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Working Paper

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Würzburg economic papers, No. 48

Provided in cooperation with:
Julius-Maximilians-Universität Würzburg

Suggested citation: Jokisch, Sabine; Halder, Gitte; Fehr, Hans (2004) : A Simulation Model for the Demographic Transition in Germany : Data Requirements, Model Structure and Calibration, Würzburg economic papers, No. 48, http://hdl.handle.net/10419/22341

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Würzburg Economic Papers

No. 46

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February 2004

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A Simulation Model for the Demographic Transition in Germany*

Data Requirements, Model Structure and Calibration

by

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February 2004

Abstract

All countries in the European Union stand at the fore of a phenomenal demographic transition. Especially Germany will realize an enormous aging of its population. The reasons for this development are twofold: On the one hand, the number of elderly will more than double over the coming decades. On the other hand, since fertility rates are projected to stay at a low level, the number of workers available to pay the elderly their government-guaranteed pension and health care benefits will decline. Due to very generous social security systems this aging process is expected to put enormous pressure on future government expenses. To address the consequences of population aging in Germany, this paper develops a dynamic, intergenerational demographic life-cycle model. The model features immigration, age-specific fertility, life span extension and life span uncertainty. Cohorts within the model differ in their human capital profiles and leave bequests arising from incomplete annuitization. We also incorporate the German pension, health care and long-term care system. After introducing the theoretical model, we simulate the transition path including reforms of the pension system imposed by the so called „Riester" reform and keeping current immigration constant. The results are presented for the case of a closed and a small open economy.

JEL classification: D58, H55,J11

Keywords: Demographic transition, overlapping generations (OLG), computable general equilibrium models (CGE)

*Research support by the Deutsche Forschungsgemeinschaft (grant: FE 377/4-1) is gratefully acknowledged. We would also like to thank Hermann Voit and Constanze Quiatowski from the Federal Statistical Office of Germany (Statistisches Bundesamt) for providing population data, Bernd Raffelhüschen for providing data on health care, long-term care and education.

List of Symbols

Indices

i, t, z	index for years during the transition
a, j, u, v	index for age of individuals
s	index for parents' age at time of birth or age at time of immigration
k	income class index
l	index for legal status (native: n, immigrant: m)

Population

$N(a, i, s, k)$	number of natives of age a in year i whose parents were age s when they were born and who belong to income class k
$M(a, i, s, k)$	number of immigrants of age a in year i who arrived at age s and who belong to income class k
$N(a, i, 22, k)$	total number of natives of age a in year i who belong to income class k
$M(a, i, 20, k)$	total number of immigrants of age a in year i who belong to income class k
$\bar{N}(a, i)$	total number of natives of age a in year i
$\bar{M}(a, i)$	total number of foreigners of age a in year i
$M^n(a, i)$	number of net-immigrants of age a in year i
$POP(i)$	total population in year i
$\Upsilon(i)$	share of immigrant children who become natives in year i
$\xi(k, i)$	income class k share in year i
$KID(a, i, k)$	number of children of a household who is a years old in year i and who belongs to income class k
$\bar{d}(a, i, k)$	unconditional death probability of an income class k agent who is a years old in year i
$d(a, i, k)$	conditional death probability
$P(a, i, k)$	survival probability of an income class k agent who is a years old in year i
$f(a, i, k)$	fertility rate of an income class k agent who is a years old in year i
$TFR(i, k)$	total fertility rate of income class k in year i
$ABA(i, k)$	average birth-giving age of income class k in year i
$LE(i, k)$	life expectancy of income class k in year i
$\eta(i)$	endogenous population growth rate in year i
$\bar{\eta}$	exogenous population growth rate after year 2050

Household sector

$U(j, t, s, k, l)$	utility function of an income class k agent at age j in year i and whose parents were age s at birth (if $l = n$) or who immigrated at age s (if $l = m$)
$V(j, t, s, k, l)$	utility an agent derives from his own consumption
$H(j, t, s, k, l)$	utility an agent derives from his children's consumption
$c(a, i, s, k, l)$	agent's consumption of goods at age a
$c_K(a, i, s, k, l)$	consumption per agent's child
$\ell(a, i, s, k, l)$	leisure consumption of an agent
$W(a, i, s, k, l)$	gross labor income of an agent

a(a, i, s, k, l)	assets of agent
$h(a, i)$	time endowment at age a in year i
$E(a, k)$	human capital profile of an agent at age a in income class k
$I(a, i, s, k, l)$	inheritance of an agent age a in year i
$T(a, i, s, k, l)$	net tax payments of agent age a in year i
θ	pure rate of time preference
γ	intertemporal elasticity of substitution
ρ	intratemporal elasticity of substitution
α	leisure preference parameter
$\kappa_0^k, \kappa_1^k, \kappa_2^k$	parameters of the human capital profile of natives and migrants
λ	rate of technological growth

Production sector

$F(\cdot)$	production function for gross output
$F_{K(i)}$	marginal product of capital in year i
$F_{L(i)}$	marginal product of labor in year i
ϕ	technology parameter
ε	capital share in production
ψ	adjustment cost coefficient

Government sector

$\bar{\tau}^w(a, i, s, k, l)$	average (individual) labor income tax rate in year i
$\tau^r(i)$,	marginal capital income tax rate in year i
$\bar{\tau}^r(a, i, s, k, l)$	average (individual) capital income tax rate in year i
$\tau^c(i), \tau^b(i)$	consumption and inheritance tax rate in year i
$\tau^p(a, i, s, k, l)$,	marginal (individual) payroll tax rate in year i
$\bar{\tau}^p(a, i, s, k, l)$	average (individual) payroll tax rate in year i
$\tau^h(i, k)$	(individual) public health care contribution rate in year i
$\tau^{lc}(i, k)$	(individual) public long-term care contribution rate in year i
$\hat{\tau}^p(i), \hat{\tau}^h(i), \hat{\tau}^{lc}(i)$	aggregate pension, health and long-term care contribution rate in year i
$z^h(a, i, k)$	(individual) lump-sum contributions to the private health care system in year i
$z^{lc}(i, k)$	(individual) lump-sum contributions to the private long-term care system
$\hat{z}^h(i), \hat{z}^{lc}(i)$	aggregate private health and long-term care contributions in year i
$\bar{\tau}^{ss}(a, i, s, k, l)$	total (average) individual social security contribution rate in year i
$\tau^{pp}(i)$	fictitious contribution rate to private pensions in year i
$T^G(i)$	aggregate tax and social security contribution revenue in year i
$G(i), g$	total and per capita public consumption in year i
$edu(a)$	age-specific profile of education costs for children
$B(i), b(i)$	stock of public debt and in relation to GDP in year i
$\Delta B(i)$	government deficit in year i
$Tr^k(a, i, k)$	child-related transfers of an agent at age a who belongs to income class k in year i
$Tr(i)$	aggregate child related transfers in year i

$tc(i)$	transfer per child in year i
$zvE(a,i,s,k,l)$	individual taxable income
$Aic(a,i,s,k,l)$	(individual) allowances for income-related expenses
$Aip(a,i,s,k,l)$	(individual) allowances for expenses of a provident nature
$Ais(i)$	savings allowances in year i
$\beta(i)$	share parameter for income-related individual allowances
$IP(\cdot), AD(\cdot),$ $BMA(\cdot), HMA(\cdot)$	variables in order to compute individual insurance allowances
$Pen(a,i,s,k,l)$	pension benefit at age a in year i
$\varrho_1(i), \varrho_2(i), \varrho_3(i)$	policy variables of the pension system
$\chi(z)$	interest income fraction of pensioners who retired in year z
$PB(i)$	aggregate pension benefits in year i
$\bar{a}(z,k)$	individual retirement age of income class k in retirement year z
$\tilde{a}(z)$	statutory retirement age set by government in retirement year z
$\bar{W}(i)$	average annual gross income of households in year i
$AF(z,k)$	adjustment factor in income class k for retirement age in year z of the pension function
$APV(i)$	actual pension value in year i
$SEP(z,s,k,l)$	sum of earning points of an agent who retires in year z
$EP(a,i,s,k,l)$	individual (normal) earning points of agent for labor income in year i
$EP^c(a,i,s,k,l)$	individual (child-related) earning points of agent for labor income in year i
$EP^f(a,i,s,k)$	individual (foreign) earning point of agent for foreign labor income in year i
$KID2(a,i,k)$	number of children below age 10 in an income class k household of age a in year i
$BBG(i)$	contribution ceiling for pension income in year i
$S^h(i), S^{lc}(i)$	social security contribution payments of the pension system in year i
$PY^p(i), PY^h(i)$	contribution base for the pension and health care system
$\varphi(a,i)$	factor for tax-benefit linkage in age a and year i
ω	parameter for tax-benefit linkage
$HB^g(i)$	aggregate public health benefits in year i
$HB^p(i)$	aggregate private health benefits in year i
$HB(i)$,	aggregate total health benefits in year i
$LCB(i)$,	aggregate total long-term care benefits in year i
$LCB^g(i)$	aggregate public long-term care benefits in year i
$LCB^p(i)$	aggregate private long-term care benefits in year i
$\mu(i)$	fraction of pension outlays financed by general taxes
$hc(a,i)$	age-specific health costs in year i
$lc(a,i)$	age-specific profile of long-term care costs
$AP(i)$	asset holdings of the public long-term care system in year i

Prices

$w(i)$	employer's wage rate in year i
$r(i)$	interest rate in year i
$q(i)$	shadow price of capital in year i
$R(i,t)$	compound interest rate

Aggregate variables

$Y(i)$	firm's marketable output in year i
$L(i)$	aggregate labor supply in year i
$K(i)$	capital stock in year i
$C(i)$	aggregate consumption in year i
$\Delta K(i)$	investment outlays in year i
$A(i)$	aggregate savings in year i
$\bar{A}(a,i,k)$	aggregate savings of age a agents in year i and income class k
$\overline{Pen}(i,k)$	aggregate pensions of income class k in year i
$TB(i)$	trade balance in year i
$B^f(i)$	stock of net foreign assets in year i
$DIV(i)$	dividend payments at the end of year i

I. Introduction

The present paper aims to describe in detail the population projections and the structure of the simulation model which is applied to analyze the economic effects of the demographic transition in Germany.

Our model follows the overlapping generation tradition of Auerbach and Kotlikoff (1987). Recent studies, such as Beetsma et al. (2003), Fehr (2000) or Kotlikoff et al. (2001) have extended this approach by introducing a demographic transition and disaggregate various income classes within each cohort. Fehr et al. (2003) introduce a multi-country model with a very detailed structure of the demographic process. Consequently, it allows for age-dependent fertility rates, accounts for immigration and includes unintended bequests. The present model builds on this previous work but focuses on Germany. This single-country-model includes a detailed structure of the German tax and social security system. It also introduces some additional features which extend the approach of Fehr et al. (2003). On the one side, since immigrants are not forced to arrive with the same asset endowments as natives of the same generation, the present model distinguishes explicitly between natives and immigrants within each generation and income class. On the other side, we include income-class-specific fertility and mortality rates. Consequently, the present model also allows for income-class-specific life expectancy and a changing income class structure of the population in the long run.

In the following, we describe our approach in detail. The first section discusses the population model, the underlying assumptions and the baseline population projections for Germany. Then we describe the structure of the simulation model. Finally, we explain the solution procedure, the calibration and some key characteristics of the baseline path.

II. Modelling Population Dynamics

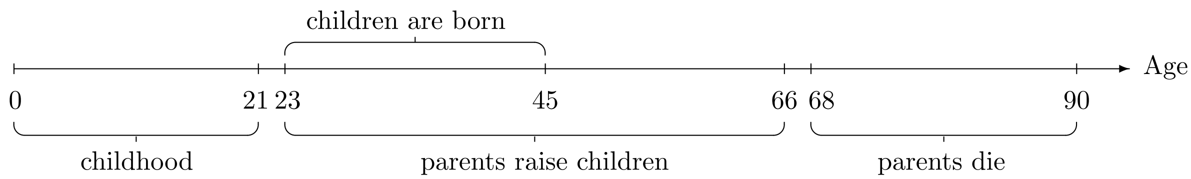
This chapter discusses our population model. We start with a description of the general model structure. Then we explain the raw data sources and necessary adjustments in the year 2000. Section 3 describes the population projections between 2000 and 2050 and section 4 the years after 2050. Finally, we compare our projections with the official ones.

