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Working Paper

Liquidity constrained exporters: Trade and futures hedging

Dresden discussion paper series in economics, No. 17/09

Provided in cooperation with:
Technische Universität Dresden

Suggested citation: Broll, Udo; Wahl, Jack E. (2009) : Liquidity constrained exporters: Trade and futures hedging, Dresden discussion paper series in economics, No. 17/09, http://hdl.handle.net/10419/39802

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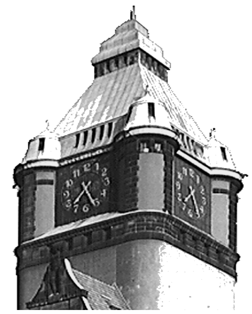

Dresden Discussion Paper Series in Economics

Liquidity constrained exporters: Trade and futures hedging

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Dresden Discussion Paper in Economics No. 17/09

ISSN 0945-4829

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Dresden Discussion Paper in Economics No. 17/09

Liquidity constrained exporters: Trade and futures hedging

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Abstract:

We present a model of risk averse exporting firm subject to liquidity constraints. The firm enters an unbiased futures market to hedge exchange rate risk and may not be able to satisfy high margin calls. Then the firm is forced to prematurely liquidate the futures position. We show that preferences and expectations become important for optimum export and hedging decisions, i.e. separation theorem and full hedge theorem are violated. Furthermore, international trade is affected, for only firms that have sufficient financial resources fully exploid gains from trade.

JEL-Classification: D81, F23, F31

Keywords: Liquidity constraint, trade, futures, hedging

1. Introduction

Recently, liquidity have become a major concern of international firms. An increasing number of articles in international trade study the relationship between the firm's liquidity constraints and export production. Different avenues of research exist: In the context of credit constraints the export decision is related to heterogeneity of international firms, to financing fixed costs to begin exporting activity, to differences in wealth endowment of firms and to equity market liberalizations (see Manova, 2008, Friberg and Wilander, 2008).

The theory of international trade under uncertainty has a long tradition (see Helpman and Razin, 1978, Grinols, 1987, Broll, Wahl and Zilcha, 1995). Under uncertainty liquidity constraints lead to interesting implications for decision making. For example, futures hedging of foreign exchange risk has to cope with an additional liquidity risk, since highly volatile futures prices may heavily move against the firm. Then management is forced to prematurely liquidate currency futures positions for liquidity reasons. We present a model of a risk averse exporting firm subject to liquidity constraints hedging foreign exchange exposure through futures contracts.

The standard futures hedging model in international trade under exchange rate uncertainty leads to the so called separation theorem and full hedge theorem (Kawai and Zilcha, 1986). Liquidity constraints for multinationals have been introduced in this framework (Lien and Wong, 2005, Meng and Wong, 2007). This paper also studies a liquidity constrained firm. We assume that management acts under a given limit of futures price upward movements. If this price threshold is exceeded management closes out the currency futures position. Our objective is to investigate on the validity of well-known results from the literature on futures hedging and international trade. We show that an underhedge may be optimal for the firm although the currency futures market is unbiased. Furthermore, overcoming financial constraints in satisfying margin calls clearly increases exports. A gradual widening of financial markets, however, may have an adverse effect on trade flows depending on the prudence aspect of preferences.

The paper is organized as follows. Section 2 lays out the model of the firm's futures hedging decision under a liquidity constraint when export production is given. We derive a 'prudent hedge theorem'. In section 3, the firm's optimal behavior regarding export production and futures hedging is analyzed under the assumption of a shortage of financial resources. If there is a positive probability that the international firm lacks sufficient liquidity then separating production from hedging is no longer optimal. A constraint on liquidity creates a risk effect upon export production and, therefore, upon international trade. Section 4 concludes.

2. Hedging model

First we focus on the hedging motive. Suppose the following hedging policy of a risk averse exporting firm: Random export revenue is hedged by futures contracts. Since the firm faces a liquidity constraint management policy calls for closing out the firm's futures position if a threshold of an unfavorable exchange rate movement is exceeded during maturity. Margin calls due to marking to market cannot be satisfied because the firm lacks sufficient liquidity.

In what follows we present some assumptions which hold throughout the paper.

