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A simulation model for the demographic transition in the OECD –

Data requirements, model structure and calibration

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December 2003

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A simulation model for the demographic transition in the OECD*

Data requirements, model structure and calibration

by

Hans Fehr, Gitte Halder, Sabine Jokisch and Laurence J. Kotlikoff

December 2003

Abstract

The developed world stands at the fore of a phenomenal demographic transition. Over the next 30 years the number of elderly in the OECD countries will more than double. At the same time, the number of workers available to pay the elderly their government-guaranteed pension and health care benefits will rise by less than 10 percent. These two demographic trends are expected to put enormous pressure on social security systems and government expenses. To address the consequences of the aging process, this paper develops a dynamic, intergenerational, and interregional demographic life-cycle model. The model has three regions - the U.S., the EU and Japan - which exchange goods and capital. The model features immigration, age-specific fertility, life span extension, life span uncertainty, bequests arising from incomplete annuitization, and intra-cohort heterogeneity. After introducing the theoretical model, we simulate the transition path for the three considered regions keeping current immigration constant, assuming the projected increase in life expectancy and the continuation of current social security and health care policies.

JEL classification: H0

Keywords: Demographic transition, overlapping generations (OLG), computable general equilibrium models (CGE)

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List of Symbols

Indices

t, i, z	index for years during the transition
a, j, u, m	index for age of individuals
s	index for parents age at time of birth
k	index for income class
x	region index which is usually omitted (i.e. $N(a,i,s)$ should be $N(a,i,s,x)$ etc.)
W	index for world

Population

$N(a,i,s,k)$	number of agents of age a in year i whose parents were age s when they were born and who belong to income class k (in region x)
$\hat{N}(a,i,k)$	total number of agents of age a in year i who belong to income class k
$\overline{N}(a,i)$	total number of agents of age a in year i
$NM(a,i,k)$	number of newly arrived net-immigrants of age a in year i in income class k
$\overline{NM}(a,i)$	number of newly arrived net-immigrants of age a in year i
$\Upsilon(k)$	share of income class k in population
$Pop(i)$	total population in year i
$KID(a,i,k)$	number of children of a household who is a years old in year i and who belongs to income class k
$\bar{d}(a,i), d(a,i)$	unconditional and conditional death probabilities
$P(a,i)$	survival probability
$f(a,i)$	fertility rate
$TFR(i)$	total fertility rate in year i
$ABA(i)$	average birth age in year i
$LE(i)$	life expectancy in year i
$n(i), \bar{n}$	endogenous and exogenous population growth rate

Household sector

$U(j,t,s,k)$	utility function of agent age j in year t whose parents were age s at birth and who belongs to income class k (in region x)
$V(j,t,s,k)$	utility of agent derived from his own consumption
$H(j,t,s,k)$	utility of agent derived from the consumption of his children
$c(a,i,s,k)$	consumption of goods of agent
$c_K(a,i,s,k)$	consumption per child of agent
$\ell(a,i,s,k)$	leisure consumption of agent
$W(a,i,s,k)$	gross labor income of agent
$\mathrm{a}(a,i,s,k)$	assets of agent
$I(a,i,s,k)$	inheritance of agent

$T(a,i,s,k)$	net tax payments of agent
θ	pure rate of time preference
γ	intertemporal elasticity of substitution
ρ	intratemporal elasticity of substitution
α	leisure preference parameter
$h(a,i)$	time endowment at age a in year i
$E(a,k)$	human capital profile of a native at age a in income class k
λ	rate of technological growth
$\xi(k)$	productivity index for income class k

Production sector

$F(\cdot)$	production function for gross output
$F_{K(i)}$	marginal product of capital in year i
$F_{L(i)}$	marginal product of labor in year i
$\Phi(\cdot)$	adjustment cost function
$\Phi_{K(i)}$	marginal adjustment cost due to the capital stock in year i
$\Phi_{\Delta K(i)}$	marginal adjustment cost due to investment in year i
ϕ	technology parameter
ε	capital share in production
σ	elasticity of substitution between capital and labor
ψ	adjustment cost coefficient
δ	rate of economic depreciation

Government sector

$\tau^w(a,i,s,k)$, $\bar{\tau}^w(a,i,s,k)$	marginal and average (individual) wage tax rate in year i
$\tau^c(i), \tau^r(i), \tau^k(i)$, $\tau^b(i)$	consumption, capital, corporate and inheritance tax rate in year i
$\tau^p(a,i,s,k)$, $\bar{\tau}^p(a,i,s,k)$	marginal and average (individual) payroll tax rate in year i
$\tau^h(a,i,s,k)$ $\bar{\tau}^h(a,i,s,k)$	marginal and average (individual) health care contributions in year i
$\tau^d(a,i,s,k)$ $\bar{\tau}^d(a,i,s,k)$	marginal and average (individual) diability insurance contributions in year i
$\hat{\tau}^p(i), \hat{\tau}^h(i), \hat{\tau}^d(i)$	aggregate pension, health care and disability insurance contribution rate in period i
$T^k(i)$	corporate tax revenues
$G(i), g$	total and per person public consumption in year i
$B(i), b(i)$	stock of public debt and in relation to GDP in year i
$\Delta B(i)$	government deficit in year i
$Pen(a,i,s,k)$	individual pension benefit
$PB(i)$	aggregate pension benefits in year i
$HB(i)$	aggregate health transfers in year i

$DB(i)$	aggregate disability transfers in year i
$PY(i)$	aggregate base for social security contributions
$\mu_1(i)$	fraction of pension outlays financed by general taxes
$\mu_2(i)$	fraction of health costs financed by general taxes
$\chi(\bar{W}(z,s,k))$	(individual) replacement rate as a function of indexed lifetime income $\bar{W}$
$\bar{a}(i)$	retirement age set by government in year i
$\hat{W}(i)$	average labor earnings in year i
Θ	contribution ceiling
$hc(a)$	age-specific profile of health costs
di	equal disability transfer
$edu(a)$	age-specific profile of education costs for children
β_0, β_1	parameter for wage tax function
ω_0, ω_1	parameter for replacement rate function

Prices

$w(i)$	gross wage rate in year i
$r(i)$	interest rate in year i
$q(i)$	shadow price of capital in year i
$R(i,t)$	gross compound interest rate

Aggregate variables

$Y(i)$	firm marketable output in year i
$L(i)$	aggregate labor supply in year i
$K(i)$	capital stock in year i
$\Delta K(i)$	investment outlays in year i
$A(i)$	aggregate savings in year i
$\mathcal{A}(i)$	aggregate assets existing at the beginning of year i
$\bar{A}(a,i,k)$	aggregate savings of age a agents of income class k in year i
$\hat{A}(i)$	aggregate savings of new immigrants in period i
$S(x)$	share of region x in the world assets
$C(i)$	aggregate consumption in year i
$TB(i)$	trade balance in year i
$B^f(i)$	stock of net foreign bonds in year i
$\Delta B^f(i)$	current account surplus in year i
$DIV(i)$	dividend payments at the end of year i
$V(i)$	value of the firm in period i

I. Introduction

The present paper aims to describe in detail the population projections and the theoretical structure of a new simulation model which is applied to analyze the economic impact of the demographic transition in the OECD.

Our project intends to provide an aggregated picture as well as detailed figures for selected regions. Consequently, our approach is twofold. In order to analyze the consequences of population aging on worldwide capital markets we develop a multi-region model consisting of the EU-15 region, the USA and Japan. In each region/country we model the population dynamics in a similar way. Although we account for immigration, natives and immigrants are not distinguished within each cohort. Immigrants arrive at different ages, but they arrive with identical asset endowments and preferences as the respective natives. Consequently, they behave like natives after arrival. Within each cohort we disaggregate three income classes similar as Fehr (2000), Beetsma et al. (2001) or Kotlikoff et al. (2001). Consequently we are able to take into account differences in the progressivity of national social security and tax systems. Such a set-up allows us to analyze issues of intra-generational as well as inter-generational and inter-national redistribution.

The next section discusses the data sources, the underlying assumptions and the baseline population projections for EU-15, USA and Japan. Then we describe the basic structure of the simulation model. Since the population projection and the theoretical model structure is identical for each region/country, we can concentrate on a representative household economy and omit a region index in the following section. Afterwards, we report the calibration issues for the simulation model and the baseline transition path.

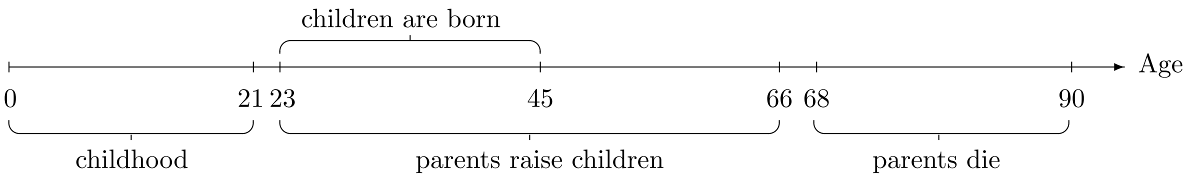
II. Modelling population dynamics

This chapter discusses our population model. We start with a description of the general model structure. Then we explain the raw data sources and necessary adjustments in the year 2000. Section 3 describes the population projections between 2000 and 2050 and section 4 the years after 2050. Finally some concluding remarks are provided.

1. Basic structure of the model population

Each economy in our model is populated by households who live up to a maximum age of 90. Consequently, we distinguish up to 91 generations within each period t. The individual life-cycle of a representative agent is described in Figure 1. Between ages 0 and 20 our households are children, who earn no money and are fed by their parents. At age 21 our agents leave their parents and start working. Between ages 23 and 45 our agents have children at the beginning of each period, i.e. children are age 0 when the parents are 23 and age 20 when the parents are 43. Between ages 46 and 66 our agents continue to raise their children. The last children who were born to age 45 parents leave their parents when the latter are age 66. Our agents die between ages 68 and 90. The probability of death is one at age 91. Consequently, the youngest child (born when the parents were 45) of parents who die early at age 68 has already reached adulthood while the oldest child (born when the parents were 23) of parents who die at age 91 is 68, i.e. parents always outlive grandparents.

Figure 1: The individual life-cycle



Each year new immigrants arrive with their children. After crossing the border, immigrants become automatically natives in an economic sense, i.e. they have identical wealth endowments, preferences and life-cycle characteristics as the natives.

The next section describes our data set for the benchmark population in the initial year 2000.

2. Benchmark population in the year 2000

The different national and supranational statistical offices provide an enormous amount of population data. The main raw data for our population model was therefore bought from the statistical office of the European Communities (EUROSTAT). Population data for the USA is provided by the bureau of census (www.census.gov./population/www/), while the data on Japan was taken from the Statistical Yearbook (www.stat.go.jp/english/) and the Institute of Population and Social Security Research (www.ipss.go.jp/English/). However, part of the raw data was only supplied in aggregates. In addition, the specific structure of our population model imposed certain restrictions on our data set. Consequently, various adjustments had to be made in order to arrive at the data set which is reported in the appendix. In the following, we explain our most important adjustments to the raw data.

Table A-1 reports the existing (native) $[\overline{N}(a, 2000)]$ population structure in the year 2000 in our three regions/countries. As much as possible, we took the original figures from the statistical offices. Our adjustments were mainly twofold. First, the newborns were calculated with the age-specific fertility rates explained below. Second, agents who are older than 90 in the original data were erased and the respective numbers for the ages between 85 and 90 were increased proportionally. Note that the cohorts in Table A-1 already include the net-immigrants of the year 2000.

We restrict immigration up to age 65 per definition mainly for simplification reasons, i.e. the oldest immigrant will at least live for some years before he dies in his new homeland. Detailed age-specific net-immigration $[\overline{NM}(a, 2000)]$ data of year 2000 was only available for the USA and Germany. For EU-15 and Japan the raw data only reports the total net-immigration in 2000. Therefore, in Table A-2 we assumed for EU-15 and Japan identical age structures to those in Germany.

Age-specific birth rates are available for all regions in our sample. However, since it is not possible to have children before age 23, the children of 15 to 22-year old mothers had to be assigned to older ones. Table A-3 reports the adjusted numbers of newborns per mother of a certain age in the year 2000 and our benchmark projection for the year 2050. Note that we do not distinguish between sexes in the model. Consequently, the fertility rate $f(a,i)$ which denotes the number of children of an agent of age a in year i is half of the respective number reported in Table A-3. For the USA detailed fertility projections are available until the year 2100. For Western Europe, Germany and Japan the United Nations (unstats.un.org/unsd/demographic/default.htm or /ww2000/table2b.htm) provide different fertility projections for the whole century. However, since most of these figures were highly aggregated, we adjusted our reported birth rates of the year 2000 in order to approach the aggregate estimates of the raw data for 2050. The resulting figures are reported in Table A-3 together with the total fertility rates per woman (TFR) and the average birth ages (ABA) for the selected years. The latter are computed according to

$$\text{TFR}(i) = \sum_{a=23}^{45} f(a,i) \qquad \text{ABA}(i) = \frac{\sum_{a=23}^{45} f(a,i)a}{TFR(i)}$$

Finally, birth rates for the years 2000 until 2050 were computed by linear interpolation.