1. Basic Structure

In each year i our model distinguishes households according to their age, their legal status, the age of their parents at birth or the age at immigration and finally their income class.

Households live up to a maximum age of 90. Consequently, we distinguish up to 91 generations within each period i. The individual life-cycle of a representative agent is described in Figure 1. Between age 0 and 20 our households are children, who earn no money and are fed by their parents. At age 21 our agents leave their parents and start working. Between ages 23 and 45 our agents give birth to children at the beginning of each period, i.e. children are age 0 when the parents are 23 and age 20 when the parents are 43. Between ages 46 and 66 our agents continue to raise their children. The last children who were born to age 45 parents leave their parents when the latter are age 66. Our agents die between ages 68 and 90. The probability of death is one at age 91. Consequently, the youngest child (born when the parents were 45) of parents who die early at age 68 has already reached adulthood while the oldest child (born when the parents were 23) of parents who die at age 91 is 68, i.e. parents always outlive grandparents.

Figure 1: The individual life-cycle



We distinguish between the native population and foreigners. In each year, new immigrants between ages 21 and 43 arrive with their children[1]. Adult immigrants can not change their nationality whereas newborn children of immigrants become automatically natives. Children between age 1 and 20 who immigrate with their parents become natives with an exogenously set probability $\Upsilon(i)$ which might change over the years due to policy reforms. When reaching adulthood at age 21, foreigners will not change their nationality until their death. Foreigners have identical life-cycle characteristics as natives of the respective income class, i.e. they have the same mortality and fertility probabilities. The economic differences between natives and immigrants are twofold: First, while adult natives receive inheritances, adult foreigners do not. Consequently, we have to disaggregate each native cohort according to the age of their parents at birth. Second, immigrants arrive without any assets when they enter the country. Therefore, we have to distinguish each foreigner cohort according to its age at arrival.

Finally, we distinguish three income classes within each native and foreigner cohort. The income class is identified by a specific human capital endowment which determines the individual wage level. Households of different income classes have the same preferences, but they might differ according to their demographic characteristics, i.e. life expectancy increases with income class and fertility might fall (or rise) with the income level.

The next section describes our data set for the benchmark population in the initial year 2002.

2. Benchmark Population in the Year 2002

While we choose the year 2002 as the initial year for our economic simulations, the population model starts in 2000 due to better availability of population data.[2] Starting point of our demographic projection is the population data of Germany in the year 2000 which was provided by the Federal Statistical Office of Germany (*Statistisches Bundesamt*). Given this population vector, the population of each following year is projected by applying age-specific fertility,

[1] That means, the oldest immigrants arrive with a 20-year old child.

[2] As a consequence, the base year population of 2002 is a projected one. Hence, the transition from the original to the artificially generated population (i.e. 2000/2001) lies outside the model´s simulation path.

mortality and immigration rates. However, the original population data was not available in the same detail as required by the economic model while at the same time the specific structure of the population model imposed certain restrictions on the data set (i.e. certain death at age 91). Consequently, various adjustments had to be made to the raw data which are explained in the following.

The Federal Statistical Office of Germany provided a vector of the total population by single-age groups, the foreign population $\bar{M}(a, 2000)$ and the net-immigrants $M^n(a, 2000)$ for the year 2000 in Germany. The native population $\bar{N}(a, 2000)$ of year 2000 is then computed by subtracting the foreign population from the total population number. Agents who are older than 90 in the original data are erased. Next, net-immigrants in the year 2000 who are older than 43 were erased and added proportionally to younger immigrants. Finally, the newborns of natives and foreigners are calculated with the age-specific fertility rates $f(a, 2000, k)$ for the year 2000, which denotes the average number of births of an agent age a in income class k. As already explained above, natives and foreigners are identical in fertility and the newborns of foreigners automatically become natives.

Since the model only allows to bear children between age 23 and 45, the original age-specific German fertility rates of year 2000 have to be adjusted by deleting the children of 15- to 22-year old mothers and assigning these children to older women. Given the birth rates for the years 2000 and 2050, the birth rates between these years have been computed by linear interpolation. Table A-2 reports the adjusted numbers of newborns per mother of a certain age in the year 2002 and our benchmark projection for 2050[3]. The figures are reported together with the total fertility rates per woman (TFR) and the average birth ages (ABA) for the selected years. The latter are income-class-specific[4] and computed according to

$$\text{TFR}(i,k) = \sum_{a=23}^{45} f(a,i,k) \qquad \text{and} \qquad \text{ABA}(i,k) = \frac{\sum_{a=23}^{45} f(a,i,k)a}{TFR(i,k)}.$$

While we assume identical fertility rates for all income classes in our benchmark projection, the mortality rates differ across income classes. However, there is very little data available on income-class-specific mortality. Referring to an estimate by Reil-Held (2000) as well as to a study by Klein (1999), we adjusted the official raw data in two steps: In the first step they were adjusted in order to fit our model restrictions. As already noted above, agents don't die in our model before age 68 and don't survive age 90. In the second step we reduced the mortality of the high income class and increased the mortality of the low income class. The probability of an income-class k agent who is age a in year i to die in this year, $d(a, i, k)$, has to be

$$d(a,i,k) \begin{cases} = 0 & \text{for} \quad 0 \leq a \leq 67 \\ > 0 & \text{for} \quad 68 \leq a \leq 90 \\ = 1 & \text{for} \quad a = 91. \end{cases}$$

[3] Note that the fertility rate $f(a, i, k)$ is half of the respective number reported in Table A-2 since we do not distinguish between sexes in the model.

[4] Since we assume the same fertility rates across income classes in the benchmark projection, the total fertility rates and average birth ages are identical in all income classes.

Table A-2 reports these so called "conditional" death probabilities $d(a,i,k)$ as well as the corresponding "unconditional" death probabilities $\bar{d}(a,i,k)$, i.e. the probabilities of an agent who is currently at an age below 68, that he will die at a certain age in the future. The unconditional death probabilities are easier to interpret and to adjust. The latter are computed from[5]

$$\bar{d}(a,i,k) = d(a,i,k)\prod_{j=1}^{a-1}[1-d(j,i,k)].$$

Given the unconditional death probabilities, the life expectancy (LE) of income class k in each period i can be computed from

$$LE(i,k) = \sum_{a=0}^{91} \bar{d}(a,i,k)a.$$

Since disaggregated data on future mortality rates is not provided in official statistics, we adjusted our original mortality rates in 2000 in order to get realistic life expectancies for the year 2050. Table A-2 reports these mortality rates in 2050. Again, the numbers of the period between 2000 and 2050 are computed by interpolation.

The next step is to disaggregate natives and foreigners in 2000 according to the restrictions of the model. Since in the economic model native children receive bequests from their parents, we have to disaggregate each cohort of the native population in year 2000 up to age 68 according to the age of their parents when they were born. In addition, we also have to distinguish different income classes within each cohort. This disaggregation was achieved by using income-class specific weights $\xi(k,2000)$ for the year 2000 and by applying past *relative* fertility shares to each cohort up to age 90 in year 2000, i.e.

$$N(a,2000,s,k) = \bar{N}(a,2000)\times\xi(k,2000)\times\frac{f(s,2000-a,k)}{TFR(2000-a,k)} \quad a = 1,\dots,67$$
$$s = 23,\dots,45, k=1,2,3.$$

For example, the cohort age 1 in year 2000 is disaggregated using the relative fertility rates of 1999, while the cohort age 40 in 2000 is disaggregated using the relative fertility rates of 1960. Due to the lack of data, we applied the same fertility rates to all income classes k. German fertility rates were available back to the 50ies. For the older cohorts we applied the latest available data. In year 2000 we assume that 20 percent of each cohort belong to the bottom ($k=3$) and top income class ($k=1$) and the remaining 60 percent belong to the middle income class ($k=2$), i.e.

$$\xi(1,2000)=0.2 \quad \xi(2,2000)=0.6 \quad \xi(3,2000)=0.2.$$

[5] An exact calculation would also take into account that the mortality rates change over time. To keep the conversion simple we assumed constant mortality rates.

We still have to compute the numbers of newborn natives in year 2000. Since we assume, per definition, that the children of immigrants who are born after arrival are natives, we get

$$N(0,2000,s,k) = [\bar{N}(s,2000) + \bar{M}(s,2000)] \times \xi(k,2000) \times f(s,2000,k) \quad \begin{array}{rcl} s & = & 23,\ldots,45, \\ k & = & 1,2,3. \end{array}$$

Summing up across all parents' ages and income classes, we receive the total number of newborn in year 2000

$$\bar{N}(0,2000) = \sum_{k=1}^{3} \sum_{s=23}^{45} N(0,2000,s,k).$$

While natives are disaggregated according to the age of their parents at birth, foreign adults are disaggregated according to the age at immigration $s = 21,\ldots,43$. Immigration age is assumed to be 21 for all immigrants between age 1 and 21, i.e.

$$M(a,2000,21,k) = \xi(k,2000) \times \bar{M}(a,2000).$$

Either immigrants arrived as children in the past and never became natives, or they just entered the country in year 2000. Similarly, all other foreigners who arrive in 2000 or in the future could be observed by their immigration age. Immigrants who arrived before year 2000, however, still have to be disaggregated according to their age at immigration. Due to the lack of statistical data, our disaggregation is fairly rough. Those 22-year old foreigners who have not arrived in 2000 could have only arrived at age 21, i.e.

$$\begin{aligned} M(22,2000,22,k) &= \xi(k,2000) \times M^n(22,2000) \\ M(22,2000,21,k) &= \xi(k,2000) \times [\bar{M}(22,2000) - M^n(22,2000)]. \end{aligned}$$

Those 23-year old foreigners who have arrived before 2000 could have arrived at age 21 or 22. We assume the same shares for the remaining immigration ages. Therefore, for each age $a = 23,\ldots,90$ we get

$$\begin{aligned} M(a,2000,a,k) &= \xi(k,2000) \times M^n(a,2000) \\ M(a,2000,s,k) &= \xi(k,2000) \times [\bar{M}(a,2000) - M^n(a,2000)]/\min[a-21;23] \end{aligned}$$

for each immigration age $s = 21,\ldots,a-1$.

This completes our calculations for the year 2000.

3. Population Projections until 2050

For the years between 2000 and 2050 population growth is computed endogenously given the population structure of year 2000 as well as exogenous future fertility, mortality rates and immigration rates. Of course, for future immigrants we have to specify the income class shares $\xi(k,i)$ as well as their number and age structure $M^n(a,i)$.

In order to calculate the vector of foreigners for a specific year, we have to distinguish between children, young and old immigrants. It was already mentioned before that children in year $i > 2000$ (i.e. those younger than age 21 in year i), who arrive in this year i or came earlier with their parents could become natives with probability $\Upsilon(i)$. Consequently, foreign children $\bar{M}(a,i)$ are computed from

$$M(a,i,21,k) = [1-\Upsilon(i)]\ [M(a-1,i-1,21,k) + \xi(k,i)M^n(a,i)] \quad a = 1,\ldots,20.$$

Note that we set $M(0,i,21,k) = 0$ because all children born in our model are supposed to be native Germans. At age 21 immigrants become adults and have to be distinguished by their age of immigration, i.e.[6]

$$M(21,i,21,k) = M(20,i-1,21,k) + \xi(k,i)M^n(21,i).$$

Of course, for foreigners between of age $a = 22,\ldots,90$ we have to make the following distinction:

a) Those who just enter in year i are distributed to the respective age group and income class, i.e.

$$M(a,i,a,k) = \xi(k,i)\ M^n(a,i) \quad a = 22,\ldots,43.$$

b) Those who entered in previous years are extrapolated from the preceding year, i.e.

$$M(a,i,s,k) = [1-d(a,i,k)]M(a-1,i-1,s,k) \quad \begin{aligned} a &= 22,\ldots,90 \\ s &= 21,\ldots,\min(a-1;43). \end{aligned}$$

Finally, we aggregate all adult immigrant cohorts

$$\begin{aligned} M(a,i,20,k) &= \sum_{s=21}^{\min[a;43]} M(a,i,s,k) \qquad \text{and} \\ \bar{M}(a,i) &= \sum_{k=1}^{3} M(a,i,20,k) \qquad a = 1,\ldots,90 \end{aligned}$$

in order to get the income class k specific and total number of foreigners of age a in year i.

