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Financing Development: The Role of Information Costs

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Financial development "accelerates economic growth and improves economic performance to the extent that it facilitates the migration of funds to the best user, i.e., to the place in the economic system where the funds will earn the highest social return," noted Raymond W. Goldsmith (1969, p. 400) some 40 years ago. Information production plays a key role in this process of steering of funds to the highest valued users. If the costs of information production drop, then financial intermediation should become more efficient with an associated improvement in economic performance.

Improvements in the efficiency of financial intermediation, due to improved information production, are likely to reduce the spread between the internal rate of return on investment in firms and the rate of return on savings received by savers. The spread between these returns reflects the costs of intermediation. This spread will include the ex post information costs of policing investments, and the costs of the misappropriation of savers' funds by management, unions, etc., that arise in a world with imperfect information. There may be no change in the rate of return earned by savers over time, because aggregate savings will adjust in equilibrium so that this return reflects savers' rates of time preference. If the wedge between the internal rate of return earned by firms and the rate of return received by savers falls, due to more effective intermediation, then both the economy's capital stock and income should rise. Additionally, if capital is redirected away from the less productive investment opportunities in the economy toward more productive ones, then the economy's output will rise further and productivity move up. In fact, empirical research strongly suggests that financial development has a causal effect on economic development—see Ross Levine (2005) for a masterful survey. Specifically, financial development leads to higher rates of growth in the capital stock, income, and productivity.

A general equilibrium model of firm finance, with competitive intermediation, is presented to address the impact that financial intermediation has on economic development. At the heart of the framework developed here is a costly-state verification paradigm that has its roots in classic work by Robert M. Townsend (1979) and Stephen D. Williamson (1986). The model here has two novel ingredients, though. First, in the standard costly-state verification paradigm, the realized return on a firm's investment activity is private information. This return can be monitored, but the *outcome* of this auditing process is deterministic: once monitoring takes place, the true state of the world is revealed with certainty. This is true whether or not a deterministic *decision rule* for monitoring is employed, as in Townsend (1979) or Williamson (1986, 1987), or a stochastic

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one is used, as in Ben Bernanke and Mark Gertler (1989) or John H. Boyd and Bruce D. Smith (1994)—see also Townsend (1979, Section 4). By contrast, in the current set-up the outcome of monitoring is random. Specifically, the probability of detecting malfeasance depends upon the amount of resources devoted to policing the returns on a project and the efficiency of the monitoring technology. In the model a project's funding and level of monitoring will be jointly determined. The information asymmetry between the firm and the intermediary gives the firm an opportunity to exploit its private knowledge about the realization of the investment return. In particular, it can extract rents from the intermediary and, hence, savers.

Second, a firm's production technology is subject to idiosyncratic randomness. This is true in the standard costly-state verification framework as well. Here, though, there is a distribution across firms over the distribution of these returns. In particular, some firms may have investment projects that offer low expected returns with little variance, while others may have projects that yield high expected returns with a large variance.

Two key features in the analysis follow from these ingredients. First, the set-up yields a finance-based theory of the equilibrium size distribution for firms. This theory derives from the facts that: (a) investment opportunities differ in the expected returns and levels of risk that they offer and (b) producing information about these returns is costly. A simple threshold rule for funding results. All firms with an expected return at least as great as the cost of raising capital are funded. Funding is increasing in a firm's expected return and is decreasing in its risk. Loan size is determinate for a given type of project because the costs of financing a project are increasing and convex in the level of the monitoring activity. The demand for monitoring services rises in step with investment in a project. Thus, high mean projects receive limited funding because the costs of monitoring will rise disproportionately with loan size. The riskier a project is, the bigger is the difference between the returns in good and bad states. This increases the incentive for a firm to low-ball its earnings in good states. Hence, more diligent monitoring will be required, which increases its financing cost. In an abstract sense, one could think that the diminishing returns in information production modeled here provide a microfoundation for Robert E. Lucas Jr.'s (1978) span of control model.

Second, the framework provides a link between the efficiency of the financial system and the level of economic activity. Such a tie was developed earlier in the models of Valerie R. Bencivenga and Smith (1991), Greenwood and Boyan Jovanovic (1990), Levine (1991), and Albert Marcet and Ramon Marimon (1992). The analysis here provides a crystal clear delineation of the Goldsmith (1969) mechanism, however. It stresses the connection between the state of technological development in the financial sector, on the one hand, and capital accumulation, along both the extensive and intensive margins, on the other. If technological improvement in the financial sector occurs at a faster pace than in the rest of the economy, then financial intermediation becomes more efficient. Loans are monitored more diligently and the rents earned by firms shrink. Additionally, lending activity will change along both extensive and intensive margins. Projects with high (low) expected returns will now receive more (less) funds. Those investments with the lowest expected returns will be cut. At high levels of efficiency in the financial sector the economy approaches the first-best equilibrium achieved in a world without informational frictions. This reallocation effect distinguishes the analysis from earlier research by Shankha Chakraborty and Amartya Lahiri (2007) and Aubhik Khan (2001) that also embed the standard costly-state verification framework into a growth model. In these frameworks all firms receive the same amount of capital.

I. The Environment

Imagine a world resting in a steady state that is made up of three types of agents: consumer/workers, firms, and financial intermediaries. In a nutshell, firms produce output using capital and labor. The consumer/worker supplies the labor, and intermediaries, the capital. All funding for capital must be raised outside of the firm. Financial intermediaries obtain the funds for capital from consumer/workers. They also use labor in their lending activity. Output is used for consumption by consumer/workers and for investment in capital by intermediaries. The behavior of firms and intermediaries will now be described in more detail. The consumer/worker plays a more passive role in the analysis, which is relegated to the background by assuming that he supplies one unit of labor and saves at some fixed interest rate, $\hat{r}$.[1]

II. Firms

Firms produce output, o, in line with the production function

$$o = \theta k^{\alpha} l^{1-\alpha},$$

where k and l represent the inputs of capital and labor used in production. The variable θ gives the productivity level of the firm's production process. Productivity is a random variable drawn from a two-point vector $\tau \equiv (\theta_1, \theta_2)$ with $\theta_1 < \theta_2$. Let $\Pr(\theta = \theta_1) = \pi_1$ and $\Pr(\theta = \theta_2) = \pi_2 = 1 - \pi_1$. The mean and variance of θ are given by $\pi_1\theta_1 + \pi_2\theta_2$ and $\pi_1\pi_2(\theta_1 - \theta_2)^2$, respectively. Thus, for a given set of probabilities these statistics differ in accordance with the values specified for θ_1 and θ_2. The realized value of θ is the firm's private information.

Now, the productivity vector, τ, differs across firms. In particular, suppose that firms in the economy are distributed over productivities in line with the distribution function $\mathrm{F} : \mathcal{T} \rightarrow [0,1]$, where $\mathcal{T} \subseteq R_+^2$ and

$$F(x, \mathrm{y}) = \Pr(\theta_1 \leq x, \theta_2 \leq y).$$

Think of this distribution as somehow specifying a trade-off between the mean and variance of project returns. Due to technological progress in the production sector of the economy, this distribution will evolve over time. Figure 1 plots an illustrative density function for F in mean/variance space.

The firm borrows capital, k, from the intermediary *before* it observes the technology shock, θ. It does this with both parties knowing its type, τ. It can employ labor, at the wage rate w, *after* it sees the realization for θ. In order to finance its use of capital the firm must enter into a contract with a financial intermediary. Last, note that a firm's production is governed by constant returns to scale. In the absence of financial market frictions no rents would be earned on production. Additionally, in a frictionless world only firms offering the highest expected return would be funded. With financial market frictions, deserving projects are underfunded, while undeserving projects are simultaneously over funded.

A. *Profit Maximization by Firms*

Consider the problem faced by a firm that receives a loan in terms of capital in the amount k. The firm hires labor after it sees the realization of its technology shock, θ. It will do this in a

[1] Think about a representative consumer with time separable preferences over consumption. The steady state interest rate will then be given by $\hat{r} = 1/\beta - 1$, where β is his discount factor.

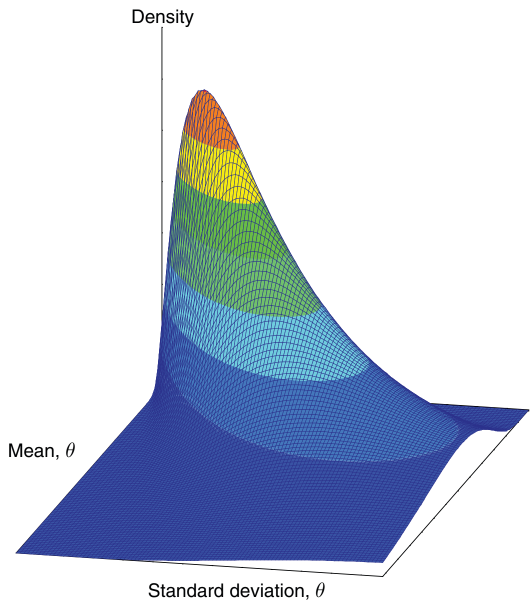

Figure 1. Illustrative Distribution for Firm Type in Mean/Variance Space

manner so as to maximize its profits. In other words, the firm will solve the maximization problem shown below.

$$R(\theta, w)k \equiv \max_{l}\{\theta k^{\alpha} l^{1-\alpha} - wl\}. \tag{P1}$$

The first-order condition associated with this maximization is

$$(1 - \alpha)\theta k^{\alpha} l^{-\alpha} = w,$$

which gives

$$l = \left[\theta \frac{(1 - \alpha)}{w}\right]^{1/\alpha} k. \tag{1}$$

Substituting the solution for l into the maximand and solving yields the unit return function, $R(\theta, w)$, or

$$r = R(\theta, w) = \alpha(1 - \alpha)^{(1-\alpha)/\alpha} w^{-(1-\alpha)/\alpha} \theta^{1/\alpha} > 0. \tag{2}$$

Think about $r_i = R(\theta_i, w)$ as giving the rate of return on a unit of capital invested in the firm given that state θ_i occurs. The expected rate of return will be $\pi_1 r_1 + \pi_2 r_2$, while the variance reads $\pi_1 \pi_2 (r_1 - r_2)^2$.