The final component for our population model are region-specific mortality rates. Again we took the official numbers from the raw data and adjusted them so that they could fit our model restrictions. As already noted above, agents don't die in our model before age 68 and don't survive age 90. Consequently, the probability of an agent who is age a in year i to die in this year, $d(a,i)$, has to be

$$d(a,i) \begin{cases} = 0 & \text{for} \quad 0 \le a \le 67 \\ > 0 & \text{for} \quad 68 \le a \le 90 \\ = 1 & \text{for} \quad a = 91. \end{cases}$$

Table A-4 reports these so called "conditional" death probabilities $d(a,i)$ as well as the corresponding "unconditional" death probabilities $\bar{d}(a,i)$, i.e. the probabilities of an agent who is currently at an age below 68, that he will die at a certain age in the future. The unconditional death probabilities are easier to interpret and to adjust. The latter are computed from[1]

$$\bar{d}(a,i) = d(a,i) \prod_{j=1}^{a-1} (1 - d(j,i)).$$

Given the unconditional death probabilities, the life expectancy (LE) in each period i could be computed from

$$LE(i) = \sum_{a=0}^{91} \bar{d}(a,i)a.$$

[1]An exact calculation would also take into account that the mortality rates change over time. To keep the conversion simple we assumed constant mortality rates.

Since disaggregated data on future mortality rates is not provided in official statistics, we adjusted our original year 2000 mortality rates in order to get realistic life expectancies for the year 2050. Note that our life expectancies at birth are higher than reported in official statistics since the model population can't die before the age of 68. However, our values come close to the life expectancies conditional on reaching age 65 in year 2000 reported by the OECD (2003). Table A-4 reports the mortality rates of the years 2000 and 2050. Again, the numbers of the period between 2000 and 2050 were filled by linear interpolation.

The next step is to disaggregate the population in year 2000 according to the structure of the economic model. Since in the economic model children receive bequests from their parents, we have to disaggregate each cohort between age 1 and up to age 68 according to the age of their parents when they were born. This disaggregation was achieved by applying past *relative* fertility shares to each cohort of age 1 to 68 in the year 2000. In addition, our population is split into three income classes so that we get

$$N(a,2000,s,k) = \Upsilon(k)\overline{N}(a,2000) \times \frac{f(s,2000-a)}{TFR(2000-a)} \quad \text{with} \quad a = 1,\dots,67,\ s = 23,\dots,45,\ k = 1,2,3.$$

For example, the cohort age 1 in year 2000 is disaggregated using the relative fertility rates of 1999, while the cohort age 40 in 2000 is disaggregated using the relative fertility rates of 1960. In most regions fertility rates have been available since the 50's. For the older cohorts we always assumed the latest available data. The parameter $\Upsilon(k)$ is the weight of the income class k in the total population. Here we assumed that 30 percent of the population belong to the lowest, 60 percent to the middle and 10 percent to the highest income class. For people older than 67 we only had to disaggregate the population in year 2000 into these income classes.

We still have to compute the numbers of newborns in year 2000 according to

$$N(0,2000,s,k) = \Upsilon(k)\overline{N}(s,2000) \times f(s,2000) \qquad s = 23,\dots,45,\ k = 1,2,3.$$

Summing up across all parents ages and income classes gives the notal number of newborn in year 2000

$$\overline{N}(0,2000) = \sum_{k=1}^{3}\sum_{s=23}^{45} N(0,2000,s,k),$$

which is reported in Table A-1.

This completes our calculations for the year 2000.

3. Population projections until the year 2050

For the years i between 2000 and 2050 population growth is computed endogenously given the exogenous fertility and mortality rates as well as the future net-immigration pattern and the population structure for the year 2000.

Consequently, the cohort age a in year i (of parents age s at time of birth) who belongs to income class k is simply computed as follows:

$$N(a,i,s,k) = (1-d(a,i))N(a-1,i-1,s,k) + NM(a,i,k) \times \frac{f(s,i-a)}{TFR(i-a)} \qquad a = 1,\ldots,90.$$

Note that we split the vector for net-immigration in year 2000 $\overline{NM}(a,2000)$ into income classes by multiplying with the same weight of the income class in total population $\Upsilon(k)$ as the existing population in year 2000.

Again, we aggregate within each cohort:

$$\hat{N}(a,i,k) = \sum_{s=23}^{45} N(a,i,s,k) \qquad a = 1,\ldots,90$$

and derive the newborn natives in year i as in year 2000

$$N(0,i,s,k) = \hat{N}(s,i,k) \times f(s,i) \quad s = 23,\ldots,45.$$

Of course, we could add up the newborn cohort across parents ages to derive $\hat{N}(0,i,k)$.

Adding up the cohorts across income classes, we derive the number of people of age a in year i as

$$\overline{N}(a,i) = \sum_{k=1}^{3} \hat{N}(a,i,k).$$

The total population in year i is computed from

$$Pop(i) = \sum_{k=1}^{3}\sum_{a=0}^{90} \hat{N}(a,i,k)$$

and is reported in the tables in subsection 5. The number of children who are fed by a household at a certain age will be important for his consumption later on. Given age a of an agent in year i who belongs to income class k this number is derived from

$$KID(a,i,k) = \sum_{j=u}^{m} \frac{N(j,i,a-j,k)}{\hat{N}(a,i,k)} \quad 23 \leq a \leq 65,$$

where $u = max(0; a-45)$ and $m = min(20, a-23)$. Agents below age 23 have no children, while after age 65 they have all left the household, i.e. $KID(a,i,k) = 0$ for $0 \leq a \leq 22$ and $66 \leq a \leq 90$.

The growth rate of the population $n(i)$ is computed from the change in the number of 21-year old compared to the previous year, i.e.

$$n(i) = \overline{N}(21,i)/\overline{N}(21,i-1) - 1.$$

Next we turn to the period after the year 2050.

4. Population projections after the year 2050

Between the period 2000 and 2050 we adjust migration as well as the fertility and mortality rates in order to model a realistic demographic transition. After the year 2050 we keep mortality constant and adjust migration and the fertility rates in order to run into a stable population structure in the future. Newborns and net-immigrants in the years after 2050 are consequently computed as follows:

$$\begin{aligned} N(0,i,s,k) &= (1+\bar{n})N(0,i-1,s,k) \qquad s=23,\dots,45,\ k=1,2,3 \\ NM(a,i,k) &= (1+\bar{n})NM(a,i-1,k) \qquad a=1,\dots,65,\ k=1,2,3 \end{aligned}$$

where $\bar{n}$ is the exogenously set population growth rate after the year 2050. It takes exactly 90 years (i.e. until 2140) until we arrive at a constant population structure in the model. Since we are not so interested in the far future, we just report the population structure for the year 2100 in the following tables where we assume $\bar{n} = 0$.

5. Summary

The following tables show the development of our model population between the years 2000 and 2100. These figures should be used as a basis for comparison with the official projections of the United Nations Population Division (2003).

Table 1: Population Projection EU-15

Year		**2000**	**2010**	**2020**	**2030**	**2040**	**2050**	**2100**
Life expectancy at birth								
Model		82.0	82.7	83.3	83.9	84.5	85.1	85.1
Official[a]		78.6	80.0	81.1	82.0	83.0	83.5	–
Fertility Rate								
Model		1.44	1.52	1.59	1.66	1.74	1.81	1.82
Official[a]		1.58	1.61	1.68	1.77	1.84	1.85	–
Average Birth Age								
Model		29.0	29.4	29.7	30.0	30.3	30.5	30.5
Official[a]		28.8	29.5	30.1	30.5	30.5	30.5	–
Total Population (in mio.)								
Model		376.3	385.7	390.5	390.9	384.4	372.9	340.2
Official[b]		377.3	383.2	384.4	382.8	377.8	369.8	-
Net-immigrants (in 1000)		450	450	450	450	450	450	450
Age Structure								
< 15	Model	16.9	15.3	14.4	14.2	14.2	14.7	16.4
	Official[b]	16.7	15.3	14.4	14.4	14.7	15.0	-
15-64	Model	67.0	66.9	64.7	60.7	57.5	56.8	59.3
	Official[b]	66.9	66.5	64.7	60.8	57.5	56.7	-
65-90	Model	16.1	17.8	20.9	25.1	28.3	28.5	24.4
	Official[b]	16.3	18.2	21.0	24.7	27.8	28.3	-

[a] UN Population Division (2003), Western Europe, Medium Variant Projections
[b] UN Population Division (2003), European Union, Medium Variant Projections

Table 2: Population Projection USA

Year		**2000**	**2010**	**2020**	**2030**	**2040**	**2050**	**2100**
Life expectancy at birth								
Model		82.1	82.8	83.5	84.2	84.9	85.6	85.6
Official[a]		77.1	78.3	79.1	79.9	81.0	81.6	-
Fertility Rate								
Model		2.04	2.06	2.07	2.09	2.10	2.12	1.81
Official[a]		2.11	2.08	2.03	1.95	1.89	1.85	-
Average Birth Age								
Model		27.3	27.2	27.2	27.2	27.1	27.1	27.1
Official[a]		26.6	26.9	27.2	27.4	27.7	27.8	-
Total Population (in mio.)								
Model		275.3	306.1	339.7	370.0	395.6	418.5	505.9
Official[a]		285.0	314.9	344.3	370.4	391.4	408.7	-
Net-immigrants (in 1000)		1000	1000	1000	1000	1000	1000	1000
Age Structure								
< 15	Model	21.3	19.9	19.8	18.9	18.9	19.3	16.4
	Official[a]	21.8	20.5	20.0	19.3	18.5	17.9	-
15-64	Model	66.1	67.2	63.7	60.3	60.0	59.8	60.6
	Official[a]	65.9	66.6	64.1	61.5	61.7	62.1	-
65-90	Model	12.6	12.9	16.5	20.7	21.1	20.9	23.0
	Official[a]	12.3	12.8	15.9	19.2	19.8	20.0	-

[a] UN Population Division (2003), Medium Variant Projections

Table 3: Population Projection Japan

Year		2000	2010	2020	2030	2040	2050	2100
Life expectancy at birth								
Model		84.0	84.6	85.2	85.8	86.4	87.1	87.1
Official		81.6	83.5	85.1	86.6	87.7	88.1	-
Fertility Rate								
Model		1.26	1.36	1.46	1.55	1.65	1.75	1.90
Official[a]		1.32	1.37	1.49	1.68	1.81	1.85	-
Average Birth Age								
Model		29.1	29.5	29.8	30.1	30.4	30.6	30.6
Official[a]		29.3	29.8	30.3	30.5	30.5	30.5	-
Total Population (in mio.)								
Model		126.7	128.4	126.2	120.6	112.9	107.0	83.1
Official[a]		127.0	128.0	125.6	121.0	115.7	109.7	-
Net-immigrants (in 1000)		54	54	54	54	54	54	54
Age Structure								
< 15	Model	14.6	13.4	12.5	11.9	12.5	12.9	16.2
	Official[a]	14.6	13.6	12.4	11.8	12.6	13.0	-
15-64	Model	67.8	64.2	59.5	58.5	55.4	52.6	56.9
	Official[a]	68.2	64.0	59.5	57.8	53.0	50.4	-
65-90	Model	17.6	22.4	28.0	29.6	32.1	34.5	27.0
	Official[a]	17.2	22.4	28.1	30.4	34.4	36.5	-

[a] UN Population Division (2003), Medium Variant Projections

III. The structure of the economic model

In this section we describe the economic model, which is used for our simulations. We begin with the household side and describe the decision problems of a representative household. Then we discuss the aggregation of the micro variables as well as the production side of the economy. Finally, the tax and transfer system of the multi-region system is explained.

1. The household sector

As already explained above, we do not distinguish between natives and immigrants in the model. The representative household leaves (unintended) bequests at the date of death due to imperfect annuitisation. All agents start to make their own economic decisions at the age of 21.