Next, we turn to the natives of year $i > 2000$. The native children of age $a = 1,\ldots,20$ are composed of previously native children and those who just changed their nationality, i.e.

$$N(a,i,s,k) = N(a-1,i-1,s,k) + \Upsilon(i)\xi(k,i)[\bar{M}(a-1,i-1) + M^n(a,i)] \times \frac{f(s,i-a,k)}{TFR(i-a,k)}.$$

Note that those foreign children, who become native have to be split according to their parents' age at birth. Again, adult natives are simply aged

$$N(a,i,s,k) = [1-d(a,i,k)]N(a-1,i-1,s,k) \quad a = 21,\ldots,90.$$

[6] For technical reasons we assumed that 20-year old immigrant children don't become natives.

Aggregating all adult native cohorts gives the income-class-k-specific and the total number of natives age a in year i:

$$\begin{aligned} N(a,i,22,k) &= \sum_{s=23}^{45} N(a,i,s,k) \qquad \text{and} \\ \bar{N}(a,i) &= \sum_{k=1}^{3} N(a,i,22,k) \qquad a = 1,\ldots,90. \end{aligned}$$

Finally, the newborn natives in year i are computed as in year 2000:

$$N(0,i,s,k) = [N(s,i,22,k) + M(s,i,20,k)] \times f(s,i,k) \quad s = 23,\ldots,45.$$

Adding up the newborn cohort across parents' ages and income classes gives

$$\bar{N}(0,i) = \sum_{k=1}^{3}\sum_{s=23}^{45} N(0,i,s,k).$$

The total population in year i is computed from

$$POP(i) = \sum_{a=0}^{90}[\bar{N}(a,i) + \bar{M}(a,i)].$$

For the base year 2002 the population structure is reported in Table A-1. The number of native kids who are fed by a household at a certain age will be important for his consumption, for government transfers and for the amount of inheritances later on. Given an income class k agent of age a in year i, the number of his children is derived from[7]

$$KID(a,i,k) = \sum_{j=u}^{v} \frac{N(j,i,a-j,k)}{N(a,i,22,k) + M(a,i,20,k)} \quad 23 \leq a \leq 65,$$

where $u = \max(0; a-45)$ and $v = \min(20; a-23)$. Agents below age 23 have no kids, while after age 66 all kids have left the household, i.e. $KID(a,i,k) = 0$ for $0 \leq a \leq 22$ and $66 \leq a \leq 90$. Note that natives and foreigners of a certain age and income class have (per definition) the same number of (native) children.

The growth rate of the population $\eta(i)$ is computed from the change in the number of 21-year old compared to the previous year, i.e.

$$\eta(i) = \frac{\bar{N}(21,i) + \bar{M}(21,i)}{\bar{N}(21,i-1) + \bar{M}(21,i-1)} - 1.$$

Next we turn to the period after the year 2050.

[7] Note that foreign kids who never become natives are never taken into account here.

4. Population Projections after 2050

Between the period 2000 and 2050 we calculate migration as well as the fertility and mortality rates endougenously. We do that in order to model a realistic demographic transition. After 2050 we keep mortality constant and adjust migration and the fertility rates in order to run into a stable population structure in the future. Newborn natives and net immigrants in the years after 2050 are consequently computed as follows:

$$\begin{aligned} N(0,i,s,k) &= (1+\bar{\eta})N(0,i-1,s,k) \quad s=23,\ldots,45 \qquad k=1,2,3 \\ M^n(a,i) &= (1+\bar{\eta})M^n(a,i-1) \quad a=0,\ldots,43 \end{aligned}$$

where $\bar{\eta}$ is the exogenously set population growth rate after year 2050. It takes exactly 90 years (i.e. until 2140) until we arrive at a constant population structure in the model. Since we are not interested in the far future, we just report the population structure until 2100 in the following tables where we assume $\bar{\eta} = 0$.

5. Summary

The following table reports the development of our model population between the years 2002 and 2100 in comparison to official population projections.

Table 1: Population Projection for Germany

Year		**2002**	**2010**	**2020**	**2030**	**2040**	**2050**	**2100**
Life expectancy								
Model, low class		78.0	79.7	79.8	80.6	81.5	82.5	82.5
Model, middle class		80.0	81.6	81.7	82.5	83.4	84.4	84.4
Model, high class		81.7	83.4	83.5	84.4	85.3	86.3	86.3
Official[b]		78.3	79.8	81.3	82.4	83.3	83.9	84.6
Fertility Rate								
Model		1.4	1.4	1.4	1.4	1.4	1.4	1.4
Official[a]		1.4	1.4	1.4	1.4	1.4	1.4	n.a.
Average Birth Age								
Model		29.0	29.0	29.0	29.0	29.0	29.0	29.0
Official[c]		(28.7)	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Total Population (in mio.)								
Official[a]		82.4	83.1	82.8	81.2	78.5	75.1	n.a.
Model		81.5	82.4	82.4	80.5	77.7	73.0	59.6
Natives		74.3	74.5	73.4	70.6	67.3	62.4	50.5
Foreigners		7.2	7.9	9.0	9.8	10.4	10.6	9.2
Net-immigrants (in 1000)		164.2	164.2	164.2	164.2	164.2	164.2	164.2
Foreigners (in % of total)		8.9	9.6	10.9	12.2	13.4	14.5	15.3
Age Structure								
< 20	Model	21.0	18.8	17.7	17.2	16.4	16.5	19.4
	Official[a]	(20.9)	18.7	17.6	17.1	16.4	16.1	n.a.
20-59	Model	55.7	56.2	53.3	48.1	47.9	47.4	47.8
	Official[a]	(55.0)	55.7	53.3	48.5	48.4	47.2	n.a.
60-90	Model	23.3	25.0	29.0	34.7	35.7	36.1	32.9
	Official[a]	(24.1)	25.6	29.2	34.4	35.2	36.7	n.a.
Dependency Ratio								
Model		41.8	44.4	54.5	72.2	74.4	76.2	68.8
Official[a]		(43.9)	46.0	54.8	70.9	72.8	77.8	n.a.

[*] Data in parenthesis refer to the year 2000 or 2001, n.a. not available.
[a] Federal Statistical Office of Germany (2003)
[b] Institut für Bevölkerungsforschung und Sozialpolitik (2003), average middle/high variant
[c] Eurostat (2003, 90)

III. The Structure of the Economic Model

In this section we describe the economic model which is applied for our simulations. We start with the household side and describe the decision problems of native and immigrant households. Then we discuss the aggregation of the micro variables as well as the production side of the economy. Finally, the tax and transfer system is explained.

1. The Household Sector

As already explained above, we distinguish between natives and immigrants in the model. Both household types leave bequests when they die since they are imperfectly annuitised. However, only native households receive inheritances per definition. In addition, immigrants have no assets when they arrive. While natives start to make their own economic decisions at age 21, adult immigrants make their decisions when entering the country.

As usual, our model assumes a preference structure that is represented by a time-separable, nested CES utility function. Within each generation we have to distinguish the legal status of natives and immigrants ($l = n, m$), different income classes k, (native) parents' ages at birth and (immigrant) ages at the time of immigration. Consequently, $U(j,t,s,k,l)$ defines remaining lifetime utility of a generation of age j at time t from income class k. In case of a native household ($l = n$), his parents were age s at time of birth, in case of a immigrant household ($l = m$), he was age s when he entered Germany. Remaining lifetime utility takes the form

$$U(j,t,s,k,l) = V(j,t,s,k,l) + H(j,t,s,k,l), \tag{1}$$

where $V(j,t,s,k,l)$ denotes the utility parents receive from their own goods and leisure consumption and $H(j,t,s,k,l)$ denotes the utility they receive from their children's consumption. The two sub-utility functions are defined as follows:

$$V(j,t,s,k,l) = \frac{1}{1-\frac{1}{\gamma}} \sum_{a=j}^{90} (1+\theta)^{j-a} P(a,i,k) \left[c(a,i,s,k,l)^{1-\frac{1}{\rho}} + \alpha \ell(a,i,s,k,l)^{1-\frac{1}{\rho}} \right]^{\frac{1-\frac{1}{\gamma}}{1-\frac{1}{\rho}}} \tag{2}$$

$$H(j,t,s,k,l) = \frac{1}{1-\frac{1}{\gamma}} \sum_{a=j}^{90} (1+\theta)^{j-a} P(a,i,k) KID(a,i,k) c_K(a,i,s,k,l)^{1-\frac{1}{\gamma}}. \tag{3}$$

where $c(\cdot)$ and $\ell(\cdot)$ denote consumption and leisure respectively and i is defined as $i = t+a-j$. The variable $c_K(\cdot)$ denotes the consumption of income class k children whose parents are a years old in year i and whose grandparents were either age s at the time of birth (of the parents) or who immigrated at age s. Since future life is uncertain, consumption in future periods is weighted with the survival probability

$$P(a,i,k) = \prod_{u=0}^{a} [1 - d(u, u-a+i, k)], \tag{4}$$

i.e. by multiplying the conditional survival probabilities from birth up to year i. The parameters θ, ρ, α and γ represent the "pure" rate of time preference, the intratemporal elasticity of substitution between consumption and leisure at each age a, the leisure preference and the intertemporal elasticity of substitution between consumption of different years respectively.

Although natives and foreigners have identical preferences, the economic differences could be observed in the budget constraint of those who start to make their own economic decisions. While for a 21-year old native from income class k in year i who's parents were age s at his birth the latter is

$$\sum_{a=21}^{90}\Big[W(a,i,s,k,n)+(1+r(i))I(a,i,s,k,n)-T(a,i,s,k,n)-c(a,i,s,k,n)- \\ KID(a,i,k)c_K(a,i,s,k,n)\Big]R(i,t)=0, \quad (5)$$

the same budget constraint for a foreign household is given by

$$\sum_{a=s}^{90}\Big[W(a,i,s,k,m)-T(a,i,s,k,m)-c(a,i,s,k,m)- \\ KID(a,i,k)c_K(a,i,s,k,m)\Big]R(i,t)=0. \quad (6)$$

Note that natives receive inheritances including interest payments while foreigners receive no inheritances at all. As already mentioned, the latter also start to make their economic decisions when they enter the country at age s in year t.

The gross labor income of native and foreign agents is defined by

$$W(a,i,s,k,l)=\frac{w(i)E(a,k)[h(a,i)-\ell(a,i,s,k,l)]}{1+0.5\bar{\tau}^{ss}(a,i,s,k,l)} \quad (7)$$

where $w(i)$ is the employers' gross wage rate in period $i=t+a-21$ and $\bar{\tau}^{ss}(\cdot)$ is the aggregate individual social security contribution rate defined below. In Germany, the employers' labor costs include half of social security contributions. Consequently, the gross individual labor income is lower than labor cost for the employers.

Similar as Kotlikoff et al. (2001) or Fehr et al. (2003) we assume that technical progress causes the time endowment $h(\cdot)$ of each successive generation to grow at the rate λ, i.e.

$$h(a,i)=(1+\lambda)h(a,i-1). \quad (8)$$

The age- and income-class-specific earnings ability profile

$$E(a,k)=e^{\kappa_0^k+\kappa_1^k(a-20)-\kappa_2^k(a-20)^2}(1+\lambda)^{a-21} \quad (9)$$

is identical for natives and foreigners, includes the income-class-specific parameters κ and is steepened by the rate of technological progress λ. For realistic values of κ we work with data estimated for Germany by Grzimek (1997).