III. Financial Intermediaries

There is a competitive intermediation sector that borrows funds from consumers and lends capital to firms. While the intermediary knows a firm's type, it cannot observe the state of a firm's business either costlessly or perfectly. That is, the intermediary cannot costlessly observe θ, o and l. The firm will make a report to the intermediary about its business situation. The intermediary can devote some resources in order to assess the veracity of this report. The payments, p, from a firm to the intermediary will be conditioned both upon the report made by the former, and the outcome of any monitoring activity done by the latter. By channelling funds through financial intermediaries consumers avoid a costly duplication of monitoring effort that would occur in an equilibrium with direct lending between them and firms—see Williamson (1986) for more detail. Likewise, in the environment under study, it is optimal for a firm to borrow from only one intermediary at a time.

Suppose a firm reports that the productivity on its project in a given period is θ_j, which may differ from the true state θ_i. The intermediary can devote resources, m_j, to verify this claim. The probability of detecting fraud is increasing in the amount of resources devoted to this activity. In particular, let $P_{ij}(m_j/k)$ denote the probability that the firm is caught cheating conditional on the following: (i) the true realization of productivity is θ_i; (ii) the firm makes a report of θ_j; (iii) the intermediary spends m_j in monitoring; (iv) the total amount of borrowing is k (which represents the size of the project). The function $P_{ij}(m_j/k)$ is assumed to be monotonically increasing in m_j/k. Additionally, let $P_{ij}(m_j/k) = 0$ if the firm truthfully reports that its type is θ_i. Any lender to the firm must monitor the *whole* project to detect cheating, because his claim to profits will depend on the total level of receipts vis à vis the total amount of disbursements paid out to others. Borrowing through a single intermediary then avoids a costly duplication of monitoring effort.

A convenient formulation for $P_{ij}(m_j/k)$ is

$$P_{ij}(m_j/k) = \begin{cases} 1 - \dfrac{1}{(\epsilon m_j/k)^{\psi}} < 1, \text{ with } 0 < \psi < 1, \\ \text{for a report } \theta_j \neq \theta_i \text{ and } m_j/k > 1/\epsilon, \\ \\ 0, \\ \text{for a report } \theta_j = \theta_i \text{ or } m_j/k \leq 1/\epsilon. \end{cases}$$

To guarantee that $P_{ij}(m_j/k) \geq 0$, this specification requires that some threshold level of monitoring, $m_j > k/\epsilon$, must be exceeded to detect cheating. Note that this threshold level of monitoring can be made arbitrarily small by picking a large enough value for ϵ. Figure 2 makes this clear, while illustrating the function $P_{ij}(m_j/k)$.

Monitoring is a produced good, measured in units of consumption. The production of monitoring is project specific. Monitoring produced for detecting fraud in one project cannot be used in a different one. Let monitoring be produced in line with the production function

$$m = z l_m^{1/\gamma}, \text{ with } 0 \leq 1/\gamma \leq 1,$$

where l_m represents the amount of labor employed in this activity. The cost function, $C(m/z; w)$, associated with monitoring is given by

$$C(m/z; w) = w(m/z)^{\gamma}.$$

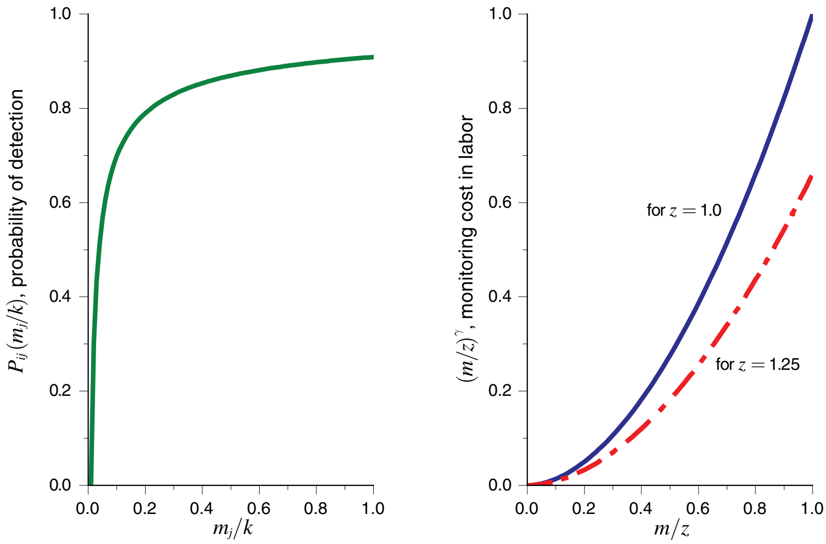

Figure 2. The Functions $P_{ij}(m_j/k)$ and $C(m/z; w)/w$.

Note: $\epsilon = 100$, $\psi = 0.52$, and $\gamma = 1.86$.

Costs are linear in wages, w. With diminishing returns to scale in production $(1/\gamma < 1)$, the cost function is increasing and convex in the amount of monitoring, m, and decreasing and convex in the state of the monitoring production technology, z. Figure 2 portrays the cost of monitoring in terms of labor, or plots $C(m/z; w)/w$.

Now, exactly which firms are funded depends on three things: (i) the firm's type, τ; (ii) the state of the monitoring production technology in the financial intermediation sector, z; (iii) the expense of monitoring effort as reflected by the wage, w. As will be seen, when the variance of a firm's project becomes larger, the informational problems associated with contracting become more severe. Therefore, high variance projects are less likely to get funded, ceteris paribus.

IV. The Financial Contract

A contract between a firm and an intermediary is summarized by the quadruple $\{k, p_j, p_{ij}, m_j\}$. Here k represents the amount of capital lent by the intermediary to the firm, p_j is the firm's payment to the intermediary if it reports θ_j and is not found cheating, p_{ij} is payment to the bank if the borrower reports θ_j and monitoring reveals that productivity is $\theta_i \neq \theta_j$, and m_j is the intermediary's monitoring effort when θ_j is reported. Denote the value of the firm's outside option by v. The value for v is determined in competitive equilibrium.

The intermediary chooses the details of the financial contract, $\{k, p_j, p_{ij}, m_j\}$, to maximize its profits. The contract is designed to have two features: (i) it entices truthful reporting by firms; (ii) it offers firms an expected return of v. The optimization problem is

$$\text{(P2)} \quad I(\tau, v) \equiv \max_{p_1, p_2, p_{12}, p_{21}, m_1, m_2, k} \{\pi_1 p_1 + \pi_2 p_2 - \tilde{r}k - \pi_1 w (m_1/z)^\gamma - \pi_2 w (m_2/z)^\gamma\},$$

subject to

$$p_1 \leq r_1 k, \tag{3}$$

$$p_2 \leq r_2 k, \tag{4}$$

$$p_{12} \leq r_1 k, \tag{5}$$

$$p_{21} \leq r_2 k, \tag{6}$$

$$[1 - P_{12}(m_2/k)](r_1 k - p_2) + P_{12}(m_2/k)(r_1 k - p_{12}) \leq r_1 k - p_1, \tag{7}$$

$$[1 - P_{21}(m_1/k)](r_2 k - p_1) + P_{21}(m_1/k)(r_2 k - p_{21}) \leq r_2 k - p_2, \tag{8}$$

and

$$\pi_1(r_1 k - p_1) + \pi_2(r_2 k - p_2) = v. \tag{9}$$

Note that the cost of capital, $\tilde{r}$, is given by $\tilde{r} = \hat{r} + \delta$; i.e., the interest paid to investors plus the depreciation on capital. The first four constraints just say the intermediary cannot demand more than the firm earns; that is, the firm has limited liability. Equations (7) and (8) are the incentive-compatibility constraints. Take (8). This simply states that the expected return to the firm from reporting state one when it actually is in state two, as given by the left-hand side, must be less than telling the truth, as represented by the right-hand side. Observe that the constraint set is not convex due to the way that m_1 enters (8). Therefore, the second-order conditions for the maximization problem are important to consider. The last constraint (9) specifies that the contract must offer the firm an expected return equal to v, its option value outside. A firm's outside option is the expected return that it could earn on a loan from another intermediary. This will be pinned down in equilibrium. Finally, note the solution for $\{p_j, p_{ij}, m_j, k\}$ is contingent upon the firm's type, $\tau = (\theta_1, \theta_2)$. To conserve on notation, this dependence is generally suppressed.