Our model assumes a preference structure that is represented by a time-separable, nested CES utility function. Remaining lifetime utility $U(j,t,s,k)$ of a generation of age j at time t whose parents were at age s at the time of birth and which belongs to income class k takes on the form

$$U(j,t,s,k) = V(j,t,s,k) + H(j,t,s,k), \tag{1}$$

where $V(j,t,s,k)$ denotes the utility parents receive from their own goods and leisure consumption and $H(j,t,s,k)$ denotes the utility they receive from their children's consumption. The two sub-utility functions are defined as follows:

$$V(j,t,s,k) = \frac{1}{1-\frac{1}{\gamma}} \sum_{a=j}^{90} \left(\frac{1}{1+\theta}\right)^{a-j} P(a,i) \left[c(a,i,s,k)^{1-\frac{1}{\rho}} + \alpha \ell(a,i,s,k)^{1-\frac{1}{\rho}}\right]^{\frac{1-\frac{1}{\gamma}}{1-\frac{1}{\rho}}} \tag{2}$$

$$H(j,t,s,k) = \frac{1}{1-\frac{1}{\gamma}} \sum_{a=j}^{90} \left(\frac{1}{1+\theta}\right)^{a-j} P(a,i) KID(a,i,k) c_K(a,i,s,k)^{1-\frac{1}{\gamma}}. \tag{3}$$

where $c(a,i,s,k)$ and $\ell(a,i,s,k)$ denote consumption and leisure respectively and i is defined as $i = t + a - j$. The children's consumption of income class k parents who are at age a at period i and whose parents were at age s at the time of their birth is defined as $c_K(a,i,s,k)$. Note that the number of children is independent of the grandparents' age when their parents were born. It varies over the life-cycle and changes in future time periods.

The uncertainties of life have forced to weigh consumption in future periods with the survival probability

$$P(a,i) = \prod_{j=0}^{a} [1 - d(j, j-a+i)], \tag{4}$$

i.e. by multiplying the conditional survival probabilities from birth up to year i. The parameters θ, ρ, α and γ represent respectively the "pure" rate of time preference, the intratemporal elasticity of substitution between consumption and leisure at each age a, the leisure preference and the intertemporal elasticity of substitution between consumption of different years.

The budget constraint of a 21-year old agent in year t whose parents were age s at his birth and who belongs to income class k is defined as follows:

$$\sum_{a=21}^{90}\Big[W(a,i,s,k)+I(a,i,s,k)-T(a,i,s,k)-c(a,i,s,k)- KID(a,i,k)c_K(a,i,s,k)\Big]R(i,t)=0 \quad (5)$$

where

$$W(a,i,s,k)=w(i)E(a,k)[h(a,i)-\ell(a,i,s,k)]$$

is the gross labor income of the agent and $w(i)$ is the gross wage rate in period $i=t+a-21$. Similarly to Altig et al. (2001) or Kotlikoff et al. (2001), we assume that technical progress causes the time endowment $h(\cdot)$ of each successive generation to grow at the rate λ, i.e.

$$h(a,i)=(1+\lambda)h(a,i-1). \quad (6)$$

The age-specific earnings ability profile

$$E(a,k)=\xi(k)e^{4.47+0.033(a-20)-0.00067(a-20)^2}(1+\lambda)^{a-21} \quad \text{with} \quad \xi(1)=0.2,\ \xi(2)=1.0,\ \xi(3)=5.0 \quad (7)$$

is taken from Auerbach and Kotlikoff (1987, 52). This profile is simply shifted by the parameter $\xi(k)$ in order to derive income class-specific profiles. Moreover, since technological change is an important determinant of secular growth in real wages during one's life cycle, we add this growth by multiplying the age-specific earnings ability profile with the term involving λ. Hence, the longitudinal age-wage profile is steepened by the rate of technological change.

The inheritance of an agent who is age a in year i and whose parents are s years older and who belongs to income class k is denoted by $I(a,i,s,k)$. Before age 68 (i.e. $a+s<68$) the probability of death is zero and consequently there are no bequests. Between age 68 and 90, a fraction of a parent cohort dies and leaves bequests which are split between their children[2]. Therefore, inheritances of their children are defined as follows:

$$I(a,i,s,k)=\frac{d(a+s)\bar{A}(a+s,i,k)}{\sum_{j=23}^{45}N(a+s-j,i,j,k)}. \quad (8)$$

The numerator defines the aggregate assets of income class k parents who die in year i at age $a+s$. The denominator defines the parent's total number of children. The inheritances are

[2]Note that those who die at age 91 leave no bequests. Consequently it's no problem when their oldest children die at the same time.

the reason why we have to disaggregate each cohort according to the age of their parents at birth. Children, who were born to older parents, receive their inheritances earlier in life, while children with young parents receive their inheritances later in life. The first children of parents (born when their parents were age 23) receive their inheritances between the ages of 45 and 67. Those children who were born when their parents were 45 receive their inheritances earlier in life (between the ages of 23 and 45).

The net-taxes of an agent age a from the income class k in year i consist of consumption, progressive wage taxes, capital taxes and social security contributions net of pensions (Pen), i.e.

$$\begin{aligned} T(a,i,s,k) &= \tau^c(i)\big[c(a,i,s,k) + KID(a,i,k)c_K(a,i,s,k)\big] + \bar{\tau}^w(a,i,s,k)W(a,i,s,k) + \\ &\quad \tau^r(i)r(i)\big[\mathrm{a}(a,i,s,k) + I(a,i,s,k)\big] + \tau^b(i)I(a,i,s,k) + \\ &\quad [\bar{\tau}^p(a,i,s,k) + \bar{\tau}^h(a,i,s,k) + \bar{\tau}^d(a,i,s,k)]W(a,i,s,k) - Pen(a,i,s,k). \end{aligned} \quad (9)$$

Note that the heir has to pay capital taxes on the interest income from his inheritance. In addition, it is also possible to levy inheritance taxes at the tax rate $\tau^b(i)$. Due to a contribution ceiling, pension, disability insurance and health care contributions may differ across agents. Pension benefits also depend on the individual income history. On the other hand, health care and disability transfers are only age- and period-specific. The tax rates τ^c, τ^r and $\bar{\tau}^w$ denote the consumption, capital and (individual) average wage tax rate, while $\bar{\tau}^p$, $\bar{\tau}^h$ and $\bar{\tau}^d$ define the (individual) average pension, health care and disability insurance contribution rate.

Finally, the discount factor with the interest rate $r(z)$ in year z is

$$R(i,t) = \begin{cases} 1 & \text{for} \quad i = t \\ \prod_{z=t+1}^{i} [1 + r(z)]^{-1} & \text{for} \quad i > t. \end{cases} \quad (10)$$

The asset accumulation of an agent age j in year i whose parents were age s at his birth and who belongs to income class k follows

$$\begin{aligned} \mathrm{a}(j+1, i+1, s, k) = \big[\mathrm{a}(j,i,s,k) + I(j,i,s,k)\big](1 + r(i)) + W(j,i,s,k) \\ - T(j,i,s,k) - c(j,i,s,k) - KID(j,i,k)c_K(j,i,s,k). \end{aligned} \quad (11)$$

Given individual consumption, leisure and assets of all agents we can compute the aggregated variables of a specific year. Aggregated consumption $C(i)$ and savings $A(i+1)$ of agents who live in period i are computed from

$$C(i) = \sum_{k=1}^{3} \sum_{a=21}^{90} \sum_{s=23}^{45} \Big[c(a,i,s,k) + KID(a,i,k)c_K(a,i,s,k)\Big] N(a,i,s,k) \quad (12)$$

and

$$A(i+1) = \sum_{k=1}^{3} \sum_{a=21}^{90} \underbrace{\sum_{s=23}^{45} \mathrm{a}(a+1, i+1, s, k) N(a, i, s, k)}_{\bar{A}(a+1,i+1,k)}. \tag{13}$$

Assets in period $i+1$ were saved by the agents who lived in period i. Since households die at the beginning of each period, we aggregate across all agents who lived in the previous period in order to compute $\bar{A}(a+1, i+1, k)$ which we need for the calculation of the bequests, see (8). If we aggregate existing assets at the beginning of period i across agents who live in period i, we get

$$\mathcal{A}(i) = \sum_{k=1}^{3} \sum_{a=21}^{90} \sum_{s=23}^{45} \mathrm{a}(a, i, s, k) N(a, i, s, k). \tag{14}$$

Note the difference between $A(i+1)$ and $\mathcal{A}(i)$. Consequently, since the former aggregates the savings of agents who lived in period i, $A(i+1)$ includes the bequests of those who die at the beginning of period $i+1$ and excludes the assets of the arriving immigrants from period $i+1$. The latter aggregates the assets of agents at the beginning of period i. Henceforth, $\mathcal{A}(i)$ excludes the assets of those who have died in the beginning of period i and includes the assets of the arriving immigrants of period i.

2. The production side

The economy is populated by a large number of competitive firms. It suffices to consider the planning problem of one representative company and normalize the number of firms to unity since they are all assumed to be identical. On the firm side we model corporate taxes and assume that all investment is financed via retained earnings. However, convex costs of adjusting the capital stock provide an incentive for smoothing investment.

The analysis of the firm's investment decision starts with an arbitrage relationship which states that in each period i the return on equity, which is composed of the dividend yield $DIV(i)$ and the capital appreciation $V(i+1) - V(i)$, equals the return on a comparable financial asset:

$$DIV(i) + V(i+1) - V(i) = r(i)V(i). \tag{15}$$

Equation (15) implies that dividends, capital gains and interest income are taxed at the same (marginal) tax rate. Integrating (15) forward in time and imposing the transversality condition

$$\lim_{T \to \infty} V(T+1)R(T,t) = 0,$$

which prevents the firm's value from becoming infinite in finite time yields the valuation of the firm by its owners

$$V(t) = \sum_{i=t}^{\infty} DIV(i)R(i,t).$$

From the cash-flow identity in period i we derive the dividend payments

$$DIV(i) = (1 - \tau^k(i))\big[Y(i) - w(i)L(i)\big] - \Delta K(i), \tag{16}$$

which links together dividends $DIV(i)$, profits (i.e. output $Y(i)$ net of wage costs $w(i)L(i)$), investment outlays $\Delta K(i)$ and corporate taxes $T^k(i)$ in period i. The latter are computed from the tax identity

$$T^k(i) = \tau^k(i)\big[Y(i) - w(i)L(i)\big], \tag{17}$$

where the corporate tax rate $\tau^k(i)$ is applied to the output net of wage costs.

The firms marketable output, $Y(i)$, is given by the difference between gross output and adjustment costs, i.e.

$$Y(i) = F(K(i), L(i)) - \Phi(\Delta K(i), K(i)) \quad \text{with} \quad F_{K(i)} > 0, F_{L(i)} > 0, \Phi_{\Delta K(i)} \geq 0, \Phi_{K(i)} \leq 0, \tag{18}$$

where $F_{K(i)} = \frac{\partial F}{\partial K(i)}$ etc. The production technology is of the linear homogeneous CES type

$$F(K(i), L(i)) = \phi \left[\varepsilon K(i)^{1-1/\sigma} + (1-\varepsilon) L(i)^{1-1/\sigma} \right]^{\frac{1}{1-1/\sigma}}, \tag{19}$$

where ε is the parameter measuring the intensity of the use of capital in production, σ is the elasticity of substitution between capital and labor in production and ϕ is a technology parameter. If $\sigma = 1$, then the technology is simplified to the Cobb-Douglas case. The following adjustment cost function is assumed (in year i)

$$\Phi(\Delta K(i), K(i)) = \frac{\psi}{2} \frac{\Delta K(i)^2}{K(i)}. \tag{20}$$

The term ψ is the adjustment cost coefficient. Larger values of ψ imply greater marginal cost of new capital goods for a given rate of investment. The installation technology is linearly homogeneous and shows an increasing marginal cost of investment (or, symmetrically, disinvestment): a faster pace of change requires a greater than proportional rise in adjustment costs (i.e. $\Phi_{\Delta K(i)} > 0$).

The objective of the firm at the beginning of period t is to maximize the firm value $V(t)$. Thereby, the firm has to take into account the financial constraint (16), the technology constraint (18) and the equation of motion for the stock of capital

$$K(i+1) = (1-\delta)K(i) + \Delta K(i), \tag{21}$$

where we assume that capital depreciates at rate δ.

In order to solve this problem, the firm maximizes the present value Hamiltonian in each period $i \geq t$

$$\mathcal{H}(i) = \{DIV(i) + q(i+1)[\Delta K(i) - \delta K(i)]\}\, R(i,t), \tag{22}$$

where $q(i)$ is the shadow price which defines the marginal increase in the firm value at time i. That is derived by adding one additional unit of capital to the firm.

The optimal path requires the following first order conditions to be satisfied:

$$\frac{\partial \mathcal{H}(i)}{\partial L(i)} = 0 => \quad w(i) = F_{L(i)} \tag{23}$$

$$\frac{\partial \mathcal{H}(i)}{\partial \Delta K(i)} = 0 => \quad q(i+1) = 1 + (1-\tau^k(i))\Phi_{\Delta K(i)} = 1 + (1-\tau^k(i))\psi\frac{\Delta K(i)}{K(i)} \tag{24}$$

$$-\frac{\partial \mathcal{H}(i)}{\partial K(i)} = R(i,t)q(i+1) - R(i-1,t)q(i)$$

$$=> \quad r(i)q(i) = (1-\tau^k(i))[F_{K(i)} - \Phi_{K(i)}] + (1-\delta)q(i+1) - q(i) \tag{25}$$

Equations (23) and (24) determine the optimal labor $L(i)$ and investment demand $\Delta K(i)$, respectively. Labor should be employed up to the point where its marginal product equals the market wage rate. The firm will invest until the marginal costs of one additional unit of investment are equal to the marginal benefits from having one additional unit of capital at the end of period i. The latter is given on the left-hand-side of equation (24) and reflects the marginal increase in the value of the firm. The firm value for the next period increases due to the addition of one unit of physical capital valued at the shadow price $q(i+1)$. Marginal cash costs on the right-hand-side consist of the marginal cash costs of acquisition plus marginal costs of installation. Equation (25) is an arbitrage condition which states that the return from investment in financial assets must be equal to the return in shares. The right-hand-side of (25) is the marginal return to an investor who bought one unit of capital at the price $q(i)$ at the end of of period $i-1$. As marginal dividends he receives the net marginal return to capital, which includes the reduction in current adjustment costs per unit of investment. Furthermore, the investor realizes a marginal return from the increase in the value of the capital unit which is net of economic depreciation. The left-hand-side of (25) gives the return if he would have invested the same amount in financial assets.