Liquidity setting. The firm has access to liquidity up to the amount L_0 denoted in domestic currency. Let f_0 denote the futures rate of the futures contract and $\tilde{f}$ the random futures price during maturity of the futures contract. Hedging volume is given by H denoted in foreign currency. The firm is forced to prematurely liquidate its futures position if the liquidity constraint $(f - f_0)H \leq L_0$ is violated, where f is a realization of $\tilde{f}$. Hence, if the observed futures price exceeds a threshold during maturity, then the margin call exceeds the amount of liquidity available to the firm. Management then closes out its futures position. We assume that threshold k depends upon liquidity, i.e. $k = k(L_0)$, $k' = dk/dL_0 > 0$. In words, the more liquidity the firm has, the higher the threshold, which is denoted in foreign currency to domestic currency. Threshold k is given by risk policy of the firm.

Economic setting. $\tilde{e}_1$ defines the random foreign exchange spot rate at maturity of the futures contract. The currency futures market is called unbiased, i.e. $f_0 = E(\tilde{e}_1)$, where E denotes the mathematical expectation's operator. e_1 is the spot rate that prevails at maturity of the futures contract.

Stochastic setting. $\tilde{e}_1 = \tilde{f} + \tilde{\epsilon}$, where $\tilde{\epsilon}$ is a zero-mean random variable conditionally independent (Ingersoll 1987) of the random futures price during maturity $\tilde{f}$.

Hence the observation f of the futures price is equal to the conditional expected value of $\tilde{e}_1$, i.e. $f = E(\tilde{e}_1|f)$ for all f. It follows that $E(\tilde{f}) = E(\tilde{e}_1)$ and, therefore, $E_{f\leq k}(f_0 - \tilde{f}) + E_{f>k}(f_0 - \tilde{f}) = f_0 - E(\tilde{e}_1)$. With an unbiased currency futures market we get $E_{f\leq k}(f_0 - \tilde{f}) = -E_{f>k}(f_0 - \tilde{f})$.

Time setting. The timing of events is shown in figure 1. For no-arbitrage reasons realizations of $\tilde{f}_1$ and $\tilde{e}_1$ must equate at the end of maturity of the futures contract, i.e. $f_1 = e_1$.

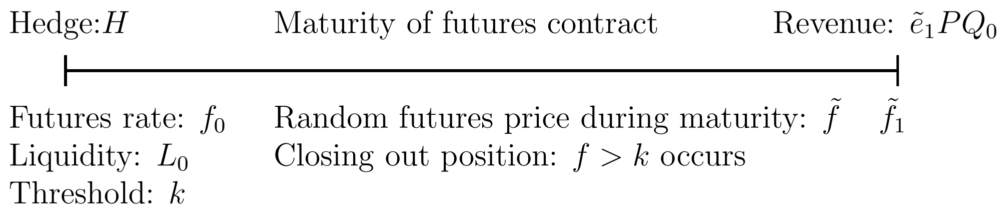

Figure 1: Time schedule

2.1 Short hedge position

Suppose an unfavorable exchange rate movement forces an exporting firm to closing out its futures position prematurely. Is there any incentive for the firm to hedge exchange rate risk if this movement occurs with some positive probability?

To examine the hedging motive we first suppose that export production Q_0 is fixed when the firm enters the futures market. The exporting firm's random profit at maturity of the futures contract is defined by:

$$\tilde{\Pi} = \tilde{e}_1 PQ_0 + (f_0 - \tilde{g})H, \tag{1}$$

where

$$\tilde{g} = \begin{cases} \tilde{e}_1, & \text{if} \quad f \leq k, \\ \tilde{f}, & \text{if} \quad f > k, \end{cases}$$

where k is the exogenously given foreign exchange futures rate threshold of the exporting firm. This threshold accounts for the liquidity constraint of the firm to carry out margin calls of the futures contract.

The firm chooses the level of its futures position, H, so as to maximize expected utility of random profits:

$$\max_H EU(\tilde{\Pi}), \tag{2}$$

where $\tilde{\Pi}$ is defined in equation (1).