The lemma below characterizes the solution to the above optimization problem.

LEMMA 1: *(Terms of the Contract) The solution to problem (P2) is described by:*

1. *The size of the loan from the intermediary to the firm, k, is*

$$k = \frac{v}{\pi_2(r_2 - r_1)[1 - P_{21}(m_1/k)]}. \tag{10}$$

2. *The amount of monitoring per unit of capital when the firm reports a bad state, m_1/k, solves the problem*

$$I(\tau, v) \equiv \max_{m_1/k}\{(\pi_1 r_1 + \pi_2 r_2 - \tilde{r})k - \frac{\pi_1 w}{z^{\gamma}} k^{\gamma}\left(\frac{m_1}{k}\right)^{\gamma} - v\}, \tag{P3}$$

where k can be eliminated using (10) above.

(*a*) *Monitoring in the bad state is simply given by*

$$m_1 = (m_1/k)k,$$

where m_1/k solves (P3).

(*b*) *The intermediary does not monitor when the firm reports a good state so that*

$$m_2 = 0. \tag{11}$$

3. *The payment schedule is*

$$p_1 = r_1 k, \tag{12}$$

$$p_2 = r_2 k - v/\pi_2, \tag{13}$$

$$p_{12} = r_1 k, \tag{14}$$

$$p_{21} = r_2 k. \tag{15}$$

PROOF:

See Greenwood, Sanchez, and Wang (2009, Appendix), which contains proofs for all of the lemmas and propositions.

It is intuitive that there are no benefits to the firm from claiming a better outcome than it actually realizes, since it will only have to pay the intermediary more. Therefore, cheating is a problem only in the high state. The intermediary would like to reduce the firm's incentive to report being in the low state when it is in a high state. So, suppose the firm reports a low state. If cheating is not detected, then the firm pays all of the revenue (minus labor cost) that would be realized in the low state—see (12). If the firm is caught cheating, then it must surrender all of the revenue (sans labor cost) that it earns in the high state—see (15). Note that due to the incentive-compatibility constraints a false report will never occur so that the payments shown by (14) and (15) do not occur in equilibrium.

The contract specifies that the intermediary should monitor the firm only when it reports a bad outcome (state 1) on its project—see (11). Monitoring in the low state is done to maximize the intermediary's profits, subject to the incentive-compatibility cum promise-keeping constraint (10), as problem (P3) dictates. Note that the higher is the value of the firm, v, the bigger must be the loan, k, to satisfy the incentive-compatibility cum promise-keeping constraint (10). This constraint (10) ensures that the contract provides the firm an expected return equal to what it would earn if it misrepresented the outcome in the good state, $\pi_2(r_2 - r_1)[1 - P_{21}(m_1/k)]k$. Furthermore, this expected return is set equal to the firm's outside option, v. The size of the loan, k, is increasing in the amount of monitoring that occurs in the low state, m_1/k. This happens because the probability of the firm not getting caught from misrepresenting its revenues, $1 - P_{21}(m_1/k)$, is decreasing in the intermediary's monitoring activity.

Now, a financial contract will be offered by an intermediary to a firm only if it yields the former nonnegative profits, $I(\tau, v) \geq 0$. Suppose that $r_1 < \tilde{r}$. A necessary condition for a contract to yield nonnegative profits is for the intermediary to devote more than the minimal level of resources per unit of funds lent, $1/\epsilon$, to monitoring a report of a bad state. If this is not done, the firm will always claim that it is in the low state, and the intermediary can earn only a loss on the contract. Briefly consider the case where $r_1 \geq \tilde{r}$. Here the firm's return on capital in its worst state of nature is at least as large as the cost of capital, $\tilde{r}$. Firms would desire to borrow an infinite amount of capital. An equilibrium will not exist.

ASSUMPTION: (*No Free Lunch*) $r_1 = R(\theta_1, w) < \tilde{r} = \hat{r} + \delta$, *for all firm types.*

Later, a lower bound on the level of productivity in the financial sector, ζ, will be imposed that guarantees that this assumption holds whenever $z > \zeta$. This lower bound ensures that the equilibrium wage, w, is high enough so that the assumption will always hold—see (2).

LEMMA 2: *(Interior solution for monitoring)* $m_1/k > 1/\epsilon$, *for all* $v > 0$.

V. Competitive Financial Intermediation

In the economy there is perfect competition in the financial sector. Consequently, an intermediary must offer a contract that maximizes a firm's value, subject to the restriction that the former does not incur a loss. If an intermediary failed to do so it would be undercut by others. The upshot is that intermediaries will make zero profits on each type of loan. Furthermore, for a firm to produce, it must make positive profits too. It is intuitive that for all this to happen a project must offer some potential for a surplus or that $\pi_1 r_1 + \pi_2 r_2 > \tilde{r}$; i.e., the expected return on capital must exceed its cost. Assume this. The value of a firm, v, is then determined by the condition

$$v = V(\tau) \equiv \arg\max_{x} \quad \{x : I(\tau, x) = 0\}. \tag{16}$$

The implications of perfect competition will now be analyzed. Key questions are: (i) What will be the loan size? (ii) Which firms will get funded?

The size of a loan for a project, k, can now be determined. To do this, substitute (10) in problem (P3) and solve for the optimal level of monitoring, m_1/k. Plug this solution for monitoring in the objective function for (P3) to obtain a formula for $I(\tau, v)$.[2] Next, compute v using condition (16). Substituting the obtained formulae for m_1/k and v into (10) yields

$$\begin{aligned} k = {} & (\psi\gamma + \gamma - \psi)^{\gamma/(\psi\gamma-\psi)} \left(\frac{1}{\psi}\right)^{-\psi/(\psi\gamma-\psi)} \left(\frac{1}{\psi\gamma+\gamma}\right)^{(\gamma+\psi)/(\psi\gamma-\psi)} \\ & \times\ (\pi_1 r_1 + \pi_2 r_2 - \tilde{r})^{(\gamma+\psi)/(\psi\gamma-\psi)} (\pi_1 w)^{-\psi/(\psi\gamma-\psi)} \left[\frac{(\epsilon z)^{\psi}}{\pi_2(r_2 - r_1)}\right]^{\gamma/(\psi\gamma-\psi)}. \end{aligned} \tag{17}$$

Equation (17) gives a determinate loan size for each type of funded project. Furthermore, funding is increasing in a project's expected return and is decreasing in its volatility.

[2] The solution obtained for $I(\tau, v)$ is

$$\begin{aligned} I(\tau, v) = {} & (\psi\gamma + \gamma - \psi)\left(\frac{1}{\psi}\right)^{-\psi/(\psi\gamma+\gamma-\psi)} \left(\frac{1}{\psi\gamma+\gamma}\right)^{(\psi\gamma+\gamma)/(\psi\gamma+\gamma-\psi)} (\pi_1 r_1 + \pi_2 r_2 - \tilde{r})^{(\psi\gamma+\gamma)/(\psi\gamma+\gamma-\psi)} \\ & \times (\pi_1 w)^{-\psi/(\psi\gamma+\gamma-\psi)} \left[\frac{(\epsilon z)^{\psi}}{\pi_2(r_2 - r_1)}\right]^{\gamma/(\psi\gamma+\gamma-\psi)} v^{\gamma/(\psi\gamma+\gamma-\psi)} - v. \end{aligned}$$

This solution presumes that $r_1 < \tilde{r}$, so there is an interior solution for monitoring, and that $\pi_1 r_1 + \pi_2 r_2 > \tilde{r}$. It is easy to see that the intermediary's profit function, $I(\tau, v)$, is $\cap$ shaped with the following properties: (i) $I(\tau, 0) = 0$; (ii) $\lim_{v \to 0} \partial I(\tau, v)/\partial v = \infty$; (iii) $\partial^2 I(\tau, v)/\partial v^2 < 0$; (iv) $\lim_{v \to \infty} I(\tau, v) = -\infty$. Therefore, there is only one $v > 0$ that solves (16).

LEMMA 3: *(Loan size) The level of investment in a firm, k, is increasing in its expected net return,* $\pi_1 r_1 + \pi_2 r_2 - \tilde{r}$*, and the state of technology in the financial sector, z, and is decreasing in the variance of the return, as proxied by* $r_2 - r_1$ *(holding the wage rate, w, fixed).*

Attention will now be directed toward determining which projects will be funded. Consider the set of firms, $\mathcal{A}(w)$, defined by

$$\mathcal{A}(w) \equiv \{\tau : \pi_1 r_1 + \pi_2 r_2 - \tilde{r} > 0\}. \tag{18}$$

Intuitively, one might expect that this set of projects will be funded in equilibrium because they offer an expected return on capital, $\pi_1 r_1 + \pi_2 r_2$, that is greater than its user cost, $\tilde{r}$. This turns out to be true. The contracting problem (P2) implies that the firm will never make negative profits, given (3) to (6). The construction of equation (17) suggests that the intermediary will be able to make a loan in this situation, and not incur a loss.