The inheritance of a native agent in income class k who is age a in year i and who's parents are s years older than himself is denoted by $I(a,i,s,k,n)$. Before parents' age 68 (i.e. $a+s<68$), the probability of death is zero and, consequently, there are no bequests. Between age 68 and 90, a fraction of a parents cohort dies and leaves bequests which are split between their (native) children[8]. Therefore, inheritances of children are computed as follows:

$$I(a,i,s,k,n) = \frac{d(a+s,k)\bar{A}(a+s,i,k)}{\sum_{j=23}^{45} N(a+s-j,i,j,k)} \quad \text{and} \quad I(a,i,s,k,m) = 0. \tag{10}$$

The numerator defines the aggregate assets of income class k parents who die in year i at age $a+s$. The denominator defines the parents' total number of native children. The inheritances are the reason why we have to disaggregate each cohort according to the age of their parents at birth. Children who were born to older parents receive their inheritances earlier in their life, while children with young parents receive their inheritances later in life. While the first children of parents (born when their parents were age 23) receive their inheritances between the ages of 45 and 67, the last children (born when their parents were age 45) receive their inheritances between the ages of 23 and 45.

The net-taxes of an agent age a in year i consist of consumption, inheritance, capital and labor income taxes as well as social security contributions net of pensions (Pen) and lump-sum transfers for children (Tr^k), i.e.

$$\begin{aligned} T(a,i,s,k,l) \;=\;& \tau^c(i)[c(\cdot) + KID(a,i,k)c_K(\cdot)] + \tau^b(i)I(\cdot) + \bar{\tau}^r(\cdot)r(i)[\mathrm{a}(\cdot) + I(\cdot)] \\ & + [\bar{\tau}^w(\cdot) + 0.5\bar{\tau}^{ss}(\cdot)]W(\cdot) + z^h(a,i,k) + z^{lc}(i,k) - Tr^k(a,i,k) \\ & - \left\{1 - \bar{\tau}^w(\cdot) - \varrho_1(i)[\tau^h(i,k) + \tau^{lc}(i,k)]\right\} Pen(\cdot). \end{aligned} \tag{11}$$

Tax rates for consumption $\tau^c(i)$ and inheritances $\tau^b(i)$ are proportional and, consequently, only indexed by the year i. Capital and labor income is taxed progressively. Therefore, $\bar{\tau}^r(a,i,s,k,l)$ and $\bar{\tau}^w(a,i,s,k,l)$ denote the average individual tax rate for capital and labor income, respectively. The German social security system consists of a pension, health care and long-term care system. Contributions to the public system are proportional to income and children are automatically insured with their parents' contributions. However, due to system-specific contribution ceilings, we have to distinguish individual marginal and average social security contributions. In addition, households above a certain income level (the so-called *Versichertenpflichtgrenze*) can decide whether they switch from the public state to the private health and long-term care system. In contrast to the public system, contributions to the private system are lump-sum and children have to be insured separately. Therefore, the decision of an agent to switch to the private system depends on his income and family status. Our model does not reflect this complex decision process. We simply assume that households in the top income class are insured in the private system, whereas middle and low-income class households are members of the state system. While we neglect a contribution ceiling for the health care and long-term care system, we include such a ceiling for the pension system. Consequently, we have to distinguish

[8] Note that those who die at age 91 leave no bequests. Consequently it's no problem when their oldest children die at the same time.

between average and marginal individual contributions $\bar{\tau}^p(a,i,s,k,l)$ and $\tau^p(a,i,s,k,l)$ only for the pension system. Marginal and average contribution rates for the public health care and long-term care system $\tau^h(i,k)$ and $\tau^{lc}(i,k)$ are identical for all agents in the low and middle income class and zero in the top income class, because they are assigned to the private system. Pensioners have to pay a fraction $\varrho_1(i)$ of public health care and long-term care contributions on pensions. The remaining fraction $1-\varrho_1(i)$ is financed by the pension budget. Lump-sum contributions of top income class households for private health care in year i, $z^h(a,i,k)$, depend on the age a since they also include contributions for the children (see below). Contributions to private long-term care $z^{lc}(i,k)$ are again independent of age since no contributions are paid for children. Note that lump-sum contributions to the private insurance systems are paid solely by the individual households. Aggregate average individual payroll social security contributions are therefore

$$\bar{\tau}^{ss}(a,i,s,k,l) = \bar{\tau}^p(a,i,s,k,l) + \tau^h(i,k) + \tau^{lc}(i,k).$$

Similar to private insurance contributions, lump-sum transfers for children only depend on the number of children and are independent of the parents' age. Finally, pension benefits depend on former contributions and, consequently, may vary according to the parents' age. Note that pensioners also pay income taxes and contributions to the health and long-term care system.

The gross discount factor with the interest rate $r(z)$ in year z is

$$R(i,t) = \begin{cases} 1 & \text{for} \quad i = t \\ \prod_{z=t+1}^{i}[1+r(z)]^{-1} & \text{for} \quad i > t. \end{cases} \tag{12}$$

The asset accumulation of an income class k agent of age j in year i who's parents were age s at his birth or who immigrated at age s follows

$$\begin{aligned} \mathrm{a}(j+1,i+1,s,k,l) = {}& [\mathrm{a}(j,i,s,k,l) + I(j,i,s,k,l)][1+r(i)] + W(j,i,s,k,l) \\ & - T(j,i,s,k,l) - c(j,i,s,k,l) - KID(j,i,k)c_K(j,i,s,k,l). \end{aligned} \tag{13}$$

Given individual consumption, leisure and assets of all native and immigrant agents we can compute the aggregated variables of a specific year. Aggregated consumption $C(i)$ and assets $A(i)$ in year i are computed from

$$\begin{aligned} C(i) = \sum_{k=1}^{3}\sum_{a=21}^{90}\Bigg\{ & \sum_{s=23}^{45}[c(a,i,s,k,n) + KID(a,i,k)c_K(a,i,s,k,n)]\,N(a,i,s,k) + \\ & \sum_{s=21}^{43}[c(a,i,s,k,m) + KID(a,i,k)c_K(a,i,s,k,m)]\,M(a,i,s,k)\Bigg\} \end{aligned} \tag{14}$$

$$\begin{aligned} A(i) &= \sum_{k=1}^{3}\sum_{a=21}^{90}\left\{\sum_{s=23}^{45}\mathrm{a}(a,i,s,k,n)\frac{N(a,i,s,k)}{1-d(a,i,k)} + \sum_{s=21}^{43}\mathrm{a}(a,i,s,k,m)\frac{M(a,i,s,k)}{1-d(a,i,k)}\right\} \qquad (15) \\ &= \sum_{k=1}^{3}\sum_{a=21}^{90}\bar{A}(a,i,k). \end{aligned}$$

Note that the assets in period i are saved by native and foreign agents who lived in period $i-1$. Since by assumption all households die at the beginning of each period, we aggregate across all agents who lived in the previous period in order to compute $\bar{A}(a,i,k)$ which we need for the calculation of the inheritances, see (10).

2. Production

The economy is populated by a large number of competitive firms. Since they are all assumed to be identical, it suffices to consider the planning problem of one representative company and normalize the number of firms to unity. On the firm side we assume that all investment is financed via retained earnings. However, convex costs of adjusting the capital stock provide an incentive for smoothing investment.

From the cash-flow identity in period i we derive the dividend payments

$$DIV(i) = Y(i) - w(i)L(i) - \Delta K(i), \tag{16}$$

which links together dividends $DIV(i)$, profits (i.e. output $Y(i)$ net of wage costs $w(i)L(i)$) and investment outlays $\Delta K(i)$ in period i.

We assume that gross output (net of depreciation) is produced using a Cobb-Douglas production technology, i.e.

$$F[K(i), L(i)] = \phi K(i)^{\varepsilon} L(i)^{1-\varepsilon}, \tag{17}$$

where $K(i)$ is aggregate capital in period i, ε is capital's share in production, and ϕ is a technology parameter. Since we posit convex capital adjustment costs, the firm's marketable output in year i, $Y(i)$, is given by the difference between gross output and adjustment costs, i.e.

$$Y(i) = F[K(i), L(i)] - 0.5\ \psi\ \Delta K(i)^2/K(i), \tag{18}$$

where the term ψ is the adjustment cost coefficient. Larger values of ψ imply higher marginal costs of new capital goods for a given rate of investment. The installation technology is linearly homogeneous and shows increasing marginal costs of investment (or, symmetrically, disinvestment): faster adjustment requires a greater than proportional rise in adjustment costs.

The objective of the firm at the beginning of period t is to maximize the present value of current and future dividends $\sum_{i=t}^{\infty} DIV(i)R(i,t)$. Thereby, the firm has to take into account the financial constraint (16), the technology constraint (18) and the equation of motion for the stock of capital:

$$K(i+1) - K(i) = \Delta K(i). \tag{19}$$

In order to solve this problem, the firm employs labor up to the point where the marginal product of labor equals the employer's wage rate $w(i)$:

$$w(i) = F_{L(i)}. \tag{20}$$

Since we abstract from any taxation at the corporate level, arbitrage between new and existing capital implies that the latter has a price per unit of

$$q(i+1) = 1 + \psi\ \Delta K(i)/K(i). \tag{21}$$

Similarly, the arbitrage condition arising from profit maximization requires identical returns to financial and real investments:

$$r(i)q(i) = F_{K(i)} + 0.5\,\psi\,(\Delta K(i)/K(i))^2 + q(i+1) - q(i). \tag{22}$$

The left side gives the return on a financial investment of amount $q(i)$, while the return on one unit of real capital investment is the net return to capital (which includes the marginal product of capital $F_{K(i)}$ plus the reduction in marginal adjustment costs) and capital gains.

3. The Government Sector

The government sector in the model represents the consolidated budget of the central, state and local governments as well as the budgets of the pension, health and long-term care system.

3.1. The Consolidated Budget

The central government issues new debt $\Delta B(i) = B(i+1) - B(i)$ and collects taxes and insurance contributions net of pensions from households and employers $T^G(i)$ in order to finance the public good $G(i)$ and the interest payments on its debt:

$$\Delta B(i) + T^G(i) = G(i) + r(i)B(i), \tag{23}$$

where

$$T^G(i) = \sum_{k=1}^{3} \sum_{a=21}^{90} \Bigg\{ \sum_{s=23}^{45} \left[T(a,i,s,k,n) + 0.5\bar{\tau}^{ss}(a,i,s,k,n)W(a,i,s,k,n)\right] N(a,i,s,k) + \sum_{s=21}^{43} \left[T(a,i,s,k,m) + 0.5\bar{\tau}^{ss}(a,i,s,k,m)W(a,i,s,k,m)\right] M(a,i,s,k) \Bigg\}$$

sums up the individual net-tax payments and the employer's social security contributions in year i.

With respect to public debt, we assume that the government keeps an exogenously fixed ratio $b(i)$ of debt to output, i.e. $B(i) = b(i)Y(i)$. The public good expenditures $G(i)$ consist of government purchases of goods and services (including government investments) and education, health and long-term care outlays. Expenditures for government purchases are identical per capita, education outlays are age-specific and only spent for children. Health and long-term care expenditures are also age-specific. Consequently, we have

$$G(i) = POP(i)g + \sum_{a=0}^{20} \left[\bar{N}(a,i) + \bar{M}(a,i)\right] \times edu(a) + HB(i) + LCB(i), \tag{24}$$

where g are the time invariant per capita outlays of general public goods, $edu(a)$ are the education outlays per child of age a and $HB(i)$ and $LCB(i)$ are the aggregate health and long-term care outlays, respectively.

