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## Dresden Discussion Paper Series in Economics



## Strategic pricing of financial options

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Dresden Discussion Paper in Economics No. 16/09

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# Strategic pricing of financial options 

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#### Abstract

: The mainstream model of option pricing is based on an exogenously given process of price movements. The implication of this assumption is that price movements are not affected by actions of market participants. However, if we assume that there are indeed impacts on the price movements it no longer possible to apply the standard pricing models. As a result we need an approach explaining interdependent actions. Game theory is in a position to offer proper olutions. This paper applies game theoretic concepts to determine option prices. Consequently, both the option price and the underlying's expiration price are endogenously determined.


JEL-Classification: G13, C72
Keywords: Game theory, Nash equilibrium, option pricing, real option

## 1 Introduction

Traditional option pricing is based on the assumption that risk management is a single person decision game. Alternatively we can say it is a game against nature. The idea behind this concept is that price movements are assumed to be governed by exogenously given event uncertainties. The price movements are independent of all actions of other decision makers. The consequence is very simple. Because the outcome is not affected by actions of the gamblers it is a simple matter of stochastic and statistic techniques to calculate the probabilities of potential outcomes. That is what Markowitz based stochastic risk management is all about.

In 1998 the hedge-fund run by Long Term Capital Management (LTCM) collapsed or near-collapsed inducing unprecedented activities to prevent a similar event from happening again. In his Financial Times article "Why risk management is not rocket science" Rene Stulz (2000) identifies reasons for the LTCM disaster (see also Stulz (2009)).

In the paper we replace the no-arbitrage equilibrium by a Nash equilibrium. To be more precise, we consider a European call option for the single period case. The two players are the seller and the buyer of the option. We are going to show how to value the call option using a binomial process according to which the underlying asset's price can take only two values at the expiration date. ${ }^{1}$

In contrast to the Cox, Ross and Rubinstein (1979) model (in the following CRR), in our model the two possible expiration date prices are not exogenously given. The seller sets the expiration prices endogenously according to his strategic goals. In other words, the seller is representing the market thus giving up the assumption of exogenously given market prices. On the opposite side of the market, the buyer is making strategic investment decisions in both the underlying and the financial option. In order to determine the outcome of the strategic interaction of the players the Nash equilibrium is adopted. The Nash equilibrium is the generally accepted concept when it comes to strategic decision making (Rubinstein (1991)).

[^0]In our approach we calculate both the current option price and the future expiration prices of the underlying asset. In the world of gambling, steered by a roulette wheel, an agent does not have any influence on the outcome. A game, defined in a broadest sense, is more than a game against nature. Briefly written, a game is any situation in which players make strategic decisions, i.e., decisions that take into account each other's actions and responses. In the light of Stultz's statement, we see clearly: strategic openness is not the same like the openness of roulette games. Stulz's critique how finance works today is essential, if we see Danielsson (2002), Jovanovic and Le Gall (2002). In one word, more insight into the real structure and the real functioning of the financial market is required.

In our study a simple option pricing game is developed and presented. If the buyer of a call option bets on what will happen and the seller of a call option decides what happens, then, without a doubt, insight in the true nature of openness (risk) is required. Given strategic behavior of both the seller and the buyer of a call option in the paper two results are presented. Firstly, we show how the seller's maximum premium is calculated. Secondly, we show that in a specific strategic setting and in the non-strategic CRR setting, the results are the same, if the objective probabilities for the sates of the world and the so-called CRR risk neutral probabilities for the sates of the world are the same.

## 2 The no-arbitrage equilibrium

A call option is a right to buy one unit of the underlying asset at the strike price at the expiration date. In economics a right is characterized by a positive price. What is the market price of the option considered? In this section we briefly review the standard approach of option pricing. There are two basic components to be taken into consideration: the concept of perfect arbitrage and the no arbitrage principle. This model is used to determine the current option price $C$. All prices of the underlying stock are assumed to be publicly known, i.e. the current stock price $S$ and the future stock price $S_{1}$.