LEMMA 4: (*The set of funded firms*) *A necessary and sufficient condition for a type-*τ *firm to be active or funded, or for* $k(\tau) > 0$, $I(\tau, V(\tau)) = 0$ *and* $V(\tau) > 0$, *is that* $\tau \in \mathcal{A}(w)$.

Therefore, as was mentioned in the introduction, a simple threshold rule exists for funding, as characterized by (18). Call $\mathcal{A}(w)$ the set of active firms. From (2) it is easy to see that

$$\pi_1 r_1 + \pi_2 r_2 - \tilde{r} = \alpha(1-\alpha)^{(1-\alpha)/\alpha} w^{-(1-\alpha)/\alpha}[\pi_1 \theta_1^{1/\alpha} + \pi_2 \theta_2^{1/\alpha}] - \tilde{r} > 0.$$

Observe that the firm's profits are decreasing in wages. A type-τ firm will operate when $w < \overline{W}(\tau)$, and will not otherwise, where the cutoff wage, $\overline{W}(\tau)$, is specified by

$$\overline{W}(\tau) \equiv \alpha^{\alpha/(1-\alpha)}(1-\alpha)\left(\frac{\pi_1 \theta_1^{1/\alpha} + \pi_2 \theta_2^{1/\alpha}}{\tilde{r}}\right)^{\alpha/(1-\alpha)}. \tag{19}$$

So, the set of active projects $\mathcal{A}(w)$ can be expressed equivalently as

$$\mathcal{A}(w) = \{\tau : w < \overline{W}(\tau)\}. \tag{20}$$

The active set depends on the wage because $r_i = R(\theta_i, w)$. It contracts (expands) with a rise (decrease) in the real wage, since $R(\theta_i, w)$ is decreasing in w. In equilibrium the wage rate, w, turns out to be increasing function of the state of technology in the financial sector, z. Hence, the active set will shrink with technological improvement in the financial sector, or a rise in z. This will become clearer in Section VII.

Figure 3 summarizes the discussion on funding. The left horizontal axes plot the expected return for a project, while the right ones give its risk. As the expected return on a project rises, more capital is allocated to it, as the first panel illustrates. (Note that the directions of the x and y axes are specific to each panel and are shown by the arrows.) The increase in funding (or the scale of the firm) is associated with higher monitoring costs given the increasing, convex form of the cost function (fourth panel). The amount of monitoring done per unit of capital is then economized on (third panel). As a consequence, the odds of detecting fraud drop (second panel). In response the expected rents earned by the firm rise (fifth panel).

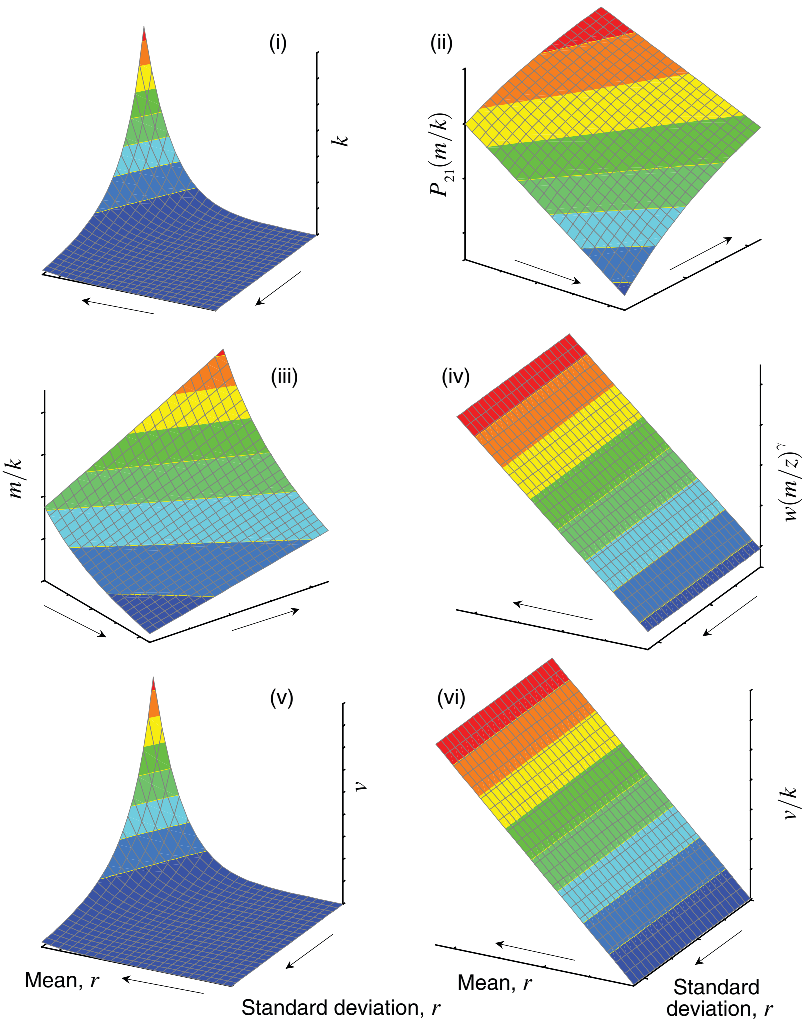

Figure 3. The Determination of Firm Size as a Function of the Mean and Standard Deviation of $r = R(\theta, \mathrm{w})$

Notes: The left horizontal axes plot the mean of r, while the right ones give its standard deviation. The direction of an axis is shown by its arrow. The parameter values used are: $\alpha = 0.33$, $\delta = 0.07$, $\epsilon = 100$, $\psi = 0.52$, $\gamma = 1.86$, $\pi_1 = \pi_2 = 0.5$, $\hat{r} = 1/0.96 - 1$, and $w = 20$.

As the risk associated with a project rises, its funding is slashed—see the first panel. When the difference between the good and bad state widens there is more incentive for the firm to falsify its earnings. The intermediary therefore monitors more per unit of capital lent (third panel). Total monitoring costs fall with risk, because the size of the loan is smaller (fourth panel). The probability of detecting malfeasance therefore moves up since monitoring per unit of capital is now higher (second panel). A firm's rents will fall with a rise in risk (fifth panel), because it receives a smaller loan and faces more vigilant policing.

VI. Stationary Equilibrium

The focus of the analysis is on stationary equilibria. First, the labor-market-clearing condition for the model will be presented. Second, a definition for a stationary equilibrium will be given. Third, it will be demonstrated that a stationary equilibrium for the model exists.

On the demand side for labor, only firms with $\tau \in \mathcal{A}(w)$ will be producing output. On the supply side, recall that the economy has one unit of labor in aggregate. The labor-market-clearing condition will then appear as

$$(21) \qquad \int_{\mathcal{A}(w)} [\pi_1 l_1(\theta_1, \theta_2) + \pi_2 l_2(\theta_1, \theta_2) + \pi_1 l_{m1}(\theta_1, \theta_2)]\, dF(\theta_1, \theta_2) = 1.$$

In the above expression, $l_1(\theta_1, \theta_2)$ and $l_2(\theta_1, \theta_2)$ represent the amounts of labor that a type-(θ_1, θ_2) firm will employ in states one and two, respectively. Likewise, $l_{m1}(\theta_1, \theta_2)$ denotes the amount of labor hired by an intermediary to monitor the firm when the latter declares that state one has occurred.

It is now time to take stock of the situation so far by presenting a definition of the equilibrium under study. It will be assumed that the economy rests in a stationary state where the cost of capital is $\tilde{r} = \hat{r} + \delta$.

DEFINITION 1: *Set the steady-state cost of capital at* $\tilde{r}$. *A stationary competitive equilibrium is described by a set of labor allocations,* l *and* l_m, *a financial contract,* $\{p_1, p_2, p_{12}, p_{21}, k, m_1, m_2\}$, *a set of active monitored firms,* $\mathcal{A}(w)$, *firm values* v, *and a wage rate,* w, *such that:*

1. *The financial intermediary offers a contract,* $\{p_1, p_2, p_{12}, p_{21}, k, m_1, m_2\}$, *which maximizes its profits,* I, *in accordance with* (P2), *given the cost of capital and wages,* $\tilde{r}$ *and* w, *and the value of firms,* v. *The intermediary hires labor for monitoring in the amount* $l_m = (m/z)^{\gamma}$.

2. *The financial contract offered by the intermediary maximizes the value of a firm,* v, *in line with* (16), *given the prices* $\tilde{r}$ *and* w.

3. *A firm is offered a contract if and only if it lies in the active set,* $\mathcal{A}(w)$, *as defined by (18), given* $\tilde{r}$ *and* w. *It hires labor,* l, *so as to maximize its profits in accordance with* (P1), *given* w *and the size of the loan,* k, *offered by the intermediary.*

4. *The wage rate,* w, *is determined so that the labor market clears, in accordance with* (21).