While education transfers depend on the number of children but are not paid directly to the households, parents receive an exogenously specified benefit payment per child $tc(i)$ (the so-called *Kindergeld*) in each year i . The aggregate child related transfers to natives and immigrant households $Tr^k(a,i,k)$ depend on the number of children under age 21. Consequently, an income class k agent of age a in year i receives

$$Tr^k(a,i,k) = KID(a,i,k)tc(i). \tag{25}$$

Summing up these transfers across all households in year i gives

$$Tr(i) = \sum_{k=1}^{3}\sum_{a=23}^{65} Tr^k(a,i,k)[N(a,i,22,k) + M(a,i,20,k)]. \tag{26}$$

Next, we turn to the progressive labor and capital income tax system. We model a dual income tax for Germany. Throughout the baseline path labor income is taxed according to the progressive tax schedule of the year 2004 (T04), while capital income is taxed with a linear income tax. In order to derive the taxable labor income $zvE(a,i,s,k,l)$ of an agent, we subtract allowances for income-connected expenses $Aic(a,i,s,k,l)$ (*Werbungskosten*) and allowances for expenses of a provident nature $Aip(a,i,s,k,l)$ (*Vorsorgeaufwendungen*) from gross wage income and pensions. Currently only the interest portion $\chi(z)$ of the pension benefit is taxable[9], therefore, we have:

$$zvE(a,i,s,k,l) = W(\cdot) + \chi(z)Pen(\cdot) - Aic(\cdot) - Aip(\cdot). \tag{27}$$

Note that we don't allow pensioners to work in or after their retirement age $\bar{a}(i,k)$. Consequently, agents can either receive wage or pension income at the same time.

We assume that income-connected expenses are a fixed fraction $\beta(i)$ of gross labor income, but households are always allowed to subtract a fixed allowance of 956 € for labor income (since 2004[10]) and 138 € for pensions without further proof, i.e.

$$Aic(a,i,s,k,l) = \begin{cases} \max[\beta(i)W(\cdot); 956] & \text{if} \quad a < \bar{a}(i,k) \\ 138 & \text{if} \quad a \geq \bar{a}(i,k). \end{cases} \tag{28}$$

Allowances for expenses of a provident nature are computed according to the complicated three stage procedure currently in practice in Germany. In our model the total insurance payments of an agent are defined by:

$$IP(a,i,s,k,l) = 0.5\bar{\tau}^{ss}(\cdot)W(\cdot) + \varrho_1(i)[\tau^h(i,k) + \tau^{lc}(i,k)]Pen(\cdot) + z^h(a,i,k) + z^{lc}(i,k).$$

At the first stage, agents are allowed to subtract an advanced deduction $AD(a,i,s,k,l)$ of 3068 € per year. Since they receive tax-free contributions to the public social security system from

[9] The interest proportion depends on the year of retirement z.

[10] The fixed allowance for labor income formerly was 1080 €. In order to partly compensate the 2004 tax cuts it has been decreased since that year.

their employers, the advance deduction is reduced by 16 percent of wage income or the pension benefit:

$$AD(a,i,s,k,l) = \max\left\{3068 - 0.16[W(\cdot) + Pen(\cdot)]; 0\right\}.$$

Expenses in excess of this amount could be subtracted at the second stage up to a basic maximum amount $BMA(\cdot)$ of 1334 €:

$$BMA(a,i,s,k,l) = \min[IP(\cdot) - AD(\cdot); 1334].$$

Finally, at the third stage, expenses in excess of these amounts are deductible by half up to the half maximum amount $HMA(\cdot)$ of 667 €:

$$HMA(a,i,s,k,l) = 0.5\min\left\{[IP(\cdot) - AD(\cdot) - BMA(\cdot)]; 1334\right\}.$$

The total allowances of a provident nature are then computed from

$$Aip(a,i,s,k,l) = AD(\cdot) + BMA(\cdot) + HMA(\cdot).$$

Taxable income determined in this manner is the basis for the assessment of the income tax according to the basic scale. Consequently, a basic allowance of 7664 € is granted on taxable income. In the first linear-progressive zone, tax rates on income in excess of the basic personal allowance rise from 16 to 24 percent on taxable income of up to 12739 €. In the second linear-progressive zone, tax is imposed at rates between 24 and 45 percent on taxable income of up to 52151 €. Finally, in the upper proportional zone of taxable income above 52151 € every increment is taxed at a constant rate of 45 percent. The average individual labor income tax rate $\bar{\tau}^w(\cdot)$ is, therefore, computed from

$$\bar{\tau}^w(a,i,s,k,l)[W(a,i,s,k,l) + Pen(a,i,s,k,l)] = T04[zvE(a,i,s,k,l)]. \tag{29}$$

Next, we turn to the taxation of capital income. Since we assume that heirs receive inheritances including interest payments, they also have to pay the tax on interest income which is levied at a uniform rate $\tau^r(i)$. The tax base for the latter is the gross interest income net of a uniform savings allowance $Ais(i)$. The individual average interest income tax rate $\bar{\tau}^r(a,i,s,k,l)$ is, therefore, computed from:

$$\bar{\tau}^r(a,i,s,k,l)r(i)[\mathrm{a}(a,i,s,k,l) + I(a,i,s,k,l)] = \tau^r(i)\big\{r(i)[\mathrm{a}(\cdot) + I(\cdot)] - Ais(i)\big\}. \tag{30}$$

Consumption and inheritance taxes are proportional. Consequently, tax rates are only year-specific and tax revenues are computed from

$$\tau^c(i)C(i) \quad \text{and} \quad \tau^b(i)\sum_{k=1}^{3}\sum_{a=21}^{66}\sum_{s=23}^{45} I(a,i,s,k,n)N(a,i,s,k),$$

respectively.

3.2. The Budget of the Pension System

We model a PAYGO-pension system in Germany. Let's assume that an agent who's parents were s years old at his birth has retired in year z at the exogenously set retirement age $\bar{a}(z,k)$. Then his pension benefits $Pen(a,i,s,k,l)$ in year $i \geq z$ when he is age $a \geq \bar{a}(z,k)$ are computed from the product of three elements:

1. The so called "adjustment factor" (AF) for pension type and retirement age,
2. the sum of "individual earning points" (SEP) which mainly reflect the retiree's relative earning position during his working time and
3. the "actual pension value" (APV) which defines the value of one earning point in €.

Therefore, we get

$$Pen(a,i,s,k,l) = AF(z,k) \times SEP(z,s,k,l) \times APV(i). \tag{31}$$

The model does not distinguish between different types of pensions. Consequently, the adjustment factor deviates from one only if the individual retirement age $\bar{a}(z,k)$ deviates from the statutory "normal retirement age" $\tilde{a}(z)$ of 65 which was introduced by the pension reform in 1992. When the complete reform is fully phased-in, benefits will be reduced by 3.6 percent for each year of earlier retirement (in addition to the effect of fewer earning points). In the baseline path individual retirement ages rise discretely from 60 to 62 between 2019 and 2035[11]. The "normal retirement age" is increased from $\tilde{a}(2002) = \tilde{a}(2003) = 62, \tilde{a}(2004) = 63, \tilde{a}(2005) = 64$ up to $\tilde{a}(z) = 65$ for $z = 2006, \ldots$. Consequently, the individual adjustment factor

$$AF(z,k) = 1 - [\tilde{a}(z) - \bar{a}(z,k)] \times 0.036$$

depends on the year of retirement. In the baseline it is one for those who retired before 2002 and it's value is reduced to 0.892 for those who retire in and after 2006.

The model distinguishes three types of earning points: normal, foreign-income-related and child-rearing-related. Normal earning points $EP(a,i,s,k,l)$ of an employee are computed from the ratio of his individual insured gross earnings to average gross earnings in each year t of service. The earning point received at age a for his annual gross labor income $W(a,i,s,k,l)$ is calculated according to the formula

$$EP(a,i,s,k,l) = \begin{cases} \min\{1.5W[(\cdot)/\bar{W}(t); 0.75]\} & \text{if} \quad W(\cdot) \leq 0.75\bar{W}(i) \\ W(\cdot)/\bar{W}(t) & \text{if} \quad 0.75\bar{W}(t) < W(\cdot) < BBG(t) \\ 2.0 & \text{if} \quad BBG(t) \leq W(\cdot). \end{cases} \tag{32}$$

[11] The high income class retirement age rises from 60 to 61 in 2019. The middle class follows in 2023, the low income class in 2025. The increase to $\bar{a}(i,k) = 62$ takes place in a similar way between 2031 and 2035.

This formulation reflects some of the redistributional features of the German pension system mentioned above. If the individual income in year t, is below 75 percent of average income

$$\bar{W}(i) = \sum_{a=21}^{\bar{a}(i,k)-1} \frac{\sum_{k=1}^{3}\left[\sum_{s=23}^{45} W(a,i,s,k,n)N(a,i,s,k) + \sum_{s=21}^{43} W(a,i,s,k,m)M(a,i,s,k)\right]}{\bar{N}(a,i)+\bar{M}(a,i)},$$

then the accounted earning point is increased up to 50 percent. If the annual individual income is above the contribution ceiling

$$BBG(i) = 2.0\bar{W}(i),$$

which exceeds the average income by 100 percent, then a maximum earning point of 2.0 is credited. Below the contribution ceiling and above the minimum threshold earning points are computed from the ratio of individual income to the average income of the respective year. Note that for working years $i < 2002$ we take the 2002 income of someone in the same age, income class and parents' age as computation base for the agent's earning points.

Foreigners who enter the country at age $s > 21$ can also receive earning points for their contributions made abroad at age $21 \geq a < s$. In the model we treat all foreigners like the German resettlers (the so-called *Vertriebene* and *Spätaussiedler*), who resettle to Germany from formerly Soviet or other Eastern Europe countries. Consequently, foreign-income-related earning points $EP^f(a,i,s,k)$ depend on the average earning points of a native of the same age and income class and the factor $\varrho_2(i)$ (which is currently 0.6) (Bundesversicherungsanstalt für Angestellte (2001)). On the other hand, foreigners don't receive pensions from their contributions they made abroad before entering Germany.

$$EP^f(a,i,s,k) = \varrho_2(i) \times \sum_{j=23}^{45} EP(a,i,j,k,n)/N(a,i,22,k) \quad \text{for} \quad a < s. \tag{33}$$

Finally, women who work in year i and have children below age 10 receive an increase of their earnings points by $\varrho_3(i)$ (currently 50 percent) which can reach a maximum of 0.33 points, i.e. 0.33/2 in the model, so that

$$EP^c(a,i,s,k,l) = \min[\varrho_3(i)KID2(a,i,k)EP(\cdot); 0.33/2], \tag{34}$$

where

$$KID2(a,i,k) = \sum_{j=u}^{v} \frac{N(j,i,a-j,k)}{N(a,i,22,k)+M(a,i,20,k)} \quad 23 \leq a \leq 65$$

with $u = \max(0; a-45)$ and $v = \min(10; a-23)$.

The sum of the earning points during working years $SEP(z,s,k,l)$ is now computed for native and foreign agents from

$$SEP(z,s,k,n) = \sum_{a=21}^{\bar{a}(z,k)-1} [EP(\cdot) + EP^c(\cdot)] \tag{35}$$

$$SEP(z,s,k,m) = \sum_{a=s}^{\bar{a}(z,k)-1} [EP(\cdot) + EP^c(\cdot)] + \min\left[\sum_{a=21}^{s-1} EP^f(a,i,s,k); 25\right], \tag{36}$$

where we have taken into account that the number of foreign-income related earning points is restricted to (currently) 25 points.

While the first two factors in (31) are kept constant in the years $i > z$ after retirement, the actual pension value is adjusted according to

$$APV(i) = APV(i-1) \times \frac{\bar{W}(i-1) \times [1 - \tau^{pp}(i-1) - \hat{\tau}^p(i-1)]}{\bar{W}(i-2) \times [1 - \tau^{pp}(i-2) - \hat{\tau}^p(i-2)]}. \tag{37}$$

Equation (37) reflects the central elements of the adjustment formula which was introduced by the Riester Reform in 2001[12]. Since then, changes in the actual pension value are related to lagged changes of an artificial income concept which is computed from the average gross income $\bar{W}(i)$ net of contributions to public pensions and fictitious contributions $\tau^{pp}(i)$ to newly introduced private pension accounts. Until 2010 the fictitious contribution rates to the private accounts increase from currently 0.5 percent to 4 percent which dampens the growth of the actual pension value.