Finally, as Hayashi (1982) has shown, the marginal value of capital equipment $q(i)$ could be also used to determine the firm value according to[3]

$$V(i) = q(i)K(i). \tag{26}$$

3. The government sector

The government sector in the model represents the consolidated budget of the central, state and local governments as well as the budgets of the pension, health and disability insurance system.

[3]A formal proof of this equation in the present context could also be found in Fehr (1999, 54).

3.1. The consolidated government budget

The central government issues new debt $\Delta B(i) = B(i+1) - B(i)$ and collects consumption, inheritance, capital and wage taxes, social security contributions net of pensions from households, and corporate taxes $T^k(i)$ from companies in order to finance the public good $G(i)$ and the interest payments on its debt:

$$\Delta B(i) + \sum_{k=1}^{3} \sum_{a=21}^{90} \sum_{s=23}^{45} T(a,i,s,k)N(a,i,s,k) + T^k(i) = G(i) + r(i)B(i). \tag{27}$$

With respect to public debt, we assume that the government keeps an exogenously fixed ratio $b(i)$ of debt to output, i.e. $\frac{B(i)}{Y(i)} = b(i)$. The public good expenditures $G(i)$ consist of government purchases of goods and services (including government investments) and education, disability and health outlays. Expenditures for government purchases are identical per capita, education outlays are age-specific and only spent for children and health. Disability outlays are also age-specific. Consequently, we have

$$G(i) = Pop(i)g + \sum_{a=0}^{20} \overline{N}(a,i) \times edu(a) + HB(i) + DB(i), \tag{28}$$

where g are the time invariant per capita outlays of general public goods, $edu(a)$ are the education outlays per child of age a and $HB(i)$ and $DB(i)$ are aggregate health and disability outlays, respectively.

The progressivity of the wage tax system is modelled as in Auerbach and Kotlikoff (1987, 113). Individual marginal wage tax rates $\tau^w(a,i,s,k)$ consequently take the form

$$\tau^w(a,i,s,k) = \beta_0 + \beta_1 W(a,i,s,k). \tag{29}$$

This yields an individual average tax rate at the tax base of

$$\bar{\tau}^w(a,i,s,k) = \beta_0 + \beta_1 \frac{W(a,i,s,k)}{2}. \tag{30}$$

Setting $\beta_1 = 0$ makes the tax system proportional. Increasing β_1 while decreasing the value of β_0 in order to keep the revenues constant makes the tax system more progressive.

3.2. The budget of the pension system

We model a PAYGO-pension system in each region. Let's assume that a k-income class agent whose parents were s years old at his birth has retired in year z at the exogenously set retirement age $\bar{a}(z)$. Then his pension benefits $Pen(a,i,s,k)$ in year $i \geq z$ when he is age $a \geq \bar{a}(z)$ depend on his average indexed earnings during his working time $\bar{W}(z,s,k)$:

$$Pen(a,i,s,k) = \chi(\bar{W}(z,s,k)) \times \bar{W}(z,s,k). \tag{31}$$

Note that the current age a in year i depends on the retirement year $z : a = \bar{a}(z) + i - z$. The replacement rate χ is a linear function of the average indexed earnings, i.e.

$$\chi(\bar{W}(z,s,k)) = \omega_0 + \omega_1 \bar{W}(z,s,k), \tag{32}$$

where the average indexed earnings are computed from

$$\bar{W}(z,s,k) = \sum_{a=21}^{\bar{a}(z)-1} \frac{W(a,t,s,k)}{\bar{a}(z)-21} \quad \text{with} \quad t = z - \bar{a}(z) + a. \tag{33}$$

The total outlays $PB(i)$ of the pension system in year i are computed from

$$PB(i) = \sum_{k=1}^{3} \sum_{a=\bar{a}(i)}^{90} \sum_{s=23}^{45} Pen(a,i,s,k) N(a,i,s,k). \tag{34}$$

The budget of the pension system must be balanced in each period. Therefore, the aggregate average contribution rate, $\hat{\tau}^p(i)$, has to be adjusted to fulfill the period budget constraint

$$\hat{\tau}^p(i) PY(i) = (1 - \mu_1(i)) PB(i), \tag{35}$$

where $PY(i)$ defines the aggregate compulsory earnings base. The compulsory earnings base is computed from

$$PY(i) = \sum_{k=1}^{3} \sum_{a=21}^{\bar{a}(i)-1} \sum_{s=23}^{45} \min(W(a,i,s,k), \Theta \hat{W}(i)) N(a,i,s,k) \tag{36}$$

where $\hat{W}(i)$ is the average labor income in period i, i.e.

$$\hat{W}(i) = \sum_{k=1}^{3} \sum_{a=21}^{\bar{a}(i)-1} \sum_{s=23}^{45} W(a,i,s,k) N(a,i,s,k) / \sum_{a=21}^{\bar{a}(i)-1} \overline{N}(a,i), \tag{37}$$

and Θ denotes the contribution ceiling.

Note that we do not allow pensioners to work after retirement in the model. Due to the contribution ceiling, individual social security tax rates might differ from the aggregate one. Marginal and average contribution rates of a k income class agent of age a in year i whose parents were age s at the time of his birth are given by

$$\tau^p(a,i,s,k) = \begin{cases} \hat{\tau}^p(i) & \text{if} \quad W(a,i,s,k) \leq \Theta \hat{W}(i) \\ 0 & \text{if} \quad W(a,i,s,k) > \Theta \hat{W}(i) \end{cases}$$

and

$$\bar{\tau}^p(a,i,s,k) = \begin{cases} \hat{\tau}^p(i) & \text{if} \quad W(a,i,s,k) \leq \Theta \hat{W}(i) \\ \hat{\tau}^p(i)\, \Theta \hat{W}(i) / W(a,i,s,k) & \text{if} \quad W(a,i,s,k) > \Theta \hat{W}(i). \end{cases}$$

The marginal social security tax is zero above the contribution ceiling and the average social security tax falls with increasing income for an individual.

3.3. The health care system

In the present model we specify an age-specific lump-sum transfer $hc(a)$ for all agents, which represents the consumption of health services financed by the health care system. The total health care outlays $HB(i)$ in year i are derived from

$$HB(i) = \sum_{a=0}^{90} hc(a)\overline{N}(a,i). \tag{38}$$

Again, since a fraction $\mu_2(i)$ is financed by general taxes, aggregate average health care contribution on wages in year i, $\hat{\tau}^h(i)$, is derived from

$$\hat{\tau}^h(i)PY(i) = (1-\mu_2(i))HB(i). \tag{39}$$

Due to the contribution ceiling, individual health care contributions might differ from the aggregate one. Marginal and average contribution rates of a k income class agent of age a in year i whose parents were age s at the time of his birth are given by

$$\tau^h(a,i,s,k) = \begin{cases} \hat{\tau}^h(i) & \text{if} \quad W(a,i,s,k) \leq \Theta\hat{W}(i) \\ 0 & \text{if} \quad W(a,i,s,k) > \Theta\hat{W}(i) \end{cases}$$

and

$$\bar{\tau}^h(a,i,s,k) = \begin{cases} \hat{\tau}^h(i) & \text{if} \quad W(a,i,s,k) \leq \Theta\hat{W}(i) \\ \hat{\tau}^h(i)\,\Theta\hat{W}(i)/W(a,i,s,k) & \text{if} \quad W(a,i,s,k) > \Theta\hat{W}(i). \end{cases}$$

3.4. The disability insurance system

Since disability insurance in the EU and Japan is guaranteed by the pension system, the insurance system in our model is adjusted for the system in the USA. Hence, we specify a lump-sum transfer di on an equal basis for all agents older than 20 and younger than 65, which represents the consumption of disability services. These expenses are financed by the disability insurance system. The total disability insurance outlays $DB(i)$ in year i are derived from

$$DB(i) = di \sum_{a=21}^{64} \overline{N}(a,i). \tag{40}$$

Similarly to the other two social security systems, aggregate average disability insurance contribution on wages in year i, $\hat{\tau}^d(i)$, is derived from

$$\hat{\tau}^d(i)PY(i) = DB(i). \tag{41}$$

People here also face a contribution ceiling and thus individual disability insurance contributions differ from the aggregate ones. Marginal and average contribution rates of a k income class agent of age a in year i whose parents were age s at the time of his birth are reached by

$$\tau^d(a,i,s,k) = \begin{cases} \hat{\tau}^d(i) & \text{if} \quad W(a,i,s,k) \leq \Theta \hat{W}(i) \\ 0 & \text{if} \quad W(a,i,s,k) > \Theta \hat{W}(i) \end{cases}$$

and

$$\bar{\tau}^d(a,i,s,k) = \begin{cases} \hat{\tau}^d(i) & \text{if} \quad W(a,i,s,k) \leq \Theta \hat{W}(i) \\ \hat{\tau}^d(i)\, \Theta \hat{W}(i)/W(a,i,s,k) & \text{if} \quad W(a,i,s,k) > \Theta \hat{W}(i). \end{cases}$$

4. Aggregation and equilibrium conditions

In general equilibrium supply has to equal demand in all markets. We start with the equilibrium condition on the (national) labor markets which states that labor demand of firms $L(i)$ in year i is equal to aggregate labor supply of agents in year i, i.e.

$$L(i) = \sum_{k=1}^{3} \sum_{a=21}^{90} \sum_{s=23}^{45} E(a,k)\,[h(a,i) - \ell(a,i,s,k)]\,N(a,i,s,k). \tag{42}$$

Next we aggregate the (individual) budget constraints (11) of the private sector

$$A(i+1) - \mathcal{A}(i) = r(i)\mathcal{A}(i) + w(i)L(i) - \sum_{k=1}^{3} \sum_{a=21}^{90} \sum_{s=23}^{45} T(a,i,s,k)N(a,i,s,k) - C(i), \tag{43}$$

where we have used the definitions (12), (13), (14) and (42). Substituting the aggregated net-taxes taken from the budget constraint of the public sector (27)

$$-\sum_{k=1}^{3} \sum_{a=21}^{90} \sum_{s=23}^{45} T(a,i,s,k)N(a,i,s,k) = \Delta B(i) + T^k(i) - G(i) - r(i)B(i), \tag{44}$$

we arrive at

$$A(i+1) - \mathcal{A}(i) = r(i)\mathcal{A}(i) + w(i)L(i) + \Delta B(i) + T^k(i) - G(i) - r(i)B(i) - C(i). \tag{45}$$

Next we turn to the (national) capital market where (national) savings in period i and the savings of the immigrants $\hat{A}$ entering at the beginning of period $i+1$ are equal to the value of the domestic capital stock, government bonds and net foreign assets B^f in period $i+1$, i.e.

$$A(i+1) + \hat{A}(i+1) = q(i+1)K(i+1) + B(i+1) + B^f(i+1). \tag{46}$$

where

$$\hat{A}(i+1) = \sum_{k=1}^{3} \sum_{a=21}^{65} \sum_{s=23}^{45} \mathrm{a}(a,i+1,s,k)NM(a,i+1,k) \times \frac{f(s,i+1-a)}{TFR(i+1-a)}.$$

Of course, if we use the definition of aggregate assets from (14), we would have

$$\mathcal{A}(i) = q(i)K(i) + B(i) + B^f(i). \tag{47}$$

Next, we substitute (46) and (47) into the consolidated budget constraint of the economy (45) and apply the definition (26):

$$V(i+1) - V(i) + \Delta B^f(i) - \hat{A}(i+1) = r(i)V(i) + r(i)B^f(i) + w(i)L(i) + T^k(i) - G(i) - C(i). \tag{48}$$

Finally, using the the arbitrage condition (15) and the definitions (16) and (17) we arrive at the national goods market equilibrium

$$Y(i) + \hat{A}(i+1) = C(i) + \Delta K(i) + G(i) + TB(i), \tag{49}$$

where we have substituted $\Delta B^f(i) = r(i)B^f(i) + TB(i)$ which simply states that the change in net foreign assets has to be equal to the net foreign interest payments and the trade balance $TB(i)$. Note that the left-hand side of this equilibrium condition equals the national income of the economy.