Let us reformulate expected utility in order to account for the liquidity constraint:

$$\begin{aligned} EU(\tilde{\Pi}) &= E_{f\leq k}U(\tilde{e}_1 PQ_0 + (f_0 - \tilde{e}_1)H) + \\ &\quad E_{f>k}U(\tilde{e}_1 PQ_0 + (f_0 - \tilde{f})H) \\ &= E_{f\leq k}E_{\epsilon}[U((\tilde{f}+\tilde{\epsilon})PQ_0 + (f_0 - (\tilde{f}+\tilde{\epsilon}))H)] + \\ &\quad E_{f>k}E_{\epsilon}[U((\tilde{f}+\tilde{\epsilon})PQ_0 + (f_0 - \tilde{f})H)]. \end{aligned} \tag{3}$$

Note that the domain $f \leq k$ implies $\tilde{\Pi} = (\tilde{f}+\tilde{\epsilon})(PQ_0 - H) + f_0 H \equiv \tilde{\Pi}_d$. However, the domain $f > k$ requires $\tilde{\Pi} = \tilde{f}(PQ_0 - H) + f_0 H + \tilde{\epsilon} PQ_0 \equiv \tilde{\Pi}_u$. We observe: $\tilde{\Pi}_u = \tilde{\Pi}_d + \tilde{\epsilon} H$. The difference in random profits comes from an additional endogenous risk which has other implications than the so called background or idiosyncratic risk (see, e.g., Franke et al. 2004).

Using expected utility from equation (3) in decision problem (2), the first-order condition which the futures position H has to satisfy in the optimum is:

$$E_{f\leq k}E_{\epsilon}[U'(\tilde{\Pi}^*)(f_0 - \tilde{e}_1)] + E_{f>k}E_{\epsilon}[U'(\tilde{\Pi}^*)(f_0 - \tilde{f})] = 0, \tag{4}$$

where E_{ϵ} is the expectation operator with respect to the probability density function of ϵ, U' denotes marginal utility of profit and an asterik indicates an optimum level. Due to risk aversion marginal utility is decreasing, i.e. $U'' < 0$.

In the following we demonstrate that although the firm is facing a liquidity constraint it nevertheless has an incentive to enter the futures market. Hence premature liquidation of the futures position during maturity does not imply that hedging policy becomes obsolete. Proposition 1 proves that the firm's optimum hedging amount is strictly positive.

Proposition 1. *Liquidity constrained exporters hedge, i.e.* $H^* > 0$.

Closing out its futures position prior to the contractual delivery date due to a variation margin to high to be financed does not destroy the exporting firm's incentive to hedge ex ante. The firm optimally chooses a short position in currency futures. This holds in general for all risk averse preferences.

Proof. We evaluate the first-order condition (4) at $H = 0$ (Mossin 1973,

p. 37):

$$\left.\frac{\partial EU(\tilde{\Pi})}{\partial H}\right|_{H=0} = E[U'(\tilde{e}_1 P Q_0)(f_0 - \tilde{e}_1)] = -\text{cov}(U'(\tilde{e}_1 P Q_0), \tilde{e}_1)),$$

which is strictly positive. Q.E.D.

Note that a vanishing hedging position, i.e. $H = 0$, drives the threshold to infinity and a liquidity problem cannot occur. Hence the random payoff from a short position in the futures contract on one currency unit is $f_0 - \tilde{e}_1$. On the other hand, notice that it is futures hedging and not international trade that causes liquidity risk. Sufficient liquidity is needed because of specific institutional rules in futures markets like margin requirements.

Now we examine the optimal hedging decision.

2.2 Optimum hedge ratio

The exporter's optimal risk management policy depends upon his preferences. Although the currency futures is unbiased, a unit hedge ratio only occurs under specific preferences. Most important is the shape of marginal utility.

Consider a full hedge. A unit hedge ratio cannot mitigate all profit risk, since residual risk emerges from prematurely liquidating the futures position during maturity. In what follows we show that it depends upon the sign of the prudence measure $U'''/(-U'')$ (Kimball 1993) how liquidity risk affects the firm's hedging position. U''' denotes the third derivative of the utility function with respect to profit and exhibits the convexity of marginal utility.

Proposition 2. (Prudent hedge theorem) *The risk averse exporter underhedges (fully hedges) [overhedges] random export revenue, if and only if prudence is positive (zero) [negative].*

Remark. Prudence is positive (zero) [negative] if and only if $U''' > (=)[<]0$. Suppose positive prudence, i.e. $U''' > 0$. Then the exporter is more sensitive to low profits than to high ones. In other words, he has an incentive to avoid low profit realizations, which occur when high margin calls emerge.