The outlays of the pension system include aggregate pension benefits $PB(i)$:

$$PB(i) = \sum_{k=1}^{3} \underbrace{\sum_{a=\bar{a}(i,k)}^{90} \left\{ \sum_{s=23}^{45} Pen(a,i,s,k,n) N(a,i,s,k) + \sum_{s=21}^{43} Pen(a,i,s,k,m) M(a,i,s,k) \right\}}_{\overline{Pen}(i,k)} \tag{38}$$

and a fraction $[1 - \varrho_1(i)]$ (currently 0.5) of the public health and long-term care contributions of pensioners, i.e.

$$S^h(i) = [1 - \varrho_1(i)] \hat{\tau}^h(i) \times \sum_{k=2}^{3} \overline{Pen}(i,k) \quad \text{and} \quad S^{lc}(i) = [1 - \varrho_1(i)] \hat{\tau}^{lc}(i) \times \sum_{k=2}^{3} \overline{Pen}(i,k). \tag{39}$$

Note that pensioners of the top income class pay their contributions to the private health and long-term care system fully by themselves.

Since the budget of the pension system must be balanced in each period, the contribution rate $\hat{\tau}^p(i)$ is computed from

$$\hat{\tau}^p(i) PY^p(i) = [1 - \mu(i)][PB(i) + S^h(i) + S^{lc}(i)], \tag{40}$$

where

$$PY^p(i) = \sum_{k=1}^{3} \sum_{a=21}^{\bar{a}(i,k)-1} \left\{ \sum_{s=23}^{45} \min[W(a,i,s,k,n); BBG(i)] N(a,i,s,k) + \right.$$
$$\left. \sum_{s=21}^{43} \min[W(a,i,s,k,m); BBG(i)] M(a,i,s,k) \right\}$$

[12] For a detailed description and an economic evaluation of this reform, see Bonin (2002).

denotes the aggregate compulsory contribution base in year i and $\mu(i)$ defines the fraction of outlays which is financed by general taxes.

The aggregate pension contribution rate $\hat{\tau}^p(i)$ which is calculated from (40) is not necessarily identical with the individual contribution rates. Due to the contribution ceiling, marginal and average contribution rates of income class k agents of age a in year i are given by

$$\tau^p(a,i,s,k,l) = \begin{cases} \hat{\tau}^p(i)(1-\varphi(a,i)) & \text{if} \quad W(\cdot) \leq BBG(i) \\ 0 & \text{if} \quad W(\cdot) > BBG(i) \end{cases}$$

and

$$\bar{\tau}^p(a,i,s,k,l) = \begin{cases} \hat{\tau}^p(i) & \text{if} \quad W(\cdot) \leq BBG(i) \\ \hat{\tau}^p(i)\, BBG(i)/W(\cdot) & \text{if} \quad W(\cdot) > BBG(i). \end{cases}$$

Above the contribution ceiling, the marginal social security tax is zero and the average social security tax falls with increasing income for an individual. The tax benefit linkage $\varphi(a,i)$ reflects the extent of redistributive elements within the pension system. If pensions would be perfectly proportional to former contributions, households would only perceive a proportion of their contributions as taxes which depends on the difference between the rate of return on the capital market and the implicit rate of return of the pension system. This proportion falls when the household approaches retirement age. In order to take this into account, we model the tax-benefit linkage as

$$\varphi(a,i,k) = e^{\omega[a-\bar{a}(i,k)]}.$$

If pensions are exactly proportional to former contributions, ω reflects the difference between the rates of return on the capital market and in the pension system. If pensions are completely independent of former contributions, ω approaches infinity i.e. $\varphi(a,i) = 0$.

3.3. The Public and Private Health Care System

In the present model we specify age-specific health costs $hc(a,i)$ which represent the consumption of health services financed by the health care system in year i. We assume that health care costs increase by 0.1 percent annually. Since all agents in the top income class are insured in the private system, total health costs $HB(i)$ consist of public ($HB^g(i)$) and private ($HB^p(i)$) costs

$$HB(i) = HB^g(i) + HB^p(i), \tag{41}$$

where

$$\begin{aligned} HB^g(i) &= \sum_{k=2}^{3}\sum_{a=0}^{90} hc(a,i)[N(a,i,22,k) + M(a,i,20,k)] \\ HB^p(i) &= \sum_{a=0}^{90} hc(a,i)[N(a,i,22,1) + M(a,i,20,1)]. \end{aligned}$$

The public health care contribution rate $\hat{\tau}^h(i)$ in year i is derived from

$$\hat{\tau}^h(i) PY^h(i) = HB^g(i), \tag{42}$$

where

$$PY^h(i) = \sum_{k=2}^{3}\sum_{a=21}^{90}\Bigg\{\sum_{s=23}^{45}[W(a,i,s,k,n) + Pen(a,i,s,k,n]N(a,i,s,k)+ \\ \sum_{s=21}^{43}[W(a,i,s,k,m) + Pen(a,i,k,s,m)]M(a,i,s,k)\Bigg\}$$

denotes the aggregate contribution base for public health care contributions in year i. Note that we do not distinguish between individual, employers and pension contributions to the health care system.

The outlays of the private system are fully financed by lump-sum payments. This lump-sum payment per adult $\hat{z}^h(i)$ is

$$\hat{z}^h(i) = \frac{HB^p(i)}{\sum_{a=21}^{90}[N(a,i,22,1) + M(a,i,20,1)] + 0.5\sum_{a=0}^{20} N(a,i,22,1)}. \tag{43}$$

We divide by half of the children because agents have to pay half of the adult payment for each child in their household (in addition to their own contributions). Individual payments are, therefore,

$$z^h(a,i,k) = \hat{z}^h(i)[1 + 0.5\ KID(a,i,k)]. \tag{44}$$

3.4. The Public and Private Long-term Care System

Similarly to the health care system, we specify age-specific long-term care costs $lc(a,i)$ for every adult agent which represents the consumption of long-term care services. Again, all agents in the top income class are insured in the private system. Total long-term care costs $LCB(i)$ consist of public $LCB^g(i)$ and private $LCB^p(i)$ costs:

$$LCB(i) = LCB^g(i) + LCB^p(i), \tag{45}$$

where

$$\begin{aligned} LCB^g(i) &= \sum_{k=2}^{3}\sum_{a=0}^{90} lc(a,i)[N(a,i,22,k) + M(a,i,20,k)] \\ LCB^p(i) &= \sum_{a=0}^{90} lc(a,i)[N(a,i,22,1) + M(a,i,20,1)]. \end{aligned}$$

Here, we again assume long-term costs to grow at an annual rate of 0.1 percent.

In contrast to the public health care system, the public long-term care system keeps accumulated assets $AP(i)$ from past contributions. Consequently, the public long-term care contribution rate $\hat{\tau}^{lc}(i)$ in year i is kept constant as long as

$$AP(i+1) = AP(i)[1 + r(i)] + \hat{\tau}^{lc}(i)PY^h(i) - LCB^g(i) \tag{46}$$

is positive. If $AP(i+1)$ turns negative we set $AP(i+1) = 0$ and compute the contribution rate $\hat{\tau}^{lc}(i)$ endogenously[13].

The outlays of the private system are fully financed by lump-sum payments, i.e. no extra payments for a household's children are levied, so that

$$\hat{z}^{lc}(i) \sum_{a=21}^{90} [N(a,i,22,1)) + M(a,i,20,1)] = LCB^{p}(i). \tag{47}$$

4. Equilibrium Conditions

In general, equilibrium supply has to equal demand in all markets. The national capital market equilibrium has to fulfill

$$A(i) + AP(i) = q(i)K(i) + B(i) + B^{f}(i), \tag{48}$$

where $B^f(i)$ denotes net foreign assets in the small open economy case. The national goods market now balances domestic supply and demand, i.e.

$$Y(i) = C(i) + \Delta K(i) + G(i) + TB(i) = Y^{D}(i), \tag{49}$$

where $TB(i)$ denotes the trade balance in the small open economy. Finally, the labor market equilibrium implies

$$L(i) = \sum_{k=1}^{3} \sum_{a=21}^{90} \Bigg\{ \sum_{s=23}^{45} E(a,k)\,[h(a,i) - \ell(a,i,s,k,n)]\,N(a,i,s,k) + \sum_{s=21}^{43} E(a,k)\,[h(a,i) - \ell(a,i,s,k,m)]\,M(a,i,s,k) \Bigg\}. \tag{50}$$

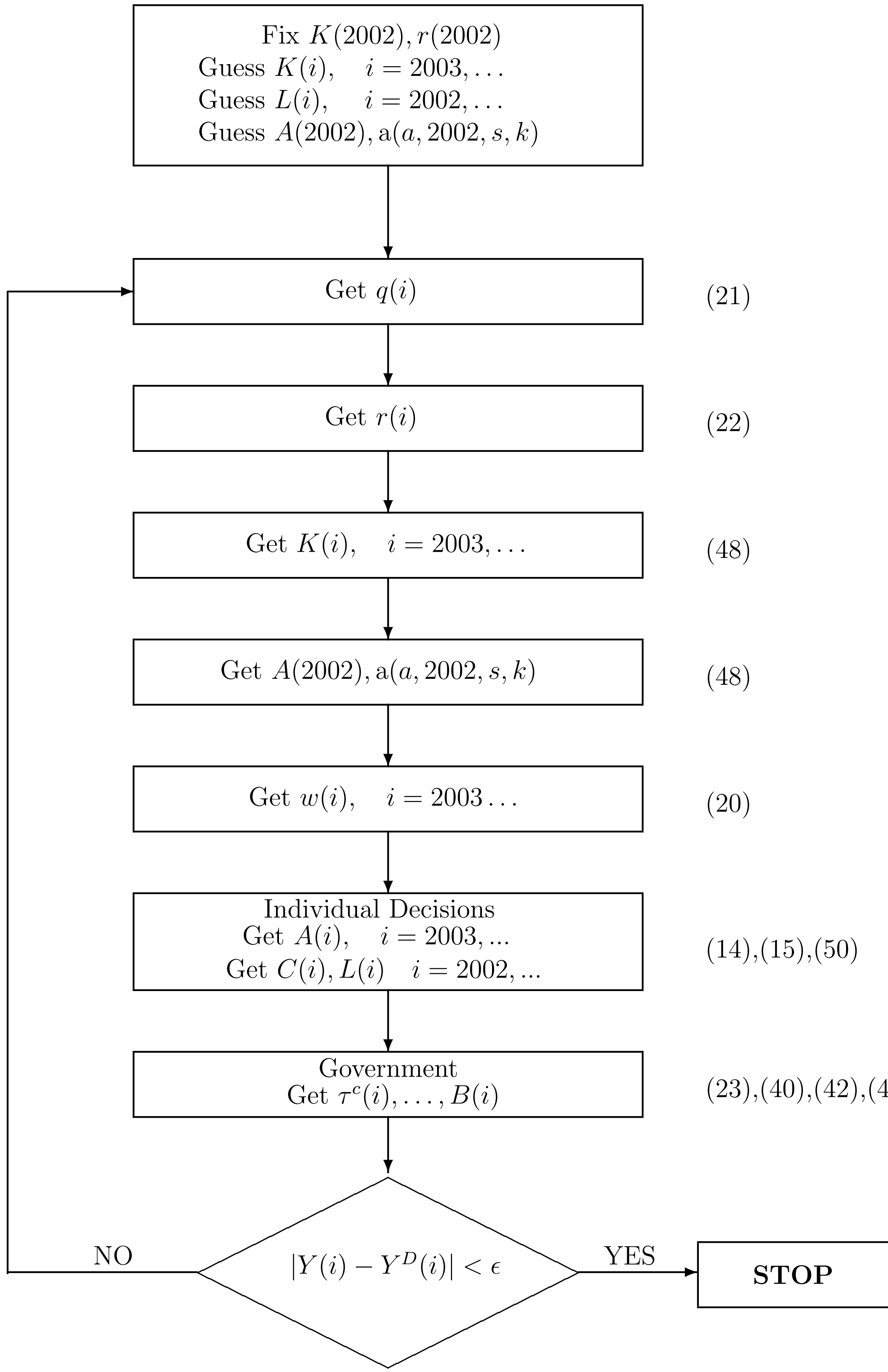
This completes the description of the model.