Naturally, trade balances as well as net foreign assets of all regions x have to add up to zero in our model, i.e.

$$\sum_{x=1}^{3} TB(i,x) = \sum_{x=1}^{3} B^f(i,x) = 0. \tag{50}$$

Consequently, in equilibrium aggregate world production and immigrant savings have to be balanced by aggregate private and public consumption and investment

$$\sum_{x=1}^{3} \Big[Y(i,x) + \hat{A}(i+1,x) \Big] = \sum_{x=1}^{3} [C(i,x) + \Delta K(i,x) + G(i,x)], \tag{51}$$

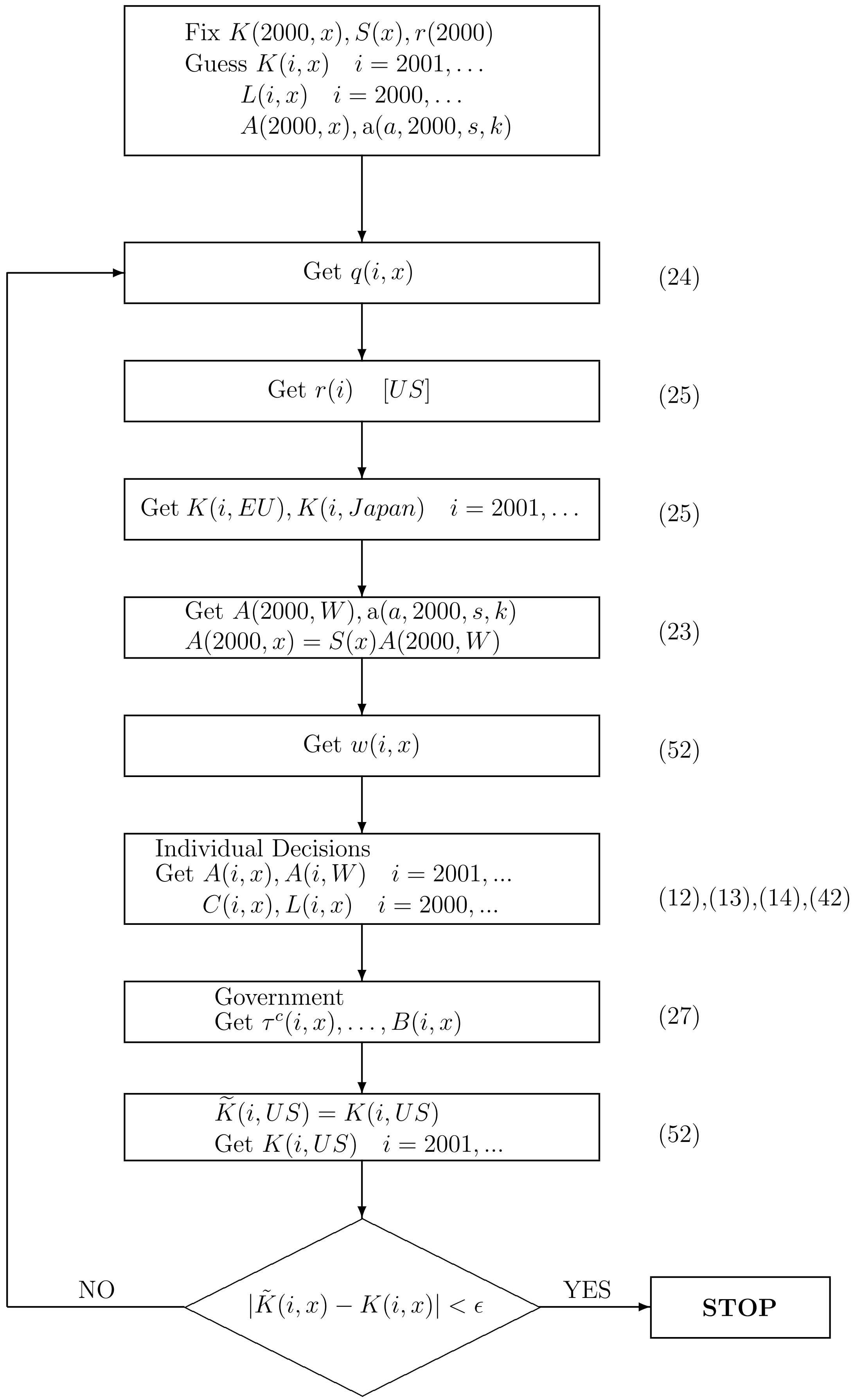
while equilibrium on the world capital market implies that world savings $\mathcal{A}(i,W)$ have to equal the worth of the world capital stock and the stock of government bonds, i.e.

$$\mathcal{A}(i,W) = \sum_{x=1}^{3} \mathcal{A}(i,x) = \sum_{x=1}^{3} \Big[q(i,x)K(i,x) + B(i,x) \Big]. \tag{52}$$

This completes the description of the model.

IV. Solving the model

The following figure gives an overview of the solution method for our simulation model which we now explain.


Fix $K(2000,x), S(x), r(2000)$
Guess $K(i,x) \quad i = 2001,\ldots$
$L(i,x) \quad i = 2000,\ldots$
$A(2000,x), \mathrm{a}(a,2000,s,k)$
Get $q(i,x)$
(24)
Get $r(i) \quad [US]$
(25)
Get $K(i,EU), K(i,Japan) \quad i = 2001,\ldots$
(25)
Get $A(2000,W), \mathrm{a}(a,2000,s,k)$
$A(2000,x) = S(x)A(2000,W)$
(23)
Get $w(i,x)$
(52)
Individual Decisions
Get $A(i,x), A(i,W) \quad i = 2001,...$
$C(i,x), L(i,x) \quad i = 2000,...$
(12),(13),(14),(42)
Government
Get $\tau^c(i,x),\ldots,B(i,x)$
(27)
$\widetilde{K}(i,US) = K(i,US)$
Get $K(i,US) \quad i = 2001,...$
(52)
NO
$|\tilde{K}(i,x) - K(i,x)| < \epsilon$
YES
STOP

Given the capital stock and world interest rate in year 2000 and the asset profiles, our model applies a Gauss-Seidel algorithm to solve for the perfect foresight general equilibrium transition path of the economy. Our initial guesses for the capital stock for the remaining years of the transition and for the aggregate labor supply and assets for all transition years are used as starting points. Next, we compute from equation (24) the path of the unit values for capital in each region. The path for the world interest rate after year 2000 is derived from the arbitrage condition (25) for the US. Given the world interest rate and the initial capital stock, we again use equation (25) in order to compute a path for the capital stock in the EU and Japan. Condition (25) is also used to update the unit value of the existing capital stock for the year 2000 in each economy. Thereafter, we derive the aggregate initial savings for the year 2000 from the world capital market equilibrium condition (52). We get the aggregated assets in each region for the initial year by applying region-specific saving shares $S(x)$ as per cent of the world assets. We update the level of the asset profile in each region according to these aggregated assets. Next, the wage rates are computed in each region which are equal to the respective marginal product of labor. Given initial assets, the time path of tax rates and factor prices, household decisions on consumption and labor supply are computed and aggregated. Then, we update the path for tax rates, social security contributions and debt given the government budget constraints (27), (35), (39) and (41). Finally, we compute a new path for the capital stock in the US using again the world capital market condition (52). The new values for capital and labor are then weighted with the initial guess of these supplies to form a new guess of the time path of these variables. The algorithm then iterates until the path of capital stock and labor converges. In the model, the transition path to the final steady state takes 300 years.

V. Calibration issues

In order to solve our model, we first need to specify the preference, technology and policy parameters to get realistic values for our starting year 2000. Table 4 reports our parameter values.

The values for the time preference rate, the inter- and intratemporal elasticity of substitution and the leisure preference parameter are simply taken from Kotlikoff et al. (2001). The same applies to the production side for the elasticity between capital and labor, the capital share in production, the adjustment cost parameter and technical progress. The technology level is computed to reach a marginal product of labor of one in the steady-state simulation. Throughout the transition, this value is kept constant.

Next, we specify the policy parameters. The consumption tax rate in the US was chosen in accordance to Kotlikoff et al. (2001), which incorporates a 8.8 percent tax on consumption expenditures and an additional 2.5 percent tax to account for indirect taxation of labor compensation in the form of pension benefits. In the EU, the indirect tax is simply derived by the unweighted average of the consumption tax rates in the member states as reported by the OECD (2001, 16). The Japanese consumption tax rate is also taken from the OECD (2001, 16). In all regions the capital tax rate is fixed at 20 percent. For the US this value seems reasonable following Kotlikoff et al. (2001, 21f.). The tax rate in Japan is close to realistic values reported

Table 4: Parameter values of the Model

Preferences and technology	**Symbol**	**Value**		
		USA	EU	Japan
Utility function				
time preference rate	θ		0.02	
intertemporal elasticity of substitution	γ		0.25	
intratemporal elasticity of substitution	ρ		0.4	
leisure preference parameter	α		1.5	
Production function				
technology level	ϕ		1.05461	
elasticity between capital and labor	σ		1.0	
capital share in production	ε		0.25	
adjustment cost parameter	ψ		10.0	
technical progress	λ		0.01	
Policy parameters				
consumption tax rate (in %)	τ^c	11.3	19.5	5.0
capital tax rate	τ^r		20.0	
wage tax parameter	β_1	0.12	0.09	0.10
debt (in % of NI)	B/Y	40.0	50.0	44.0
replacement rate parameters	ω_0	0.80	0.75	0.75
	ω_1	-0.335	-0.192	-0.265
age of retirement	$\bar{a}(j)$	63	60	60
contribution ceiling	Θ	2	2	1.68
capital shares	S	0.350	0.484	0.166

by the Ministry of Finance (1999). Assuming a capital tax rate of 20 percent in the EU is also reasonable if one compares the taxation of interest income from government bonds in EU countries as presented by Joumard (2001, 30f.). These values range between 12.5 percent and 30 percent. In some EU countries, net-interest income is taxed at the individual's marginal income tax rate. However, on average the tax rate should not exceed 20 percent. With regards to the progressive wage tax, we specify the progressive parameter β_1 in each region in order to get realistic average and marginal tax rates. The proportional term β_0 is computed endogenous to balance the government budget by the wage tax. The resulting average and marginal tax rates in the US in the initial year of the transition are respectively 4.5 percent and 6.1 percent in the low income class, 8.1 percent and 13.2 percent in the middle income class and 16 percent and 29.1 percent in the high income class. The average and marginal wage tax rates in the EU are respectively 9.8 percent and 11.0 percent in the low income class, 12.6 percent and 16.6 percent in the middle income class and 19.1 percent and 29.5 percent in the high income class. Compared to the EU, the average and marginal tax rates in Japan take similar values of 9.4

percent and 10.7 percent, respectively, in the low, 12.4 percent and 16.8 percent in the middle and 19.4 percent and 30.8 percent in the high income class.

On first sight, the government debt as per cent of national income seems to be too low since the European Commission (2003) reports values of 69.5 percent of national income for the US, 72.8 percent for the EU and 150.4 percent for Japan. However, we tried to get realistic interest payments of the government also reported by the European Commission (2003). Therefore, we chose these values for the government debt. In order to get realistic per capita (of children) education costs, we took the German age-specific profile for all regions. This profile was rescaled to yield observed education outlays in 2000. The per capita outlays of general public goods g were adjusted to get realistic government purchases of goods and services as per cent of national income for the year 2000 as reported by the European Commission (2003).

Next, we turn to the social security systems. The retirement ages for the US and EU are taken from Blöndal and Scarpetta (1999, 53). Since we only model pension benefits that depend on the labor income received during the individual's working life, we choose the retirement age for the earnings-related pension in Japan according to Whitehouse (2002, 24). The progressive and proportional parameters for the pension replacement rate were calculated to approximately get the replacement rates relative to individual lifetime earnings as reported by Whitehouse (2002, 55). However, we had to adjust these values to get realistic aggregated pension benefits. Due to this calibration, the average replacement rates in 2000 for the low, middle and high income class in the US are 71.4 percent, 52.7 percent and 18.3 percent, respectively. In Japan the resulting replacement rates are respectively 69.9 percent, 52.3 percent and 18.5 percent for the low, middle and high income class which is somewhat less progressive compared to the US pension system. The replacement rates in the EU are flatter than in the other two countries. Here, the low, middle and high income class receive 69.9 percent, 58.8 percent and 36.4 percent of their average indexed earnings. The contribution ceiling for Japan is also taken from Whitehouse (2002). For the US and EU we assume a contribution ceiling of twice the average indexed earnings as reasonable. As far as health care systems go, we take age-specific profiles for health care in Japan and Germany (for the EU) which are rescaled in order to yield realistic health outlays. Disability insurance in these two regions is covered by their pension systems. Following Kotlikoff et al. (2001, 24), the health and disability insurance in the US only applies to households older and younger than age 65. The health and disability costs are assumed to be uniform per capita lump-sum transfers.