Proof. We evaluate the first-order condition (4) at $H = PQ_0$, which is called a full hedge of random export revenue:

$$\begin{aligned}
\left.\frac{\partial EU(\tilde{\Pi})}{\partial H}\right|_{H=PQ_0} &= E_{f\leq k}E_\epsilon[U'(f_0PQ_0)(f_0-(\tilde{f}+\tilde{\epsilon}))] + \\
&\quad E_{f>k}E_\epsilon[U'((f_0+\tilde{\epsilon})PQ_0)](f_0-\tilde{f}) \\
&= E_{f\leq k}(f_0-\tilde{f})U'(f_0PQ_0) + \\
&\quad E_{f>k}(f_0-\tilde{f})E_\epsilon[U'(f_0PQ_0+\tilde{\epsilon}PQ_0)] \\
&= E_{f>k}(f_0-\tilde{f})\{E_\epsilon[U'(f_0PQ_0+\tilde{\epsilon}PQ_0)] - U'(f_0PQ_0)\}.
\end{aligned}$$

Due to $L_0 > 0$ we observe that $f_0 < k < f$. Hence $E_{f>k}(f_0 - \tilde{f}) < 0$. Therefore,

$$\begin{aligned}
\operatorname{sgn}\left(\left.\frac{\partial EU(\tilde{\Pi})}{\partial H}\right|_{H=PQ_0}\right) &= \operatorname{sgn}\{U'(f_0PQ_0) - E_\epsilon[U'(f_0PQ_0+\tilde{\epsilon}PQ_0)]\} \\
&= -\operatorname{sgn} U'''
\end{aligned}$$

by Jensen's inquality. This implies for optimum hedge:

$$\operatorname{sgn}(PQ_0 - H^*) = \operatorname{sgn} U''',$$

since expected utility is monotone in the amount of hedging. Q.E.D.

Though the futures exchange rate f_0 is perceived as unbiased by the exporter the full hedge theorem does not hold. Liquidity risk creates an additional profit risk. This risk is endogenous because its magnitude depends upon the hedging amount. A unit hedge ratio, which is optimal for quadratic utility (i.e. $U''' = 0$), reveals this fact: $\operatorname{var}(\tilde{\Pi}_d) = 0$, whereas $\operatorname{var}(\tilde{\Pi}_u) = \operatorname{var}(\tilde{\epsilon})(H^*)^2$ is strictly positive. (For the definitions, see section 2.1.) Hence a unit hedge ratio does not assure riskless export profits.

Corollary *(i) If the utility function exhibits nonincreasing absolute risk aversion, then the exporting firm underhedges its foreign exchange exposure, i.e.* $H^* < PQ_0$. *(ii) A quadratic utility function implies a unit hedge ratio, i.e.* $H^* = PQ_0$.

Proof. Absolute risk aversion is measured by $-U''/U'$. (i) $(-U''/U')' \leq 0$ implies $U''' > 0$. (ii) U' is a linear function of export profit. Therefore, $U''' = 0$. Q.E.D.

Overhedging export revenue, i.e. $H^* > PQ_0$, can only be optimal, if the exporter's utility function exhibits increasing absolute risk aversion.

Without any liquidity problems of the exporting firm our economic setting implies full hedging as optimum hedging policy. The economic intuition for an optimal underhedge in case of a liquidity constraint is as follows. Introducing some positive probability of premature liquidation of the futures position creates an additional risk. Then, even in an unbiased currency futures market the beforehand riskless profit becomes risky. To regain its maximal expected utility, at least partially, the exporter reduces the hedging volume in order to lessen the loss in expected profit when closing out the position. In other words, the threshold level increases and, therefore, the probability of premature liquidation decreases.

A smaller hedge ratio means that some diversification is lost. This reasoning is captured by the substitution effect between return and risk. But there is also an income (wealth) effect. With nonincreasing absolute risk aversion the wealth effect strengthens the substitution effect leaving the firm with an hedge ratio less than unity.

3. Export production and hedging

In the following sections export production is endogeous. We demonstrate that under our liquidity constraint the so-called separation theorem is no longer valid.

The cost function of export production Q is denoted by $C(Q)$ and exhibits the usual properties: marginal cost C' is strictly positive and increasing, $C'' > 0$.

3.1 No separation

The exporting firm's random profits read:

$$\tilde{\Pi} = \tilde{e}_1 PQ - C(Q) + (f_0 - \tilde{g})H, \tag{5}$$

where $\tilde{g}$ is defined as before in equation (1).

The exporter maximizes expected utility of profits $EU(\tilde{\Pi})$ by choosing Q and H, and $\tilde{\Pi}$ is defined in equation (5).

