assume $t_s > t_h$. An increase in differential taxation, such that $dt_s + dt_h = 0$, implies that the tax-adjusted speculative demand for futures contracts increases with the sales tax rate.

Proof Let $\Delta_{t_s,t_h} = (1-t_s)s - (1-t_h)h$ denote the tax-adjusted speculative demand for futures contracts. It follows from (10) that

$$\frac{\Delta^*_{t_s,t_h}}{1-t_h} = s\,\frac{1-t_s}{1-t_h} - h^*_{t_s,t_h} = \frac{1}{(1-t_s)(1-t_h)}\,\Delta^*,$$

such that subject to $dt_s = -dt_h$ we obtain

$$\frac{\partial}{\partial t_s}\frac{|\Delta^*_{t_s,t_h}|}{1-t_h} = \frac{t_s - t_h}{(1-t_s)^2(1-t_h)^2}\,|\Delta^*| > 0. \tag{11}$$

Result 4 (Tax-sensitivity and risk) Assume $t_s > t_h$ and an increase in differential taxation, such that $dt_s + dt_h = 0$. Then the increase in the tax-adjusted speculative demand for futures contracts due to an increase in the sales tax rate lessens when return on sales becomes more risky.

Proof From equation (11) we get $\partial^2|\Delta^*_{t_s,t_h}|(1-t_h)^{-1}/\partial t_s\partial\sigma_r < 0.$

Since the random return on sales is perfectly negatively correlated to the random return of the hedge increasing differential taxation by increasing the positive spread between sales and hedge tax rates allows to use the intuition of the unified taxation case. Our condition $dt_s + dt_h = 0$ implies that the changes in both tax rates cancel out in a way. It, therefore, provides a straight investigation on the impact of differentiability in tax rates on corporate hedging volume and its tax sensitivity. In general, it is not the change in tax rates per se what matters but it is the change in the spread of the tax rates.

Another example that helps focussing on the tax rates' spread is the case of $t_s > t_h = 0$. If only corporate sales are taxed results 3 and 4 also hold: $\partial|\Delta^*_{t_s,t_h}|/\partial t_s = |\Delta^*|/(1-t_s)^2 > 0$ and $\partial^2|\Delta^*_{t_s,t_h}|/\partial t_s\partial\sigma_r < 0$. Hence, increasing the tax rate of corporate sales holding corporate gains from hedge untaxed produces an incentive to increase the tax-adjusted speculative futures position and decreases tax-sensitivity to risk.

Finally, note that tax-sensitivity of corporate hedging regarding expected return on sales is positive in both, the unified and differential taxation scheme. Hence, a higher return works in favor of the well-known tax effect on the tax-adjusted speculative demand for futures contracts.

4. Concluding Remarks

We have analyzed a risk averse corporate management which hedges risky return by selling futures contracts and which follows a generalized form of the Roy preference function. With unified taxation the full-hedge theorem holds. If the gains out of the hedge position are taxed differently than risky operating returns the full-hedge theorem is violated.

Most importantly, pure hedge and speculative hedge, as a function of the risk premium, have to be considered separately. A unified tax rate does not alter pure hedge, but influences the speculative position. It increases with the tax rate, where this increase lessens with risk. If tax rates differ, then the pure hedge as well as the speculative hedge depend upon both rates.

Suppose operating returns are taxed higher than hedge gains. Increasing the tax differential makes hedging less attractive under full loss-offset. But tax-sensitivity depends upon risk. The higher the riskiness of operating returns the smaller tax-sensitivity.

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