Our model also requires an initial distribution of assets by age and income class for each region. These profiles are taken from each region's data.[4] We made a linear interpolation of the data to distribute assets on all ages since these data were only provided as averages for certain age groups. In addition, these average values for all households had to be distributed among the different income classes. Therefore, the profiles were shifted by the same parameter as the earnings ability profile. We also need to fix the shares S to explain how the aggregate world

[4]Data on Japanese net worth were provided by Charles Horioka, while the European profiles were adjusted to German data provided by Reinhold Schnabel. US Data were derived from the 1998 Survey of Consumer Finances.

assets are distributed across regions. In order to get these shares, we made a simulation for all three regions as closed economies and computed the world assets in the initial year 2000. Then, we calculated each region's share in the world assets.

Finally, we have to specify the capital stock and the initial interest rate in the starting year 2000 for each region. Here we take the resulting values from an iteration of the model without adjustment costs.

VI. Initial equilibrium and baseline path

In this section, we report the simulation results for the baseline path of our model. Changing variables during the transition are only due to the different aging processes in the three regions. Actual policies are held fixed throughout the transition.

1. The initial year 2000

The following table shows the macroeconomic structure in the initial year 2000 of the transition. For this initial year we tried to replicate a realistic macroeconomic structure and highlight the differences in the structure of the public sector. Of course, due to the restrictions of the theoretical model our data will deviate from reality.

These restrictions already become obvious from the national income shares reported in the upper part of Table 5. The government shares seem to be too high, but, according to the definition of our variable, they include health and, in the case of the US, disability benefits which are usually reported separately. As soon as the latter are subtracted, the public sector shares in national income in the US and EU come very close to those reported in European Commission (2003). However, if one compares the government purchases in Japan in our model to the realistic values, one can observe a big difference. The reason is that we set government purchases in Japan in order to get realistic overall tax revenues of about 20.7 percent of national income in year 2000 as reported in European Commission (2003). Those government expenditures are financed by an exogenously calculated average tax rate of 14.1 percent, which is too high compared to the values reported by OECD (2002b). However, if we would have set government purchases to 26.6 percent as reported by the European Commission (2003), this tax rate would have been much higher. During the transition payroll taxes in Japan rise to more than 50 percent of wage income and also the wage tax rate rises to a value of more than 20 percent. Therefore, with a higher wage tax rate in the initial year, it was impossible to get a solution for the transition path so that we concentrated on the tax revenues. One could argue that the government debt in our model is too low. Hence, wage tax rates are higher than observed in reality. One has to also keep in mind that interest payments on government debt in our model would be higher than in reality since the world interest rate is higher than the observed interest rate in Japan. Due to an increasing interest rate in the baseline path of our model with constant government debt, the wage tax rate would have to increase much more than it does now. Thus, even with higher government debt it would be impossible to get a baseline path for Japan. This should suffice to justify our adjustments.

Table 5: The year 2000 of the baseline path

	Model			Official		
	USA	EU	Japan	USA	EU	Japan
National Income						
private consumption	77.4	69.4	78.7	77.6	67.8	67.8
government purchases of goods and services	22.8	32.9	22.4	23.0	32.1	33.4
current account	0.2	-1.2	3.1	-4.6	-0.4	3.0
Government indicators						
aggregate education outlays	5.9	6.0	4.4	5.9	6.0	4.3
aggregate pension benefits	5.9	11.4	10.8	5.7	11.6	10.8
aggregate health benefits	2.1	6.4	5.2	2.5	6.2	6.8
aggregate disability benefits	1.3	-	-	0.9	-	-
pension contribution rate (in %)	8.8	16.9	16.5	10.6	-	17.3
health care contribution rate (in %)	2.8	9.6	8.0	2.9	-	8.0
disability insurance contribution rate (in %)	1.9	-	-	1.9	-	-
Tax revenues	22.3	30.3	20.9	26.6	32.5	20.7
direct taxes	13.6	16.8	17.0	17.9	16.5	10.5
indirect taxes	8.7	13.5	3.9	8.7	16.0	10.2
interest rate (in %)		9.0			-	

* in per cent of national income if not stated different

The private consumption expenditures in the US and EU are very close to the values reported in European Commission (2003). The same cannot be said of Japan. The national saving rate in the US is also very realistic. However, in the EU and Japan they are somewhat smaller than observed in reality. Next, we turn to the current accounts. In Japan the current account has a realistic value for the year 2000 whereas the deficit in the EU is somewhat too high compared to the value in European Commission (2003). In the US, it seems to be highly unrealistic. However, in our model we consider trade only between our three regions. Trade with China and South East Asia makes a big contribution to the current account deficit. Hence, the small surplus in the US is acceptable.

The reported shares in education are very close to the realistic levels reported by the OECD (2002a). Next, we consider the government insurance systems. For the US we tried to replicate the payroll tax rates like in Kotlikoff et al. (2001) which are very close to the real values. Therefore, the shares of pension, disability and health benefits are also close to the values reported by the Social Security Agency (2001) and the U.S. Department of Health and Human Services (2002). In the case of the EU, pension benefits were set in order to reach the values reported by the European Commission (2001). The health benefits were set to the average value for the member states in the EU reported by OECD (2003). The payroll taxes are then calculated endogenously. The reached contribution rates of 16.9 percent for the pension system and 9.6 percent for the health care system are reasonable values for the EU. In Japan we set the contribution rate for the health insurance system to the actual value in 2000. The endogenous value for the health benefits is lower than reported by the IPSS (2003). Concerning the pension system we also set first the contribution rate to 17.3 percent. However, pension benefits were much higher than reported by the IPSS (2003) since pension system revenues are used to build up a capital stock. Since we don't model this in our simulation model, we tried to get realistic pension benefits and calculated the contribution rate endogenously.

The reported tax revenues come close to the reported levels as in European Commission (2003) for all three regions. Even the distribution on direct and indirect taxes is realistic for the US and EU. In the US the initial average and marginal wage tax rates are 10 percent and 17 percent, respectively, and in the EU 14.2 percent and 19.7 percent, respectively. The values in both regions are in the range of the reported levels in OECD (2002b). In Japan the revenue of indirect taxes is very low compared to the official level while the wage tax rates are too high as already discussed above.

Finally, the world interest rate is fairly high. On the other hand, the capital-output ratios are realistic. With values of 3.2 in the US and EU and 3.3 in Japan, they are very close together due to the fact that we took the capital stock from a simulation without adjustment costs.

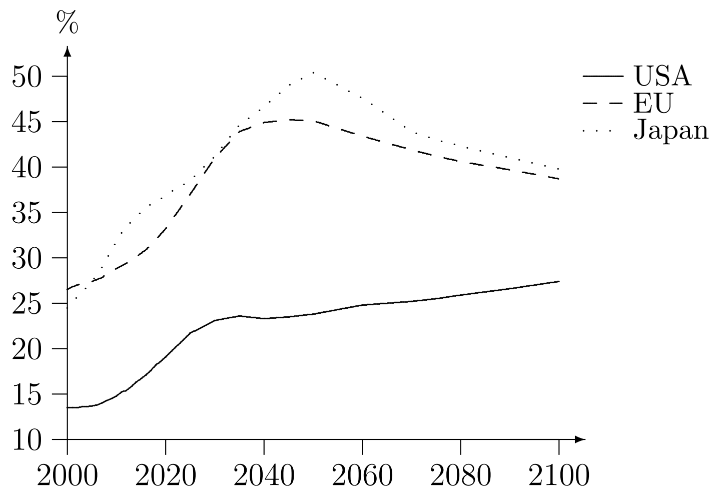
2. The baseline transition path

Next, we turn to the transition path of the baseline simulation (see also Table A-5). As already mentioned above, the dynamics are solely due to the changing population level and structure in the three regions. The existing policy in year 2000 is held fix during the transition. Figure 2 shows the dynamics of the social security contribution rates (including pension, health care and disability insurance) between 2000 and 2100.

As one would expect from the development of the population structure in our three regions (see Tables 1-3), the strongest increase over the next 50 years will take place in Japan. There the contribution rates rise from 24.5 percent in 2000 to 50.4 percent in 2050. After the peak of the aging process, they fall and reach a value of 39.8 percent in 2100 which is still much higher than in the initial year. In Europe, the payroll taxes increase to a somewhat lower extent than in Japan from 26.5 percent in year 2000 to a maximum of 45.1 percent in year 2050. This is also followed by a reduction to 38.7 percent in 2100. The increase of the social security contribution rates in the US is more modest than in the other two regions since population ageing is not as strong. Here it increases from 13.5 percent in 2000 to 23.8 percent in 2050. After year 2050, however, since the aging process in the US takes longer, payroll taxes keep rising but to a lower extent than in the years before so that they amount to a value of 27.4 percent in the long-run.

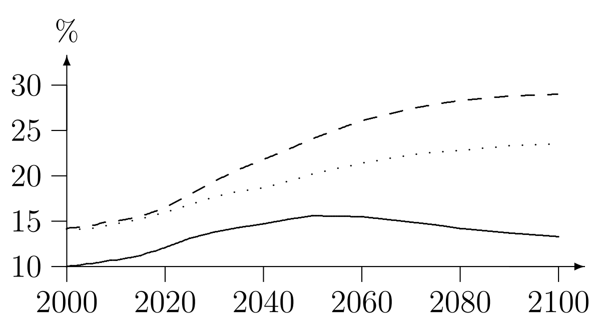
The average wage tax rate (see Figure 3) that is computed endogenously shows a strong increase in the EU from 14.2 percent in 2000 to 29 percent in 2100. This is due to high government

Figure 2: Social security contribution rates



expenses per capita and the reduction of the number of active persons. The increase in Japan is lower since government expenses are less generous. Here the average tax rate rises from 14.1 percent in 2000 to 23.5 percent in 2100. In the US, the development is different. While in the other two regions the wage tax rate increases during the whole transition, it increases in the US from 10 percent to a maximum of 15.6 percent in 2050. Afterwards, it falls to a long-run level of 13.3 percent. This is due to the fact that effective labor supply rises strongly after year 2050. Furthermore, government purchases are lower than in the EU which leads to a smaller increase of the wage tax rate.

Figure 3: Average wage tax rates



Figures 4 and 5 show the capital stock and labor changes during the next century relative to the initial year 2000. At first, it might seem strange that labor supply in all regions is increasing until 2100 although the economies are aging. However, one has to keep in mind that the assumed labor-augmenting technical change raises the time endowments of successive cohorts by one per cent. Therefore, effective labor supply is rising over time. The highest increase is in the US where the aging process is not as strong as in the other two regions and where immigration is higher. Hence, effective labor supply in 2100 is 5.06 times higher than in 2000.

Japan, however, with an enormous aging population, has the lowest increase in effective labor supply - it is only 76 percent higher in 2100 compared to the initial year. The development in the EU falls between the two previously discussed extremes. Here, labor supply is 176 percent higher in the long-run than in 2000.

Figure 4: Aggregate capital stock (relative to 2000)

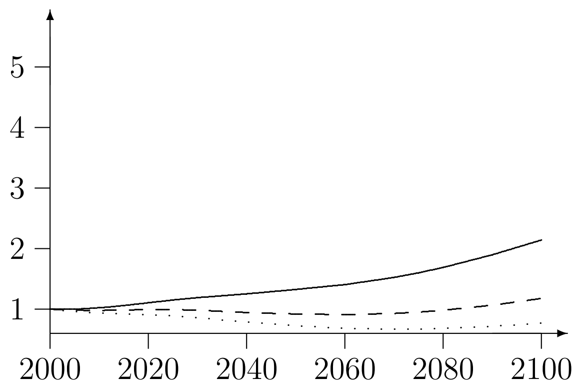
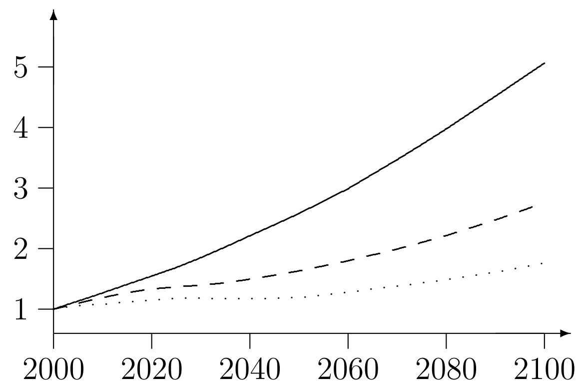


Figure 5: Aggregate labor supply (relative to 2000)



The capital stock, on the other hand, increases in the US while it falls in the EU until 2060 and in Japan until 2070. Afterwards, there is a slight increase of the capital stocks in the EU and Japan. However, while the long-run capital stock in the US is 2.14 times higher and in the EU 1.18 times higher compared to the initial year, it is 23 percent lower in Japan than in 2000. During the whole transition the capital-output ratio falls since growth in labor supply exceeds growth in the capital stock in all three regions. One reason why the capital stock in the US is not rising as much as the work force and in the EU and Japan even falls during the first part of the transition, is the existence of the social security systems. As seen above, the contribution rates to these insurance systems rise strongly over the whole period. Even if the national saving rate rises over the whole period this increase is not high enough to increase the capital stock as much as the work force in the US due to the rising payroll taxes. In the EU and Japan the social security systems are more generous and population aging is more severe. Therefore, contribution rates rise much more than in the US which leads at first to a reduction in the capital stock in both regions. This reduction is strongest in Japan. Another reason is the increase in the wage tax rates to finance the government expenses. Since the workers have less disposable income out of which to save, higher capital accumulation is prevented. However, we also repeated our simulations for all regions modelled as closed economies. There we found that the capital stock rises much more in the US and less in the EU and Japan compared to the here presented open economy case. These differences are due to capital flows between the regions which we want to highlight below.