The first-order conditions for an optimum are:

$$E[U'(\tilde{\Pi}^*)(\tilde{e}_1 P - C'(Q^*))] = 0, \tag{6}$$

$$E[U'(\tilde{\Pi}^*)(f_0 - \tilde{g})] = 0. \tag{7}$$

Proposition 3. *If the exporting firm faces a liquidity constraint to satisfy margin calls from its futures position then the firm's export production decision cannot be separated from the firm's hedging policy and depends upon risk attitude and expectations.*

Proof. Multiply equation (7) by P and add the result to equation (6). Rearranging terms and using the definition $\hat{U}'(\Pi) \equiv U'(\Pi)/EU'(\tilde{\Pi})$ we obtain:

$$C'(Q^*) = f_0 P + E[\hat{U}'(\tilde{\Pi}^*)(\tilde{e}_1 - \tilde{g})]P. \tag{8}$$

The second term on the RHS violates the separation theorem. Q.E.D.

The intuition of this result is that the futures contract does not exactly mimic the foreign exchange exposure. In other words, random export revenue, i.e. $\tilde{e}_1 P Q^*$, is imperfectly correlated to the random gain of the futures position, i.e. $(f_0 - \tilde{g})H$. Hence, in principle, a perfect hedge is excluded.

3.2 Risk effect on trade

How does the necessity to closing out the firm's hedging position because of liquidity problems affect the exporter's optimum export production? Let us compare this scenario with the case in which there is no liquidity constraint.

Proposition 4. *Introducing a liquidity constraint induces the exporting firm to reduce export production.*

Proof. (i) From equation (8), the definition of $\tilde{g}$ in equation (1) and the stochastic setting $\tilde{e}_1 = \tilde{f} + \tilde{\epsilon}$, where $E(\tilde{\epsilon}|f) = 0$, we get:

$$E[U'(\tilde{\Pi}^*)(\tilde{e}_1 - \tilde{g})] = E_{f>k}\text{cov}_\epsilon[U'(\tilde{\Pi}^*), \tilde{\epsilon}] < 0, \tag{9}$$

since the covariance term is negative. Hence from equation (8) the inequality $C'(Q^*) < f_0 P$ holds. (ii) Let Q^{**} denote optimum export production when the liquidity constraint is not binding. Then $C'(Q^{**}) = f_0 P$. (iii) The convexity of the cost function then implies: $Q^{**} > Q^*$. Q.E.D.

The negative covariance in equation (9) reveales a risk effect: covariance is negative for the whole domain $f > k$, which describes that the liquidity constraint is binding. In this domain profit is increasing in ϵ, the deviation between the random future spot rate at the end of maturity and the random futures price during maturity (see figure 1). Last but not least marginal utility is (generally) decreasing in profit. Taking the partial expected value from negative covariances results in a negative magnitude.

Proposition 5. *Suppose prudence is nonpositive, i.e.* $U''' \leq 0$*. If liquidity constraints become less severe, then optimum export production increases.*

Remark. Common preferences in the literature of Economics and Finance, for example CARA, DARA and CRRA (see, e.g., Battermann, Broll and Wahl, 2008), imply a positive prudence, i.e. $U''' > 0$. In this case a gradual deepening and widening of financial markets in order to facilitate the acquisition of liquidity may have an adverse effect on international trade.
Proof. See appendix.

To sum up, the occurance of liquidity constraints for exporting firms is negatively affecting internationale trade. Real transactions in the open economy are impeded. Therefore, some gains from trade may be lost.

4. Conclusions

In this paper we have studied the behavior of a competitive risk averse exporting firm (exporter) under exchange rate risk. In order to hedge exchange rate risk exposure, the firm has access to an unbiased currency futures market. Futures contracts are marked-to-market and thus require interim cash settlement of gains and losses of the futures position. Margin calls, however, may require tremendous financial resources if the futures price is very volatile.

In the presence of liquidity constraints the exporting firm may be forced to closing out its futures position before maturity. A liquidity risk occurs and destroys well-known results from the literature on futures hedging and international trade.

First, the full hedge theorem is no longer valid. Still the firm has an incentive to hedge. But futures hedging cannot perfectly scope with profit risk coming from liquidity risk. The exporter may want to avoid that liquidity supply fails which occur when high margin calls have to be satisfied. It depends on prudence how the exporter weights low realizations of profits when the firm's futures position must be liquidated. We show that if the utility function exhibits positive prudence then the firm's optimum hedge ratio is less than unity despite of an unbiased futures market. For less hedging means lower margin calls when the futures price moves against the firm.