When will an equilibrium exist for the economy under study? To address this question, let $\overline{\theta}_1 \equiv \max\{\theta_1 : (\theta_1, \theta_2) \in \mathcal{T}$, over all $\theta_2\}$. Next, define the constant ω by the equation

$$(22) \qquad R(\overline{\theta}_1, \omega) = \tilde{r}.$$

The constant ω specifies a lower bound on the feasible equilibrium wage rate.[3] When $w = \omega$ for a type-$(\overline{\theta}_1, \theta_2)$ project it will happen that $r_1 = \tilde{r}$. In this situation the intermediary could simply ask for a payment of $r_1 k$ in both states of the world and engage in no monitoring. It would earn zero

[3] The function $R(\theta, w)$ is continuous and strictly decreasing in w, with $\lim_{w \to 0} R(\theta, w) = \infty$, and $\lim_{w \to \infty} R(\theta, w) = 0$; hence, ω is well defined.

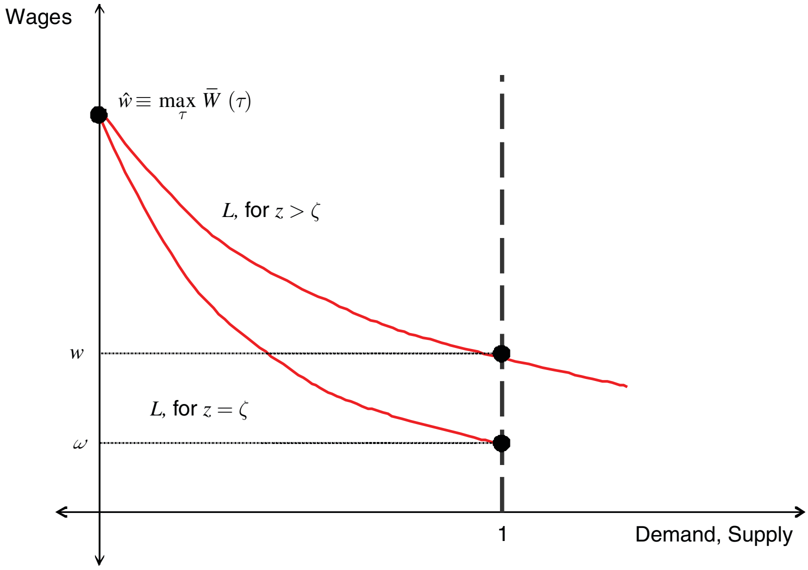

FIGURE 4. EXISTENCE, DEMAND AND SUPPLY FOR LABOR

profits. The firm's profits would be $\pi_2(r_2 - r_1)k$. It would desire a loan of infinite size. Therefore, as the equilibrium wage, w, approaches ω from above the equilibrium defined above will eventually become tenuous.

The situation is portrayed in Figure 4, which graphs the demand and supply for labor. The demand for labor is portrayed by the solid line labeled L. The properties of this demand schedule are established during the course of the proof for Lemma 5—see Greenwood, Sanchez, and Wang (2009, Appendix). Demand is downward sloping in w. The question is whether or not it will cross the vertical supply schedule for labor. Now, at a given wage rate, loan size increases with more efficient intermediation. This leads to more labor being demanded when intermediation improves. In other words, it can be shown that the demand for labor schedule rotates rightward with an upward movement in z. Now, define ζ as the level of z such that the demand curve intersects the supply curve at the point $(1, \omega)$.

LEMMA 5: (*Existence of an equilibrium*) *There is a constant ζ such that for all $z > \zeta$ there exists a stationary equilibrium for the economy.*

VII. The Impact of Technological Progress on the Economy

The primary goal of the analysis is to understand how technological advance in the financial sector affects the economy. To this end, the impact that technological progress has, in either the financial or production sector, on the portfolio of funded projects will be characterized. To develop some intuition for the economy under study, four special cases will be examined.

A. *Balanced Growth*

In the first special case, technological progress in the financial sector proceeds in balance with the rest of the economy. Specifically, the economy may move along a balanced growth path where the

$\theta_i^{1/\alpha}$'s grow at the common rate $g^{1/\alpha}$ and z grows at rate $g^{1/(1-\alpha)}$.[4] Therefore, $F_{t+1}(\theta_1, \theta_2) = F_t(\theta_1/g, \theta_2/g)$. The salient features of this case are summarized by the proposition below.

PROPOSITION 1: (*Balanced growth*) *Let the* $\theta_i^{1/\alpha}$*'s grow at rate* $g^{1/\alpha}$ *and* z *increase at rate* $g^{1/(1-\alpha)}$. *There exists a balanced growth path where the capital stock, k, wages, w, and rents, v, will all grow at rate* $g^{1/(1-\alpha)}$. *The amount of resources devoted to monitoring per unit of capital,* m_1/k, *remains constant.*

In this situation the financial sector is not becoming more efficient over time, relative to the rest of the economy. The amount of monitoring done per unit of capital invested remains constant over time. Thus, the probability of a firm getting caught by misrepresenting a high level of earnings, $P_{21}(m_1/k)$, is constant over time too. For any particular project type, the spread between the return on capital (net of labor costs) and its user cost, $\pi_1 r_1 + \pi_2 r_2 - \tilde{r}$, is fixed over time. The existence of a balanced growth path results from the fact that the probability of detection, $P_{21}(m_1/k)$, depends on the employment of monitoring services relative to the size of the loan.

B. *Unbalanced Growth*

The above case suggests that for technological progress in the financial sector to have an impact it must outpace advance in the rest of the economy. Suppose that this is the case. Then, one would expect that as monitoring becomes more efficient those projects offering the lowest expected return will be cut.

PROPOSITION 2: (*Technological progress in financial intermediation*) *Consider* z *and* z' *with* $z < z'$. *Let* w *and* w' *be the wage rates associated with* z *and* z', *respectively. Then,* $\mathcal{A}(w') \subset \mathcal{A}(w)$. *Additionally, if* $\tau = (\theta_1, \theta_2) \in \mathcal{A}(w) - \mathcal{A}(w')$ *and* $\tau' = (\theta_1', \theta_2') \in \mathcal{A}(w')$ *then* $\pi_1(\theta_1)^{1/\alpha} + \pi_2(\theta_2)^{1/\alpha} < \pi_1(\theta_1')^{1/\alpha} + \pi_2(\theta_2')^{1/\alpha}$.

An increase in z makes financial intermediation more efficient. For any given wage rate, w, the aggregate demand for labor will increase for two reasons. First, more capital will be lent to each funded project. Second, more labor will also be hired by the intermediary to monitor the project. Since the demand for labor rises, the wage rate must move up to clear the labor market. This increase in wages causes the set of active projects, $\mathcal{A}(w)$, to shrink, with the projects offering the lowest expected return being culled.

Alternatively, technological advance could occur in the production sector and not the financial one. Here, the lack of development in the financial sector will hinder growth in the rest of the economy. Specifically, technological advance in the production sector of the economy will drive up wages. This leads to the costs of monitoring rising. Therefore, less is done. This lack of scrutiny by intermediaries now allows firms with marginal projects offering low expected returns to receive funding.

PROPOSITION 3: (*Technological progress in production*) *Suppose all the* $\theta_i^{1/\alpha}$*'s increase by the factor* $g^{1/\alpha}$, *holding* z *fixed. Then, the set of active projects,* $\mathcal{A}(w)$, *expands with the new projects offering lower expected returns than the old ones.*

[4] In order to get a fixed interest rate assume that the consumer/worker has isoelastic preferences over consumption. Then, in standard fashion, along a balanced growth path, $\hat{r} = g^{\iota/(1-\alpha)}/\beta - 1$, where ι is the coefficient of relative risk aversion.

C. *Efficient Finance*

An extreme example of Proposition 2 would be to assume that z grows forever. Then, the financial sector will become infinitely efficient relative to the rest of the economy. This leads to the fourth special case.

PROPOSITION 4: *(Efficient finance) Suppose $\mathcal{T}$ is a compact and countable subset of R_+^2, with a positive measure of projects for each type, $\tau = (\theta_1, \theta_2)$. Then,*

1. $\lim_{z\to\infty} A(w) = A^* \equiv \arg\max_{\tau=(\theta_1,\theta_2)\in\mathcal{T}} [\pi_1(\theta_1)^{1/\alpha} + \pi_2(\theta_2)^{1/\alpha}]$,
2. $\lim_{z\to\infty} m_1/z = 0$, *for* $\tau \in A^*$,
3. $\lim_{z\to\infty} m_1/k = \infty$ *and* $\lim_{z\to\infty} P_{21}(m_1/k) = 1$, *for* $\tau \in A^*$,
4. $\lim_{z\to\infty} p_2 = r_2 k$, *for* $\tau \in A^*$,
5. $\lim_{z\to\infty} v = 0$, *for* $\tau \in A^*$,
6. (23) $\lim_{z\to\infty} w = w^* \equiv \alpha^{\alpha/(1-\alpha)}(1-\alpha)\{\max_{\tau=(\theta_1,\theta_2)\in\mathcal{T}} [\pi_1(\theta_1)^{1/\alpha} + \pi_2(\theta_2)^{1/\alpha}]/\tilde{r}\}^{\alpha/(1-\alpha)}$,
7. (24) $\lim_{z\to\infty} \int_{A(w)} k dF = \mathbf{k}^* \equiv \left(\frac{\alpha}{\tilde{r}}\right)^{1/(1-\alpha)} \{\max_{\tau=(\theta_1,\theta_2)\in\mathcal{T}} [\pi_1(\theta_1)^{1/\alpha} + \pi_2(\theta_2)^{1/\alpha}]\}^{\alpha/(1-\alpha)}$.