A simple stochastic process is the binomial process. With the binomial world the stock price moves up and down over time, but the stock price can take only two outcomes at $t=1$.

$$
S_{1}=\left\{\begin{array}{l}
S^{+}=(1+u) S  \tag{1}\\
S^{-}=(1+d) S
\end{array}\right.
$$

where $u=$ percentage increase (up) of the stock price in $t=1$ with $u>0$ and $d$ percentage decrease (down) of the stock price in $t=1$ with $-1 \leq d<0$. We assume that the states are known to all participants in the market place.

Comparable to $S_{1}$ the expiration price of the call option can take two outcomes only

$$
C_{1}= \begin{cases}C^{+} & \text {if } S_{1}=S^{+}  \tag{2}\\ C^{-} & \text {if } S_{1}=S^{-}\end{cases}
$$

The expiration price (2) is the larger of zero and the difference between the stock price at the expirations date and the strike price $X$

$$
\begin{equation*}
C_{1}=\max \left\{S_{1}-X, 0\right\} \tag{3}
\end{equation*}
$$

The basic idea of perfect arbitrage portfolios is as follows: It is possible to compose a portfolio consisting of the underlying stock and a risk free bond that perfectly matches the future cash flows of the option. As a result, in a no arbitrage situation the option must have the same current price as the arbitrage portfolio. In other words, a perfect arbitrage portfolio is a com-bination of securities that perfectly replicates the future cash flows of the derivative security.

The next step is to calculate the optimal combination of the underlying stock and the risk free bond in the perfect arbitrage portfolio at $t=0$. Assuming a bond price of one the volume of the bond investment is $F$ and the risk free rate is $r$, with $u>r>0>d$. The volume of the stock investment is the product of the number of stocks $Y$ times the stock price $S$. The result is $S Y$. In our simple binomial world, the no arbitrage equilibrium holds if the following conditions are satisfied at $t=1$

$$
\begin{align*}
& S^{+} Y+(1+r) F=C^{+}  \tag{4a}\\
& S^{-} Y+(1+r) F=C^{-} \tag{4b}
\end{align*}
$$

From (4a) and (4b) we get

$$
\begin{gather*}
Y=\frac{C^{+}-C^{-}}{(u-d) S}  \tag{5}\\
F=-\frac{1}{1+r} \frac{(1+d) C^{+}-(1+u) C^{-}}{(u-d)} \tag{6}
\end{gather*}
$$

The present value of (4a) and (4b) is given by

$$
\begin{equation*}
S Y+F=C^{*} \tag{7}
\end{equation*}
$$

From (5) and (6) we calculate the current option price

$$
\begin{equation*}
C^{*}=\frac{1}{1+r}\left(q^{*} C^{+}+\left(1-q^{*}\right) C^{-}\right) \tag{8}
\end{equation*}
$$

where

$$
\begin{equation*}
q^{*}=\frac{r-d}{u-d} \tag{9}
\end{equation*}
$$

and

$$
\begin{equation*}
1-q^{*}=\frac{u-r}{u-d} \tag{10}
\end{equation*}
$$

The probabilities (9) and (10) are often called quasi-probabilities or risk neutral probabilities. For the remainder of the paper without loss of generality $S=1$ is assumed.

## 3 The non-cooperative equilibrium

This section develops the concept of strategic option pricing. We continue to consider the case of a call option for a single period horizon as analysed in section 2. However, now the option seller is setting the prices of the underlying in both states along the lines of strategic reasoning. In addition, we assign probabilities to the states up and down. The state $u$ occurs with the probability $q$ and the state $d$ occurs with the probability $1-q$. Again, the risk free rate is $r$. As an additional constraint we assume $u>r>0>d>-1$.

We consider an individual investor (the buyer) who can choose from two alternatives at $t=0$ denoted by (i) B: to buy the stock at the rate $r$ and (ii) NB: not to buy the stock.