Output growth in all regions is positive despite this development of the capital stocks. In the US this growth is the strongest so that the national income is more than four times higher in year 2100 compared to year 2000. The lowest output growth is observed in Japan where the national income grows by only 43 percent over the next 100 years. In the EU the long-run national income is 2.23 times higher than in the initial year.

Of course, the paths of capital and labor determine in turn the paths of wages, capital prices, current accounts and the interest rate. As shown in Figure 6, wages fall in all three regions over the whole period. This reduction during the transition is strongest in the US since labor supply rises stronger than in the other two regions. In the EU and Japan the fall of the wage rates is slower. However, until the year 2100 the wage rates are reduced by 19 percent in all regions compared to the initial year.

Figure 6: Wage rates (relative to 2000)

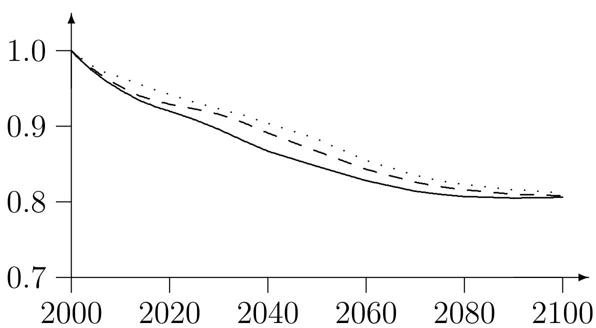
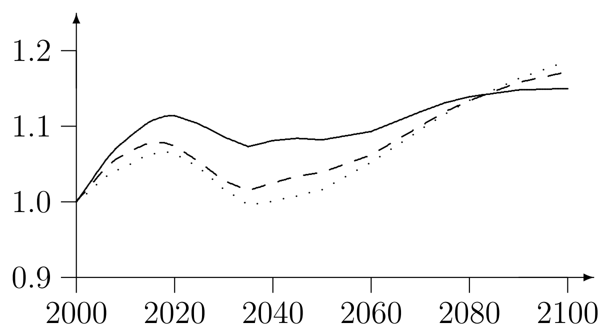


Figure 7: Prices of capital (relative to 2000)

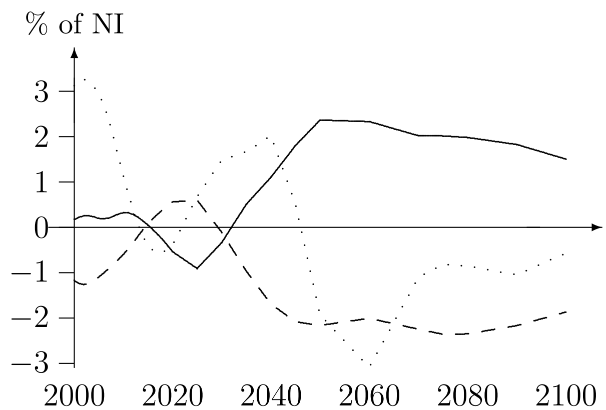


The prices of capital (see Figure 7) increase in the US until 2020 and in the EU and Japan until 2018. They begin to fall when the aging process starts to get severe. In the US, however, this reduction is the smallest since population aging is more modest compared to the other two regions where capital prices are reduced strongly. After 2035 the prices rise again. This increase is more modest in the US since the aging process is slower. In the EU and Japan the increase is so steep that, starting in 2090, the capital prices are even higher than in the US.

These differences in the transitional path of the capital prices also influence capital flows between our three open economies and hence the development of the current accounts which is represented in Figure 8. During the first years of the transition the initial current current account deficit in the EU lowers and turns into a surplus with the maximum value in 2025. This development is due to the fact that capital from the EU is invested in the US where capital prices rise stronger. That, in turn, makes investments more profitable. At the same time the initial current account surplus in Japan lowers and becomes a deficit with the maximum value in 2018. Since the national saving rate in Japan falls, capital invested abroad is shifted back to Japan where this capital is needed. After 2018 however the saving rate starts to rise so that capital flows out of the country again since investments in foreign countries lead to higher returns. In the US the initial current account surplus lightly increases during the first years of the transition. Afterwards, it turns into a deficit which mirrors the capital inflows from the EU and later from Japan since capital prices in the US rise stronger than in the other two regions. However, when the impact of the aging process in the US starts to grow, the current account deficit lowers and runs into a surplus again. This indicates capital flows from the US into the

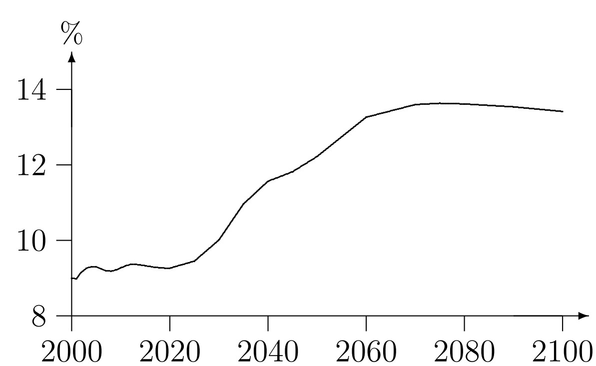
EU and Japan where capital prices are rising much stronger. The current account is still in surplus in the long-run while it shows a deficit in the EU and Japan since the aging process in the US is slower than in the other two regions. In Japan, however, the deficit lowers until 2100. The increase in the national saving rates in the EU and Japan and these strong capital inflows in the long-run leads to the increase in the capital stock observed in the EU after 2050 and in Japan after 2075.

Figure 8: Current accounts



Finally, the development of the capital prices and the capital stocks will also affect the world interest rate (see Figure 9). In the first years of the transition, the interest rate remains relatively stable. When population aging in the three regions starts to get severe after 2020 there is a strong increase in the world interest rate until about 2075 where a maximum value of 13.6 percent is reached. Afterwards, there can be observed a light reduction in the interest rate so that in 2100 it amounts to a value of 13.5 percent. Overall, there is an increase of 4.5 percentage points over the transition.

Figure 9: World interest rate



This completes our overview of the baseline path of the most important variables in our simulation model during the demographic transition in the coming 100 years.

VII. Conclusion

In this paper we have presented a new simulation model which allows us to analyze the interaction of aging in the three major industrialized world regions. The simulation of a baseline path, which keeps current policies fixed, shows that the aging process will have a drastic impact on several variables. However, population aging is not identical in all three regions. It is more profound in Europe and Japan and more modest in the US. Therefore, payroll taxes in the EU and Japan have to rise much stronger than in the US to balance the budgets of the social security systems. Due to high fertility rates, high immigration and our assumed technical progress, labor supply in the US rises stronger during the remaining century than in the other two world regions. This causes a steady increase in the capital stock in the US. It decreases in the first part of the transition in the EU and Japan. That is followed by an increase in the long-run. However, due to international capital flows, this increase in the US is smaller and in the EU and Japan stronger than in a closed-economy simulation. The aging process has also a strong impact on the world interest rate and the asset prices - both will increase significantly. Furthermore, wage rates will be reduced by 19 percent in the next 100 years. Future living standards and the welfare of the population are expected to decline due to this reduction and the high payroll tax burden.

References

Altig, D., A.J. Auerbach, L.J. Kotlikoff, K.A. Smetters and J. Walliser (2001): Simulating Fundamental Tax Reform in the United States, American Economic Review 91, 574-595.

Auerbach, A.J. and L.J. Kotlikoff (1987): Dynamic Fiscal Policy, Cambridge.

Beetsma, R., L. Bettendorf and P. Broer (2001): The Budgeting and Economic Consequences of Ageing in the Netherlands, CESifo Working Paper No. 630, Munich.

Blöndal, S. and S. Scarpetta (1999): The retirement decision in OECD countries, OECD Economics Department Working Paper No. 202, Paris.

European Commission (2001): Budgetary challenges posed by ageing populations: the impact on public spending on pensions, health and long-term care for the elderly and possible indicators of the long-term sustainability of public finances, Brussels.

European Commission (2003): Statistical Annex to European Economy No. 4, Brussels.

Fehr, H. (1999): Welfare effects of dynamic tax reforms, Tübingen.

Fehr, H. (2000): Pension Reform during the Demographic Transition, Scandinavian Journal of Economics 102, 419-443.

Hayashi, F. (1982): Tobin's Marginal Q and Average Q: A Neoclassical Interpretation, *Econometrica 50*, 213-224.

Institute of Population and Social Security Research (IPSS) (2003): The cost of social security in Japan, Statistical Report No. 13, Tokyo.

Joumard, I. (2001): Tax systems in European Union Countries, OECD Economics Department Working Paper No. 301, Paris.

Kotlikoff, L.J., K.A. Smetters, J. Walliser (2001): Finding a way out of America's demographic dilemma, NBER Working Paper No. 8258, Cambridge.

Ministry of Finance (1999): Japanese tax system, Tokyo.

OECD (2001): Consumption tax trends, Paris.

OECD (2002a): Education at a glance, Paris.

OECD (2002b): Taxing Wages 2000-2001, Paris.

OECD (2003): Health Data, 4th ed., Paris.

Social Security Agency (2001): Fast Facts and Figures about Social Security, Washington D.C.

United Nations Population Division (2003): World Population Prospects: The 2002 Revision, New York.

U.S. Department of Health and Human Services (2002): CMS Statistics, Washington D.C.

Whitehouse, E. (2002): Pension systems in 15 countries compared: the value of entitlements, Discussion Paper 02/04, Centre for Pensions and Superannuation, Sydney.

Appendix

The following tables in this appendix show our age-specific distribution of the population and immigrants for all three regions in the initial year 2000. Age-specific fertility rates, the total fertility rates and the average birth ages as well as the mortality rates and life expectancy are also reported. Furthermore, we report the development of the most important variables during the transition of our baseline simulation.

Table A-1: Population in 2000

Age	EU-15	USA	Japan
0	4.016.593	3.761.366	1.142.303
1	4.039.194	3.778.878	1.157.266
2	4.051.224	3.777.910	1.166.500
3	4.066.273	3.791.496	1.176.192
4	4.091.145	3.817.928	1.185.624
5	4.133.381	3.853.875	1.193.710
6	4.193.783	3.896.863	1.200.809
7	4.264.405	3.949.172	1.208.377
8	4.329.908	4.003.247	1.219.562
9	4.378.894	4.043.429	1.236.451
10	4.406.124	4.048.954	1.260.192
11	4.416.677	4.019.948	1.289.340
12	4.417.558	3.974.382	1.322.759
13	4.419.488	3.938.160	1.359.858
14	4.431.238	3.921.727	1.400.574
15	4.462.448	3.926.118	1.442.117
16	4.514.123	3.944.943	1.478.969
17	4.579.322	3.974.051	1.506.759
18	4.641.379	3.995.603	1.526.957
19	4.689.189	3.987.213	1.545.673
20	4.721.889	3.927.543	1.570.080
21	4.753.609	3.822.257	1.603.192
22	4.800.551	3.697.031	1.646.624
23	4.875.684	3.585.891	1.699.710
24	4.982.191	3.506.686	1.761.550
25	5.117.305	3.470.788	1.824.982
26	5.271.843	3.483.963	1.878.661
27	5.432.383	3.556.493	1.909.253
28	5.584.173	3.668.488	1.911.772
29	5.717.028	3.782.916	1.890.661
30	5.829.740	3.855.559	1.853.426
31	5.926.926	3.889.323	1.805.648
32	6.011.740	3.919.156	1.747.784
33	6.079.295	3.987.070	1.691.846
34	6.116.245	4.095.298	1.647.290
35	6.110.562	4.220.238	1.624.994
36	6.059.583	4.332.891	1.613.056
37	5.973.571	4.429.809	1.604.068
38	5.867.285	4.508.320	1.590.006
39	5.751.939	4.565.710	1.577.142
40	5.634.489	4.584.069	1.566.772
41	5.519.027	4.563.689	1.561.130
42	5.410.227	4.511.872	1.562.278
43	5.310.361	4.441.307	1.576.303
44	5.220.873	4.349.265	1.607.117
45	5.144.848	4.234.100	1.655.019