As the cost of monitoring borrowers drops, the intermediation sector becomes increasingly efficient. The financial intermediary can then perfectly police loan payments without devoting a significant amount of resources in terms of labor to this activity, as points (2) and (3) in the proposition make clear. Since firms are operating constant-returns-to-scale production technologies, no rents will accrue on their activity—see point (5). Firms must pay the full marginal product of capital to the intermediary—point (4). That is, the spread between a firm's internal rate of return (before depreciation), $\pi_1 r_1 + \pi_2 r_2$, and the user cost of capital, $\tilde{r} = \hat{r} + \delta$, vanishes, where the latter is made of the interest paid to savers, $\hat{r}$, and the rate of depreciation, δ. In this world only projects with the highest return are financed, as point (1) states, even though they may be the most risky. In the aggregate any idiosyncratic project risk washes out. Therefore, in the absence of a contracting problem, only the mean return on investment matters. And, with constant-returns-to-scale technologies everything should be directed to the most profitable opportunity. The wage rate, w^*, and aggregate capital stock, $\mathbf{k}^*$, in the efficient economy are determined in standard fashion by the conditions that the expected marginal product of capital for the most profitable projects must equal the user cost of capital, $\tilde{r}$, and the fact that the labor market must clear. These two conditions yield (24) and (23). (By comparison, consider the standard deterministic growth model with the production technology $o = \theta k^\alpha l^{1-\alpha}$ and one unit of aggregate labor. Here $w^* \equiv \alpha^{\alpha/(1-\alpha)}(1-\alpha)(\theta^{1/\alpha}/\tilde{r})^{\alpha/(1-\alpha)}$ and $\mathbf{k}^* \equiv (\alpha/\tilde{r})^{1/(1-\alpha)}(\theta^{1/\alpha})^{\alpha/(1-\alpha)}$. The differences in the formulae are due to two facts that pertain to the current setting: (i) the best projects from a portfolio $\mathcal{T}$ are chosen; (ii) there is uncertainty in θ.)

VIII. Conclusions

What is the link between the state of financial intermediation and economic development? This question is explored here by embedding a costly-state verification framework into the standard neoclassical growth paradigm. The model has two novel ingredients. As in the standard costly-state verification paradigm, the ex post return on a project is private information, and an intermediary can audit the reported return. The first new ingredient is that likelihood of a successful audit is increasing and concave in the amount of resources devoted to monitoring. The cost of auditing is increasing and convex in the amount of resources spent on this activity. Second, there is a distribution over firm type, each type offering a different combination of risk and return.

Two key features follow from these ingredients. First, a financial theory of firm size results. All firms are funded that earn an expected return greater than the cost of raising capital from savers. Funding is increasing in a project's expected return and decreasing in its variance. The size of a firm is limited by diminishing returns in information production.

Second, a Goldsmithian (1969) link is created between the state of financial development and economic development. The presence of informational frictions leads to a distortion between the expected marginal product of capital and its user cost, the interest paid to savers plus capital consumption. This distortion is modeled endogenously here. As the efficacy of auditing increases, due to technological progress in the financial sector, the size of this distortion shrinks. The upshot is an increase in the economy's income. Intuitively, the rise in income derives from three effects: (a) as the spread shrinks there is more overall capital accumulation in the economy; (b) capital is redirected toward the most productive investment opportunities in the economy; (c) less labor is required to monitor loans, which frees up resources for the economy.

There are two natural extensions of the above framework. The first would be to allow for long-term contracts. On this, Wang (2005) presents a dynamic costly-state verification model, while Anthony A. Smith Jr. and Wang (2006) embed a long-term contracting framework into a model of financial intermediation. The properties of dynamic contracting for firm finance in worlds with private information have been examined by Gian Luca Clementi and Hugo A. Hopenhayn (2006) and Vincenzo Quadrini (2004). The use of dynamic contracts could mitigate the informational problems discussed here. How much is an open question. In a competitive world, such contracts may be severely limited by the ability of each party to leave the relationship at any point in time and seek a better partner.

Second, a dynamic contracting version of the developed framework could be taken to the cross-country data to measure the importance of financial intermediation in economic development. Greenwood, Sanchez, and Wang (2009) do this for the static contracting case emphasized here. They find that financial intermediation plays a quantitatively important role in economic development. This finding is in accord with recent quantitative work by others using a variety of different models; e.g., Pedro S. Amaral and Erwin Quintin (forthcoming); Francisco J. Buera, Joseph P. Kaboski, and Yongseok Shin (2009); Rui Castro, Clementi, and Glenn MacDonald (2009); Andres Erosa and Ana Hidalgo Cabrillana (2008); and Townsend and Kenichi Ueda (2006).[5] Such an extension could be melded together with a Hopenhayn and Richard Rogerson (1993) industry-dynamics model. An interesting question is whether such a framework is capable of delivering large resource misallocations across firms, as emphasized by Diego Restuccia and Rogerson (2008).

[5] Castro, Clementi, and MacDonald (2009) undertake a quantitative analysis within the context of a costly-state falsification model. It is particularly interesting for the analysis undertaken here. They explain why per-capita income covaries positively with the PPP-adjusted investment rate and negatively with the relative price of capital goods. This happens because the capital goods sector is risky, implying that the costs of finance are high in countries with poor investor protection. The current framework would generate a similar prediction.

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IX Appendix–Proofs for Lemmas and Propositions in Greenwood, Sanchez and Wang (2010)

Proof for Lemma 1. First, substitute the promise-keeping constraint (9) into the objective function to rewrite it as

$$(\pi_1 r_1 + \pi_2 r_2 - \widetilde{r})k - \pi_1 w(m_1/z)^\gamma - \pi_2 w(m_2/z)^\gamma - v.$$

Next, it is almost trivial to see that optimality will dictate that $p_{12} = r_1 k$ and $p_{21} = r_2 k$, since this costlessly relaxes the incentive constraints (7) and (8). Next, drop the incentive constraint (7) from problem (P2) to obtain the auxiliary problem now displayed:

(P4) $$\widetilde{I}(\tau, v) \equiv \max_{p_1, p_2, m_1, k} \{(\pi_1 r_1 + \pi_2 r_2 - \widetilde{r})k - \pi_1 w(m_1/z)^\gamma - v\},$$

subject to

(25) $$p_1 \leq r_1 k,$$

(26) $$p_2 \leq r_2 k,$$

(27) $$[1 - P_{21}(m_1/k)](r_2 k - p_1) \leq r_2 k - p_2,$$

and

(28) $$\pi_1(r_1 k - p_1) + \pi_2(r_2 k - p_2) = v.$$

The strategy will be to solve problem (P4) first. Then, it will be shown that (P2) and (P4) are equivalent. Problem (P4) will now be solved. To this end, note the following points:

1. The incentive constraint (27) is binding. To see why, suppose not. Then, reduce m_1 to increase the objective.

2. The constraint (26) is not binding. Assume, to the contrary, it is. Then, (27) is violated. This happens because the right-hand side is zero. Yet, the left-hand side is positive, given that $p_1 \leq r_1 k < r_2 k$, so that $[1 - P_{21}(m_1/k)](r_2 k - p_1) > 0$.

3. The constraint (25) is binding. Again, suppose not, so that $p_1 < r_1 k$. It will be shown that exists a profitable feasible deviation from any contract where this constraint is slack. Specifically, consider increasing k very slightly by $dk > 0$ while adjusting p_1 and p_2 in the following manner so that (27) and (28) still hold. Also, hold m_1 fixed. The implied perturbations for p_1 and p_2 are given by

$$\begin{bmatrix} dp_1 \\ dp_2 \end{bmatrix} = \begin{bmatrix} -[1 - P_{21}(m_1/k)] & 1 \\ \pi_1 & \pi_2 \end{bmatrix}^{-1} \times \begin{bmatrix} r_2 P_{21}(m_1/k) - (r_2 k - p_1)(m_1/k^2) dP_{21}/d(m_1/k) \\ \pi_1 r_1 + \pi_2 r_2 \end{bmatrix} dk.$$

Note that such an increase in k will raise the objective function.

It will now be demonstrated that the optimization problems (P2) and (P4) are equivalent. First, note that $\widetilde{I}(\tau, v) \geq I(\tau, v)$, because problem (P4) does not impose the constraint (7). It will now be established that $\widetilde{I}(\tau, v) \leq I(\tau, v)$. Consider a solution to problem (P4). It will be shown that this solution is feasible for (P2). On this, note that point 1 implies that

$$r_2 k - p_2 = [1 - P_{21}(m_1/k)](r_2 k - p_1) \leq r_2 k - p_1,$$

so that

$$p_1 \leq p_2.$$

Now, set $m_2 = 0$ in (P2), which is feasible but not necessarily optimal. Then, constraint (7) becomes $p_1 \leq p_2$, which is satisfied by the solution to (P4). Therefore, $\widetilde{I}(\tau, v) \leq I(\tau, v)$.