Assuming the outcome in the state $u$ is equal to $(1+u)$ and in the state $d$ the outcome is equal to $(1+d)$ we find, buying the stock means choosing the lottery $L_{B}=[(u-r),(d-r) ; q,(1-q)]$. Alternatively, not buying results in choosing a degenerated lottery with a sure payoff of zero. We assume that the buyer is risk neutral and, consequently, the buyer is planning to maximize the expected payoff.

Now the seller enters the stage and offers the buyer to decide on buying or not buying the stock when the financial market state will be disclosed at $t=1$. This
comes to offering the buyer a call option with the strike price $X=1+r$. For the buyer holding this option means being in the situation of the lottery $L^{*}$ given by

$$
\begin{equation*}
L^{*}=[(u-r), 0 ; q,(1-q)] . \tag{11}
\end{equation*}
$$

We label $L^{*}$ the (call) option lottery. What is the maximum premium the buyer is willing to pay for this option?

First, the prices of the underlying are no more predetermined as the seller is able to set these prices. In other words, the seller decides what prices the underlying will take at the expiration date $t=1$. We assume that the set of strategies of the seller, $A$, is given by

$$
\begin{equation*}
A=\{a: \max \{r-u,|d|-1\} \leq a \leq r-d\} \tag{12}
\end{equation*}
$$

If the seller chooses any $a \in A$, then $S_{1}$, the value of the underlying at the end of the period, takes on one of the two possible values

$$
S_{1}=\left\{\begin{array}{l}
S^{+}(a)=1+(u+a)  \tag{13}\\
S^{-}(a)=1+(d+a)
\end{array}\right.
$$

For keeping our model still simple we only consider a 1-dimensional price strategy for the seller. A price change specific to each state $(u(+)$ or $d(-))$ does not come under consideration. More formally, in our model the seller only chooses $a=a^{+}=$ $a^{-}$instead of $a^{+}=a^{-}$.

There are three possible strategies: The choice of $a=0$ means choosing inactivity, which is the roulette situation. Now, the market exclusively determines the prices of the underlying in both states. ${ }^{2}$ Alternatively, $a>0$ means the choice of a price increasing strategy for the underlying. The price of the underlying will be raised in both states of the market. Finally, $a<0$ means the choice of a price reducing strategy. The price of the underlying will be cut in both states. Condition (12) prevents negative prices occurring.

By assumption, the buyer cannot observe the seller's choice. Both the buyer and the seller are assumed to be risk neutral. The seller's expected payoff depends on which alternative the buyer will choose at $t=0$ in the absence of the call option offer. The payoff to the seller is assumed to be the difference of the expected value of the call option lottery and the buyer's expected payoff given alternative

[^1]actions. The implicit assumption is that the seller is a premium maximizer. By our assumption both the seller and the buyer are making simultaneous choices, i.e. there is no different timing of actions, our model seems to be restrictive. Nevertheless, we find a set of application areas, especially in institutional settings, that are indeed characterized by simultaneous moves. As an example we mention a call option with credit default swaps (CDS) as the underlying and a strategically induced reduction of the credit quality caused by the option buyer, e.g., private equity firms. A more general example for simultaneous moves we refer to the valuation of strategic investment pattern with the real option theory.

Hence, the expected payoff to the buyer playing her (pure) strategy B (or NB) is given by

$$
\begin{gather*}
A_{2}(a, B)=q((u+a)-r)+(1-q)((d+a)-r)  \tag{14}\\
A_{2}(a, N B)=0 . \tag{15}
\end{gather*}
$$

Referring to the buyer's choice at $t=0$ and note that $E^{*}$ is the fair value of $L^{*}$ what means that the seller does not offer $L^{*}$ for a price smaller than $E^{*}$ the expected payoff to the seller is given by

$$
\begin{gather*}
A_{1}(a, B)=E^{*}-A_{2}(a, B)  \tag{16}\\
A_{1}(a, N B)=E^{*} . \tag{17}
\end{gather*}
$$

Let the expected payoff of the option lottery $L^{*}$ denoted by $E^{*}$, i.e.,

$$
\begin{equation*}
E^{*}=q((u+a)-r) \tag{18}
\end{equation*}
$$

we obtain

$$
\begin{align*}
A_{1}(a, B) & =(1-q)(-r(d+a))  \tag{4}\\
A_{1}(a, N B) & =q((u+a)-r) . \tag{5}
\end{align*}
$$

We refer to the preceding setting as the one period hypothetical option game with risk neutral agents. Next, we ask the question: What choice of $a \in A$ will be made by the seller when aiming at a maximum premium for the call option?