[13] For the transition period i from positive to negative accumulated assets, we use the rest of the assets in i to reduce the long-term care system deficit in the same period and then calculate the endogenous contribution rate $\hat{\tau}^{lc}(i)$.

IV. Solving the Model

Figure 2 gives an overview of the solution method for our simulation model which we now explain.
Given the capital stock, the interest rate and the asset profiles in year 2002, our model applies a Gauss-Seidel algorithm to solve for the perfect foresight general equilibrium transition path of the economy. Starting points are initial guesses for the capital stock for the remaining years of the transition and for the aggregate labor supply and assets for all transition years. The path for the interest rate after year 2002 is derived from the arbitrage condition (22). Applying equation (48), we get the capital stocks from 2003 onwards as well as the updated aggregate savings in 2002. According to these aggregated assets, we update the level of the initial asset profile. Next, the wage rate which is equal to the marginal product of labor is computed. Given initial assets, the time path of factor prices, household decisions on consumption and labor supply are computed and aggregated. Then, we update the path for tax rates, social security contributions and debt given the government budget constraints (23), (40), (42) and (46). Finally, we check if supply equals demand for all years, using equilibrium condition (49). The algorithm then iterates until the path of capital stock and labor converges, i.e. until markets are balanced. In the model, the transition path to the final steady state takes 300 years. In the open economy case the algorithm is quite similar. However, we keep the interest rate fixed at the year 2002 level of the closed economy. Capital prices are then computed applying equation (22) and the path of the capital stock results from equation (21). In contrast to the closed economy, we do not adjust initial aggregate savings but net foreign assets according to equation (48).

Figure 2: Solution method for the simulation model



V. Calibration Issues

In order to solve our model, we first need to specify the preference, technology and policy parameters to get realistic values for our starting year 2002. Table 2 reports our parameter values.

Table 2: Parameter values of the Model

Preferences and technology	**Symbol**	**Value**
Utility function		
Time preference rate	θ	0.015
Intertemporal elasticity of substitution	γ	0.25
Intratemporal elasticity of substitution	ρ	0.8
Leisure preference parameter	α	1.5
Production function		
Technology level	ϕ	5.5
Capital share in production	ε	0.25
Adjustment cost parameter	ψ	10.0
Technical progress	λ	0.01
Policy parameters		
Capital tax rate	τ^r	0.14
Inheritance tax rate	τ^b	0.028
Deductable income-connected expenses (in % of gross labor income)	β	0.11
Debt (in % of GDP)	B/Y	0.6
Retirement age in 2002	$\bar{a}$	60
APV (per month) in 2002 (in €)	APV	25

The values for the inter- and intratemporal elasticity of substitution, the leisure preference parameter and the time perference rate are taken from Auerbach and Kotlikoff (1987) or Kotlikoff et al. (2001). The same applies on the production side for the elasticity between capital and labor, the capital share in production, the adjustment cost parameter and technical progress. The time endowment in the year 2002 is set to 4000 hours. The technology level ϕ is then specified in order to yield a realistic gross annual income level in the lowest income class.

Next, we specify the policy parameters. The per capita outlays of general public goods g were adjusted to get realistic government purchases of goods and services as per cent of GDP in year 2002 as reported by the Deutsche Bundesbank (2003). Age-specific education costs were provided by Bernd Raffelhüschen. The original data was slightly adjusted in order to get realistic GDP shares which are reported by Institut der deutschen Wirtschaft (2003). The same applies to child allowances.

Next, we turn to the social security system. Age-specific profiles for health and long-term care

are again provided by Bernd Raffelhüschen. As before, they are adjusted in order to yield realistic health and long-term care contribution rates and GDP shares. With respect to the pension system, the applied Actual Pension Value from the year 2002 is sufficient to yield a realistic contribution rate and GDP share in 2002.

Our model also requires an initial distribution of assets by age and income class. These profiles are generated by an artificial steady state simulation. In addition we also had to specify the initial capital stock in the base year 2002. Capital stock and asset endowments were adjusted in order to yield realistic saving rates and a capital coefficient in the base year.

VI. Initial Equilibrium and Baseline Path

In this section, we report the simulation results for the baseline path of our model. Changing variables during the transition are due to the aging process as well as to the above mentioned changes in the social security system. All other actual policies are held fix throughout the transition, but the consumption tax is adjusted to balance the budget in each year.

1. The Initial Year 2002

The following tables show the macroeconomic structure in the initial year 2002 of the transition. For this year, we tried to replicate a realistic macroeconomic structure and highlight the differences in the structure of the public sector. Of course, due to the restrictions of the theoretical model our data will sometimes deviate from reality.

Table 3: The year 2002 of the baseline path

Indicator	Model		Official*
	Closed	Open	
National Income			
Private consumption	71,3	70,6	69,0
Government purchases of goods and services	24,0	23,7	22,5
Investment	2,8	1,4	3,4
Trade balance	0,0	2,2	4,9
Government indicators (in % of GDP			
Aggregate pension benefits	12,2	12,1	13,0
Aggregate health benefits	6,7	6,7	6,8
Aggregate long-term care benefits	0,8	0,8	0,8
Aggregate education outlays	4,0	4,0	4,0
Aggregate child allowances	1,9	1,9	1,6
Public debt	60,0	60,0	60,1
Interest payments on public debt	4,0	4,0	3,3
Total tax revenue	21,6	21,6	19,6
Wage tax	8,6	8,6	6,6
Interest income	1,2	1,2	1,1
Consumption tax	11,6	11,6	11,8
Inheritance tax	0,1	0,1	0,1
Tax and contribution rates (in %)			
Consumption tax rate	16,3	16,4	16,0
Average wage tax rate	12,4	12,4	app. 15
Marginal wage tax rate	28,4	28,5	app. 30
Pension contribution rate	19,4	19,3	19,1
Health care contribution rate	14,4	14,4	14,0
Long-term care contribution rate	1,7	1,7	1,7
Capital coefficient	3,7	3,7	3,5
Interest rate (in %)	6,8	6,8	-
National saving rate (in %)	10,7	11,7	10,4

All data in per cent of GDP if not stated different.

*Source: Deutsche Bundesbank (2003), Deutsche Bank Research (2003), Institut der deutschen Wirtschaft Köln (2003)

Table 4: Income distribution in the year 2002 of the baseline path

Indicator	**Model**		**Official***
	Closed	**Open**	**2001**
Fractions of average disposable income			
1. Quintile	8,4	8,4	8,5
2. Quintile	14,5	14,7	14,3
3. Quintile	19,8	19,7	18,2
4. Quintile	21,5	21,5	23,1
5. Quintile	35,7	35,7	35,9
Average disposable income (in €)			
1. Quintile	9.492	9.542	8.272
2. Quintile	16.417	16.559	13.857
3. Quintile	22.373	22.389	17.669
4. Quintile	24.248	24.247	22.425
5. Quintile	40.247	40346.	34.714
Average disposable income (in €)	22.563	22.625	19.388
Average gross labor earnings (in €)	26.895	26.946	28.518
Gini-Coefficient before tax	28,3	28,2	-
Gini-Coefficient after tax	25,9	25,8	-

*Source: Grabka et. al. (2003)

For almost all government and macroeconomic indicators, we come very close to the official values. Wage tax revenues and interest payments on public debt deviate from realistic values since we do not consider unemployment and social benefit payments (*Sozialhilfe*). In addition, our nominal interest rate is fairly high.

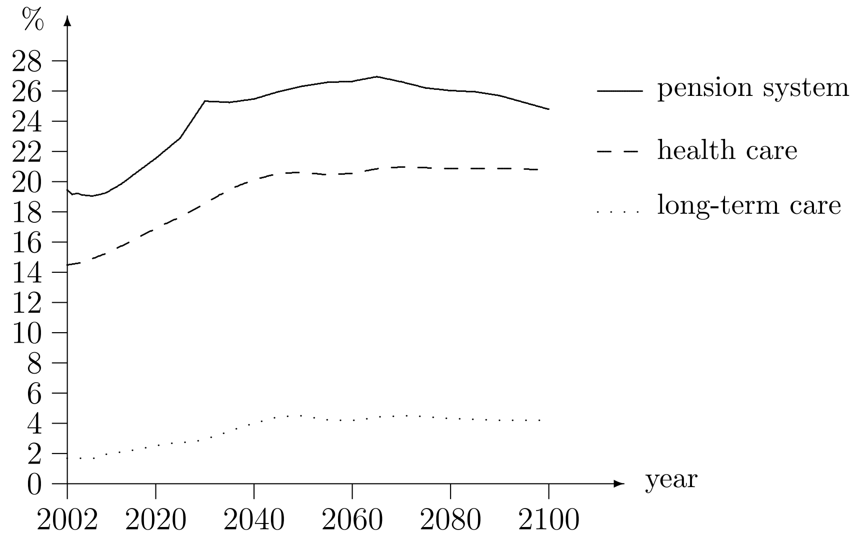
Table 4 indicates that gross-labor earnings are lower than officially, while disposable income is higher than in reality. Again, this is due to government programs which are not included in the model. However, the upper part of Table 4 shows that the model reproduces the relative income distribution in the base year quite well.

2. The Baseline Transition Path

Next, we turn to the transition path of the baseline simulation. Figure 3 shows the dynamics of the social security contribution rates between 2002 and 2100.

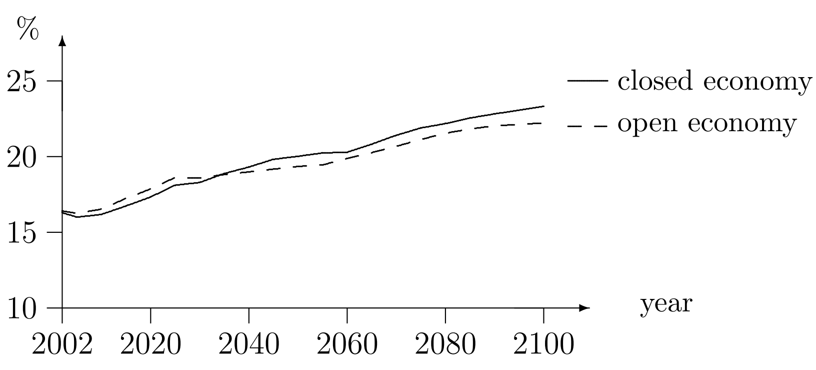
While the 2001 pension reform aimed at flattening the rise of the contribution rate and even preventing it to exceed 22 percent until 2030, our model suggests an increase up to 25.4 in 2030 and further to 26.8 percent in 2065. The first "boost" of the contribution rate goes along with the accelerated aging process in the 2020ies and 30ies. Demographic aging continues but looses speed in the 2040ies which leads to the further, minor increase of the contribution rate.

Figure 3: Social security contribution rates



After that, the contribution rate falls and reaches a value of 24.6 percent in 2100 which is still much higher than in the initial year. Due to the aging process, health and long-term care contribution rates also increase substantially until 2050. The increase in health care and long-term care contribution rates takes place although those population groups with the highest life expectancies and hence the highest health and long-term care costs are not even insured in the public system.

Figure 4: Consumption tax rate



Since the consumption tax is the only endogenous tax rate it rises in the future due to decreasing revenues from labor and interest income taxes as well as to rising government expenditures.

Figures 5 and 6 show the development of capital stock and labor during the next century relative to the initial year 2002. Labor supply is increasing although Germany is aging at high speed due to the assumed labor-augmenting technical progress which raises the time endowments of successive cohorts by one per cent.