Last, with the above facts in hand, recast the optimization problem as

$$I(\tau, v) \equiv \max_{p_2, m_1, k} \{(\pi_1 r_1 + \pi_2 r_2 - \widetilde{r})k - \pi_1 w(m_1/z)^\gamma - v\},$$

subject to

$$r_2 k - p_2 = (r_2 - r_1)k[1 - P_{21}(m_1/k)], \quad \text{cf (27)},$$

and

$$r_2 k - p_2 = v/\pi_2, \text{ cf (28)}.$$

The above two constraints collapse in the single constraint (10), by eliminating $r_2k - p_2$, that involves just m_1 and k. The problem then appears as (P3). ■

Proof for Lemma 2. Suppose that the solution dictates that $m_1/k \leq 1/\epsilon$. Then, from (P3) it is clear that the optimal solution will dictate that $m_1/k = 0$. This happens because $P_{21}(m_1/k) = 0$ for all $m_1/k \leq 1/\epsilon$, yet monitoring costs are positive for all $m_1/k > 0$. Next, by substituting (10) into (P3) it is easy to deduce that the intermediary's profit function can be written as

$$
\begin{aligned}
I(\tau, v) &= [\frac{\pi_1 r_1 + \pi_2 r_2 - \widetilde{r}}{\pi_2(r_2 - r_1)} - 1]v = \frac{r_1 - \widetilde{r}}{\pi_2(r_2 - r_1)}v \\
&\lesseqqgtr 0 \text{ as } r_1 \lesseqqgtr \widetilde{r}.
\end{aligned}
\tag{29}
$$

Therefore profits are negative if $r_1 < \widetilde{r}$ and $v > 0$. Hence, a contract will not be offered when $m_1/k \leq 1/\epsilon$. ■

Proof for Lemma 3. Clear from equation (17).■

Proof for Lemma 4. *Necessity*: From problem (P3) it is clear that the intermediary will incur a loss when $\pi_1 r_1 + \pi_2 r_2 - \widetilde{r} \leq 0$ and $v > 0$, for any $\tau \in \mathcal{A}(w)$, because $m_1/k > 1/\epsilon$ by Lemma 2.

Sufficiency: Suppose that $\pi_1 r_1 + \pi_2 r_2 - \widetilde{r} > 0$ for some $\tau \in \mathcal{T}$. By equation (17) it is immediate that the intermediary will issue a loan $k > 0$. By construction it will earn zero profits on this loan. Recall that the derivation of (17), discussed in the text, used a solution for v. This solution is

(30)

$$
\begin{aligned}
v = V(\tau) = (\psi\gamma + \gamma - \psi)^{(\psi\gamma+\gamma-\psi)/(\psi\gamma-\psi)}(\frac{1}{\psi})^{-\psi/(\psi\gamma-\psi)}(\frac{1}{\psi\gamma+\gamma})^{(\psi\gamma+\gamma)/(\psi\gamma-\psi)} \\
\times (\pi_1 r_1 + \pi_2 r_2 - \widetilde{r})^{\frac{\gamma\psi+\gamma}{\gamma\psi-\psi}}[\pi_2(r_2 - r_1)]^{\frac{-\gamma}{\gamma\psi-\psi}}(\pi_1 w)^{\frac{-\psi\gamma}{\gamma\psi-\psi}}(z/\epsilon)^{\frac{\psi\gamma}{\gamma\psi-\psi}}.
\end{aligned}
$$

Therefore, the firm will earn positive rents too. ■

Proof for Lemma 5. To begin with let $k = K(w; \tau, z)$ represent capital stock that will be employed by a type-τ firm when productivity in the financial sector is z and the wage rate is $w > \omega$. Similarly, let $m_1/z = M(w; \tau, z)$ denote the amount of monitoring services, relative to z, that will be devoted to this project. Given this notation, the expected demand for labor by both the firm and intermediary for a project of type τ is

$$L(w;\tau,z) \equiv (\pi_1\theta_1^{1/\alpha} + \pi_2\theta_2^{1/\alpha})[\frac{(1-\alpha)}{w}]^{\frac{1}{\alpha}} K(w;\tau,z) + \pi_1 M(w;\tau,z)^{\gamma}. \tag{31}$$

Note that this demand is only specified for $\tau \in \mathcal{A}(w)$ and $w > \omega$, where ω is defined by (22). In order to characterize $L(w;\tau,z)$, the properties of its components $K(w;\tau,z)$ and $M(w;\tau,z)$ must be developed when $\tau \in \mathcal{A}(w)$ and $w > \omega$. First, take $K(w;\tau,z)$. On this, rewrite equation (17) as

(32)

$$\begin{aligned} k &= K(w;\tau,z) \\ &= \Upsilon(\pi_1\Theta_1 + \pi_2\Theta_2 - \widetilde{r}w^{(1-\alpha)/\alpha})^{(\gamma+\psi)/(\psi\gamma-\psi)}[\frac{\epsilon^{\psi}}{\pi_2(\Theta_2-\Theta_1)}]^{\gamma/(\psi\gamma-\psi)} z^{\gamma/(\gamma-1)} w^{-1/[\alpha(\gamma-1)]}, \end{aligned}$$

where $\Theta_i \equiv \alpha(1-\alpha)^{(1-\alpha)/\alpha}\theta_i^{1/\alpha}$ and $\Upsilon \equiv (\psi\gamma+\gamma-\psi)^{\gamma/(\psi\gamma-\psi)}(\frac{1}{\psi})^{-\psi/(\psi\gamma-\psi)}(\frac{1}{\psi\gamma+\gamma})^{(\gamma+\psi)/(\psi\gamma-\psi)}$. Note the following things about this solution for k: (i) The level of investment in a firm, k , is continuous and strictly decreasing in w; (ii) $k \to 0$ as $w \to \overline{W}(\tau) = [(\pi_1\Theta_1 + \pi_2\Theta_2)/\widetilde{r}]^{\alpha/(1-\alpha)}$ (when z is finite); (iii) k is continuous and strictly increasing in z.

Now, switch attention to the second term in $L(w;\tau,z)$. A formula for $M(w;\tau,z)$ can be derived in the same manner as the one for $K(w;\tau,z)$. It is

$$m_1/z = M(w;\tau,z) = \Delta(\pi_1\Theta_1 + \pi_2\Theta_2 - \widetilde{r}w^{(1-\alpha)/\alpha})^{(\psi+1)/(\psi\gamma-\psi)} w^{-1/[\alpha(\gamma-1)]} z^{1/(\gamma-1)}, \tag{33}$$

where $\Delta \equiv \Upsilon[(\psi\gamma+\gamma-\psi)/(\psi\gamma+\gamma)]^{-1/\psi}\{\epsilon^{\psi}/[\pi_2(\Theta_2-\Theta_1)]\}^{1/(\psi\gamma-\psi)}$. Note the following things about this solution for m_1/z: (i) m_1/z is continuous and strictly decreasing in w; (ii) $m_1/z \to 0$ as $w \to \overline{W}(\tau) \equiv [(\pi_1\Theta_1 + \pi_2\Theta_2)/\widetilde{r}]^{\alpha/(1-\alpha)}$ (when z is finite); (iii) m_1/z is continuous and strictly increasing in z.

Thus, the demand for labor by a type-τ project has the following properties: (i) $L(w;\tau,z)$ is continuous and strictly decreasing in w; (ii) $\lim_{w\to\overline{W}(\tau)} L(w;\tau,z) = 0$; (iii) $L(w;\tau,z)$ is continuous and strictly increasing in z. Define the function

$$\widetilde{L}(w;\tau,z) = \begin{cases} L(w;\tau,z), & \text{for } w \le \overline{W}(\tau), \\ 0, & \text{for } w > \overline{W}(\tau), \end{cases} \tag{34}$$

for $w > \omega$. The aggregate demand for labor can be expressed as $\int_{\mathcal{A}(w)} L(w;\tau,z)dF(\tau) = \int_{\mathcal{T}} \widetilde{L}(w;\tau,z)dF(\tau)$. Now, determine the constant ζ by $\lim_{w\downarrow\omega} \int_{\mathcal{T}} \widetilde{L}(w;\tau,\zeta)dF(\tau) = 1$,where again the lower bound on wages ω is given by (22). From the simple closed-form solutions for (32) and (33) it is easy to deduce that such a ζ must exist. To summarize, the aggregate demand for labor, $\int_{\mathcal{T}} \widetilde{L}(w;\tau,z)dF(\tau)$, has the following properties:

1. $\int_{\mathcal{T}} \widetilde{L}(w;\tau,z)dF(\tau)$ is continuous and strictly decreasing in w for $w \in (\omega,\infty)$;
2. $\int_{\mathcal{T}} \widetilde{L}(w;\tau,z)dF(\tau)$ is continuous and strictly increasing in z for $z \in (\zeta,\infty)$;
3. $\int_{\mathcal{T}} \widetilde{L}(\widehat{w};\tau,z)dF(\tau) = 0$, where $\widehat{w} = \max_{\tau\in\mathcal{T}} \overline{W}(\tau)$;
4. $\int_{\mathcal{T}} \widetilde{L}(\omega;\tau,\zeta)dF(\tau) = 1$.