First, the seller will take into consideration that his choice of increasing or reducing the price of the underlying will be reflected by the action the buyer will decide upon at $t=0$ given the choice $a$. Second, we observe that in equilibrium the buyer will randomize between her two actions if

$$
\begin{equation*}
0<q<\frac{1+r}{u-d} \tag{21}
\end{equation*}
$$

holds true for the exogenously given probability $q$ at the state $u$.

To explain (21) the line of reasoning is as follows: Suppose to the contrary that the buyer in equilibrium chooses a pure strategy, that is to say, her equilibrium choice were either B or NB: (i) say, it were B. In equilibrium, the choice of the seller must be a best reply to the choice of the buyer. We conclude from (19) that the seller's choice must be $a=|d|-1$ or $a=r-u$, (ii) now say, the buyer's equilibrium choice were NB. Then, (20) tells us that the seller's best response is $a=r-d$ and (iii) however, in either case it is easy to see that the respective choice of the buyer is not an optimal answer to that choice of the price of the underlying of the seller. This, however, must be the case in equilibrium: If, for instance, the equilibrium choice of the seller was $a=|d|-1$ or $a=r-u$, then the buyer's best response is, accord-ing to (14) and (15), given by $N B$. Note that in this case, according to (14), the buyer's payoff is given by $A_{2}(|d|-1, B)=q(u-d)-(1+r)$ or $A_{2}(r-u, B)=(1-q)(d-u)$. We conclude that $A_{2}(|d|-1, B)<A_{2}(|d|-1, N B)$ or $A_{2}(r-u, B)<A_{2}(r-u, N B)$ must hold true (see (21)). Thus, neither $(|d|-1, B)$ nor $(r-u, B)$ can be an equilibrium. If, on the other hand, the equilibrium choice of the seller was $a=r-d$, then the buyer's best response is, according to (14) and (15), given by B: The payoff to the buyer at NB equals $A_{2}(r-d, N B)$. From (21) we obtain $A_{2}(r-d, N B)<A_{2}(r-d, B)$. Thus $(r-d, N B)$ cannot be an equilibrium.

As a result, we know that in equilibrium the buyer will employ a mixed strategy. Each of her alternatives must generate the same payoff to her in equilibrium, for otherwise she could do better by not randomizing and choosing a pure strategy instead, that is to say, by choosing either B or NB. Therefore in equilibrium it must hold true that ${ }^{3}$

$$
\begin{equation*}
A_{2}\left(a^{*}, B\right)=A_{2}\left(a^{*}, N B\right) \tag{22}
\end{equation*}
$$

with $a^{*}$ denoting the seller's equilibrium strategy. From this we obtain

$$
\begin{equation*}
a^{*}=r-(q u+(1-q) d) \tag{23}
\end{equation*}
$$

If the seller employs his equilibrium strategy, then the value of the underlying at the end of the period, $S_{1}$, takes on one of the following two prices (see (13))

$$
S_{1}=\left\{\begin{array}{l}
S^{+}\left(a^{*}\right)=(1+r)+(1-q)(u-d)  \tag{24}\\
S^{-}\left(a^{*}\right)=(1+r)+q(u-d)
\end{array}\right.
$$

Furthermore, from (22) we conclude that

$$
\begin{equation*}
A_{1}\left(a^{*}, B\right)=A_{1}\left(a^{*}, N B\right) \tag{25}
\end{equation*}
$$

[^2]must hold true (see (16), (17)). In particular, we have
\[

$$
\begin{equation*}
A_{1}\left(a^{*}, B\right)=E^{*}-A_{2}\left(a^{*}, N B\right), \tag{26}
\end{equation*}
$$