Table A-1 continued

Age	EU-15	USA	Japan
46	5.086.603	4.102.250	1.718.945
47	5.052.213	3.975.331	1.801.920
48	5.040.041	3.864.726	1.905.994
49	5.041.768	3.775.251	2.026.142
50	5.030.727	3.690.008	2.132.587
51	4.977.221	3.596.330	2.182.303
52	4.856.614	3.465.435	2.135.068
53	4.684.995	3.298.697	2.006.290
54	4.501.262	3.107.546	1.855.759
55	4.355.660	2.938.442	1.758.895
56	4.265.669	2.797.866	1.734.933
57	4.229.966	2.680.483	1.749.554
58	4.225.800	2.555.162	1.749.776
59	4.228.373	2.428.899	1.712.295
60	4.210.118	2.311.363	1.654.335
61	4.159.013	2.219.608	1.603.561
62	4.080.647	2.149.356	1.577.449
63	3.989.944	2.094.575	1.565.785
64	3.894.977	2.044.047	1.551.739
65	3.801.749	1.993.197	1.524.076
66	3.716.696	1.943.001	1.487.647
67	3.649.538	1.903.196	1.449.515
68	3.594.243	1.878.473	1.411.086
69	3.537.256	1.865.520	1.368.102
70	3.461.704	1.851.226	1.318.476
71	3.367.831	1.825.583	1.264.430
72	3.261.258	1.785.638	1.208.199
73	3.150.596	1.735.595	1.148.572
74	3.037.631	1.682.030	1.082.242
75	2.924.761	1.628.546	1.008.431
76	2.807.758	1.575.363	931.508
77	2.670.597	1.517.209	857.154
78	2.483.318	1.446.491	789.219
79	2.231.694	1.358.134	725.173
80	1.939.208	1.255.088	663.794
81	1.668.439	1.147.799	604.525
82	1.474.081	1.045.571	552.270
83	1.368.957	951.405	507.137
84	1.317.013	862.388	467.387
85	1.269.226	776.295	427.465
86	1.187.566	693.506	385.530
87	1.052.546	615.878	341.719
88	849.985	544.112	298.142
89	578.461	477.310	255.824
90	259.817	413.313	214.655
$\sum$	376.339.250	275.262.239	126.715.967

Table A-2: Immigration in 2000

Age	**EU-15**	**USA**	**Japan**
0	0	0	0
1	4.476	14.584	537
2	3.105	12.722	373
3	1.058	11.131	127
4	762	10.337	91
5	1.497	12.246	180
6	359	11.782	43
7	178	12.849	21
8	2.903	13.302	348
9	3.421	13.828	410
10	4.163	16.603	500
11	5.191	17.545	623
12	5.915	19.263	710
13	7.115	20.867	854
14	7.343	22.925	881
15	9.135	24.338	1.096
16	12.191	26.711	1.463
17	13.683	30.267	1.642
18	18.035	35.677	2.164
19	29.034	34.615	3.484
20	36.799	35.896	4.416
21	31.825	20.377	3.819
22	34.941	22.482	4.193
23	33.028	24.041	3.963
24	29.895	29.085	3.587
25	23.901	31.842	2.868
26	17.586	32.395	2.110
27	14.609	29.957	1.753
28	11.550	27.991	1.386
29	9.048	26.094	1.086
30	6.271	25.327	753
31	3.917	24.962	470
32	2.135	21.876	256
33	1.538	20.716	185
34	1.551	19.030	186
35	1.349	17.108	162
36	1.962	16.328	235
37	1.801	14.675	216
38	2.401	13.528	288
39	2.966	12.303	356
40	3.369	12.160	404
41	3.769	10.967	452
42	3.514	11.092	422
43	3.969	9.828	476
44	2.977	8.283	357
45	3.607	9.039	433

Table A-2 continued

Age	**EU-15**	**USA**	**Japan**
46	3.492	8.018	419
47	2.952	8.266	354
48	3.253	7.182	390
49	3.341	6.722	401
50	2.916	6.715	350
51	1.965	5.484	236
52	1.612	5.161	193
53	1.370	5.205	164
54	444	5.341	53
55	550	5.322	66
56	095	5.169	11
57	470	5.178	56
58	807	5.186	97
59	2.032	5.091	244
60	32	5.102	4
61	635	4.565	76
62	1.216	4.849	146
63	1.545	4.404	185
64	859	4.225	103
65	574	3.841	69
$\sum$	450.000	1.000.000	54.000

Table A-3: Age-specific fertility rates

Woman Age	EU-15		USA		Japan	
	2000	**2050**	**2000**	**2050**	**2000**	**2050**
23	0.1102	0.0173	0.3597	0.3750	0.0683	0.0119
24	0.1166	0.0702	0.3284	0.3429	0.0631	0.0465
25	0.1214	0.1259	0.2663	0.2790	0.0778	0.0818
26	0.1239	0.1489	0.2048	0.2157	0.1076	0.1038
27	0.1248	0.1535	0.1439	0.1531	0.1375	0.1312
28	0.1241	0.1557	0.1131	0.1215	0.1565	0.1681
29	0.1204	0.1556	0.1113	0.1196	0.1498	0.1975
30	0.1130	0.1525	0.1073	0.1154	0.1240	0.2046
31	0.1021	0.1455	0.0744	0.0816	0.0940	0.1858
32	0.0888	0.1342	0.0645	0.0715	0.0639	0.1504
33	0.0744	0.1194	0.0531	0.0598	0.0457	0.1115
34	0.0600	0.1024	0.0424	0.0487	0.0441	0.0810
35	0.0461	0.0845	0.0328	0.0338	0.0406	0.0649
36	0.0346	0.0671	0.0225	0.0180	0.0307	0.0573
37	0.0260	0.0515	0.0346	0.0304	0.0210	0.0484
38	0.0192	0.0388	0.0261	0.0217	0.0127	0.0362
39	0.0137	0.0289	0.0196	0.0151	0.0078	0.0242
40	0.0094	0.0211	0.0139	0.0092	0.0057	0.0153
41	0.0061	0.0149	0.0093	0.0044	0.0044	0.0101
42	0.0037	0.0101	0.0058	0.0008	0.0031	0.0072
43	0.0021	0.0065	0.0033	0.0000	0.0019	0.0053
44	0.0011	0.0039	0.0018	0.0000	0.0009	0.0036
45	0.0004	0.0022	0.0009	0.0009	0.0003	0.0021
TFR	1.4422	1.8109	2.0397	2.1183	1.2613	1.7487
ABA	29.01	30.5	27.28	27.07	29.11	30.6

TFR=Total fertility rate (female), ABA=Average birth age

Table A-4: Mortality rates

Age	EU-15				USA			
	2000		**2050**		**2000**		**2050**	
	$\bar{d}(\cdot)$	$d(\cdot)$	$\bar{d}(\cdot)$	$d(\cdot)$	$\bar{d}(\cdot)$	$d(\cdot)$	$\bar{d}(\cdot)$	$d(\cdot)$
⋮	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
68	0.023	0.023	0.002	0.002	0.026	0.026	0.002	0.002
69	0.024	0.025	0.004	0.004	0.028	0.028	0.003	0.003
70	0.026	0.027	0.005	0.005	0.029	0.030	0.005	0.005
71	0.027	0.029	0.007	0.007	0.030	0.033	0.006	0.006
72	0.029	0.032	0.009	0.009	0.032	0.036	0.007	0.007
73	0.031	0.035	0.011	0.012	0.033	0.038	0.009	0.009
74	0.033	0.039	0.015	0.016	0.034	0.042	0.012	0.012
75	0.035	0.043	0.020	0.022	0.035	0.045	0.016	0.017
76	0.036	0.047	0.026	0.028	0.036	0.048	0.022	0.023
77	0.038	0.051	0.030	0.034	0.037	0.052	0.027	0.029
78	0.039	0.056	0.034	0.039	0.038	0.055	0.032	0.036
79	0.040	0.061	0.036	0.043	0.038	0.059	0.035	0.041
80	0.041	0.067	0.038	0.048	0.038	0.063	0.038	0.046
81	0.042	0.073	0.040	0.053	0.038	0.067	0.040	0.050
82	0.042	0.079	0.042	0.058	0.038	0.071	0.041	0.055
83	0.042	0.084	0.044	0.064	0.037	0.075	0.042	0.060
84	0.041	0.090	0.045	0.071	0.036	0.079	0.043	0.065
85	0.039	0.094	0.047	0.079	0.034	0.080	0.044	0.071
86	0.037	0.098	0.048	0.088	0.029	0.076	0.045	0.077
87	0.033	0.099	0.049	0.098	0.022	0.062	0.045	0.084
88	0.027	0.088	0.049	0.110	0.014	0.041	0.045	0.093
89	0.016	0.059	0.050	0.124	0.007	0.021	0.045	0.102
90	0.007	0.026	0.049	0.141	0.003	0.009	0.045	0.112
91	0.002	1.000	0.048	1.000	0.001	1.000	0.044	1.000
LE	**82.05**		**85.13**		**82.06**		**85.63**	

LE= Life expectancy, $\bar{d}(\cdot), d(\cdot)$ death probabilities

Table A-4 continued

Age	Japan			
	2000		2050	
	$\bar{d}(\cdot)$	$d(\cdot)$	$\bar{d}(\cdot)$	$d(\cdot)$
⋮	0.000	0.000	0.000	0.000
68	0.013	0.013	0.001	0.001
69	0.014	0.014	0.001	0.001
70	0.015	0.016	0.001	0.001
71	0.017	0.018	0.002	0.002
72	0.018	0.020	0.002	0.002
73	0.020	0.021	0.002	0.002
74	0.021	0.023	0.002	0.002
75	0.023	0.026	0.002	0.002
76	0.025	0.029	0.002	0.002
77	0.027	0.033	0.002	0.002
78	0.029	0.036	0.003	0.003
79	0.031	0.039	0.003	0.003
80	0.034	0.046	0.003	0.003
81	0.039	0.055	0.004	0.004
82	0.043	0.064	0.025	0.026
83	0.045	0.072	0.046	0.049
84	0.047	0.080	0.068	0.075
85	0.047	0.088	0.089	0.107
86	0.047	0.095	0.110	0.148
87	0.045	0.102	0.131	0.207
88	0.043	0.107	0.152	0.303
89	0.040	0.111	0.131	0.374
90	0.043	0.136	0.110	0.501
91	0.024	1.000	0.087	1.000
LE	**84.00**		**87.05**	

Table A-5: Simulation results

	Year	National Income	Capital Stock	Effective Labor Supply	Current Account	Before-Tax Wage	Capital Price	Interest Rate	OASHDI Cost Rate	Average Wage Tax
US	2000	1.00	1.00	1.00	.002	1.00	1.000	.090	.135	.100
	2005	1.10	1.00	1.13	.002	0.97	1.046	.093	.137	.103
	2010	1.20	1.02	1.27	.003	0.95	1.081	.093	.148	.107
	2020	1.42	1.11	1.55	-.005	0.92	1.114	.093	.191	.121
	2030	1.65	1.19	1.85	-.003	0.90	1.086	.100	.231	.138
	2050	2.18	1.33	2.58	.024	0.85	1.082	.122	.238	.156
	2075	3.00	1.60	3.72	.020	0.81	1.131	.136	.255	.146
	2100	4.06	2.14	5.06	.015	0.81	1.150	.134	.274	.133
EU	2000	1.00	1.00	1.00	-.012	1.00	1.000	.090	.265	.142
	2005	1.07	0.98	1.10	-.011	0.97	1.038	.093	.274	.145
	2010	1.13	0.98	1.19	-.006	0.95	1.063	.093	.288	.150
	2020	1.24	0.99	1.33	.006	0.93	1.074	.093	.332	.165
	2030	1.28	0.98	1.40	-.001	0.92	1.028	.100	.410	.194
	2050	1.41	0.92	1.63	-.022	0.87	1.039	.122	.451	.241
	2075	1.72	0.95	2.10	-.024	0.82	1.119	.136	.412	.279
	2100	2.23	1.18	2.76	-.019	0.81	1.172	.134	.387	.290
Japan	2000	1.00	1.00	1.00	.031	1.00	1.000	.090	.245	.141
	2005	1.03	0.96	1.05	.029	0.98	1.028	.093	.274	.142
	2010	1.05	0.94	1.08	.011	0.97	1.047	.093	.318	.147
	2020	1.09	0.91	1.15	-.004	0.94	1.064	.093	.369	.160
	2030	1.09	0.86	1.18	.015	0.92	1.017	.100	.412	.177
	2050	1.05	0.73	1.19	-.019	0.88	1.016	.122	.504	.202
	2075	1.19	0.67	1.43	-.008	0.83	1.116	.136	.430	.226
	2100	1.43	0.77	1.76	-.006	0.81	1.186	.134	.398	.235

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