Therefore by the intermediate theorem for all $z > \zeta$ there will exist a single value of w that sets labor demand equal to labor supply (or 1)—see Figure 4. ■

Proof for Proposition 1. Express the labor-market-clearing condition as

$$\int_{\mathcal{A}(w)} \{(\pi_1\theta_1^{1/\alpha} + \pi_2\theta_2^{1/\alpha})[\frac{(1-\alpha)}{w}]^{1/\alpha} K(w;\tau,z) + \pi_1 M(w;\tau,z)^{\gamma}\}dF = 1. \tag{35}$$

Let the $\theta_i^{1/\alpha}$'s grow at the common rate $g^{1/\alpha}$ and z grow at rate $g^{1/(1-\alpha)}$. Recall that exists a solution to the model without growth, as demonstrated by Lemma 5. It is easy to construct a balanced growth path using this solution. The solution implies that there will be a wage rate that solves (35). Conjecture that along a balanced growth path wages, w, will grow at rate $g^{1/(1-\alpha)}$. From (32) it can be deduced that $K(w;\tau,z)$ will grow at rate $g^{1/(1-\alpha)}$. Therefore, the first term in braces in (35) will be constant. Equation (33) implies that $M(w;\tau,z)$, or the second term, will be constant too. The active set $\mathcal{A}(w)$ will not change—equation (20). Therefore, labor demand remains constant. Hence, the conjectured solution for the rate of growth in wages is true. Using (30) is easy to calculate that v will grow at rate $g^{1/(1-\alpha)}$. Last, since $M(w;\tau,z)$ is constant it must be the case that m_1 is growing at the same rate as z, or $g^{1/(1-\alpha)}$. Therefore, m_1/k will remain unchanged along a balanced growth path. ■

Proof for Proposition 2. First, point 2 in the proof of Lemma 5 established that the aggregate demand for labor is continuous and strictly increasing in z. Therefore, at a given wage rate the demand for labor rises as z moves up. In order for equilibrium in the labor market to be restored, wages must increase, since the demand for labor is decreasing in wages—point 1. Last, recall from (19) that a type-τ project will only be funded when $w < \overline{W}(\tau) = \alpha^{\alpha/(1-\alpha)}(1-\alpha)[(\pi_1\theta_1^{1/\alpha} + \pi_2\theta_2^{1/\alpha})/\widetilde{r}]^{\alpha/(1-\alpha)}$. It's trivial to see that as w rises the set of $\tau \in \mathcal{T}$ satisfying this restriction, or $\mathcal{A}(w)$, shrinks; if $\tau = (\theta_1, \theta_2)$ fulfills the restriction for some wage it will meet it for all lower ones too, yet there will exist a higher wage that will not satisfy it. Furthermore, observe that $\overline{W}(\tau)$ is strictly increasing in $\pi_1\theta_1^{1/\alpha} + \pi_2\theta_2^{1/\alpha}$. Therefore, those τ's offering the lowest expected return will be cut first as w rises, because they have the lowest threshold wage. ■

Proof for Proposition 3. Let the $\theta_i^{1/\alpha}$'s increase by the common factor $g^{1/\alpha} > 1$. Suppose that wages increase in response by the proportion $g^{1/(1-\alpha)}$. Will the labor-market-clearing condition (35) still hold? The answer is no, because the demand for labor will fall. Take the first term behind the integral, which gives the demand for labor by a firm. From (32) it is clear that $K(w;\tau,z)$ will rise by a factor less than $g^{1/(1-\alpha)}$, when z is held fixed. Therefore, the first term in braces in (35) will decline. Turn to the second term. From (33) it is easy to see that $M(w;\tau,z)$ will drop under the conjecture solution. Therefore, wages must rise by less than $g^{1/(1-\alpha)}$, since the demand for labor is decreasing in w (as was established in the proof of Lemma 5). The active set, $\mathcal{A}(w)$, will therefore expand, because $\pi_1 r_1 + \pi_2 r_2$ increases–see (18). ■

Proof for Proposition 4. The set of projects in $\mathcal{T}$ offering the highest expected return is given by

$$\mathcal{A}^* = \underset{\tau=(\theta_1,\theta_2)\in\mathcal{T}}{\arg\max}[\,\pi_1(\theta_1)^{1/\alpha} + \pi_2(\theta_2)^{1/\alpha}].$$

By assumption $\int_{\mathcal{A}^*} dF > 0$. Take *any* equilibrium wage w. From (20) it is immediate that if $\tau \in \mathcal{A}^*$ then $\tau \in \mathcal{A}(w)$, since $\pi_1 r_1 + \pi_2 r_2 - \widetilde{r} = \alpha(1-\alpha)^{(1-\alpha)/\alpha} w^{-(1-\alpha)/\alpha}(\pi_1\theta_2^{1/\alpha} + \pi_2\theta_2^{1/\alpha}) - (\widehat{r} + \delta)$ is increasing in $\pi_1\theta_1^{1/\alpha} + \pi_2\theta_2^{1/\alpha}$. Hence, $\mathcal{A}^* \subseteq \mathcal{A}(w)$ for all w. In equilibrium the wage will be a function of z, so denote this dependence by $w = W(z)$. Now, let $z \to \infty$. It will be shown that $w = W(z) \to w^*$, where

$$w^* \equiv \alpha^{\alpha/(1-\alpha)}(1-\alpha)\{\max_{\tau=(\theta_1,\theta_2)\in\mathcal{T}}[\pi_1(\theta_1)^{1/\alpha} + \pi_2(\theta_2)^{1/\alpha}]/\widetilde{r}\}^{\alpha/(1-\alpha)}. \tag{36}$$

To see why, suppose alternatively that $w \to \widetilde{w} \neq w^*$. First, presume that $\widetilde{w} < w^*$. Then, by (19) all projects of type $\tau \in \mathcal{A}^*$ will be funded since their cutoff wage is $\overline{W}(\tau) = w^* > \widetilde{w}$. From equations (31), (32) and (34) it is clear that $\lim_{z\to\infty} \widetilde{L}^d(W(z);\tau,z) = \infty$, for $\tau \in \mathcal{A}^*$. Since, $\int_{\mathcal{A}^*} dF > 0$, this implies that $\lim_{z\to\infty} \int_{\mathcal{T}} \widetilde{L}^d(W(z);\tau,z)dF = \infty$. Therefore, such an equilibrium cannot exist because the demand for labor will exceed its supply. Second, no firm can survive at a wage rate bigger than w^*, by (19). Here, $\lim_{z\to\infty} \int_{\mathcal{T}} \widetilde{L}^d(W(z);\tau,z)dF = 0$. This establishes (23). Last, note that $\mathcal{A}(w^*) = \mathcal{A}^*$.

It is immediate that $\mathcal{A}^* \subseteq \lim_{w\uparrow w^*} \mathcal{A}(w)$, because $\tau \in \mathcal{A}^*$ is viable for *all* wages $w \leq w^* = \overline{W}(\tau)$ by (19). It is also true that $\lim_{w\uparrow w^*} \mathcal{A}(w) \subseteq \mathcal{A}^*$, since from (19) any project $\tau \notin \mathcal{A}^*$ requires an upper bound on wages $\overline{W}(\tau) < w^*$ to survive; that is, for any $\tau \notin \mathcal{A}^*$ there will exist some high enough wage w such that $\overline{W}(\tau) < w < w^*$. Therefore, $\lim_{w\uparrow w^*} \mathcal{A}(w) = \mathcal{A}^* = \mathcal{A}(w^*)$. This establishes point 1 of the Proposition.

To have an equilibrium it must be the case that $m_1/z < \infty$ for $\tau \in \mathcal{A}^*$, otherwise the demand for labor would be infinitely large. From equation (33) this can only happen when $\widetilde{r}w^{(1-\alpha)/\alpha} \to \pi_1\Theta_1 + \pi_2\Theta_2$, or equivalently when $\pi_1 r_1 + \pi_2 r_2 \to \widetilde{r}$. Solve problem (P3) for the optimal level of monitoring, m_1/k, and then use (30) to solve out for v to obtain

$$m_1/k = (\frac{\psi\gamma + \gamma - \psi}{\psi\gamma + \gamma})^{-1/\psi}[\frac{\epsilon^{\psi}(\pi_1 r_1 + \pi_2 r_2 - \widetilde{r})}{\pi_2(r_2 - r_1)}]^{-1/\psi}.$$

It is apparent that $\lim_{z\to\infty} m_1/k = \infty$; because $\pi_1 r_1 + \pi_2 r_2 \to \widetilde{r}$. Consequently, a false report by a firm will be caught with certainty, or $\lim_{z\to\infty} P_{21}(m_1/k) = 1$. The contracting problem (P2) then requires $\lim_{z\to\infty} p_2 = r_2 k$ and $\lim_{z\to\infty} v = 0$. A comparison of (30) and (33) leads to the conclusion that in fact $\lim_{z\to\infty} m_1/z = 0$, when $\lim_{z\to\infty} v = 0$. Using this result and (36), in conjunction with the labor-market-clearing condition, $\int_{\mathcal{T}} \widetilde{L}(w;\tau,z)dF = 1$, then gives (24). ∎