\]

which gives

$$
\begin{equation*}
A_{1}\left(a^{*}, B\right)=q(1-q)(u-d) . \tag{27}
\end{equation*}
$$

Discounting (27) gives the strategically determined option premium $C_{s}$

$$
\begin{equation*}
C_{s}=\frac{1}{1+r} q(1-q)(u-d) . \tag{28}
\end{equation*}
$$

Now, recall that the quasi-probability in the one period option pricing formula $q^{*}$, is given by (see (9))

$$
\begin{equation*}
q^{*}=\frac{r-d}{u-d} . \tag{29}
\end{equation*}
$$

The CRR quasi-probability of the single period case, $q^{*}$, satisfies condition (21) if the borderline case $d=-1$ is excluded. In particular, if the exogenously given probability for the state $u, q$, equals $q^{*}$, we conclude from (28) that

$$
\begin{equation*}
C_{s}=\frac{1}{1+r}\left(\frac{r-d}{u-d}(u-r)\right) \tag{30}
\end{equation*}
$$

must hold true. Furthermore, the no-arbitrage pricing formula derived in CRR entails for the single period case the option premium $C^{*}$

$$
\begin{equation*}
C^{*}=\frac{1}{1+r} q^{*}(u-r) . \tag{31}
\end{equation*}
$$

Drawing a comparison with (8), we see that $C^{+}=u-r$ and $C^{-}=0$ must be hold in addition. We obtain the result.

Proposition 1 The strategic option premium of the one period option game with risk neutral agents equals the single period no-arbitrage (CRR) premium if, and only if, the exogenously given probability for the state $u$ equals the quasiprobability in the one period option pricing formula, i.e. $C_{s}=C^{*}$ if and only if $q=q^{*}$.

## 4 Properties of the solution

The restrictions, made above, keeping in mind in this section we explain details of the solution. Some of the features are important to the understanding of our model's rationale.
(i) Recall that the buyer in equilibrium chooses a mixed strategy denoted by $Y^{*}=\left(y^{*}, 1-y^{*}\right)$. As the buyer chooses $B$ with the probability $y^{*}$ we get