Figures 7 and 8 show that the average wage rate is almost constant until 2050 and then decreases

Figure 5: Aggregate capital stock (relative to 2002)

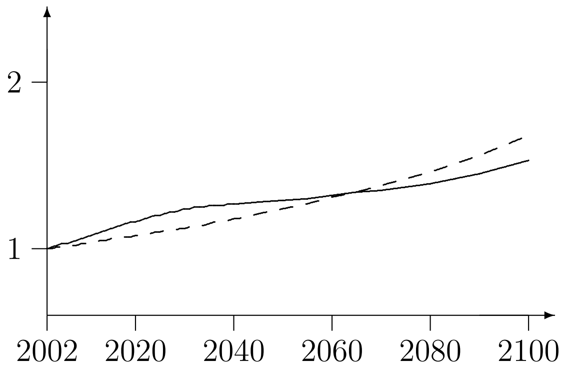
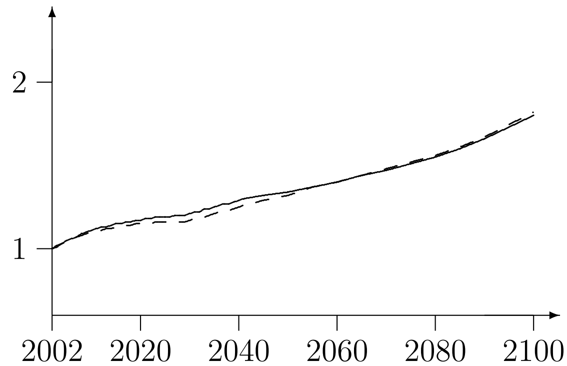


Figure 6: Aggregate labor supply (relative to 2002)



slightly by 4 percentage points. This effect is even weaker in the open economy because of the constant interest rate. The long-term decrease of the wage rate is due to the development of the effective labor supply that exceeds the growth of the capital stock in the long run.

Figure 7: Wage rates (relative to 2002)

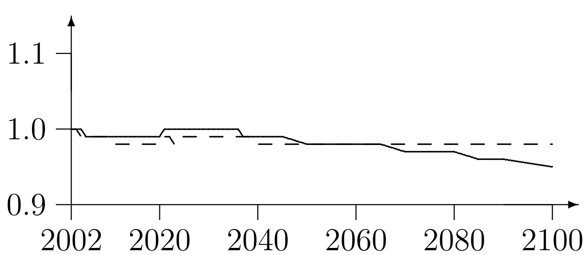
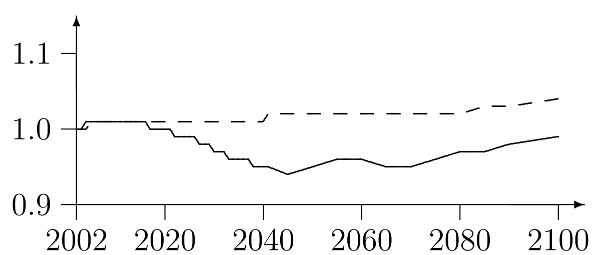
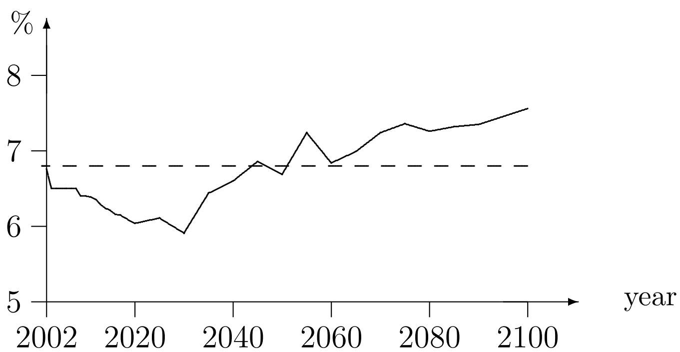


Figure 8: Price of capital (relative to 2002)



In the closed economy case the price of capital (see Figure 8) is almost constant until 2015. It begins to fall when the aging process starts to get severe and recovers after 2050. As the share of the elderly increases, the demand for capital decreases, because they reduce their savings in order to consume. The interest rate decreases when the demographic situation deteriorates, it recovers after 2030 and increases again after the initial level.

Figure 9: Interest rate



This completes the overview of the development path of the most important variables in our simulation model during the demographic transition in the coming 100 years.

Appendix

The appendix reports in detail our benchmark population data, such as population by age groups, fertiliy and mortality rates.

Table A-1: Population structure in Germany in 2002

Age	Native Population $\bar{N}(a, 2002)$	Foreign Population $\bar{M}(a, 2002)$	Net-immigrants $M^n(a, 2002)$
0	746.646	0.000	0.000
1	765.385	1.551	1.633
2	787.800	2.550	1.133
3	699.284	78.777	0.386
4	712.116	81.129	0.278
5	718.357	84.039	0.546
6	715.166	81.741	0.131
7	708.326	77.476	0.065
8	711.310	80.314	1.059
9	729.699	83.451	1.248
10	756.652	85.171	1.519
11	804.994	84.411	1.894
12	840.618	86.170	2.158
13	875.566	86.812	2.596
14	874.626	87.071	2.679
15	874.603	84.748	3.333
16	857.022	81.734	4.448
17	843.427	81.158	4.992
18	834.987	84.355	6.580
19	845.884	91.399	10.593
20	853.254	105.259	13.426
21	854.505	121.618	11.611
22	832.184	136.868	12.748
23	812.143	148.311	12.050
24	789.851	154.823	10.907
25	778.594	160.486	8.720
26	760.029	167.385	6.416
27	749.399	173.510	5.330
28	750.418	176.925	4.214
29	791.375	178.630	3.301
30	864.137	180.307	2.288
31	954.572	177.204	1.429
32	1038.015	181.358	1.896
33	1114.837	176.404	1.678
34	1185.394	170.543	1.683
35	1245.220	157.994	1.609
36	1273.021	159.612	1.833
37	1300.581	158.306	1.774
38	1316.558	152.247	1.993
39	1317.668	142.924	2.199
40	1302.633	134.114	2.346
41	1283.487	119.939	2.492
42	1244.602	126.003	2.399
43	1209.296	113.418	2.565
44	1169.830	107.163	0

Table A-1 continued

Age	$\bar{\mathbf{N}}(\mathbf{a},\mathbf{2002})$	$\bar{\mathbf{M}}(\mathbf{a},\mathbf{2002})$	$\mathbf{M^n}(\mathbf{a},\mathbf{2002})$
45	1135.464	100.127	0
46	1105.278	102.291	0
47	1083.023	98.692	0
48	1060.014	94.210	0
49	1050.989	88.271	0
50	1030.870	92.079	0
51	1034.393	86.628	0
52	1007.543	97.796	0
53	973.186	95.013	0
54	915.389	93.782	0
55	833.638	90.937	0
56	754.869	82.541	0
57	775.396	74.760	0
58	836.445	70.268	0
59	919.224	66.531	0
60	984.424	67.714	0
61	1072.848	61.890	0
62	1151.792	65.895	0
63	1146.780	58.058	0
64	1089.830	54.679	0
65	1033.678	48.600	0
66	991.533	43.077	0
67	951.171	36.749	0
68	832.875	33.505	0
69	728.976	29.667	0
70	673.118	25.180	0
71	684.245	21.538	0
72	684.048	20.924	0
73	679.063	17.372	0
74	642.175	15.389	0
75	606.875	13.729	0
76	568.830	12.563	0
77	531.798	11.300	0
78	493.119	10.243	0
79	460.698	9.051	0
80	443.500	8.069	0
81	422.549	6.924	0
82	361.270	6.045	0
83	266.944	4.470	0
84	181.114	3.099	0
85	134.326	2.403	0
86	127.092	2.160	0
87	127.084	2.111	0
88	112.150	1.956	0
89	76.086	1.132	0
90	30.590	0.508	0
$\sum$	74.300.380	7.235.340	164.180

Table A-2: Fertility and mortality rates in 2002

Fertility rates	
Age	2002-2050
23	0.1066
24	0.1128
25	0.1175
26	0.1199
27	0.1208
28	0.1201
29	0.1165
30	0.1093
31	0.0988
32	0.0859
33	0.0720
34	0.0581
35	0.0446
36	0.0334
37	0.0251
38	0.0186
39	0.0133
40	0.0091
41	0.0059
42	0.0036
43	0.0020
44	0.0010
45	0.0004
TFR	**1.4**
ABA	**29.01**

TFR= Total fertility rate,
ABA= Average birth age

Low class mortality rates				
Age	2002		2050	
	$\bar{d}(\cdot)$	$d(\cdot)$	$\bar{d}(\cdot)$	$d(\cdot)$
68	0.026	0.026	0.002	0.002
69	0.028	0.028	0.005	0.005
70	0.029	0.031	0.010	0.010
71	0.031	0.034	0.018	0.018
72	0.033	0.038	0.026	0.027
73	0.036	0.042	0.032	0.034
74	0.038	0.047	0.035	0.039
75	0.041	0.052	0.037	0.043
76	0.043	0.059	0.039	0.047
77	0.046	0.066	0.042	0.052
78	0.048	0.074	0.044	0.059
79	0.050	0.083	0.047	0.066
80	0.052	0.094	0.049	0.074
81	0.053	0.106	0.051	0.084
82	0.055	0.122	0.053	0.095
83	0.055	0.141	0.055	0.109
84	0.055	0.163	0.057	0.125
85	0.054	0.190	0.058	0.147
86	0.052	0.226	0.059	0.174
87	0.049	0.277	0.059	0.211
88	0.046	0.359	0.058	0.264
89	0.042	0.514	0.057	0.350
90	0.030	0.749	0.055	0.516
91	0.010	1.000	0.051	1.000
LE	**78.01**		**82.49**	

LE= Life expectancy,
$\bar{d}(\cdot)$ = unconditional death probability,
$d(\cdot)$ = conditional death probability

Table A-2 continued

Age	Middle class mortality rates				High class mortality rates			
	2002		2050		2002		2050	
	$\bar{d}(\cdot)$	$d(\cdot)$	$\bar{d}(\cdot)$	$d(\cdot)$	$\bar{d}(\cdot)$	$d(\cdot)$	$\bar{d}(\cdot)$	$d(\cdot)$
68	0.022	0.026	0.001	0.001	0.016	0.018	0.002	0.002
69	0.024	0.028	0.005	0.005	0.018	0.020	0.004	0.004
70	0.026	0.031	0.008	0.008	0.019	0.023	0.005	0.005
71	0.028	0.034	0.010	0.010	0.021	0.025	0.006	0.006
72	0.030	0.038	0.010	0.010	0.022	0.027	0.007	0.007
73	0.032	0.042	0.011	0.011	0.024	0.030	0.008	0.008
74	0.034	0.047	0.011	0.012	0.026	0.034	0.008	0.008
75	0.036	0.052	0.012	0.012	0.028	0.038	0.008	0.009
76	0.039	0.059	0.013	0.013	0.030	0.043	0.008	0.009
77	0.041	0.066	0.014	0.015	0.033	0.049	0.009	0.009
78	0.043	0.074	0.014	0.016	0.035	0.054	0.009	0.010
79	0.045	0.083	0.015	0.017	0.038	0.061	0.009	0.010
80	0.047	0.094	0.019	0.021	0.040	0.070	0.010	0.011
81	0.050	0.106	0.030	0.036	0.042	0.079	0.012	0.013
82	0.054	0.122	0.051	0.061	0.045	0.090	0.018	0.020
83	0.058	0.141	0.074	0.096	0.050	0.106	0.030	0.035
84	0.062	0.163	0.101	0.144	0.055	0.123	0.050	0.060
85	0.065	0.190	0.127	0.211	0.060	0.143	0.077	0.097
86	0.064	0.226	0.134	0.283	0.066	0.171	0.107	0.149
87	0.059	0.277	0.117	0.344	0.069	0.206	0.133	0.217
88	0.053	0.359	0.091	0.408	0.071	0.259	0.144	0.301
89	0.046	0.514	0.066	0.501	0.069	0.344	0.135	0.405
90	0.032	0.749	0.043	0.653	0.064	0.511	0.113	0.569
91	0.012	1.000	0.023	1.000	0.058	1.000	0.086	1.000
LE	**80.03**		**84.38**		**81.73**		**86.29**	

LE= Life expectancy, $\bar{d}(\cdot), d(\cdot)$ unconditional and conditional death probabilities

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