Second, the separation theorem is violated. Expectations and preferences also determine export production. Premature liquidation impedes the ability of the futures contract to mimic the random cash flow from exports. Hence hedging policy cannot alone take care of profit risk. Optimum production and hedging must be find simultaneously.

Third, we show that a shortage of financial resources hinders international trade. When a liquidity constraint becomes binding the exporting firm undoubtedly reduces export production. The important economic policy question, whether or not a gradual deepening and widening of financial markets has a positive impact on trade flows (as Chaney 2005 suggests in a certainty trade model), has been answered in our paper. Under uncertainty Chaney's result may not be true and heavily depends upon preferences. This may explain why empirical research on this matter is inconclusive.

Appendix: Proof of proposition 5

We start from equation (8). Rearranging terms we get:

$$\begin{aligned} 0 &= (f_0 P - C'(Q^*))EU'(\tilde{\Pi}^*) + E_{f>k}E_\epsilon[U'(\tilde{\Pi}^*)\tilde{\epsilon}]P \\ &\equiv F(L_0, Q^*(L_0), H^*(Q^*(L_0))). \end{aligned} \tag{10}$$

Taking the total derivative of equation (10) leads to:

$$\frac{\partial F}{\partial L_0} + \frac{dQ^*}{dL_0}\left(\frac{\partial F}{\partial Q^*} + \frac{\partial F}{\partial H^*}\frac{dH^*}{dQ^*}\right) = 0.$$

Since the term in brackets is negative we obtain:

$$\text{sgn}\frac{dQ^*}{dL_0} = \text{sgn}\frac{\partial F}{\partial L_0}. \tag{11}$$

Partially differentiating equation (10) with respect to L_0 yields:

$$\frac{\partial F}{\partial L_0} = (f_0 P - C'(Q^*))\frac{\partial}{\partial L_0}EU'(\tilde{\Pi}^*) + \frac{\partial}{\partial L_0}\int_k^\infty E_\epsilon[U'(\tilde{\Pi}^*)\tilde{\epsilon}]Pd\Phi(f), \tag{12}$$

where $\Phi(f)$ denotes the cumulative density function of the random futures price.
(i) $f_0 P - C'(Q^*) > 0$, from the proof of proposition 4, step (i).

(ii) $\frac{\partial}{\partial L_0}\int_k^\infty E_\epsilon[U'(\tilde{\Pi}^*)\tilde{\epsilon}]d\Phi(f) = -\text{cov}_\epsilon[U'(\tilde{\Pi}_u^*),\tilde{\epsilon}|f=k]\,\text{Prob}(\tilde{f}=k)\,k' > 0$, since Π_u (see section 2.1) is increasing in ϵ, marginal utility of profits is decreasing and $k' > 0$.
(iii) $\frac{\partial}{\partial L_0}EU'(\tilde{\Pi}^*) = \frac{\partial}{\partial L_0}\int_0^k E_\epsilon[U'(\tilde{\Pi}_d^*)]d\Phi(f) + \frac{\partial}{\partial L_0}\int_k^\infty E_\epsilon[U'(\tilde{\Pi}_u^*)]d\Phi(f) = E_\epsilon[U'(\tilde{\Pi}_d^*) - U'(\tilde{\Pi}_u^*)|f=k]\,\text{Prob}(\tilde{f}=k)\,k' > (=)[<]\,0$ by Jensen's inequality, if and only if U' is concave (linear) [convex] in profit, i.e. $U''' < (=)[>]\,0$. The reason for this result is that random profits $\tilde{\Pi}_u$ are more volatile than random profits $\tilde{\Pi}_d$.
Combining the results from step (i), step (ii) and step (iii) for the case of non-positive prudence ($U''' \leq 0$) from equation (12) we get $\frac{\partial F}{\partial L_0} > 0$. Therefore, from equation (11) we have the comparative static result $\frac{dQ^*}{dL_0} > 0$. Q.E.D.

Remark. $U''' = 0$ is implied by quadratic utility. For positive prudence ($U''' > 0$) the comparative static analysis is inconclusive. If positive prudence is strong enough ($U''' >> 0$), we may have an adverse risk effect, i.e. $\frac{dQ^*}{dL_0} < 0$. Hence optimum export production is not monotonically increasing or decreasing in liquidity.

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