$$
\begin{equation*}
y^{*}=q . \tag{32}
\end{equation*}
$$

According to $\max \{r-u,|d|-1\}<a^{*}<r-d$ the seller's equilibrium choice $a^{*}$ is an interior solution. $A_{1}\left(a, Y^{*}\right)$ denotes the expected payoff of the seller given the mixed strategy of the buyer. Then, observe that $\frac{\partial}{\partial a} A_{1}\left(a, Y^{*}\right)$ is constant and conclude from $\frac{\partial}{\partial a} A_{1}\left(a, Y^{*}\right)=0$ that (32) holds.
(ii) In case $1>q>(r+1) /(u-d)$ and $\max \{r-u,|d|-1\}=|d|-1$ hold true, $(|d|-1, B)$ constitutes a Nash equilibrium. With (19), we see that the seller's best response to $B$ is $a=|d|-1$. Then (14) and (15) tell us that $B$ is the buyer's best response to $|d|-1$ if, and only if, (33) $q[(u|d|-1)-r]+(1-q)[(d+|d|-1)-r]>0$ holds true. The latter is the case if, and only if, (34) $q>(r+1) /(u-d)$ holds true. Equation (34) in particular gives $q>q^{*}($ if $d \neq-1)$; the quasi-probability in the one period option pricing formula by CRR allows of the representation $q_{*}=(r-d) /(u-d)$. However, in general plausible values of $r, u$ and $d$ imply $(r+1) /(u-d)>1$. Hence, the necessary condition for $(|d|-1, B)$ being a Nash equilibrium of the one period hypothetical option game is not satisfied. Thus, condition (21) is not very restrictive.
(iii) In case $1>q>(r+1) /(u-d)$ holds but $\max \{r-u,|d|-1\}$ equals $r-$ $u$ (instead of $|d|-1$ ), then neither $(r-u, B)$ nor $(r-d, N B)$ can be a Nash equilibrium. Again, the equilibrium is given by $\left(a^{*}, Y^{*}\right)$ (see (23), (32)).
(iv) Given $q$ fulfils (21) we have seen, that $C_{s}=C^{*}$ holds if $q$ equals the risk neutral probability $q^{*}$ (assuming $|d|<1$ ). We distinguish the cases (a) $q^{*}>.5$ and (b) $q^{*}<.5$. In case (a) we observe that $C_{s}>C^{*}$ holds if, and only if, $1-q^{*}<q<q^{*}$ is valid. In case (b) we obtain $C_{s}>C^{*}$ if, and only if, $q^{*}<q<1-q^{*}$ is true. If $q^{*}=.5$ holds, we have $C_{s}<C^{*}$ for all $q \neq q^{*}$.
(v) Again, $q$ is satisfying (21) is assumed. Given $q=q^{*}$ we find the result $q^{*}=$ 0 . Intuitively the result can be described as follows: If the exogenously given expectation of state $u$ is identical with the CRR quasi-probability of state $u$ we find that the seller is completely passive. He is accepting the exogenously given expiration date prices without considering any change.
(vi) Like above, $q$ fulfils (21) is assumed. The expiration date prices of the underlying are $\left(1+u+a^{*}\right)$ and $\left(1+d+a^{*}\right)$ in $u, d$. Calculating the CRR quasi-probability,
$\tilde{q}^{*}$, we find $\tilde{q^{*}}=\left(r-d-a^{*}\right) /(u-d)$ (see (29)). As a result, we find the CRR premium $\tilde{C^{*}}=q(1-q)(u-d) /(1+r)$ (see (31)). Alternatively, we can say that the strategic option premium, $C_{s}$, is identical with the CRR premium taking care of the adjusted data situation (see (28)).
(vii) Finally, we would like to emphasize that the above analysis is exclusively based on the well known CRR scenario which answers the following question: What is the maximum premium the option buyer is willing to pay for the call option in a one period setting? As explained earlier in detail the strategic reformulation of the CRR model is considering an active option seller who is setting different outcomes of the states of nature. By assumption both the option buyer and the option seller make their decisions at the same point of time. Unfortunately, simultaneous decision making is not a proper description of the institutional setting of today's options markets. However, we consider our analysis as a first step in order to investigate strategic option pricing.

## 5 Conclusion

Traditional option pricing is based on the assumption that risk management is a single person decision game. Alternatively we can say it is a game against nature. The idea behind this concept is that price movements are assumed to be governed by exogenously given event uncertainties. The price movements are independent of all actions of other decision makers. A simple analogy is taken from gambling at the roulette table: Whatever bets are placed by gamblers there is never any impact on the outcome of the spin. The consequence is very simple. Because the outcome is not affected by actions of the gamblers it is a simple matter of stochastic and statistic techniques to calculate the probabilities of potential outcomes. That is what Markowitz based stochastic risk management is all about.

Of course, there are very sophisticated techniques developed in the recent past to cope with some new problems. Nevertheless the basic idea is that outcomes are predictable to a certain degree. In our paper we assume that expiration date prices are changed as a result of actions of the option sellers. Clearly the option buyers will take new expectations about these actions into consideration. As a consequence they will also change their actions. Now it is the option seller's turn to react one more time. In other words, the uncertainty is strategic in the sense of the game theoretic approach. We consider the paper as a first step in the area
of strategic pricing of derivatives securities. Further research must take different institutional settings into consideration.

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[^0]:    ${ }^{1}$ For using game theory to model option pricing from a strategic point of view see for example Rubinstein (1991), Thakor (1995) and Ziegler (2004).

[^1]:    ${ }^{2}$ Note that (1) is exactly the special case of (13) with $a=0$

[^2]:    ${ }^{3}$ See (i) in section 4 that $a^{*}$ according to (32) $y^{*}=q$ is indeed a best reply for the seller.

